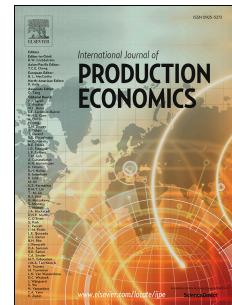


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# Two Echelon Vehicle Routing Problem with Drones in Last Mile Delivery

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## Abstract

In recent years, drone routing and scheduling has become a highly active area of research. This research introduces a new routing model that considers a synchronized truck-drone operation by allowing multiple drones to fly from a truck, serve one or multiple customers, and return to the same truck for a battery swap and package retrieval. The model addresses two levels (echelons) of delivery: primary truck routing from the main depot to serve assigned customers and secondary drone routing from the truck, which behaves like a moveable intermediate depot to serve other sets of customers. The model takes into account both trucks' and drones' capacities with the objective of finding optimal routes of both trucks and drones that minimizes the total arrival time of both trucks and drones at the depot after completing the deliveries. The problem can be solved by formulated mixed integer programming (MIP) for the small-size problem, and two efficient heuristic algorithms are designed to solve the large-size problems: Drone Truck Route Construction (DTRC) and Large Neighborhood Search (LNS). Numeric results from the experiment compare the performance of both heuristics against the MIP method in

small/medium-size instances from the literature. A sensitivity analysis is conducted to show the delivery time improvement of the proposed model over the previous truck-drone routing models.

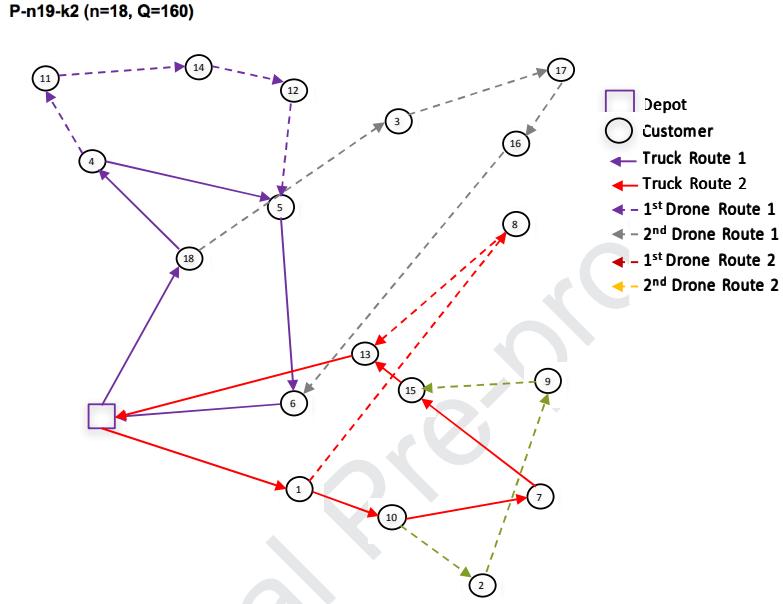
## **1. Introduction**

Over the past few years, the use of unmanned aerial vehicles (UAVs) or drone(s) has grown rapidly across different applications including real time monitoring, providing wireless coverage, remote sensing, searching and rescuing, inspecting, and delivering goods (Otto et al., 2018). As the e-commerce market has grown, many technology companies as well as traditional logistics providers have been experimenting with drone delivery, with the goal of cutting costs and providing cheaper, yet faster and more efficient service. In 2013, Amazon, the world's largest e-commerce company, announced the idea of using drones to fulfill orders by offering drone delivery services which could deliver customers' orders within 30 minutes through its Prime Air delivery program (Meola, 2017). Following the announcement, many technology companies, such as Google, Alibaba, and JD as well as traditional logistics providers such as UPS, DHL and FedEx have been experimenting with and adopting drone delivery services. Flirtey, a drone delivery company, and DHL successfully completed the first fully autonomous delivery with a package that included bottled water, emergency food, and other urgently-needed goods to people located in an area where most traditional delivery options are not available (Hern, 2014). Drones have also been successfully used to deliver food like pizza by Flirtey and Chipotle by Alphabet X, Google (Volkman, 2018). In Asia, JD.com, a giant e-commerce company in China, has been aggressively developing its drone delivery service with at least seven different types of delivery drones in testing or operation across four provinces in China (Beijing, Sichuan, Shaanxi, and Jiangsu) (Vincent, 2017).

Drones are considered as a potential fit for the last mile delivery because of their high travel speed and ability to access areas regardless of road infrastructure, thus avoiding traffic congestion. They use electric batteries, which generate less air pollution. Nonetheless, drones' main disadvantages are their small load capacity and their short battery endurance, which can be offset by pairing them with trucks' large load capacity and long endurance range. The academic routing community has acknowledged an interest in combining drones with bigger vehicles (trucks) to increase the efficiency and effectiveness of the last mile delivery. The model of combining trucks and drones was first introduced by Murray and Chu (2015) as the "Flying Sidekick Traveling Salesman Problem" (FSTSP), inspired by the Vehicle Routing Problem (VRP) with synchronization constraints (Drexl, 2012). In this proposed model, a drone is attached on top of the delivery truck, simultaneously moves along with the truck, and flies from the truck to make a delivery while the truck continues to serve its customers in different locations. Once the drone completes its service for one customer, it needs to fly back to the truck at the current delivery location or along the route to the next delivery location.

This paper extends the FSTSP by allowing multiple drones and multiple trucks to make deliveries with the consideration of both trucks' and drones' capacities. The drone is allowed to carry multiple packages to make multiple deliveries to different customers before merging with the truck. We call this model "Two Echelon Vehicle Routing Problems with Drones" (2EVRPD). The proposed model aims at finding a set of both drone and truck routes such that the demands of all customers are satisfied while the total delivery time is minimized. Successful implementation of this type of delivery in the industry could bring about cost efficiency and reduce the total delivery time of the last mile delivery. Figure 1 represents a simple 2EVRPD and its solution in which two trucks and four drones are employed for delivery. As illustrated, each

truck contains two drones and each drone can fly to serve multiple customers before finally returning to the truck. The routes belonging to the truck's delivery are represented as solid lines, whereas the routes belonging to the drone's delivery are depicted as dotted lines.



**Fig. 1.** Illustration of the Two Echelon Vehicle Routing Problems with Drones (2EVRPD)

In the classical 2EVRP, the delivery is divided into two levels: the trucks operate on the first level between a central depot and selected intermediate distribution facilities, called satellites and the secondary vehicles at satellites serve all customers at the second level (Cuda et al., 2015). In the first echelon of 2EVRPD, a fleet of trucks must leave a depot, visit customers along the route and return to the same depot. The route constructed by each truck is referred as *a truck route*. When each drone makes multiple deliveries per trip prior to returning to the truck, it creates a *drone route* in each drone trip, which is categorized as a second echelon delivery level. In 2EVRPD, trucks behave like intermediate depots where they can launch and retrieve drones at one of the delivery locations. Each truck is equipped with drones, which belong to that specific truck. The drone can be launched from the truck and make multiple deliveries before returning to

the truck. After launching the drone, the truck can simultaneously make other deliveries along the route while the drone is active.

In this paper, we propose a new MIP model and heuristic algorithms to solve a new problem, the Two Echelon Vehicle Routing Problems with Drones (2EVRPD). The main contributions of this paper are the following. First, we introduce a “Mixed Integer Program” (MIP) formulation for the 2EVRPD. The model might be solved by any standard MILP solver, e.g., GAM and IBM CPLEX. They are able to handle small-size problems. Second, we propose two heuristics to solve 2EVRPD: Drone Route Construction (DTRC) and a metaheuristic based on Large Neighborhood Search (LNS). In particular, DTRC generates an initial 2EVRPD solution from the VRP solution. The LNS will repeatedly search for a better solution using three destroy operators and three repair operators. Third, we conduct numerical experiments and sensitivity analyses on different types of problems using both MIP and heuristics approaches. The small-size problems can be solved directly by the MIP solver and the medium/large-size problems can be solved by DTRC and LNS. The results are compared with the classical VRP optimal solutions and other TSP/VRP with drone routing models on the CVRP benchmark problems.

The remainder of the paper is structured as follows. Section 2 provides the literature reviews of the related work. Section 3 presents the formal definition of the 2EVRPD model and its mathematical MIP formulation. Section 4 presents the two heuristics approaches: Drone Truck Route Construction and Large Neighborhood Search, to solve the proposed model along with the pseudocodes in detail. In Section 5, we provide the results of the numerical experiments on the different test instances and benchmark problems with the sensitivity analysis of the performance of the algorithm. Section 6 concludes the paper and provides discussions for future research.

## 2. Literature review

The Two Echelon Vehicle Routing Problems with Drones (2EVRPD) is a variant of the classical Vehicle Routing Problem with the implementation of drones in the operation (Toth & Vigo, 2014). Due to the rapidly growing popularity, many works related to the drone routing optimization problems are found in the literature. This section presents the most relevant papers in this area, which can be categorized into two variants of the classical routing models: the Traveling Salesman Problem (TSP), in which only one truck is employed, and the Vehicle Routing Problem (VRP), in which multiple trucks are employed.

### *2.1. Drone routing as an extension of TSP*

Murray and Chu (2015) introduced the FSTSP and provided a mathematical programming formulation and a simple heuristic for the problem of coordinating a single traditional delivery truck with a single drone. The same authors also proposed another model called the “Parallel Drone Scheduling Traveling Salesman Problem” (PDSTSP) in which trucks and drones work independently to serve all customers. Ponza (2015) examined the FSTSP in detail and applied the simulated annealing technique to search for good solutions. Jeong et al. (2019) modified the FSTSP to consider the effect of the payload on the UAV energy consumption and restricted flying areas.

Agatz et al. (2018) studied a similar problem called the “Traveling Salesman Problem with Drone” (TSP-D), in which the authors presented the new MIP model and provided efficient heuristics based on local search and dynamic programming. Ha et al. (2018) proposed the min-cost TSP-D with the objective to minimize the total transportation cost using two algorithms: TSP-LS based on local search and a Greedy Randomized Adaptive Search Procedure (GRASP).

Yurek and Ozmutlu (2018) provided an iterative optimization algorithm for the TSP-D based on the decomposition of the problem into two components: (1) finding a truck route and (2) finding the optimal drone routes within the truck route by solving a mixed integer linear programming. Bouman et al. (2018) recently developed an exact solution for the TSP-D based on dynamic programming. Marinelli et al. (2017) extended the TSP-D by allowing a drone to be launched and merge with a truck at any location along a route arc (en-route).

Mathew et al. (2015) proposed a new drone and truck problem called the “Heterogeneous Delivery Problem” (HDP), which shares similar features with TSP-D and FSTSP. Ferrandez et al. (2016) proposed an optimization model of a truck-drone system in tandem delivery networks by using the K-means algorithms to find the most efficient launch locations as well as using a genetic algorithm to assign the truck route between those launch locations. Kim and Moon (2019) developed the “TSP with a drone station” (TSP-DS), which has similar features to the PDSTSP but includes a drone station as the facility where drones and charging devices are stored. Tu et al. (2018) proposed the new problem as an extension of the TSP-D problem in which a truck travels with multiple drones called “TSP-mD.”

Kitjacharoenchai et al. (2019) proposed the “Multiple Traveling Salesman Problem with Drones” (mTSPD), which has the same operation as FSTSP but utilizes multiple trucks and drones and allows a drone to be retrieved by any truck that is nearby and not necessarily the same truck that it is launched from. Murray and Raj (2019) introduced the “Multiple Flying Sidekicks Traveling Salesman Problem” (mFSTSP), which is an extension of their previous work (FSTSP) with the consideration of an arbitrary number of heterogeneous UAVs that may be deployed from the depot or from the delivery truck. The authors provided the “Mixed Integer Linear Programming” (MILP) formulation along with the three-phased heuristic solution

approach.

