



Mathematical  
Olympiads  
Discord  
Server

## 2020 May Intermediate Contest

Saturday and Sunday, 9–10 May 2020

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**Problem 1.** Prove that for any positive integer  $n$  greater than  $1010^4$ , there exists at most one pair of positive integers whose product is  $n$  and whose difference is not greater than 2020.

**Problem 2.** Let  $S$  be a set of 2020 points in the plane with integer coordinates. What is the largest possible number of triangles with non-integer area that have vertices in  $S$ ?

**Problem 3.** Let  $ABC$  be an isosceles triangle with  $AB = AC$  and  $\angle BAC < 60^\circ$ . Let  $M$  be the midpoint of  $AB$  and  $\Gamma$  be the circumcircle of  $MBC$ . Let  $D$  be a point on  $\Gamma$ . Suppose that the circle centred at  $D$  passing through  $A$  intersects  $\Gamma$  at distinct points  $X$  and  $Y$ . Let  $P$  and  $Q$  lie on  $XY$  such that  $PB$  and  $QC$  are tangent to  $\Gamma$ , and denote by  $R$  the intersection of  $PB$  and  $QC$ .

Prove that regardless of the choice of  $D$ , the triangle  $PQR$  has constant perimeter.

**Problem 4.** Let  $\mathbb{R}$  denote the set of real numbers. Find all triples of functions  $p, q, r : \mathbb{R} \rightarrow \mathbb{R}$  such that for all real  $x$  and  $y$ ,

$$p(x^3 + y) + q(x + y^3) = r(xy).$$

*Language: English*

*Time: 4 hours  
Each problem is worth 7 points*