

# 【DS】 Day1

☰ Tags	
📅 Date	@May 19, 2022
☰ Summary	

## 【Week1】 Union-Find

Steps to developing a usable algorithm:

- Model the problem
- Find an algorithm to solve it
- Fast enough? Fit in memory?
- If not, figure out why.
- Find a way to address the problem
- Iterate until satisfied

### 1.1 Dynamic Connectivity

Given a set of N objects.

- **Union command**: Connect two objects
- **Find/connected query**: Is there a path connecting the two objects?

```

union(4, 3)
union(3, 8)
union(6, 5)
union(9, 4)
union(2, 1)
connected(0, 7) ✗
connected(8, 9) ✓

```



### Modelling the Objects

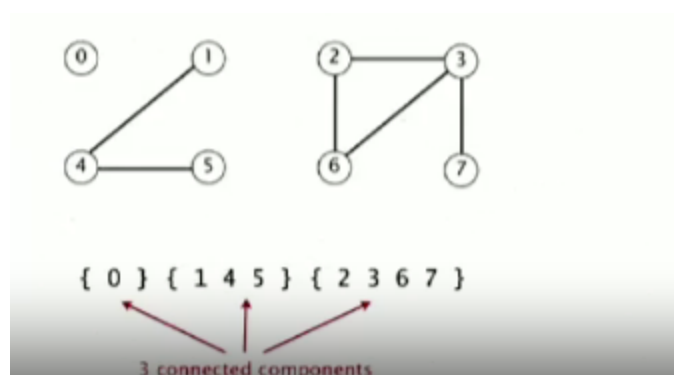
- Computers in the network
- Friends
- Pixels on a photo

### Modelling the connections

We assume “is connected to” is an equivalence relation:

- **Reflexive**: p is connected to p
- **Symmetric**: If p is connected to q, then q is connected to p
- **Transitive**: If p is connected to q and q is connected to r, then p is connected to r.

**Connected Components**: Maximal set of objects that are mutually connected



## Implementing the Operations

Find query: Check if two objects are in the same component

Union command: Replace components containing two objects with their union.

## Union-find Data Type(API)

Goal: Design efficient data structure for union-find

- **Number of objects**  $N$  can be huge
- **Number of operations**  $M$  can be huge
- Find queries and union commands may be intermixed

```
public class UF
{
    UF(int N)           initialize union-find data structure with
                        N objects (0 to N - 1)
    void union(int p, int q)  add connection between p and q
    boolean connected(int p, int q) are p and q in the same component?
    int find(int p)         component identifier for p (0 to N - 1)
    int count()             number of components
}
```

Test client: test if each operation does what we expect

```
public static void main(String[] args) {
    int N = StdIn.readInt();
    UF uf = new UF(N);

    // 当StdIn输入不为空时, 进行判断
    while (!StdIn.isEmpty()) {
        int p = StdIn.readInt();
        int q = StdIn.readInt();

        // 检查数组p和q在uf中是否相连
        // 若已经相连, 不做操作. 若未相连, 将他们连起来
        if (!uf.connected(p, q)) {
            uf.union(p, q);
            StdOut.println(p + " " + q);
        }
    }
}
```

```
}  
}  
]
```