

Saturday, December 13, 2019

Problem 1. Let S be the set of polynomials of the form

$$a_{2019}x^{2019} + \cdots + a_1x + a_0$$

where $a_i \in \{-1, 0, 1\}$ for all $i \in \{0, 1, \dots, 2019\}$. Prove that at least $7/8$ ths of the polynomials in S have at least one real root.

Problem 2. Let \mathbb{N}^+ denote the set of positive integers. Find all $b \in \mathbb{N}^+$ such that for all $c \in \mathbb{N}^+$ there exists an $n \in \mathbb{N}^+$ satisfying

$$1 + b + b^2 + \cdots + b^c = \frac{n(n-1)}{2}.$$

Problem 3. Let M be a Discord server with 2019 people such that exactly one third of the pairs of people know each other. A *dynamic* of M labels a subset of the pairs of people who know each other as exactly one of the three following types of relationships: friends, pending, or blocked. (We consider S and the empty set to both be subsets of a set S .) The dynamic is called *tanyushi* if and only if each person has either an odd number of each type of relationship, or an even number of each type of relationship.

Let $f(M)$ be the number of distinct tanyushi dynamics M can have. Find all possible values of $f(M)$.

Language: English

*Time: 4 hours and 30 minutes
Each problem is worth 7 points*