Homework 2: Recursion **hw02.zip (hw02.zip)**

Due by 11:59pm on Thursday, September 24

Instructions

Download hw02.zip (hw02.zip). Inside the archive, you will find a file called hw02.py (hw02.py), along with a copy of the `ok` autograder.

Submission: When you are done, submit with `python3 ok --submit`. You may submit more than once before the deadline; only the final submission will be scored. Check that you have successfully submitted your code on okpy.org (https://okpy.org/). See Lab 0 (/~cs61a/fa20/lab/lab00#submitting-the-assignment) for more instructions on submitting assignments.

Using Ok: If you have any questions about using Ok, please refer to this guide. (/~cs61a/fa20/articles/using-ok.html)

Readings: You might find the following references useful:

- Section 1.7 (http://composingprograms.com/pages/17-recursive-functions.html)
- Section 2.3 (http://composingprograms.com/pages/23-sequences.html)

Grading: Homework is graded based on correctness. Each incorrect problem will decrease the total score by one point. There is a homework recovery policy as stated in the syllabus. **This homework is out of 2 points.**

Required questions

Q1: Num eights

Write a recursive function `num_eights` that takes a positive integer `x` and returns the number of times the digit 8 appears in `x`. *Use recursion - the tests will fail if you use any assignment statements.*

```
def num_eights(x):
    """Returns the number of times 8 appears as a digit of x.

    >>> num_eights(3)
    0
    >>> num_eights(8)
    1
    >>> num_eights(88888888)
    8
    >>> num_eights(2638)
    1
    >>> num_eights(86380)
    2
    >>> num_eights(12345)
    0
    >>> from construct_check import check
    >>> # ban all assignment statements
    >>> check(HW_SOURCE_FILE, 'num_eights',
    ...       ['Assign', 'AugAssign'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Watch the hints video below for somewhere to start:

Use Ok to test your code:

```
python3 ok -q num_eights
```

Q2: Ping-pong

The ping-pong sequence counts up starting from 1 and is always either counting up or counting down. At element `k`, the direction switches if `k` is a multiple of 8 or contains the digit 8. The first 30 elements of the ping-pong sequence are listed below, with direction swaps marked using brackets at the 8th, 16th, 18th, 24th, and 28th elements:

Index	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**[8]**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**[16]**	**17**	**[18]**	**19**	**20**	**21**	**22**	**23**

PingPong Value	1	2	3	4	5	6	7	[8]	7	6	5	4	3	2	1	[0]	1	[2]	1	0	-1	-2	-3

Index (cont.)	**[24]**	**25**	**26**	**27**	**[28]**	**29**	**30**
PingPong Value	[-4]	-3	-2	-1	[0]	-1	-2

Implement a function `pingpong` that returns the nth element of the ping-pong sequence *without using any assignment statements*.

You may use the function `num_eights`, which you defined in the previous question.

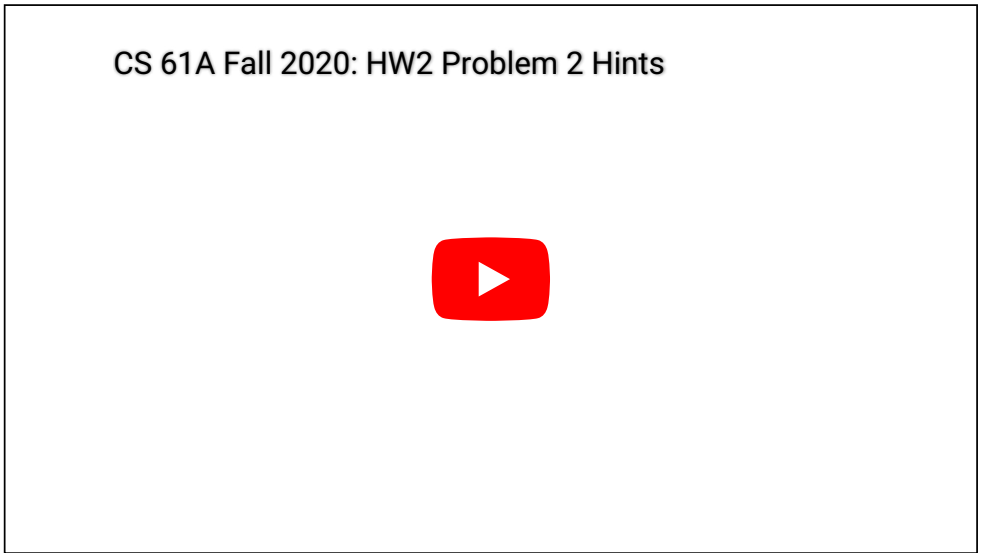
Use recursion - the tests will fail if you use any assignment statements.

> *Hint*: If you're stuck, first try implementing `pingpong` using assignment statements and a `while` statement. Then, to convert this into a recursive solution, write a helper function that has a parameter for each variable that changes values in the body of the while loop.

