HARVARD
School of Engineering
and Applied Sciences

Cache performance measurement and optimization

CS61, Lecture 13

Prof. Stephen Chong

October 14, 2010

Announcements

- Lab 3
  - Design checkpoint deadline tonight
  - Final submission deadline Tuesday 19th

Topics for today

- Cache performance metrics
- Discovering your cache's size and performance
- The "Memory Mountain"
- Matrix multiply, six ways
- Blocked matrix multiplication
- Exploiting locality in your programs

Cache Performance Metrics

- Miss Rate
  - Fraction of memory references not found in cache (# misses / # references)
  - Typical numbers:
    - 3-10% for L1
    - Can be quite small (e.g., < 1%) for L2, depending on size and locality.
- Hit Time
  - Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
  - Typical numbers:
    - 1-2 clock cycles for L1
    - 5-20 clock cycles for L2
- Miss Penalty
  - Additional time required because of a miss
    - Typically 50-200 cycles for main memory

Wait, what do those numbers mean?

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory
- Would you believe 99% hits is twice as good as 97%?
  - Consider:
    cache hit time of 1 cycle
    miss penalty of 100 cycles
- Average access time:
  - 97% hits: 1 cycle + 0.03 * 100 cycles = 4 cycles
  - 99% hits: 1 cycle + 0.01 * 100 cycles = 2 cycles
- This is why “miss rate” is used instead of “hit rate”

Writing Cache Friendly Code

- Repeated references to variables are good (**temporal locality**)
- Stride-1 reference patterns are good (**spatial locality**)
- Examples:
  - cold cache, 4-byte words, 4-word cache blocks

```
int sum_array_rows(int a[M][N]) {
  int i, j, sum = 0;

  for (i = 0; i < M; i++)
     for (j = 0; j < N; j++)
        sum += a[i][j];
  return sum;
}
```

Miss rate = 1/4 = 25%

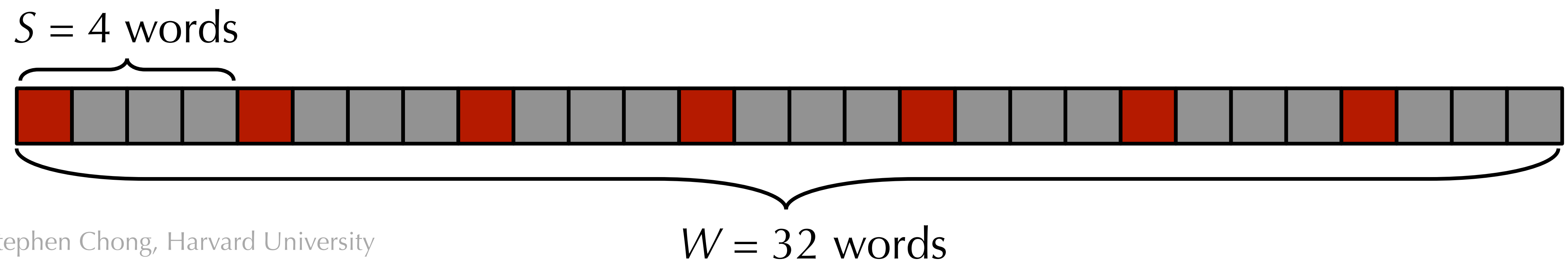
```
int sum_array_cols(int a[M][N]) {
  int i, j, sum = 0;

  for (j = 0; j < N; j++)
     for (i = 0; i < M; i++)
        sum += a[i][j];
  return sum;
}
```

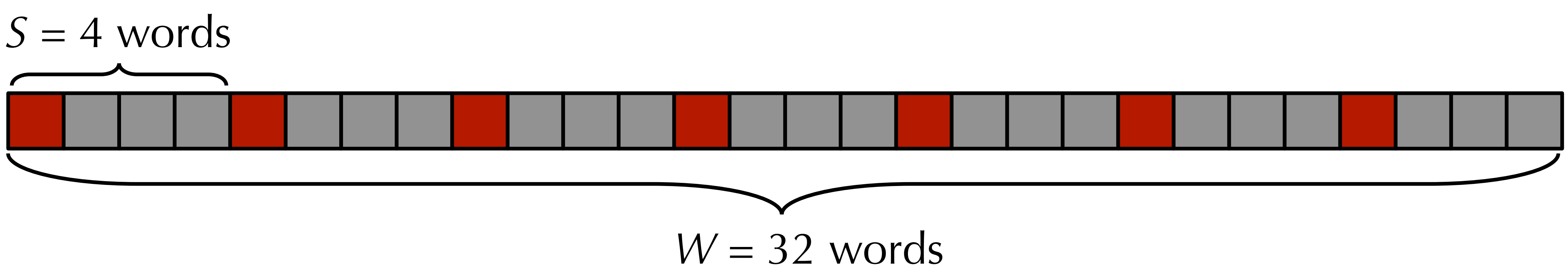
Miss rate = 100%

Determining cache characteristics

- Say you have a machine but don't know its cache size or speeds.
- How would you figure these values out?
- Idea: Write a program to measure the cache's behavior and performance.
  - Program needs to perform memory accesses with different locality patterns.
- Simple approach:
  - Allocate array of size *W* words
  - Loop over the array with stride index *S* and measure speed of memory accesses
  - Vary *W* and *S* to estimate cache characteristics



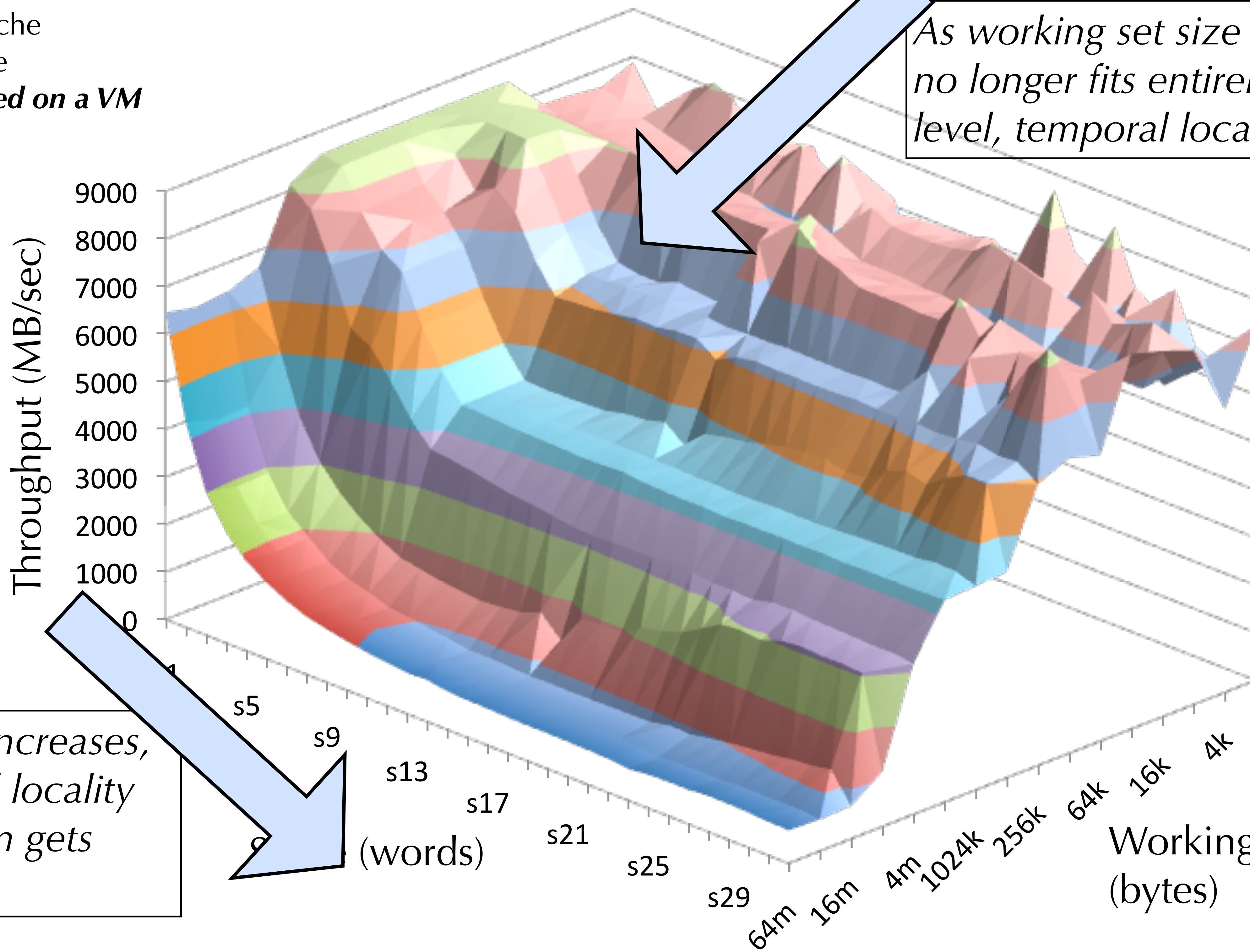
Determining cache characteristics



- What happens as you vary W and S?
- Changing W varies total amount of memory accessed by program
  - As W gets larger than one cache level, performance of program will drop
- Changing S varies the spatial locality of each access.
  - If S is less than the size of a cache line, sequential accesses will be fast.
  - If S is greater than the size of a cache line, sequential accesses will be slower.
- See end of lecture notes for example C program to do this.

