Lancaster University

PHYS451 Masters Project Report

A Manned Mission to Mars - Simulations of Transfer Trajectories to the Red Planet

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Abstract

Presented is a study of orbital transfers a manned mission to Mars might take and the associated fuel costs and transfer times. A program designed to simulate and search for these trajectories was devised and operated to provide data for five launch opportunities in the next ten years. The results of this program have been used along with some preliminary mass estimates to calculate the total mass of the manned portion of the mission. The mission layout suggested involves sending unmanned cargo vehicles ahead of the crew along an efficient trajectory to await the crew's arrival at Mars, the crewed vehicle will then rendezvous after travelling along one of the more efficient routes discussed. A selection of propulsion choices have also been discussed and their implications on the final total mass have been considered. The report suggests the use of Nuclear Thermal Rocket engines as propulsion for the manned portion of the mission giving the vehicle a total mass of ≈ 200 T to be launched from Low Earth Orbit. The vehicle would likely be constructed in orbit using multiple launches.

May 18, 2015

1 Introduction

A manned mission to Mars is seen by many to be the next big step in human space exploration and a manned mission to our nearest neighbour would certainly reveal a wealth of new science. Mars is seen as one of the most likely candidates for having harboured ancient life and a manned mission could vastly improve our chances of discovering it. Humans have the benefit of being able to work autonomously and intelligently on the Martian surface in a way which cannot be mimicked by unmanned rovers, which need periodic instruction from operators on Earth. There can be delays in signals of up to 22 minutes when Mars is at its furthest from Earth and this results in slow progress for Mars' current robotic explorers.

In this report I discuss a model created to simulate trajectories a spacecraft may take on a journey to Mars and I look to explore the relationship between the fuel cost (Delta-V, ΔV) and transfer times for a range of upcoming launch windows, with a focus on minimising the transfer times. Reducing this time as much as possible is important due to the high radiation environment astronauts will face in interplanetary space. During the 253 day transit the Mars Science Laboratory (AKA Curiosity) undertook on its way to Mars, the Radiation Assessment Detector (RAD) instrument made measurements of the radiation environment experienced by the spacecraft. This lead to the unsettling conclusion that the astronauts embarking on a manned mission to Mars would likely receive more than their current career limit doses on the round trip [1]. Other reasons for the desire to keep the transfer time short include both the negative psychological effects of being in an isolated, confined space for a long time and the physiological effects of long term weightlessness on the human body.

In the first section of this report I will discuss the mechanics of orbital transfers which form the basis of the trajectory simulations performed by the program. I then go on to outline the function of the model and how it uses the mathematics of orbital mechanics to generate trajectories a spacecraft may take between the Earth and Mars. The results section follows and presents the differences in fuel costs, transfer times and launch dates for a selection of launch windows available in the next ten years. Finally, I explore the implications of these results and go on to outline a possible mission design making some preliminary mass and fuel estimates for a future manned mission to Mars.

2 Orbital Mechanics

The motion of the planets, their moons and all other bodies travelling through the solar system (and indeed, the universe) are governed predominantly by gravitational attraction. Kepler first described the motion of the planets in 1619, he had noticed that all the planets orbited elliptically with the Sun at one focus of the ellipse. Newton described his law of gravitation in 1687 and we now know the motions of the planets to be governed by their mutual gravitational attraction. The maths used to describe the three-dimensional motions of bodies in orbits is called orbital mechanics and is briefly outlined in this section as it is a necessary prelude to calculating orbital trajectories.

2.1 Orbital Elements

Any orbit around a central body can be completely described by six parameters known as the Keplerian Orbital Elements (see Figures 1 and 2 for diagrams)

- Eccentricity, e
- Semi-Major axis, a
- Inclination, i
- Longitude of Ascending node, Ω
- Argument of Periapsis, ω
- True Anomaly, ν

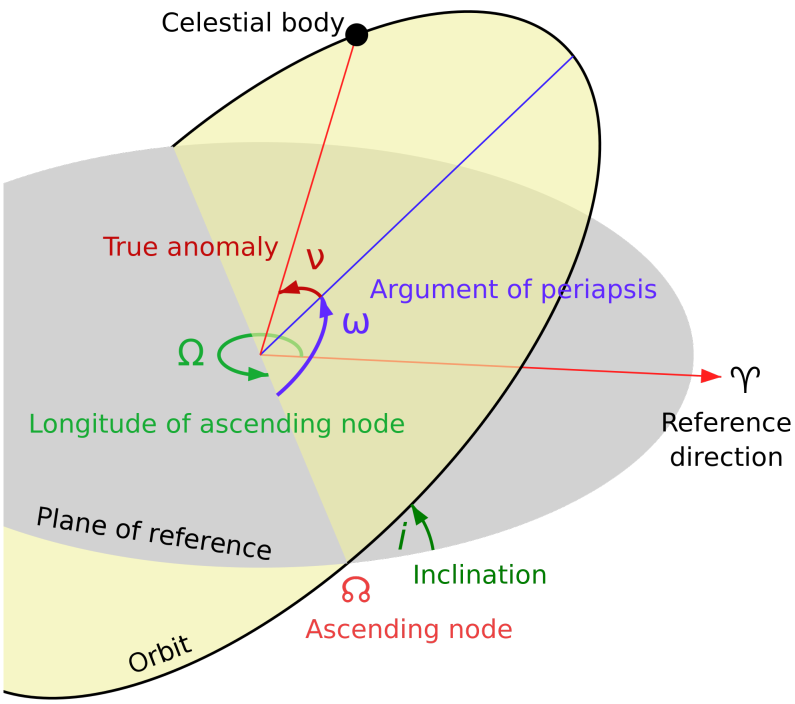

Figure 1: Diagram showing the how the orbital elements define a celestial body's orbit around a central body with respect to a predetermined reference direction.

The point where the orbit is closest to its parent body is called the periapsis and the point at which it is further away is called the apoapsis. For a spacecraft in orbit, the current direction of travel is known as the prograde direction and the direction opposite is known as retrograde.

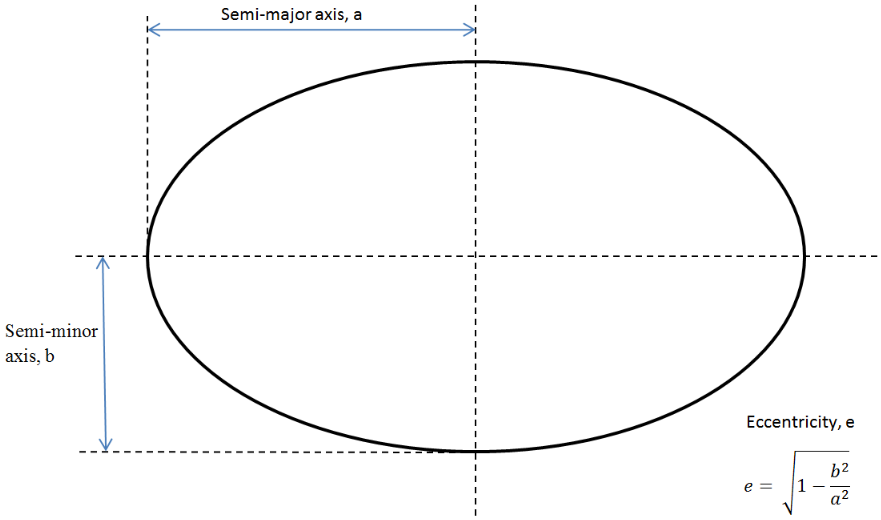

Figure 2: Diagram showing how the semi-major, semi-minor axis and eccentricity are defined for an elliptical orbit.

These parameters restrict the six degrees of freedom associated with (non-uniform) motion in 3-dimensional space allowing the position of a body (e.g. a planet or spacecraft) in a stable orbit to be calculated [2]. To find this position we must calculate the true anomaly, which can only be found by first calculating the mean anomaly and eccentric anomaly. The difference between these are pictured in Figure 3.

The mean anomaly, M, is defined as the fraction of an orbital period which has elapsed since periapsis.

$$M - M_0 = n(t - t_0) \tag{1}$$

Where M_0 is the mean anomaly at a time t_0 and the mean motion, $n = \sqrt{\frac{GM}{a^3}}$

The eccentric anomaly, E is then defined as:

$$M = E - e \sin E \tag{2}$$

Which is a transcendental equation and cannot be solved analytically, however it can be easily solved numerically using Newton's iterative method, where each step in the iteration is defined as in (Equation 3).

$$E_{n+1} = E_n - \frac{f(E_n)}{f'(E_n)} \tag{3}$$

Which leads to Equation 4. This means the eccentric anomaly can be found by first estimating it as equal to the mean anomaly as they are approximately the same and performing a number of iterations until convergence.

$$E_{n+1} = E_n - \frac{E - e\sin E - M}{1 - e\cos E} \tag{4}$$

This then lets us find the true anomaly, ν, which is the angle between the direction of periapsis and the position of the body with respect to the main focus of the elliptical orbit. It is defined in Equation 5 and rearranged into Equation 6 for use for calculation as this form retains quadrant information.

$$\begin{aligned} \nu &= \cos^{-1}\left(\frac{\cos E - e}{1 - e\cos E}\right) && (5) \\ &= 2\arctan\left[\sqrt{1-e}\cos\frac{E}{2}, \sqrt{1+e}\sin\frac{E}{2}\right] && (6) \end{aligned}$$

The distance from the main focus of the orbit, r, is given by:

$$r = \frac{a(1-e^2)}{1 + e\cos\nu} \tag{7}$$

Therefore using the results of Equations 6 and 7, the position of the body can be found.

