ELE 503

Advanced Computer Programming and Statistics

Week#4: Numerical Iteration Procedures and Techniques

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Week 04.

Numerical Iteration Procedures and Techniques

Introduction to Numerical Iteration Methods

Numerical Iteration Methods

Newton-Raphson Method

Gauss-Seidel Method

Applying Iterative Methods to Nonlinear Equations

Convergence Criteria and Error Analysis

C# Programming for Iterative Methods

Visualizing Convergence of Iterative Method

Hands-On Exercises

Q&A

Closing Take away

Efosa's Introduction

Engineer | Programmer | Innovator

Technical Authority

Shell Nigeria

Subject Mater Expert (EMEA) for Process Automation & Control (PACO)-Subsea control systems and Subsea Distribution

Containerization 85 %

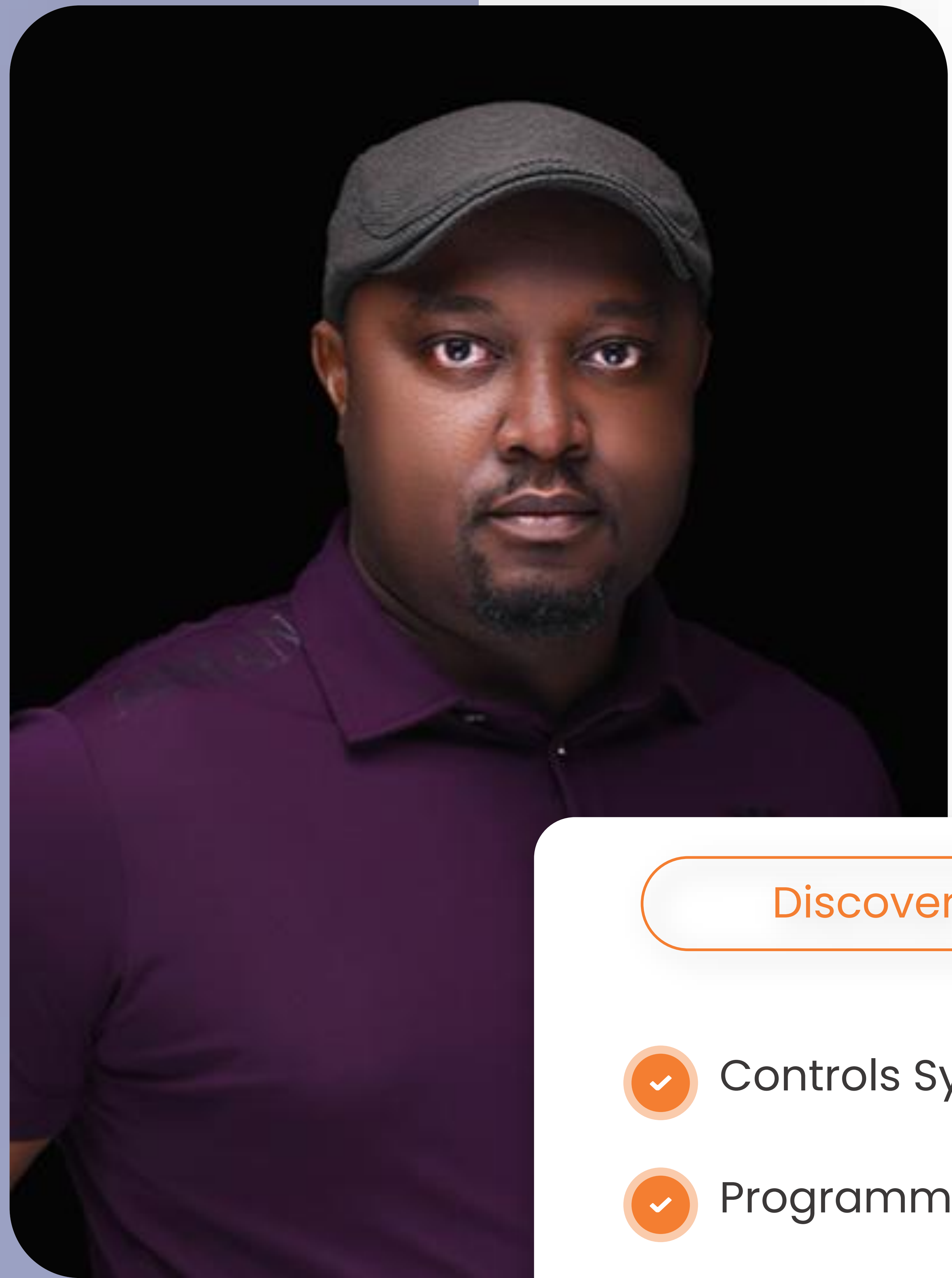
AI & ML 80 %

Renewable Energy 90 %

Innovator, VC

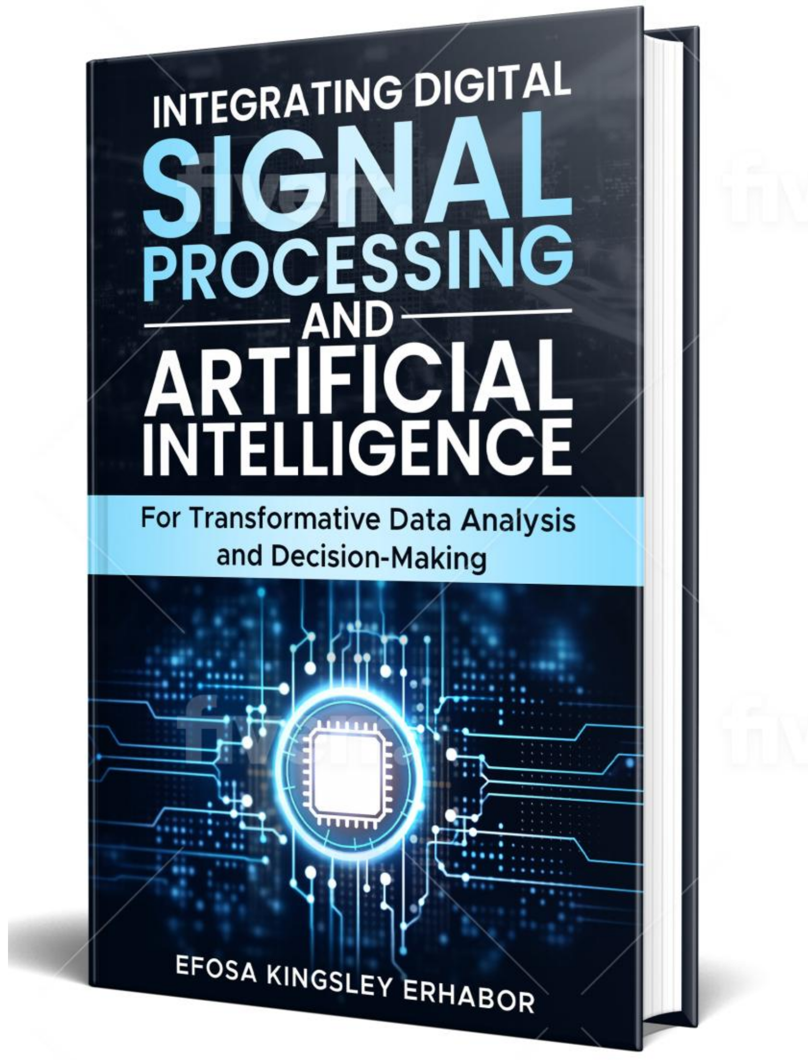
Katharos Technologies

Linux, Devops, AI and Software SME, Innovator and enterprenur



Discover

- Controls System
- Programmer
- Subsea Engineer
- AI and ML



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Part 1:

Introduction to Numerical Iteration Methods

Learning Objectives

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•**Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

•**Apply** iterative methods to solve nonlinear equations and simulate engineering systems.

•**Comprehend** convergence criteria and perform error analysis in numerical computations.