The Memory Mountain

Intel Core i7
2.7 GHz
32 KB L1 d-cache
256 KB L2 cache
8MB L3 cache
CAVEAT: Tested on a VM

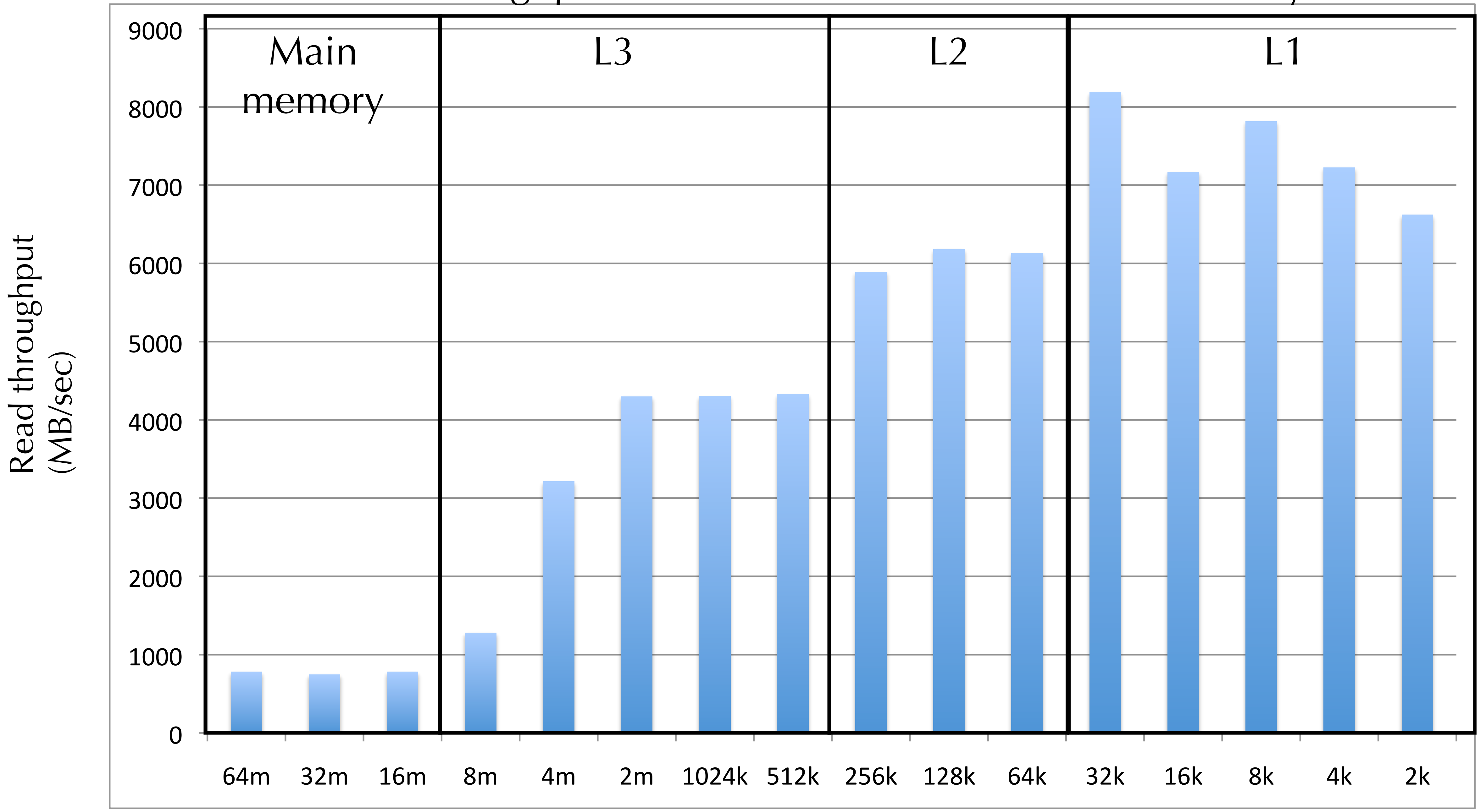


As working set size increases, no longer fits entirely in a cache level, temporal locality worsens.

As stride increases, the spatial locality of program gets worse

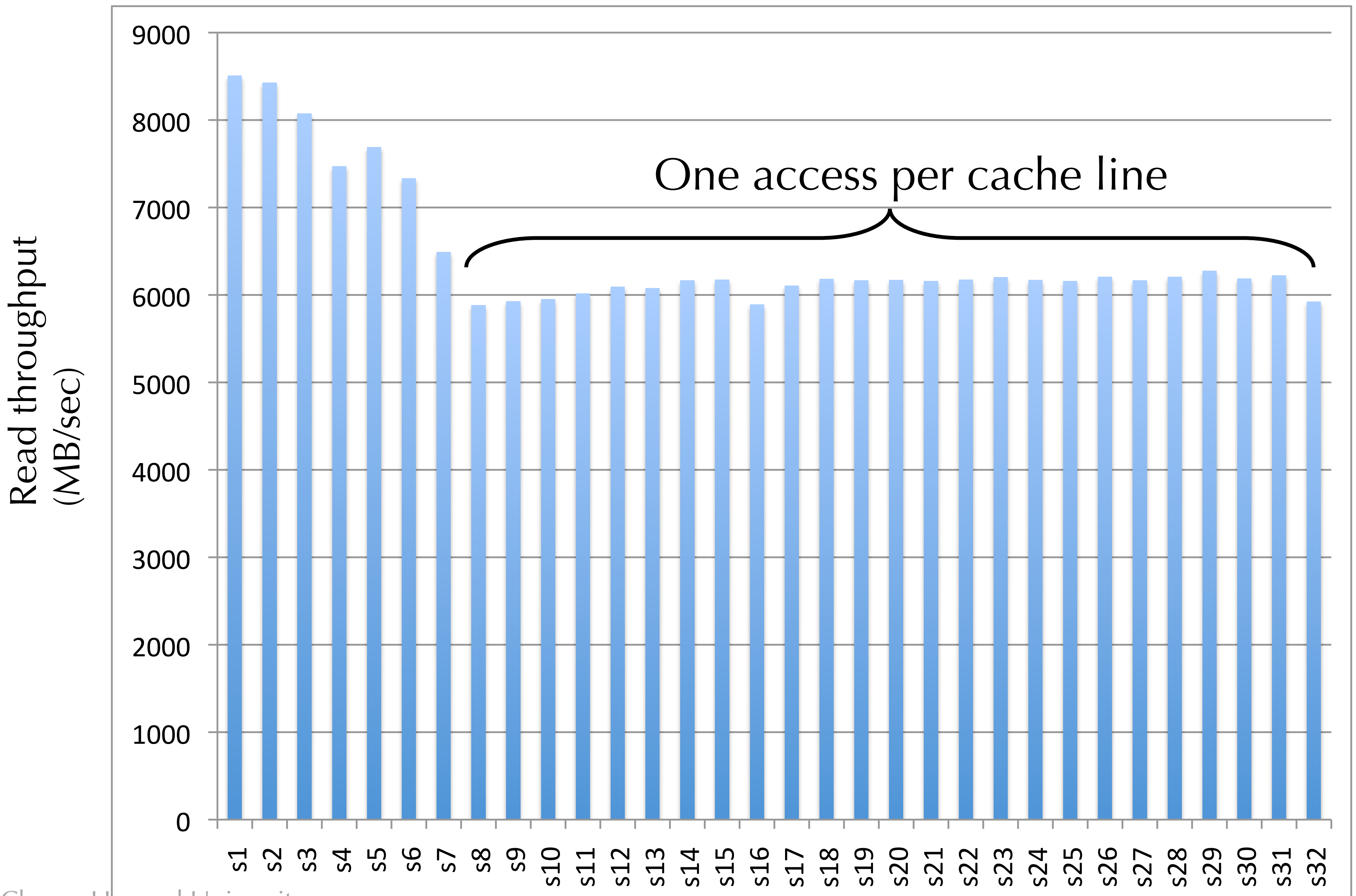
Varying Working Set

- Keep stride constant at $S = 16$ words, and vary W from 1KB to 64MB
  - Shows size and read throughputs of different cache levels and memory

