

Let  $(x_1, x_2, \dots, x_n)$  be sample of size  $n$

Mean  $\rightarrow \theta_1$       Var  $\rightarrow \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

take log

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2$$

for  $\theta_1$  diff  $\log(L(\theta_1, \theta_2))$  wrt  $\theta_1$  and set it to 0

$$\frac{d \log(L)}{d \theta_1} = -\frac{1}{\theta_2} \sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^n x_i$$

MLE of  $\theta_1$  is sample mean  
for  $\theta_2$  diff wrt  $\theta_2$  and put zero.

$$\theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \theta_1)^2$$

## ② Binomial Distribution

$n$  = no. of trials

$\theta = (0, 1)$  prob of success

$$L(\theta) = \prod_{i=1}^n f(x_i, n, \theta)$$

PMF

$$f(x, n, \theta) = {}^n C_x \theta^x (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^n ({}^n C_{x_i}) \cdot \theta^{x_i} (1-\theta)^{n-x_i}$$

take log

$$\frac{d \log(\ell)}{d\theta} = \frac{1}{\theta} \sum_{i=1}^m x_i - \frac{1}{1-\theta} \sum_{i=1}^m (m-x_i) = 0$$

$$= \frac{1}{\theta} \sum_{i=1}^m x_i - \frac{1}{1-\theta} \sum_{i=1}^m (m-x_i)$$

Multiply by  $\theta(1-\theta)$

$$\Rightarrow (1-\theta) \sum_{i=1}^m x_i = \theta \sum_{i=1}^m (m-x_i)$$

$$\Rightarrow \left[ \theta = \frac{\sum_{i=1}^m x_i}{m} \right]$$

Jayant Singh / 102103556 / 3COE20  
PARAMETER ESTIMATION  
Data Science Assignment