### *2.2. Drone routing as an extension of VRP*

Wang et al. (2016) provided worst-case analyses for the “Vehicle Routing Problem with Drones” (VRPD) problem by studying how the two parameters—the number of drones per truck and the speed of the drones—can affect the maximum savings from using drones. Poikonen et al. (2017) considered the same work by finding upper bounds on the amount of time saved through drone operations with an integration of the drone’s battery life, cost metrics, and fixed cost of deploying drones. Schermer et al. (2018) formulated a mixed integer linear program for VRPD and proposed an algorithm based on the well-known “Variable Neighborhood Search” (VNS) to solve the problem. Pugliese and Guerriero (2017) introduced the “Vehicle Routing Problem with Drones and Time Windows” (VRPDTW), which includes the time window constraints in the model.

Campbell et al. (2017) proposed continuous approximation (CA) models to obtain the optimal number of truck and drone deliveries per route, the optimal number of drones per truck, and the total cost of operation in the hybrid truck-drone delivery problem. Similarly, Carlsson and Song (2017) implemented a CA technique to determine the best set of parameters that results in the minimum completion of all truck-drone deliveries in the Euclidean plane. Dorling et al. (2017) in another perspective, proposed the VRP based models: one on the total delivery costs subject to a delivery time limit and another on the overall delivery time subject to a budget constraint.

Ulmer and Thomas (2017) proposed the “Same-Day Delivery Routing Problems with Heterogeneous Fleets” (SDDPHF) and modeled it as a Markov decision process. Cheng et al. (2018) proposed a “Multi-Trip Drone Routing Problem” (MTDRP) in the drone-only system in

which the drone can visit multiple customers per trip. Dayarian et al. (2017) presented a “Vehicle Routing Problem with Drone Resupply” (VRPDR) in which a fleet of drones and a fleet of vehicles collaboratively perform home deliveries of online orders from a fulfillment center. Ham (2018) extended the PDSTSP by considering two different types of drone tasks: drop and pickup. Hong et al. (2017) developed a heuristic model to obtain the optimal location of drone recharging stations by connecting the stations and delivery locations based on a continuous space shortest path.

Luo et al. (2017) proposed a “Two-Echelon cooperated Routing Problem for the Ground Vehicle (GV) and its carried unmanned aerial vehicle (UAV)” (2E-GU-RP). The problem is very similar to VRPD proposed by Schermer et al. (2018) but allows drones to make multiple deliveries in one trip. Karak and Abdelghany (2019) presented the “Hybrid Vehicle-Drone Routing Problem” (HVDRP) for pick-up and delivery services in which multiple drones can be dispatched from a mothership to perform dozens of pick-ups and deliveries simultaneously. Wang and Sheu (2019) presented the “Vehicle Routing Problem with Drones” (VRPD) with a distinctive feature that allows drones to make multiple deliveries per trip and return to any available truck in the fleet. The authors proposed a mixed integer programming model and developed a branch-and-price algorithm to solve VRPD for the exact solution. Poikonen and Golden (2020) recently developed the “k-Multi-visit Drone Routing Problem” (k-MVDRP), which considers a tandem between a truck and k drones allowing a drone to deliver one or more packages to customers.

### **3. Mathematical Definition and Formulation**

The 2EVRPD covers two echelon routing levels where the first level deals with the delivery made by trucks and the second level deal with the delivery made by drones. It is defined on a

directed graph  $G = (V, E)$ , where  $V$  is the set of  $n$  nodes representing customers with one depot and  $E$  is the set of arcs. Unlike the typical 2EVRP where the set of edges  $E$  is divided into two subsets, representing the first and the second echelon respectively, both echelon levels use only one set of edges  $E$ . Both vehicles need to start from and return to the depot to complete the operation. Each truck is equipped with certain number of drones where a drone can independently make its own delivery i.e. serving multiple customers prior returning to the truck. A truck can simultaneously make its own delivery and behaves like an intermediate movable depot. The important assumptions are summarized as follows:

- each truck has limited space to carry only a specific number of drones. A drone has its own capacity and so does a truck. A drone has a limited amount of battery capacity, which determines how long it can travel before having to return to a truck for a battery swap or a recharge.
- We assume that multiple drones are not allowed to be launched or retrieved at the same node in any given time.
- Drones can only merge with a truck at a customer node and are not allowed to merge with a truck in any intermediate location.
- The time of both trucks and drones at the customer locations must be adjusted to be the same. In the other words, they must wait for each other whenever one arrives at the customer node before the other.
- The set-up and recovery times will also be included when the drone is launched or retrieved at a particular node.

- The drone must go back to the truck from which it was launched (it cannot be merged with different trucks) and it is important to keep track of when the drones are available for the launch.

Let  $\tau_{i,j}^T$  be a truck travel time associated with  $E$ ,  $(i,j) \in E$  and  $\tau_{i,j}^D$  be a drone travel time associated with  $E$ ,  $(i,j) \in E$ , differentiating the travel times for the truck and drone accounts for each vehicle's unique travel speed. The 2EVRPD is said to be symmetric if  $\tau_{i,j}^T = \tau_{j,i}^T$  and  $\tau_{i,j}^D = \tau_{j,i}^D$  and asymmetric otherwise. The travel times for truck and drone matrix satisfies the triangle inequality  $\tau_{i,k}^T + \tau_{k,j}^T \geq \tau_{i,j}^T$ .

A fleet of  $K$  homogeneous trucks, defined as a set of  $K = \{1, 2, 3, 4, \dots, k\}$ , with capacity  $Q$  is located at the depot. The maximum number of  $KD$  homogeneous drones, defined as a set of  $KD = \{1, 2, 3, 4, \dots, kd\}$ , with the capacity  $QD$  is attached along with each truck. The total length of a drone route in each launch does not exceed a preset limit  $B$  (Battery life). In our model, the fleet size of truck and the number of drones in each truck are given a priori. We denote the set of customer nodes by  $C = \{1, 2, 3, 4, 5, 6, \dots, n\}$ . As for the depot, we assign it to two unique node numbers at  $0(s)$ , the starting depot, and  $0(r)$ , the ending depot. Set  $C_0 = C \cup \{0(s)\}$  is the set of customer nodes including the starting depot, and set  $C_+ = C \cup \{0(r)\}$  is the set of customer nodes including the ending depot. Each customer  $i$  ( $i = 1, 2, 3, \dots, n$ ) is associated with a known nonnegative demand,  $D_i$ , to be delivered, and the depot has a fictitious demand  $D_o = 0$ . The customer demand is measured by weight unit and can be satisfied by either truck or drone delivery.

We define the following decision variables: Let  $x_{i,j}^k$  be equal to 1 if a truck  $k$  is traveled along the arc  $(i, j) \in E$  and 0 otherwise. This refers to the situation when the truck travels from node  $i \in C_0$  to  $j \in C_+$  where  $i \neq j$ . Let  $y_{i,j}^{kd,k}$  be equal to 1 if a drone  $kd$  of truck  $k$  travels along the arc  $(i,$

$j) \in E$  and 0 otherwise. This refers to the situation when a drone is launched from node  $i \in C$  to node  $j \in C$  (making a delivery at a customer node) or when a drone is launched from node  $i \in C$  to merge with a truck at node  $j \in C$  (recharging/swapping a battery at truck). Let  $yt_i^k$  be equal to 1 if a truck  $k$  serves customer node  $i$  and 0 otherwise. Similarly, let  $yd_i^{kd,k}$  be equal to 1 if a drone  $kd$  of truck  $k$  serves customer node  $i$  and 0 otherwise.

We denote the variable  $tt_j^k$  as the truck  $k$  arrival time at node  $j \in C_+$  and  $dt_j^{kd,k}$  as the drone  $kd$  of truck  $k$  arrival time at node  $j \in C_+$ .  $tt_j^k$  and  $dt_j^{kd,k}$  are the arrival times of the truck and drones at node  $j$  respectively. Lastly, we define other the auxiliary decision variables including: 1)  $u_i^k$  which is used in the VRP subtour elimination constraints (Desrochers & Laporte, 1991), 2)  $la_i^{kd,k}$  which is used to indicate the status of whether a drone  $kd$  of truck  $k$  can be launched from node  $i$  or not, and 3)  $zd_1$  &  $zd_2$  which is used to indicate whether there is any drone arc coming out from node  $i$  and entering node  $i$  accordingly.

The proposed MIP formulation of 2EVRPD is presented as follows:

Objective

$$\text{minimize } z \quad (1)$$

Subject to

$$\sum_{k \in K} tt_{0(r)}^k = z \quad (2)$$

$$\sum_{k \in K} \sum_{kd \in KD} yd_i^{kd,k} + \sum_{k \in K} yt_i^k = 1; \forall i \in C \quad (3)$$

$$\sum_{i \in C_+} x_{0(s),i}^k = 1; \forall k \in K \quad (4)$$

$$\sum_{i \in C_0} x_{i,0(r)}^k = 1; \forall k \in K \quad (5)$$

$$\sum_{j \in C_+} x_{i,j}^k = \sum_{j \in C_0} x_{j,i}^k = yt_i^k; \forall i \in C, \forall k \in K \quad (6)$$

$$yd_j^{kd,k} (\sum_{i \in C} yd_{j,i}^{kd,k} - 1) = 0; \forall j \in C, \forall k \in K, \forall kd \in KD \quad (7)$$

$$yd_j^{kd,k}(\sum_{i \in C} y_{i,j}^{kd,k} - 1) = 0; \forall j \in C, \forall k \in K, \forall kd \in KD \quad (8)$$

$$\sum_{kd \in KD} \sum_{k \in K} \sum_{j \in C} y_{i,j}^{kd,k} (zd1_i - 1) = 0; \forall i \in C \quad (9)$$

$$\sum_{kd \in KD} \sum_{k \in K} \sum_{j \in C} y_{j,i}^{kd,k} (zd2_i - 1) = 0; \forall i \in C \quad (10)$$

$$\sum_{j \in C} \sum_{kd \in KD} y_{i,j}^{kd,k} \geq 1 - M(2 - yt_i^k - zd1_i); \forall i \in C, \forall k \in K \quad (11)$$

$$\sum_{j \in C} \sum_{kd \in KD} y_{j,i}^{kd,k} \geq 1 - M(2 - yt_i^k - zd2_i); \forall i \in C, \forall k \in K \quad (12)$$

$$y_{i,j}^{kd,k} \leq 2 - (yt_i^k + yt_j^k); \forall i, j \in C, \forall k \in K, \forall kd \in KD \quad (13)$$

$$(x_{i,j}^k) \left( \sum_{\substack{s \in C \\ s \neq i}} y_{s,j}^{kd,k} \right) la_j^{kd,k} = 0; \forall i, \forall j \in C, \forall k \in K, \forall kd \in KD \quad (14)$$

$$la_i^{kd,k} (\sum_{j \in C} y_{i,j}^{kd,k}) = 0; \forall i \in N, \forall k \in K, \forall kd \in KD \quad (15)$$

$$(x_{i,j}^k) \left( \sum_{\substack{q \in C \\ q \neq j}} y_{i,q}^{kd,k} \right) (1 - \sum_{\substack{s \in C \\ s \neq i}} y_{s,j}^{kd,k}) (1 - la_j^{kd,k}) = 0; \forall i, \forall j \in C, \forall k \in K, \forall kd \in KD \quad (16)$$

$$(x_{i,j}^k) (la_i^{kd,k}) (1 - \sum_{\substack{s \in C \\ s \neq i}} y_{s,j}^{kd,k}) (1 - la_j^{kd,k}) = 0; \forall i, \forall j \in C, \forall k \in K, \forall kd \in KD \quad (17)$$