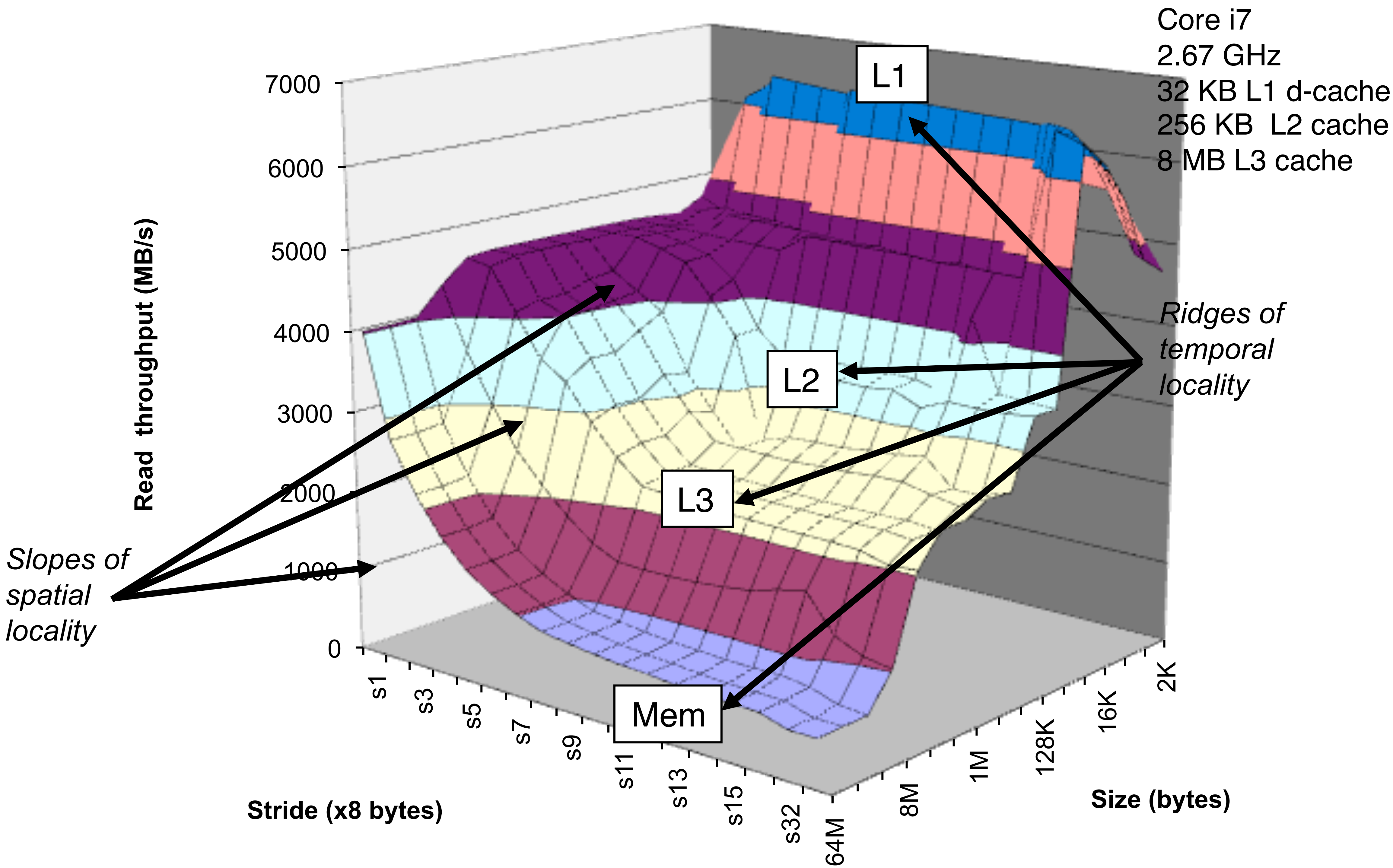


Varying stride

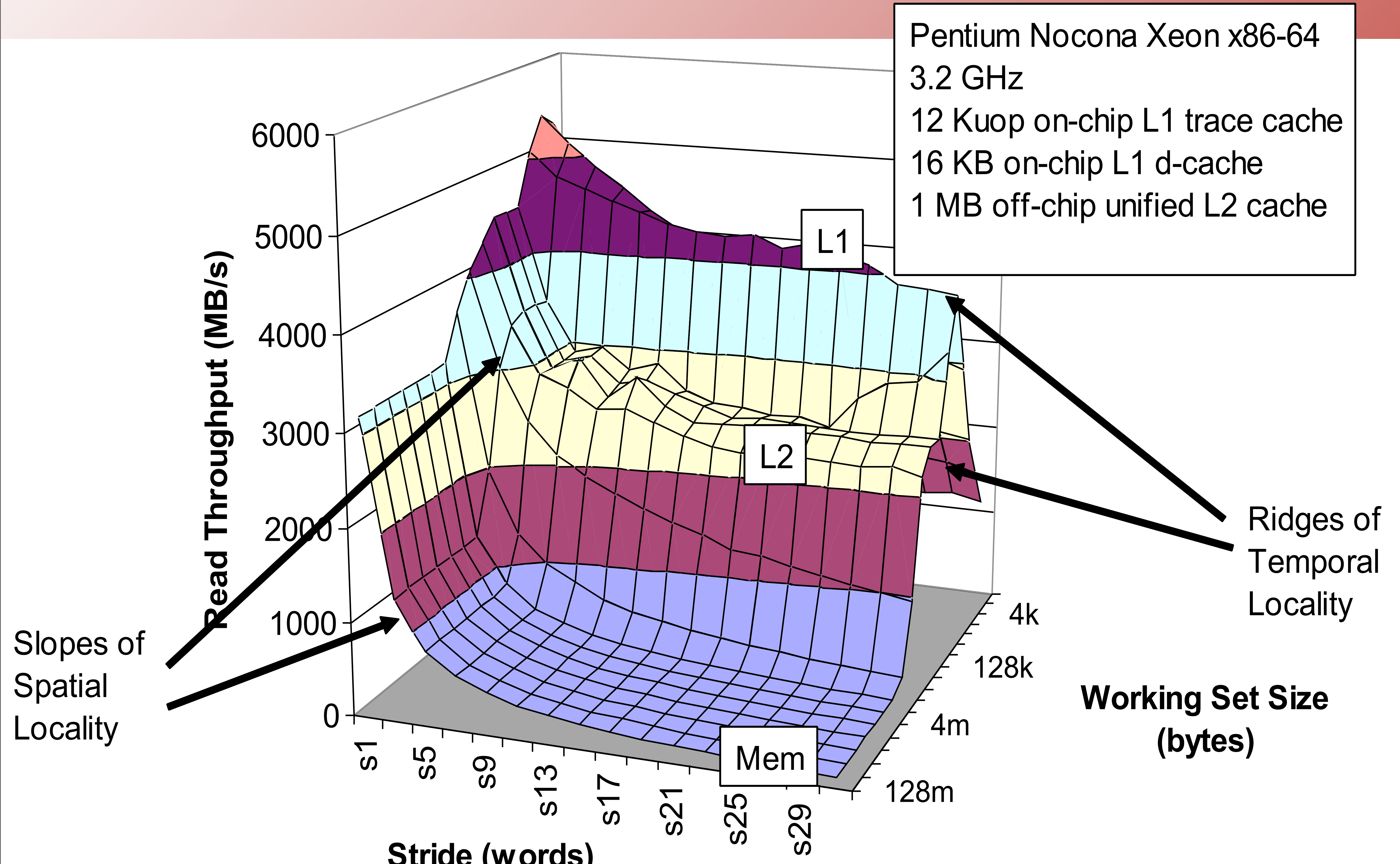
- Keep working set constant at W = 256 KB, vary stride from 1-32 words



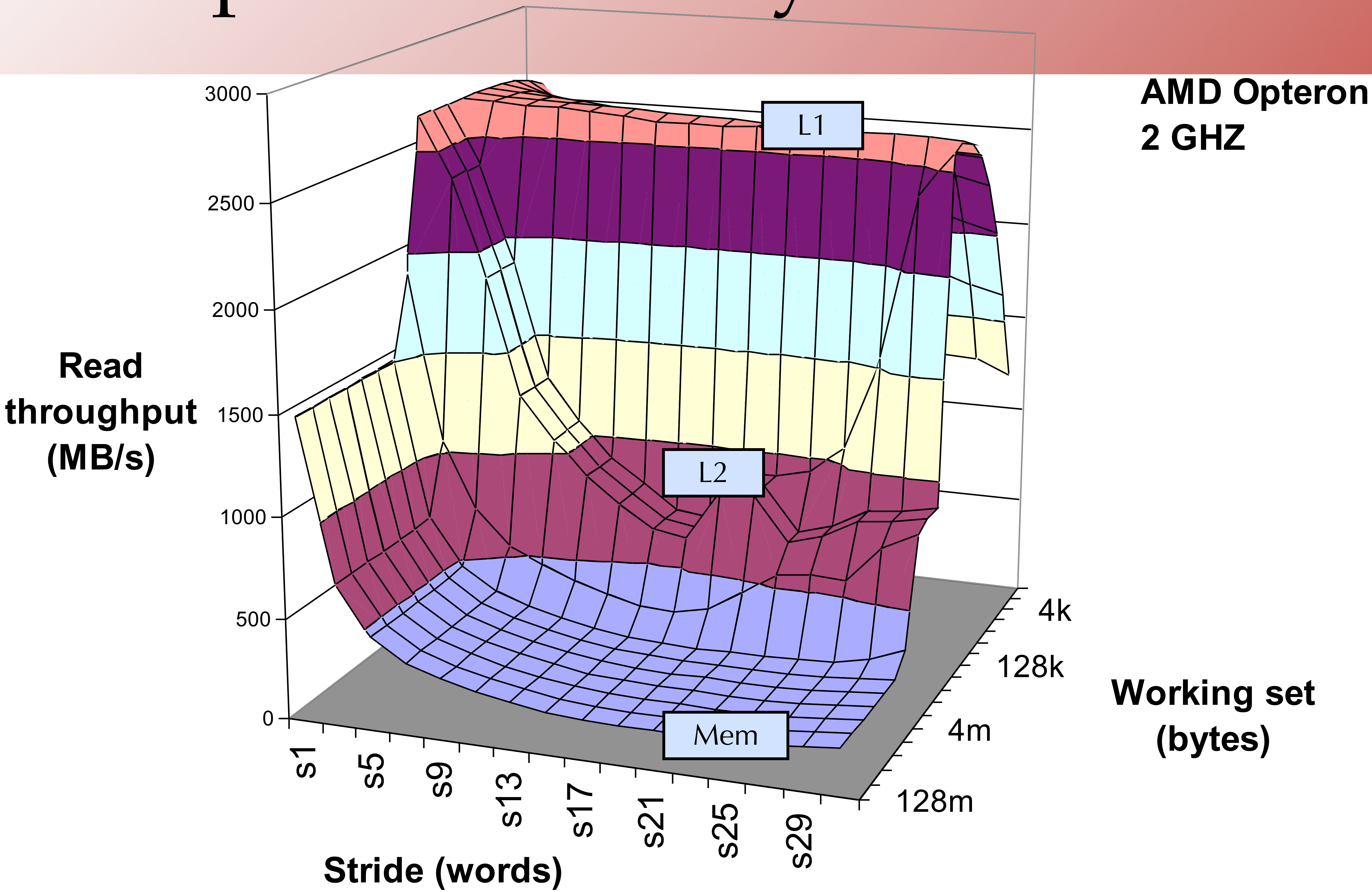
Core i7


Core i7
2.67 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
L1
L2
L3
Mem
Ridges of temporal locality
Slopes of spatial locality
Read throughput (MB/s)
7000
6000
5000
4000
3000
2000
1000
0
Stride (x8 bytes)
s1
s3
s5
s7
s9
s11
s13
s15
s32
Size (bytes)
64M
8M
1M
128K
16K
2K

Pentium Xeon



Opteron Memory Mountain



Topics for today

- Cache performance metrics
- Discovering your cache's size and performance
- The “Memory Mountain”
- Matrix multiply, six ways
- Blocked matrix multiplication
- Exploiting locality in your programs

Matrix Multiplication Example

- Matrix multiplication is heavily used in numeric and scientific applications.
  - It's also a nice example of a program that is highly sensitive to cache effects.
- Multiply two N x N matrices
  - $O(N^3)$ total operations
  - Read N values for each source element
  - Sum up N values for each destination

```
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    /* ijk */
    for (i=0; i<n; i++)  {
      for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
          sum += a[i][k] * b[k][j];
        c[i][j] = sum;
      }
    }
```