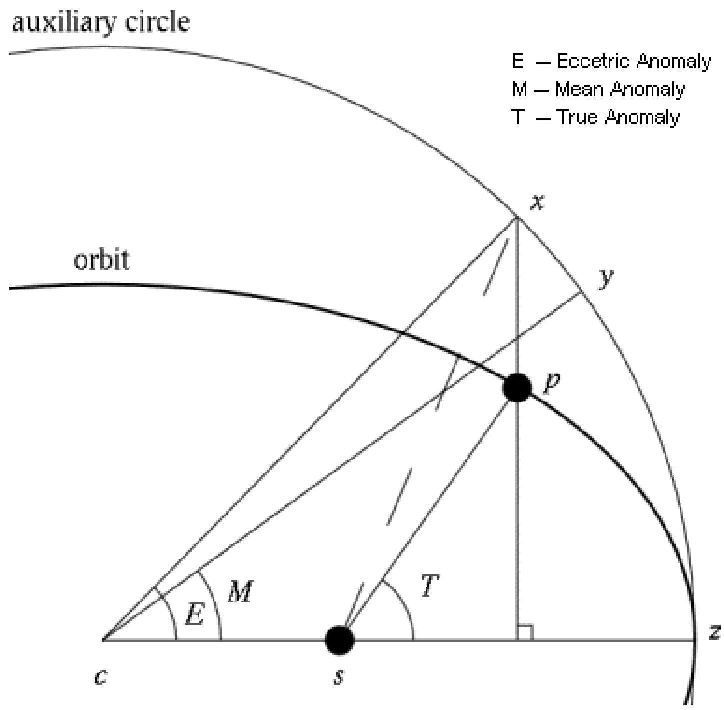

Figure 3: Diagram showing the mean anomaly, eccentric anomaly and true anomaly are defined and how they differ from one another. The diagram shows this for the different between

2.2 Delta-V ΔV & Specific Impulse I_{SP}

The velocity of a body at any point in its orbit can be found by equating its energy (kinetic + potential) to half the potential energy at the average distance, a.

$$\begin{aligned}\frac{1}{2}mv^2 - \frac{GMm}{r} &= -\frac{GMm}{2a} &(8)\\ \frac{1}{2}v^2 &= \frac{GM}{r} - \frac{GM}{2a} &(9)\\ v^2 &= GM\left(\frac{2}{r} - \frac{1}{a}\right) &(10)\end{aligned}$$

Where G is Newton's Gravitational constant, M is the mass of the central body and r is the current distance of the orbiting body from the central body. Equation 10 is called the vis-viva equation and from it we can see that changing the velocity of a body at a distance r from the central body will have an effect on the semi-major axis, a of its orbit and therefore modify the eccentricity, e as well.

Therefore, in order to move from one orbit to another, the velocity of the spacecraft must be changed. The amount of velocity change which is required for a given manoeuvre is called the ΔV.

The amount of ΔV produced by a propulsion system is given by Tsiolkovsky's Rocket Equation (Equation 11):

$$\Delta V = v_{\text{e}} \ln \frac{M_0}{M_1} \tag{11}$$

Where, v_{e} is the exhaust velocity and $\frac{M_0}{M_1}$ is the fraction of the mass when fully fuelled and when empty. The specific impulse I_{SP} is commonly used as a measure of engine efficiency, engines with a higher I_{SP} having a higher efficiency. It is defined as

$$v_e = gI_{\text{SP}} \tag{12}$$

Where $g = 9.81\text{ms}^{-1}$ [3]. I will not go into detail on the subject of the rocket equation and specific impulse as they were covered in my literature review, but they must be stated here as they are necessary for the estimations made later in this report.

2.3 Orbital Transfers

Orbital transfers are transitions between two orbits around a central body. In order to change the orbit of a body, its velocity must be changed by the application of thrust.

As we will be dealing with interplanetary transfers between Earth and Mars, we will need to use a transfer type which works well with transfers between two near-circular orbits. For these kind of transfers, the Hohmann Transfer (shown in Figure 4) orbit has been shown to be the most efficient provided the ratio of radii of the larger to smaller orbit is less than 11.8 [4] [5], this is applicable for an Earth-Mars transfer.

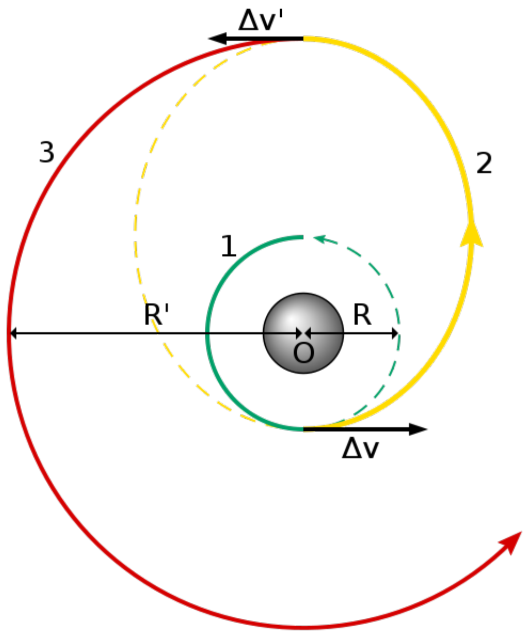

Figure 4: Diagram of a Hohmann Transfer from orbit 1 to orbit 3 taking path 2. ΔV is initially applied in orbit 1, then when the spacecraft reaches the apoapsis it applies $\Delta V'$ to circularise.

The ΔV required for a Hohmann transfer can be easily calculated provided a number of simplifying assumptions are made:

- The orbits of Earth and Mars are circular
- They orbit in the same plane so have no relative inclination

The transfer must begin at the orbit of Earth and end at Mars' orbit, therefore the semi-major axis of the transfer will be given by the expression:

$$a_{\text{Transfer}} = \frac{r_{\text{Earth}} + r_{\text{Mars}}}{2} = \frac{1.496 \times 10^{11}\ \text{m} + 2.279 \times 10^{11}\ \text{m}}{2} = 1.886 \times 10^{11}\ \text{m} \tag{13}$$

For the case of circular orbits where $r = a$ throughout the whole orbit, the vis-viva equation (Equation 10) for a planet orbiting the sun becomes:

$$v = \sqrt{\frac{GM_{\text{Sun}}}{r}} \tag{14}$$

We can therefore make estimates of the ΔV required in order to make the manoeuvre from Earth's orbit into a Mars' transfer orbit.

We start by calculating the initial velocity of a spacecraft in Earth's orbit (orbit around the Sun at the same distance as Earth).

$$v_E = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{Earth}}}} = 29,786\ \text{ms}^{-1} \tag{15}$$

We then calculate the velocity the spacecraft must have in order to enter the correct transfer trajectory.

$$v_T = \sqrt{GM_{\text{Sun}}\left(\frac{2}{r_{\text{Earth}}} - \frac{2}{a_{\text{Transfer}}}\right)} = 32,732 \text{ ms}^{-1} \tag{16}$$

Therefore the change in velocity, the ΔV, will be given by:

$$\Delta V_1 = v_T - v_E = 2946 \text{ ms}^{-1} \tag{17}$$

This gives us our estimate for the ΔV required to place the spaceship in the interplanetary transfer.

When the spacecraft reaches the orbit of Mars, it must again change its velocity to match that of Mars. If this were to be done using an entirely propulsive manoeuvre, then we can repeat the calculation for a second burn.

$$v_T = \sqrt{GM_{\text{Sun}}\left(\frac{2}{r_{\text{Mars}}} - \frac{2}{a_{\text{Transfer}}}\right)} = 21,478 \text{ ms}^{-1} \tag{18}$$

$$v_M = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{Mars}}}} = 24,152 \text{ ms}^{-1} \tag{19}$$

$$\Delta V_2 = v_M - v_T = 2674 \text{ ms}^{-1} \tag{20}$$

2.4 Launch Windows

Spacecraft cannot easily launch on missions to other planets at any time. A launch window is the span of time available for the spacecraft to undertake the transfer. This is because Earth and Mars are moving around the Sun at different distances and different orbital speeds, so opportunities to transfer between them are limited. The spacecraft must arrive at Mars' orbit when Mars is there in order for it to attempt to be captured.

If we assume that Earth and Mars are in circular orbits, then it is quite simple to estimate the interval between times when Earth and Mars are at the same relative angle. This interval is called the synodic period and for superior planets is given by Equation 21:

$$\frac{1}{P_{\text{Syn}}} = \frac{1}{P_{\text{Earth}}} - \frac{1}{P_{\text{Mars}}} \tag{21}$$

Where P_{Earth} and P_{Mars} are the orbital periods of Earth and Mars respectively. Using this equation, $P_{\text{Syn}} = 779$ days.

However, this only gives us the time between the same angle but not the angle needed for the transfer (called the phase angle, θ). To estimate this we can use a basic Hohmann transfer between circular approximations of Earth and Mars' orbits.

As calculated before in Equation 13, a transfer orbit between these will have the semi-major axis of 1.886×10^{11} m and using Kepler's 3rd Law (Equation 22), we can estimate the time the transfer will take:

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM} \tag{22}$$

$$T = \sqrt{a^3 \frac{4\pi^2}{GM}} \tag{23}$$

Where T is the orbital period, a is the semi-major axis and M is the mass of the central body. Using Equation 23 we can find the orbital period of the transfer orbit, dividing this in half will give us the time taken to transfer from Earth's orbit, to Mars'.

$$T = \frac{1}{2}\sqrt{(1.886 \times 10^{11} \text{ m})^3 \frac{4\pi^2}{G \times 1.99 \times 10^{30} \text{ kg}}} = 22.3 \times 10^6 \text{ seconds} \tag{24}$$
$$= 258 \text{ days}$$

The spacecraft will therefore take around 258 days to reach Mars' orbit on the opposite side of the Sun. So the spacecraft should launch when Mars is 258 days in its orbit before reaching that point (see Figure 5).