$$\sum_{j \in C} D_j (yd_j^{kd,k}) \leq QD; \forall i \in N, \forall k \in K, \forall kd \in KD \quad (18)$$

$$\sum_{i \in C} \sum_{j \in C} \tau_{i,j}^D (y_{i,j}^{kd,k}) \leq B; \forall k \in K, \forall kd \in KD \quad (19)$$

$$\sum_{i \in C} D_i (yt_i^k) + \sum_{i \in C} \sum_{kd \in KD} D_i (yd_i^{kd,k}) \leq Q; \forall k \in K \quad (20)$$

$$\sum_{j \in C} y_{i,j}^{kd,k} (tt_i^k - dt_i^{kd,k}) = 0; \forall i \in C_0, \forall k \in K, \forall kd \in KD \quad (21)$$

$$\sum_{j \in C} y_{j,i}^{kd,k} (tt_i^k - dt_i^{kd,k}) = 0; \forall i \in C, \forall k \in K, \forall kd \in KD \quad (22)$$

$$tt_j^k \geq tt_i^k + \tau_{i,j}^T - M(1 - x_{i,j}^k) + SL(\sum_{h \in C} \sum_{kd \in KD} y_{i,h}^{kd,k}) + SR(\sum_{p \in C} \sum_{kd \in KD} y_{p,j}^{kd,k}); \\ \forall i \in C_0, \forall j \in C_+, \forall k \in K \quad (23)$$

$$dt_j^{kd,k} \geq dt_i^{kd,k} + \tau_{i,j}^D - M(1 - y_{i,j}^{kd,k}) + SL(\sum_{h \in C_+} x_{i,h}^k) + SR(\sum_{p \in C_0} x_{p,j}^k); \\ \forall i, \forall j \in C, \forall k \in K, \forall kd \in KD \quad (24)$$

$$u_i^k - u_j^k + Q(x_{i,j}^k) \leq Q - D_j; \forall i, \forall j \in C \cup C_0 \cup C_+, \forall k \in K \quad (25)$$

$$D_i \leq u_i^k \leq Q ; \forall i, \forall j \in C \cup C_0 \cup C_+, \forall k \in K \quad (26)$$

$$x_{i,j}^k, y_{i,j}^{kd,k}, yt_i^k, yd_i^{kd,k}, la_i^{kd,k}, zd1_i, zd2_i \in \{0,1\}, tt_j^k, dt_j^{kd,k} \geq 0, dt_j^{kd,k}$$

$$\forall i, \forall j \in C, \forall k \in K, \forall kd \in KD \quad (27)$$

The objective function (1) minimizes the total truck arrival time of trucks at the depot. Constraint (2) straightforwardly represents the sum of each truck's arrival time at the depot. Constraints (3) ensure that each customer will receive the package either by a drone or a truck. It restricts each customer to be visited exactly once by one vehicle. Constraints (4) and (5) impose that each truck must depart from and arrive to the depot. Constraints (6) ensure the flow conservation of the truck route at each node  $i$ , which guarantees that whenever the truck  $k$  arrives at a node, it must depart from the node as well. Constraints (7) and constraints (8) define that if the drone  $kd$  of the truck  $k$  makes a delivery at node  $j$ , there must be exactly one arc from this assigned drone entering node  $j$  and exactly one arc from this drone departing node  $j$ . These sets of constraints ensure the flow conservation of the drone arc at the customer node  $j$ .

Constraints (9) impose that if a drone departs from node  $i$ , the auxiliary variable  $zd1_i$  must be equal to 1. Similarly, constraints (10) impose that if a drone enters node  $i$ , the auxiliary variable  $zd2_i$  must be equal to 1 as well. Constraints (11) impose that if a customer is served by a truck at node  $i$ , and there is a drone arc leaving node  $i$ , node  $i$  is categorized as a *launching node*, where a drone is launched from. Constraints (12) impose that if a customer is served by a truck at node  $i$ , and there is a drone arc entering node  $i$ , node  $i$  is categorized as a *landing node*, where a drone lands. While constraints (7) and (8) preserve the flow conservation of the customer node  $i$  that is served by a drone, constraints (9) to (12) ensure that a drone can land at and be launched from a truck, which represents the scenario in which a drone merges with a truck to retrieve a new load and gets a battery swapped. Constraints (13) ensure that a drone is

not allowed to travel directly from node  $i$  to node  $j$  where node  $i$  and node  $j$  are already served by a truck.

The following sets of constraints (14) to (17) guarantee the correct order of launching and landing operation. Basically, a drone can only be launched if it has never been launched before or was previously launched and returned to receive a service at the truck. The constraints ensure that the same drone cannot be launched from the same truck if it was previously launched and has not returned yet. Constraints (14) and (15) impose that a drone is not allowed to be launched or land at node  $i$  once the auxiliary variable  $la_i^{kd,k}$  is equal to 1 and vice versa. Constraint (14) specifically ensures that if a drone returns to node  $j$  where the truck  $k$  serves its customer, then the auxiliary variable  $la_j^{kd,k}$  must be equal to 0, the state in which an arc drone can leave or enter node  $j$ . In constraints (16), the auxiliary variable  $la_j^{kd,k}$  must be equal to 1 when a drone is launched from node  $i$ , and a truck travels from node  $i$  to node  $j$  at which the drone has not yet returned. Similarly, constraints (17) deal with the case when a drone was previously launched (not able to be launched at node  $i$  again) and has not returned to node  $j$ .

Constraints (18) ensure that the amount of a drone's load must be less than the drone's capacity ( $QD$ ) in any given delivery node. Constraints (19) ensure that the amount of battery consumption of each drone must be less than its drone's battery limit at any point in time. Constraints (20) enforce that the total delivery loads of both truck and drone combined must be less than the truck capacity at each truck route  $k$ .

Constraints (21) and (22) adjust the departure time and the arrival time of both drone and truck to be the same once the two vehicles merge. Constraints (23) keep track of the arrival time of the truck at every node. It adds the truck travel time to the previous customer node when the truck travels from one customer node to another customer node. The service time (SR/SL) is

included when a drone lands at or is launched from this specific node accordingly. Similarly, constraints (24) keep track of the arrival time plus recovery/service time of the drone at the node to which the drone returns after making a delivery.

Constraints (25) and (26) are sets of the Desrochers and Laporte (DL) subtour elimination constraint, which ensures that there is no subtour in all tours of the trucks (Desrochers & Laporte, 1991). Constraints (27) specify the types and ranges of the variables. Note that the M value must be large enough. Thus, we can use the total time of all the delivery routes made by trucks alone, i.e., solve a regular CVRP.

The 2EVRPD is a generalization of the classical VRP/TSP and is thus by nature an NP-hard problem. Mixed integer programming formulation was developed to obtain an optimal solution that works for the small-size problems. Because of the NP-hardness of the 2EVRPD, a heuristic approach is implemented to find solutions quickly for the larger-size problems.

#### **4. Solution method**

Mixed integer programming (MIP) formulations (1) through (27) for 2EVRPD have been developed and presented in the previous section. Although we are able to obtain the exact optimal solutions to small-size problems (less than 10 nodes) with the CPLEX solver, solving the larger-size problems could take too much time to obtain the exact solution. In this section, we describe how to solve the defined problem within a practical timeframe using two proposed heuristics: Drone Truck Route Construction (DTRC) and Large Neighborhood Search (LNS).

##### *4.1. Drone Truck Route Construction Heuristic*

The Drone Truck Route Construction Heuristic is considered as a constructive heuristic which gradually builds a feasible solution while keeping an eye on the solution cost. The structure of the algorithm is composed of two phases. The first phase includes the process of

building a solution for the Capacitated VRP using a fast and efficient classical heuristic. The second phase is to construct a feasible drone sub route within the truck main route while maintaining the feasibility constraints such as battery life, load capacity, and the order of launching and landing operations. Algorithm 1 shows the basic structure of the proposed method.

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**Algorithm 1** DTRC.
 

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**Input:** Trucks, Drones, All Nodes, Demand, Travel Time, Capacity, Battery

**Output:**  $S^{2EVRPD}$

1. Generate  $S^{CVRP}$  using Savings (Truck, Nodes, Demand, Travel Time)
2. **For** Route  $\in S^{CVRP}$
3.     Set  $RemainN = \{All\ nodes\ in\ Route\}$
4.     Initialize  $I^{st}\ Launch\ Node = I^{st}\ Truck\ Node\ in\ Route$ ,  $LandingN = []$ ;
5.     **While**  $RemainN \notin \emptyset$
6.         Select Drone from Available Drones = {1,2,...kd}
7.         **If** Available Drones ==  $\emptyset$ , proceed to line (17), **else**
8.             Update Available Drones = Available Drones \ Drone
9.         **Repeat**
10.         Choose Next Drone Delivery Node  $\leftarrow \min \vartheta$
11.         Update Drone Bat, Drone Load, Route
12.         Update RemainN = RemainN - {Next Drone Delivery Node}
13.         **Until** Drone Load  $\geq$  Drone Capacity || Drone Bat  $\geq$  Battery
14.         Update LandingN = LandingN  $\cup$  Last Drone Delivery Node
15.     **End If**
16.     Choose Next Truck Delivery Node  $\leftarrow \min \mathcal{D}$  from RemainN  $\cup$  LandingN
17.     Update Route, RemainN = RemainN - Next Truck Delivery Node

18.       **If**  $\text{Next Truck Delivery Node} == \text{LandingN}$
19.       Reset  $\text{Drone Load}_{\text{Drone}}$ ,  $\text{Drone Bat}_{\text{Drone}} = 0$
20.        $\text{Available Drones} = \text{Available Drones} \cup \text{Drone}$
21.       **End If**
22.       **End while**
23.       Return  $S^{2\text{EVRPD}}$  for each  $\text{Route}$
24. **End For**
25. **Return**  $S^{2\text{EVRPD}}$

#### 4.1.1. CVRP solution

The Capacitated Vehicle Routing Problem (CVRP) is a vehicle routing problem with additional constraints on the capacities of the vehicles. To quickly obtain the CVRP solution, we adopt the Clarke and Wright Saving Algorithm (Clarke & Wright, 1964), which is by far the best-known approach and yet conceptually simple, yielding reasonably good solutions to the CVRP problem (Laporte, 2009). After obtaining the CVRP solution from the Saving Algorithm, we apply some well-known local searches such as, 2 opt, simple relocate, and swap move (Gendreau, 2008) to improve the solution quality. This completes the first phase of the algorithm (Line 1.)

#### 4.1.2. Drone Route Construction

Using the CVRP solution obtained from the Saving Algorithm, we gradually build up the new truck routes and drone sub routes to generate a feasible 2EVRPD solution. The Drone Route Construction steps can be provided in detail as follows:

**Step 1:** For each truck route, select the initial truck node where the drone can be

launched. The initial node is usually the node which is located closest to the depot (Line 2-4).

Step 2: Select one of available drones to be launched and then select one of the truck nodes as a drone delivery node using the lowest ratio of  $\vartheta = \frac{\mathcal{D}}{\mathcal{W}}$  where  $\mathcal{D}$  is the distance between the current drone node and an unvisited node, and  $\mathcal{W}$  is the current drone load plus the demand of an unvisited node. We use this ratio because it is intuitively more efficient for a drone to drop a heavy package at the closer distance to preserve more energy, which should be prioritized first (Line 10).

