【Discrete Math】Day1

【Ch2】Let us count

2.2 Sets and the like

Any collection of things, called elements, is a set.

For mathematics, various sets of numbers are important: the set of real numbers, denoted by R; the set of rational numbers, denoted by Q; the empty set, denoted by $\emptyset$

The number of elements of a set A is denoted by $|A|$.

We may specify a set by listing the elements between braces; so

$$P = \{12, 23, 27, 33\}$$

Often we specify a set by a property that singles out the elements from a large universe like real numbers. We then write this property inside the braces, but after a colon.

$$\{x \in Z : x \geq 0\}$$

is the set of non-negative integers.

$$\{x \in P : x \geq 25\} = \{27, 33\}$$

A set A is a subset of a set B, if every element of A is also an element of B.

We allow A consists of all elements of B(in which case A = B), or none of them(in which case $A = \emptyset$) So the empty set is a subset of every set.

The relation that A is a subset of B is denoted by

$$A \subset B$$

The intersection of two sets is the set consisting of those elements that elements of both sets. It is denoted by

$$A \cap B$$

Two sets whose intersection is the empty set(in other words, have no element in common) are called disjoint.

The union of any set of sets consists of those elements which are elements of at least one of the sets. It is denoted by

$$A \cup B$$

Exercise

2.16 We form the union of two sets. We know that one of them has n elements and the other has m elements. What can we infer for the cardinality of the union?

The cardinality of the union is at least the larger of n and m and at most n+m.

2.18 We form the intersection of two sets. We know that one of them has n elements and the other has m elements. What can we infer for the cardinality of the intersection?

The cardinality of the intersection is at most the minimum of n and m.

2.19 Prove that $|A \cup B| + |A \cap B| = |A| + |B|$.

The common elements of A and B are counted twice on both sides; the elements in either A or B but not both are counted only once on both sides.