Name: Roll No: 23BCS3

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computer Science and Engineering

END SEMESTER EXAMINATION- EVEN 2024-'25

APRIL 2025

ICS221 Theory of Computation

Time: 9:30 AM - 12:30 PM **Max marks:** 100

Course Instructor: : Dr. Nandini J Warrier, Dr. P. Chakradhar, Dr. Krishnendhu SP. **Semester:IV**

Answer all Questions.

1. Convert the following grammar into Chomsky Normal Form (CNF):

$$S \to aA \mid a \mid B \mid C$$
$$A \to aB \mid \epsilon$$
$$B \to aA$$
$$C \to cCD$$
$$D \to abd$$

Where A, B, C are non-terminals and a, ϵ are terminals. [4]

2. Does $A^* = B^*$ imply that $A = B$? Find a counter example or provide a proof. [5]

3. Let $C = \{w \in \Sigma^* \mid n_a(w) \bmod 4 = 1\}$ where $\Sigma = \{a, b\}$ and $n_a(w)$ is the number a's in the string w. For example, $n_a(\text{babaabb}) = 3$. Determine whether this language is:

(A) Regular

(B) Context-free, but not regular

(C) Turing-decidable, but not context-free

Justify your answer. [5]

4. If L is a finite language with k elements, show that L^2 has atmost k^2 elements. For each positive integer k, is it possible to find a language over a one letter alphabet such that L^2 has k^2 elements? [6]

5. Create an epsilon NFA that accepts the language generated by $((0+01)^*11^*(0+11)^*)^*$. Reduce the NFA to minimal DFA. [7]

6. Show that $E_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \varnothing\}$ is decidable. [8]

7. Consider the set of all DFA's with k states over a binary language. Prove that the number of different languages recognized by DFA in this set is atmost $k^{(2k+1)} * 2^k$ [10]

8. Identify the language generated by the following CFG:

a. $S \to abB \quad B \to bbAa \quad A \to aaBb \mid \epsilon$

b. $S \to aA \quad A \to a \mid B \quad B \to bB \mid b$ [5+5]

9. Show that the class of Pushdown Automata with two stacks is equivalent to the class of Turing Machines. [10]

P.T.O.

10. Let $\Sigma = \{\#, 0, 1\}$. Provide an implementation-level description of an input-output Turing Machine (TM) that computes the function

$$f(\# < x >) = \begin{cases} \#\langle x/2 \rangle, & \text{if } x \text{ is even} \\ \#\langle 3x + 1 \rangle, & \text{otherwise} \end{cases}$$

Where $\langle x \rangle$ stands for the binary representation of the number x.

(For example: If the TM starts with #100 on the tape, it should halt with #10 on the tape. If it starts with #11, it should halt with #1010) You may use a TM with more than one tape in this case, the output should be written on the first tape. [15]

11. For the language $L = \{a^n b^m c^k \mid n = m \text{ or } m = k,\ n, m, k \geq 1\}$

a. Construct a Pushdown Automaton (PDA) for the language L.

b. Using the PDA constructed in (a), give the instantaneous descriptions for the strings where :

(i) $n = m$ (ii) $m = k$

(**Note :** An Instantaneous description of a TM is a single-step snapshot showing the current state, tap content, and head position of the machine.)

c. Write a Context-Free Grammar (CFG) for the language L [10 + 4 + 6]

* * * **Best wishes** * * *

Roll No:.....

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INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM
Department of Computer Science and Engineering

MAKEUP END SEM EXAMINATION- ~~JULY~~ JUNE, 2022

COURSE TITLE: ICS221 THEORY OF COMPUTATION

Time: 2 PM – 5 PM | Max. Marks: 100

Course Instructor: DR DIVYA SINDHU LEKHA | Batch: I & II

Answer all Questions

PART A

(6 x 4 = 24)

1. What is an automaton? Give any 3 abstract models of computers and the languages recognized by them.
2. Give a formal definition of the concatenation of 2 languages. Check if the following languages over $\Sigma = \{0,1\}$ are *finite* OR *infinite and countable* OR *infinite and uncountable*.