```
def pingpong(n):
    """Return the nth element of the ping-pong sequence.

    >>> pingpong(8)
    8
    >>> pingpong(10)
    6
    >>> pingpong(15)
    1
    >>> pingpong(21)
    -1
    >>> pingpong(22)
    -2
    >>> pingpong(30)
    -2
    >>> pingpong(68)
    0
    >>> pingpong(69)
    -1
    >>> pingpong(80)
    0
    >>> pingpong(81)
    1
    >>> pingpong(82)
    0
    >>> pingpong(100)
    -6
    >>> from construct_check import check
    >>> # ban assignment statements
    >>> check(HW_SOURCE_FILE, 'pingpong', ['Assign', 'AugAssign'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Watch the hints video below for somewhere to start:



Use Ok to test your code:

```
python3 ok -q pingpong
```

Q3: Missing Digits

Write the recursive function `missing_digits` that takes a number `n` that is sorted in increasing order (for example, `12289` is valid but `15362` and `98764` are not). It returns the number of missing digits in `n`. A missing digit is a number between the first and last digit of `n` of a that is not in `n`. *Use recursion - the tests will fail if you use while or for loops.*

```
def missing_digits(n):
    """Given a number a that is in sorted, increasing order,
    return the number of missing digits in n. A missing digit is
    a number between the first and last digit of a that is not in n.
    >>> missing_digits(1248) # 3, 5, 6, 7
    4
    >>> missing_digits(1122) # No missing numbers
    0
    >>> missing_digits(123456) # No missing numbers
    0
    >>> missing_digits(3558) # 4, 6, 7
    3
    >>> missing_digits(35578) # 4, 6
    2
    >>> missing_digits(12456) # 3
    1
    >>> missing_digits(16789) # 2, 3, 4, 5
    4
    >>> missing_digits(19) # 2, 3, 4, 5, 6, 7, 8
    7
    >>> missing_digits(4) # No missing numbers between 4 and 4
    0
    >>> from construct_check import check
    >>> # ban while or for loops
    >>> check(HW_SOURCE_FILE, 'missing_digits', ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Watch the hints video below for somewhere to start:

Use Ok to test your code:

