Name:_______________ Roll No: 23BCS3

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computational Science and Humanities

END SEMESTER EXAMINATION- NOVEMBER 2024

IMA211 Probability, Statistics and Random Process

Date & Time: 22-11-2024, 09:30 AM - 12:30 PM **Max marks: 100**

Course Instructors: Dr. T. Bakkyaraj/Dr. Murugan D/Dr. Asha Sebastian **All Batches**

Answer all Questions

1. (i) An aircraft emergency locator transmitter (ELT) is a device designed to transmit a signal in the case of a crash. The manufacturing company I makes 80% of the ELTs, the company II makes 15% of them, and company III makes 5%. The ELTs made by company I have a 4% rate of defects, company II have a 6% rate of defects, and the company III have 9% rate of defects. If a randomly selected ELT is then tested and is found to be defective, find the probability that it was made by the company I. [5]

   (ii) If X is a random variable having probability mass function

$$f(x) = \begin{cases} kx, & for \;\; x=1,2,3 \\ 0 & \text{elsewhere.} \end{cases}$$

   a. Find k.
   b. Variance of X.
   c. Moment generating function of X.
   d. $E(4X^3 + 3X + 11)$.

   [5]

2. If X is the amount of money (in INR) that a salesperson spends on gasoline during a day and Y is the corresponding amount of money (in INR) for which he or she is reimbursed, the joint density of these two random variables is given by

$$f(x,y) = \begin{cases} k\left(\frac{20-x}{x}\right), & 10 \le x \le 20, \; \frac{x}{2} \le y \le x, \\ 0 & \text{elsewhere.} \end{cases}$$

   a. Find k.
   b. The marginal density function of X. [10]
   c. Conditional density of Y given $X = 12$.
   • d. The probability that the salesperson will be reimbursed atleast Rs.8 when spending Rs.12.

3. • (i) A and B shoot independently until each has hit his own target. The probabilities of hitting the target at each shot are 3/5 and 5/7 respectively. Find the probability that B will require more shots than A. [5]

   • (ii) A distribution with unknown mean μ has variance equal to 1.5. Find how large a sample should be taken from the distribution in order that probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean. [5]

4. • (i) If X,Y and Z are uncorrelated random variables with mean 0, and standard deviations 5, 12, 9 respectively, and if $U = X + Y, V = Y + Z$, find the covariance between U and V. [5]

   × (ii) A discrete random variable X takes the values $-1, 0, 1$ with probabilities $\frac{1}{8}, \frac{3}{4}, \frac{1}{8}$ respectively. Evaluate $p[|X - \mu| \ge 2\sigma]$ and compare it with the upper bound given by chebyshev's inequality. [5]

5. (i) On the basis of data reported in the article, it was concluded that the ferritin distribution in the elderly had a smaller variance than in the younger adults. For a sample of 28 elderly men, the sample standard deviation of serum ferritin (mg/L) was 52.6; for 26 young men, the sample standard deviation was 82.4 . Does this data support the conclusion as applied to men? Test the hypothesis at .01 level of significance ($F_{0.99,27,25} = 0.394$). [5]

(ii) The average adult male height in a certain country is 170 cm. We suspect that the men in a certain city in that country might have a different average height due to some environmental factors. We pick a random sample of size 9 from the adult males in the city and obtain the following values for their heights (in cm): $\bar{X} = 165.8, S^2 = 68.01$. Assume that the height distribution in this population is normally distributed. Based on the observed data, is there enough evidence to reject $H_0(\mu = 170)$ at significance level $\alpha = 0.05$? [5]

6. (i) If $x_1, x_2, \cdots x_n$ are the values of the random sample from an exponential population, find the maximum likelihood estimator of its parameter θ. [5]

(ii) Measurements of the diameters of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of 0.824 inch and a standard deviation of 0.042 inch. Find 98% confidence limits for the mean diameter of the ball bearings . [5]

