Quiz 6:

Question 1. Consider the following axioms:

a. All hounds howl at night.

b. Anyone who has any cats will not have any mice.

c. Light sleepers do not have anything which howls at night.

d. John has either a cat or a hound.

e. (Conclusion) If John is a light sleeper, then John does not have any mice. Use Resolution to prove (or disprove) the Conclusion

- Let Hound(x) denote "x has a hound."
- Let Cat(x) denote "x has a cat."
- Let Mice(x) denote "x has a mouse."
- Let LightSleeper (x) denote "x is a light sleeper."
- Let HowlsAtNight(x, y) denote "x has an animal y howls at night."

Now, we can represent the axioms:

a. ∀x Hound(x) → HowlsAtNight (x, Hound(x))

b. ∀x Cat(x) → ¬Mice(x)

c. ∀x ∀y LightSleeper(x) → ¬HowlsAtNight(x,y)

d. Hound(John) ∨ Cat(John)

Conclusion: Light_sleeper(John) → ¬Mice(John)

Now, let's use resolution to see if the conclusion follows from the premises:

1. ∀x Hound(x) → HowlsAtNight (y, Hound(x))
   Become: ¬Hound(a) ∨ HowlsAtNight(a, Hound(a))
2. ∀xCat(x)→¬Mice(x)
   Become: ¬Cat(b) ∨ ¬Mice(b)
3. ∀x ∀y LightSleeper(x) → ¬HowlsAtNight(x,y)
   Become: ¬LightSleeper(c) ∨ ¬HowlsAtNight(c, d)

4. Hound(John) ∨ Cat(John)

5. Conclusion: ¬(Light_sleeper(John)→¬Mice(John)) (Negate conclusion)
   Become: Light_sleeper(John) ∧ Mice(John)
   Become: Light_sleeper(John)
   Mice(John)

6. Resolve ($\neg$Cat(b) ∨ $\neg$Mice(b)) and (Mice(John)): (John/b)
   Cat(John).
7. Resolve ($\neg$LightSleeper(c) ∨ $\neg$HowlsAtNight(c, d)) and (Light_sleeper(John)): (John/c)
   $\neg$ HowlsAtNight (John, b)
8. Resolve ($\neg$HowlsAtNight (John, b)) and ($\neg$Hound(a) ∨ HowlsAtNight(a, Hound(a))): (John/a, b/Hound(a))
   $\neg$Hound(John)
9. Resolve (Hound(John) ∨ Cat(John)) and ($\neg$Hound(John)):
   Cat(John)
10. Resolve (6) and (9): Hence the conclusion is disproved

Question 2. Consider the following axioms:

a. Every child loves Santa.

b. Everyone who loves Santa loves any reindeer.

c. Rudolph is a reindeer, and Rudolph has a red nose.

d. Anything which has a red nose is weird or is a clown.

e. No reindeer is a clown.

f. Scrooge does not love anything which is weird.

g. (Conclusion) Scrooge is not a child.

Use Resolution to prove (or disprove) the Conclusion

- Let Child(x) denote "x is a child."
- Let Loves(x, y) denote "x loves y."
- Let Reindeer(x) denote "x is a reindeer."
- Let HasRedNose (x) denote "x has red noise."
- Let Weird(x) denote "x is weird."
- Let Clown(x) denote "x is clown."

Now, we can represent the axioms:

a. $\forall$x (Child(x) $\rightarrow$ Loves(x,Santa))

b. $\forall$x ((Loves(x,Santa)) $\rightarrow$ ($\forall$y (Reindeer(y) $\rightarrow$ Loves(x,y))))

c. Reindeer(Rudolph) ∧ HasRedNose(Rudolph)

d. $\forall$z (HasRedNose(z) $\rightarrow$ (Weird(z) ∨ Clown(z)))

e. $\exists$w $\neg$(Reindeer(w) ∧ Clown(w))

f. $\forall$v (Loves(Scrooge,v) $\rightarrow$ $\neg$Weird(v))

Conclusion: ¬Child(Scrooge)