Importance of Numerical Iteration Methods in Engineering

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- **Solving Complex Equations:** Essential for equations without analytical solutions.

- **Simulation of Systems:** Models dynamic and nonlinear engineering systems.

- **Optimization:** Enhances design and operational efficiency.

- **Reliability:** Ensures accurate and reliable engineering solutions.

Part 2:

Numerical Iteration Methods

Introduction to Numerical Iteration Methods

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- **Definition:** Iterative methods are techniques used to obtain successive approximations to the solutions of mathematical problems.

- **Applications:** Root finding, linear and nonlinear system solving, optimization problems.

- **Advantages:** Applicable to a wide range of problems, especially where analytical solutions are infeasible.

- **Challenges:** Convergence issues, computational cost, sensitivity to initial guesses.

Newton-Raphson Method - Overview

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- **Purpose:** Find successively better approximations to the roots (or zeroes) of a real-valued function.

- **Application:** Solving nonlinear equations, optimization problems.

- **Key Features:** Quadratic convergence near the root, requires derivative computation

Newton-Raphson Method – Theory and Derivation

•**Starting Point:** Given a function f(x) and its derivative f′(x)

•**Iteration Formula:**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

•**Graphical Interpretation:** Tangent line intersects the x-axis closer to the root.

•**Convergence:** Rapid convergence when starting close to the actual root.

Newton-Raphson Method - Step-by-Step Algorithm

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- **Initial Guess (xo):** Choose an initial approximation close to the expected root.

- **Compute Function and Derivative:** Evaluate f(xn) and f′(xn)

- **Update Estimate:**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- **Check for Convergence:** If $|x_{n+1} - x_n|$ is below a predefined tolerance, stop.

- **Repeat:** Set n=n+1 and return to step 2.

Newton-Raphson Method - Convergence Criteria

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- **Tolerance Level (ϵ - epsilon):** A small threshold to determine when to stop iterations.

- **Maximum Iterations:** Prevents infinite loops if convergence is not achieved.

- **Absolute vs. Relative Error:** Decide based on the problem's sensitivity.

- **Derivative Considerations:** Avoid points where $f'(X_n)=0$ to prevent division by zero.

Gauss-Seidel Method – Overview

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- **System of Equations:**

$$A\mathbf{x} = \mathbf{b}$$

- **Iteration Formula for xi:**

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$$

- **Convergence Requirement:** Matrix AAA must be diagonally dominant or symmetric positive definite.

Gauss-Seidel Method - Step-by-Step Algorithm

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•**Initial Guess (x(0))**Start with an initial vector of guesses.

•**Update Variables Sequentially:** For each equation, solve for one variable using the latest available values.

•**Check for Convergence:** If the difference between successive iterations is below a predefined tolerance, stop.

•**Repeat:** Continue updating until convergence criteria are met.

Gauss-Seidel Method - Convergence Criteria

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- **Tolerance Level (ϵ - epsilonϵ):** Determines when the solution is sufficiently accurate.

- **Maximum Iterations:** Limits the number of iterations to prevent infinite loops.

- **Matrix Conditions:** Ensure that the coefficient matrix is suitable (e.g., diagonally dominant).

Part 3:

Applying Iterative Methods to Nonlinear Equations

Applying Iterative Methods to Nonlinear Equations

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- **Nonlinear Equations:** Equations where the unknown appears with an exponent other than one or inside a non-linear function.

- **Iteration Approach:** Use methods like Newton-Raphson to linearize and solve step-by-step.

- **Engineering Applications:** Thermodynamics, fluid dynamics, structural analysis

Simulating Engineering Systems with Iterative Methods

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- **Dynamic Systems:** Use iterative methods to simulate time-dependent behavior.

- **Nonlinear Simulations:** Handle complexities in system responses.

- **Examples:**
  - Finite Element Analysis (FEA)
  - Computational Fluid Dynamics (CFD)
  - Electrical circuit simulations

Convergence Criteria and Error Analysis

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- **Convergence:** The process by which successive iterations approach a stable solution.

