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# Parameter Estimation

Q1 Let $(x_1, x_2 \ldots)$ be a random sample of size $n$ taken from a normal pop with parameter $\theta_1$ & $\theta_2$

$\theta_1$ = Mean

$\theta_2$ = Variance. Find MLE

Sol) PDF : $f(x) = \frac{1}{\sqrt{\theta_2 2\pi}} e^{-\frac{1}{2}\left(\frac{x_1-\theta_1}{\sqrt{\theta_2}}\right)^2}$

$$L = \prod_{1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-1/2\left(\frac{x_i-\theta_1}{\theta_2}\right)^2}$$

Taking log B/S

$$\log(L) = -\frac{n}{2}\log(2\pi\theta_2) + \left(\frac{-1}{2\theta_2}\right)\sum_{1}^{n}(x_1-\theta_1)^2$$

Diff b/s w.r.t $\theta_1$

$$\frac{\partial L}{\partial \theta_1} = \frac{-1}{2\theta_2}\sum_{1}^{n} 2(x_1-\theta_1)(-1)$$

$$= \frac{1}{2\theta_2}\sum_{1}^{n} 2(x_i-\theta_1)$$

$$\frac{\partial L}{\partial \theta_1} = 0$$

$L = 0$ or $\frac{1}{2\theta_2}\sum_{1}^{n} 2(x_i-\theta_1) = 0$

$$\sum_{1}^{n} 2\theta_1 = \sum_{1}^{n} 2x_i$$

$$n\theta_1 = \sum_{1}^{n} x_i$$

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$$\boxed{\theta_1 = \frac{\sum^n x_i}{n}}$$

Diff w.r.t $\theta_2$

$$\frac{\partial L}{\partial \theta_2}\left(\frac{1}{L}\right) = -\frac{n}{2}\frac{2\pi}{2\pi\theta_2} + \sum_1^n (x_i - \theta_1)^2 \frac{1}{2\theta_2^2} = 0$$

$$\frac{-n}{2\theta_2} + \sum_1^n (x_1-\theta_1)^2 \frac{1}{2\theta_2^2} = 0$$

$$\sum_1^n (x_i - \theta_1)^2 = n\theta_2$$

$$\boxed{\theta_2 = \frac{\sum (x_i-\theta_1)^2}{n}}$$

Q2 Let $X_1, X_2 \ldots X_n$ be a random sample from $B(m,\theta)$ distribution where $\theta \in (0,1)$ is unknown and $m$ is a known positive integer compute value of $\theta$ using MLE

Sol PMF of $B(m,\theta)$

$$P(X=k) = {}^mC_k\, \theta^k (1-\theta)^{m-k}$$

$$L(\theta) = \prod_1^n {}^mC_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i}$$

Taking log b/s

$$\log L = \log\left(\prod_1^n {}^mC_{x_i}\, \theta^{x_i}(1-\theta)^{m-x_i}\right)$$

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Performing differentiation

$$\frac{1}{L}\frac{dL}{d\theta} = \frac{1}{\theta}\sum_{1} x_i - \frac{1}{1-\theta}\sum_{1}^{n}(m - x_i)$$

$$\frac{\partial L}{\partial \theta} = 0$$

$$\frac{1}{\theta}\sum x_i = \frac{1}{1-\theta}\sum_{1}^{n} m - x_i$$

$$(1-\theta)\sum x_i = \theta n m - \theta \sum x_i$$

$$\boxed{\theta = \frac{\sum x_i}{nm}}$$