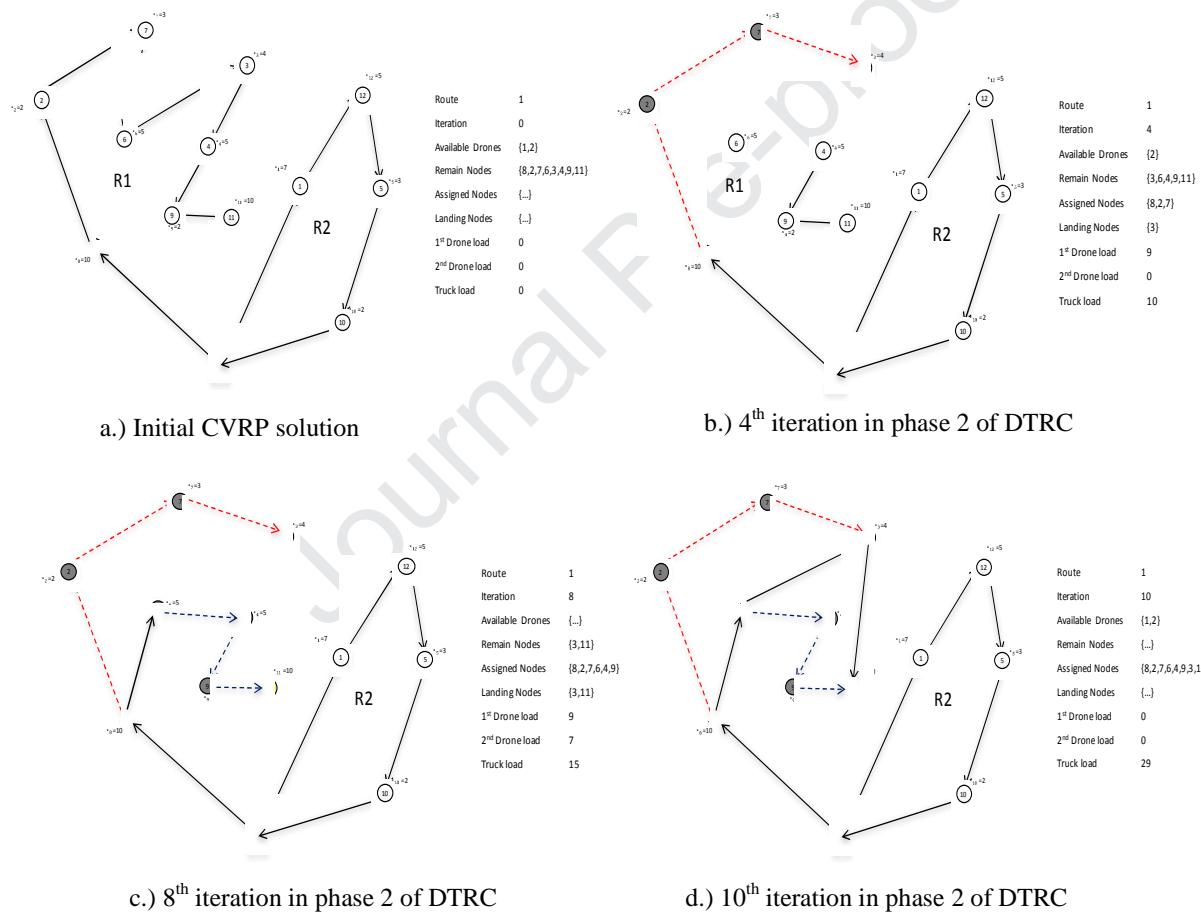
Step 3: Remove the selected drone delivery node from the truck route and add it to the current drone route. Update the drone's battery consumption and load (Line 11,12)

Step 4: Repeat steps 2 & 3 for the same drone until no other nodes can be selected due to either the drone's load exceeding its capacity or the drone's energy consumption exceeding its battery capacity. The last customer node in a drone route is called the "landing node."

Step 5: At the current truck node, select the next available customer node for the next truck delivery from the remaining nodes that have not been previously selected to construct the drone route yet or the landing node from any drone route. If the latter is selected, we obtain a complete drone route and this drone will be available for the new selection in the next iteration (Line 18-20). Otherwise, we can select the next customer node to be served by the truck using the closest distance to the current truck node ( $\min \mathcal{D}$ ).

Step 6: Repeat steps 2 – 5 until no drone can be selected, and we have a complete truck route return back to the depot. Complete the same steps for all the routes in the CVRP solution.

Figures 2a to 2d represent the graphical examples of steps 1-6 in phase 2 of the DTRC heuristic.



**Fig. 2.** Example of the phase 2 of the DTRC algorithm

#### 4.2 Large Neighborhood Search

LNS is based on a process of continual relaxation and re-optimization. An initial feasible solution of the problem is destroyed and repaired iteratively to gradually improve the solution quality. The framework of LNS has recently been used to successfully solve multiple variants of vehicle routing problems (Pisinger & Ropke, 2010). LNS offers a large move that could expand the solution search space by disintegrating a large part of the previous solution and giving the freedom to create a new one (Schriftpf et al., 2000). Since our proposed model has many side constraints, such as the order of launching and landing operations as well as the time adjustment for both trucks and drones, it would be more applicable to implement an LNS to our problem rather than using standard local search move operators. Algorithm 2 shows the basic structure of the proposed LNS for 2EVRPD.

---

**Algorithm 2.** LNS-2EVRPD
 

---

**Input:** Trucks, Drones, All Nodes, Demand, Travel Time, Capacity, Battery

**Output:**  $S^{Best}$

---

1. Generate  $S^{Best} \leftarrow S^{Current} \leftarrow DTRC(\text{Truck}, \text{Drones}, \text{Nodes}, \text{Demand}, \text{Travel Time})$
  2. Initialize  $time = 0, i = 0$
  3. **Repeat**
  4.   **While**  $i \leq i_{max}$
  5.     Enter *Destroy Phase*
  6.        $S^{TempD1} \leftarrow \text{Call Drone node removal } (S^{Current})$
  7.        $S^{TempD2} \leftarrow \text{Call Truck node removal } (S^{TempD1})$
  8.        $S^{TempD3} \leftarrow \text{Call Sub-drone route removal } (S^{TempD2})$
  9.     Enter *Repair Phase*
  10.       $S^{TempR1} \leftarrow \text{Call Drone node insertion } (S^{TempD3})$
-

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11.  $S^{TempR2} \leftarrow \text{Call } Truck \text{ node insertion } (S^{TempD3})$   
 12.  $S^{TempR3} \leftarrow \text{Call } Drone \text{ route creation } (S^{TempD3})$   
 13. Select  $S^{Temp} = \min (S^{TempR1}, S^{TempR2}, S^{TempR3})$   
 14. **If** Cost ( $S^{Temp}$ ) < Cost ( $S^{Current}$ )  
 15.  $S^{Current} \leftarrow S^{Temp}$   
 16.  $i = 0$   
 17. **Else**  
 18.  $i = i + 1$   
 19. **End If**  
 20. **End While**  
 21. **If** Cost ( $S^{Current}$ ) < Cost ( $S^{Best}$ )  
 22.  $S^{Best} \leftarrow S^{Current}$   
 23.  $i = 0$   
 24. **Else**  
 22.  $S^{Current} \leftarrow DTRC (Truck, Drones, Nodes, Demand, Travel Time)$   
 23.  $i = 0$   
 24. **End If**  
 26. **Until**  $time \geq time_{max}$   
 27. **Return**  $S^{Best}$

---

The proposed algorithm starts by generating the initial solution from DTRC. This initial solution will be stored as the current solution as well as the global best solution (Line 1). At each iteration, a partial solution destruction is performed sequentially using three destroy operators on the current solution routes, then the destroyed solution is repaired by the repair operators (Line 6 – Line 12). The best repair solution will be selected and recorded as the temporary solution (Line

13). If the objective of the temporary solution is lower than the one from current solution, we accept the new current solution and the index  $i$  is reset to 0 (Line 14-16). Otherwise, we still keep the old current solution and perform another round of destroy-repair operations to the temporary solution. We keep improving the current solution until the index  $i$  reaches  $i_{max}$ . Then, we compare the objective between the current solution and the global best solution. If the current solution provides a lower objective value, we accept and update the new global best solution (Line 21-23). If there is no objective improvement after a large number of iterations, then the algorithm will restart from a new initial solution (Line 22). We repeat the same steps until the  $time \geq time_{max}$  and return the best solution. All the different types of destroy operators and repair operators will be described in the next sub section.

#### 4.2.1 Destroy operators

Our algorithm relies on three different destroy operators, which are invoked at each iteration in sequential order. We denote  $p_1$ ,  $p_2$  and  $p_3$  as the percentage of drone only nodes, truck only nodes, and sub routes accordingly. The operators are presented in the sequential order of execution as follow.

*Drone node removal:* This operator removes  $\lceil p_1 \cdot C_{Only\ Drone} \rceil$  nodes from the current solution, with  $C_{Only\ Drone}$  being the set of customer nodes who receive deliveries by drone only. If all nodes in a sub drone route are removed, the original launching and landing node becomes available for launching and retrieving any available drone.

*Truck node removal:* This operator removes  $\lceil p_2 \cdot C_{Only\ Truck} \rceil$  nodes from the current solution, with  $C_{Only\ Truck}$  being the set of customer nodes who receive deliveries by truck only (excluding the launching and landing node). If all customer nodes are removed from the truck route, the solution for that truck route becomes empty and the truck stays at the depot.

*Sub-drone route removal:* This operator removes one or multiple sub-drone routes (level two) from the truck route (level one). Each sub-drone route includes all customer nodes who are currently served by drones. We allow  $[p_3 \cdot R_{Sub\ Route}]$  routes to be removed, with  $R_{Sub\ Route}$  being the set of sub drone routes. If all sub routes are removed from the main truck route, the remaining customers will be served by truck alone and the solution route is reduced to the level 1 delivery, which is equivalent to the solution from regular CVRP.

#### 4.2.2 Repair operators

At the repair phase, each node would go through all three repair operators in parallel. Each repair operator returns the new routing solution with the selected node already inserted in the route. We compare the solutions obtained from three operators. The best repaired solution will be selected and become the current solution of the problem. We perform the repair process until no node is left in the re-insert list and reevaluate the current solution with the global solution. The repair operators are described as follow.

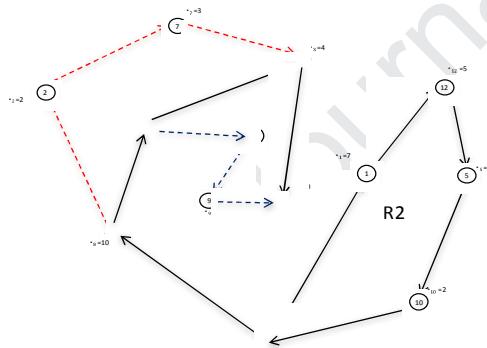
*Drone node insertion:* The operator takes a node input and inserts it into one of the existing drone sub routes (level 2) in the solution. The route to insert the node into is not restricted to the one that this node is removed from and can be a sub drone route in any truck route. The insertion is achieved with a simplified cheapest insertion heuristic. The insertion is prohibited in the drone route if the new drone route after the insertion process causes 1.) the drone's load to exceed its capacity and 2.) the drone's battery consumption to exceed its battery limitation.

*Truck node insertion:* The operator inserts the selected node into one of the truck routes (level 1) at the current solution. The operator searches for all the feasible positions to insert the node into and selects the one with the lowest increase in total cost. If the current capacities of

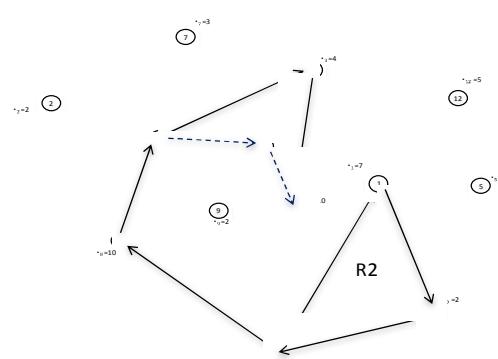
trucks for all routes are full, the node can be inserted into an empty route, which creates one more truck route in the solution.