- **Error Metrics:**
  - **Absolute Error:** $|x_{n+1} - x_n|$
  - **Relative Error:** $\frac{|x_{n+1} - x_n|}{|x_{n+1}|}$
  - **Residual Error:** | f(xn) |for root-finding

- **Error Analysis:** Assess the accuracy and reliability of the iterative solution.

Example Slide 1 – Newton–Raphson Method in C#

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- **Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

- **Apply** iterative methods to solve nonlinear equations and simulate engineering systems.

- **Comprehend** convergence criteria and perform error analysis in numerical computations.

Example Slide 2 - Gauss–Seidel Method in C#

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•**Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

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Example Slide 3 – Manual Iteration Computation (Newton-Raphson)

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Problem: Find the root of $f(x) = x^2 - 4$ using Newton-Raphson

Given:

- Initial guess $x_0 = 3$
- $f(x) = x^2 - 4$
- $f'(x) = 2x$

Iterations:

1. Iteration 1:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{3^2 - 4}{2 \times 3} = 3 - \frac{5}{6} = 2.1667$$

2. Iteration 2:

$$x_2 = 2.1667 - \frac{(2.1667)^2 - 4}{2 \times 2.1667} \approx 2.1667 - \frac{0.6944}{4.3334} \approx 2.1667 - 0.1602 = 2.0065$$

3. Iteration 3:

$$x_3 = 2.0065 - \frac{(2.0065)^2 - 4}{2 \times 2.0065} \approx 2.0065 - \frac{0.0261}{4.013} \approx 2.0065 - 0.0065 = 2.0000$$

Result: $x \approx 2.0000$

- **Initial Guess (xo):** Choose an initial approximation close to the expected root.
- **Compute Function and Derivative:** Evaluate f(xn) and f′(xn)
- **Update Estimate**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- **Check for Convergence:** If $|x_{n+1} - x_n|$ is below a predefined tolerance, stop.
- **Repeat:** Set n=n+1 and return to step 2.

Slide 19: Example Slide 4 – Manual Iteration Computation (Gauss-Seidel)

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Problem: Solve the system using Gauss-Seidel

System:

$$\begin{cases} 3x + y = 9 \\ x + 2y = 8 \end{cases}$$

Initial Guess: $x^{(0)} = 0, y^{(0)} = 0$

Iterations:

1. Iteration 1:

$$x^{(1)} = \frac{9 - y^{(0)}}{3} = \frac{9-0}{3} = 3$$

$$y^{(1)} = \frac{8 - x^{(1)}}{2} = \frac{8-3}{2} = 2.5$$

2. Iteration 2:

$$x^{(2)} = \frac{9 - y^{(1)}}{3} = \frac{9-2.5}{3} = 2.1667$$

$$y^{(2)} = \frac{8 - x^{(2)}}{2} = \frac{8-2.1667}{2} = 2.9167$$

3. Iteration 3:

$$x^{(3)} = \frac{9 - y^{(2)}}{3} = \frac{9-2.9167}{3} = 2.3611$$

$$y^{(3)} = \frac{8 - x^{(3)}}{2} = \frac{8-2.3611}{2} = 2.8194$$

4. Continue Iterations Until Convergence

Result: $x \approx 2.0, y \approx 3.0$

•**Initial Guess (x(0))**Start with an initial vector of guesses.

•**Update Variables Sequentially:** For each equation, solve for one variable using the latest available values.

•**Check for Convergence:** If the difference between successive iterations is below a predefined tolerance, stop.

•**Repeat:** Continue updating until convergence criteria are met.

Slide 20: Example Slide 5 – Visualizing Convergence (Newton-Raphson)

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Visualization: Convergence of Newton-Raphson for $f(x) = x^3 - x - 2$

Steps:

1. **Function and Derivative:**

$$f(x) = x^3 - x - 2$$

$$f'(x) = 3x^2 - 1$$

2. **Initial Guess:** $x_0 = 1.5$
3. **Iterations:**
   - Compute $x_1, x_2, \ldots$ using Newton-Raphson formula.
4. **Plot:** Graph $f(x)$ with iteration points converging to the root.

