Strategic Timing and Resource Allocation for Optimal Isolation and Travel Restrictions in Infectious Disease Control

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Motivation

- In public health, strategies to reduce the impacts of infectious disease spread are elimination, suppression, mitigation, and circuit breaker, where these strategies differ in their objectives, the timing and magnitude of public health interventions, and the resultant epidemiology (Table 1).
- Optimal control is a branch of mathematics that determines the timing of control measures for biological dynamics, such as those described by epidemiological models, to minimize disease incidence [Hansen and Day, 2011] or achieve another objective.

Objectives

The aim of our study is to:

1. Identify when different public health strategies are optimal, as defined by optimal control theory.

2. Extend existing results to consider imported infections and travel measures.

Problem Description

We extend the epidemiological model of Hansen and Day (2011) to consider disease importation. Specifically,

$$\frac{dS}{dt} = -\beta S(I_1 + cI_2) \tag{1}$$
$$\frac{dI_1}{dt} = \beta S(I_1 + cI_2) - (\mu + u_1)I_1 \tag{2}$$
$$\frac{dI_2}{dt} = \theta - (\gamma + u_2)I_2 \tag{3}$$

I_1 and I_2 are infection prevalence in the local community and travellers, respectively, β is the transmission rate, μ is the per capita loss rate of infected community members through both mortality and recovery, θ is the baseline number of infected non-resident travellers per unit time, c is the relative transmissibility of travellers, γ is the removal rate of non-resident travellers.

Post-Arrival Travel Measures (u_2)

- u_2: Represents the post-arrival travel isolation measure.
  - Implemented after travelers arrive to reduce the spread of infection from incoming cases.
  - Includes actions like quarantine, isolation, and testing of travelers.
  - u_2 is integrated into the model as a control variable affecting the rate of change of I_2 (infected non-resident travelers).

Other Travel Measures & formulation in the model

- **Pre-Arrival Screening/Travel Restrictions:**
  - Could be represented as a reduction in θ, the rate of infected travelers entering the community.
  - Example: Enhanced screening at borders to prevent infected individuals from entering.
  - Implementing travel bans or restrictions on high-risk regions.

In keeping with Hansen and Day (2011), we assume that resources are limited, such that,

$$\int_0^T u_1 I_{1[u_1,u_2]} \, dt \leq U_{1max} \tag{4}$$

and

$$\int_0^T u_2 I_{2[u_1,u_2]} \, dt \leq U_{2max} \tag{5}$$

The aim of public health measures is assumed to minimize the number of new infections,

$$\min J = \int_0^T \beta S_{[u_1,u_2]}(I_{1[u_1,u_2]} + I_{2[u_1,u_2]}) \, dt \tag{6}$$

Hansen and Day (2011) defined an outbreak as over at $t = T$ if prevalence is less than a minimum value ($I_{min} = 0.5$ is assumed). The reason for defining an outbreak end-point in this way is to prevent a second wave of infection arising from a fractional individual [Hansen and Day, 2011]. Defining an outbreak end-point in this way is necessary to consider elimination strategies as a possible recommended strategy [Martignoni et al., 2024].

The optimal controls are bang-bang

Our control strategies, u_1 and u_2, involve switching between two extreme values, typically represented as $u_{\max}$ and 0. This defines the concept of bang-bang controls.

$$u_1(t) = \begin{cases} u_{1max}, & \text{maximum rate of community isolation} \\ 0, & \text{no community isolation} \end{cases} \tag{7}$$

$$u_2(t) = \begin{cases} u_{2max}, & \text{maximum rate of traveller isolation} \\ 0, & \text{no traveller restrictions} \end{cases} \tag{8}$$

Problem	Description	Special values of parameters
1	Community member self-isolation only, no importations	$\theta = 0, I_2(0) = 0, U_{2max} = 0$
2	Community member self-isolation, with importations	$U_{2max} = 0.$
3	Travel measures only	$U_{1max} = 0.$
4	Both community member self-isolation and travel measures	None

