Strategic Timing and Resource Allocation for Optimal Isolation and Travel Restrictions in Infectious Disease Control

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Motivation

- In public health, strategies to reduce the impacts of infectious disease spread are elimination, suppression, mitigation, and circuit breaker, where these strategies differ in their objectives, the timing and magnitude of public health interventions, and the resultant epidemiology (Table 1).
- Optimal control is a branch of mathematics that determines the timing of control measures for biological dynamics, such as those described by epidemiological models, to minimize disease incidence (Hansen and Day, 2011) or achieve another objective.

Objectives

The aim of our study is to:

1. Identify when different public health strategies are optimal, as defined by optimal control theory.

2. Extend existing results to consider imported infections and travel measures.

Definitions of Public Health Strategies

Public Health Strategy	Description	Our Definition
Elimination	Strict public health measures reduce infection prevalence to zero locally, but not in all regions, such that there remains a risk of disease importation (Baker, Wilson, and Blakely 2020; Metcalf et al. 2021)	(a) The outbreak is eliminated by public health measures not infected-derived immunity, i.e., $U_{1[u_{1\max}]}(T) < U_{1\max}$ and/or $U_{2[u_{2\max}]}(T) < U_{2\max}$. (b) $\frac{dI_1}{dt} < 0$ shortly after $u_1^*(t)$ and/or $u_2^*(t)$ are implemented.
Suppression	Infection is kept at low levels (Baker, Wilson, and Blakely 2020)	(a) The outbreak is eliminated by public health measures not infected-derived immunity, i.e., $U_{1[u_{1\max}]}(T) < U_{1\max}$ and/or $U_{2[u_{2\max}]}(T) < U_{2\max}$; (b) $\frac{dI_1}{dt} > 0$ shortly after $u_1^*(t)$ and/or $u_2^*(t)$ are implemented.
Circuit Breaker	Public health measures are intermittent with breaks in between	The optimal control involves at least two switches between public health measures of different intensity.

Table 1: Definitions of public health strategies

We extend the epidemiological model of Hansen and Day (2011) to consider disease importation. Specifically,

$$\frac{dS}{dt} = -\beta S(I_1 + cI_2) \tag{1}$$

$$\frac{dI_1}{dt} = \beta S(I_1 + cI_2) - (\mu + u_1)I_1 \tag{2}$$

$$\frac{dI_2}{dt} = \theta - (\gamma + u_2)I_2 \tag{3}$$

with $S(0) > 0, I_1(0) \geq 0, I_2(0) \geq 0,\ \beta, \mu, \theta, \gamma \geq 0$, where β is the transmission rate, μ is the per capita loss rate of infected community members through both mortality and recovery, θ is the baseline number of infected non-resident travellers per unit time, c is the relative transmissibility of travellers, γ is the removal rate of non-resident travellers.

In keeping with Hansen and Day (2011), we assume that resources are limited, such that,

$$\int_0^T u_1 I_{1[u_1,u_2]} \, dt \leq U_{1max} \tag{4}$$

and

$$\int_0^T u_2 I_{2[u_1,u_2]} \, dt \leq U_{2max} \tag{5}$$

The aim of public health measures is assumed to be to minimize the number of new infections,

$$\min J = \int_0^T \beta S_{[u_1,u_2]}(I_{1[u_1,u_2]} + I_{2[u_1,u_2]}) \, dt \tag{6}$$

$T = \inf\{t | I_{1[u_1,u_2]}(t) = 0.5\}, (u_1(t), u_2(t)) \in [0, u_{1max}] \times [0, u_{2max}]$ for all $t \in [0, T]$, $u_{1,2max} \in (0, \infty)$.

We assume that infected non-resident travellers have contracted the infection before entering the community. These individuals have a temporary stay and are promptly either recovered or removed from the community; therefore, $\gamma > \mu$.

Hansen and Day (2011) defined an outbreak as over at $t = T$ if prevalence is less than a minimum value ($I_{min} = 0.5$ is assumed). The reason for defining an outbreak end-point in this way, is to prevent a second wave of infection arising from a fractional individual (Hansen and Day 2011). Defining an outbreak end-point in this way is necessary to consider elimination strategies as a possible recommended strategy (Martignoni, Arino, and Hurford 2024).

