Mathematical
Olympiads
Discord
Server

2020 March Advanced Contest

Problem 1. In terms of a, b, and a prime p, find an expression which gives the number of $x \in \{0, 1, \ldots, p-1\}$ such that the remainder of ax upon division by p is less than the remainder of bx upon division by p.

Problem 2. An acute triangle ABC has circumcircle Γ and circumcentre O. The incentres of AOB and AOC are I_b and I_c respectively. Let M be the the point on Γ such that $MB = MC$ and M lies on the same side of BC as A. Prove that the points M, A, I_b, and I_c are concyclic.

Problem 3. A *simple polygon* is a polygon whose perimeter does not self-intersect. Suppose a simple polygon $\mathcal{P}$ can be tiled with a finite number of parallelograms. Prove that regardless of the tiling, the sum of the areas of all rectangles in the tiling is fixed.

Note: Points will be awarded depending on the generality of the polygons for which the result is proven.

Problem 4. Let $\mathbb{Z}^2$ denote the set of points in the Euclidean plane with integer coordinates. Find all functions $f : \mathbb{Z}^2 \to [0, 1]$ such that for any point P, the value assigned to P is the average of all the values assigned to points in $\mathbb{Z}^2$ whose Euclidean distance from P is exactly 2020.

Language: English

Time: 4 hours
Each problem is worth 7 points
