HW 6

ankit gupta

June 2023

1 Basic Systems of Linear Equations

1.1 a

Let's denote 'A' as the coefficient matrix and 'b' as the load vector.

When a = 0, the determinant of 'A' is -9 which is not equal to 0. This implies that there is a unique solution.

When a = -1, the determinant of 'A' becomes 0, and the load vector 'b' does not belong to the null space of 'A'. Hence, there are no solutions in this case.

When a = 1, the determinant of 'A' is again 0, but in this case, the load vector 'b' lies within the null space of 'A'. Therefore, there are infinitely many solutions.

If 'b' equals 0, then 0 always stands as a trivial solution. When a = 0, the unique solution is also 0. When a = -1 or a

1.2 b

when is approximately 1, we encounter a situation where we subtract two numbers that are very close to each other. This can lead to a loss of many significant digits, resulting in a large error.

2 Gaussian Elimination in Matlab

Result
(i)

error1 =
1.0e-12 *
0.3880 -0.8888 0.5058 -0.1044 0.0071
error2 =
1.0e-13 *
0.2109 -0.4774 0.3353 -0.0799 0.0056
Errors are smaller with pivoting
(ii)

error1 =
0.0012 -0.0059 0.0119 -0.0136 0.0094 -0.0042 0.0011 -0.0002 0.0000 -0.0000
error2 =
1.0e-09 *
-0.0340 0.1668 -0.3486 0.4050 -0.2864 0.1268 -0.0348 0.0057 -0.0005 0.0000
Errors are smaller with pivoting compared to (i)

(iii)

error1 =
1.0e+15 *
Columns 1 through 19
-0.1003 0.5476 -1.3221 1.8613 -1.6973 1.0525 -0.4526 0.1347 -0.0270 0.0034 -0.0002 -0.0000 0.0000 -0.0000 0.0000 -0.0000 0.0000 -0.0000 0.0000
Column 20
-0.0000
error2 =
Columns 1 through 19
-0.0019 0.0193 -0.0892 0.2542 -0.5017 0.7270 -0.8015 0.6870 -0.4638 0.2484 -0.1057 0.0357 -0.0095 0.0020 -0.0003 0.0000 -0.0000 0.0000 -0.0000
Column 20
0.0000

Please observe that in the last case, the solution vector exhibits a significantly large error, despite the use of pivoting. Without the incorporation of pivoting, obtaining a meaningful result would be entirely impossible. This phenomenon is due to the fact that the Van der Monde matrix becomes extremely ill-conditioned as 'n' increases.

When a matrix is ill-conditioned, it implies that the matrix's condition number is high. A high condition number indicates a high degree of sensitivity to changes in the input (in this case, the elements of the matrix). This can lead to significant errors or instability in the solution when numerical methods are applied, as we observe in our problem with the large 'n' values.

3 Application of System of Linear Equations

f1 =

-26.8701
19.0000
10.0000
-28.0000
12.7279
19.0000
0
-28.0000
8.4853
22.0000
0
-16.0000
-8.4853
22.0000
16.0000
-22.6274
16.0000

f2 =

-19.7990
14.0000
15.0000

-13.0000
-1.4142
14.0000
0
-13.0000
1.4142
12.0000
0
-11.0000
-1.4142
12.0000
11.0000
-15.5563
11.0000

f3 =

-22.6274
16.0000
10.0000
-22.0000
8.4853
16.0000
0
-22.0000
-8.4853
28.0000
20.0000
-14.0000
-19.7990
28.0000
14.0000
-19.7990
14.0000

f4 =

-14.1421
10.0000
0
-20.0000
14.1421
10.0000
0
-20.0000
0
20.0000
10.0000
-10.0000
-14.1421
20.0000
10.0000

-14.1421
10.0000

4 X System of Linear Equations

Pseudo code in Matlab file
Result
x =

1
1
1
1
1
1
1
1
1
1
1