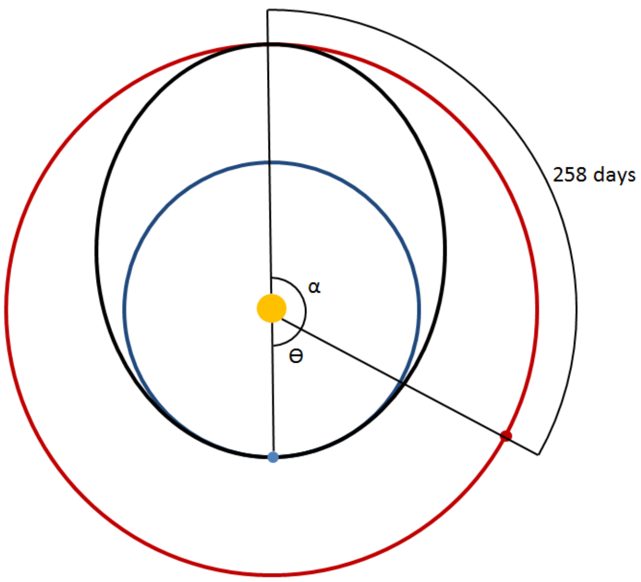

Figure 5: Figure showing how the positions of the Earth and Mars at the beginning of the transfer in order for the spacecraft to rendezvous with Mars. The spacecraft must launch when Mars is 258 days behind the rendezvous point in it's orbit. θ is known as the phase angle and is the relative angle between the planets when the transfer begins in order to properly rendezvous at the end of the transfer.

Therefore, as one Martian year is 687 days, α and θ can be determined.

$$\alpha = \frac{258 \text{ days}}{687 \text{ days}} \times 360^\circ = 135^\circ \tag{25}$$

$$\theta = 180^\circ - 135^\circ = 45^\circ \tag{26}$$

The next time this angle occurs between the Earth and Mars is in January 2016 [6], and so a selection of launch windows can be roughly determined as this phase angle will occur every synodic period from then:

- January 2016
- March 2018
- May 2020
- June 2022
- August 2024

These estimates are extremely rough as the orbits of the Earth and Mars are eccentric not circular but they should suffice as a starting point for the model to search for transfers across multiple launch windows.

3 The Model

The model was created by the author using GDL, an open source version of the Interactive Data Language (IDL) and the code is attached in Appendix A. While not the ideal programming language for running simulations like this on a large scale, it is easy to use and sufficient for the the purpose of the running our program satisfactorily.

3.1 Aims

The main aim was to model trajectories a spacecraft may take between Earth and Mars with a variety of departure dates across multiple launch windows. The transfer times and ΔV required must then be recorded in order to explore the relationship between these and to discuss the implications in relation to a manned mission to Mars. I have called the types of transfers being tested Hohmann-like in that they apply the ΔV in the direction of travel and encounter Mars on the opposite side of the Sun, however they may encounter Mars at an earlier time if more ΔV is applied (this is shown in Figure 6), this leads to faster transfers which are of particular interest.

The goal was to start with a simple model and gradually build complexity in order to remove assumptions and simplifications.

1. Two dimensional, circular orbits around a central body and transfers between orbits
2. Transfers between two bodies in circular orbits around a central body
3. Transfers between the Earth and Mars' actual eccentric orbits

The model must then be able to procedurally search for valid trajectories using a range of criteria and then output the results in a way which can be usefully analysed.

There were a number of aims which time constraints did not allow us to explore which would further reduce the assumptions made.

4. Model transfers between Mars' and Earth to fully consider the round trip
5. Fully generalise into a three dimensional model

The inclinations of Earth and Mars were not taken into account as it is a 2D model, though this should have a relatively small effect as Earth and Mars' inclinations only differ by $\approx 2°$.

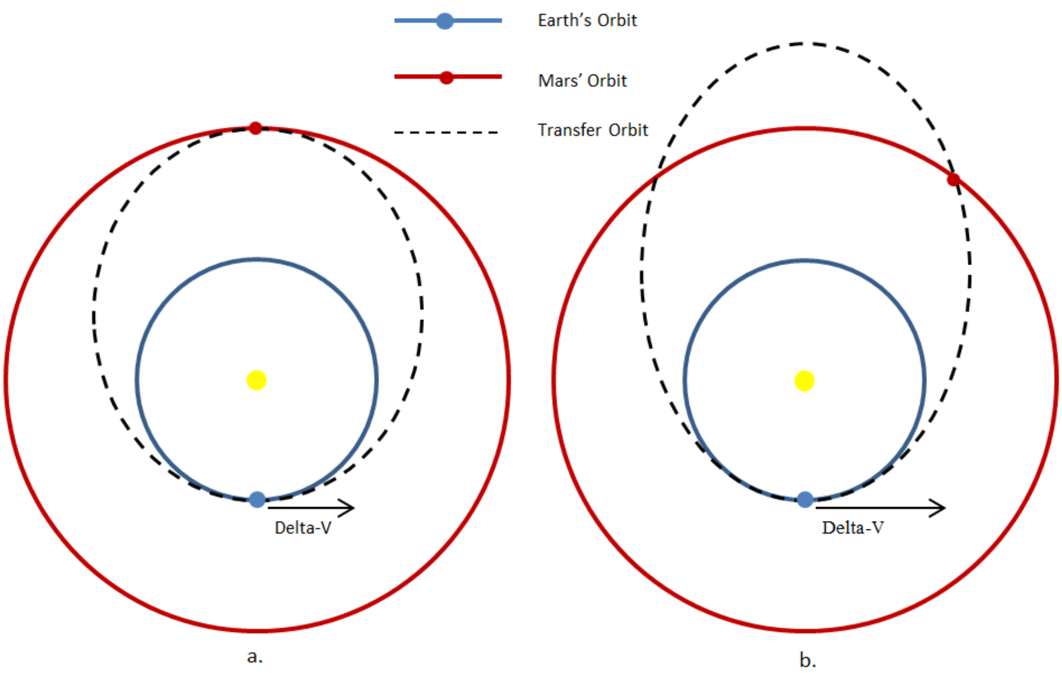

Figure 6: Figure showing the Hohmann-like trajectories the program models. a. shows a Hohmann transfer where the ΔV application is made to perfectly align with Mars at apoapsis. b. Diagram showing a Hohmann-like transfer where more ΔV is applied in order to encounter Mars at an earlier time.

3.2 Model Design

The model uses the maths discussed in Section 2.1 to iteratively simulate trajectories. The process is shown in Figure 7

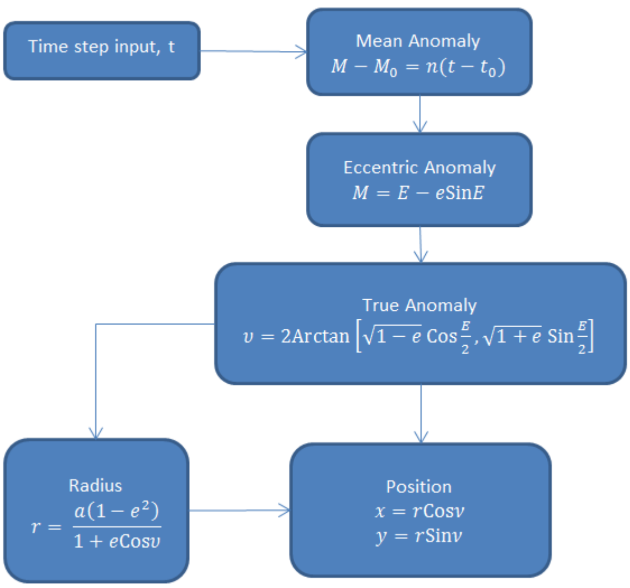

Figure 7: Figure showing the process the model uses to model trajectories in a gravitational field. The program takes a time-step argument and iterates over that time-step, calculating the new position at each step.

Although the model was able to simulate trajectories across interplanetary distances, due to the eccentricities of Earth and Mars' orbits, it is impossible to know analytically what the semi-major axis a (from which the ΔV can be calculated) the trajectory should have in order to rendezvous with Mars.

We needed to simulate a large number of trajectories with varying semi-major axis and test whether they are valid transfers. To test this I have used the condition of whether the trajectory enters the sphere of influence (SOI) of Mars. The SOI is defined as the region of space surrounding a gravitational body in which it is the primary gravitational influence and given by Equation 27.

$$r_{\mathrm{SOI}} = a\left(\frac{M}{m}\right)^{\frac{2}{5}} \tag{27}$$

Where a and m are the semi-major axis and mass of the orbiting body and M is the mass of the central body. For a planet like Mars, the Sun is the central body and Mars is the orbiting body. Using this equation, the SOI of Mars is $\approx 5.77 \times 10^8$ m.

The model also needs input for the positions of Earth and Mars at times in the future, to do this I have used the NASA JPL Horizons ephemeris system to provide the initial inputs of the planet positions. The model then simulates the trajectories of both the planets and trials trajectories which begin at Earth and end within the SOI of Mars.

The program includes three embedded loops to conduct its search:

1. An iteration of the date to be tested within the time period being modelled, passes down the position of the Earth and so start of the trajectory.
2. An iteration of the starting ΔV, passes this to the initial step of the simulation loop.
3. The iteration loop follows the process in Figure 7 which is iterated over a time period which allows the full transfer. If the resulting trajectory passes within the SOI of Mars it records the result.

The result of this is that for each day in the launch window selected, a large range of different initial ΔV is tested to search for transfers.

4 Results

4.1 Minimising Transfer Time

The program was run for 5 future launch windows in 2016, 2018, 2020, 2022 and 2024. It selects the transfers which resulted in the spacecraft entering Mars' SOI, recording the launch date, total ΔV and transfer time. The results of each launch window are shown in Figure 8.