Visualization:

Note: Replace the placeholder image with an actual plot showing the convergence.

- **Root** the graph f(x) with iteration points converging to the root of the function when its plotted.

Part 4:
C# Programming for Iterative Methods

C# Programming for Iterative Methods

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- **Purpose:** Implement and automate iterative methods for solving engineering problems.

- **Key Aspects:**
  - Algorithm implementation
  - Handling convergence and divergence
  - Error analysis and reporting

- **Tools and Libraries:** Utilize built-in C# functions and libraries for mathematical operations and data handling

Implementing Newton-Raphson in C#

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- **Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

Implementing Newton-Raphson in C#

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- **Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

Visualizing Convergence of Iterative Methods

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Plotting Convergence: Newton-Raphson Method

Steps:

1. **Define the Function and Its Derivative:**

$$f(x) = x^3 - x - 2$$

$$f'(x) = 3x^2 - 1$$

2. **Implement Newton-Raphson in C# with Iteration Logging.**

3. **Collect Iterative Values:**

   - Store x_n at each iteration.

4. **Plot x_n vs. Iteration Number:**

   - Visualize how x_n approaches the root.

Visualization Example:

Note: Replace the placeholder image with an actual plot generated from the C# implementation.

Hands-On Exercise 1 – Manual Iteration Computation

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Exercise: Apply Newton-Raphson to $f(x) = x^2 - 3$

Given:

- Initial Guess $x_0 = 2$
- Function $f(x) = x^2 - 3$
- Derivative $f'(x) = 2x$

Tasks:

1. **Iteration 1:**

$$x_1 = 2 - \frac{2^2 - 3}{2 \times 2} = 2 - \frac{1}{4} = 1.75$$

2. **Iteration 2:**

$$x_2 = 1.75 - \frac{(1.75)^2 - 3}{2 \times 1.75} \approx 1.75 - \frac{0.0625}{3.5} \approx 1.75 - 0.0179 = 1.7321$$

3. **Iteration 3:**

$$x_3 = 1.7321 - \frac{(1.7321)^2 - 3}{2 \times 1.7321} \approx 1.7321 - \frac{0.0003}{3.4642} \approx 1.7321 - 0.0001 = 1.7320$$

Result: $x \approx 1.7320$

- **Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

Hands-On Exercise 2 – Coding Iterative Methods

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Exercise: Implement Gauss-Seidel for 4x + y + z = 7, x + 3y + z = -1, 2x + y + 5z = 10

Tasks:

1. Set Up the System:

$$\begin{cases} 4x + y + z = 7 \\ x + 3y + z = -1 \\ 2x + y + 5z = 10 \end{cases}$$

2. Implement Gauss-Seidel in C# using the provided template.
3. Choose Initial Guess (e.g., $x = 0, y = 0, z = 0$).
4. Run the Program and Observe Convergence.
5. Analyze the Solution and Convergence Behavior.

- **Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

Hands-On Exercise 3 – Visualizing Iterative Convergence

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•**Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

•**Apply** iterative methods to solve nonlinear equations and simulate engineering systems.

•**Comprehend** convergence criteria and perform error analysis in numerical computations.

Hands-On Exercise 3 – Visualizing Iterative Convergence

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Exercise: Plotting Convergence of Gauss-Seidel Method

Tasks:

1. **Modify the Gauss-Seidel Implementation:**
   - Store values of x, y, and z at each iteration.
2. **Use a Visualization Library (e.g., OxyPlot) to Plot:**
   - Iteration Number vs. Variable Values.
3. **Analyze the Plot:**
   - Observe how variables converge to their final values.
   - Identify the rate of convergence.

Example Visualization:

Note: Replace the placeholder image with an actual plot generated from the C# implementation.

- **Understand** standard numerical iteration methods such as Newton-Raphson and Gauss-Seidel.