Problem Description

Problem	Description	Special values of parameters
1	Community member self-isolation only, no importations	$\theta = 0, I_2(0) = 0, U_{2max} = 0$
2	Community member self-isolation, with importations	$U_{2max} = 0.$
3	Travel measures only	$U_{1max} = 0.$
4	Both community member self-isolation and travel measures	None

Table 2: The four problems that we analyze

Problem 1

This problem has previously been solved in Hansen and Day (2011).

Theorem (1)

(Optimal Isolation Policy): *If* $U_{1[u_{1max}]}(T) \leq U_{1max}$*, then the optimal isolation policy for Problem 1 is* $u_1^* = u_{1max}$*. If* $U_{1[u_{1max}]}(T) > U_{1max}$*, then the optimal policy* u_1^* *is any control* u_1 *such that* $U_{1u_{1max}}(T) = U_{1max}$*.*

Rephrasing Theorem 1 in terms of public health terminology, the optimal isolation strategy is:

- **Elimination:** if $U_{1[u_{1max}]}(T) < U_{1max}$ and community infections decline shortly after the implementation of public health measures (Figure 1A).

- **Suppression:** if $U_{1[u_{1max}]}(T) \leq U_{1max}$ and community infections increase shortly after the implementation of public health measures (Figure 1B)

- **Suppression or circuit-breaker:** if $U_{1[u_{1max}]}(T) = U_{1max}$ (Figure 1C)

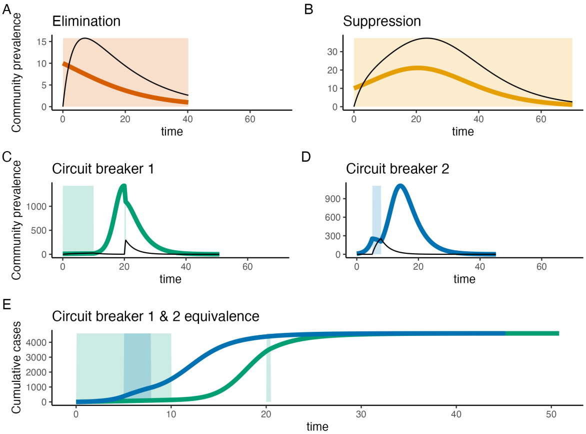
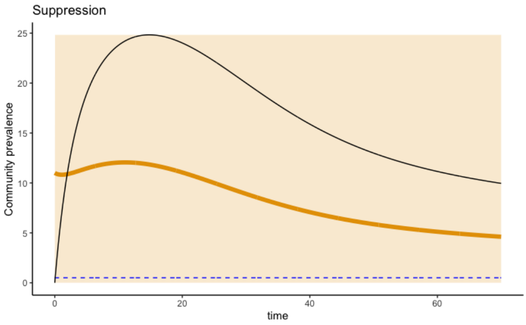


Figure 1. Optimal control for the isolation-only model (Hansen and Day 2011) described in terms of public health strategies. The optimal control is to isolate community members at the maximum rate (shaded region) or not at all (unshaded regions), and the outbreak is over when community infection prevalence (solid lines in A-D) is sufficiently low ($I_1(T) = 0.5$). Infected community members currently in isolation are shown with the black line (A-D). In A and B, there are sufficient resources to implement the control for the entire outbreak. In C-D, resource limitations mean isolation measures cannot remain in place for the entire outbreak. Here, as shown in Hansen and Day, 2011, the cumulative number of cases in the outbreak does not depend on when isolation requirements are implemented, and any strategy that uses all the resources is equivalent (E). Parameters are $u_{1max} = 0.7$ (A) or 0.6 $(B - E)$; $U_{1max} =$ 500 (A, B) or 400 $(C - E)$ and for all panels $\beta = 0.002$, $\mu = 0.334$, $S(0) = 5000$ and $I_1(0) = 10$, and all other initial conditions are zero.