1. ∀x (Child(x) → Loves(x,Santa))
   Become: ¬Child (a) ∨ Loves (a, Santa)

2. ∀x ((Loves(x,Santa)) → (∀y (Reindeer(y) → Loves(x,y))))
   Become: ∀x(¬Loves(x,Santa) ∨ (∀y (¬Reindeer(y) ∨ Loves(x,y))))

   Become: ¬Loves(b,Santa) ∨ ¬Reindeer(c) ∨ Loves(b,c)

3. Reindeer(Rudolph) ∧ HasRedNose(Rudolph)
   Become: Reindeer(Rudolph)
   HasRedNose(Rudolph)

4. ∀z (HasRedNose(z) → (Weird(z) ∨ Clown(z)))
   Become: ¬HasRedNose(d) ∨ Weird(d) ∨ Clown(d)

5. ∃w ¬(Reindeer(w) ∧ Clown(w))
   Become: ¬Reindeer(e) ∨ ¬Clown(e)

6. ∀v (Loves(Scrooge,v) → ¬Weird(v))
   Become: ¬ Loves(Scrooge,f) ∨ ¬Weird(f)

7. Conclusion: ¬Child(Scrooge)
   Become: Child(Scrooge) (Negate conclusion)

8. Resolve (Child(Scrooge)) and (¬Child (a) ∨ Loves (a, Santa)): (Child/a)
   Loves(Scrooge, Santa)
9. Resolve (Loves(Scrooge, Santa)) and ¬Loves(b,Santa) ∨ ¬Reindeer(c) ∨ Loves(b,c): (Scrooge,b)
   ¬Reindeer(c) ∨ Loves(Scrooge,c)
10. Resolve (¬Reindeer(c) ∨ Loves(Scrooge,c)) and (Reindeer(Rudolph)): (Rudolph / c)
    Loves(Scrooge, Rudolph)
11. Resolve (¬HasRedNose(d) ∨ Weird(d) ∨ Clown(d)) and (HasRedNose(Rudolph)): (Rudolph / d)
    Weird(Rudolph) ∨ Clown(Rudolph)
12. Resolve (¬Reindeer(e) ∨ ¬Clown(e)) and (Reindeer (Rudolph)): (Rudolph / e)
    ¬Clown(Rudolph)
13. Resolve (Weird(Rudolph) ∨ Clown(Rudolph)) and (¬Clown (Rudolph)): (Rudolph / e)
    Weird (Rudolph)
14. Resolve (¬ Loves(Scrooge,f) ∨ ¬Weird(f)) and (Weird (Rudolph)): (Rudolph / f)
    ¬ Loves (Scrooge , Rudolph)
15. Resolve (¬Loves(b,Santa) ∨ ¬Reindeer(c) ∨ Loves(b,c)) and (Loves(Scrooge, Rudolph)) and (Reindeer(Rudolph)): (Rudolph/c, Scrooge,b)
    Loves (Scrooge , Rudolph)

16. Resolve (15) ∨ (16): Hence the conclusion is proved

Question 3. There are three suspects for a murder: Adams, Brown, and Clark.
Adams says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him."
Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all the week."
Clark says "I didn't do it. I saw both Adams and Brown downtown with the victim that day; one of them must have done it."
Assume that the two innocent men are telling the truth, but that the guilty man might not be. Write out the facts as sentences in Propositional Logic, and use propositional resolution to solve the crime.

A	=	Adams did it
B	=	Brown did it
C	=	Clark did it
p	=	Brown knew victim
q	=	Brown was in town
R	=	Clark was in town
T	=	Adams was in town
S_A	=	$(\neg A \land p)$
S_B	=	$(\neg B \land \neg p \land \neg q)$
S_C	=	$(\neg C \land q \land t)$

We have 3 version that we need to check:

Adams is lying and the others are telling the truth:
$\neg S_A = \neg(\neg A \land p) = (A \lor \neg p)$