   (a) Σ^4 (b) Σ^* (c) 2^{Σ^*}
3. Give a formal definition of NFA. How an NFA is different from a DFA?
4. Write the context free grammar for the language of balanced paranthesis, by providing explicitly all its elements.
5. Show the transitions in a PDA recognizing the language $L = \{1^n 0^n \mid n \geq 0\}$.
6. Consider the following TM. Write the configurations of the TM if the input tape contains 00.

State	0	1	X	Y	B
q0	(q1, X, R)	-	-	(q3, Y, R)	-
q1	(q1, 0, R)	(q2, Y, L)	-	(q1, Y, R)	-
q2	(q2, 0, L)	-	(q0, X, R)	(q2, Y, L)	-
q3	-	-	-	(q3, Y, R)	(q4, B, R)
q4	-	-	-	-	-

P.T.O

PART B

(5 x 8 = 40)

1. Construct a DFA for a Language $L_3 = \{a, b\}^*\{abb\}$
2. Prove or disprove that the language $L = \{ 1^p | p \ is \ a \ prime \ number\}$ is regular.
3. Write CFG for $L = \{ww^R | \ w \in \Sigma^*\}, \Sigma = \{0,1\}$ [Hint: Even Palindrome]
4. Convert the following CFG to Chomsky Normal Form: $S \rightarrow abS \mid baS \mid \lambda$
5. Prove that the Halting problem is undecidable

Part C

(3 x 12=36)

1. Give the algorithm for constructing Finite Automata from regular grammar. Write regular grammar and construct FA from grammar of

   $$L = \{w \in \{a, b\}^* | \ w \ contains \ substring \ abb\}$$

2. Show that the language L is context free by specifying a context free grammar that generates it. Be precise in the specification of the grammar, by providing explicitly all its elements. $L = \{a^n b^m | n \neq 2m\}$
3. Give the formal definition of a Turing Machine. How is it different from a Finite Automaton? Explain Universal Turing Machine.

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Roll No:.....

Name:.....

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM
Department of Computer Science and Engineering

REPEAT END SEM EXAMINATION- ~~JULY~~ JUNE 2022

COURSE TITLE: ICS221 THEORY OF COMPUTATION

Time: 2 PM – 5 PM **Max. Marks: 100**

Course Instructor: DR DIVYA SINDHU LEKHA **Batch: I & II**

Answer all Questions

PART A

(6 x 4 = 24)

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2. Give a formal definition of the concatenation of 2 languages. Check if the following languages over $\Sigma = \{0,1\}$ are *finite* OR *infinite and countable* OR *infinite and uncountable*.

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q0	(q1, X, R)	-	-	(q3, Y, R)	-
q1	(q1, 0, R)	(q2, Y, L)	-	(q1, Y, R)	-
q2	(q2, 0, L)	-	(q0, X, R)	(q2, Y, L)	-
q3	-	-	-	(q3, Y, R)	(q4, B, R)
q4	-	-	-	-	-

P.T.O

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Roll No:.....

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INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computer Science & Engineering

END SEMESTER EXAMINATION- JUNE, 2023

COURSE TITLE: ICS221 Theory of Computation

Time: 9:30 AM to 12:30 PM (3 hrs) **Max. Marks: 100**

Course Instructor: Dr Divya, Dr Cinu **Batch: 2021 (CSE, CY)**

Answer all Questions

PART A

1. Give a binary encoding of the given TM:

$$M = \{\{q_1, q_2\}, \{0,1\}, \{0,1,B\}, \{\delta(q_1, 0) = (q_2, 0, R), \delta(q_1, 1) = (q_2, 1, R), \delta(q_1, B) = (q_2, B, L)\}, B, \{q_2\}\}$$

(5 marks)

2. Show that the language L is context free by specifying a context free grammar that generates it. Be precise in the specification of the grammar, by providing explicitly all its elements.

$$L = \{a^n b^m c^{2(n+m)} | n \geq 0, m \geq 0\}$$

(5 marks)

3. Construct an NFA that accepts all binary strings having occurrences of 0 multiples of two apart. (5 marks)
4. Explain Church-Turing thesis. Differentiate between Turing-decidable and Turing-recognizable languages. (5 marks)
5. Prove that the following language is not regular using pumping lemma.

$$L = \{a^{k^2} | k \geq 0\}$$

(5 marks)

Part-B

6. A state government has a website for knowing the Driving License details. The site has a specific page to know your driving license status. Driving Licence number can be entered in any of the following formats:

   DL-1420110012345 OR ***DL 1420110012345***

   Total number of input characters should be exactly 16 (including space - can be represented by using "$" symbol or '-'). The format is *SS-RRYYYYNNNNNNN* where

- *SS* - Two character State Code (like RJ for Rajasthan, TN for Tamil Nadu etc)
- *RR* - Two digit RTO Code
- *YYYY* - 4-digit Year of Issue
- Rest of the numbers are to be given in 7 digits

(A) Design a finite-state automata to accept the valid driving license numbers.