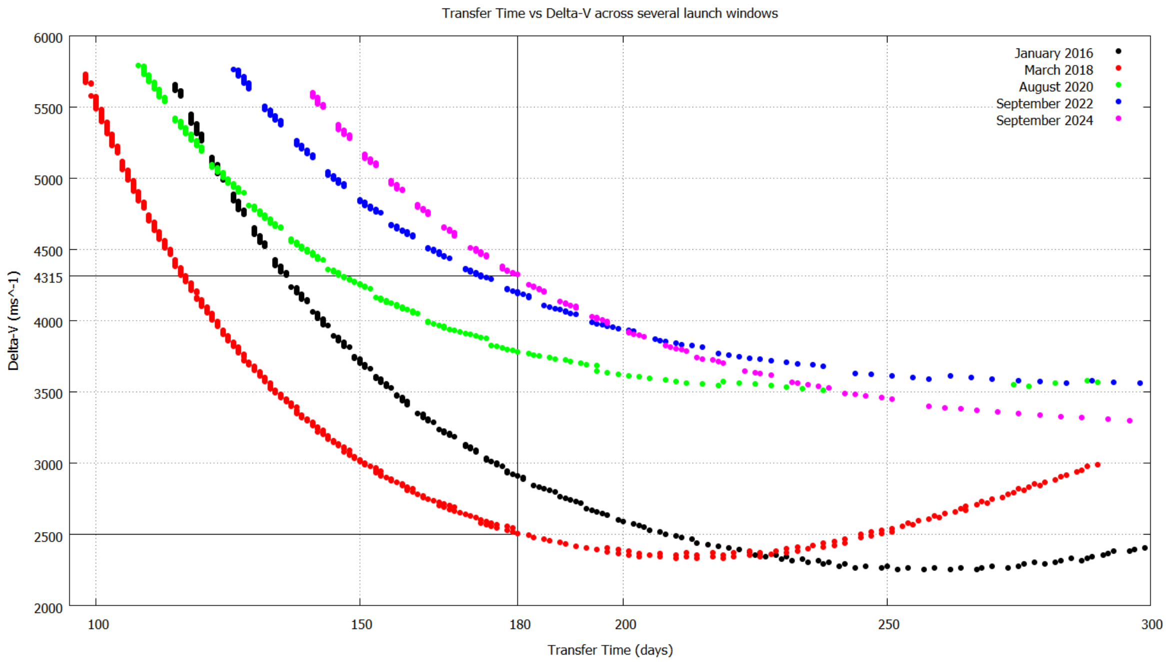

Figure 8: Figure showing the ΔV plotted against transfer time in days for launch windows in the next 10 years. Marked with the solid black lines is a reference transfer time of 180 days in order to highlight how the required ΔV varies across launch windows.

From this, it is clear that the amount of ΔV required for the transfer varies greatly between launch windows. The NASA Design Reference Architecture 5.0 [7] describes a proposed transfer with a 6 month duration so I have used that here as benchmark. Using this 180 day transfer as a guide, the amount of ΔV required for the transfer varies between $2500 - 4315$ ms^{-1} with the fastest transfers available in the March 2018 window.

4.2 Launch Date

Figures 9 - 13 show the Δ_V required for each day in the launch window, with the colour gradient showing the transfer times associated.

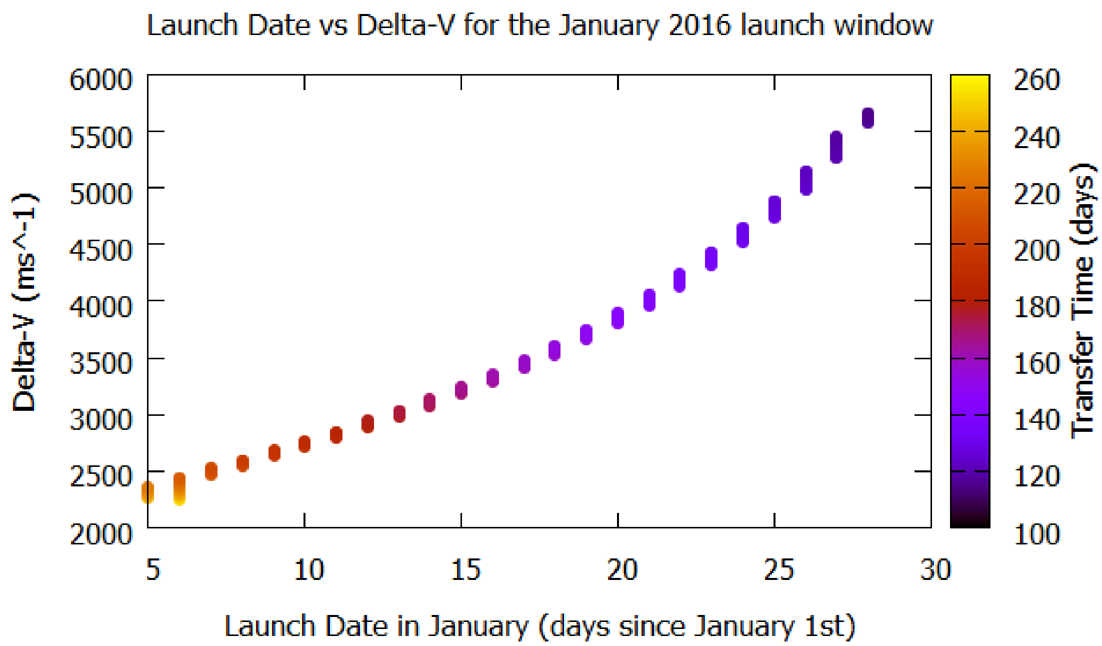

Figure 9: Figure showing the transfer ΔV plotted against the launch date in January 2016. The colour gradient shows the associated transfer time.

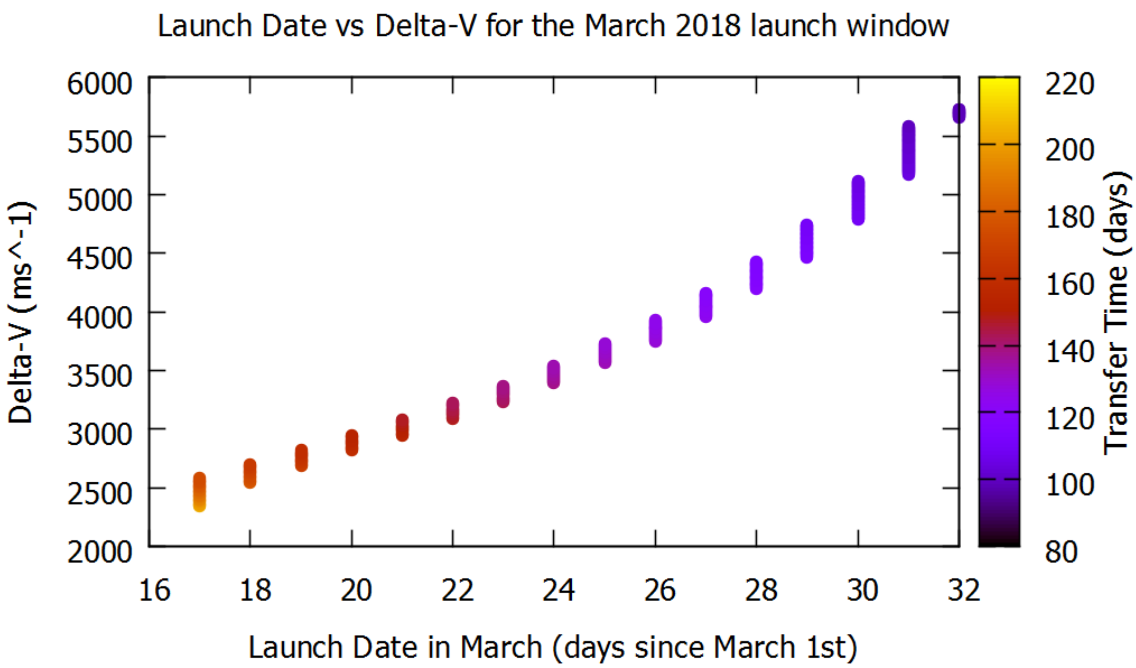

Figure 10: Figure showing the transfer ΔV plotted against the launch date in March 2018. The colour gradient shows the associated transfer time.

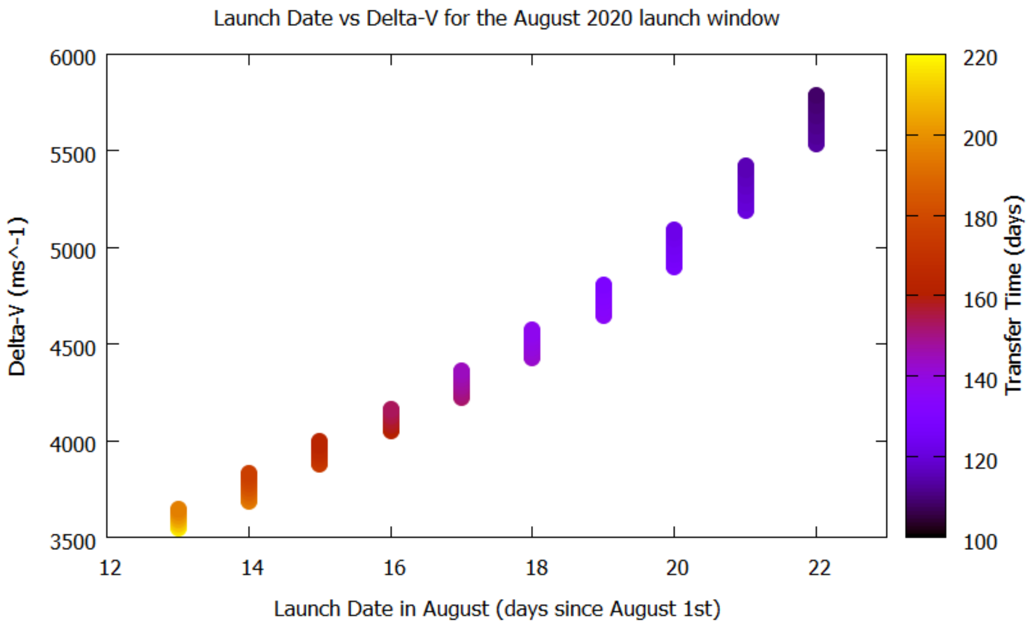

Figure 11: Figure showing the transfer ΔV plotted against the launch date in August 2020. The colour gradient shows the associated transfer time.