7. (i) Find the characteristic function of the distribution with pdf $f_X(x) = \dfrac{1}{\pi(1+x^2)}$ for all $x \in \mathbb{R}$. You may use the integral formula $\int_0^\infty \dfrac{\cos(tx)}{b^2+x^2}dx = \dfrac{\pi e^{-tb}}{2b}, t \geq 0$ [5]

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform distribution $U[0, \theta]$. What is the maximum likelihood estimator of upper bound θ. [5]

8. (i) Let $\{N(t), t \in [0, \infty)\}$be a Poisson Process with rate λ. Find the probability that there are two arrivals in $(0, 2]$ or three arrivals in $(4, 7]$. [5]

(ii) Let $\{X(n),\ n \in \mathbb{Z}\}$ be a WSS discrete-time random process with $\mu_X(n) = 1$ and $R_X(m, n) = e^{\{-(m-n)^2\}}$. Define the random process $Z(n)$ as $Z(n) = X(n) + X(n-1)$, for all $n \in \mathbb{Z}$.

(a) Find the mean function of $Z(n)$.

(b) Find the autocorrelation function of $Z(n)$.

(c) Is $Z(n)$ a WSS random process? Justify your answer. [5]

9. (i) In a fair coin experiment, we define the process $\{X(t)\}$ as follows:

$$X(t) = \begin{cases} \sin \pi t, & \text{if head shows,} \\ 2t & \text{it tail shows.} \end{cases}$$

Find $E\{X(t)\}$ and $F_{X(t)}(x, t)$ for t=0.25. [5]

(ii) If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and autocovariance $C(t_1, t_2) = 16 \exp^{-|t_1 - t_2|}$. Find the probability that (i) $X(10) \leq 8$ and (ii) $|X(10) - X(6)| \leq 4$. [5]

10. Consider a Markov Chain with three states $\{1, 2, 3\}$ with the state transition matrix $P = \begin{bmatrix} \frac{1}{4} & 0 & \frac{3}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$. Suppose $P(X_1 = 1) = \frac{1}{2}$ and $P(X_1 = 2) = \frac{1}{4}$.

(a) Draw the state transition diagram.

(b) Find $P(X_1 = 3, X_2 = 2, X_3 = 1)$

(c) Find $P(X_1 = 3, X_3 = 1)$ [10]

****Best Wishes****

t table

df \ α	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	318.3
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	22.33
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	10.21
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	7.173
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	5.893
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	5.208
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.785
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	4.501
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	4.297
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	4.144
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	4.025
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.930
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.852
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.787

Standard normal table: $P(0 < z < z_1)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Name: Sam Ruben Abraham Roll No: 2023BCD0002

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Indian Institute of Information Technology Kottayam

Department of Computational Science and Humanities

FIRST MID SEMESTER EXAMINATION- SEPTEMBER, 2024

IMA211: Probability, Statistics and Random Processes

Date & Time: 24-09-2023, 02:30 PM - 04:00 PM **Max marks:** 50
Course Instructors: Dr. Bakkyaraj T./Dr. Murugan D/Dr. Asha Sebastian **Batch:** 2023

Answer all Questions

1. A student buys 1000 chips from supplier A, 2000 chips from supplier B, and 3000 chips from supplier C. He tested the chips and found that the probability that a chip is defective depends on the supplier from where it was bought. Specifically, given that a chip came from supplier A, the probability that it is defective is 0.05; given that a chip came from supplier B, the probability that it is defective is 0.10; and given that a chip came from supplier C, the probability that it is defective is 0.10. If the chips from the three suppliers are mixed together and one of them is selected at random,

   (i) What is the probability that it is defective?

   (ii) If a randomly selected chip is defective, what is the probability that it came from supplier A? **(8 Marks)**

2. (a) Let a random variable X have range $[0, 3]$ and density function $f_X(x) = kx^2$. Let $Y = X^3$.

   (i) Find k.