Variable sum held in register

Matrix Multiplication Example

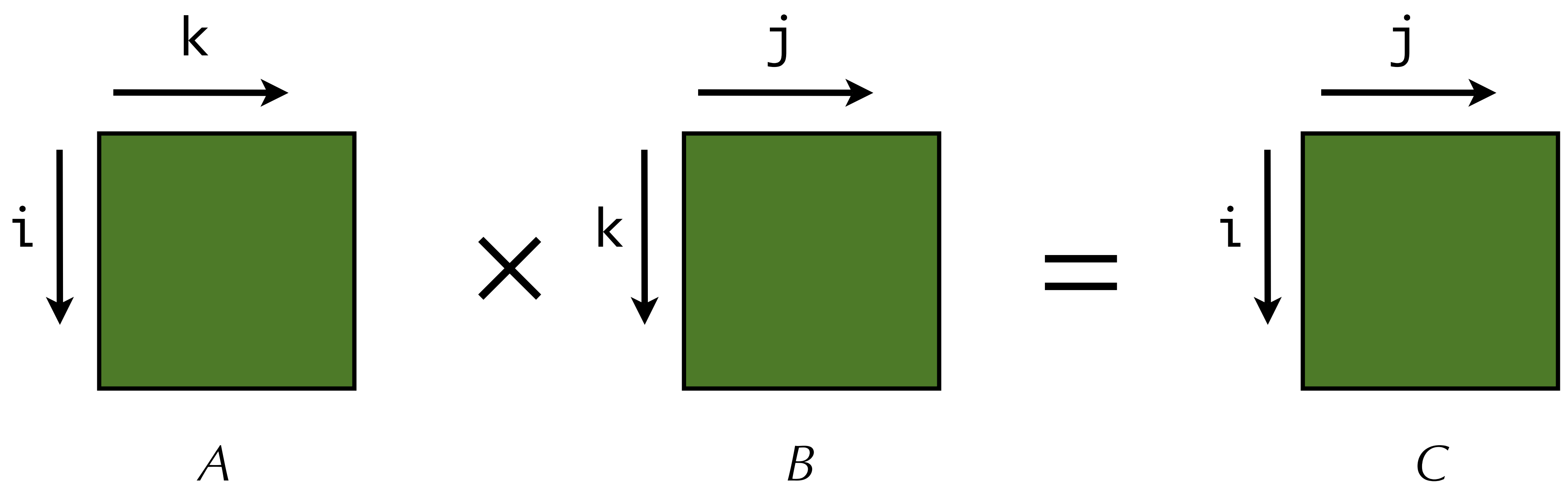
```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

$$4\times3 \; + \; 2\times2 \; + \; 7\times5 = 51$$

$$\begin{bmatrix} 4 & 2 & 7 \\ 1 & 8 & 2 \\ 6 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 & 1 \\ 2 & 4 & 5 \\ 5 & 9 & 1 \end{bmatrix} = \begin{bmatrix} 51 & & \\ & & \\ & & \end{bmatrix}$$

Miss Rate Analysis for Matrix Multiply

- Assume:
  - Line size = 32B (big enough for four 64-bit "double" values)
  - Matrix dimension N is very large
  - Cache is not big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



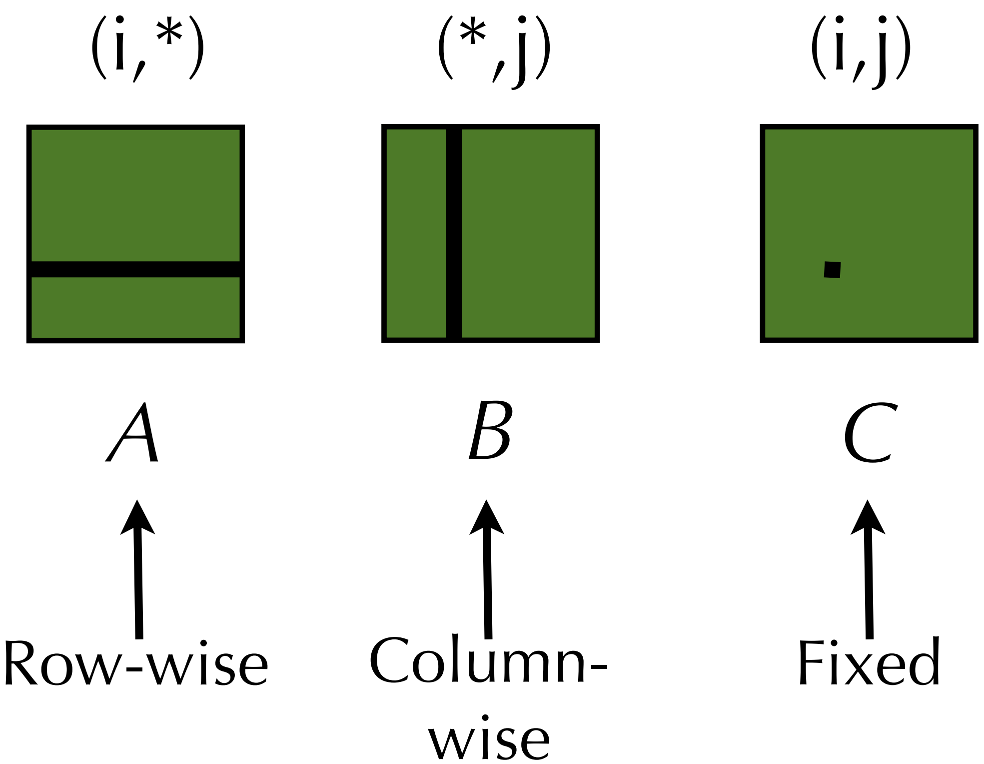
Layout of C Arrays in Memory (review)

- C arrays allocated in **row-major** order
  - Each row in contiguous memory locations
- Stepping through columns in one row:
  - ```
    for (i = 0; i < N; i++)
          sum += a[0][i];
    ```
  - Accesses successive elements
  - Compulsory miss rate: (8 bytes per double) / (block size of cache)
- Stepping through rows in one column:
  - ```
    for (i = 0; i < n; i++)
          sum += a[i][0];
    ```
  - Accesses distant elements — no spatial locality!
  - Compulsory miss rate = 100%

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



- 2 loads, 0 stores per iteration
- Assume cache line size of 32 bytes, so 4 doubles per line
- Misses per iteration:

A = 0.25 B = 1 C = 0 Total: 1.25

Cache miss analysis

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:

(i,*) (*,j) (i,j)

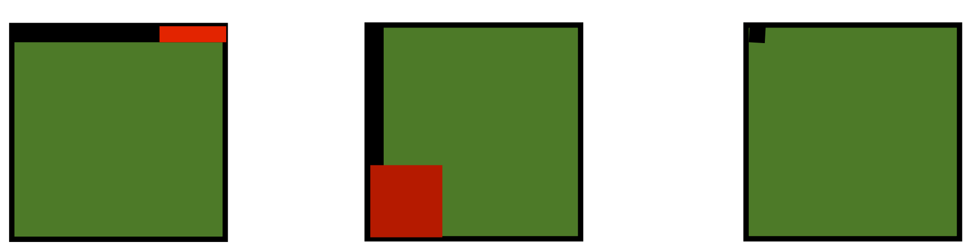
A *B* *C*

Row-wise Column-wise Fixed

First iteration:

A *B* *C*

After first iteration in cache (schematic):

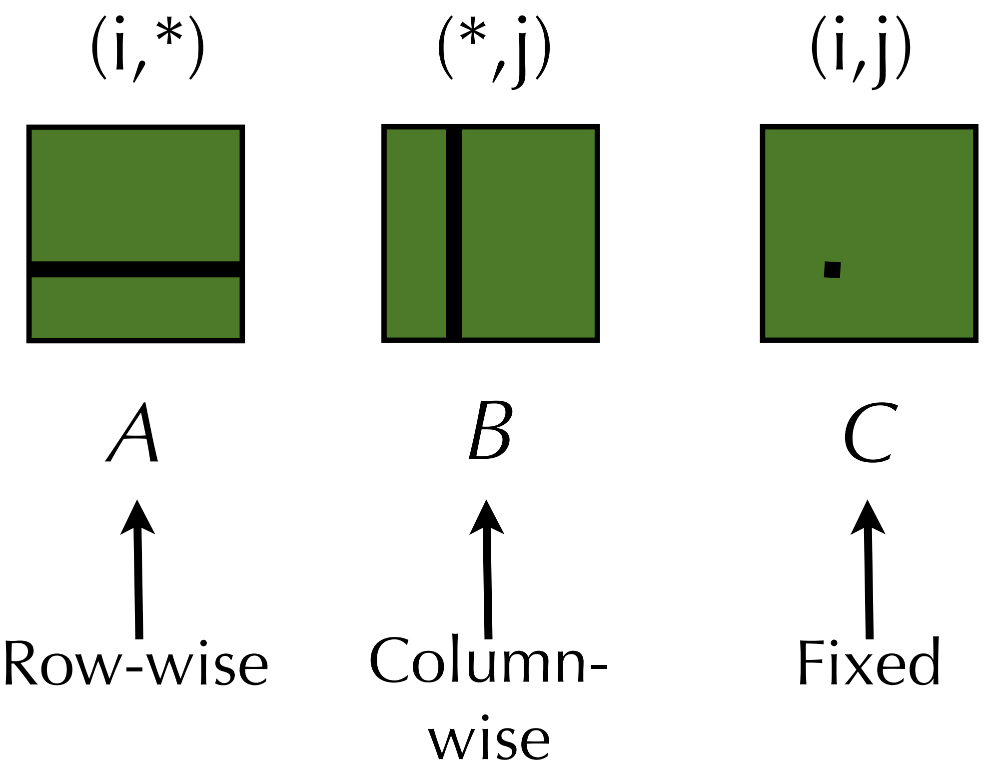


4 doubles wide

Cache miss analysis

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

