HW4

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1 Trapezoid Rule

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1	0.818731	0.670320	0.548812	0.449329

1.1 a

n=4

$$\Delta x = (b-a)/n = 4$$

$$x_0 = 0.x_1 = 0.2, x_2 = 0.4, x_3 = 0.6, x_4 = 0.8$$

$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4))$$

$$\int_a^b f(x)\,dx = \frac{0.2}{2}(1 + 2(0.818731 + 0.670320 + 0.548812) + 0.44932) = 0.552505$$

1.2 b

n=4

$$\Delta x = (b-a)/n = 4$$

$$x_0 = 0.x_1 = 0.2, x_2 = 0.4, x_3 = 0.6, x_4 = 0.8$$

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4))$$

$$\int_a^b f(x)\,dx = \frac{0.2}{3}(1 + 4(0.818731 + 0.548812) + 2(0.670320) + 0.44932) = 0.550675$$

1.3 c

$$\int_a^b f(x)\,dx = \int_0^{0.8} f(x)\,dx = 0.550671$$

Absolute Error by Trapezoidal Rule = 0.001834
Absolute Error by Simpson's Rule = 0.000004
Simpson's Rule Better

1.4 d

Trapezoidal Rule

$$a = 0, b = 0.8, E_r = 10^{-4}, f(x) = e^{-x}$$

Calculator

$$n \geq 17$$

Simpson's Rule

$$a = 0, b = 0.8, E_r = 10^{-4}, f(x) = e^{-x}$$

Calculator

$$n \geq 2$$

so it is 5 (2n+1)

2 Simpson's Rule

$$\int_0^1 f(x)\,dx$$

x	0.0	0.25	0.5	0.75	1
$f(x)$	0	0.06	0.24	0.51	0.84

2.1 a

$$h = 0.25$$

$$x_0 = 0.x_1 = 0.25, x_2 = 0.50, x_3 = 0.75, x_4 = 1$$

$$\int_0^1 f(x)\,dx = \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4))$$

$$\int_0^1 f(x)\,dx = \frac{0.25}{3}(0 + 4(0.06) + 2(0.24) + 4(0.51) + 0.84) = 0.3$$

2.2 b

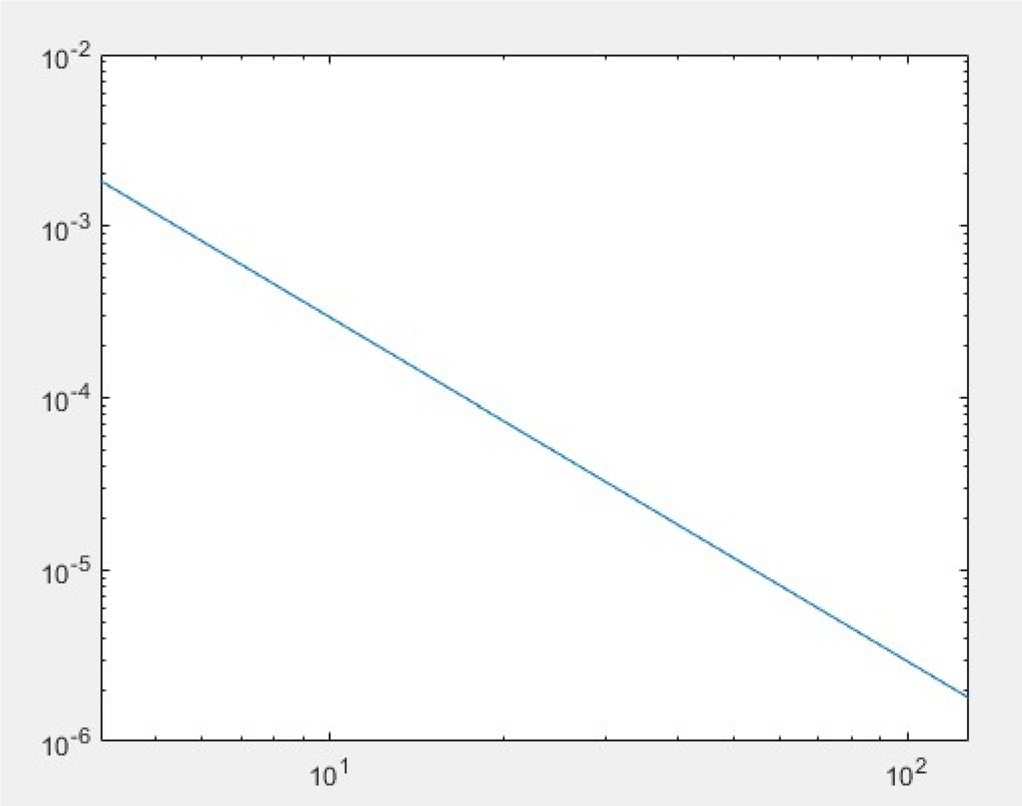
$$a = 0, b = 1, E_r = 10^{-6}, f^{(4)}(0.5) = -3$$