$(S_B \land S_C) \land \neg S_A$

1)¬B

2)¬p

3)¬q

4)¬C

5)q

6)t

7){A, ¬p}

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8){}(3,5) Contradiction

Brown is lying and the others are telling the truth:
$S_B = \neg(\neg B \land \neg p \land \neg q) = \neg(B \lor p \lor q)$

$(S_A \land S_C) \land \neg S_B$

1)$\neg A$

2)p

3)C

4)q

5)t

6)$\{B, p, q\}$

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7)No Contradiction

Clark is lying and the others are telling the truth:
$\neg S_C = \neg(\neg C \land q \land t) = (C \lor \neg q \lor \neg t)$

$(S_A \land S_B) \land \neg S_C$

1)$\neg A$

2)p

3)$\neg B$

4)$\neg p$

5)$\neg q$

6)$\{C, \neg p, \neg q\}$

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7)$\{\}$(2,4) Contradiction

Since (Adam) and (Clark) have contradictions, only (Brown) can be true.

Conclusion: Brown is lying, and is the murderer.
Question 4. Consider this Knowledge Base in propositional logic:

KB = $\{A, B, A \lor C, K \land E \leftrightarrow A \land B, \neg C \rightarrow D, E \lor F \rightarrow \neg D\}$

Check if these sentences in entailed by the KB:

a) $B \land C$?

b) $C \lor E \rightarrow F \land B$?

a) Negate $(B \land C)$: $\neg(B \land C) \equiv \neg B \lor \neg C$

1. A
2. B
3. $A \lor C$

4. $K \wedge E \Leftrightarrow A \wedge B$
   $\equiv (K \wedge E \rightarrow A \wedge B) \wedge (A \wedge B \rightarrow K \wedge E) \equiv (\neg K \vee \neg E \vee (A \wedge B)) \wedge (\neg A \vee \neg B \vee (K \wedge E))$
5. $\neg C \rightarrow D \equiv C \vee D$
6. $E \vee F \rightarrow \neg D \equiv (\neg E \wedge \neg F) \vee \neg D \equiv (\neg E \vee \neg D) \wedge (\neg F \vee \neg D)$

We have KB = $\{A, B, A \vee C, (\neg K \vee \neg E \vee (A \wedge B), \neg A \vee \neg B \vee (K \wedge E), C \vee D, (\neg E \wedge \neg F) \vee \neg D\}$

Now, add the negation of $B \wedge C$: 7. $\neg B \vee \neg C$ (negation of $B \wedge C$)

KB = $\{A, B, A \vee C, (\neg K \vee \neg E \vee (A \wedge B)), (\neg A \vee \neg B \vee (K \wedge E)), C \vee D, (\neg E \wedge \neg F) \vee \neg D, \neg B \vee \neg C\}$

$(\neg K \vee \neg E \vee (A \wedge B)) \equiv (\neg K \vee \neg E \vee A) \wedge (\neg K \vee \neg E \vee B)$

$(\neg A \vee \neg B \vee (K \wedge E)) \equiv (\neg A \vee \neg B \vee K) \wedge (\neg A \vee \neg B \vee E)$

KB = $\{A, B, A \vee C, (\neg K \vee \neg E \vee A), (\neg K \vee \neg E \vee B), (\neg A \vee \neg B \vee K), (\neg A \vee \neg B \vee E), C \vee D, (\neg E \vee \neg D), (\neg F \vee \neg D), \neg B \vee \neg C\}$

The resolution process is as follows:

1. Resolve (A) from KB:

   KB = $\{B, (C \vee \neg B \vee K \vee E), (\neg K \vee \neg E), C \vee D, \neg E \vee \neg D, \neg F \vee \neg D, \neg B \vee \neg C\}$

2. Resolve (B) from KB:

   KB = $\{(C \vee K \vee E), (\neg K \vee \neg E), C \vee D, \neg E \vee \neg D, \neg F \vee \neg D, \neg C\}$

3. Resolve (C) from KB:

   KB = $\{K \vee E, \neg K \vee \neg E, D, \neg E \vee \neg D, \neg F \vee \neg D\}$

