EMS726U/P
Engineering Design Optimisation and Decision Making

CW2 - Paper Helicopter:

Multi-objective Experiment Optimum Engineering Design

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1. Problem description

In the helicopter design industry, autorotation is the rotation of helicopter rotors in response to downward motion when no power is available, it is critical to the survival of pilots and passengers when a helicopter is in an emergency engine-out condition. To maximise the effectiveness of autorotation, manufacturer requires a large investment of intellectual and material resources, which increases the costs of assembling. That is, managing the autorotation effectiveness is weighed against controlling the assembling costs. This coursework requires participants to formulate a multi-objective engineering design problem, using a paper helicopter as a simplified and abstractive model to get insights of the tradeoff between maximising autorotation effectiveness and minimising assembling costs. The autorotation effectiveness is measured by the time it takes a paper helicopter to reach the ground from an initial altitude (~2.5 meters), $i.e.$, the longer the time, the better the effectiveness. The assembling cost corresponds to the entire area of the paper helicopter, the larger the entire area, the higher the cost.

A paper helicopter is made from a sheet of A4 size paper ($210\text{mm} \times 297\text{mm}$) and consists of two rotors, one body, one tail, and one paper clip placed on the bottom of the tail. Paper clip plays a role as the stabiliser of the paper helicopter. **Figures** 1 and 2 show the 2D and 3D templates. The **length of rotors** and the tail area (both **tail height and width**) are three independent variables that need to be optimised.

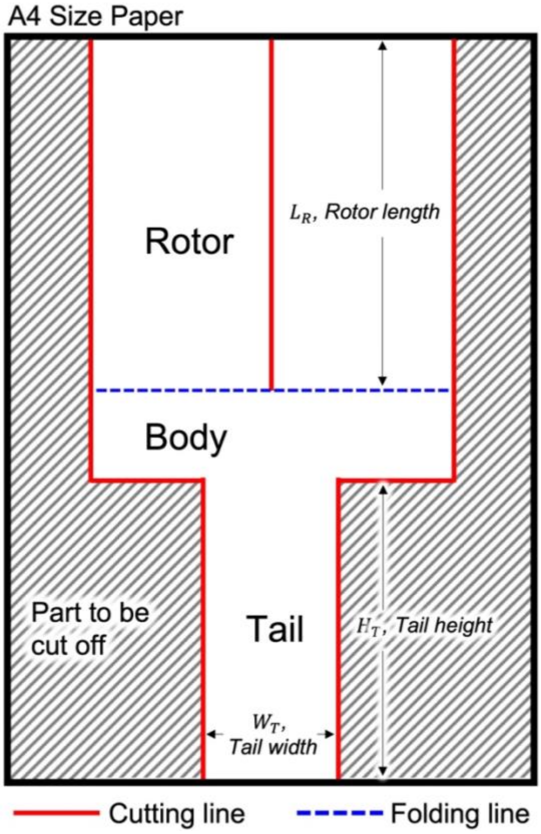

Figure 1 *Paper helicopter 2D template*

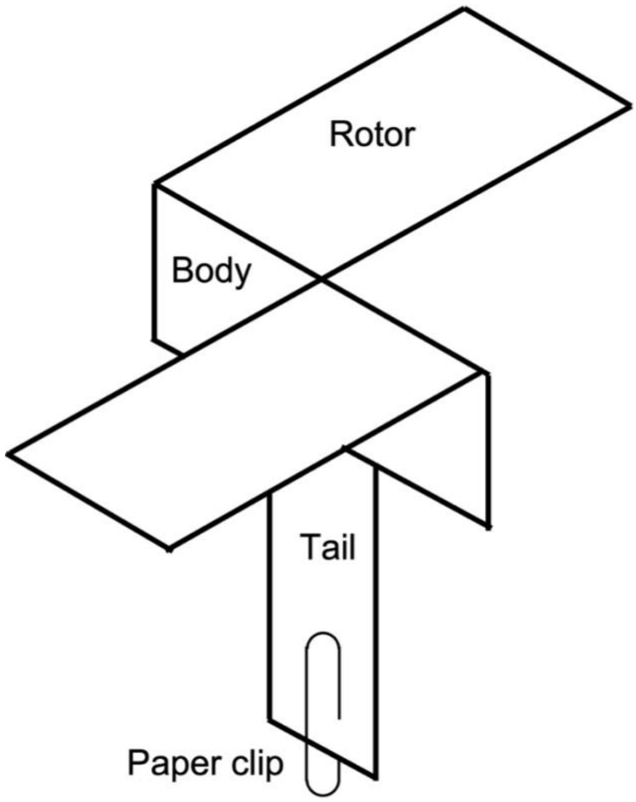

Figure 2 *Paper helicopter 3D template*

There are **7** tasks to be completed in order to deliver a group report and a group presentation (see Section 4):

1. Mathematical analysis on the relationship between the independent variables and optimisation objectives. Based on the equations given in **Section 2**, please derive two equations showing how the autorotation effectiveness and assembling cost are affected by the independent variables respectively.