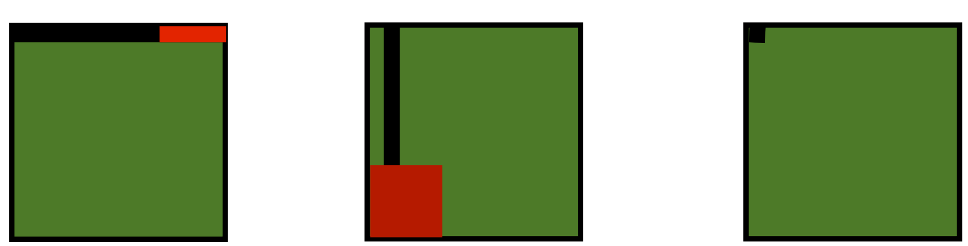
Inner loop:



First iteration:

A B C

After first iteration in cache (schematic):

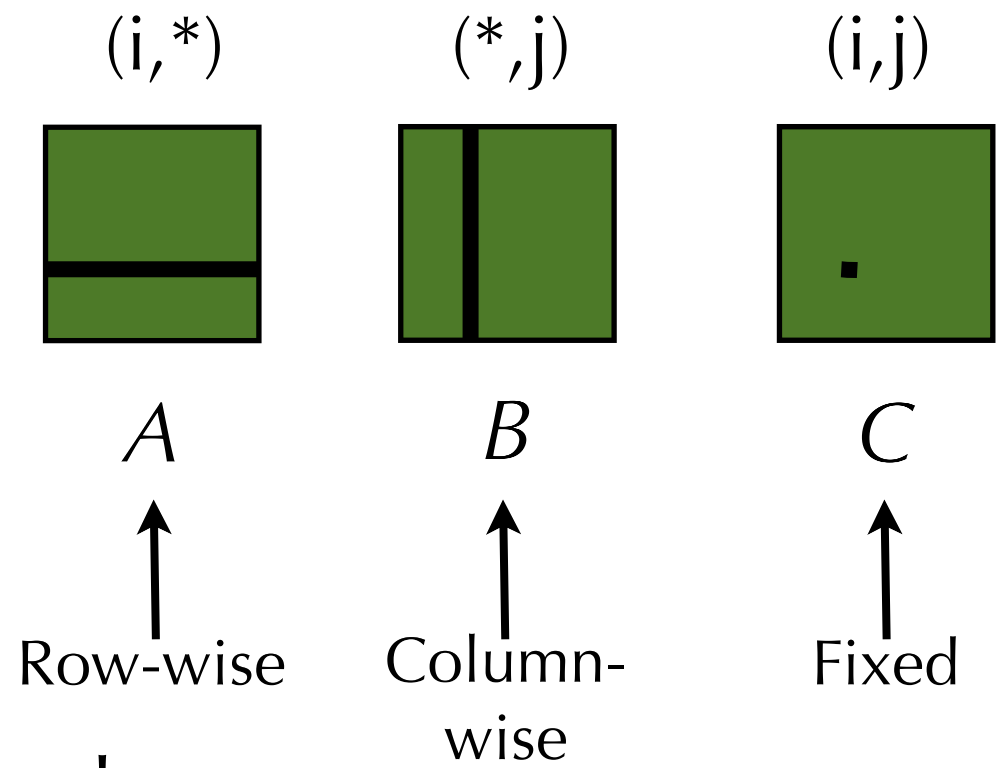


4 doubles wide

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++)  {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:

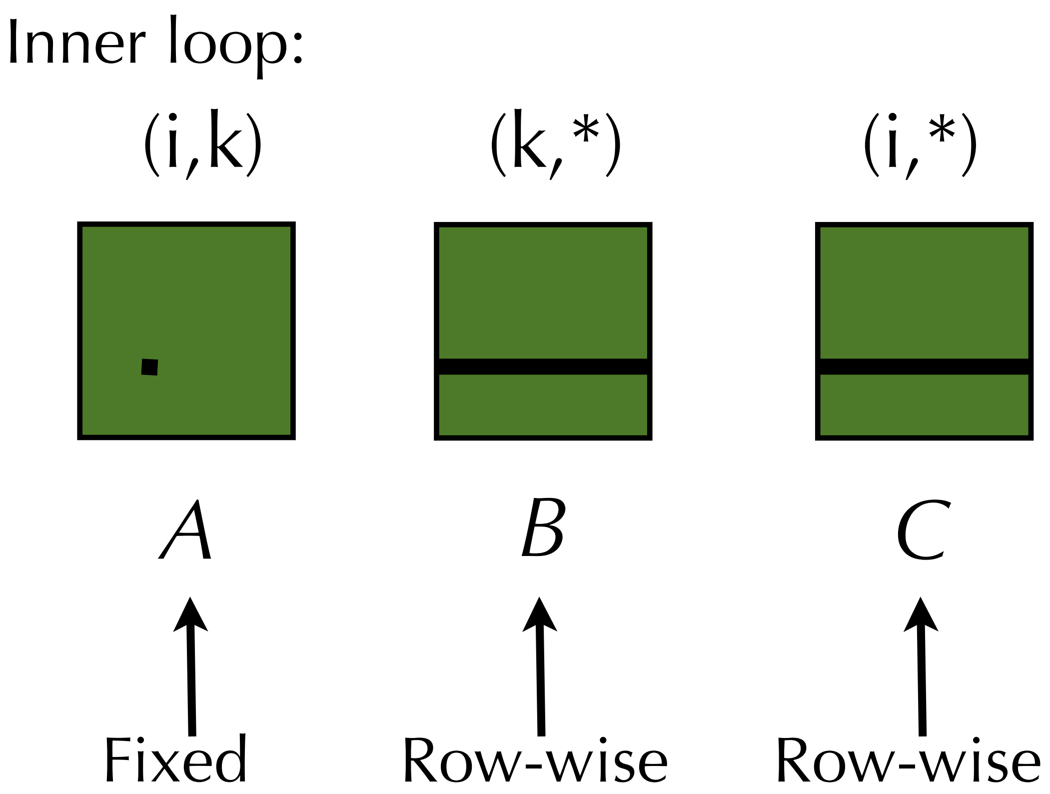


- Same as ijk, just swapped order of outer loops
- 2 loads, 0 stores per iteration
- Assume cache line size of 32 bytes, so 4 doubles per line
- Misses per iteration:

A = 0.25 B = 1 C = 0 Total: 1.25

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

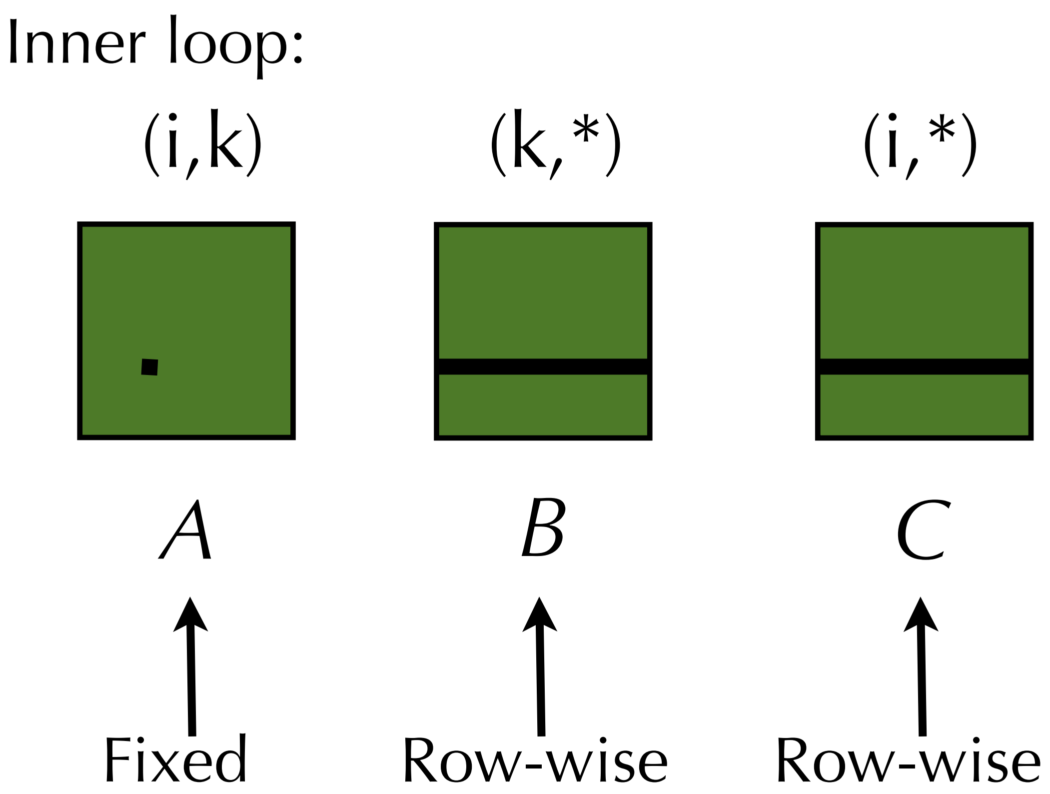


- 2 load, 1 store per iteration
- Assume cache line size of 32 bytes, so 4 doubles per line
- Misses per iteration:

A = 0 B = 0.25 C = 0.25 Total: 0.5

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```



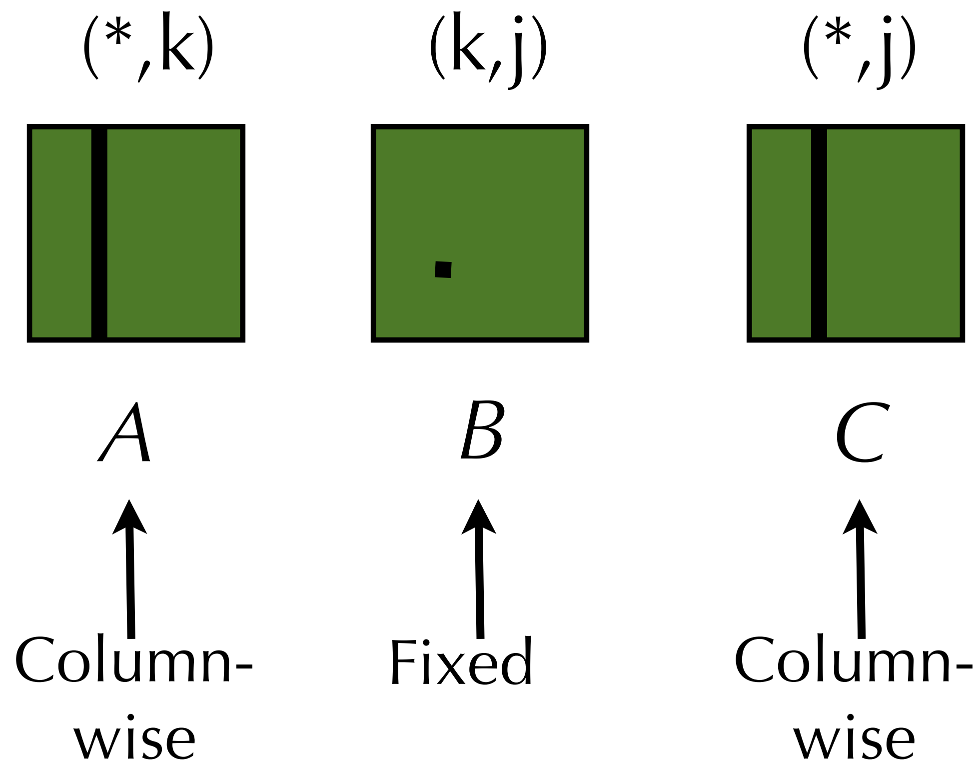
- Same as kij, just swapped order of outer loops
- 2 load, 1 store per iteration
- Assume cache line size of 32 bytes, so 4 doubles per line
- Misses per iteration:

A = 0 B = 0.25 C = 0.25 Total: 0.5

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



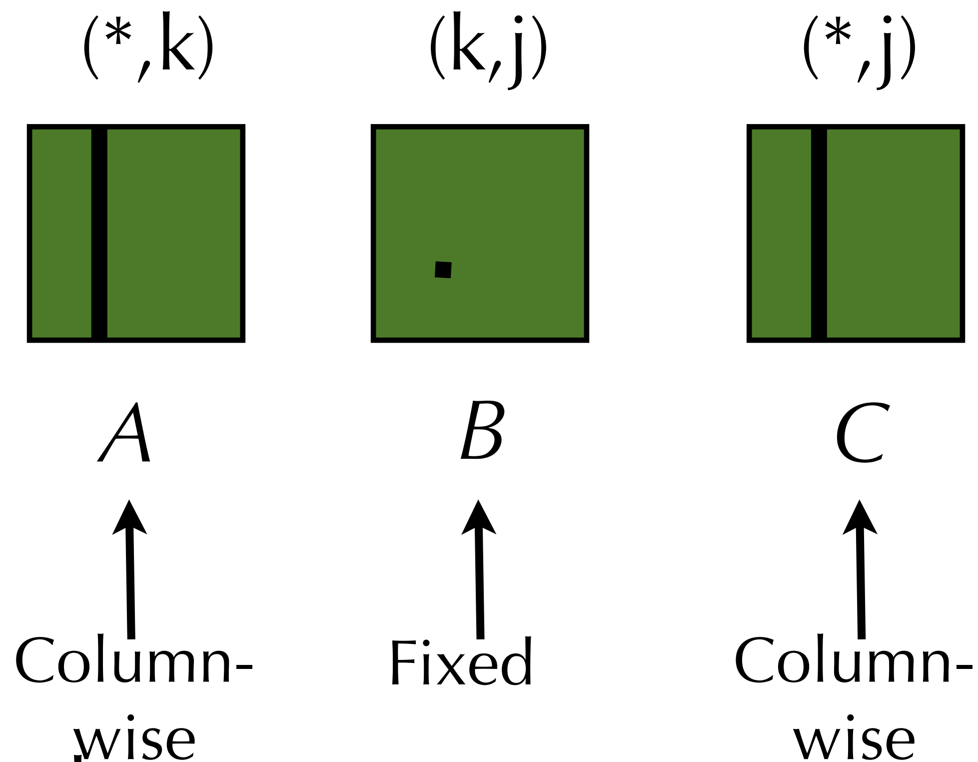
- 2 load, 1 store per iteration
- Assume cache line size of 32 bytes, so 4 doubles per line
- Misses per iteration:

A = 1 B = 0 C = 1 Total: 2

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



- Same as kji, just swapped order of outer loops
- 2 load, 1 store per iteration
- Assume cache line size of 32 bytes, so 4 doubles per line
- Misses per iteration:

A = 1 B = 0 C = 1 Total: 2

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
 for (j=0; j<n; j++) {
  sum = 0.0;
  for (k=0; k<n; k++)
    sum += a[i][k] * b[k][j];
  c[i][j] = sum;
 }
}
```