(10 marks)

(B) Write regular grammar for the language. Be precise in the specification of the grammar, by providing explicitly all its elements. (5 marks)

7. (A) Design a Turing Machine that transforms a string containing only **a**'s, **b**'s and **c**'s by replacing each letter following '**b**' with '**c**'. For example, the string '**acca**' would remain unchanged while '**abacba**' would change to '**abccbc**'. (10 marks)

(B) Give a description of the Universal Turing Machine. (5 marks)

8. Discuss any 3 variants of Turing Machine. (15 marks)

9. (A) Construct regular expression for the following language: (3 marks)

*the set of strings over the alphabet {**a, b, c**} that contain the substring '**aa**' starting at an odd position and the substring '**bb**' starting at an even position*

(B) Construct NFA accepting the language. (8 marks)

(C) Also, find the equivalent regular grammar G. Be precise in the specification of the grammar, by providing explicitly all its elements. (4 marks)

10. (A) Construct a Pushdown Automata that accepts the set of all strings of properly nested parentheses. (8 marks)

(B) Write the equivalent Context Free Grammar. Be precise in the specification of the grammar, by providing explicitly all its elements. (4 marks)

(C) Convert the CFG to Chomsky Normal Form (3 marks)

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Roll No:.....

Name:.....

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computer Science and Engineering

END SEM EXAMINATION- MAY. 2022

COURSE TITLE: ICS221 THEORY OF COMPUTATION

Time: 10:00 AM - 1:00 PM **Max. Marks: 100**

Course Instructor: DR DIVYA SINDHU LEKHA **Batch: I & II**

Answer all Questions

PART A

(6 x 4 = 24)

1. What are the 3 central concepts in the theory of computation? What is an automaton?
2. Are the regular expressions $E_1 = a(a+b)^* + c \; and \; E_2 = \; c + a(a+b)^*$ equivalent to each other? Explain why?
3. State pumping lemma for regular languages. Give its application.
4. Differentiate between Greibach Normal Form and Chomsky Normal Form for CFGs
5. Every decidable language is Turing recognizable. True or false? Justify.
6. Give a binary encoding of the given TM:

$$M = \{\{q_1, q_2\}, \{0,1\}, \{0,1,B\}, \{\delta(q_1, 0) = (q_2, 0, R), \delta(q_1, 1) = (q_2, 1, R), \delta(q_1, B) = (q_2, B, L)\}, B, \{q_2\}\}$$

PART B

(5 x 8 = 40)

1. Construct an NFA that accepts all binary strings that contain at least two 0's and at most one 1.
2. Prove or disprove that the language $L = \{0^{p^2}1^{p^2} | p \geq 0\}$ is regular.

P.T.O

3. Write CFG for $L = \{a^n b^m c^k | k = n + m\}, \Sigma = \{a, b, c\}$
4. Design a Turing Machine for computing 1's complement of a given binary number
5. Prove that the given problem is undecidable: PROBLEM - "Given a TM, does it accept a given input word?"

Part C

(3 x 12=36)

1. Give the algorithm for constructing Finite Automata from regular grammar. Write regular grammar and construct FA from grammar of

$$L = \{w \in \{a, b\}^* | if\ w\ contains\ substring\ aa, then\ |w| is\ odd\}$$

2. Show that the language L is context free by specifying a context free grammar that generates it. Be precise in the specification of the grammar, by providing explicitly all its elements. $L = \{a^n b^m c^{2(n+m)} | n \geq 0, m \geq 0\}$
3. Discuss 3 variants of Turing Machine.

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Register No:.....

