Spatial GEV Regression

Bayesian Storm Surge Risk Modeling

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Data

- Storm surges (non-tidal residuals), at gauges provided by UCF Coastal Risks and Engineering lab
- ERA5-Interim - wind speed, pressure, precipitation at gauge, 1979 - present
- Approach - Model the yearly surge maxima as GEV distributed
- Regression for GEV parameters at each location, assuming spatial correlation weighted by distance

Notation

- Anywhere we see an s represents a specific location in space (i.e. a tide gauge)
- S then represents a list of all of the locations we use
- i represents the different times the surges occur at (i.e. which year for annual maxima)

GEV

$$p(y_{is}) = \frac{1}{\sigma_s}\left(1 + \xi_s\left(\frac{y_{is} - \mu_s}{\sigma_s}\right)\right)^{-(1+\frac{1}{\xi_s})} \cdot$$
$$\exp\left(-\left(1 + \xi_s\left(\frac{y_{is} - \mu_s}{\sigma_s}\right)\right)^{-\frac{1}{\xi_s}}\right)$$

- Models maxima of iid samples
- Assume $\xi_s = \frac{\sigma_s}{\mu_s}$
  - i.e. the distribution has a minimum value of 0

Regression

$$\ln(\sigma_s) = \beta^\top X_s + a_s + \epsilon_{\sigma,s} \qquad \epsilon_{\sigma,s} \sim N(0, \sigma_\sigma^2)$$
$$\mu_s = \gamma^\top X_s + b_s + \epsilon_{\mu,s} \qquad \epsilon_{\mu,s} \sim N(0, \sigma_\mu^2)$$

- Assumes these parameters do not change over time
- At location, one X_s is associated to one σ_s and μ_s, which are in turn associated with many actual measurements y_{is}
- Use Gaussian process prior for overall a, b vectors
- Approach inspired by (Boumis et al., 2023) and (He and Huang, 2024). This work: (Scott and Huang, 2025)

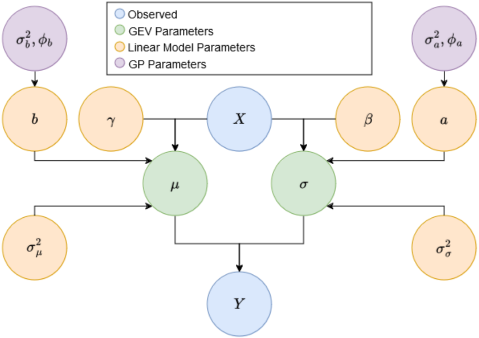
Spatial Random Effects

$$a \sim N(0, \sigma_a^2 K_{\phi_a}(S, S)) \qquad \sigma_a^2 \sim \text{Gamma}(\alpha, \theta)$$

$$K_\phi(s, s') = e^{-\left(\text{dist}(s,s')^2\right)\phi}$$

- Assumes that spatial correlations fall off according to the right half of some gaussian curve
  - Height of gaussian curve controlled by σ_a^2
  - Spread controlled by ϕ
- b (random effect for μ) will be modeled similarly

Model Visualization



$$p(. \mid y) \propto \underbrace{p(y|\mu_s, \sigma_s)}_{\text{GEV likelihood}}$$

$$\cdot \underbrace{p(\sigma_s|\beta, a, \sigma_\sigma^2) \cdot p(\beta, \sigma_\sigma^2) \cdot p(a|\phi_a, \sigma_a^2) \cdot p(\phi_a, \sigma_a^2)}_{\text{Model for } \sigma_s}$$

$$\cdot \underbrace{p(\mu_s|\gamma, b, \sigma_{\mu_s}^2) \cdot p(\gamma, \sigma_\mu^2) \cdot p(b|\phi_b, \sigma_b^2) \cdot p(\phi_b, \sigma_b^2)}_{\text{Model for } \mu_s}$$

Results

Regression Coefficient	95 % C.I.
Intercept	$[0.178, 0.773]$
Sea Level Pressure	$[-0.199, -0.038]$
Wind	$[-0.060, 0.016]$
Precipitation	$[-0.066, 0.030]$

Table: 95% credible intervals for the model coefficients for μ.

- For every 146.7 pascal decrease in the mean annual minimum sea level pressure at a location, we expect somewhere from a 0.038 meter to 0.199 meter increase in the GEV location parameter μ.

Results

Parameter	95 % C.I.
ϕ	$[3.03, 133.7]$
σ_b^2	$[0.0061, 0.34]$

Table: 95% credible intervals for the GP parameters for μ.

- Intercept correlation at locations less than 50 miles apart are 0.2 or greater.
- The correlation decays to 0.2 at a distance of 50 to 300 miles.
- The error variance for μ is between 0.001 and 0.009; spatial correlations may* explain 1.027 to 141 times as much variance in μ between locations than the error term.
  - * Simulations show that this estimate is not always reliable.

Results

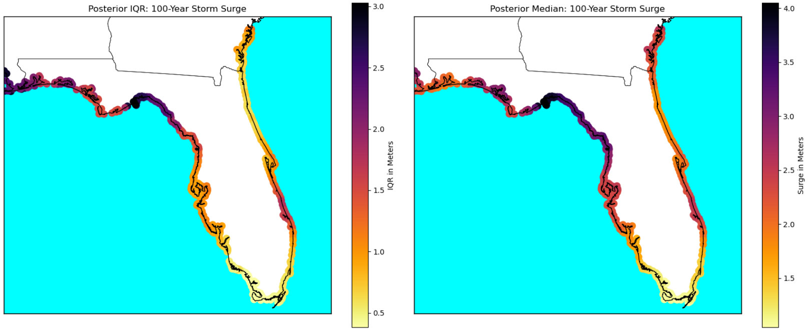


Figure: Left: Upper bound for 100-year storm surges based on data from 1979 to 2019. Right: Posterior median for the same.

Completeness of Dataset

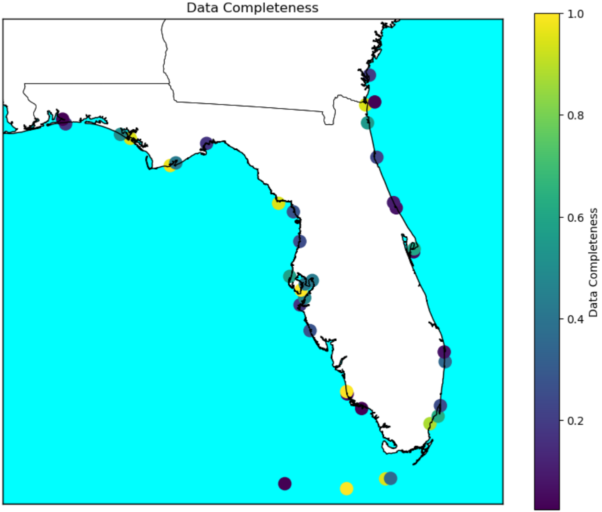


Figure: Proportion of years (1979-2020) for which there are measured maxima at each location

Direction

- Update using now-released ERA5 back to 1940, expand region of interest
  - Check for temporal effects with expanded data
- Improve computational speed for better usability in practice
- Further investigate ways to constrain the support to include 0

References

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