Mathematical Olympiads Discord Server

2020 February Intermediate Contest

Problem 1. How many triples (n, k, m) of integers with $0 < n, k \leq 20$ satisfy

$$\left(n+\frac{1}{1}\right)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{3}\right)\cdots\left(n+\frac{1}{k}\right)=m?$$

Problem 2. Brainy Smurf has two beakers, each of capacity 150 units. In the first beaker there are 100 units of a green liquid and in the second beaker there are 100 units of a red liquid. Brainy Smurf can perform a series of moves, where in each move he pours any amount of the mixture in one beaker into the other, without overflowing, and then stirs both beakers thoroughly to ensure a uniform mixture. Is it possible for Brainy Smurf to have 50 units of each liquid in each beaker after a finite number of moves?

Problem 3. Let $a_1, a_2, \ldots, a_n$ be positive real numbers. Is it necessarily true that

$$\frac{1}{a_1}+\frac{2}{a_1+a_2}+\cdots+\frac{n}{a_1+a_2+\cdots+a_n}<\frac{2}{a_1}+\frac{2}{a_2}+\cdots+\frac{2}{a_n}?$$

Problem 4. A square $ABCD$ has side length 1 and centre O. A point P, distinct from O, is chosen in the interior of $ABCD$. Let P_a lie on the ray AP such that $AP_a \cdot AP = 1$. Define P_b, P_c, and P_d similarly. Suppose that P_aPP_c and P_bPP_d are non-degenerate triangles. Prove that their circumcircles intersect on line OP at a point other than P.

Language: English

Time: 4 hours
Each problem is worth 7 points
