Lab 4: Recursion, Tree Recursion, Python Lists **lab04.zip (lab04.zip)**

Due by 11:59pm on Tuesday, September 22.

Starter Files

Download lab04.zip (lab04.zip). Inside the archive, you will find starter files for the questions in this lab, along with a copy of the Ok (ok) autograder.

Topics

Consult this section if you need a refresher on the material for this lab. It's okay to skip directly to the questions and refer back here should you get stuck.

Recursion | Tree Recursion | List Introduction

Required Questions

Lists Practice

Q1: List Indexing

Use Ok to test your knowledge with the following "List Indexing" questions:

```
python3 ok -q list-indexing -u
```

For each of the following lists, what is the list indexing expression that evaluates to `7`? For example, if `x = [7]`, then the answer would be `x[0]`. You can use the interpreter or Python Tutor to experiment with your answers. If the code would cause an error, type `Error`.

```
>>> x = [1, 3, [5, 7], 9]
______

>>> x = [[3, [5, 7], 9]]
______
```

What would Python display? If you get stuck, try it out in the Python interpreter!

```
>>> lst = [3, 2, 7, [84, 83, 82]]
>>> lst[4]
______

>>> lst[3][0]
______
```

Recursion

Q2: Skip Add

Write a function `skip_add` that takes a single argument `n` and computes the sum of every other integer between `0` and `n`. Assume `n` is non-negative.

```
this_file = __file__

def skip_add(n):
    """ Takes a number n and returns n + n-2 + n-4 + n-6 + ... + 0.

    >>> skip_add(5)  # 5 + 3 + 1 + 0
    9
    >>> skip_add(10) # 10 + 8 + 6 + 4 + 2 + 0
    30
    >>> # Do not use while/for loops!
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(this_file, 'skip_add',
    ...       ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q skip_add
```

Q3: Summation

Now, write a recursive implementation of `summation`, which takes a positive integer `n` and a function `term`. It applies `term` to every number from `1` to `n` including `n` and returns the sum of the results.

```
def summation(n, term):

    """Return the sum of the first n terms in the sequence defined by term.
    Implement using recursion!

    >>> summation(5, lambda x: x * x * x) # 1^3 + 2^3 + 3^3 + 4^3 + 5^3
    225
    >>> summation(9, lambda x: x + 1) # 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10
    54
    >>> summation(5, lambda x: 2**x) # 2^1 + 2^2 + 2^3 + 2^4 + 2^5
    62
    >>> # Do not use while/for loops!
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(this_file, 'summation',
    ...       ['While', 'For'])
    True
    """
    assert n >= 1
    "*** YOUR CODE HERE ***"
```

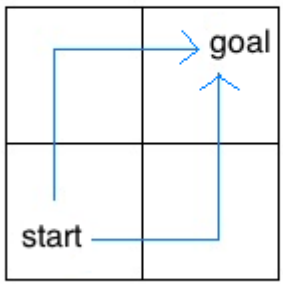
Use Ok to test your code:

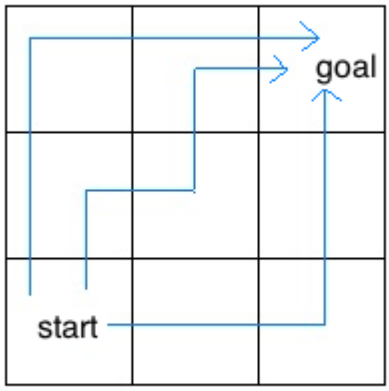
```
python3 ok -q summation
```

Tree Recursion

Q4: Insect Combinatorics

Consider an insect in an *M* by *N* grid. The insect starts at the bottom left corner, *(0, 0)*, and wants to end up at the top right corner, *(M-1, N-1)*. The insect is only capable of moving right or up. Write a function `paths` that takes a grid length and width and returns the number of different paths the insect can take from the start to the goal. (There is a closed-form solution (https://en.wikipedia.org/wiki/Closed-form_expression) to this problem, but try to answer it procedurally using recursion.)





