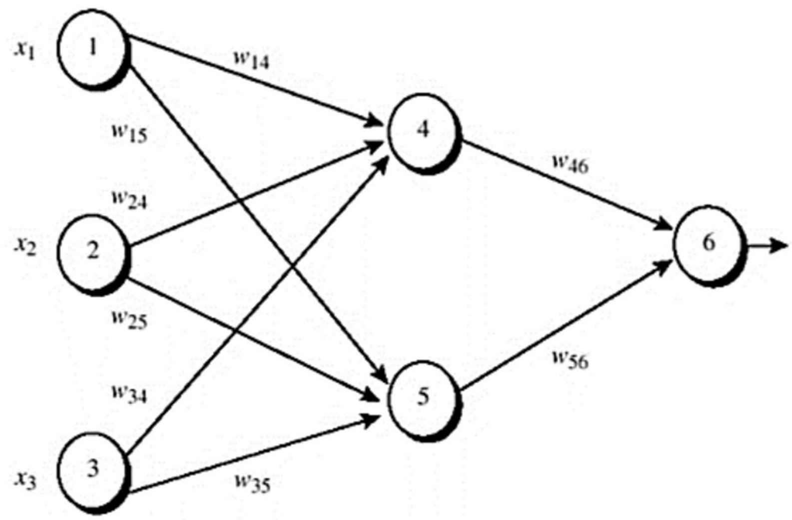
Introduction to Artificial Intelligence

Quiz 8

1. Problem:

Consider the following neuron network, which includes 3 input neurons, 2 hidden neurons and 1 output neurons.



Initial input, weight and bias values are

x_1	x_2	x_3	w_{14}	w_{15}	w_{24}	w_{25}	w_{34}	w_{35}	w_{46}	w_{56}	θ_4	θ_5	θ_6
1	0	1	0.2	−0.3	0.4	0.1	−0.5	0.2	−0.3	−0.2	−0.4	0.2	0.1

The expected output value is 1. The learning rate is 0.9 Knowing that the actual output at some neuron j is calculated as follows.

$$y_j(p) = \text{sigmoid}\left[\sum_{i=1}^{n} x_i(p) \times w_{ij}(p) + \theta_j\right]$$

where n is the number of inputs of neuron j, wij is the corresponding link from a neuron i in the previous layer to neuron j, and θj is the bias at neuron j. Present all calculations required to perform the backpropagation once (i.e., one forward pass and one backward pass) on the given neural network in the following cases

$\text{Output} = \text{Activation Function}\ (\sum_{i=0}^{n} \text{input}_\text{i} * \ \text{weight}_\text{i})$

$\text{Output} = \frac{1}{1 + e^{-\left(\sum_{i=0}^{n} \text{input}_\text{i} * \text{weight}_\text{i}\right)}}$

a) Ignore all biases (precision to 3 decimal places).

Ignore all biases – Forward

Sum4 $= x_1 * \text{ weight}_{14} + x_2 * \text{ weight}_{24} + x_3 * \text{ weight}_{34} = 1 * 0.2 + 0 * 0.4 + 1 * (-0.5) = -0.3$

Output4 $= \frac{1}{1+e^{-(-0.3)}} = 0.425$

Sum5 $= x_1 * \text{ weight}_{15} + x_2 * \text{ weight}_{25} + x_3 * \text{ weight}_{35} = 1 * (-0.3) + 0 * 0.1 + 1 * 0.2 = -0.1$

Output5 $= \frac{1}{1+e^{-(-0.1)}} = 0.475$

Sum6 $= \text{ output}_4 * \text{ weight}_{46} + \text{ output}_5 * \text{ weight}_{56} = 0.425 * (-0.3) + 0.475 * (-0.2) = -0.223$

Output6 $= \frac{1}{1+e^{-(-0.1)}} = 0.444$

Ignore all biases – Backward
Error gradient at neuron 6 $= 0.444 * (1 - 0.444) * (1 - 0.444) = 0.137$

Error gradient at neuron 5 $= (0.425) * (1 - 0.425) * (-0.3 * 0.137) = -0.01$
Error gradient at neuron 4 $= (0.475) * (1 - 0.475) * (-0.2 * 0.137) = -0.0068$
Updated weight:
weight46 $= (-0.3) + 0.9 * 0.137 * 0.425 = -0.247$
weight56 $= (-0.2) + 0.9 * 0.137 * 0.475 = -0.141$
weight14 $= (0.2) + 0.9 * (-0.0068) * 1 = 0.194$
weight24 $= (0.4) + 0.9 * (-0.0068) * 0 = 0.4$
weight34 $= (-0.5) + 0.9 * (-0.0068) * 1 = -0.506$
weight15 $= (-0.3) + 0.9 * (-0.01) * 1 = -0.309$
weight25 $= (0.1) + 0.9 * (-0.01) * 0 = 0.1$
weight35 $= (-0.2) + 0.9 * (-0.01) * 1 = -0.209$

b) Consider all biases such that each bias is treated as a neuron and thus it will be also updated (precision to 3 decimal places).
Sum 4 $= x_1 * \text{ weight}_{14} + x_2 * \text{ weight}_{24} + x_3 * \text{ weight}_{34} + \text{bias}_4 = 1 * 0.2 + 0 * 0.4 + 1 * (-0.5) + (-0.4) = -0.7$

Output4 $= \frac{1}{1+e^{-(-0.7)}} = 0.332$

Sum5 $= x_1 * \text{ weight}_{15} + x_2 * \text{ weight}_{25} + x_3 * \text{ weight}_{35} + \text{bias}_5 = 1 * (-0.3) + 0 * 0.1 + 1 * 0.2 + 0.2 = 0.1$

Output5 $= \frac{1}{1+e^{-(0.1)}} = 0.524$

Sum6 $= \text{ output}_4 * \text{ weight}_{46} + \text{ output}_5 * \text{ weight}_{56} + \text{bias}_6 = 0.332 * (-0.3) + 0.524 * (-0.2) + 0.1 = -0.104$

Output6 $= \frac{1}{1+e^{-(-0.104)}} = 0.474$

Error gradient at neuron 6 $= 0.474 * (1 - 0.474) * (1 - 0.474) = 0.131$

Error gradient at neuron 5 $= (0.524) * (1 - 0.524) * (-0.2 * 0.131) = -0.006$

Error gradient at neuron 4 $= (0.332) * (1 - 0.332) * (-0.3 * 0.131) = -0.009$

Updated weight:

weight46 $= (-0.3) + 0.9 * 0.131 * 0.332 = -0.261$

weight56 $= (-0.2) + 0.9 * 0.131 * 0.524 = -0.138$

weight14 $= (0.2) + 0.9 * (-0.009) * 1 = 0.192$

weight24 $= (0.4) + 0.9 * (-0.009) * 0 = 0.4$

weight34 $= (-0.5) + 0.9 * (-0.009) * 1 = -0.508$

weight15 $= (-0.3) + 0.9 * (-0.006) * 1 = -0.305$

weight25 $= (0.1) + 0.9 * (-0.006) * 0 = 0.1$

weight35 $= (-0.2) + 0.9 * (-0.006) * 1 = -0.205$