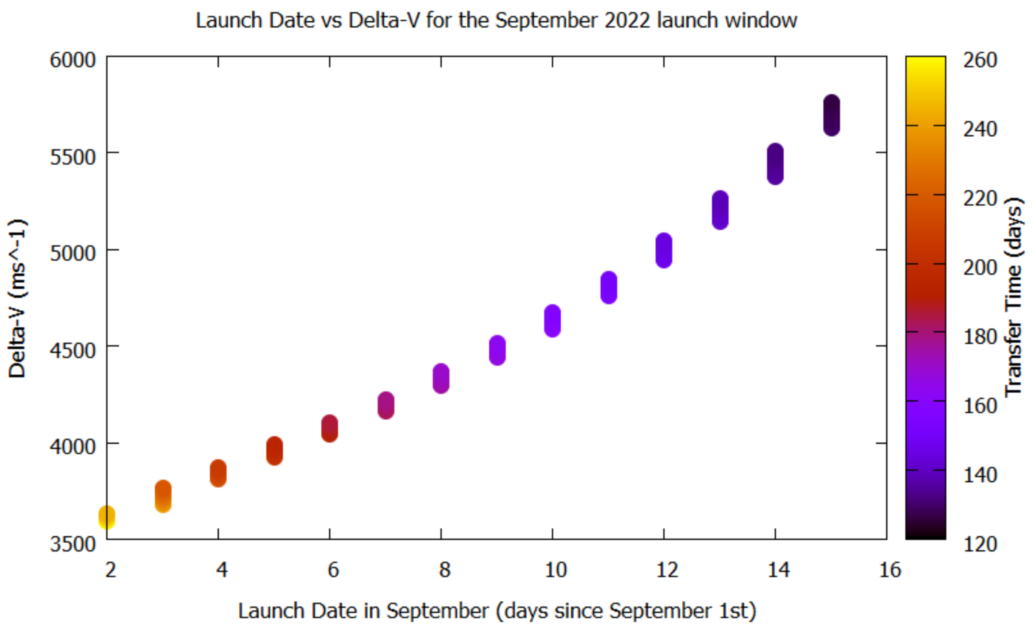

Figure 12: Figure showing the transfer ΔV plotted against the launch date in September 2022. The colour gradient shows the associated transfer time.

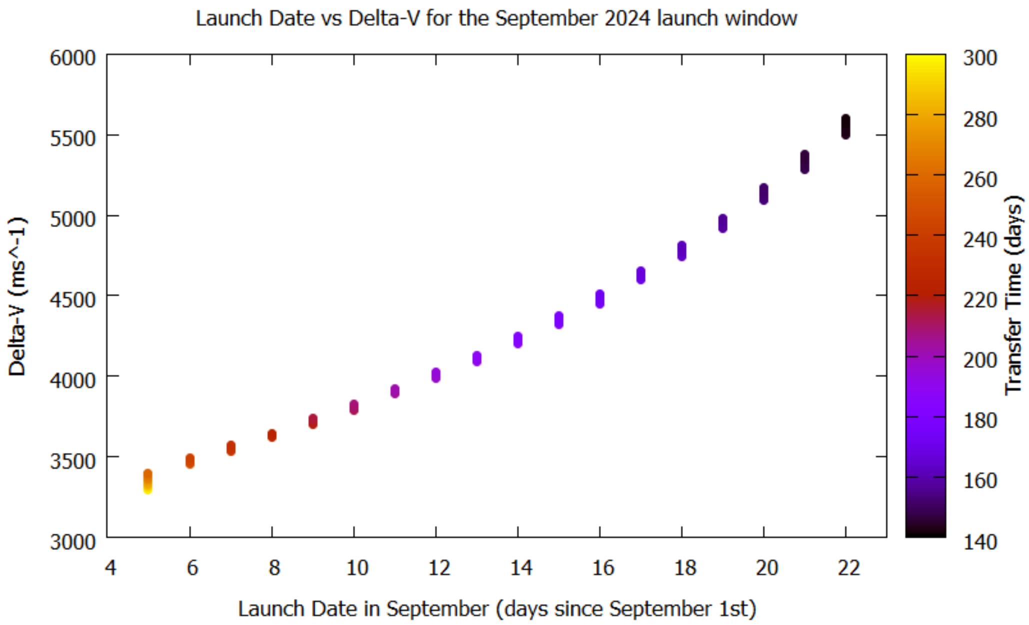

Figure 13: Figure showing the transfer ΔV plotted against the launch date in September 2024. The colour gradient shows the associated transfer time.

These all show similar trends in that the more fuel efficient, longer transfers are available towards the beginning of the window. The faster, less efficient transfers occur in the latter part of the window.

Each also shows linear behaviour for approximately the first half of the window before the gradient starts to increase in the latter part of the window, this is most prominent in Figures 9, 10 and 13.

5 Conclusions & Discussion

5.1 Window variation

It is obvious from Figure 8 that the amount of ΔV required varies between launch windows. This is because although Earth and Mars are at the same angle required for a Hohmann transfer every 779 days, their eccentricities mean that their distance from each other will vary between windows. There should be some longer timescale on which this pattern will repeat based on the consideration of eccentricity as well as phase angle.

For a reference transfer time of 180 days, the ΔV cost varies between $2500 - 4315$ ms^{-1} over the ten year period examined. This shows that if a specific year is to be chosen then significant fuel savings can be made, in the next ten years then the best window to chose would be the March 2018 window. In the event manned exploration becomes commonplace on Mars then this could be taken advantage of by sending higher mass missions during these periodic low fuel cost windows.

We can also compare these results to the simple, circular orbit case we calculated in Section 2.3 and 2.4 of 2946 ms^{-1} of ΔV with a transfer time of 258 days (Table 1).

Table 1: Table comparing each modelled launch window with the analytical circular case at a reference transfer time of 258 days (or as close to it as is possible with the data produced)

Launch Window	Transfer Time (days)	Transfer ΔV (ms^{-1}
Circular orbit case	**258**	**2946**
January 2016	257	2251
March 2018	258	2606
August 2020	274	3550
September 2022	258	3591
September 2024	258	3400

It is now clear that the task of calculating interplanetary trajectories cannot be reasonably simplified to a circular case problem as the required ΔV varies quite considerably either side of the analytical value. It is possible that selecting the launch window carefully, significant fuel may be saved over the cost the circular case predicts and if the window is chosen poorly then the fuel costs could be much greater.

5.2 Date of Launch

Examining Figures 9 - 13, it can be seen that as the launch window progresses the gradient of the relationship increases, especially towards the end of the window where the shortest transfers can be found. This could be important when considering the launch date for a mission designed for a short duration mission, if it is delayed by a few days then it is possible it will no longer have enough fuel to make the transfer at all.

It is also worth noting how the launch windows found by the program differ from the predicted launch windows found using the synodic period in Section 2.4. As the transfers move further away from optimal the time difference between launch windows gets shorter. This further hints at a deeper complexity than the circular orbit approximation takes into account.

5.3 Example Mission Design

We will now use our results to discuss the implications for a possible manned mission to Mars. The mission will require the spacecraft being launched from the Earth's surface into Low Earth Orbit, for this reason the mass may be an issue as this is limited by current launch vehicles. We will assume that the spacecraft bound for Mars may feasibly be constructed in orbit using multiple launches if necessary as orbital construction has already been demonstrated with the International Space Station (ISS). We will explore the example of an 180 day journey either way with a crew size of 6.

As discussed before in Section 2.4, the times in which a transfer between the Earth and Mars is possible are limited to short launch windows approximately every 779 days, this condition also applies for the return journey from Mars back to Earth. For this reason, there are two classes of mission design; conjunction - long stay and opposition - short stay. The conjunction class involves a stay of up to 500 days at Mars in order to wait for the next launch window to open. The opposition class involves shorter stays of just 30 days with a more complicated return trajectory involving a gravitational assist from Venus. In this example design, we will consider only the conjunction class mission for a number of reasons:

- The program simulates only Hohmann-like trajectories.
- The Hohmann-like transfers are more fuel efficient.
- A longer stay mission is scientifically preferable.
- Trajectories involving gravitational assists have not been modelled.

Throughout our mission design it is important to consider any options which may allow us to reduce the total mass of the spacecraft as this is likely to be an area where large fuel savings and cost savings can be made.

5.3.1 Patched Conics

The model specifically simulates the trajectory between Earth and Mars' orbit, however we want to estimate the ΔV required from Low Earth Orbit (LEO) to Low Mars Orbit (LMO). To do this, we will employ a technique called patched conics where the trajectory is modelled as a series of two body problems and 'patched' together to produce a full trajectory.

For the outbound journey, the sections are:

- LEO to edge of Earth SOI: two-body problem between the spacecraft and Earth

- Edge of Earth SOI to edge of Mars SOI: two-body problem between the spacecraft and the Sun
- Edge of Mars SOI to LMO: two- body problem between the spacecraft and Mars

5.3.2 Escaping from Earth

In order to escape the gravitational influence of a body, an object must have enough relative velocity, this is called the escape velocity, v_{esc}. It is found by equating an orbiting spacecraft's kinetic energy with its gravitational potential energy.

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \tag{28}$$
$$v_{\text{esc}}^2 = \frac{2GM}{r} \tag{29}$$
$$= 2v_{\text{circ}}^2 \tag{30}$$

Where v_{circ} is the velocity in a circular orbit.

If we were to give our spacecraft the escape velocity it will escape Earth's gravity ending up in solar orbit in an orbit similar to the Earth's. However we know from our results that the spacecraft must leave the Earth's SOI with an excess velocity of 2500 - 4315 ms^{-1}.