   (ii) Compute $E(Y), Var(Y)$. **(4 Marks)**

   (b) Suppose that the random variable X has cumulative distribution function

$$F_X(x) = \begin{cases} 0, & x \leq 0; \\ 1 - e^{-x^2}, & x > 0. \end{cases}$$

   What is the probability that X exceeds 1? **(4 Marks)**

3. (a) State the axioms of the probability. **(2 Marks)**

   (b) The IQ of a randomly selected individual is often supposed to follow a normal distribution with a mean of 100 and a Standard deviation of 15. Find the probability that an individual has an IQ **(8 Marks)**

   (i) above 140.

   (ii) between 120 and 130.

   (iii) find a value x such that 99% of the population has IQ at least x.

4. (a) If X represents the outcomes, where fair die is tossed, find the Moment generating function of X and hence find the mean and variance of X. **(4 Marks)**

   (b) Name a discrete distribution that has the memoryless property. Justify your answer with a computation. **(4 Marks)**

5. (a) Let X denote the distance that an animal moves from its birth site to the first territorial vacancy it encounters. Suppose that for banner tailed kangaroo rats, X has an exponential distribution with parameter $\lambda = 0.01386$. (5 Marks)

   (i) What is the probability that the distance is at most 100m? Between 100 and 200m?

   (ii) What is the probability that the distance exceeds the mean distance by more than two standard deviations?

   (b) A life insurance company insures the lives of 5000 men of age 42 years. The probability of a 42 year old man dying in a given year is 0.001. What is the probability that the company will have to pay 4 claims in a given year? (3 Marks)

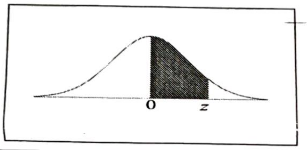
6. Assume that each of your calls to a popular radio station has a probability of 0.03 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent. (8 Marks)

   (i) What is the probability that your first call that connects is your tenth call?

   (ii) What is the probability that it requires more than five calls for you to connect?

   (iii) What is the mean number of calls needed to connect?

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
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0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
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1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
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1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Gilles Cazelais. Typeset with LaTeX on April 20, 2006

*** **Best wishes** ***

Abhinav Bhagwat Roll No: 2022BCS0019

Name:

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computational Science and Humanities

Indian Institute of Information Technology Kottayam

FIRST MID TERM EXAMINATION- SEPTEMBER 2023

IMA211 Probability, Statistics and Random Process

Max marks: 50
Batch: 1, 2

Date & Time: 01-09-2023, 09:30 AM - 11:00 AM
Course Instructors: Dr. Murugan D/Dr. Asha Sebastian

Answer all Questions

1. There are two labs A and B. A contains n desktops and 2 laptops, and B contains 2 desktops and n laptops. One of the two labs are selected at random and two items are drawn from it without replacement. If both the items drawn are desktops and the probability that Lab A was chosen is 6/7. Find the value of n. [8]

2. A random variable X has probability density function given by

$$f(x) = \begin{cases} ke^{-2x}, & if\ x \geqslant 0 \\ 0, & otherwise \end{cases}$$

Find

1. Find k.
2. Find moment generating function and use it to find it's mean and variance.

[8]

3. Let X and Y be two random variables having the joint probability mass function $f(x, y) = k(x+2y)$ where x and y can assume only the integer values $0, 1, 2$. Find the marginal distribution of X and Y, and all conditional distributions of $f(y/x)$ and $f(x/y)$. [8]

4. If X is the proportion of persons who will respond to one kind of mail order solicitation, Y is the proportion of persons who will respond to another kind of mail order solicitation, and the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} k(x + 4y), & if\ 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$$

(a) Find k.
(b) Find the probability that atleast 30 percent will respond to the first kind of mail order solicitation.
(c) Find the probability that atmost 50 percent will respond to the second kind of mail order solicitation given that there has been a 20 percent response to the first kind of mail order solicitation.

[10]

(P.T.O.)

