HW-4

4.7 $$f(t) = \begin{cases} \frac{\sin(t)}{t}, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

$$J = \int_0^1 f(t)\,dt \qquad n = 1, 3, 9$$

a) $$\int_a^b f(t)\,dt = \frac{\Delta t}{2}\left[f(t_0) + 2f(t_1) + 2f(t_2) + 2f(t_3) + 2f(t_{n-1}) + f(t_n)\right]$$

$n = 1$

$$\Delta t = \frac{b-a}{n} = \frac{1-0}{1} = 1$$

$$\int_0^1 f(t)\,dt = \frac{1}{2}\left[f(0) + 2f(0.5) + f(1)\right]$$

$$= \frac{1}{2}\left[1 + 2\frac{\sin(0.5)}{0.5} + \frac{\sin(1)}{1}\right] = 1.8759$$

$n = 3$ $\qquad \Delta t = \frac{1}{3}$

$$\int_0^1 f(t)\,dt = \frac{1}{3}\left[f(0) + 2f(0.3) + 2f(0.6) + 2f(0.9) + f(1)\right]$$

$$\int_0^1 f(t)\,dt = 1.93612$$

$n = 9$ $\qquad \Delta t = \frac{1}{9}$

$$\int_0^1 f(t) = \frac{1}{9}\left[f(0) + 2f(0.1) + 2f(0.2) + 2f(0.3) \ldots \ldots 2f(0.9) + f(1)\right]$$

$$\int_0^1 f(t)\,dt = 1.946081$$

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~~b)~~
a)

$x_1 = -0.339981$ | $x_3 = 0.339981$

$x_2 = -0.861136$ | $x_4 = 0.861136$

$a_1 = 0.652145 = a_3$ | $a_2 = 0.347855 = a_4$

approx)

$$\int_{-1}^{1} f(x) \cdot dx \simeq a_1 f(x_1) + a_2 f(x_2) + a_3 f(x_3) + a_4 f(x_4)$$

$$\int_{-1}^{1} 1\, dx = 2$$

$$\int_{-1}^{1} f(x)\, dx \simeq a_1 [f(x_1) + f(x_3)] + a_2 [f(x_2) + f(x_3)]$$

$$\int_{-1}^{1} x\, dx = \underline{0}$$

$$\int_{-1}^{1} x^2 dx \approx 0.652145 [x_1^2 + x_3^2] + 0.347855 [2x_2^2]$$

$$\int_{-1}^{1} x^2 dx = \underline{0.6658}$$

$$\int_{-1}^{1} x^3 dx = \underline{0}$$

$$\int_{-1}^{1} x^4 dx \approx a_1 [2x_1^4] + a_2 [2x_2^4] = \underline{0.392}$$

$$\int_{-1}^{1} x^5 dx = \underline{0}$$

$$\int_{-1}^{1} x^6 dx = a_1 [2x_1^6] + a_2 [2x_2^6] = \underline{0.28497}$$

$$\int_{-1}^{1} x^7 dx = \underline{0}$$

True

b) $\int_{-1}^{1} f(x)\,dx \approx a_1 f(-0.5) + a_2 f(0) + a_3 f(0.5)$

↳ Valid for $1, x, x^2$

$\int_{-1}^{1} 1\,dx = a_1 + a_2 + a_3 = 2$

$\int_{-1}^{1} x\,dx = -a_1 0.5 + 0.5 a_3 = 0 \qquad a_1 = a_3$

$\int_{-1}^{1} x^2\,dx = \frac{a_1}{4} + \frac{a_3}{4} = \frac{2}{3} \qquad a_1 + a_3 = \frac{8}{3} \qquad a_1 = \frac{4}{3}$

$\boxed{a_1 = \frac{4}{3},\ a_3 = \frac{4}{3},\ a_2 = -\frac{2}{3}} \qquad a_2 = 2 - \frac{4}{3} - \frac{4}{3} = -\frac{2}{3}$

c) $\int_{-1}^{1} f(x)\,dx = w f(-a) + w f(a) \rightarrow 1, x, x^2, x^3$

$\int_{-1}^{1} dx = w f(-a) + w f(a) = 2 = 2w \quad | \quad w = 1$

$\int_{-1}^{1} x\,dx = \cancel{wf} -aw + wa = 0$

$\int_{-1}^{1} x^2\,dx = a^2 w + w a^2 = \frac{2}{3} = 2a^2 = \frac{2}{3} \quad a = \sqrt{\frac{1}{3}}$

$w = 1, \quad a = \sqrt{\frac{1}{3}}$