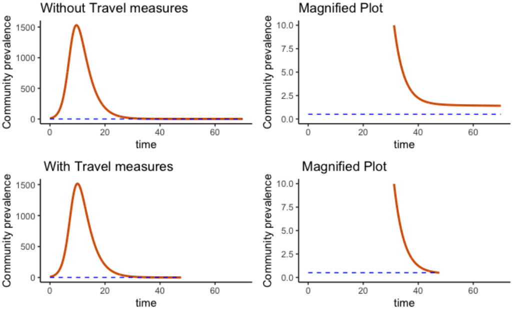
Problem 2

The optimal control is similar to Problem 1; however, the terminal condition that $I_1(T) = I_{\min} = 0.5$ may never be met if a substantial amount of infection is spread from travellers to community members. In this case, elimination cannot be achieved, and a finite choice of T is appropriate (Figure 2).



Problem 3

The optimal control is similar to that of Problem 2. We note that high u_{2max} and U_{2max} can make it possible to achieve the terminal condition $I(T) = I_{\min} = 0.5$, when otherwise it may not have been possible as the endemic equilibrium without any public health measures exceeds 0.5 (Figure 3).



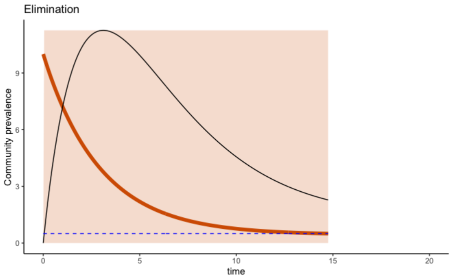
Problem 4

The optimal control is similar to Problems 2 and 3, where elimination can be an optimal strategy (Figure 4A). If one of the controls is used up before elimination occurs, we assume that under biologically reasonable parameter values, community infections

$$\frac{dI_1}{dt}$$

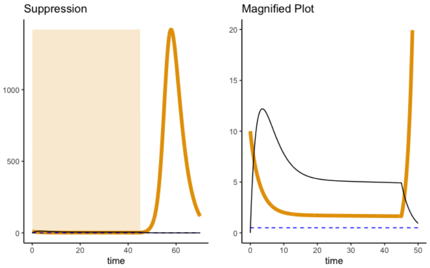
would then increase, such that the optimal control is then either suppression (Figure 4B) or suppression or circuit-breaker (Figure 4C).

(A)



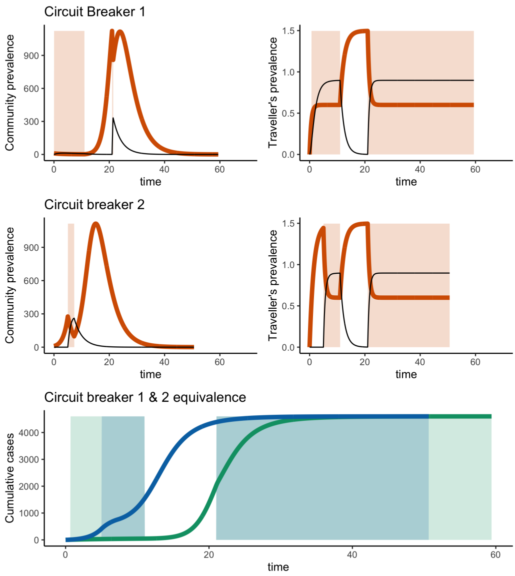
Elimination
Community prevalence
0
3
6
9
0
5
10
15
20
time

(B)



Suppression
0
500
1000
0
20
40
60
time
Magnified Plot
0
5
10
15
20
0
10
20
30
40
50
time

(C)



Circuit Breaker 1
Community prevalence
900
600
300
0
0
20
40
60
time
Traveller's prevalence
1.5
1.0
0.5
0.0
0
20
40
60
time
Circuit breaker 2
Community prevalence
900
600
300
0
0
20
40
60
time
Traveller's prevalence
1.5
1.0
0.5
0.0
0
20
40
60
time
Circuit breaker 1 & 2 equivalence
Cumulative cases
4000
3000
2000
1000
0
0
20
40
60
time

Figure 5A should be a plot that shows T and how it changes with u_{1max} (x-axis) and u_{2max} (y-axis).

Could do the same in Figure 5B but with U_{1max} and U_{2max}.

References I