- **Apply** iterative methods to solve nonlinear equations and simulate engineering systems.

Case Study 1 – Solving Nonlinear Equations in Structural Analysis

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- **Problem:** Determine the load-deflection relationship in a nonlinear beam.

- **Approach:**
  - Model the nonlinear equation governing deflection.
  - Apply Newton-Raphson to solve for deflection under varying loads.

- **Outcome:** Accurate prediction of beam behavior, informing design adjustments to enhance structural integrity.

Case Study 2 – Simulating Electrical Circuits with Gauss-Seidel

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- **Problem:** Analyze voltage distribution in a complex electrical network.
- **Approach:**
  - Formulate the system of equations based on Kirchhoff's laws.
  - Utilize Gauss-Seidel to solve for node voltages iteratively.
- **Outcome:** Efficient circuit analysis, facilitating the design of reliable and optimized electrical systems.

Summary of Key Concepts

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- **Iterative Methods:** Essential for solving complex and nonlinear engineering problems.

- **Newton-Raphson:** Efficient for root finding with rapid convergence near the root.

- **Gauss-Seidel:** Effective for large linear systems, especially with diagonally dominant matrices.

- **Convergence and Error Analysis:** Critical for ensuring the accuracy and reliability of solutions.

- **C# Programming:** Facilitates the implementation and automation of iterative methods.

- **Visualization:** Enhances understanding of convergence behavior and solution stability

Importance of Iterative Methods in Engineering

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- **Flexibility:** Applicable to a wide range of engineering disciplines and problems.
- **Scalability:** Efficiently handles large and complex systems.
- **Integration:** Combines with other numerical and computational techniques for comprehensive analysis.
- **Innovation:** Drives advancements in simulation, optimization, and system design through robust numerical solutions

Q&A Session

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- **Open Floor:** Address any questions or clarifications.

- **Discussion Points:**
  - Challenges faced during manual computations and coding.
  - Insights from case studies.
  - Best practices for ensuring convergence in iterative methods.
  - Further exploration of advanced iterative techniques.

Homework Assignment

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1. **Programming Task:**
   - **Objective:** Implement Newton-Raphson and Gauss-Seidel methods.
   - **Tasks:**
     - **Newton-Raphson:**
       - Solve $f(x) = x^3 - 6x^2 + 11x - 6$ with an initial guess of $x_0 = 3$.
       - Track and report each iteration's x_n value until convergence.
     - **Gauss-Seidel:**
       - Solve the system:

$$\begin{cases} 5x + 2y - z = 12 \\ -3x + 9y + 2z = -25 \\ 2x - y + 7z = 3 \end{cases}$$

       - Use an initial guess of $x = 0$, $y = 0$, $z = 0$.
       - Report the solution after convergence.

2. **Data Analysis Project:**
   1. **Objective:** Apply iterative methods to a real-world engineering problem.
   2. **Tasks:**
      1. Select an engineering-related nonlinear equation or system of equations.
      2. Implement the appropriate iterative method in C#.
      3. Solve the equation/system and analyze the convergence behavior.
      4. Present your findings with visualizations illustrating the convergence process.

3. **Error Analysis Report:**
   1. **Objective:** Understand and evaluate the accuracy of iterative solutions.
   2. **Tasks:**
      1. Choose an iterative method implemented during exercises.
      2. Calculate the absolute and relative errors at each iteration.
      3. Discuss the convergence rate and factors affecting accuracy.
      4. Suggest improvements or alternative methods to enhance convergence.

Closing Remarks

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- **Mastering Iterative Methods:** Crucial for solving complex engineering problems with precision.

- **Programming Proficiency:** Enhances capability to implement and customize numerical algorithms.

- **Continuous Learning:** Explore advanced iterative techniques and their applications in various engineering fields.

- **Real-World Impact:** Apply iterative methods to drive innovations, optimize systems, and ensure reliability in engineering solutions.

- **Support:** Utilize spare hours, peer discussions, and online resources for further assistance and knowledge enhancement.