[DS] Day2(2)

■ Summary	Mathematical Model
□ Date	@May 21, 2022
:≣ Tags	

[Week1] Analysis of Algorithms

1.7 Mathematical Model

Total running time: sum of cost * frequency for all operations

Cost of Basic Operations

operation	example	nanoseconds †
integer add	a + b	2.1
integer multiply	a * b	2.4
integer divide	a / b	5.4
floating-point add	a + b	4.6
floating-point multiply	a * b	4.2
floating-point divide	a / b	13.5
sine	Math.sin(theta)	91.3
arctangent	Math.atan2(y, x)	129.0

† Running OS X on Macbook Pro 2.2GHz with 2GB RAM

Example: 1-sum

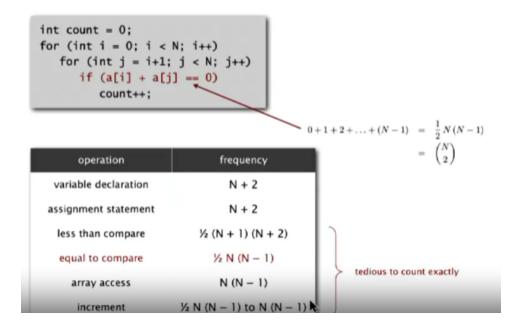
```
int count = 0;
for (int i = 0; i < N; ++i) {
  if (a[i] == 0)</pre>
```

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```
count++;
}
```

operation	frequency
variable declaration	2
assignment statement	2
less than compare	N + 1
equal to compare	N
array access	N
increment	N to 2 N

Example: 2-sum



Simplification 1: cost model

Use some basic operation as a proxy for running time.

Simplification 2: Tilde notation

Ignore lower order terms.

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- When N is large, terms are negligible
- When N is small, we don't care.

operation	frequency	tilde notation
variable declaration	N + 2	~ N
assignment statement	N + 2	~ N
less than compare	½ (N + 1) (N + 2)	~ 1/2 N2
equal to compare	½ N (N − 1)	~ 1/2 N2
array access	N (N - 1)	~ N ²
increment	½ N (N − 1) to N (N − 1)	~ ½ N ² to ~ N ²

Estimating a Discrete Sum

Replace the sum with an integral, and use calculus.

$$\sum_{i=1}^{N} i \sim \int_{x=1}^{N} x \, dx \sim \frac{1}{2} N^2$$

Ex 2.
$$1 + 1/2 + 1/3 + ... + 1/N$$
.

$$\sum_{i=1}^{N} \frac{1}{i} \sim \int_{x=1}^{N} \frac{1}{x} dx = \ln N$$

$$\sum_{i=1}^{N} \sum_{j=i}^{N} \sum_{k=j}^{N} \ 1 \ \sim \ \int_{x=1}^{N} \int_{y=x}^{N} \int_{z=y}^{N} \ dz \, dy \, dx \ \sim \ \frac{1}{6} \, N^3$$