To calculate the velocity needed from LEO, we use a simple energy conservation argument, where the kinetic energy of the spacecraft after it has left on the transfer is equal to the total energy needed minus the energy cost to escape.

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - \frac{1}{2}mv_{\text{esc}}^2 \tag{31}$$
$$v^2 = v_0^2 - v_{\text{esc}}^2 \tag{32}$$

Where v is the velocity needed for the Earth-Mars transfer, our 2500 - 4315 ms^{-1}. v_0 is the initial velocity required and v_{esc} is the escape velocity of Earth from LEO.

Therefore, we can form an expression for the velocity required to achieve the excess needed for the transfer.

$$v_0 = \sqrt{v^2 + 2v_{\text{circ}}^2} \tag{33}$$

Using Equation 10 the velocity in LEO can be calculated.

$$v_{\text{circ}} = \sqrt{\frac{GM_{\text{Earth}}}{R_{\text{Earth}} + 400\text{km}}} = 7690 \text{ ms}^{-1} \tag{34}$$

Using this information we can now calculate a range for v_0:

$$v_0 = \sqrt{(2500 \text{ ms}^{-1})^2 + 2 \times (7690 \text{ ms}^{-1})^2} = 11,159 \text{ ms}^{-1} \tag{35}$$
$$v_0 = \sqrt{(4315 \text{ ms}^{-1})^2 + 2 \times (7690 \text{ ms}^{-1})^2} = 11,700 \text{ ms}^{-1} \tag{36}$$

Taking into account the fact that the spacecraft is already moving at the orbital speed in LEO of 7690 $^{-1}$, this gives us a new ΔV requirement for departure from LEO of 3469 - 4010 ms^{-1}.

5.3.3 Capturing & Aerobraking

Once the spacecraft reaches Mars, it must capture into Mars orbit else it will continue to fly past remaining in an elliptical solar orbit, it must reduce its relative velocity to be captured. This is usually involves performing extra burns to enter orbit. However, a mission to Mars could potentially utilise aerobraking or aerocapture in order to reduce the amount of fuel required to capture into orbit around Mars. Aerobraking is a manoeuvre whereby the spacecraft uses atmospheric friction by passing into the atmosphere of a planet or moon in order to reduce its relative velocity. Aerocapture is the term used to describe a manoeuvre which uses aerobraking to capture into orbit of a body from a hyperbolic entry orbit.

Mars has a thin atmosphere, atmospheric pressure on the surface is around 0.6 % of Earth's. For this reason it may be suspected that the atmosphere would not contribute enough friction to significantly slow down a spacecraft. However, due to Mars' low gravity, the atmosphere extends further into space than Earth's. For this reason, the atmosphere can be used for braking at high altitude over a long period of time to achieve enough velocity change to be significant.

Aerobraking provides a huge opportunity for reducing the fuel needed to slow the relative velocity when arriving at Mars (and when returning to Earth). However, there are a number of difficulties and risks associated with aerobraking and aerocapture. Because the manoeuvre uses friction between the spacecraft and the atmosphere and the velocities involved are large, there can be considerable heat associated with aerobraking. This means the spacecraft must be designed to survive the heat and the altitude of the spacecraft's trajectory must be closely controlled. If the altitude is too high then the atmosphere may be too thin to have enough of a slowing effect and the spacecraft may fail to achieve orbit, if the spacecraft passes too deeply into the atmosphere then the spacecraft may heat too much and burn up or deorbit. This was the cause for the loss of the Mars Climate Orbiter in 1999 as it passed too deeply into the atmosphere and disintegrated [8]. If these difficulties can be overcome, it is reasonable to assume that most, if not, all of the velocity change needed to capture around Mars could be gained passively through the use of aerocapture. [9]

5.3.4 Mass Estimate

Estimating the total mass payload which needs to be taken to Mars is a critical component in the mission design and the minimising of this mass is crucial.

The first step of this process is to consider how many separate vehicles may feasibly be used. The desire to use multiple vehicles for different tasks stems from the premise that anything which is no longer necessary for the mission should be left behind as carrying it with you means more fuel must be used to accelerate this extra mass.

Therefore, we consider a mission which consists of five distinct phases;

- Crew transportation to Mars orbit
- Landing on the surface of Mars
- Habitation, equipment and supplies for stay on surface
- Ascent to Mars orbit
- Crew return to Earth

It is convenient and logical to use the same vehicles for crew transportation to and from Mars and that the landing/ascent vehicle be one and the same as they will share hardware and engines optimised for their operating environments.

As the mission class being considered is conjunction and involves a long stay on the surface, the astronauts will require a large amount of supplies and equipment as well as habitation. The equipment and habitation should be left on the surface when they leave the surface and the ascent vehicle should be left in orbit around Mars when the astronauts depart for Earth.

This leads to the division of the mission into three parts;

- Crew Transportation Vehicle - Carries crew to Mars and returns them to Earth.
- Descent/Ascent Vehicle - Transports crew from Mars orbit to the surface and back again, this can be left in orbit around Mars.
- Surface Support Vehicle - Sustains the crew on the surface and contains scientific instruments, equipment, supplies and habitation, may be left on the surface.

We will focus of the Crew Transportation Vehicle for this report as it is the part which will be inhabited by the crew on their journey between the Earth and Mars. We may assume that the other parts of the mission may be launched separately to the crewed mission. To make an estimate of the total mass of this vehicle we must first estimate the dry mass (excluding fuel) of the vehicle (Table 2).

Table 2: A simple breakdown of the mass estimated for the Crew Transportation Vehicle.

Crew Transportation Vehicle	Mass (T)	Comments
Supplies	20	
Habitat	35	In which the crew live during the transit
Crew Return Capsule	10	For returning crew to the Earth's surface
Total	**65**	

Therefore, we will use a mass of 65 metric tonnes to help estimate the fuel requirements.

5.3.5 Engines

Although the liquid propellant rocket system has been by far the most used form of space propulsion, there have been a large variety of proposed systems ranging from solar sails to antimatter powered rockets. We will focus on a few of the better developed of these to examine how their usage may effect a mission to Mars.

Liquid-propellant engines are currently by far the most used form of in-space propulsion. In general they generate thrust by mixing fuel with oxidiser in the combustion chamber in an endothermic chemical reaction and directing the exhaust gases through a rocket engine nozzle, this produces a force via Newton's 3rd Law (see Figure 14) Liquid-propellant engines have been heavily developed since their inception and current efficiencies can reach up to an I_{SP} of 450 s, the specific impulse generated by the Space Shuttle's main engines fuelled by liquid oxygen and liquid hydrogen. These engines produce a lot of thrust but it is their inefficiency which leads to them being widely considered inadequate for the future of space travel. It is currently unclear whether further technological improvements may extend their useful range but their eventual redundancy is assured when considering the vast distances and velocities associated with travel to the outer solar system.

liquid_engine.jpg

Figure 14: Diagram showing a basic schematic of a liquid propellant rocket engine. The fuel and oxidiser are combined explosively together in the combustion chamber, producing thrust from the expulsion of exhaust gases through the nozzle.

The next type of engine to consider is the Nuclear Thermal Rocket (NTR) engine. Rather than using a chemical reaction, the NTR engine uses the energy generated by a nuclear fission reactor to heat a propellant. The propellant thermally expands through a rocket nozzle generating thrust in the same way as the exhaust does in the liquid propellant engine (see Figure 15).

Although this concept has been thoroughly explored and tested on the ground, it is not currently a flight tested technology. In the aftermath of the successful Apollo program to land on the Moon, NASA turned its sights on Mars and begun developing NTRs in its Nuclear Engine for Rocket Vehicle Application (NERVA) program in 1963. Unfortunately however, due to massive budget cuts, the program was shelved and NASA started to focus on missions to LEO with the Space Shuttle and International Space Station. The program, nevertheless, provided a proof of concept and showed than NTRs could generate a significant amount of specific impulse ($\approx$ 800 - 1000 s) more than double what liquid-propellant engines can produce.

Although much more efficient than their liquid fuel counterparts, NTRs suffer from a number of drawbacks. The first being a lack of thrust, some 10 times lower than liquid-propellant engines, this means that when burning it is slower to produce a given change in velocity, resulting in longer burn times. This is not necessary a problem as the burn can simply be carried out over a longer period of time, this should not be a problem when dealing with interplanetary travel as the burn times are still extremely small compared to the coasting time. Secondly, the radioactive nature of the engine presents a potential danger to the crew if proper radiation shielding is not implemented. [9]

NTR.png

Figure 15: Diagram showing a basic schematic of a nuclear thermal rocket engine. The reactor heats the hydrogen propellant which expands through the nozzle, producing thrust. The reflector directs neutrons back into the reactor to sustain the fission reaction, the reaction can be shut down by removing the reflector or extracting the fuel rods.

Next we consider electric propulsion which uses electrical energy to produce thrust. This can be done in a number of ways: electrostatically or electrothermally. Electrostatic propulsion first ionises the propellant gas using an electric discharge before accelerating the ions through the Coulomb force via a set of charged grids, directing the accelerated particles backward into space (Figure 16). Electrothermal propulsion relies on the electric heating of the propellant which then expands through the nozzle generating thrust as in a conventional rocket.

These types of engines typically have very high exhaust velocities and so have a large I_{SP} of up to ≈ 3000 s. However, they are limited by having a low mass flow rate which limits them to very low thrust. This low thrust means that the spacecraft would take a long time to escape the Earth's gravity, having to slowly spiral outwards, the long time is probably not suited to manned space travel. Another drawback is that the engine require a source of electrical power, typically in the form of batteries and solar panels, which add to the weight and complexity. Solar panels also suffer from reduced power output when operating further from the Sun like out where Mars' orbits. [9]

IonEngine.png

Figure 16: Diagram showing a basic schematic of an Ion Engine. The propellant is ionised by an electron gun in a magnetic field before being accelerated electrostatically by a set of parallel charged plates. The exhaust is then neutralised by an electron gun in order to avoid a build-up of charge on the engine.