ijk or jik:
2 loads, 0 stores
misses/iter = 1.25

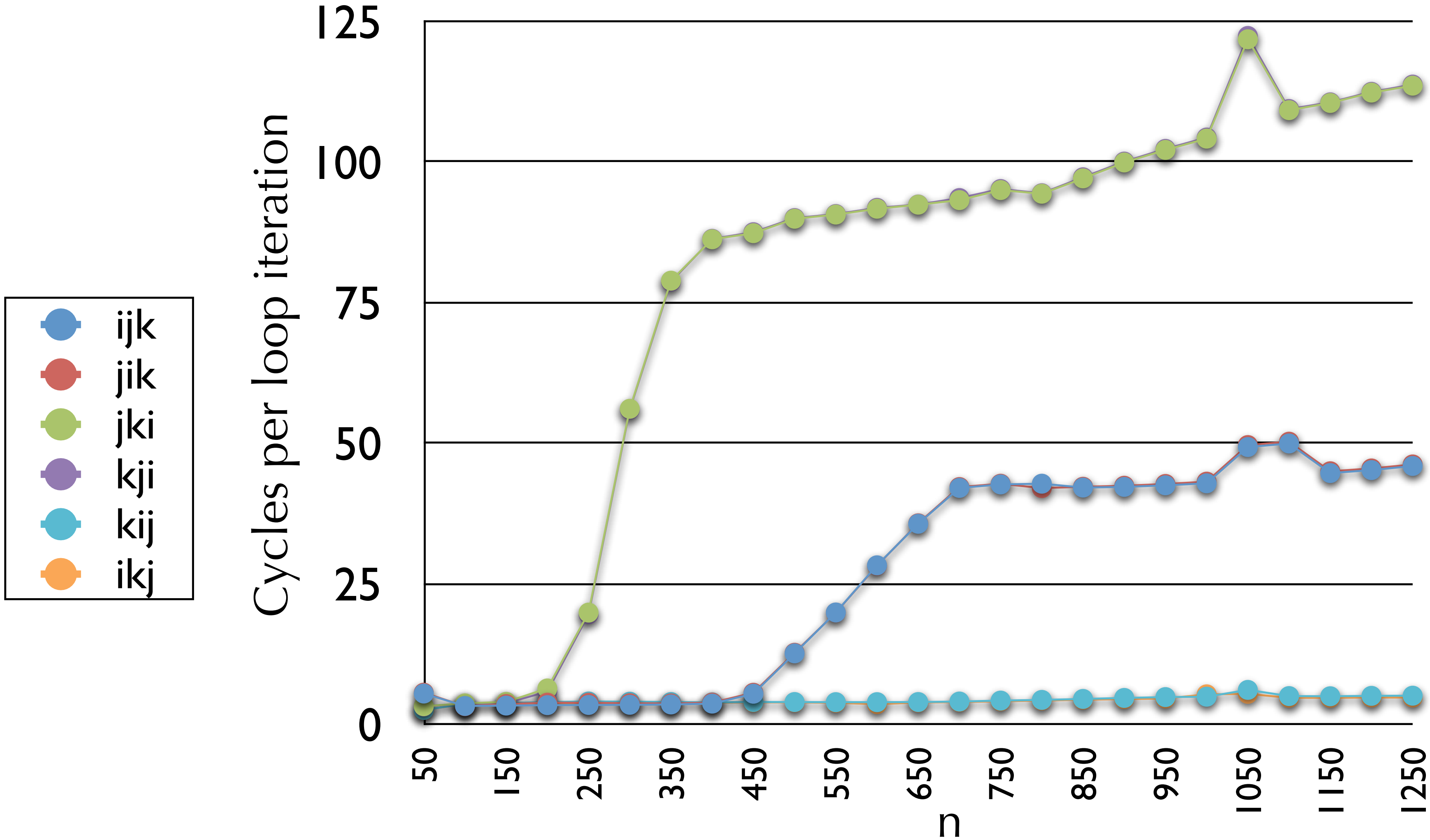
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij or ikj:
2 loads, 1 store
misses/iter = 0.5