5. If X and Y are independent random variables with probability density functions $f(x) = e^{-x}, \quad x \geqslant 0$ and $f(y) = e^{-y}, \quad y \geqslant 0$ respectively. Check whether

$$U = \frac{X}{X+Y} \quad \text{and} \quad V = X + Y \quad \text{are independent.}$$

[8]

6. If the trivariate cumulative distribution function of X_1, X_2, X_3 is given by

$$F(x_1, x_2, x_3) = \begin{cases} -\frac{1}{2}(x_1^2 x_2 + x_1 x_2^2)e^{-x_3}, & if\ 0 < x_1 < 1, 0 < x_2 < 1, x_3 > 0 \\ 0, & otherwise \end{cases}$$

Find $P((X_1, X_2, X_3) \in A)$ where A is the region

$$\{(x_1, x_2, x_3) : 0 < x_1 < 1/2, 1/2 < x_2 < 1, x_3 < 1\}.$$

[8]

$* * *$ **Best wishes** $* * *$

Name: Abhinav Bhagwat Roll No: 2022BCS0019

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Indian Institute of Information Technology Kottayam

Department of Computational Science and Humanities

SECOND MID TERM EXAMINATION- OCTOBER 2023

IMA211 Probability, Statistics and Random Process

Date & Time: 06-10-2023, 09:30 AM - 11:00 AM **Max marks: 50**

Course Instructors: Dr. Murugan D/Dr. Asha Sebastian **Batch: 1, 2**

Answer all Questions (Calculators are permitted)

1. Suppose that 30% of all students who have to buy a text for a particular course want a new copy whereas the other 70% want a used copy. Consider randomly selecting 25 purchasers. [8]

   a. What are the mean value and standard deviation of the number who want a new copy of the book?

   b. What is the probability that the number who want new copies is atmost 3?

   c. Suppose that new copies cost Rs100 and used copies cost Rs70. Assume the bookstore currently has 50 new copies and 50 used copies. What is the expected value of total revenue from the sale of the next 25 copies purchased?

2. Suppose that only 0.10% of all computers of a certain type experience CPU failure during the warranty period. Consider a sample of 10,000 computers. [4]

   a. What is the probability that more than 10 sampled computers have the defect?

   b. What is the probability that no sampled computers have the defect?

3. Let X be a continuous random variable defined on $[a, b]$. It's probability density function is inversly proportional to length of the interval. Find mean and variance of X. [4]

4. Suppose the diameter at breast height (inches) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8. [10]

   a. What is the probability that the diameter of a randomly selected tree will be at least 10 inches?

   b. What is the probability that the diameter of a randomly selected tree will exceed 20 inches?

   c. What is the probability that the diameter of a randomly selected tree will be between 5 and 10 inches?

   d. What value c is such that the interval (8.8-c, 8.8+c) includes 98% of all diameter values?

5. Suppose the time spent by a randomly selected student who uses a terminal connected to a local time-sharing computer facility has a gamma distribution with mean 20 min and variance 80 min^2. [4]

   a. What is the probability that a student uses the terminal for at most 24 min?

   b. What is the probability that a student spends between 20 and 40 min using the terminal?

6. Let $X_1, X_2 \cdots X_n$ be independent and identically distributed Poisson variates with parameter λ. Use central limit theorem to estimate $P(120 \leqslant S_n \leqslant 160)$, where [4]

$$S_n = X_1 + X_2 + \cdots + X_n;\ \lambda = 2 \text{ and } n =$$

(P.T.O.) $U=\lambda$

7. Data collected at Cochin International Airport suggests that an exponential distribution with mean value 2.725 hours is a good model for rainfall duration. [8]

   a. What is the probability that the duration of a particular rainfall event at this location is at most 3 hours? Between 2 and 3 hours?

   b. What is the probability that the duration of a particular rainfall event at this location is at least 2 hours given that it has exceeded 90 minutes?

   c. What is the probability that rainfall duration exceeds the mean value by more than 2 standard deviations? What is the probability that it is less than the mean value by more than one standard deviation?

8. (i) A random sample of size 5 is drawn from a normal population of unknown mean. Consider the following estimators of mean: [8]

$$T_1 = X_1 + X_2 - X_3,\; T_2 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5},\; T_3 = \frac{2X_1 + X_2 + kX_3}{5}$$

   a. Are T_1 and T_2 unbiased?

   b. Find the value of k such that T_3 is unbiased.

   c. Which is the best estimator?