I have selected some of the most efficient examples of each type of propulsion currently or recently in use to provide us with a baseline for different propulsion technologies (Table 3).

Table 3: A comparison of engine specific impulses, taken from a selection of examples of each propulsion technology.

Vehicle	Specific Impulse, I_{SP}	Source
Liquid-Propellant	450 s	Space Shuttle's main engines [10]
Nuclear Thermal Rocket	$\approx$ 1000 s	NASA NERVA Program [11]
Electric Propulsion	$\approx$ 3000 s	NASA Dawn Space Probe [12]

5.3.6 Fuel Estimate

Now we have an estimate of the dry mass of the vehicle and have discussed the properties of possible propulsion systems we can make an estimate of the total fuel mass required using each type of engine in order to produce the required amount of ΔV found using the program.

The spacecraft must travel to and from Mars so we will take our ΔV estimate of 3469-4010 ms^{-1} and double it to account for the return journey.

To make our fuel estimate, we must use Tsiolkovsky's Rocket Equation (Equation 11) and the definition of specific impulse (Equation 12) to formulate an expression for the fuel mass required:

$$\Delta V = g I_{SP} \ln \frac{M_0}{M_1} \tag{37}$$

$$\frac{\Delta V}{g I_{SP}} = \ln \frac{M_0}{M_1} \tag{38}$$

$$M_1 = M_0 \, e^{\frac{\Delta V}{g I_{SP}}} \tag{39}$$

Where M_0 is the dry mass (mass of payload without fuel), I_{SP} is the specific impulse of the engine, $g = 9.81$ ms^{-1}.

Using this equation along with the ΔV estimate gives us a range of fuel requirements collated in Table 4.

Engine	Specific Impulse, I_{SP}	Fuel Required (T)	Total Vehicle Mass (T)
Liquid-Propellant	450 s	313 - 400	378 - 465
Nuclear Thermal Rocket	$\approx$ 1000 s	132 - 147	197 - 212
Electric Propulsion	$\approx$ 3000 s	82 - 85	147 - 150

Table 4: Table showing the fuel mass and total vehicle masses in metric tonnes of the Crew Transportation Vehicle for a selection of engine types.

There clearly some large mass savings to be made when using the more efficient engines, significantly reducing the total mass of the vehicle and therefore the cost and complexity of the mission.

5.3.7 Mission Layout & Discussion

We can now go on to outline a simplified mission scenario for a mission to Mars, including a few mass and fuel estimates. A basic timeline of the mission is given below.

1. Unmanned spacecraft deliver Descent/Ascent Vehicle to Mars orbit and the Surface Support Vehicle(s) to the surface in preparation for crew arrival. These unmanned missions take fuel efficient routes in order to save on mass.
2. In the following launch window ($\approx$ 2 years later), the Crew Transportation Vehicle launches for Mars along a shorter duration trajectory.
3. Crew Transportation Vehicle utilises aerobraking to capture at Mars and docks with Descent/Ascent Vehicle waiting in orbit.
4. Crew board Descent/Ascent Vehicle and begin descent, leaving the Crew Transportation Vehicle in orbit in a dormant state.
5. Crew land in close proximity to Surface Support Vehicle(s) and begin the science mission using the supplies already deployed on Mars for their stay.
6. After a 500 day stay, crew launch back into Mars orbit in the ascent stage of the landing vehicle and rendezvous with the Crew Transportation Vehicle, leaving all equipment on the surface except for some surface samples collected.
7. Crew Transportation Vehicle leaves for Earth along another fast trajectory, leaving the ascent vehicle in Mars Orbit.

We will now look specifically at the manned portion of the transportation to and from Mars without going into much further detail about the landing or surface occupation as this exceeds the scope of the study.

The Crew Transportation Vehicle should be propelled using Nuclear Thermal Rocket engines as they can potentially save nearly half of the mass required compared to liquid-propellant engines, giving it a mass of around 200 T. While having a lower thrust, this should not be a problem as the burns will just take longer to complete, the distances involved mean this should not be a problem. NTRs also have the problem of being highly radioactive and requiring adequate shielding, this too should not be problematic as the crew will need to be shielded from the high radiation associated with solar flares and solar activity anyway and shielding from the engine could perhaps double as a solar radiation shield if necessary. While also highly efficient, electric propulsion currently produces too little thrust to be effective for manned spaceflight as the spacecraft would take a long time to reach Mars, it would have to spiral outwards on a constant burn which would result in significantly longer travel times than would be considered acceptable. They could however be considered for use in unmanned cargo missions perhaps delivering surface supplies or the landing equipment in advance of the crew vehicle.

The mission would almost certainly require the vehicle to be constructed in orbit as even the in development Space Launch System (SLS) by NASA will only be capable of delivering 70-130 tonnes into LEO. This should not be problematic given the experience of constructing the International Space Station. The other vehicles of the mission may also need to be constructed in space, as these are likely to have similar masses to the crew vehicle.

It is clear from Figure 8 that transfer times can be further reduced at large costs to the amount of fuel and therefore mass used. It is possible that more efficient engines could allow manned spacecraft to take these less efficient trajectories in order to shorten their journey even further than the 180 days discussed in this report. It is also possible that developments in launching vehicles into LEO could result in the mass being a less tight constraint, however for now one of the main costs in any manned mission would be the launching of the hardware required into space initially.

5.4 Future Work

5.4.1 Limitations & Improvements

There are a number of performance related improvements which could be made. One is to create a a more 'intelligent' searching algorithm to close in on transfer solutions much faster than the 'brute force' method used currently. Due to computing time restraints, the program was only used to search around previously estimated launch windows. With enhanced performance, possibly using a more powerful coding language, the program could search using many more starting dates and could be run much further into the future to explore how the relationship between launch windows varies on a longer timescale than is discussed in this report.

There are also a number of improvements which could be made to expand the model, the first of which being its expansion to three dimensions so as to take into account the relative inclinations of the Earth and Mars. The second is to expand on the types of transfers being simulated. The program only searches for Hohmann-like transfers, that is, transfers in which the ΔV is applied entirely in the direction of travel as pictured in Figure 17.a. The thrust can however be applied off the the prograde direction to form orbits in which Earth is not at the apoapsis as shown in Figure 17.b. These orbits have the potential to provide lower energy and/or faster transfer solutions and further work could investigate if this is the case.

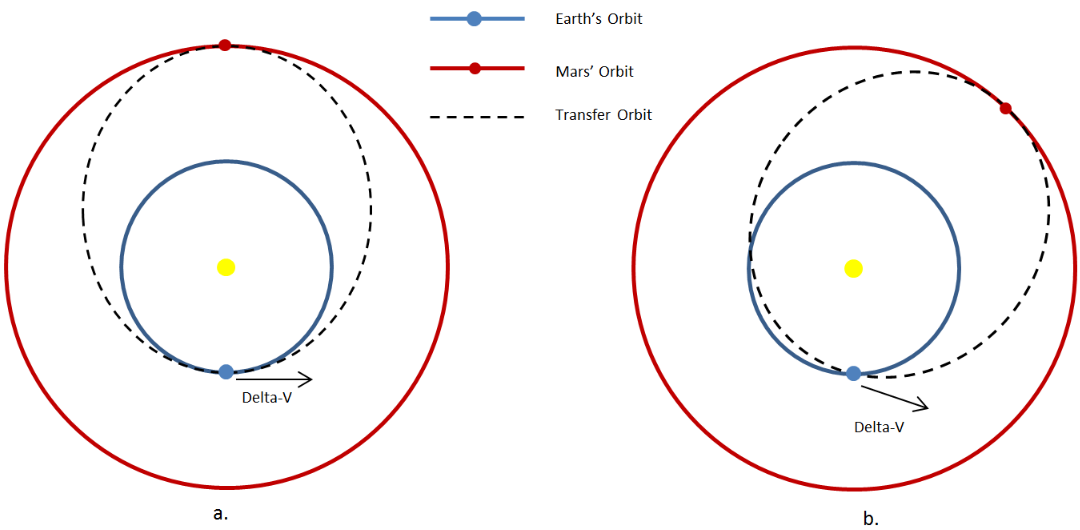

Figure 17: a. Diagram showing the Hohmann type transfers the model simulates where the ΔV application is made in the direction of travel - prograde. b. Diagram showing the resulting trajectory when the application of ΔV is made off the prograde direction and how it may result in an encounter with Mars at an earlier date.

5.4.2 Further Work

An effort should also be made to thoroughly investigate the return journey from Mars as we have used the assumption that it will simply require the same ΔV to travel back. While this is an energetically sound assumption (gravitational fields are conservative), there may be alternative trajectories which could be considered. It is possible that using other celestial bodies to perform gravitational slingshots could change the transfer time and fuel cost.

The physics of the model could be easily used to simulate trajectories to any of the other planets, though the fuel costs and transfer times to these are likely to be much greater than those to Mars. It is clear that some new technology may be needed in order to feasibly send people to the outer solar system, nuclear propulsion could be the first step towards this.

Work of this nature is important for the future of space exploration as the issues of long term spaceflight are becoming apparent and the need to explore alternative routes towards solving this issue is pressing.