Table 1: The four problems that we analyze

Definitions of Public Health Strategies

Public Health Strategy	Description	Our Definition
Elimination	Strict public health measures reduce infection prevalence to zero locally, but not in all regions, such that there remains a risk of disease importation (Baker, Wilson, and Blakely 2020; Metcalf et al. 2021)	(a) The outbreak is eliminated by public health measures not infected-derived immunity, i.e., $U_{1[u_{1\max}]}(T) < U_{1\max}$ and/or $U_{2[u_{2\max}]}(T) < U_{2\max}$. (b) $\frac{dI_1}{dt} < 0$ shortly after $u_1^*(t)$ and/or $u_2^*(t)$ are implemented.
Suppression	Infection is kept at low levels (Baker, Wilson, and Blakely 2020)	(a) The outbreak is eliminated by public health measures not infected-derived immunity, i.e., $U_{1[u_{1\max}]}(T) < U_{1\max}$ and/or $U_{2[u_{2\max}]}(T) < U_{2\max}$; (b) $\frac{dI_1}{dt} > 0$ shortly after $u_1^*(t)$ and/or $u_2^*(t)$ are implemented.
Circuit Breaker	Public health measures are intermittent with breaks in between	The optimal control involves at least two switches between public health measures of different intensity.

Table 2: Definitions of public health strategies

Problem 1: Community Isolation with no importation

This problem has previously been solved by Hansen and Day (2011).

Theorem (1)

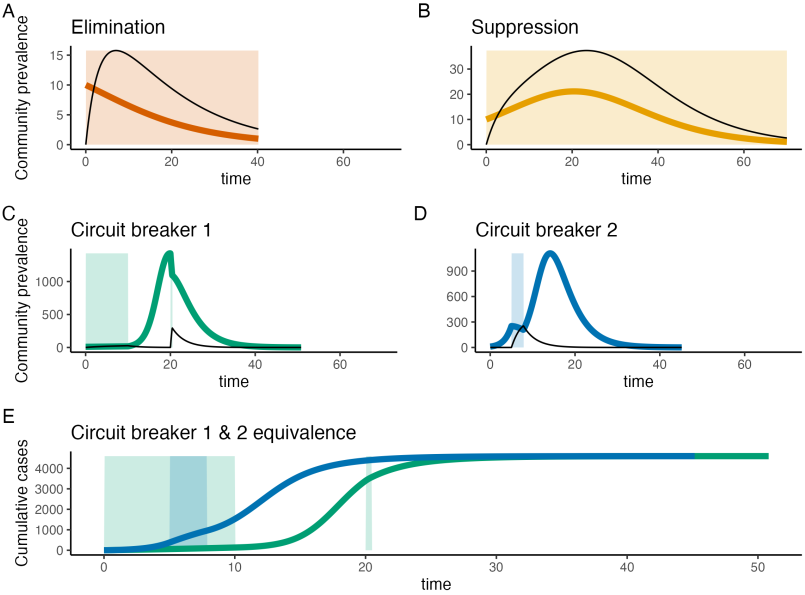
(Optimal Isolation Policy): *If* $U_{1[u_{1max}]}(T) \leq U_{1max}$*, then the optimal isolation policy for Problem 1 is* $u_1^* = u_{1max}$*. If* $U_{1[u_{1max}]}(T) > U_{1max}$*, then the optimal policy* u_1^* *is any control* u_1 *such that* $U_{1u_{1max}}(T) = U_{1max}$*.*

Rephrasing Theorem 1 in terms of public health terminology, the optimal isolation strategy is:

- **Elimination:** if $U_{1[u_{1max}]}(T) < U_{1max}$ and community infections decline shortly after the implementation of public health measures (Figure 1A).

- **Suppression:** if $U_{1[u_{1max}]}(T) \leq U_{1max}$ and community infections increase shortly after the implementation of public health measures (Figure 1B)

- **Suppression or circuit-breaker:** if $U_{1[u_{1max}]}(T) = U_{1max}$ (Figure 1C)

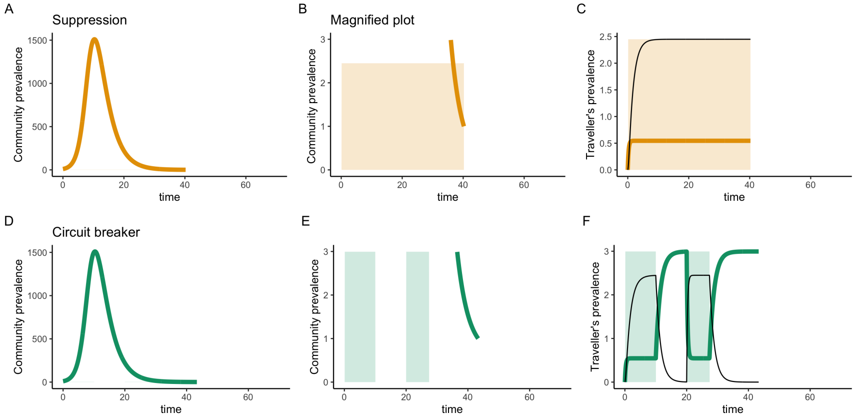

A
Elimination
Community prevalence
15
10
5
0
0
20
40
60
time
B
Suppression
30
20
10
0
0
20
40
60
time
C
Circuit breaker 1
Community prevalence
1000
500
0
0
20
40
60
time
D
Circuit breaker 2
900
600
300
0
0
20
40
60
time
E
Circuit breaker 1 & 2 equivalence
Cumulative cases
4000
3000
2000
1000
0
0
10
20
30
40
50
time