   (ii) Let $\overline{X}$ denote the mean of a random sample of size n from a distribution with mean μ and variance 10. Find n so large that the level of confidence is approximately 95.4% that the random interval $(\overline{X} - 0.5, \overline{X} + 0.5)$ includes μ.

$\mu \in \bar{X} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$1.88 \cdot \frac{\sqrt{10}}{\sqrt{n}} = \frac{1}{2}$ $\quad 1-\alpha = 0.954$

$z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 0.5$

$n = 3.76\sqrt{10}$ $\quad \alpha = 1 - 0.954$

$= 0.046$

$\frac{\alpha}{2} = 0.023$

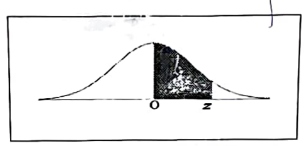
$n = 12$

$0.5 - 0.023$

0.477

1.88

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Gilles Cazelais. Typeset with LaTeX on April 20, 2006

* * * Best wishes * * *

Roll No:......2021BCY0016...

Name:......Parisi Priyatham..

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computational Science and Humanities

Indian Institute of Information Technology Kottayam

END SEMESTER EXAMINATION- February, 2023

IMA 211 (Probability, Statistics, and Random Processes)

Time: 10:00 AM – 1:00 PM | Max. Marks:100

Date: 15/2/2023

All questions are compulsory.

Part A: Each question carries 2 marks. $10 \times 2 = 20$

1: Which of the following can be the state transition matrix of a Markov chain?

(a) $P = \begin{bmatrix} 0.65 & 0.35 \\ 0.20 & 0.80 \end{bmatrix}$ (b) $P = \begin{bmatrix} 0.65 & 0.38 \\ 0.21 & 0.83 \end{bmatrix}$

(c) Both (a) and (b) (d) Neither (a) nor (b).

2: If $\widehat{\Theta}$ is the estimator of the parameter θ, then $\widehat{\Theta}$ is called unbiased if

(a) $E(\widehat{\Theta}) = \theta$ (b) $E(\widehat{\Theta}) \neq \theta$

(c) $E(\widehat{\Theta}) > \theta$ (d) None of the above.

3: If the population standard deviation σ is known and the sample size n is greater than 30, then the confidence interval for the population mean μ is

(a) $\overline{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ (b) $\overline{X} \pm t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

(c) $\overline{X} \pm Z_{\frac{\alpha}{2}} \times \sigma$ (d) None of the above.

4: If p value of the test statistic is less than 0.05 then

(a) H_0 is rejected with 95% confidence level (b) Both (a) and (c) are true

(c) H_0 is rejected at 5% level of significance (d) None of the above.

5: Consider a hypothesis $H_0: \mu_0 = 5$ against $H_1: \mu_0 > 5$. The test is

(a) Left tailed test (b) Right tailed test

(c) Cannot be determined (d) None of the above.

6: A test is conducted for H_0: μ = 50 vs. H_1: μ > 50. The value of test statistic (Z or Z_{stat}) is 2.36. The p-value is

(a) $P(Z > 2.36)$ (b) $P(Z < 2.36)$

(c) $P(Z > 1.18)$ (d) None of the above.

7: The random variables X and Y both have variance 0.5. Let $Z = 4X - 2Y$. The variance of Z is

(a) 3 (b) 7

(c) 10 (d) None of the above.

$4^2 0.5 - 2^2 0.5$

6 8-2

PTO

8: If a random variable X has the density function

$$f_X(x) = \begin{cases} \frac{1}{8}, & -4 < x < 4 \\ 0, & otherwise. \end{cases}$$

$P(2X + 3 > 5)$ is:

(a) 0

(b) $\frac{1}{8}$

(c) $\frac{3}{8}$

or D

?

(d) None of the above.