```
python3 ok -q missing_digits
```

Q4: Count coins

Given a positive integer `total`, a set of coins makes change for `total` if the sum of the values of the coins is `total`. Here we will use standard US Coin values: 1, 5, 10, 25 For example, the following sets make change for `15`:

- 15 1-cent coins
- 10 1-cent, 1 5-cent coins
- 5 1-cent, 2 5-cent coins
- 5 1-cent, 1 10-cent coins
- 3 5-cent coins
- 1 5-cent, 1 10-cent coin

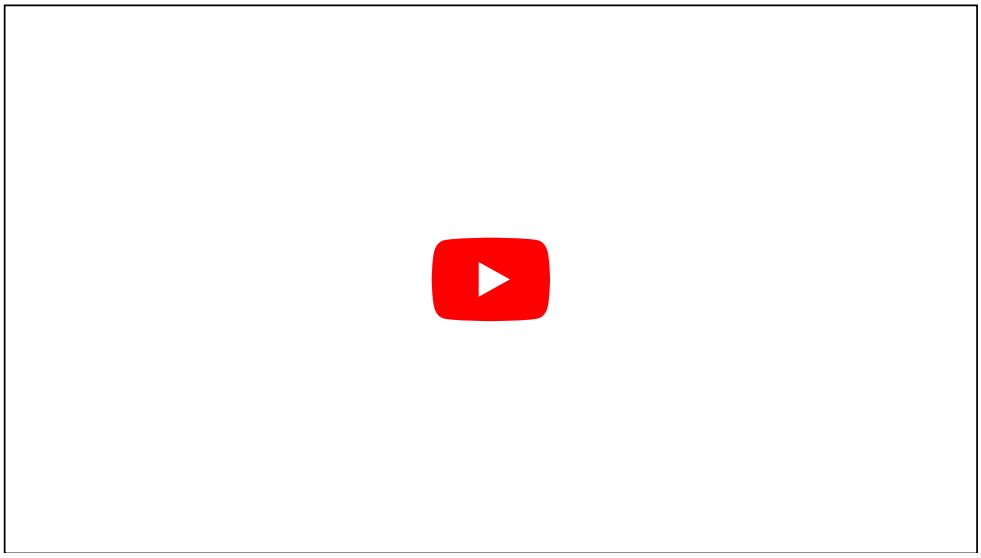
Thus, there are 6 ways to make change for `15`. Write a recursive function `count_coins` that takes a positive integer `total` and returns the number of ways to make change for `total` using coins. Use the `next_largest_coin` function given to you to calculate the next largest coin denomination given your current coin. I.e. `next_largest_coin(5) = 10`.

> *Hint:* Refer the implementation (http://composingprograms.com/pages/17-recursive-functions.html#example-partitions) of `count_partitions` for an example of how to count the ways to sum up to a total with smaller parts. If you need to keep track of more than one value across recursive calls, consider writing a helper function.

```
def next_largest_coin(coin):
    """Return the next coin.
    >>> next_largest_coin(1)
    5
    >>> next_largest_coin(5)
    10
    >>> next_largest_coin(10)
    25
    >>> next_largest_coin(2) # Other values return None
    """
    if coin == 1:
        return 5
    elif coin == 5:
        return 10
    elif coin == 10:
        return 25

def count_coins(total):
    """Return the number of ways to make change for total using coins of value of 1, 5, 10, 25.
    >>> count_coins(15)
    6
    >>> count_coins(10)
    4
    >>> count_coins(20)
    9
    >>> count_coins(100) # How many ways to make change for a dollar?
    242
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(HW_SOURCE_FILE, 'count_coins', ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Watch the hints video below for somewhere to start:



Use Ok to test your code:

```
python3 ok -q count_coins
```

Submit

Make sure to submit this assignment by running:

```
python3 ok --submit
```

Just for Fun Questions

> This question demonstrates that it's possible to write recursive functions without assigning them a name in the global frame.

Q5: Anonymous factorial

The recursive factorial function can be written as a single expression by using a conditional expression (http://docs.python.org/py3k/reference/expressions.html#conditional-expressions).

```
>>> fact = lambda n: 1 if n == 1 else mul(n, fact(sub(n, 1)))
>>> fact(5)
120
```

However, this implementation relies on the fact (no pun intended) that `fact` has a name, to which we refer in the body of `fact`. To write a recursive function, we have always given it a name using a `def` or assignment statement so that we can refer to the function within its own body. In this question, your job is to define fact recursively without giving it a name!

Write an expression that computes `n` factorial using only call expressions, conditional expressions, and lambda expressions (no assignment or def statements). *Note in particular that you are not allowed to use `make_anonymous_factorial` in your return expression.* The `sub` and `mul` functions from the `operator` module

are the only built-in functions required to solve this problem:

```
from operator import sub, mul

def make_anonymous_factorial():
    """Return the value of an expression that computes factorial.

    >>> make_anonymous_factorial()(5)
    120
    >>> from construct_check import check
    >>> # ban any assignments or recursion
    >>> check(HW_SOURCE_FILE, 'make_anonymous_factorial', ['Assign', 'AugAssign', 'FunctionDef', 'Recurs
    True
    """
    return 'YOUR_EXPRESSION_HERE'
```

Use Ok to test your code:

```
python3 ok -q make_anonymous_factorial
```

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