4. Resolve (D) from KB:

   KB = $\{K \vee E, \neg K \vee \neg E, \neg E, \neg F\}$

5. Resolve (E) from KB:

   KB = $\{K, \neg K, \neg F\}$

6. Resolve (K) from KB:

   KB = $\{\neg F\}$ hence $B \wedge C$ is not entailed from KB

b) Negate $\neg(C \vee E \rightarrow F \wedge B) \equiv \neg(\neg(C \vee E) \vee (F \wedge B)) \equiv (C \vee E) \wedge (\neg F \vee \neg B)$

Add $(C \vee E) \wedge (\neg F \vee \neg B)$ to KB

KB = {A, B, (A ∨ C), (¬K ∨ ¬E ∨ A), (¬K ∨ ¬E ∨ B), (¬A ∨ ¬B ∨ K), (¬A ∨ ¬B ∨ E), (C ∨ D), (¬E ∨ ¬D), (¬F ∨ ¬D), (C ∨ E), (¬F ∨ ¬B)}

1. Resolve (A) from KB:

   KB = {B, (C ∨ ¬ B ∨ K ∨ E), (¬K ∨ ¬E), (C ∨ D), (¬E ∨ ¬D), (¬F ∨ ¬D), (C ∨ E), (¬F ∨ ¬B)}

2. Resolve (B) from KB:

   KB = {(C ∨ K ∨ E), (¬K ∨ ¬E), (C ∨ D), (¬E ∨ ¬D), (¬F ∨ ¬D), (C ∨ E), ¬F}

3. Resolve (C) from KB:

   KB = {(K ∨ E), (¬K ∨ ¬E), D, ¬E ∨ ¬D, ¬F ∨ ¬D, E, ¬F}

4. Resolve (D) from KB:

   KB = {(K ∨ E), ¬K ∨ ¬E, ¬E, ¬F , E, ¬F}

5. Resolve (E) from KB:

   KB = {K, ¬K, ¬F, ¬F}

6. Resolve (K) from KB:

   KB = {¬F, ¬F}

7. Resolve (F) from KB:

   KB = null => KB entails (C ∨ E→ F ∧ B)

Question 5. Consider the following knowledge base of definite clauses.

1. C ∧ D → Y

2. R ∧ Z → C

3. ¬B ∨ D

4. ¬D ∨ ¬R ∨ Z

5. B

6. R → D

7. D → R

Prove Y using backward chaining and forward chaining. In forward chaining, we only trigger a rule once for simplicity.

Backward chaining

1. **Goal: Y**
2. **Rule 1: $C \land D \rightarrow Y$**
    - This rule suggests that if $C \land D$ is true, then Y is true.
    - So, let's try to prove $C \land D$.
3. **Rule 2: $R \land Z \rightarrow C$**
    - This rule suggests that if $R \land Z$ is true, then C is true.
    - We need to prove $R \land Z$.
4. **Rule 4: $\neg D \lor \neg R \lor Z$**
    - To prove $\neg D \lor \neg R \lor Z$, we need to consider the following subgoals:
        - $\neg B \lor D$ (Rule 3)
        - B (Rule 5)
5. **Rule 3: $\neg B \lor D$**
    - This rule suggests that if $\neg B$ is false, then D is true.
    - Since B (Rule 5) is known, $\neg B$ is false, so we can use the rule to conclude D.
6. **Rule 6: $R \rightarrow D$ and Rule 7: $D \rightarrow R$**
    - This rule suggests $D \leftrightarrow R$.
    - We have already proven D, so we can use this rule to conclude R.
7. **Now that we have D and R**, we can go back to subgoal 4 and use modus ponens on Rule 6 and 7 to conclude Z is true.
8. **Now that we have $R \land Z$**, we can go back to subgoal 3 and use modus ponens on Rule 2 to conclude C is true.
9. **Now that we have $C \land D$**, we can go back to the original goal and use modus ponens on Rule 1 to conclude Y.

