Strategic Timing and Resource Allocation for Optimal Isolation and Travel Restrictions in Infectious Disease Control

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Motivation

- In public health, strategies to reduce the impacts of infectious disease spread are elimination, suppression, mitigation, and circuit breaker, where these strategies differ in their objectives, the timing and magnitude of public health interventions, and the resultant epidemiology (Table 1).
- Optimal control is a branch of mathematics that determines the timing of control measures for biological dynamics, such as those described by epidemiological models, to minimize disease incidence [Hansen and Day, 2011] or achieve another objective.

Objectives

The aim of our study is to:

1. Identify when different public health strategies are optimal, as defined by optimal control theory.

2. Extend existing results to consider imported infections and travel measures.

Problem Description

We extend the epidemiological model of Hansen and Day (2011) to consider disease importation. Specifically,

$$\frac{dS}{dt} = -\beta S(I_1 + cI_2) \tag{1}$$
$$\frac{dI_1}{dt} = \beta S(I_1 + cI_2) - (\mu + u_1(t))I_1 \tag{2}$$
$$\frac{dI_2}{dt} = \theta - (\gamma + u_2(t))I_2 \tag{3}$$

I_1 and I_2 are infection prevalence in the local community and travellers, respectively, β is the transmission rate, μ is the per capita loss rate of infected community members through both mortality and recovery, θ is the baseline number of infected non-resident travellers per unit time, c is the relative transmissibility of travellers, γ is the removal rate of non-resident travellers.

Post-Arrival Travel Measures ($u_2(t)$)

- $u_2(t)$: Represents the post-arrival travel isolation measure.
  - Implemented after travelers arrive to reduce the spread of infection from incoming cases.
  - Includes actions like quarantine, isolation, and testing of travelers.
  - $u_2(t)$ is integrated into the model as a control variable affecting the rate of change of I_2 (infected non-resident travelers).

Resource constraints

In keeping with Hansen and Day (2011), we assume that resources are limited, such that,

$$U_{1[u_1,u_2]}(T) = \int_0^T u_1(t) I_{1[u_1,u_2]}\, dt \leq U_{1max} \tag{4}$$

and

$$U_{2[u_1,u_2]}(T) = \int_0^T u_2(t) I_{2[u_1,u_2]}\, dt \leq U_{2max} \tag{5}$$

The aim of public health measures is assumed to minimize the number of new infections,

$$J = \int_0^T \beta S_{[u_1,u_2]}(I_{1[u_1,u_2]} + cI_{2[u_1,u_2]})\, dt \tag{6}$$

Without constraints

When we have no resource limitations (constraints) on the controls, we get:

$$U_{1[u_{1max},u_2^*]}(T) = \int_0^T u_1(t) I_{1[u_1,u_2]} \, dt \tag{7}$$

and

$$U_{2[u_1^*,u_{2max}]}(T) = \int_0^T u_2(t) I_{2[u_1,u_2]} \, dt \tag{8}$$

Defining an Outbreak End-Point

- Hansen and Day (2011) defined an outbreak as over at $t = T$ if prevalence is less than a small value, I_{min}. This approach prevents a second wave of infection arising from a fractional individual [Hansen and Day, 2011].

- Defining an outbreak end-point in this way is necessary to consider elimination strategies as a possible recommended strategy [Martignoni et al., 2024].

The optimal controls are bang-bang

Hansen and Day (2011) show that the optimal control for this problem is bang-bang. Bang-bang control involve switching between two exteme values, typically represented as u_{max} and 0.

$$u_1^*(t) = \begin{cases} u_{1max}, & \text{maximum rate of community isolation} \\ 0, & \text{no community isolation} \end{cases} \tag{9}$$

$$u_2^*(t) = \begin{cases} u_{2max}, & \text{maximum rate of traveller isolation} \\ 0, & \text{no traveller restrictions} \end{cases} \tag{10}$$

Problem	**Description**	**Special values of parameters**
1	Community member self-isolation only, no importations	$\theta = 0, I_2(0) = 0, U_{2max} = 0$
2	Community member self-isolation, with importations	$U_{2max} = 0.$
3	Travel measures only	$U_{1max} = 0.$
4	Both community member self-isolation and travel measures	None