2. Basic data collection. Please first check the necessary basic information listed in **Table 1**, then search and fill in the missing parameter values using information available online or from industrial standards.
3. Paper helicopter design. Use the derived equations obtained from **Task 1** and parameters from **Task 2** to model the multi-objective paper helicopter optimisation problem in Pymoo (covered in the IT class of Week 9). Choose one of the multi-objective optimisation algorithms from Pymoo to design the helicopter. Justify your choice.
4. Paper helicopter making. According to the configurations obtained from **Task 3**, use the weighted decision matrix method to select several configurations from the Pareto front. Select and justify appropriate weights to obtain configurations from the desired region of the Pareto front. For each configuration selected, please make 2 paper helicopters, where one for tests and the other for **Task 6**.
5. Performance testing. The test priority is to check if the designed configurations can make paper helicopters rotate effectively during the whole falling, rather than drifting down and swaying around. If some configurations fail to reach this goal, please analyse the reason, and then go back to **Tasks 3** and **4**.
6. Experiments. Use paper helicopters with configurations that have passed the tests in **Task 5**, record performance values on two objectives (when counting the rotating time, please carry out at least 5 runs for each configuration and calculate the average) and take one video of each configuration experiment.
7. Write a detailed report and prepare slides for the presentation based on the results of **Tasks 1~6**. Please refer to **Section 4 Assessment** for more information.

2. Analytical design

The downward rotation process of the paper helicopter is almost a uniform linear motion, where the upward drag force is balanced with the downward gravity.

$$G = D = \frac{1}{2}\rho_{air}V^2C_DS \tag{1}$$

where G and D denote the entire gravity and drag respectively, ρ_{air} is the air density, V is the downward velocity, C_D denotes the air drag coefficient, and S is the area spanned by the rotors and $S = \pi {L_R}^2$, in which L_R is the length of rotor.

Therefore, the steady-state velocity V is obtained via **Eq.2**.

$$V = \sqrt{\frac{2G}{\rho_{air}C_D\pi {L_R}^2}} \tag{2}$$

This steady-state velocity V can be used as a surrogate objective, since minimising it is equivalent to maximising the descent time, $i.e.$, maximising the autorotation effectiveness. When doing **Tasks 1**, **3** and **5** in **Section 1**, please use **minimising the steady-state velocity** and **minimising the assembling costs** as the objectives, in order to make the Pareto front a convex curve.

The entire gravity G is the sum of the gravities of the rotors, body, tail, and paper clip, as shown in **Eq.3**.

$$G = G_{rotors} + G_{body} + G_{tail} + G_{clip} \tag{3}$$

The gravities of the body and tail are calculated by multiplying their masses by gravitational acceleration, where each mass is determined by multiplying its area by the paper density. The gravity of rotors is assumed to increase as the cube of its radius, to reflect strength and stiffness requirements in a real helicopter.

$$G_{rotors} = G_{rotors\_0}\left(L_R/L_{R\_0}\right)^3 \tag{4}$$

where G_{rotors_0} is the initial gravity of rotors with the initial length L_{R_0}. The value of L_{R_0} is in **Table 1**.

Altogether, the gravity in **Eq.3** can be expressed in **Eq.5**, where W and H denote the values of width and height respectively, with B and T indicating the body and tail respectively.

$$G = G_{rotors\_0}\left(L_R/L_{R\_0}\right)^3 + W_B H_B \rho_{paper} g + W_T H_T \rho_{paper} g + G_{clip} \tag{5}$$

By **Eq.5**, **Eq.2** can be written in an equivalent form in **Eq.6**.

$$V^2 {L_R}^2 = f_1(L_R, W_T, H_T) = a_1 {L_R}^3 + a_2 W_T H_T + a_3 \tag{6}$$

Please derive the constants a_1, a_2 and a_3 in **Eq.6** to complete **Task 1** in Section 1. Note that, the final results of **Task 1** should be in the form of **Eqs.7** and **8** below.

$$V = f_2(\cdot) = \cdots \tag{7}$$

$$Cost = f_3(\cdot) = \cdots \tag{8}$$

Please note f_1, f_2 and f_3 in **Eqs.6-8** denote functions, instead of implying there three objective functions.