*Drone route creation:* The operator creates a new sub drone route by inserting the selected node between a pair of truck nodes. Considering a pair of  $(i,k)$  nodes where node  $i$  must precede node  $k$ , we insert node  $j$  between node  $i$  and node  $k$  to create a new drone sub route (level 2). The drone is launched from node  $i$ , makes a delivery at node  $j$  and returns to another truck, node  $k$ . The operator searches for the cheapest pair  $(i,k)$  among all possible combinations to construct a new drone route with the lowest increase in total cost.

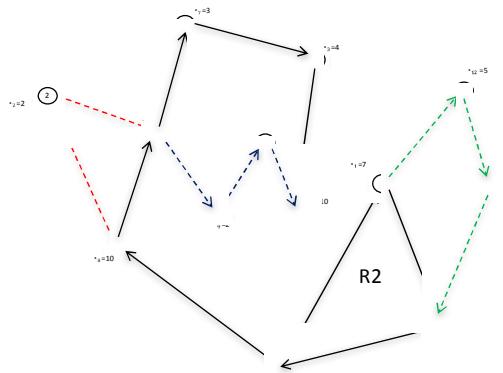
Figures 3a, 3b, and 3c represent the current solution, the solution after all three destroy operators are executed and the solution after the all three repair operators are executed accordingly.



a.) Initial 2EVRPD solution



b.) Solution after the destroy operators: Drone node removal {9}, Truck node removal {12,5}, Sub-drone route removal {2,7}



- c.) Repair operators on 2EVRPD solution: Drone node insertion {9},  
 Truck node insertion {7}, Drone route creation {2}, {12,5}

**Fig. 3.** Example of LNS heuristic on 2EVRPD solution

## 5. Computational examples and results

This section examines the formulated MIP problem and the proposed algorithms using numerical examples. We conducted our experiments from different sets of instances, taken from four classic sets of the CVRP benchmark from Augerat et al. (sets A, B, and P) and Christofides and Elion (set E). The input data are available online at the Capacitated Vehicle Routing Problem Library (Augerat et al., 1995). We assume that the travel time can be represented by the cost metric associated with the benchmark problem. We set the truck travel time to be 1.5 time units longer than the drone travel time ( $\tau_{j,i}^T = 1.5 \tau_{i,j}^D$ ) since the drone speed is roughly about 1.5 times faster than the truck speed (Brar et al., 2015). The service times for the launch and recovery (SL/SR) are set to be 1 time unit. The truck capacity and the number of trucks for each instance are excerpted from the instance input as well. Other parameters are currently assigned randomly and are subject to more calibrations in the future work. For the headers of all the tables presented in this section, we refer to  $n$  as the number of customer nodes,  $K$  as the number of trucks,  $KD$  as the number of drones,  $Q$  as the truck capacity, and  $QD$  as the drone capacity. We assume that both truck and drone travel in Euclidean space. All the algorithms were executed in Matlab on a computer with 2.7GHz Intel Core i5 with 8GB ram RAM running Windows 7 64-bit mode. All the mixed integer linear programming models were solved using GAMS 23.51 with CPLEX solver.

*5.1 Heuristics performance and comparison of 2EVRPD-MIP model and CVRP-MIP model on small-size problems*

In this experiment, we compare the solution obtained from the suggested heuristics with the optimal solution from the CPLEX solver on 2EVRPD and compare the optimal solution between the proposed 2EVRPD and the classical CVRP on small-size problems. Exact solutions are obtained for both models using the CPLEX solver for comparison. We ran LNS and DTRC heuristics 10 times for each instance and report the Average Objective, Average Runtime (second) and Average GAP (%), which is the difference between the Average objective of both heuristics and the Optimal objective in term of percentage. The goal of this experiment is threefold: First, we want to evaluate cost (time) saving by implementing the multiple drops by drone operation with trucks under the small-size problem circumstance. Second, we want to evaluate the performance of both heuristics when comparing with the exact solution from the MIP solver. Third, we want to get an estimation of how long it would take to solve the 2EVRPD using the MIP method to obtain an optimal solution and compare it with the computational times from LNS/DTRC. To simplify and reduce the amount of runtime, we modified the benchmark problems by reducing the number of customer nodes to be 8 and limit the number of trucks to be 2. The results are shown in Table 1.

**Table 1.** Comparison of the results between MIP-2EVRPD and MIP-CVRP on small-size instance with the setting: n=8, K = 2, Q = 100, KD = 2, QD = 40

Instance	2EVRPD						CVRP Optimal	GAP % (2EVRPD V.S. CVRP)		
	Heuristics			MIP						
	LNS	Avg % Gap	Time (sec)	DTRC	Avg % Gap	Time (sec)				
A1-n8-k2	293	0.0	30.15	337.00	15.0	0.24	293	4524.67		
A2-n8-k2	232	0.9	30.60	258.60	12.4	0.31	230	7861.05		
A3-n8-k2	179	0.0	30.35	201.00	12.3	0.29	179	5119.56		
B1-n8-k2	316	0.0	30.86	322.00	1.9	0.34	316	4208.31		
B2-n8-k2	180	0.6	30.31	184.60	3.1	0.29	179	9615.45		
P1-n8-k2	124	5.1	30.29	130.00	10.2	0.31	118	4258.62		
P2-n8-k2	130	2.4	30.22	133.00	4.7	0.35	127	2861.38		
Average	<b>1.3</b>	<b>30.40</b>		<b>8.5</b>	<b>0.30</b>		<b>5492.72</b>	<b>13.20</b>		

From Table 1, the LNS heuristic performs well in all small-size problems while consuming significantly less computational time than the CPLEX. The Avg % Gap column shows that the LNS can find near optimal/optimal solutions for most instances (< 1% for instance A-B) with the mean of 1.3% for all instances. However, the DTRC heuristic returns mixed results of both small and big relative gap ranging from 1% to 15% on various instances with the mean of 8.5% for all instances. When comparing the optimal solution of 2EVRPD and CVRP, the results show that implementing the drones with the multiple drops feature could reduce the objective approximately by 13.20% (5.29% to 24.59%) depending on the instance. In general, we expect the saving to be lower than the classical CVRP but varied by the location of the customer nodes.

In terms of runtime, both heuristics use significantly less computational time than CPLEX while still returning a satisfying result with the mean of 30.40 seconds and 0.3 seconds for LNS and DTRC accordingly. As the solver takes a significant amount of time (as showing 5492.72 seconds on average) to generate the results even for the small-size problem, it might not be worth the time to compute for an exact solution on the larger-size problems. Therefore, we conducted more experiments to solve 2EVRPD using the proposed heuristics described in Section 4.

### *5.2 Comparison of the proposed heuristics and the 2EVRPD-MIP model*

In this section, we tested our proposed heuristics with the larger problem size that CPLEX cannot obtain the exact solutions within a reasonable amount of time. We retrieved some of the instances from the benchmark problems and tested them with our heuristics. We ran each heuristic 10 iterations for each instance and reported the best objective among the iterations, the average objectives, and the average runtime. We also computed the GAP of the average objective between the one from CPLEX and the other two from both heuristics. The negative

GAP indicates that the solutions from heuristic algorithms perform better. In addition, we ran the CPLEX for one hour for each instance and compare the results with the proposed algorithms. The numerical results are listed in Table 2.

**Table 2.** Results of the proposed heuristic and CPLEX on the larger instances

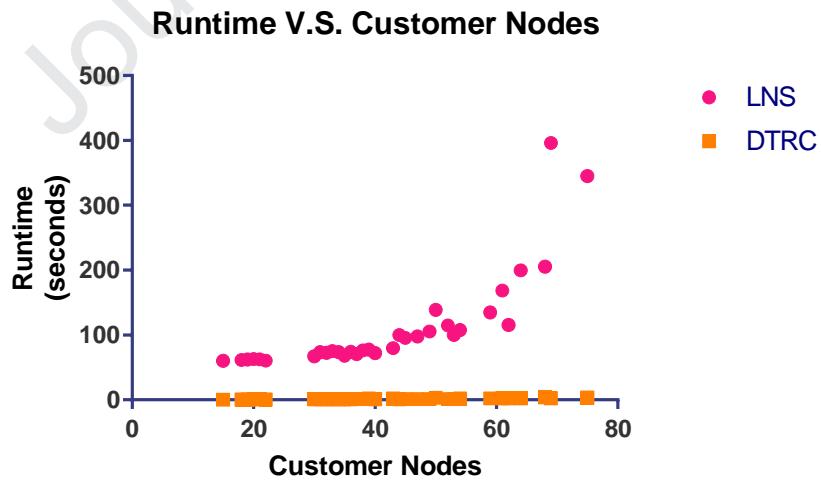
Instance	n	K	Q	KD	QD	CPLEX			LNS				DTRC			
						Obj	Duality GAP	Time (Sec)	Best Obj	Avg Obj	Avg Gap	Avg Time	Best Obj	Avg Obj	Avg Gap	Avg Time
A-n32-k5	31	5	100	2	35	1726	100	3600	677	713.20	<b>-58.68</b>	73.37	790	791.6	<b>-54.14</b>	0.86
A-n33-k5	32	5	100	2	35	1408	100	3600	546	568.50	<b>-59.62</b>	73.71	643	655.9	<b>-53.42</b>	1.05
A-n33-k6	32	6	100	2	35	1529	100	3600	629	636.40	<b>-58.38</b>	71.66	708	708	<b>-53.70</b>	0.91
B-n31-k5	30	5	100	2	35	1348	100	3600	655	657.30	<b>-51.24</b>	67.37	674	675.7	<b>-49.87</b>	0.92
B-n34-k5	33	5	100	2	35	1523	100	3600	740	751.10	<b>-50.68</b>	77.62	772	775.2	<b>-49.10</b>	1.19
P-n16-k8	15	8	35	2	20	450	61	3600	444	444.00	<b>-1.33</b>	60.41	469	469	4.22	0.36
P-n19-k2	18	2	160	2	40	236	100	3600	146	155.60	<b>-34.07</b>	61.51	203	210	<b>-11.02</b>	0.83
P-n20-k2	19	2	160	2	40	216	100	3600	152	160.00	<b>-25.93</b>	62.11	201	207.5	<b>-3.94</b>	0.74
P-n21-k2	20	2	160	2	40	245	100	3600	144	149.70	<b>-38.90</b>	63.07	215	218.8	<b>-10.69</b>	0.96
P-n22-k2	21	2	160	2	40	240	100	3600	154	159.30	<b>-33.63</b>	62.74	208	209	<b>-12.92</b>	0.92
						Average			<b>-41.25</b>	67.36					<b>-29.46</b>	0.87

As the results show, both LNS and DTRC algorithms can obtain better results in significantly less time compared to CPLEX. On average, we see -41.25% average GAP from CPLEX for the LNS and -29.46% average GAP from CPLEX for the DTRC. Please note that the objective reported from CPLEX is the current solution found in one hour with the reported duality gap by CPLEX. For most instances, the duality gap is 100% after one hour, which indicates that it was extremely time consuming for CPLEX to solve this complex combinatorial problem and the solver can only find one or two feasible solutions before it got terminated. The results demonstrate that the two heuristics can return reasonably good quality solutions when compared to the solutions of CPLEX within one hour. In terms of computational time, DTRC can return the solution using the lowest CPU runtime with the average of only 0.87 seconds followed by LNS with the average of 67.36 seconds and CPLEX with the fixed time at 3600 seconds respectively.