Table 1: The four problems that we analyze

Definitions of Public Health Strategies

Public Health Strategy	**Description**	**Our Definition**
Elimination	Strict public health measures reduce infection prevalence to zero locally, but not in all regions, such that there remains a risk of disease importation (Baker, Wilson, and Blakely 2020; Metcalf et al. 2021)	(a) The outbreak is eliminated by public health measures, i.e., $U_{1[u_{1\max}]}(T) \leq U_{1\max}$ and $U_{2[u_{2\max}]}(T) \leq U_{2\max}$. (b) $\frac{dI_1}{dt} < 0$ shortly after $u_1^*(t)$ and/or $u_2^*(t)$ are implemented.
Suppression	Infection is kept at low levels (Baker, Wilson, and Blakely 2020)	(a) The outbreak is eliminated by public health measures, i.e., $U_{1[u_{1\max}]}(T) \leq U_{1\max}$ and/or $U_{2[u_{2\max}]}(T) \leq U_{2\max}$; (b) $\frac{dI_1}{dt} \geq 0$ shortly after $u_1(t)$ and/or $u_2(t)$ are implemented.
Circuit Breaker	Public health measures are intermittent with breaks in between	An optimal control involves at least two switches between public health measures of different intensity.

Table 2: Definitions of public health strategies

Problem 1: Community Isolation with no importation

This problem has previously been solved by Hansen and Day (2011).

Theorem (1)

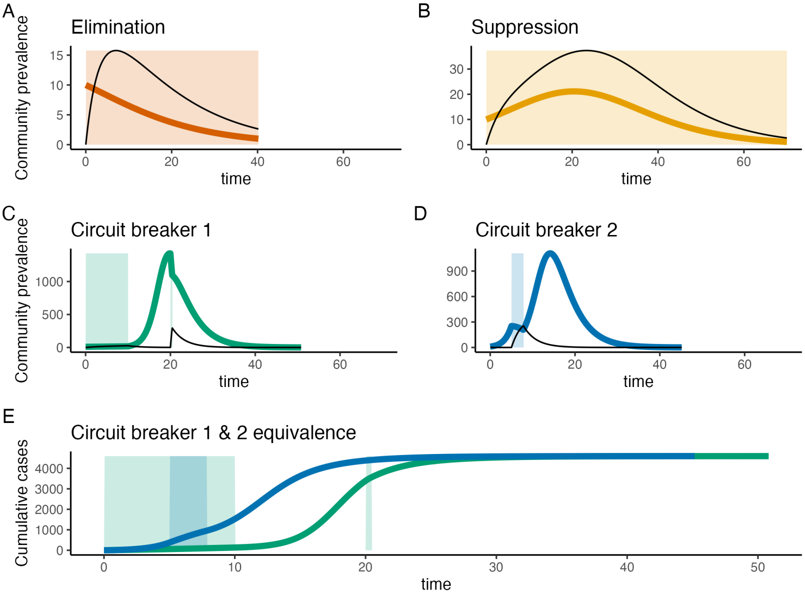
(Optimal Isolation Policy): *If* $U_{1[u_{1max}]}(T) \leq U_{1max}$*, then the optimal isolation policy for Problem 1 is* $u_1^*(t) = u_{1max}$*. If* $U_{1[u_{1max}]}(T) > U_{1max}$*, then the optimal control* $u_1^*(t)$ *is any bang-bang control* $u_1(t)$ *such that* $U_{1[u_1^*]}(T) = U_{1max}$*.*

Rephrasing Theorem 1 in terms of public health terminology, the optimal isolation strategy is:

- **Elimination:** if $U_{1[u_{1max}]}(T) \leq U_{1max}$ and community infections decline shortly after the implementation of public health measures (Figure 1A).

- **Suppression:** if $U_{1[u_{1max}]}(T) \leq U_{1max}$ and community infections increase shortly after the implementation of public health measures (Figure 1B)

- **Suppression or circuit-breaker:** if $U_{1[u_{1max}]}(T) > U_{1max}$ (Figure 1C)