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
  r = b[k][j];
  for (i=0; i<n; i++)
   c[i][j] += a[i][k] * r;
 }
}
```

jki or kji:
2 loads, 1 store
misses/iter = 2.0

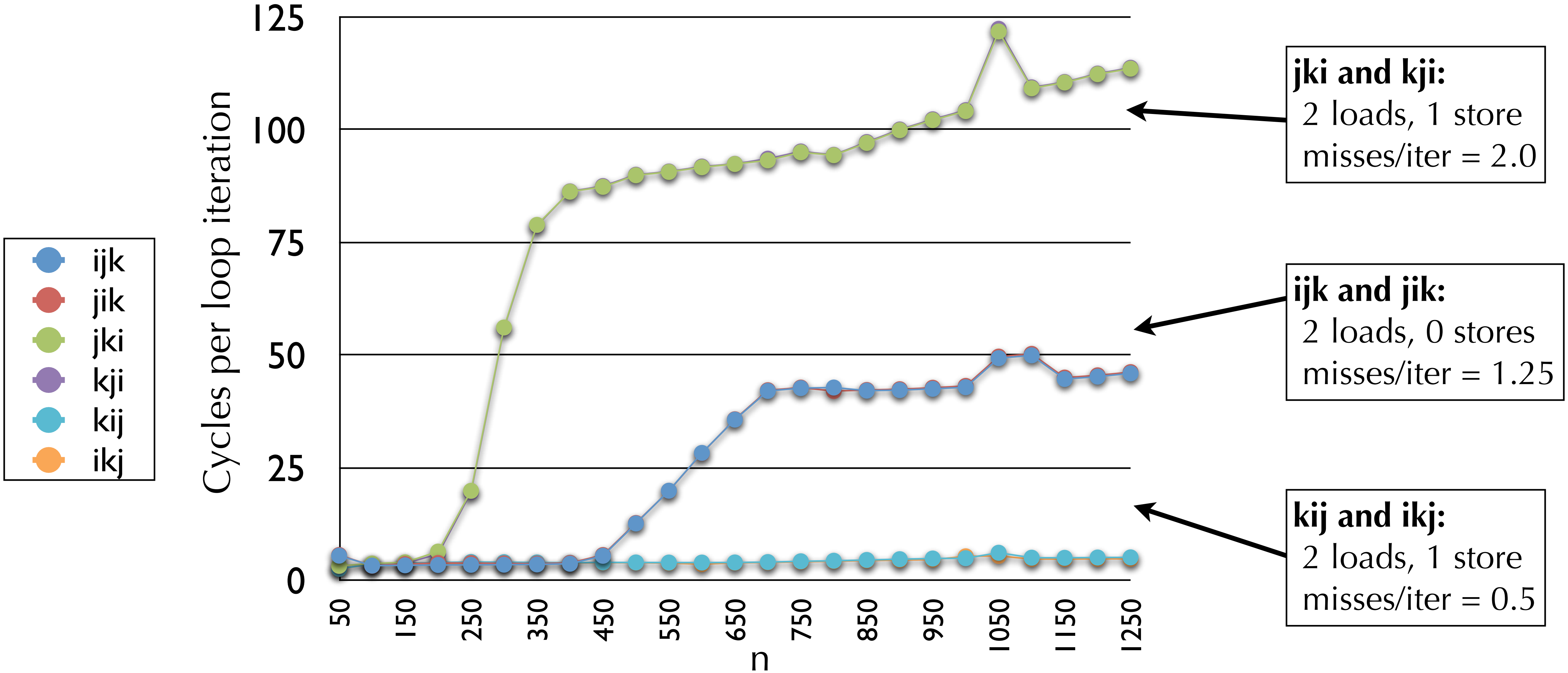
Matrix Multiply Performance



On cs61-login-2

- Each implementation doing same number of arithmetic operations, but ~20× difference!
- Pairs with same number of mem. references and misses per iteration almost identical

Matrix Multiply Performance



- Miss rate better predictor or performance than number of mem. accesses!
- For large N, kij and ikj performance almost constant.
  Due to **hardware prefetching**, able to recognize stride-1 patterns.

Topics for today

- Cache performance metrics
- Discovering your cache's size and performance
- The "Memory Mountain"
- Matrix multiply, six ways
- Blocked matrix multiplication
- Exploiting locality in your programs

Using blocking to improve locality

- Blocked matrix multiplication
  - Break matrix into smaller blocks and perform independent multiplications on each block.
  - Improves locality by operating on one block at a time.
  - Best if each block can fit in the cache!
- Example: Break each matrix into four sub-blocks

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., A_{xy}) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiply

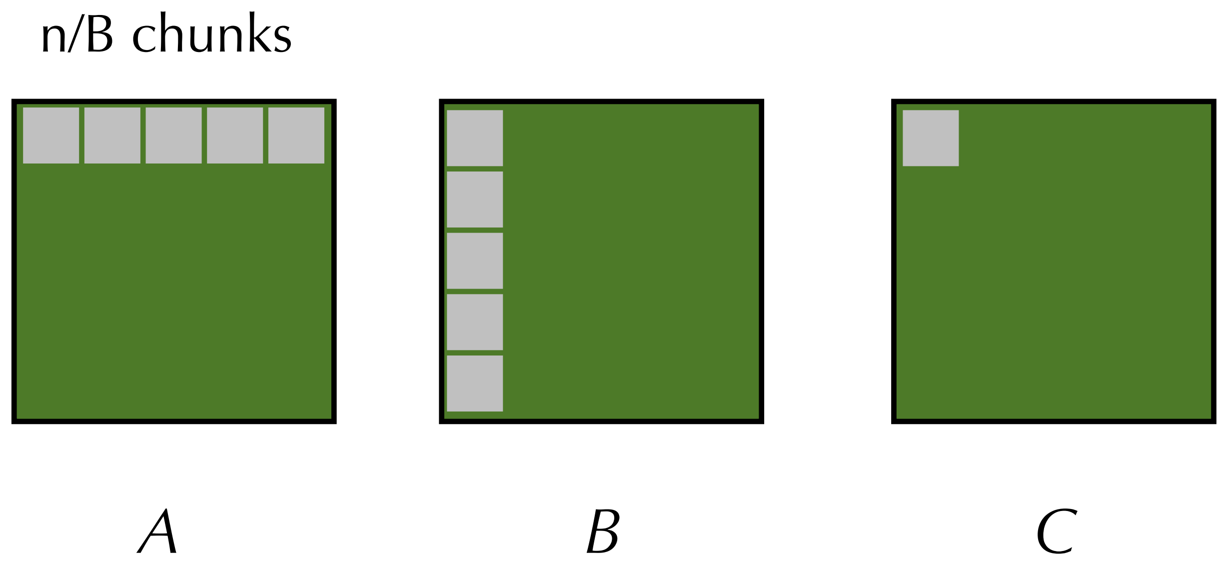
```
void bmmm(int n, double a[n][n], double b[n][n], double c[n][n]) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1][j1] += a[i1][k1] * b[k1][j1];
}
```

Code becomes harder to read!
Is it worth it?
Tradeoff between performance and maintainability...

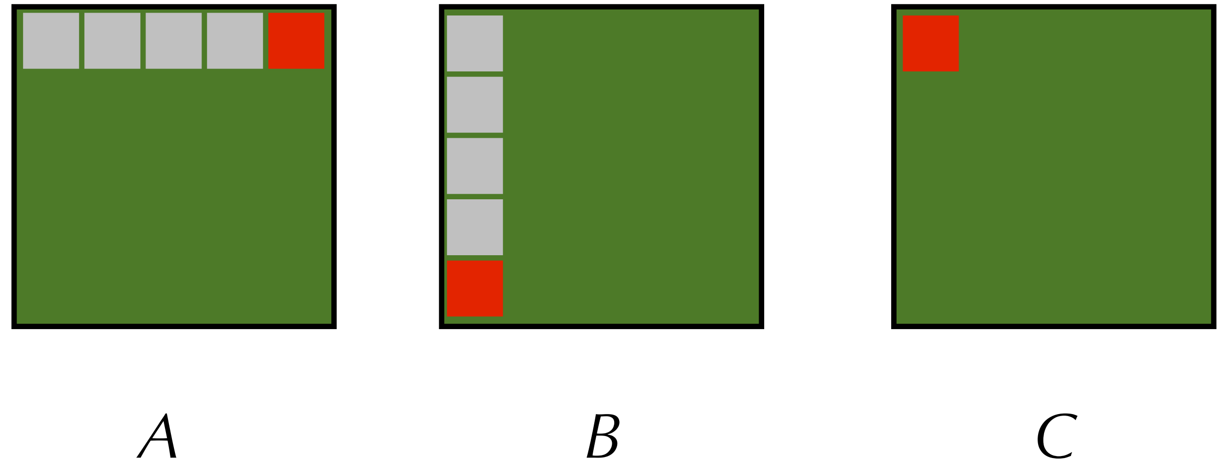
- Partition arrays into *bsize* × *bsize* chunks
- Innermost (i1, j1, k1) loop pair multiplies an *A* chunk by a *B* chunk and accumulates result in a *C* chunk

Blocked matrix multiply

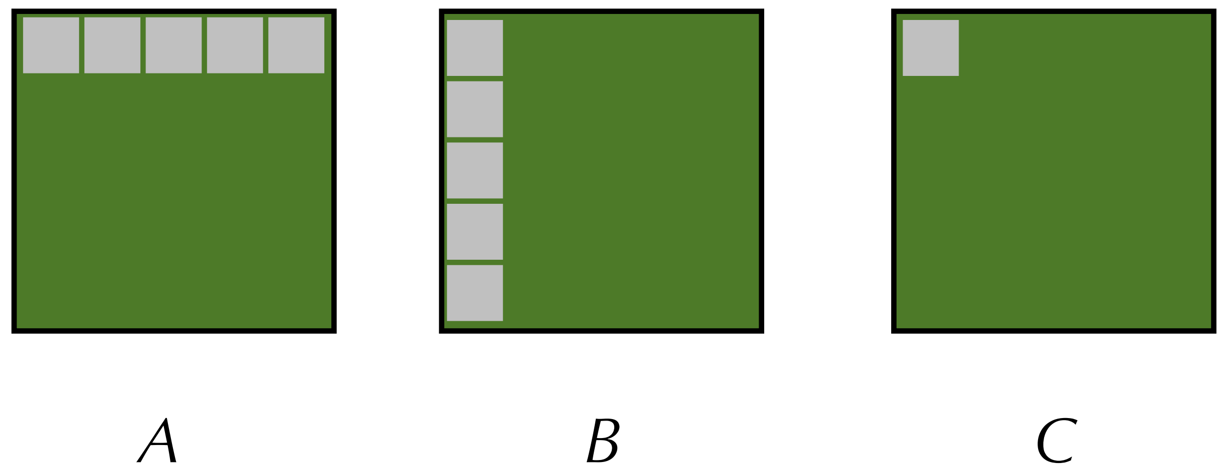
- Assume 3 chunks can fit into the cache, i.e., $3bsize^2 < C$
- First block iteration



- After first iteration in cache (schematic)

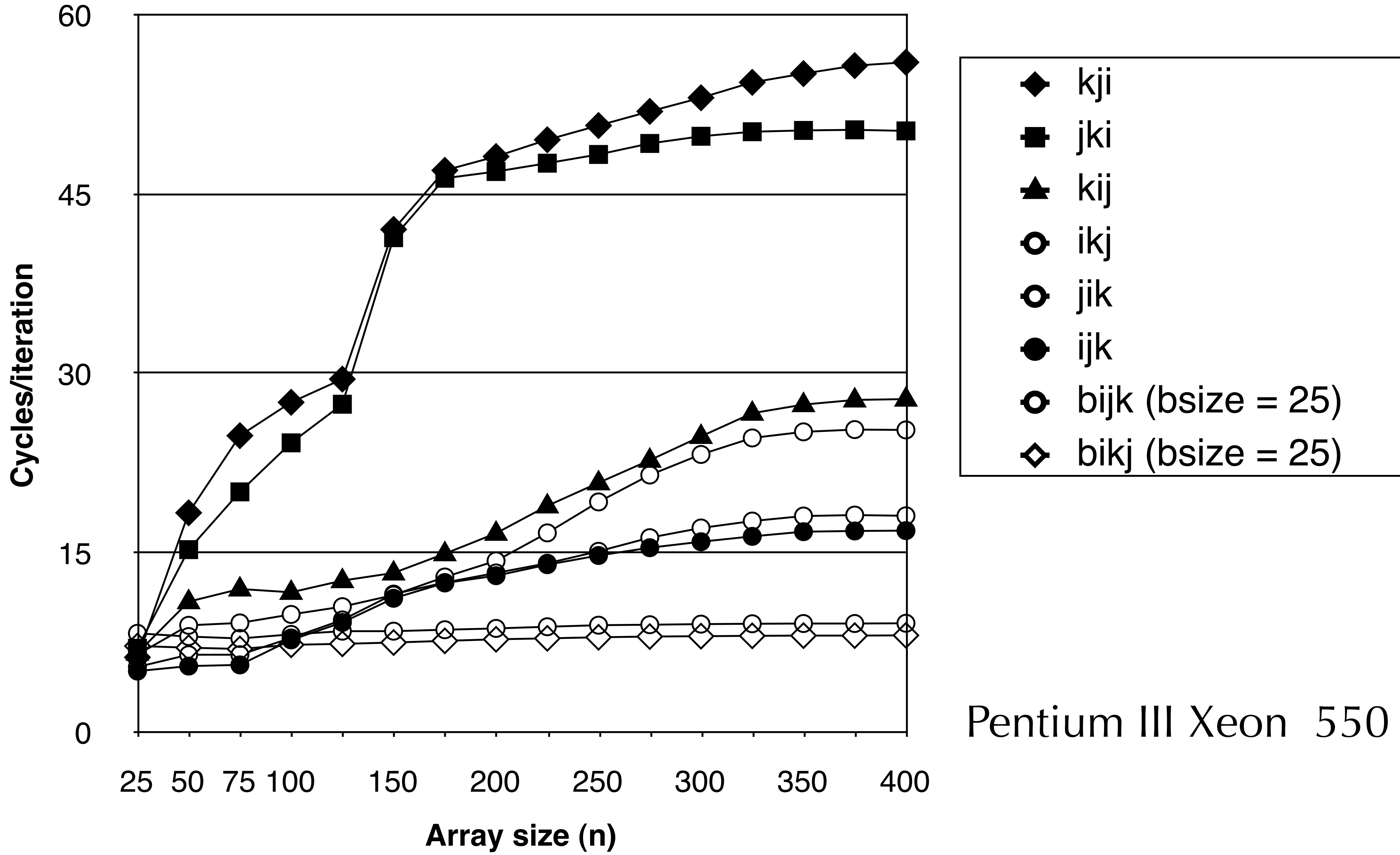


Cache miss analysis



- Assume 3 chunks can fit into the cache
- Assume *bsize* is a multiple of 4
- $bsize^2/4$ misses per chunk, so $3/4 * bsize^2$ misses per chunk iteration
- $(n/bsize)^3$ chunk iterations
