Optimal Control

Alternate Project

Spring 2022

Date: 5 April 2022

Materials Allowed During Examination

You are permitted to use any and all resources you want for this exam, but you are not permitted to communicate with anyone except myself during the course of completing this exam. All solutions must be typed; no handwritten solutions are permitted.

Guidelines for Solutions

As I have stressed in class, communication is an extremely important part of demonstrating that you understand the material. To this end, the following guidelines are in effect for all problems on the examination:

- Your solutions must be clearly explained. Any unclear explanations will be marked as incorrect.
- You must be crystal clear with every step of your solution. In other words, any step in a derivation or statement you write must be unambiguous (that is, every step must have one and only one meaning). If it is ambiguous as to what you mean in a step, then I will assume the step is incorrect.

In short, please write your solutions in a orderly fashion so that somebody else can make sense of what you are doing and saying.

Point Distribution

The exam consists of two questions and the value of each question clearly indicated. Unless otherwise stated, full credit will be given for a proper application of a relevant concept (for example, proper description of kinematics and kinetics, understanding how to compute a transfer function). Contrariwise, no credit will be given for a concept applied incorrectly, ***even if the final answer is correct***.

University of Florida Honor Code

On your exam you must state and sign the University of Florida honor pledge as follows:

I pledge on my honor that I did not violate University of Florida honor code during any portion of this exam.

Name: **UF-ID:**

Signature: **Date:**

Thought Before Starting Exam

You say : "’Ere thrice the sun done salutation to the dawn"
And you claim these words as your own
But I’ve read well, and I’ve heard them said
A hundred times (maybe less, maybe more)
If you must write prose/poems
The words you use should be your own
Don’t plagiarise or take "on loan"
’Cause there’s always someone, somewhere
With a big nose, who knows
And who trips you up and laughs
When you fall
Who’ll trip you up and laugh
When you fall

1 Description

This document serves as an alternate project for course EML6934. The problem described in this document can be used in place of the original course project.

2 Objective

Consider the orbit transfer problem from the midterm exam. In this project the goal will be to develop a solution to a modification of the original problem from the midterm using Legendre-Gauss-Radau collocation. The instructions follow in the next section.

3 Problem

In the orbit transfer problem given on the midterm, the only control was the thrust angle β. The formulation of this problem is modified so that both the thrust magnitude, T, and the thrust angle, β are controls. Using this formulation, the differential equations remain the same as those obtained for the problem on the midterm, the only difference being that now the formulation contains *two* controls. While the thrust angle is free, the thrust magnitude is constrained as follows:

$$0 \leq T \leq T_{\max}$$

where $T_{\max}$ is the value of the thrust that was used for the midterm exam (which, for completeness, is $T_{\max} = 0.1405$). Using this modification to the formulation of the problem, consider the following two objective functionals:

- minimize t_f
- maximize $m(t_f)$ (which is equivalent to minimize $-m(t_f)$).

Unlike the formulation given on the midterm, in this case time minimization and mass maximization are *not* the same problem because the thrust magnitude is not constant (which was the case on the midterm).

4 Numerical Solution of Optimal Control Problem

Solve both aforementioned formulations of the orbit transfer optimal control problem using multiple-interval Legendre-Gauss-Radau collocation. It is permitted to use the code I gave you as a starting point, but any modifications you make to this code must be performed independently (that is, without anyone else's assistance). Solve the problem for the following discretizations of Legendre-Gauss-Radau collocation:

(a) a polynomial degree of three in each interval with the number of intervals set to $K = (2, 4, 8, 16, 32)$

(b) a polynomial degree of four in each interval with the number of intervals set to $K = (2, 4, 8, 16, 32)$

You may observe that the problem is quite difficult to solve for the cases above (that is, IPOPT may not converge that easily). In addition, you may observe that the thrust angle is not well behaved. In order to make the problem more tractable, consider the following substitutions:

$$\begin{aligned} u_1 &= \sin\beta, \\ u_2 &= \cos\beta \end{aligned}$$

When making the substitution above, it is necessary to add the following equality path constraint:

$$u_1^2 + u_2^2 = 1$$

The reason that the path constraint is necessary is because of the trigonometric identity $\sin^2\beta + \cos^2\beta = 1$. Using this alternate parameterization of the control, re-solve the cases given in the itemized list above.

Compare the quality of the solutions obtained when the angle β is used as a control as compared to when (u_1, u_2) is used in conjunction with the additional path constraint. Analyze the results you obtain and discuss why using the second parameterization of the control may be more helpful than using the original parameterization of the control.