9: An analyst is conducting a hypothesis test to determine if the mean time spent on investment research is different from 3 hours per day. The test is performed at the 1% level of significance and uses a random sample of 64 portfolio managers, where the mean time spent on research is found to be 2.7 hours. The population standard deviation is 1.5 hours. What is the value of the test statistic in this case?

(a) Z (or Z_{stat}) = −2.67

(b) t (or t_{stat}) = −2.67

(c) Z (or Z_{stat}) = −1.60

(d) None of the above.

$\alpha = 0.01$ $\quad -0.3 \;\; 01.$

$\frac{8 \times 2}{3}$ $\quad \frac{2.7-3}{\frac{1.5}{8}}$

10: If X be a random variable and $F_X(x)$ is its CDF. Then $\lim_{x \to -\infty} F_X(x)$ is

(a) 0

(b) 1

(c) ∞

(d) None of the above.

Part B: Each question carries 5 marks. **$6 \times 5 = 30$**

1: A population distribution is known to have standard deviation 20. Determine the p-value of a test of the hypothesis that the population mean is equal to 50, if the average of a sample of 64 observations is (a) 52.5; (b) 55.0; (c) 57.5.

Hint: $\emptyset(1) = 0.8413,\ \emptyset(2) = 0.9772,\ \emptyset(3) = 0.9987.$

2: It is claimed that a vacuum cleaner expends 46 kWh per year. A random sample of 12 homes indicates that vacuum cleaners expend an average of 42 kWh per year with (sample) standard deviation 11.9 kWh. At a 0.05 level of significance, does this suggest that, on average, vacuum cleaner expend less than 46 kWh per year?

Hint: $t_{0.05}$ at 11 df$= 1.796$.

3: Professor Symons either walks to school, or he rides his bicycle. If he walks to school one day, then the next day, he will walk or cycle with equal probability. But if he bicycles one day, then the probability that he will walk the next day is 1/4. Model this process through Markov chain.

(a) Draw its State transition diagram.

(b) Calculate its transition matrix.

(c) If it is assumed that the first day is Monday, write a matrix that gives probabilities of a transition from Monday to Wednesday.

4: If $X \sim binomial(n, p), where\ n = 10, p = 0.4$. Find the approximation of $P(4 \leq X \leq 8)$ by using the

(a) Normal distribution,

(b) Poisson distribution.

5: Let X be a discrete uniform random variable which consider all the integers with range $[-2\pi, 2\pi]$. Let $Y = f(X) = X + 5$. Then,

(a) Find $P_X(x)$ and $P_Y(y)$. $\frac{1}{13}$

(b) Find $E[X]$ and $E[Y]$. 0, 5

6: If you have tossed a coin thrice. Let X be the number of observed tails. Find the CDF $F_X(x)$ of X.

Part C: Each question carries 10 marks. $10 \times 5 = 50$

1: A manufacturer Glucosamine capsules claims that each capsule contains on the average 500 mg of glucosamine with standard deviation of 8.5. To test this claim 40 capsules were selected and amount of glucosamine measured in each capsule with average mean 496.3 mg. Test the claim under (a) 95% confidence interval (b) 80% confidence interval.

Hint: $Z_{\frac{\alpha}{2}} = 1.96$ for $1 - \alpha = 0.95$, NO, NO

$Z_{\frac{\alpha}{2}} = 1.28$ for $1 - \alpha = 0.80$.

2: The following are scores on IQ tests of a random sample of 18 students at a large university.

130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142.

Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university. Assume the population to be normally distributed.

Hint: $t_{0.025}$ at 17 df $= 2.11$. (131.11, 135.33)

3: Suppose that X is a discrete random variable with the following probability mass function: where $0 \leq \theta \leq 1$ is a parameter.

X	0	1	2	3
$P(X)$	$\frac{2\theta}{3}$	$\frac{\theta}{3}$	$\frac{2(1-\theta)}{3}$	$\frac{1-\theta}{3}$

The 10 independent observations $(x_1 = 3, x_2 = 0, x_3 = 2, x_4 = 1, x_5 = 3, x_6 = 2, x_7 = 1, x_8 = 0, x_9 = 2, x_{10} = 1)$ were taken from such a distribution. What is the maximum likelihood estimate of θ?