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INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

END SEMESTER EXAMINATION- APRIL, 2019

COURSE TITLE: ICS221 THEORY OF COMPUTATION

Time: 09:30-12:30 pm **Max. Marks: 50**

Instructor: Dr Ebin Deni Raj

Note: Show the steps clearly for all questions.
The Question paper consists of **three** printed pages.

Answer all Questions

1. Is it possible to frame an algorithm for the following puzzle? Pairs A, B, C of puzzle pieces are shown; select some sequence of pairs of pieces, forming a top row and a bottom row of pieces that must fit together.

   a. Is this problem decidable? **[1]**

   b. Is it possible to solve the above problem using an algorithm, prove it? **[2]**

2. Assume that you are appointed as the chief algorithm designer of *Amazon*. The chief marketing officer (CMO) has noticed that there are lot of misinformation (information that wrongly describes the product and sometimes over rates or under rate a product) in the user reviewer section of majority of the products. The CMO wants you to design and implement an algorithm that can automatically detect and remove the wrong information. **[2]**

   a. Is it possible to find an algorithm for this problem? Explain

   b. Is the problem NP?

   c. Is this an NP-complete problem? Justify and elaborate

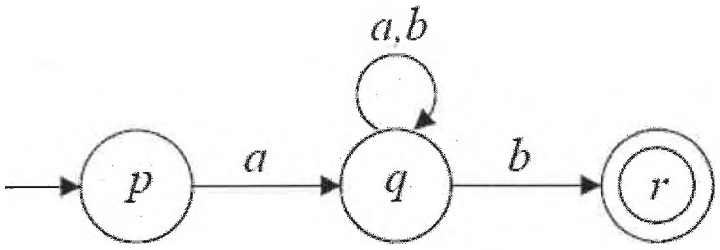
3. Consider the language L, $L = a^i b^j c^k d^l \mid i, j, k, l \geq 0, i + j \leq k$

   a. Write a CFG G with L(G)=L **[3]**

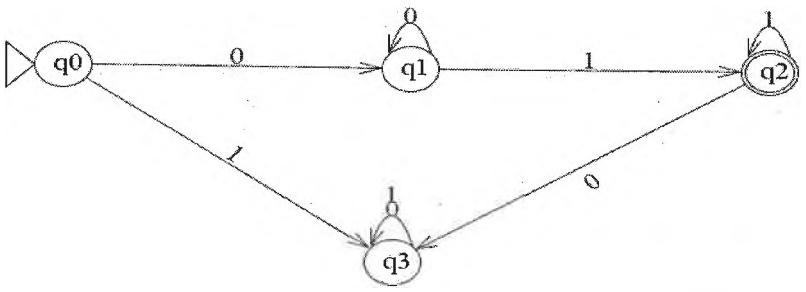
   b. Design a Push Down automata L(M)=L **[3]**

4. Use the subset-construction procedure to convert the following NFA (over {a, b}) to an equivalent DFA. Mark the states of your DFA by subsets of {p, q, r}. Indicate which states of your DFA are accessible. No credit will be given if you

design your DFA by any method other than the subset-construction procedure. **[3]**



5. Explain about Church-Turing thesis and how is it relevant even today? **[2]**
6. Differentiate Turing machine with stay option, Semi-infinite tape and Non deterministic Turing machine. **[2]**
7. Construct a Turing machine to implement simple Binary adder. **[2]**
8. Convert to Regular Expression using Arden's theorem **[2]**



9. Check the equivalence of the following Regular Expressions (L+M)* and (L*M*)*. [Hint: Construct the Minimized DFA of the regular expressions] **[4]**
10. The answers to these problems should be short and not complicated. **[3]**
    a. If an NFA M accepts the empty string (i.e., ε), does M's start state have to be an accepting state? Why or why not?
    b. Is every finite language regular? Why or why not?
    c. Let L1 = L(S) and L2 = L(T), where the non-terminal symbols S and T satisfy the productions:
       S → a | b | abSS.
       T → a | b | TTab.
       Find examples of strings *a* of *length* forty (40) such that: a ∈ L1 and a ∈ L2.

