

HW-7

7.1

a)

$$\begin{aligned} 4x_1 + 3x_2 &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= -24 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 4 & 3 & 0 & 24 \\ 3 & 4 & -1 & 30 \\ 0 & -1 & 4 & -24 \end{array} \right]$$

$$R_2 - 3R_1 \rightarrow R_2$$

$$\frac{R_1}{4} \rightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0.75 & 0 & 6 \\ 3 & 4 & -1 & 30 \\ 0 & -1 & 4 & -24 \end{array} \right]$$

$$R_2 - 3R_1 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 0.75 & 0 & 6 \\ 0 & 1.75 & -1 & 12 \\ 0 & -1 & 4 & -24 \end{array} \right]$$

$$\frac{R_2}{1.75} \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0.75 & 0 & 6 \\ 0 & 1 & -4/7 & 48/7 \\ 0 & -1 & 4 & -24 \end{array} \right]$$

$$R_1 - 0.75R_2 \rightarrow R_1, R_3 + R_2 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & 3/7 & 6/7 \\ 0 & 1 & -4/7 & 48/7 \\ 0 & 0 & 24/7 & -120/7 \end{array} \right]$$

$$\frac{R_3}{24} \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & 6/7 \\ 0 & 1 & -4/7 & 48/7 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

$$R_1 - \frac{3}{7}R_3 \rightarrow R_1$$

$$R_2 + \frac{4}{7}R_3 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$$

$$b) \quad x_1 = x \quad | \quad x_2 = y \quad | \quad x_3 = z$$

$$x_{k+1} = \frac{1}{4} (24 - 3x_k - 0z_k)$$

$$y_{k+1} = \frac{1}{4} (30 - 3x_k + z_k)$$

$$z_{k+1} = \frac{1}{4} (-24 - 0x_k + y_k)$$

$$1st \quad (0, 0, 0)$$

$$x_1 = 6 \quad | \quad y_1 = 7.5 \quad | \quad z_1 = 6$$

$$2nd \quad (6, 7.5, 6)$$

$$x_2 = 0.375 \quad | \quad y_2 = 1.5 \quad | \quad z_2 = -4.125$$

$$3rd \quad (0.375, 1.5, -4.125)$$

$$x_3 = 4.875 \quad | \quad y_3 = 6.1875 \quad | \quad z_3 = -5.625$$

$$c) \quad A = \begin{bmatrix} 4 & 0 & 0 \\ 3 & 3 & 0 \\ 0 & -1 & 4 \end{bmatrix} \quad U = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad L^{-1} = \begin{bmatrix} 0.25 & 0 & 0 \\ -0.125 & 0.25 & 0 \\ 0.0625 & 0.0625 & 0.25 \end{bmatrix}$$

$$T = L^{-1} \cdot U = \begin{bmatrix} 0 & -0.75 & 0 \\ 0 & 0.5625 & 0.25 \\ 0 & 0.1406 & 0.0625 \end{bmatrix}$$

$$C = L^{-1} \cdot \begin{bmatrix} 24 \\ 30 \\ -24 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -5.25 \end{bmatrix} = x^0$$

e)

$$\cancel{x_1 = 24}$$

$$x_{k+1} = \frac{1}{4} (24 - 3y_{k+1}) \quad \left| \quad y_{k+1} = \frac{1}{4} (30 - x_{k+1} + z_k) \right.$$

$$z_{k+1} = \frac{1}{4} (24 + y_{k+1})$$

$$(6, 7.5, -6)$$

1st

$$x_1 = 1.5 \quad \left| \quad x_2 = 7.125 \quad \right| \quad x_3 = -4.21875$$

2nd

$$x_1 = 2.59375 \quad \left| \quad x_2 = 7.039 \quad \right| \quad x_3 = -4.24$$

$$\cancel{x^1 = T x^0 = \begin{bmatrix} 3.75 \\ 3.375 \\ -5.1563 \end{bmatrix} \quad \cancel{x^2 = T x^1 = \begin{bmatrix} 3.4688 \\ 3.6094 \\ -5.6977 \end{bmatrix}}$$

d) SOR $\omega = 1.25$ $x_1 = 6$, $x_2 = 7.5$, $x_3 = -6$

$$x_{k+1} = (1-\omega)x_k + \frac{\omega}{4}(24 - 3y_k)$$

$$y_{k+1} = (1-\omega)y_k + \frac{\omega}{4}(30 - 3x_{k+1} + z_k)$$

$$z_{k+1} = (1-\omega)z_k + \frac{\omega}{4}(-24 + y_{k+1})$$

$$\begin{cases} x_1 = x \\ x_2 = y \\ x_3 = z \end{cases}$$

~~1st~~
 ~~$x_1 = 1.03 \mid y_1 = 6.59 \mid z_1 = 3.94$~~

~~2nd~~
 ~~$x_2 = 1.58 \mid y_2 = 8.31 \mid z_2 = 3.91$~~

1st
 $x_1 = 0.75 \mid y_1 = 7.59375 \mid z_1 = -4.51$

2nd
 $x_2 = 1.488 \mid y_2 = 6.489 \mid z_2 = -4.67$

By looking at calculations, c and d appears to converge faster

7.2)

$$\begin{aligned} 5x + 4y - 2z &= 2 \\ -2x + 8y - 3z &= 6 \\ x + y - 7z &= 5 \end{aligned}$$

$$x_0 = y_0 = z_0 = 1$$

Jacobi

$$x_1 = \frac{(2 - 4 + 2)}{5} = 0 \quad y_1 = \frac{(6 + 2 + 3)}{8} = 1.25 \quad z_1 = \frac{(5 - 1 - 1)}{-7} = -0.43$$

Gauss

$$x_1 = \frac{(2 - 4 + 2)}{5} = 0 \quad y_1 = \frac{(6 + 2 + 3)}{8} = 1.125 \quad z_1 = \frac{(5 - 1.125)}{-7} = -0.55$$

7.5

$$a) \quad x_i^{k+1} = (1-w)x_i^k + \frac{w}{a_{ii}} \left[b_i - \sum_{j \neq i} (a_{ij} \cdot x_j^k) \right]$$

$$b) \quad x^{k+1} = (1-w)x^k + w \cdot D^{-1} \left[G - (L+U) \cdot x^k \right]$$