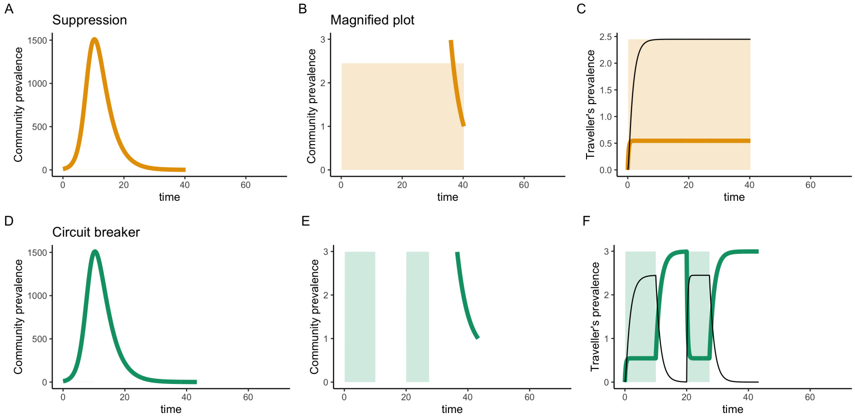

A
Elimination
Community prevalence
15
10
5
0
0
20
40
60
time
B
Suppression
30
20
10
0
0
20
40
60
time
C
Circuit breaker 1
Community prevalence
1000
500
0
0
20
40
60
time
D
Circuit breaker 2
900
600
300
0
0
20
40
60
time
E
Circuit breaker 1 & 2 equivalence
Cumulative cases
4000
3000
2000
1000
0
0
10
20
30
40
50
time

Problem 2: Community isolation when importations occur

- The optimal control is similar to Problem 1 [Hansen and Day, 2011].
- With importations elimination requires a higher community isolation rate (u_{1max}).

Problem 3: Post-arrival travel measures

The optimal control is similar to Theorem 1 [Hansen and Day, 2011]. We note that high u_{2max} and U_{2max} can make it possible to achieve the terminal condition $I_1(T) = I_{\min} = 1$.



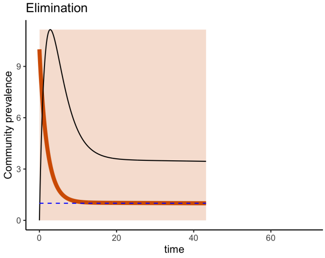
Problem 4: Combined strategies

Theorem (2)

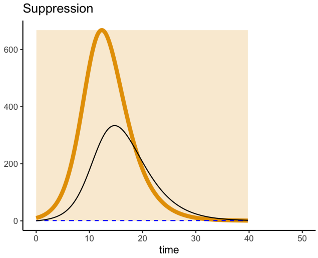
(Optimal Mixed Strategies):

If $U_{1[u_{1max},u_{2max}]} \leq U_{1max}$ *and* $U_{2[u_{1max},u_{2max}]} \leq U_{2max}$ *then the optimal control is* $u_1^*(t) = u_{1max}$ *and* $u_2^*(t) = u_{2max}$. *If* $U_{1[u_{1max},u_{2max}]} > U_{1max}$ *or* $U_{2[u_{1max},u_{2max}]} > U_{2max}$ *then the optimal control is any bang-bang* $u_1^*(t)$ *and* $u_2^*(t)$ *such that* $U_{1[u_1^*,u_2^*]} = U_{1max}$ *and* $U_{2[u_1^*,u_2^*]} = U_{2max}$.

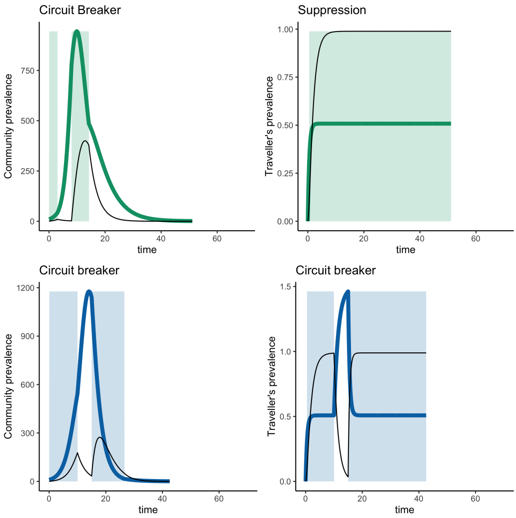
(A)


Elimination
Community prevalence
0
3
6
9
0
20
40
60
time
Traveller's prevalence
0.00
0.25
0.50
0.75
1.00
0
20
40
60
time