Table 1 *Experiment parameters*

Variable	Description	Value
W_R	The width of one rotor	6 cm
L_{R_0}	The initial length of the rotors	8 cm
L_R	The length of the rotors, to be optimised	8 cm $\leq L_R \leq$ 15 cm
W_B	The width of the body	12 cm
H_B	The height of the body	4 cm
W_T	The width of the tail, to be optimised	2 cm $\leq W_T \leq$ 5 cm
H_T	The height of the tail, to be optimised	5 cm $\leq H_T \leq$ 10.6 cm
ρ_{air}	Air density	To be searched and filled in
C_D	Air drag coefficient	
g	Gravitational acceleration	
ρ_{paper}	A4 paper density in experiments	
G_{clip}	The gravity of one paper clip in experiments	
G_{rotors_0}	The gravity of rotors with the initial length L_{R_0}	To be calculated

3. Safety notice

The safety of all participants is uppermost. The potential danger mainly involves:

- Sharp tools that may be used during hand-making process, like scissors and knife to cut papers.
- Risk of falling from height. **Tasks 5** and **6** in **Section 1** need to release paper helicopters from a height of 2.5 meters. These two tasks can be either completed on campus or somewhere safe agreed by all group members. If participants would like to do the two tasks on campus, the Whitehead Aeronautical Laboratory is available from 13:00 to 14:00 on Monday 9 Dec. and from 15:00 to 16:00 on Wednesday 11 Dec. (**Week 12**). If preferring somewhere else, please be aware of the potential risks as mentioned above during the whole test/experiment process.

Whitehead Aeronautical Laboratory address: ENG-110, Engineering Building (east part), Mile End Campus.

Note that, the time slot of Whitehead Aeronautical Laboratory is only for you to carry out experiments by yourselves. One demonstrator will be there to maintain the order and ensure safety, but **no coursework-related questions will be answered**. Should participants have any questions, please make sure that you attend the **Week 10** Problem-Solving Class. **Please get all necessary materials ready before visiting Whitehead Aeronautical Laboratory for tests and experiments to be efficient and productive.**

4. Assessment

The deliverables of this coursework (30% of the overall module assessment) are a group report and a group presentation, both of which are delivered in the **Week 12** and submitted through QM+. Note that the score for each member in a group might be different, depending on the contributions claimed.

The report (70% of CW2) should be concise and limited to 10 pages, including the title page, the table of contents, main body of the report, appendices, and references. **The presentation** (30% of CW2) will take 5 minutes with the aid of the prepared slides. Please check the detailed marking scheme in **Table 2** for what should be included in the report and presentation/slides.

Table 2 *Marking scheme for CW2*

	Main parts	**Guidance & marking**
Report	Problem modelling (**Task 1**) – 8%	• A correct equation showing the ready-state velocity V calculation. 5%. • A correct equation showing the entire costs $Cost$ calculation. 3%.
	Information collection (**Task 2**) – 4%	• Values of variables in **Table 1** and their reference resources. The calculated value of G_{rotors_0} and its calculation process. 4%.
	Engineering design (**Tasks 3** & **4**) – 25%	• Justifying the choice of the algorithm and at least one plot showing the whole Pareto front (with the selected solutions being marked on the plot) as the result of the chosen algorithm. 10%. • Tabulated values of independent variables corresponding to all solutions on the Pareto front (with the selected solutions being marked in the table). 3%. • Description of the employed multi-criteria decision-making method, justification of parameters. 6%. • Tabulated programming codes. 6%.
	Result analysis (**Tasks 5** & **6**) – 18%	• Experimental results tabulated. 5%. • Description of the potential limitations of the employed optimisation algorithm and reasons of the difference in results obtained from experiments and theoretical analysis (please find some relevant reference papers if necessary). What improvements can be made in theoretical analysis and experiments. 9%. • Review the whole coursework to check which variable(s) affect(s) the autorotation effectiveness of paper helicopters most. 4%.
	Report Writing – 15%	• Report layout. 3%. • Discussion in logical engineering style. 4%. • Format. 8%. (Equation 2 %. Table 2 %. Plot 2 %. Reference 2 %.)

Table 3 *Continued*

Presentation	Technical argument – 15%	• Description of the paper helicopter design: justifying your choice of the algorithm, theoretical performance of objectives. 4%. • Description of the potential limitations of the employed optimisation algorithm and reasons of the difference in performances between theoretical analysis and experimental results. 4%. • Analysis of variable(s) which affect(s) the autorotation effectiveness most. 4%. • Description of improvements can be made regarding the whole coursework implementation. 3%.
	Visual presentation – 8%	• Appropriate plots, *e.g.,* Pareto front. 3%. • Tabulated experimental results. 2%. • Videos showing the rotating status of different paper helicopters. 3%.
	Engagement and pitch to audience – 7%	• Details of individual contributions to the coursework. 3%. • Draws viewers into the argument in a compelling way from the start. 2%. • At an appropriate technical level throughout. 2%.

5. Timetable

Date	Missions	Time	Place
Week 9 Wednesday 20 Nov.	Coursework release	15:00 – 16:00	Queens: QB-212 PC Lab
Week 10 Wednesday 27 Nov.	Problem solving	15:00 – 16:00	Queens: QB-212 PC Lab
Week 12 Monday 9 Dec. **Week 12** Wednesday 11 Dec.	Paper helicopter tests (**optional**)	13:00 – 14:00 15:00 – 16:00	Aero Lab: ENG-110
Week 12, Friday 13 Dec.	Group presentation assessment	15:00 – 17:00	Peter Landin: PL 3.02
Week 12, Sunday 15 Dec.	Group report submission	Before 23:59	QM+