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A Model Code

A.1 Functions

The `Functions` code contains all the functions used within the model for calculating each step in the trajectory simulation. The process is shown diagrammatically in Section 3.2, Figure 7.

```
Function MEAN_ANOM, t, M_body, a ; Function for calculating the Mean Anomaly
given a time, t, semi-major axis, a, and the mass of the body being orbited,
M_body.

G = 6.67384e-11 ; Gravitational Constant

n = sqrt((G*M_body)/(a^3))  ; finds the mean motion, n, in rad/s
M = n*t ; converts this into an angle in radians for a given time#

return, M  ; returns M, the Mean Anomaly

end

;----------------------------------

Function MEAN_ANOM2, M_0, t_0, t, M_body, a ; Same as MEAN_ANOM, but takes an
initial mean anomaly and time

G = 6.67384e-11 ; Gravitational Constant

n = sqrt((G*M_body)/(a^3))  ; finds the mean motion, n, in rad/s

M = M_0 + n*(t-t_0)

return, M  ; returns M, the Mean Anomaly

end

;----------------------------------

Function ECC_ANOM, ecc, M ; Function for calculating the Eccentric Anomaly, E
given the eccentricity and the Mean Anomaly, M using Newton's iterative method.

;Uses the equation - f(E) = E - ecc*SIN(E) - M = 0

;Newton's iterative method given by E_n+1 = E_n - F(E_n)/F'(E_N)

E = M  ; Initially estimates E = M
x = 10 ; Iterates x times, testing shows that 4 times is enough to converge for
very large eccentricity, up to about 0.99
```

```
For i = 0, (x-1) DO begin
E = E - (E-ecc*SIN(E)-M)/(1-ecc*COS(E)) ;iterates x times
endfor

return,E ;retuns E, the Eccentric Anomaly

end

;-----------------------------------

Function TRUE_ANOM, ecc, E ;Function for calculating the True Anomaly from
the Eccentric Anomaly and the eccentricity.

theta = 2*ATAN(sqrt(1-ecc)*COS(E/2),sqrt(1+ecc)*SIN(E/2))

return, !pi - theta
end

;-----------------------------------

Function RADIUS, ecc, a, theta ; Function for calculating the distance of the
orbit from the centre of the body being orbited given the eccentricity,
semi-major axis and the True Anomaly

r = (a*(1-ecc^2))/(1+ecc*COS(theta))

return, r

end

;-----------------------------------

Pro POLAR_PLOT, r, theta

window, xsize=600, ysize=600  ;sets window size

x = r*COS(theta) ;polar coordinate conversion
y = r*SIN(theta)

device, decomposed=0

plot, x, y ,BACKGROUND = 255, COLOR = 0, $
xrange = [-1.2e7,1.2e7], yrange = [-1.2e7,1.2e7], $
xtitle = "X position (m)", ytitle = "Y position (m)"

im=tvrd()
write_jpeg,'POLAR_PLOT.jpeg',im
```

```
end

;----------------------------------

Function orbital_period, a, M_body ; finds the orbital period of an orbit

G = 6.67384e-11 ; Gravitational Constant

P = 2*!pi*sqrt(a^3/(G*M_body))

return, P

end

;----------------------------------

Function orbital_speed, r, a, M_body ; finds the orbital speed at a given point
in the orbit

G = 6.67384e-11 ; Gravitational Constant

v = sqrt(G*M_body*((2.0/r)-(1.0/a)))

return, v

end
```

A.2 IPTransfer

The IPTransfer program contains the loops and program structure needed to produce the results in an output file. It uses the equations defined in the

```
PRO iptransfer6

COMPILE_OPT IDL2

G = 6.67384e-11 ; Gravitational Constant in m3 kg-1 s-2
AU = 149597870700.0 ; AU in meters

day = 86400.0 ; Length of a day in seconds
year = 3.15569e7 ; Length of a year in seconds

M_Earth = 5.972e24 ; Mass of Earth in kg
M_Mars = 6.4185e23 ; Mass of Mars in kg
M_Sun = 1.989e30 ; Mass of Sun in kg

Earth_Peri16 = 2457391.500000 ; Julian Date of Earth's periapsis in 2016 (4-Jan)
Mars_Peri16 = 2457690.500000 ; Julian Date of Mars' periapsis in 2016 (30-Oct)
```

```
Earth_Peri18 = Earth_Peri16 + 720*1
Mars_Peri18 = Mars_Peri16 + 1364*1

Earth_Peri20 = Earth_Peri16 + 720*2
Mars_Peri20 = Mars_Peri16 + 1364*2

Earth_Peri22 = Earth_Peri16 + 720*3
Mars_Peri22 = Mars_Peri16 + 1364*3

Earth_Peri24 = Earth_Peri16 + 720*4
Mars_Peri24 = Mars_Peri16 + 1364*4

Mars_soi = 5.77E8

;EARTH AT 1 JAN 2016

;Earth_JD = 2457388.500000000 ;Julian Date (days)
;Earth_W = 1.147253230802430E02*(!pi/180) ;Argument of Periapsis
;Earth_T = (Earth_Peri16 - Earth_JD)*day ;Sets the time until periapsis

;----------------

;MARS AT 1 JAN 2016

;Mars_JD = 2457388.500000000 ;Julian Date (days)
;Mars_W = 3.332860835330721E02*(!pi/180) ;Argument of Periapsis
;Mars_T = (Mars_Peri16 - Mars_JD)*day ;Sets the time until periapsis

Earth_ecc = 0.01671123
Earth_A = AU

Mars_ecc = 0.093315
Mars_A = 1.523679*AU

;EARTH AND MARS LOOPS

runtime = 2*year ; Length of trajectory time
timestep = 1000 ; Timestep for calculation

LOOP = 1 ;Chooses whether to plot planet trajectories
IF(LOOP EQ 1) THEN BEGIN

openw, Earth_file, 'EarthTrajectory.txt', /GET_LUN ; open file to write
Earth's trajectory
openw, Mars_file, 'MarsTrajectory.txt', /GET_LUN ; open file to write
Mars' trajectory

FOR i = 0, runtime, timestep DO BEGIN
```

```
t = i

Earth_M = MEAN_ANOM2(0,Earth_T,t,M_Sun,Earth_A)
Earth_E = ECC_ANOM(Earth_ecc, Earth_M)
Earth_theta = TRUE_ANOM(Earth_ecc,Earth_E)
Earth_angle = Earth_theta + Earth_W
Earth_r = RADIUS(Earth_ecc,Earth_A,Earth_theta)

Earth_v = orbital_speed(Earth_r,Earth_A,M_Sun)

Earth_x = Earth_r*COS(Earth_angle)
Earth_y = Earth_r*SIN(Earth_angle)

printf, Earth_file, Earth_x, Earth_y, Earth_angle, Earth_r

;-----------------------------------------------------------

Mars_M = MEAN_ANOM2(0,Mars_T,t,M_Sun,Mars_A)
Mars_E = ECC_ANOM(Mars_ecc, Mars_M)
Mars_theta = TRUE_ANOM(Mars_ecc,Mars_E)
Mars_angle = Mars_theta + Mars_W
Mars_r = RADIUS(Mars_ecc,Mars_A,Mars_theta)

Mars_v = orbital_speed(Mars_r,Mars_A,M_Sun)

Mars_x = Mars_r*COS(Mars_angle)
Mars_y = Mars_r*SIN(Mars_angle)

printf, Mars_file, Mars_x, Mars_y, Mars_angle

ENDFOR

close, Earth_file
close, Mars_file
free_lun, Earth_file
free_lun, Mars_file

ENDIF

;TRANSFER LOOPS

openw, Results_file, 'Results.txt', /GET_LUN

aruntime = 60
astep = 1
astart = 0
```

```
FOR a = astart, aruntime, astep DO BEGIN

print, "Day", a

jruntime = 500
jstep = 1

E_FILE = 'EarthTrajectory.txt'
M_FILE = 'MarsTrajectory.txt'

Mlines = FILE_LINES('MarsTrajectory.txt')

openr, earthfile, E_FILE, /GET_LUN

kruntime = a*day
kstep = 1000

IF Kruntime EQ 0 THEN readf, earthfile, Ex , Ey , ETheta, Er

FOR k = 1, kruntime, kstep DO BEGIN
readf, earthfile, Ex , Ey , ETheta, Er
ENDFOR

VEL_1 = orbital_speed(Er, Earth_A, M_Sun)

FOR j = 1, jruntime, jstep DO BEGIN

difference = -3e10 + j*2e8

transfer_a = (Earth_A + Mars_A)/2 + difference

transfer_ecc = 1 - (Er/transfer_a)
transfer_w = ETheta
transfer_T = 0

VEL_2 = orbital_speed(Er, transfer_a, M_Sun)

openr, marsfile, M_FILE, /GET_LUN

MinDistance = 1*AU
TransferTime = 0

IF Kruntime EQ 0 THEN readf, marsfile, Mx , My , MTheta

FOR k = 1, kruntime, kstep DO BEGIN
readf, marsfile, Mx , My , MTheta
ENDFOR
```

```
FOR i = 1, runtime - (kruntime+kstep), timestep DO BEGIN

readf, marsfile, Mx , My , MTheta

t = i

transfer_M = MEAN_ANOM2(0,transfer_T,t,M_Sun,transfer_a)
transfer_E = ECC_ANOM(transfer_ecc, transfer_M)
transfer_theta = TRUE_ANOM(transfer_ecc,transfer_E)
transfer_angle = transfer_theta + transfer_w
transfer_r = RADIUS(transfer_ecc,transfer_a,transfer_theta)

transfer_v = orbital_speed(transfer_r,transfer_a,M_Sun)

transfer_x = transfer_r*COS(transfer_angle)
transfer_y = transfer_r*SIN(transfer_angle)


distance = SQRT((transfer_x-Mx)^2+(transfer_y-My)^2)

IF (distance LT MinDistance) THEN BEGIN
MinDistance = distance
TransferTime = t
ENDIF

ENDFOR

DeltaV = Vel_2 - Vel_1

IF (MinDistance LT Mars_soi) THEN BEGIN
printf, Results_file, (a-astart)+1 , MinDistance, TransferTime/86400,
transfer_a, DeltaV
ENDIF

close, marsfile
free_lun, marsfile

ENDFOR

close, earthfile
free_lun, earthfile

ENDFOR

close, Results_file
free_lun, Results_file

END
```