11. Write short description about: **[3]**
    a. Chomsky Hierarchy

b. BPP and RP

c. Recursively enumerable and Recursive

12. Using the pumping lemma, prove that the language $L_3 = \{a^i b^j \mid i, j > 0, \text{ and } |i - j| \text{ is a prime}\}$ is not regular. (Note that 1 is not treated as a prime.) **[2]**

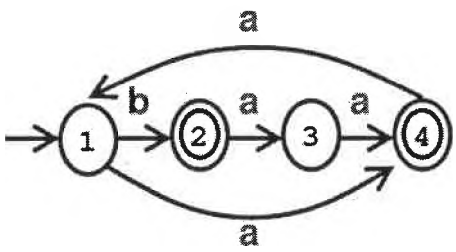
13. Design a DFA that accepts the language

$L(M) = \{w \in \{0,1\}^* : w \text{ does not contain } 3 \text{ consecutive } 1's\}$. Derive the equivalent Regular expression using state elimination method. **[2+2]**

14. Answer the following:

a. Construct an ε-NFA equivalent to the regular expression $((aa+bb+\varepsilon)(ab+ba)^*+a)^*+b$ **[3]**

b. Apply Table filling method and Partitioning method to minimize the following DFA **[5]**



15. With respect to context free language answer the following: **[4]**

a. Write a context-free grammar for the language L_2 where,

$L_2 = \{\alpha \in \{a, b, c\}^* \mid \#a(\alpha) + \#b(\alpha) = \#c(\alpha)\}$. Here, $\#d(\alpha)$ means the number of occurrences of the symbol d in the string α, where $d \in \{a, b, c\}$. Write only the productions in your grammar, and mention which is the start symbol.

b. Convert the grammar of Part (a) to Chomsky normal form. Show all the relevant steps briefly.

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Register No:.....

Name:.....

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

END SEMESTER REPEAT EXAMINATION –MAY, 2019

COURSE TITLE: ICS 221 Theory of Computation

Time: 9:30-12:30 pm **Max. Marks: 50**

Instructor: Dr Ebin Deni Raj

Note: This Question paper consists of three printed pages

Answer all Questions

1. Assume that the HR of a *Fortune 500* company has approached you to be their consultant. The company is trying to develop a solution for *halting problem*. As a consultant, answer the following questions:
   a. Is halting problem solvable? Give reasons [2]
   b. Describe in detail the Chomsky hierarchy of languages with a diagram. [2]

2. Imagine that you have designed a Turing machine for adding two numbers. Suppose you want to design the same Turing machine for multiplying two numbers, what are the steps to be followed?
   a. Is there any alternate solution other than "hardwiring" the Turing machine? [1]
   b. If the answer is 'yes' for part (a), then explain in detail about that solution. [2]

3. Explain about Church-Turing thesis? [1]

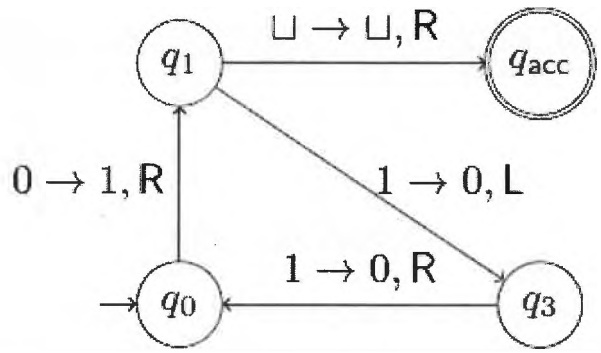
4. Find a 1-1 correspondence (one-to-one, onto function) between these two sets:
   A = {positive odd integers}
   B = {positive multiples of 3} [2]

5. What language does the following TM recognize? Describe. [4]

```
1. Sweep from left to right.
   IF there is any a after b, REJECT.
2. Move head to start.
   Search for a.
IF found
        Cross it. (Replace by X)
        Search for a.
        IF found
                Cross it.
        ELSE
                REJECT.
  ELSE
        Go to 5.
3. Search for b.
   IF found
        Cross it.
  ELSE
        REJECT.
4. Go to 2.
5. Move head to start.
   Search for b.
 IF found
        REJECT.
  ELSE
        ACCEPT.
```

P.T.O

6. Consider the following Turing machine M on input alphabet {0, 1}. All transitions not shown in the diagram below are assumed to go to the reject state q_{rej}.