### 5.3 Comparison of the proposed heuristics and the optimal solution of CVRP on various instances

In this section, we want to test the performance of the proposed heuristics on different variations of instances. A total of 50 test benchmark CVRP instances were used in this experiment. Each benchmark instance was run 10 times independently. We compare the performance of our two heuristics, LNS and DTRC, with the CVRP optimal solutions for each instance from the literature. The results are shown in Table 3.

In terms of solution quality, the LNS returns better objective values than the DTRC for all instances with an average GAP about 10%. However, the computational time of the DTRC heuristic is about 98% lower on average. Figure 4 shows the variable number growth for different problem sizes and the time increase as problem size increases. The LNS plot shows that the number of variables grows exponentially (once greater than 40) as the number of customer nodes increases linearly while the DTRC plot does not show an obvious correlation.



**Fig. 4.** The line scatter plot showing the runtime of different problem size

When comparing with the optimal CVRP solutions, both LNS and DTRC heuristics provide lower objective values by 12.06% and 2.32% respectively, as shown in Table 3 (Negative GAP

on the table means better objective). Considering the relatively moderate GAP difference between the LNS and optimal CVRP solutions on the different problem sets, LNS is quite effective and can be reliable for solving 2EVRPD with the appropriate parameters tuning. Although the heuristic might not return the optimal solution, the high average gap from the optimal CVRP solution still shows that it is more beneficial to implement multiple drones in the operation.

**Table 3.** Results of the proposed heuristics and optimal CVRP on the various instances

Instance	n	K	Q	KD	QD	CVRP (Optimal)	Algorithm Method				
							Large Neighborhood search (LNS)				
							GAP (From Truck)	Best	Average	Time	
A-n32-k5	31	5	100	2	35	784	<b>-13.65</b>	677	713.2	73.37	<b>0.77</b>
A-n33-k5	32	5	100	2	35	661	<b>-17.40</b>	546	568.5	73.71	<b>-2.72</b>
A-n33-k6	32	6	100	2	35	742	<b>-15.23</b>	629	636.4	71.66	<b>-4.58</b>
A-n34-k5	33	5	100	2	35	778	<b>-17.22</b>	644	655.5	72.50	<b>-5.14</b>
A-n36-k5	35	5	100	2	35	799	<b>-18.40</b>	652	689.4	68.05	<b>-5.97</b>
A-n37-k5	36	6	100	2	35	669	<b>-20.48</b>	532	553.2	80.35	<b>-3.89</b>
A-n37-k6	36	6	100	2	35	949	<b>-12.12</b>	834	855.7	67.26	<b>-8.11</b>
A-n38-k5	37	5	100	2	35	730	<b>-19.32</b>	589	646.5	68.19	<b>0.27</b>
A-n39-k5	38	5	100	2	35	822	<b>-16.18</b>	689	723.9	73.23	<b>-1.09</b>
A-n39-k6	38	6	100	2	35	831	<b>-17.93</b>	682	718.2	81.10	<b>-0.72</b>
A-n44-k6	43	6	100	2	35	937	<b>-13.55</b>	810	848.7	79.62	<b>-3.52</b>
A-n45-k6	44	6	100	2	35	944	<b>-11.86</b>	832	863	94.52	<b>-1.06</b>
A-n46-k7	45	7	100	2	35	914	<b>-14.99</b>	777	807.6	95.53	<b>-1.97</b>
A-n48-k7	47	7	100	2	35	1073	<b>-9.79</b>	968	984.4	97.89	<b>-0.75</b>
A-n53-k7	52	7	100	2	35	1010	<b>-8.71</b>	922	950.8	114.61	<b>-3.47</b>
A-n54-k7	53	7	100	2	35	1167	<b>-10.45</b>	1045	1080.8	100.27	<b>-3.17</b>
A-n55-k9	54	9	100	2	35	1073	<b>-7.83</b>	989	1010.8	125.71	<b>-0.84</b>
A-n62-k8	61	8	100	2	35	1288	<b>-6.83</b>	1200	1246.2	168.36	<b>-1.32</b>
A-n63-k10	62	10	100	2	35	1314	<b>-4.19</b>	1259	1282.8	115.62	<b>-1.29</b>
A-n65-k9	64	9	100	2	35	1174	<b>-6.98</b>	1092	1121	197.38	<b>-0.43</b>
A-n69-k9	68	9	100	2	35	1159	<b>-10.70</b>	1035	1089.7	205.74	<b>-6.30</b>
B-n31-k5	30	5	100	2	35	672	<b>-2.53</b>	655	657.3	67.37	<b>0.30</b>
B-n34-k5	33	5	100	2	35	788	<b>-6.09</b>	740	751.1	77.62	<b>-2.03</b>
B-n35-k5	34	5	100	2	35	955	<b>-6.81</b>	890	895	73.51	<b>-0.21</b>
B-n38-k6	37	6	100	2	35	805	<b>-11.55</b>	712	745	73.06	<b>-0.25</b>
B-n39-k5	38	5	100	2	35	549	<b>-9.11</b>	499	510.1	74.49	<b>-0.73</b>
B-n41-k6	40	6	100	2	35	829	<b>-2.29</b>	810	832	72.32	<b>5.55</b>
E-n51-k5	50	5	160	2	50	521	<b>-18.23</b>	426	455.6	139.10	<b>-6.53</b>
E-n76-k7	75	7	220	2	55	682	<b>-7.48</b>	631	645.9	370.69	<b>-12.90</b>
E-n76-k8	75	8	180	2	45	735	<b>-9.39</b>	666	687.6	363.38	<b>-9.52</b>
E-n76-k10	75	10	140	2	40	830	<b>-5.54</b>	784	803.5	378.17	<b>-1.57</b>
E-n76-k14	75	14	100	2	35	1021	<b>-4.02</b>	980	1016.2	268.62	<b>-0.69</b>
P-n16-k8	15	8	35	2	20	450	<b>-1.33</b>	444	444	60.41	<b>4.22</b>
P-n19-k2	18	2	160	2	40	212	<b>-31.13</b>	146	155.6	61.51	<b>-4.25</b>
P-n20-k2	19	2	160	2	40	216	<b>-29.63</b>	152	160	62.11	<b>-6.94</b>
P-n21-k2	20	2	160	2	40	211	<b>-31.75</b>	144	149.7	63.07	<b>1.90</b>
P-n22-k2	21	2	160	2	40	216	<b>-28.70</b>	154	159.3	62.74	<b>-3.70</b>
P-n23-k8	22	8	40	2	20	529	<b>-3.78</b>	509	512.8	60.56	<b>-0.38</b>
P-n40-k5	39	5	140	2	40	458	<b>-19.87</b>	367	383.1	77.73	<b>-1.53</b>
P-n45-k5	44	5	150	2	40	510	<b>-21.57</b>	400	413.9	106.31	<b>-4.90</b>
P-n50-k7	49	7	150	2	40	554	<b>-15.16</b>	470	511.2	103.22	<b>-0.72</b>
P-n50-k8	49	8	120	2	40	631	<b>-8.72</b>	576	585.8	120.93	<b>-4.12</b>
P-n50-k10	49	10	100	2	35	696	<b>-6.90</b>	648	675.4	92.55	<b>0.43</b>

P-n55-k7	54	7	170	2	45	568	<b>-11.80</b>	501	533.2	120.99	<b>-2.46</b>	554	559.9	2.04
P-n55-k10	54	10	115	2	40	694	<b>-3.60</b>	669	679.3	106.81	<b>0.29</b>	696	698.6	1.89
P-n55-k15	54	15	70	2	38	989	<b>-7.58</b>	914	941.1	77.41	<b>-5.36</b>	936	944	1.49
P-n60-k10	59	10	120	2	40	744	<b>-6.72</b>	694	721.4	140.93	<b>-2.15</b>	728	733.3	2.40
P-n60-k15	59	15	80	2	30	968	<b>-4.24</b>	927	946.9	128.51	<b>-0.41</b>	964	970	1.78
P-n65-k10	64	10	130	2	40	792	<b>-8.21</b>	727	749.3	202.15	<b>-2.40</b>	773	791.5	2.40
P-n70-k10	69	10	135	2	40	827	<b>-5.68</b>	780	803.8	396.18	<b>0.48</b>	831	842.1	2.53
Average						<b>-12.06</b>			122.54		<b>-2.32</b>			1.71

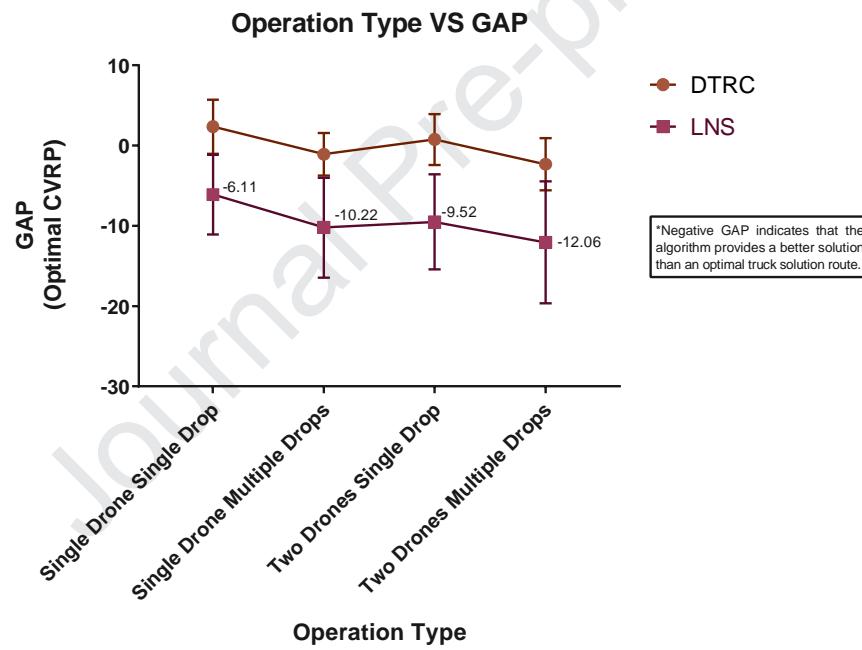
#### 5.4 Sensitivity Analysis

Sensitivity analysis was conducted to evaluate the impact of major parameters and components of the method. We are specifically interested in making a comparison between different numbers of drones and see how it would affect the performance of the 2EVRPD solution. In addition, we want to see the impact of our new multiple drops feature when comparing with the typical single drop proposed by Murray and Chu (2015). Four different operational settings are evaluated, including Single drone / Single drop; Single drone / Multiple drops; Two drones / Single drop; and Two drone / Multiple drops. All four types of operations are compared with the optimal solutions from the classical truck only delivery (CVRP).

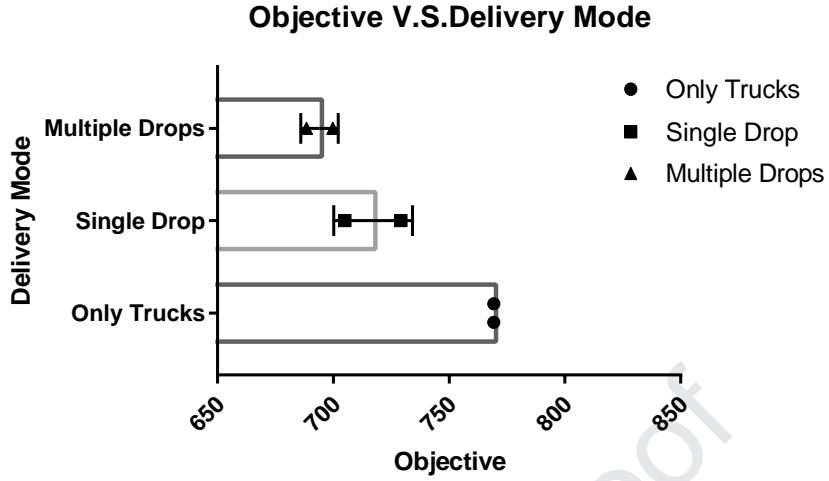
Using the same benchmark instances as Section 5.3, we conducted the experiment using our proposed heuristics for four types of operations. Each instance was run 10 times and we recorded all the experimental results in Table 4. We use these results to generate the statistical plot in Figure 5 showing the percentage GAP between each operation objective and CVRP objective for all instances with the mean and standard deviation.