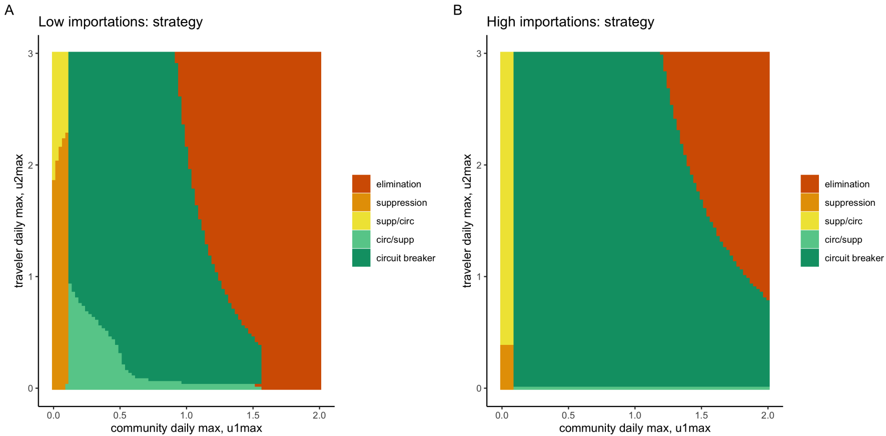
(B)

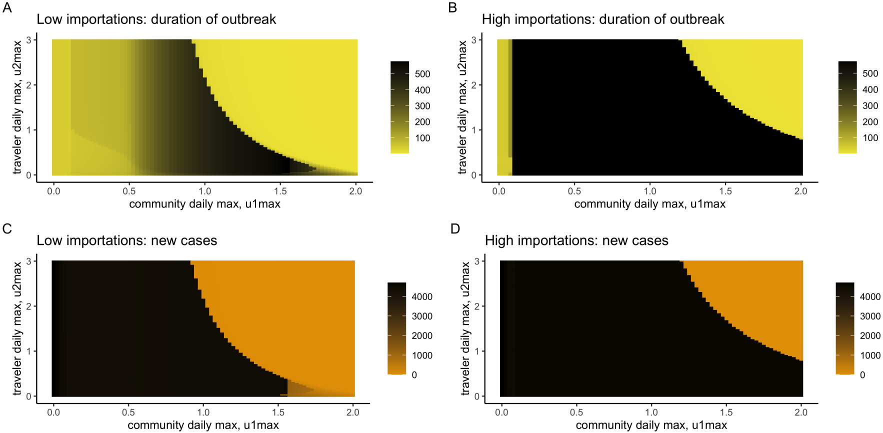

Suppression
0
200
400
600
0
10
20
30
40
50
time
Traveller's prevalence
0.00
0.25
0.50
0.75
1.00
0
10
20
30
40
50
time

(C)



$u_{1max} = 1.15$ for elimination (A), $u_{1max} = 0.2$ for suppression and circuit breaker (B-C). $u_{2max} = 1.3$ for elimination, suppression and circuit breaker (A-C). $C_{1max} = 1500$, $C_{2max} = 50$ for elimination and suppression (A-B), $C_{1max} = 1000$, $C_{2max} = 40$ for circuit breaker (C).


A
Low importations: strategy
B
High importations: strategy
traveler daily max, u2max
community daily max, u1max
0
1
2
3
0.0
0.5
1.0
1.5
2.0
elimination
suppression
supp/circ
circ/supp
circuit breaker


A
Low importations: duration of outbreak
B
High importations: duration of outbreak
C
Low importations: new cases
D
High importations: new cases
traveler daily max, u2max
community daily max, u1max
0.0
0.5
1.0
1.5
2.0
0
1
2
3
500
400
300
200
100
4000
3000
2000
1000
0

- Elimination can be achieved with adequate isolation resources and maximum efforts, resulting in a shorter outbreak duration and fewer new cases.
- For Suppression and supp/circ strategies:
  - Public health measures have a small effect and do not delay infections.
  - This causes the epidemic to grow quickly and end swiftly.
- For Circuit breaker and circ/supp strategies:
  - The total number of cases is similar for suppression and circuit breaker strategies, and substantially more than the number of cases when the maximum daily isolation rates are sufficient to achieve elimination.

Conclusion I

- Hansen and Day (2011) discovered that if resources are insufficient to maintain isolation for the entire outbreak, then any strategy that maximizes the use of available resources is optimal.
- We built upon their work by considering the importation of cases from infected travellers and implementing travel restrictions as a control measure.
- Our results indicate that:
  - If sufficient resources are available, it is best to isolate with maximum effort beginning immediately.
  - Larger values of u_{1max} and u_{2max} are necessary to achieve elimination when the importation rate, θ, is higher.

Conclusion II

- Despite the importation of cases, the optimal course of action mirrors that of [Hansen and Day, 2011].
- Small increases in u_{1max} and u_{2max} may make elimination possible, which substantially reduces the duration of the outbreak and the number of cases in the outbreak.

References I

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References II

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