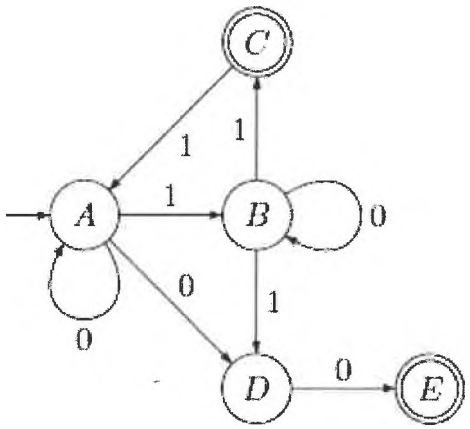
a. Give the formal definition of M as a tuple. [1]
b. Describe the computation of M on the input 0111 formally, as a sequence of instantaneous descriptions/configurations. [2]
c. Is there any input on which M does not halt? If so, give an example string. [1]

9. Give a brief description about: [5]
a. PSPACE and NPSPACE
b. L and NL
c. BPP and RP
d. ZPP and IP
e. Recursively Enumerable and Recursive

10. Design a Turing machine to compute $f(x)=3x$ [2]

11. Using the pumping lemma, prove that the language L3 = $\{a^i b^j \mid i, j > 0$, and $|i - j|$ is a prime$\}$ is not regular. (Note that 1 is not treated as a prime.) [2]

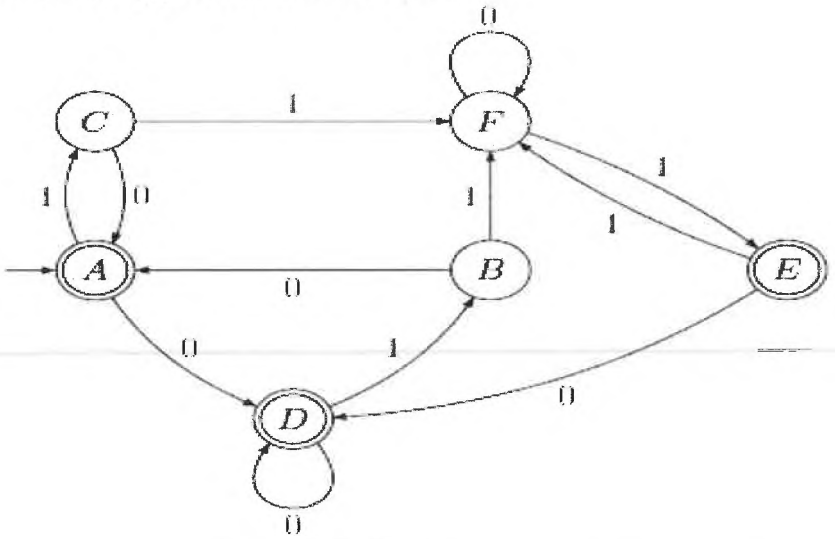
12. Using Subset construction method convert the following NFA to DFA. [4]



13. Convert the following grammar over the alphabet {a, b, c, d} to the Chomsky normal form. [3]

$$S \to aSd \mid T$$
$$T \to bTc \mid \epsilon$$

14. Minimize the DFA using partition based method [3]



15. Consider the language L, $L = a^i b^j c^k d^l \mid i,j,k,l \geq 0, i+j \leq k$

a. Write a CFG G with L(G)=L [3]

b. Design a Push Down automata L(M)=L [3]

16. One of the following two languages is context-free, and the other is not. Identify which one is what and justify (Draw PDA). [3]

a. $L_a = \{a^l b^m c^n \mid l,m, n \geq 0, l + m \geq n\}$.

b. $L_b = \{a^l b^m c^n \mid l,m, n > 0, l \geq n \text{ and } m \geq n\}$.

17. Company XYZ decided to place a coffee vending machine for their customers. How can you design a vending machine using Moore and Mealy machine? Assume that each coffee cost you Rs 5 /-. The machine is capable of accepting Re 1, Rs 2. and Rs 5 coins. (The machine is **not** capable of giving back change **nor** can it accept more than Rs 5 [i.e if three Rs 2 coins are inserted, the machine will not accept it]. Design a Moore and Mealy machine for the vending machine. [4]

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