Referring to Table 4, all four types of operations' solutions solved by the LNS algorithm provide the GAP from an optimal CVRP solution much lower than the ones solved by the DTRC algorithm. Among the types of operations, the Two drones / Multiple drops operation provides the best solution (with the mean GAP of -13.25%), and the Single drone / Single drop operation provides the worst (with the mean GAP of -7.44%) as visualized by Figure 5. As we expect, having more drones leads to lower total route delivery time. In addition, we reported the solution

objectives among different types of delivery features as shown in Figure 6 with the lowest (best) mean objective values from using trucks and drones with the multiple drops feature followed by using trucks and drones with the single drop feature, and only trucks accordingly. It can also be observed that the Single drone / Multiple drops operation provides a slightly better solution (with the mean GAP of -11.23%) than the Two Drones / Single drops operation (with the mean GAP of -10.22%). The result from this experiment demonstrates the potential benefit of implementing truck and drones together with the multiple drops feature in order to significantly reduce the total travel time of the entire delivery.



**Fig. 5.** The comparison of different operational types using the GAP from a truck alone delivery. Negative GAP indicates an improvement over the truck alone operation



**Fig. 6.** The comparison of objective values among different delivery modes

## 6. Conclusions and future work

In this paper, we propose a new routing model, the Two Echelon Vehicle Routing Problems with Drones (2EVRPD), which implements both trucks and drones in the last mile delivery. The model is a variation of the classic CVRP problem and the extension of the previous FSTSP model. We generalize our previous work mTSPD, in which we allow multiple drones and multiple trucks to perform deliveries, to 2EVRPD, which includes capacity constraints and a multiple drops feature. The MIP formulation is mathematically constructed to model the 2EVRPD to solve for an optimal solution for the small-size instances, for which the results are shown in Section 5.1. To solve the large-size instances, we develop two heuristics: Drone Truck Route Construction (DTRC), which is a constructive heuristic used for creating an initial 2EVRPD solution from an empty route, and LNS, which iteratively improves the 2EVRPD solution using destroy and repair principles. Using these heuristics, we are able to solve and obtain better solutions than the CPLEX solver under the same computational time as shown in Section 5.2. The LNS heuristic generates higher quality than DTRC when tested on the various

CVRP benchmark problems. LNS also returns the solutions better than the reported optimal CVRP on the same instances (Section 5.3), which implies that using this new approach provides shorter delivery time than simply using trucks alone in the operation. Lastly, the results from the sensitivity analysis in Section 5.4 shows that implementing a multiple drops feature improves the solution quality from the typical single drop feature.

This research work can also be extended by using different types of both vehicles (heterogeneous fleets of both trucks and drones) as well as multiple depots to expand the range of the operation. It would also be interesting to study how the payload (weight) and the battery consumption can affect the drone travel performance and prioritize which customer orders should get served first. In addition, future works may include designing the system which integrates the proposed model with drone only delivery as a unified last mile delivery system. From an algorithmic perspective, designing other meta-heuristic algorithms could be an effective way to solve the 2EVRPD problem with a better objective value and lower computational time. Lastly, the research direction can be shifted to the logistics problem with drones which involves more strategic decisions such as the location of depots as well as both tactical and operational which include assigning customers to the opened depots and the organization of the vehicles with drones routing (LRPD)

**Table 4.** Results of the proposed heuristics and optimal CVRP in different types of operations

Instance	n	K	Q	Qd	CVRP (Optima- l)	Single Drone												Two Drones											
						Single Drop						Multiple Drops						Single Drop						Multiple Drops					
						DTRC			LNS			DTRC			LNS			DTRC			LNS			DTRC			LNS		
						Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time
A-n32-k5	31	5	100	35	784	857.00	861.00	0.63	750.00	764.20	71.12	783.00	794.30	0.68	699.00	725.80	68.36	822.00	834.80	0.89	714.00	736.30	68.52	790.00	791.60	0.86	677.00	713.20	73.37
A-n33-k5	32	5	100	35	661	696.00	701.10	0.88	593.00	601.10	66.95	646.00	656.70	0.79	561.00	571.80	68.36	688.00	690.40	1.13	567.00	579.10	69.76	643.00	655.90	1.05	546.00	568.50	73.71
A-n33-k6	32	6	100	35	742	706.00	706.00	0.69	653.00	657.10	10.06	701.00	701.00	0.69	634.00	645.30	63.53	708.00	708.00	0.93	631.00	637.90	74.32	708.00	708.00	0.91	629.00	636.40	71.66
A-n34-k5	33	5	100	35	778	772.00	775.50	0.84	671.00	691.00	70.23	752.00	756.00	0.86	648.00	660.90	66.14	761.00	769.40	1.05	651.00	672.00	69.37	738.00	745.40	1.02	644.00	655.50	72.50
A-n36-k5	35	5	100	35	799	818.00	822.20	0.95	717.00	736.80	66.31	759.00	760.50	1.11	669.00	688.70	68.63	791.00	797.70	1.32	682.00	704.50	77.11	751.30	751.00	1.21	652.00	689.40	68.05
A-n37-k6	36	6	100	35	669	670.00	670.00	0.89	577.00	596.50	69.95	661.00	668.00	1.25	516.00	549.50	68.40	647.00	648.70	1.26	545.00	574.20	88.44	643.00	646.60	1.08	532.00	553.20	80.35
A-n37-k6	36	6	100	35	949	926.00	928.20	1.03	861.00	884.70	66.20	914.00	914.00	1.01	837.00	850.80	66.45	906.00	917.00	1.25	857.00	872.30	73.74	872.00	872.60	1.24	834.00	855.70	67.26
A-n38-k5	37	5	100	35	730	770.00	770.00	1.03	654.00	679.40	68.28	721.00	721.00	1.03	636.00	648.10	70.57	770.00	771.90	1.29	634.00	667.30	76.62	732.00	732.30	1.19	589.00	646.50	68.19
A-n39-k5	38	5	100	35	822	887.00	901.20	2.17	742.00	775.20	73.15	833.00	847.50	2.35	698.00	732.30	66.38	876.00	893.50	3.00	714.00	747.20	76.98	813.00	824.70	2.42	689.00	723.90	73.23
A-n39-k6	38	6	100	35	831	830.00	833.10	1.16	758.00	776.20	71.07	849.00	849.00	1.12	692.00	741.20	76.10	803.00	813.80	1.41	717.00	740.20	70.44	825.00	825.00	1.33	682.00	718.20	81.10
A-n44-k6	43	6	100	35	937	931.00	933.60	1.86	906.00	920.60	73.57	914.00	914.70	1.78	825.00	860.20	72.64	922.00	924.70	2.51	839.00	869.60	92.63	904.00	905.10	2.06	810.00	848.70	79.62
A-n45-k6	44	6	100	35	944	967.00	968.00	0.85	902.00	919.50	79.17	943.00	943.00	0.92	846.00	869.40	85.52	958.00	960.70	1.20	854.00	893.30	90.76	934.00	934.00	1.08	832.00	863.00	94.52
A-n45-k7	44	7	100	35	1146	1168.00	1179.60	1.42	1125.00	1147.20	75.78	1168.00	1187.20	1.43	1102.00	1143.30	70.91	1125.00	1153.10	1.26	1110.00	1129.80	71.81	1153.00	1168.10	1.61	1084.00	1121.00	76.97
A-n46-k7	45	7	100	35	914	951.00	953.10	1.20	828.00	848.30	80.15	905.00	906.50	1.27	810.00	828.60	90.40	914.00	920.80	1.46	804.00	837.50	86.98	896.00	897.00	1.41	777.00	807.60	95.53
A-n48-k7	47	7	100	35	1073	1089.00	1102.80	1.29	998.00	1023.40	90.15	1075.00	1079.10	1.34	973.00	996.50	73.54	1087.00	1094.00	1.55	961.00	1003.40	121.79	1065.00	1066.30	1.53	968.00	984.40	97.89
A-n53-k7	52	7	100	35	1010	1073.00	1078.70	1.04	964.00	999.10	107.78	1000.00	1006.40	1.02	952.00	983.10	95.63	1060.00	1066.40	1.44	949.00	988.90	144.23	975.00	984.40	1.25	922.00	950.80	114.61
A-n54-k7	53	7	100	35	1167	1158.00	1164.20	1.17	1103.00	1122.40	81.69	1143.00	1146.80	1.58	1057.00	1100.60	91.10	1129.00	1143.70	1.52	1076.00	1095.90	120.12	1130.00	1133.90	1.44	1045.00	1080.80	100.27
A-n55-k9	54	9	100	35	1073	1094.00	1095.80	1.49	1031.00	1049.10	90.48	1057.00	1059.70	1.53	981.00	1011.00	94.46	1084.00	1084.00	2.11	1007.00	1027.50	142.06	1064.00	1071.70	1.82	989.00	1010.80	125.71
A-n60-k9	59	9	100	35	1354	1368.00	1371.90	1.98	1317.00	1331.30	119.00	1334.00	1342.30	1.93	1270.00	1298.00	138.87	1377.00	1384.70	2.32	1271.00	1299.60	203.44	1336.00	1341.60	2.41	1237.00	1282.60	213.00
A-n61-k9	60	9	100	35	1034	1367.00	1372.50	1.98	1301.00	1328.70	101.13	1331.00	1339.40	1.92	1287.00	1314.00	102.64	1378.00	1387.30	2.77	1274.00	1314.60	142.50	1331.00	1339.00	2.77	1263.00	1301.00	217.58
A-n62-k8	61	8	100	35	1288	1319.00	1321.90	2.66	1266.00	1290.60	112.05	1287.00	1299.20	3.22	1241.00	1274.60	118.51	1289.00	1295.00	3.82	1236.00	1260.70	260.83	1271.00	1275.60	2.84	1200.00	1246.20	168.36
A-n63-k9	62	9	100	35	1616	1645.00	1648.70	2.73	1629.00	1654.90	109.16	1619.00	1619.00	2.88	1573.00	1593.60	90.99	1624.00	1626.30	3.30	1570.00	1613.90	142.07	1615.00	1615.90	2.95	1567.00	1598.30	145.77
A-n63-k10	62	10	100	35	1314	1340.00	1342.50	2.29	1293.00	1314.60	114.32	1305.00	1306.50	2.26	1267.00	1281.30	93.83	1351.00	1355.50	3.01	1241.00	1301.80	178.51	1297.00	1297.90	2.71	1259.00	1282.80	115.62
A-n64-k9	63	9	100	35	1401	1455.00	1459.00	1.53	1420.00	1441.90	104.95	1420.00	1425.80	1.48	1366.00	1426.50	115.74	1414.00	1418.40	1.93	1402.00	1411.60	116.15	1420.00	1423.70	1.81	1358.00	1410.20	242.81
A-n65-k9	64	9	100	35	1174	1222.00	1231.80	2.81	1159.00	1183.80	155.56	1180.00	1183.60	2.96	1080.00	1135.40	145.90	1216.00	1398.40	2.25	1109.00	1144.00	212.95	1169.00	1173.10	3.18	1092.00	1121.00	197.38
A-n69-k9	68	9	100	35	1159	1158.00	1162.60	2.45	1100.00	1157.40	153.99	1078.00	1089.10	2.40	1079.00	1121.60	152.80	1147.00	1151.00	3.48	1100.00	1126.50	247.68	1086.00	1093.20	3.80	1035.00	1089.70	205.74
B-n21-k5	30	5	100	35	672	673.00	675.20	0.70	662.00	664.20	63.60	672.00	674.60	0.72	653.00	657.00	66.35	676.00	677.20	0.98	660.00	662.00	66.12	674.00	675.70	0.92	655.00	657.30	67.37
B-n34-k5	33	5	100	35	788	792.00	794.60	0.92	759.00	768.50	70.35	769.00	777.70	0.87	748.00	752.10	69.87	791.00	796.80	1.20	754.00	764.80	67.41	772.00	775.20	1.19	740.00	751.10	77.62
B-n35-k5	34	5	100	35	955	974.00	976.00	0.71	914.00	916.90	71.94	954.00	960.40	0.71	896.00	902.30	67.05	972.00	977.00	1.04	896.00	901.90	74.21	953.00	959.60	0.93	890.00	895.00	73.51
B-n38-k5	37	6	100	35	805	828.00	829.90	1.12	735.00	752.30	69.73	803.00	805.90	1.11	723.00	741.50	69.44	814.00	818.10	1.39	723.00	747.30	82.26	803.00	804.50	1.37	712.00	745.00	73.06
B-n39-k5	38	5	100	35	549	546.00	548.30	0.95	518.00	523.40	73.33	548.00	548.80	0.97	497.00	505.60	75.09	545.00	548.10	1.30	512.00	518.30	71.48	545.00</td					