Problem 2: Community isolation when importations occur

- The optimal control is similar to Problem 1 [Hansen and Day, 2011].
- We found that achieving elimination necessitates a higher isolation rate (u_{1max}).

Problem 3: Post-arrival travel measures

The optimal control is similar to Theorem 1 [Hansen and Day, 2011]. We note that high u_{2max} and U_{2max} can make it possible to achieve the terminal condition $I_1(T) = I_{\min} = 1$.



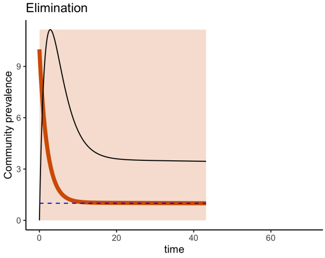
Problem 4: Combined strategies

The optimal control is similar to Problems 2 and 3, where elimination can be an optimal strategy (Figure 3A). If one of the controls is used up before elimination occurs, we assume that under biologically reasonable parameter values (values that are based on empirical data or observations from real-world scenarios), community infections

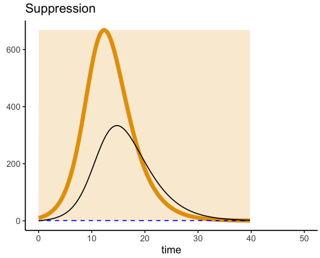
$$\frac{dI_1}{dt}$$

would then increase, such that the optimal control is then either suppression (Figure 3B) or suppression or circuit-breaker (Figure 3C).

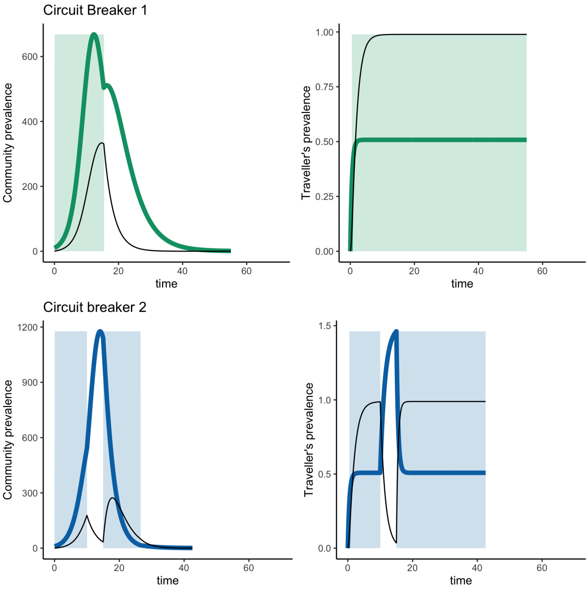
(A)

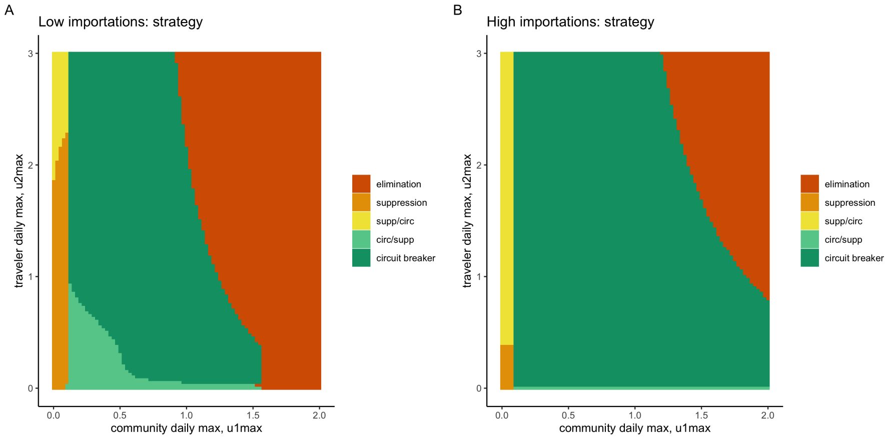

Elimination
Community prevalence
0
3
6
9
time
0
20
40
60
Traveller's prevalence
0.00
0.25
0.50
0.75
1.00
time
0
20
40
60