Calculator

$$n \geq 6$$

So it is 13 (2n+1)

3 Trapezoid Rule in Matlab



the error is decreasing when n gets higher(doubled) and we expected this from the error estimate.

4 Trapezoid rule and Romberg algorithm.

4.1 a

$$\int_{-1}^{1} 3x^2\,dx$$

$$n = 1$$

$$\Delta x = 2$$

$$\int_{-1}^{1} 3x^2\,dx = \frac{2}{2}(f(-1) + f(1)) = 6$$

$$n = 2$$

$$\Delta x = 1$$

$$\int_{-1}^{1} 3x^2\,dx = \frac{1}{2}(f(-1) + 2f(0) + f(1)) = 3$$

4.2 b

$$n = 1$$

calculator

$$R_1 = 6$$

$$n = 2$$

Calculator

$$R_2 = 3$$

same value,because the number of intervals are so small

5 Romberg Algorithm in Matlab

5.1 b

```
>> R=romberg1('@(x) sin(x)',0,pi,4)

R =

    0.0000         0         0         0
    1.5708    2.0944         0         0
    1.8961    2.0046    1.9986         0
    1.9742    2.0003    2.0000    2.0000

>> R=romberg1('@(x) sqrt(x)',0,1,4)

R =

    0.5000         0         0         0
    0.6036    0.6381         0         0
    0.6433    0.6565    0.6578         0
    0.6581    0.6631    0.6635    0.6636

fx >> |
```

I used the command window to do the B part

5.2 c

the integral in ii) is the integral of the square root function over the interval [0, 1]. The square root function, sqrt(x), has a vertical tangent at x = 0. In other words, it's not differentiable at x = 0, making it non-smooth at that point.

When we are dealing with a non-smooth function like this, the assumptions underpinning the error reduction in Romberg's method are not satisfied, and the method performs poorly.

The extrapolation technique used by Romberg's method is intended to eliminate the higher-order terms in the error approximation, but this process assumes that the function and its derivatives are continuous and exist over the entire interval of integration.

So, if a function has a point of non-differentiability (like the sqrt(x) function at x=0), or has discontinuities, then the effectiveness of the Romberg method (and other similar methods) is significantly reduced.

5.3 d

quadl have smaller error compared to quad

Workspace

Name	Value
a1	0
a2	0
ans	4x4 double
approximation1_quad	2.0000
approximation1_quadl	2.0000
approximation2_quad	0.6667
approximation2_quadl	0.6667
b1	3.1416
b2	1
error1_quad	8.7486e-13
error1_quadl	3.3307e-15
error2_quad	5.0886e-09
error2_quadl	1.0203e-09
f1	@(x)sin(x)
f2	@(x)sqrt(x)
R	4x4 double
tol	1.0000e-09

6 Computing Pi with Romberg

```
R =

    2.8284         0         0         0         0         0
    3.0600    3.1371         0         0         0         0
    3.1209    3.1412    3.1415         0         0         0
    3.1364    3.1416    3.1416    3.1416         0         0
    3.1403    3.1416    3.1416    3.1416    3.1416         0
    3.1413    3.1416    3.1416    3.1416    3.1416    3.1416

fx >>
```

7 Numerical Integration and Extrapolation

check the other pdf

8 Gaussian Quadrature and Beyond

check the other pdf