Forward chaining

1. **Initial Facts:**
    - Rule 5: B (given in the knowledge base)
2. **Apply Rule 3: $\neg B \lor D$**
    - Since B is true, we can use this rule to derive D is true.
3. **Apply Rule 7: $D \rightarrow R$**
    - Using modus ponens, we can now derive R is true.
4. **Apply Rule 6: $R \rightarrow D$**
    - Using this we can conclude $D \leftrightarrow R$.
5. **Apply Rule 4: $\neg D \lor \neg R \lor Z$**
    - We can now apply this rule since we have D and R, therefore, Z is true.
6. **Apply Rule 2: $R \land Z \rightarrow C$**
    - Using modus ponens, we can derive C is true.
7. **Apply Rule 1: $C \land D \rightarrow Y$**
    - Since we have C and D, we can use modus ponens to derive Y.

Question 6. Consider the following KB.

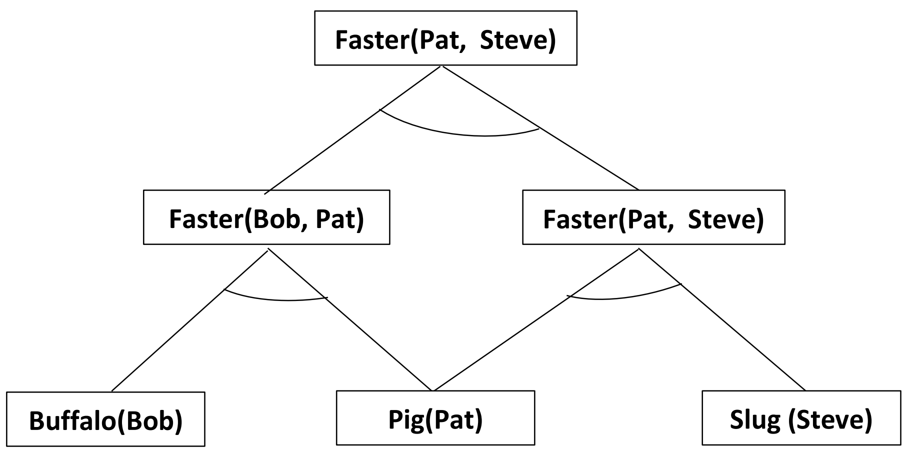
1. $Buffalo(x) \land Pig(y) \rightarrow Faster(x,y)$	4. $Buffalo(Bob)$
2. $Pig(y) \land Slug(z) \rightarrow Faster(y,z)$	5. $Pig(Pat)$
3. $Faster(x,y) \land Faster(y,z) \rightarrow Faster(x,z)$	6. $Slug(Steve)$

Use forward chaining in first-order logic to prove Faster (Bob, Steve). If several rules apply, use the one with the smallest number. Do not forget to indicate the unification at every step.

Convert to cnf form:

1. $\forall x\ \forall y\ (Buffalo(x) \land Pig(y)) \rightarrow Faster(x,y)$
2. $\forall y\ \forall z\ (Pig(y) \land Slug(z)) \rightarrow Faster(y,z)$
3. $\forall x\ \forall y\ \forall z(Faster(x,y) \land Faster(y,z)) \rightarrow Faster(x,z)$
4. Buffalo(Bob)
5. Pig(Pat)
6. Slug(Steve)

Conclusion: Faster(Bob, Steve)
Forward chaining



1. **Initial Facts:**
   - Buffalo(Bob) (Rule 4)
   - Pig(Pat) (Rule 5)
   - Slug(Steve) (Rule 6)
2. **Apply Rule 1: $\forall x\ \forall y\ (Buffalo(x) \land Pig(y)) \rightarrow Faster(x, y)$**
   - Unify with x=Bob and y=Pat.
   - Result: Faster(Bob, Pat)
3. **Apply Rule 3: $\forall x\ \forall y\ \forall z\ (Faster(x, y) \land Faster(y, z)) \rightarrow Faster(x, z)$**
   - Unify with x=Bob, y=Pat, and z=Steve.
   - Result: Faster(Bob, Steve)