



## 2020 June Advanced Contest

Saturday and Sunday, 13–14 June 2020

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**Problem 1.** A tuple of real numbers  $(a_1, a_2, \dots, a_m)$  is called *stable* if for each  $k \in \{1, 2, \dots, m-1\}$ ,

$$\left| \frac{a_1 + a_2 + \dots + a_k}{k} - a_{k+1} \right| < 1.$$

Does there exist a stable  $n$ -tuple  $(x_1, x_2, \dots, x_n)$  such that for any real number  $x$ , the  $(n+1)$ -tuple  $(x, x_1, x_2, \dots, x_n)$  is not stable?

**Problem 2.** Let  $p$  be a prime number. At a school of  $p^{2020}$  students it is required that each club consist of exactly  $p$  students. Is it possible for each pair of students to have exactly one club in common?

**Problem 3.** Let a *lattice tetrahedron* denote a tetrahedron whose vertices have integer coordinates. Given a lattice tetrahedron, a *move* consists of picking some vertex and moving it parallel to one of the three edges of the face opposite the vertex so that it lands on a different point with integer coordinates. Prove that any two lattice tetrahedra with the same volume can be transformed into each other by a series of moves.

**Problem 4.** Let  $c$  be a positive real number. Alice wishes to pick an integer  $n$  and a sequence  $a_1, a_2, \dots$  of distinct positive integers such that  $a_i \leq ci$  for all positive integers  $i$  and

$$n, \quad n + a_1, \quad n + a_1 - a_2, \quad n + a_1 - a_2 + a_3, \quad \dots$$

is a sequence of distinct nonnegative numbers. Find all  $c$  such that Alice can fulfil her wish.

Language: English

Time: 4 hours  
Each problem is worth 7 points