- Total of $(n/bsize)^3 * 3/4 * bsize^2$ misses $= n^3 * 3/(4 * bsize)$
- Compare with $n^3 * 1/2$ total misses for kij algorithm

Blocked Matrix Multiply Performance



Pentium III Xeon 550 Mhz

- Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)
    - Relatively insensitive to array size.

Blocked Matrix Multiply Performance

Intel Core i7
2.7 GHz
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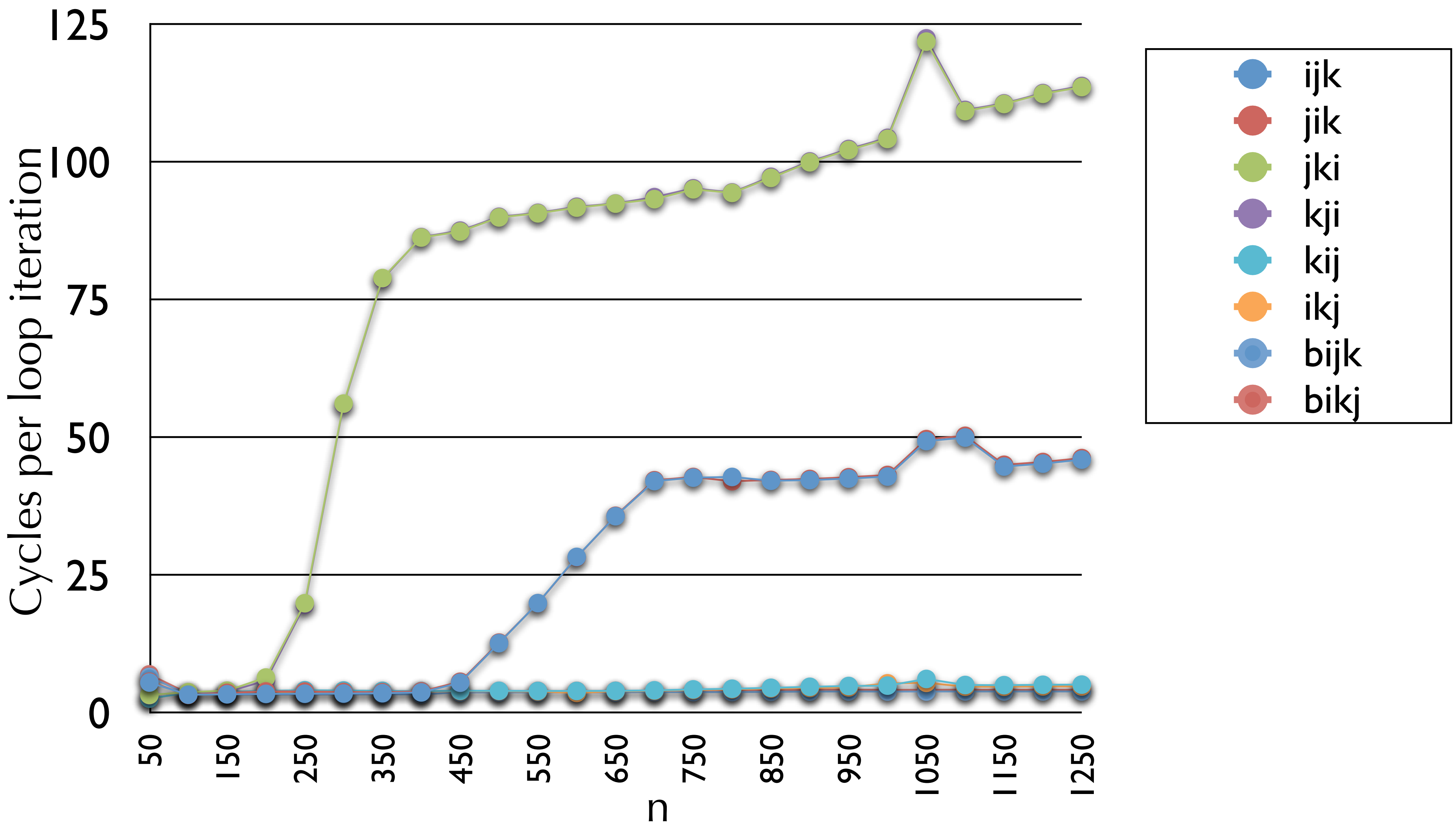
256 KB L2 cache
8MB L3 cache
CAVEAT: Tested on a VM



Blocked Matrix Multiply Performance

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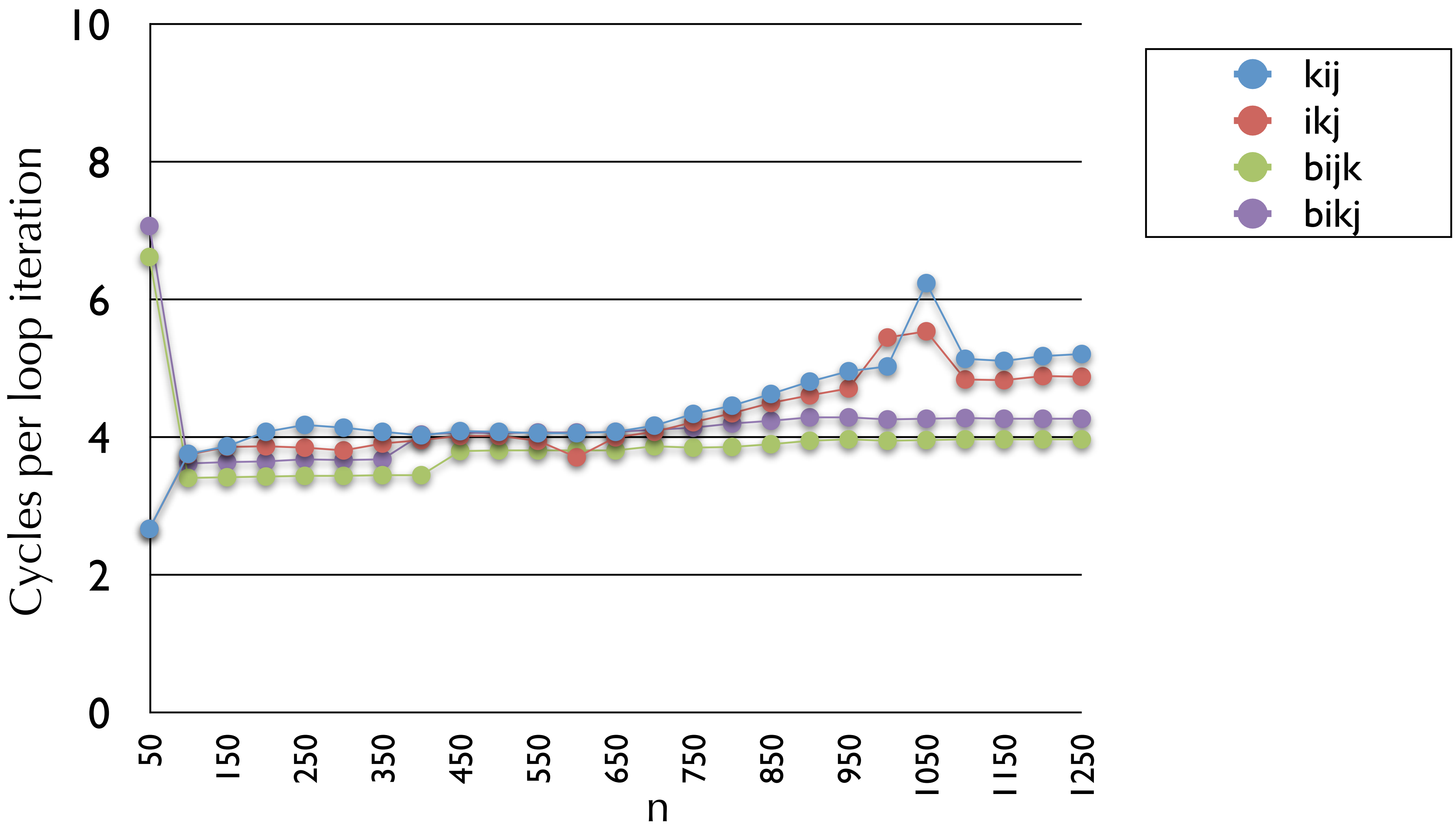

Exploiting locality in your programs

- Focus attention on inner loops
  - This is where most computation and memory accesses in your program occurs
- Try to maximize spatial locality
  - Read data objects sequentially, with stride 1, in the order they are stored in memory
- Try to maximize temporal locality
  - Use a data object as often as possible once it has been read from memory

Next lecture

- Virtual memory
  - Using memory as a cache for disk

Cache performance test program

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride)
{
    uint64_t start_cycles, end_cycles, diff;
    int elems = size / sizeof(int);

    test(elems, stride);                    /* warm up the cache */
    start_cycles = get_cpu_cycle_counter(); /* Read CPU cycle counter */
    test(elems, stride);                    /* Run test */
    end_cycles = get_cpu_cycle_counter();   /* Read CPU cycle counter again */
    diff = end_cycles - start_cycles;       /* Compute time */
    return (size / stride) / (diff / CPU_MHZ);  /* convert cycles to MB/s */
}
```

Cache performance main routine

```
#define CPU_MHZ 2.8 * 1024.0 * 1024.0; /* e.g., 2.8 GHz */
#define MINBYTES (1 << 10)  /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23)  /* ... up to 8 MB */
#define MAXSTRIDE 16        /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS];         /* The array we'll be traversing */

int main()
{
    int size;        /* Working set size (in bytes) */
    int stride;      /* Stride (in array elements) */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride));
        printf("\n");
    }
    exit(0);
}
```