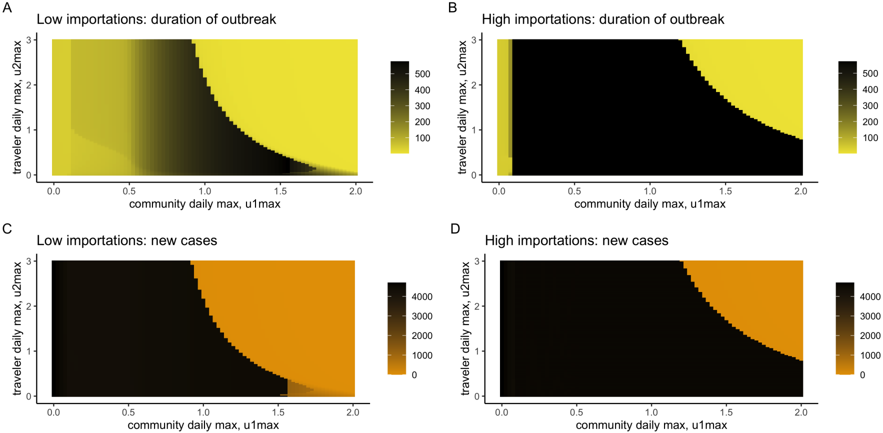
(B)


Suppression
0
200
400
600
time
0
10
20
30
40
50
Traveller's prevalence
0.00
0.25
0.50
0.75
1.00
time
0
10
20
30
40
50

(C)


Circuit Breaker 1
Community prevalence
0
200
400
600
0
20
40
60
time
Traveller's prevalence
0.00
0.25
0.50
0.75
1.00
0
20
40
60
time
Circuit breaker 2
Community prevalence
0
300
600
900
1200
0
20
40
60
time
Traveller's prevalence
0.0
0.5
1.0
1.5
0
20
40
60
time


A
Low importations: strategy
B
High importations: strategy
traveler daily max, u2max
community daily max, u1max
0
1
2
3
0.0
0.5
1.0
1.5
2.0
elimination
suppression
supp/circ
circ/supp
circuit breaker


A
Low importations: duration of outbreak
B
High importations: duration of outbreak
C
Low importations: new cases
D
High importations: new cases
traveler daily max, u2max
community daily max, u1max
0.0
0.5
1.0
1.5
2.0
0
1
2
3
500
400
300
200
100
4000
3000
2000
1000
0

- Elimination can be achieved with adequate isolation resources and maximum efforts, resulting in a shorter outbreak duration and fewer new cases.

- For Suppression and supp/circ strategies:
  - The outbreak duration is shorter due to delays in rapidly isolating infections.
  - This causes the epidemic to grow quickly (resulting in a higher number of new cases and few or no susceptibles) and end swiftly.

- For Circuit breaker and circ/supp strategies:
  - Similar results are observed as with the suppression strategy, indicating that timing does not significantly affect the outcome.

Conclusion I

- Hansen and Day (2011) discovered that if resources are insufficient to maintain isolation for the entire outbreak, then any strategy that maximizes the use of available resources is optimal.

- We built upon their work by considering the importation of cases from infected travellers and implementing travel restrictions as a control measure.

- Our results indicate that:
  - If sufficient resources are available, it is best to isolate with maximum effort.

Conclusion II

- Larger values of u_{1max} and u_{2max} are necessary to achieve elimination when the importation rate, θ, is higher.

- Despite the importation of cases, the optimal course of action mirrors that of [Hansen and Day, 2011].

- Small increases in u_{1max} and u_{2max} may make elimination feasible, which substantially reduces the duration of the outbreak and the number of cases in the outbreak.

References I

Baker, M., Wilson, N., and Blakely, T. (2020).
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BMJ, 371:m4907.

Hansen, E. and Day, T. (2011).
Optimal control of epidemics with limited resources.
Journal of Mathematical Biology, 62(3):423–451.

Martignoni, M. M., Arino, J., and Hurford, A. (2024).
Is SARS-CoV-2 elimination or mitigation best? Regional and disease characteristics determine the recommended strategy.
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References II

Metcalf, C. J., Andriamandimby, S., Baker, R., Glennon, E., Hampson, K., Hollingsworth, T., Klepac, P., and Wesolowski, A. (2021).
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