subject to

$$x_1 + 2x_2 \leq 15$$
$$x_1 + x_2 \leq 12$$
$$5x_1 + 3x_2 \leq 45$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

3.1-3.* A manufacturing firm has discontinued the production of a certain unprofitable product line. This act created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table:

Machine Type	Available Time (in Machine Hours per Week)
Milling machine	500
Lathe	350
Grinder	150

The number of machine hours required for each unit of the respective products is as follows:

Productivity Coefficient (in Machine Hours per Unit)

Machine Type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week. The unit profit would be \$50, \$20, and \$25, respectively, on products 1, 2, and 3. The objective is to determine how much of each product the firm should produce to maximize profit.

(*a*) Formulate a linear programming model for this problem.

A* (*b*) Use a computer to solve this model by the simplex method.

3.1-4. A television manufacturing company has to decide on the number of 27- and 20-inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of the 20-inch sets can be sold per month. The maximum number of work hours available is 500 per month. A 27-inch set requires 20 work hours, and a 20-inch set requires 10 work hours. Each 27-inch set sold produces a profit of \$120, and each 20-inch set produces a profit of \$80. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research.

(*a*) Formulate a linear programming model for this problem.

D, I* (*b*) Solve this model graphically.

A* (*c*) Use a computer to solve this model by the simplex method.

3.1-5. A company produces two products requiring resources P and Q. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of resource P and 2 units of resource Q are required. For each unit of product 2, 3 units of resource P and 2 units of resource Q are required. The company has 200 units of resource P and 300 units of resource Q. Each unit of product 1 gives a profit of \$1, and each unit of product 2, up to 60 units, gives a profit of \$2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.

(*a*) Formulate a linear programming model for this problem.

D, I* (*b*) Solve this model graphically. What is the resulting total profit?

A* (*c*) Use a computer to solve this model by the simplex method.

3.1-6. An insurance company is introducing two new product lines: special risk insurance and mortgages. The expected profit is 5 per unit on special risk insurance and 2 per unit on mortgages.

Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work Hours per Unit		
Department	Special Risk	Mortgage	Work Hours Available
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

(*a*) Formulate a linear programming model for this problem.

D, I* (*b*) Use the graphical method to solve this model.

3.1-7. You are the production manager for a manufacturer of three types of spare parts for automobiles. The manufacture of each part requires processing on each of two machines, with the following processing time (in hours):

	Part		
Machine	A	B	C
1	0.02	0.03	0.05
2	0.05	0.02	0.04

Each machine is available 40 hours per month. Each part manufactured will yield a unit profit as follows:

	Part		
	A	B	C
Profit	50	40	30

You want to determine the mix of spare parts to produce in order to maximize total profit.

(*a*) Formulate a linear programming model for this problem.

A* (*b*) Use a computer to solve this model by the simplex method.

All the assumptions of linear programming hold.

(*a*) Formulate a linear programming model for this problem.

D, I* (*b*) Solve this model graphically.

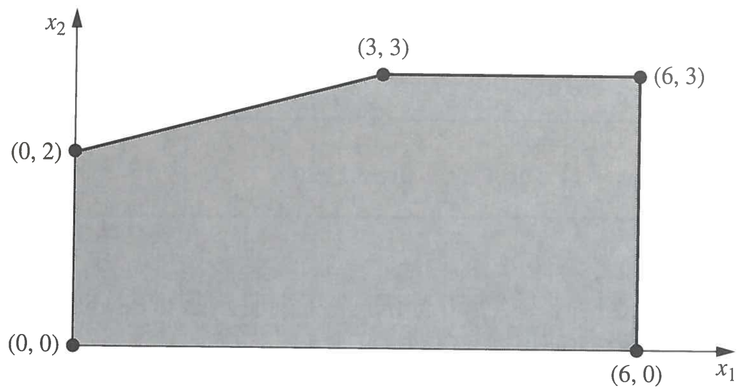
A* (*c*) Use a computer to solve this model by the simplex method.

3.2-2. The shaded area in the following graph represents the feasible region of a linear programming problem whose objective function is to be maximized. Label each of the following statements as true or false, and then justify your answer based on the graphical method. In each case, give an example of an objective function that illustrates your answer.

(*a*) If (3, 3) produces a larger value of the objective function than (0, 2) and (6, 3), then (3, 3) must be an optimal solution.

(*b*) If (3, 3) is an optimal solution and multiple optimal solutions exist, then either (0, 2) or (6, 3) must also be an optimal solution.

(*c*) The point (0, 0) cannot be an optimal solution.



3.2-3.* Suppose you have just inherited \$6,000 and you want to invest it. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of \$5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be \$4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any fraction of a full partnership you would like; your share of the profit would be proportional to this fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one friend's or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

(*a*) Describe the analogy between this problem and the Wyndor Glass Co. problem discussed in Sec. 3.1. Then construct and fill in a table like Table 3.3 for this problem, identifying both the activities and the resources.

(*b*) Formulate a linear programming model for this problem.

D, I* (*c*) Solve this model graphically. What is your total estimated profit?

3.3-1. Reconsider Prob. 3.2-3. Indicate why each of the four assumptions of linear programming (Sec. 3.3) appears to be reasonably satisfied for this problem. Is one assumption more doubtful than the others? If so, what should be done to take this into account?

3.3-2. Consider a problem with two decision variables, x_1 and x_2, which represent the levels of activities 1 and 2, respectively. For each variable, the permissible values are 0, 1, and 2, D, I **3.4-5.** Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\text{Maximize} \quad Z = c_1x_1 + 2x_2,$$

subject to

$$4x_1 + x_2 \leq 12$$
$$x_1 - x_2 \geq 2$$

and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1.

3.4-6. You are given the following nutritional and cost information regarding steak and potatoes:

	Grams of Ingredient per Serving		
Ingredient	Steak	Potatoes	Daily Requirement (Grams)
Carbohydrates	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	\$4	\$2	

You wish to determine the number of daily servings (it may be fractional) of steak and potatoes that will meet these requirements at a minimum cost.

(*a*) Formulate a linear programming model for this problem.

D, I* (*b*) Solve this model graphically.

A* (*c*) Use a computer to solve this model by the simplex method.

3.4-7. A farmer is raising pigs for market and wishes to determine the quantities of the available types of feed (corn, tankage, and alfalfa) that should be given to each pig. Since pigs will eat any mix of these feed types, the objective is to determine which mix will meet certain nutritional requirements at a *minimum cost.* The number of units of each type of basic nutritional ingredient contained within 1 kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs:

Nutritional Ingredient	Kilogram of Corn	Kilogram of Tankage	Kilogram of Alfalfa	Minimum Daily Requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (¢)	84	72	60	

(*a*) Formulate a linear programming model for this problem.

A* (*b*) Solve this model by the simplex method.