For example, the 2 by 2 grid has a total of two ways for the insect to move from the start to the goal. For the 3 by 3 grid, the insect has 6 diferent paths (only 3 are shown above).

```
def paths(m, n):
    """Return the number of paths from one corner of an
    M by N grid to the opposite corner.

    >>> paths(2, 2)
    2
    >>> paths(5, 7)
    210
    >>> paths(117, 1)
    1
    >>> paths(1, 157)
    1
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q paths
```

Q5: Maximum Subsequence

A subsequence of a number is a series of (not necessarily contiguous) digits of the number. For example, 12345 has subsequences that include 123, 234, 124, 245, etc. Your task is to get the maximum subsequence below a certain length.

```
def max_subseq(n, t):
    """
    Return the maximum subsequence of length at most t that can be found in the given numb
    For example, for n = 20125 and t = 3, we have that the subsequences are
        2
        0
        1
        2
        5
        20
        21
        22
        25
        01
        02
        05
        12
        15
        25
        201
        202
        205
        212
        215
        225
        012
        015
        025
        125
    and of these, the maxumum number is 225, so our answer is 225.

    >>> max_subseq(20125, 3)
    225
    >>> max_subseq(20125, 5)
    20125
    >>> max_subseq(20125, 6) # note that 20125 == 020125
    20125
    >>> max_subseq(12345, 3)
    345
    >>> max_subseq(12345, 0) # 0 is of length 0
    0
    >>> max_subseq(12345, 1)
    5
    """
    "*** YOUR CODE HERE ***"
```

There are two key insights for this problem:

- You need to split into the cases where the ones digit is used and the one where it is not. In the case where it is, we want to reduce `t` since we used one of the digits, and in the case where it isn't we do not.
- In the case where we are using the ones digit, you need to put the digit back onto the end, and the way to attach a digit `d` to the end of a number `n` is `10 * n + d`.

Use Ok to test your code:

```
python3 ok -q max_subseq
```

Submit

Make sure to submit this assignment by running:

```
python3 ok --submit
```

Optional Questions

> While "Add Characters" is optional, it is good practice for the Cats project and is thus highly recommended!

Q6: Add Characters

Given two words, `w1` and `w2`, we say `w1` is a subsequence of `w2` if all the letters in `w1` appear in `w2` in the same order (but not necessarily all together). That is, you can add letters to any position in `w1` to get `w2`. For example, "sing" is a substring of "ab**s**orb**ing**" and "cat" is a substring of "**c**ontr**a**s**t**".

Implement `add_chars`, which takes in `w1` and `w2`, where `w1` is a substring of `w2`. This means that `w1` is shorter than `w2`. It should return a string containing the characters you need to add to `w1` to get `w2`. **Your solution must use recursion**.

In the example above, you need to add the characters "aborb" to "sing" to get "absorbing", and you need to add "ontrs" to "cat" to get "contrast".

The letters in the string you return should be in the order you have to add them from left to right. If there are multiple characters in the `w2` that could correspond to characters in `w1`, use the leftmost one. For example, `add_words("coy", "cacophony")` should return "acphon", not "caphon" because the first "c" in "coy" corresponds to the first "c" in "**c**ac**o**phon**y**".

```
def add_chars(w1, w2):
    """
    Return a string containing the characters you need to add to w1 to get w2.

    You may assume that w1 is a subsequence of w2.

    >>> add_chars("owl", "howl")
    'h'
    >>> add_chars("want", "wanton")
    'on'
    >>> add_chars("rat", "radiate")
    'diae'
    >>> add_chars("a", "prepare")
    'prepre'
    >>> add_chars("resin", "recursion")
    'curo'
    >>> add_chars("fin", "effusion")
    'efuso'
    >>> add_chars("coy", "cacophony")
    'acphon'
    >>> from construct_check import check
    >>> # ban iteration and sets
    >>> check(LAB_SOURCE_FILE, 'add_chars',
    ...       ['For', 'While', 'Set', 'SetComp']) # Must use recursion
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q add_chars
```

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