0.75

$0.4\theta + 0.3\theta + 0.6 - 0.3\theta \quad 0.2 - 0.1\theta$

$0.2\theta + 0.8 = 1$

$\theta = 1$

$(0.8 - 0.1\theta) = 3$

$0.8 - 3 = 0.1\theta$

$\theta = \frac{0.8 - 3}{0.1} = -22$

PTO

I
2

4: Let X and Y be two jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6}, & 0 \le x \le 1, 0 \le y \le 2 \\ 0, & otherwise. \end{cases}$$

Find,

(a) $P\left(X < \frac{1}{2} \middle| Y = y\right)$, (b) $E[X|Y = 1]$, (c) $var\left(X \middle| Y = \frac{1}{2}\right)$.

0.243 0.1186

5: If X is a continuous random variable with PDF

$$f_X(x) = \begin{cases} k(x^2 + x), & 0 < x < 1 \\ 0, & otherwise. \end{cases}$$

(a) Find the value k that makes $f_X(x)$ a probability density function (PDF), 1.2
(b) Find the cumulative distribution function (CDF),
(c) Find $P\left(X < \frac{1}{2}\right)$. 0.2

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Name:__________ Roll No:__________

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computational Science and Humanities

MAKE UP/REPEAT EXAMINATION-JANUARY 2024

IMA211 Probability, Statistics and Random Process

Date & Time: 05-01-2024, 05:00 PM - 08:00 PM **Max marks: 100**
Course Instructors: Dr. Murugan D/Dr. Asha Sebastian/ Dr. Riyasudheen T. K. **All Batches**

Answer all Questions

1. Let us consider the function
$$P_X(n) = \frac{c}{n^2}, \text{ for the values of } X = n = 1, 2, 3, ...,$$
Can $P_X(n)$ be a probability mass function for some constant c? Justify your claim, and comment on the variance $Var(X)$. [10]

2. For a certain binary communication channel, the probability that a transmitted zero is received as a zero is 0.95, and the probability that a transmitted one is received as one is 0.9. If the probability that a zero is transmitted is 0.4, find the probability that
(i) one is received,
(ii) one was transmitted given that a one was received. [10]

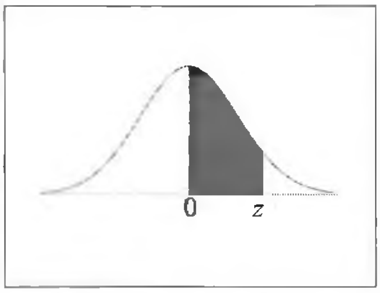
3. A component has an exponential time to failure distribution with mean of $10,000$ hours
(i) The Component has already been in operation for its mean life. What is the probability that it will fail by 15000 hours?
(ii) If the component has been operational for 15000 hours, What is the probability that is will operate for another 5000 hrs?

4. A certain machine makes electrical resistors having mean resistance of 40 Ohms and standard deviation of 2 Ohms. Assuming that the resistance follows a normal distribution and can be measured to an degree of accuracy
(i) What percentage of resistors have resistances that exceed 43 Ohms.
(ii) What percentage of resistors will have resistances below 43 Ohms. [5+5=10]

5. Consider the Joint p.d.f at x and y given by
$$f(x,y) = \begin{cases} \frac{x^3y^3}{16}; & 0 \leq x \leq 2, 0 \leq y \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$
(i) Find the marginal densities of x and y,
(ii) Find the cumulative distribution functions at x and y. [10]

6. Consider a Markov system that can be in one of two possible states, $S = \{0, 1\}$. In particular, suppose that the transition matrix is given by
$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$
Suppose that the system is in state 0 at time $n = 0$, i.e., $X_0 = 0$.
i. Draw the state transition diagram,
ii. Find the probability that the system is in state 1 at time $n = 3$.

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
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2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998