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Instance	n	K	Q	Qd	(Optima l)	CVRP						Single Drone						Two Drones								
						Single Drop			Multiple Drops			Single Drop			LNS			DTRC			LNS					
						Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time	Best	Average	Time			
A-n32-k5	31	5	100	35	784	857.00	861.00	0.63	750.00	764.20	71.12	783.00	794.30	0.68	699.00	725.80	68.36	822.00	834.80	0.89	714.00	736.30	68.52	790.00	791.60	0.86
A-n33-k5	32	5	100	35	661	696.00	701.10	0.88	593.00	601.10	66.95	646.00	656.70	0.79	561.00	571.80	68.36	688.00	690.40	1.13	567.00	579.10	69.76	643.00	655.90	1.05
A-n33-k6	32	6	100	35	742	706.00	706.00	0.69	653.00	657.10	10.06	701.00	701.00	0.69	634.00	645.30	63.53	708.00	708.00	0.93	631.00	637.90	74.32	708.00	708.00	0.91
A-n34-k5	33	5	100	35	778	772.00	775.50	0.84	671.00	691.00	70.23	752.00	756.00	0.86	648.00	660.90	66.14	761.00	769.40	1.05	651.00	672.00	69.37	738.00	745.40	1.02
A-n35-k5	35	5	100	35	799	818.00	822.20	0.95	717.00	736.80	66.31	759.00	760.50	1.11	669.00	688.70	68.63	791.00	797.70	1.32	682.00	704.50	77.11	751.30	751.00	1.21
A-n37-k5	36	6	100	35	669	670.00	670.00	0.89	577.00	596.50	69.95	661.00	668.00	1.25	516.00	549.50	68.40	647.00	648.70	1.26	545.00	574.20	88.44	643.00	646.60	1.08
A-n37-k6	36	6	100	35	949	926.00	928.20	1.03	861.00	884.70	66.20	914.00	914.00	1.01	837.00	850.80	66.45	906.00	917.00	1.25	857.00	872.30	73.74	872.00	872.60	1.24
A-n38-k5	37	5	100	35	730	770.00	770.00	1.03	654.00	679.40	68.28	721.00	721.00	1.03	636.00	648.10	70.57	770.00	771.90	1.29	634.00	667.30	76.62	732.00	732.30	1.19
A-n39-k5	38	5	100	35	822	887.00	901.20	2.17	742.00	752.30	73.15	833.00	847.50	2.35	698.00	732.30	66.38	876.00	893.50	3.00	714.00	747.20	76.98	813.00	824.70	2.42
A-n39-k6	38	6	100	35	831	830.00	833.10	1.16	758.00	776.20	71.07	849.00	849.00	1.12	692.00	741.20	76.10	803.00	813.80	1.41	717.00	740.20	70.44	825.00	825.00	1.33
A-n44-k6	43	6	100	35	937	931.00	933.60	1.86	906.00	920.60	73.57	914.00	914.70	1.78	825.00	860.20	72.64	922.00	924.70	2.51	839.00	869.60	92.63	904.00	909.10	2.06
A-n45-k6	44	6	100	35	944	967.00	968.00	0.85	902.00	919.50	79.17	943.00	943.00	0.92	846.00	869.40	85.52	958.00	960.70	1.20	854.00	893.30	90.76	934.00	934.00	1.08
A-n45-k7	44	7	100	35	1146	1168.00	1179.60	1.42	1125.00	1147.20	75.78	1168.00	1187.20	1.43	1102.00	1143.30	70.91	1125.00	1153.10	1.26	1110.00	1129.80	71.81	1153.00	1168.10	1.61
A-n46-k7	45	7	100	35	914	953.10	953.10	1.20	828.00	848.30	80.15	905.00	906.50	1.27	810.00	828.60	90.40	914.00	920.80	1.46	804.00	837.50	86.98	896.00	897.00	1.41
A-n48-k7	47	7	100	35	1073	1089.00	1102.80	1.29	998.00	1023.40	90.15	1075.00	1079.10	1.34	973.00	996.50	73.54	1087.00	1094.00	1.55	961.00	1003.40	121.79	1065.00	1066.30	1.53
A-n53-k7	52	7	100	35	1010	1073.00	1078.70	1.04	964.00	999.10	107.78	1000.00	1006.40	1.02	952.00	983.10	95.63	1060.00	1065.40	1.44	949.00	988.90	144.23	975.00	984.40	1.25
A-n54-k7	53	7	100	35	1167	1158.00	1164.20	1.17	1103.00	1122.40	81.69	1143.00	1146.80	1.58	1057.00	1100.60	93.10	1129.00	1143.70	1.52	1076.00	1095.90	120.12	1130.00	1133.90	1.44
A-n55-k9	54	9	100	35	1073	1094.00	1095.80	1.49	1031.00	1049.10	90.48	1057.00	1057.90	1.53	981.00	1011.00	94.46	1084.00	1084.00	2.11	1007.00	1027.50	142.06	1064.00	1071.70	1.82
A-n60-k9	59	9	100	35	1354	1368.00	1371.90	1.98	1317.00	1331.30	119.00	1334.00	1342.30	1.93	1270.00	1298.00	138.87	1377.00	1384.70	2.32	1271.00	1299.60	203.44	1336.00	1341.60	2.41
A-n61-k9	60	9	100	35	1034	1367.00	1372.50	1.98	1301.00	1328.70	101.13	1331.00	1339.40	1.92	1287.00	1314.00	102.64	1378.00	1387.30	2.77	1274.00	1314.60	142.50	1331.00	1339.00	2.77
A-n62-k8	61	8	100	35	1288	1319.00	1321.90	2.66	1266.00	1290.60	112.05	1287.00	1293.20	3.32	1241.00	1274.60	118.51	1289.00	1295.50	3.82	1236.00	1260.70	260.83	1271.00	1275.60	2.84
A-n63-k9	62	9	100	35	1616	1645.00	1648.70	2.73	1629.00	1654.90	109.16	1619.00	1619.90	2.88	1573.00	1593.60	90.99	1624.00	1626.30	3.30	1570.00	1613.90	142.07	1615.00	1615.90	2.95
A-n63-k10	62	10	100	35	1314	1340.00	1342.50	2.29	1293.00	1314.60	114.32	1305.00	1306.50	2.26	1267.00	1281.30	93.83	1351.00	1355.50	3.01	1241.00	1301.80	178.51	1297.00	1297.90	2.71
A-n64-k9	63	9	100	35	1401	1455.00	1459.90	1.53	1420.00	1425.80	1.48	1420.00	1426.50	115.74	1366.00	1426.50	115.74	1414.00	1418.40	1.93	1402.00	1423.70	1.81	1358.00	1410.20	242.81
A-n65-k9	64	9	100	35	1174	1222.00	1231.80	2.81	1159.00	1183.80	155.56	1180.00	1183.60	2.96	1080.00	1135.40	145.90	1216.00	1398.40	2.25	1109.00	1144.00	212.95	1169.00	1173.10	3.18
A-n69-k9	68	9	100	35	1159	1158.00	1162.60	2.45	1100.00	1157.40	153.99	1078.00	1089.10	2.40	1079.00	1121.60	152.80	1147.00	1151.00	3.48	1100.00	1126.50	247.68	1086.00	1093.20	3.80
B-n31-k5	30	5	100	35	672	673.00	675.20	0.70	662.00	664.20	63.60	672.00	674.60	0.72	653.00	657.00	66.35	676.00	677.20	0.98	660.00	662.00	66.12	674.00	675.70	0.92
B-n34-k5	33	5	100	35	788	792.00	794.60	0.92	759.00	768.50	70.35	769.00	777.70	0.87	748.00	752.10	69.87	791.00	796.80	1.20	754.00	764.80	67.41	772.00	775.20	1.19
B-n35-k5	34	5	100	35	955	974.00	979.60	0.71	914.00	954.00	960.40	0.71	896.00	903.20	67.05	972.00	977.00	1.04	896.00	901.90	74.21	953.00	959.60	0.93		
B-n38-k6	37	6	100	35	805	828.00	829.90	1.12	735.00	752.30	69.73	803.00	805.90	1.11	723.00	741.50	69.44	814.00	818.10	1.39	723.00	747.30	82.26	803.00	804.50	1.37
B-n39-k5	38	5	100	35	549	546.00	548.30	0.95	518.00	524.30	73.33	548.00	548.80	0.97	497.00	505.60	75.09	545.00	548.10	1.30	512.00	518.30	71.48	545.00	546.40	1.21
B-n41-k6	40	6	100	35	829	893.00	896.00	0.77	843.00	855.50	73.86	875.00	880.70	0.82	813.00	838.00	82.07	893.00	895.20	1.06	824.00	843.80	74.85	875.00	880.20	1.02
E-n51-k5	50	5	160	50	521	571.00	581.20	2.88	490.00	508.60	90.18	502.00	511.80	2.86	452.00	470.50	94.20	533.00	535.90	3.38	451.00	472.40	162.48	487.00	504.30	3.16
E-n77-k5	75	7	220	55	682	712.00	747.50	3.75	641.00	656.20	3.87	622.00	651.80	3.87	622.00	651.80	150.79	662.00	667.10	4.64	640.00	679.80	698.69	699.00	716.90	4.24
E-n76-k8	75	8	180	45	735	738.00	753.30	4.01	745.00	748.00	711.70	218.46	664.00	711.90	233.76	730.00	744.10	5.03	689.00	726.30	364.22	665.00	687.50	4.19		
E-n76-k10	75	10	140	40	830	874.00	877.60	3.02	826.00	870.60	205.25	827.00	832.80	3.06	795.00	818.40	289.56	855.00	857.40	3.39	805.00	828.40	413.21	817.00	827.30	3.53
E-n76-k14	75																									



## Author Contribution Statement

**Patchara Kitjacharoenchai:** Methodology, Software, Validation, Visualization, Writing - Original Draft, Writing - Review & Editing

**Byung-Cheol Min:** Conceptualization, Resources, Supervision

**Seokcheon Lee:** Supervision, Project administration