2020 April Beginner Contest

Saturday and Sunday, 11–12 April 2020

Problem 1. Find all triples (x, y, z) of non-negative integers satisfying $2^x = 3^y + 6^z$.

Problem 2. Let ABC be a triangle with $\angle BAC$ not a right angle. Let X be the point on ray BC such that $BA = BX$, and Y be the point on ray CB such that $CA = CY$. Let $P \neq A$ be the point on AX such that $CA = CP$, and $Q \neq A$ be the point on AY such that $BA = BQ$. Show that CP, BQ and the perpendicular bisector of BC are concurrent.

Problem 3. Find all sequences $a_1, a_2, \ldots$ of real numbers such that $a_{i+2020} = a_i$ and

$$a_i + 2a_{i+2} \geq a_{i+1}^2 + a_{i+1} + 1$$

for all positive integers i.

Problem 4. The participants at the Polyglot Mathematics Olympiad each speak some of 4 languages. It is found that any two participants can communicate in a common language with each other. Prove that there exists a language spoken by at least 60% of the participants.

Language: English

Time: 4 hours
Each problem is worth 7 points
