hw3

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1 Working on Splines

1.1 a

S(x) is a piece-wise linear polynomial
Continuity check-
At x= 0.5
both sides (left and right) are the same value of 0.5.
At x=2
both sides (left and right) are the same value of 3.5.

So, S(x) is a linear spline.

1.2 b

$$S_0(x) = ax^3 + x^2 + cx$$

$$S_1(x) = bx^3 + x^2 + dx$$

knots

$$x_0 = -1$$

$$x_1 = 0$$

$$x_2 = 1$$

Continuity check for S(x)

$$S_0(0) = S_1(0)$$

S'(x) must also be continuous

$$S_0'(0) = c$$

$$S_1'(0) = d$$

$$c = d$$

S"(x) must also be continuous

$$S_0''(0) = 2$$

$$S_1''(0) = 2$$

Boundary conditions

$$S_0''(-1) = 0, a = 1/3$$

$$S_1''(1) = 0b = -1/3$$

So

$$a = 1/3, b = -1/3, c = d$$

1.3 c

knots

$$x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2$$

functions

$$f_0(x) = 1 + 2(x+1) + (x+1)^3$$
$$f_1(x) = 3 + 5x + 3x^2$$
$$f_2(x) = 11 + (x-1) + 3(x-1)^2 + (x-1)^3$$

Continuity check for f(x)

$$f_0(0) = 4, f_1(0) = 3$$

$$f_0(0) \neq f_1(0)$$

f(x) is not a cubic spline

1.4 d

knots

$$x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$$

S(x) has to be continuous, So

$$x = -1, 0 = -b, b = 0$$
$$x = 0, a = 0 - 3, a = -3$$
$$x = 1, c = 4$$
$$a = -3, b = 0, c = 4$$

1.5 e

knots

$$x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$$

S(x) has to be continuous, So

$$x = -1, 2 = - - b, b = -2$$
$$x = 0, a = -2 - 3, a = -2$$
$$x = 1, c = 5$$
$$a = -2, b = 0 - 2, c = 5$$

2 Simple cases of natural cubic spline

2.1 a

knots

$$t_0, t_1$$

To be a cubic spline

$$Q''(t_0) = Q''_(t_1) = Q''(t) = 0$$

So, the formula is

$$Q(x) = y_0 + \frac{y_1 - y_0}{t_1 - t_0} x$$

where x lies

$$[t_0, t_1]$$

2.2 b

knots

$$h_i = h$$

$$H\vec{z} = \vec{v}$$

Answer So

$$v_i = \frac{6}{h}(y_{i+1} - 2y_i + y_{i-1})$$

2.3 c

$$f(x) = x^6$$

knots (0,1,2)

$$f(0) = 0, f(1) = 1, f(2) = 64$$

$$h = 1$$

using the equation from b

$$z_0 = 0, z_1 = 93, z_2 = 0$$

Then using formula

$$S_0(x) = \frac{31x^2 - 29x^2}{2}, 0 \leq x \leq 1$$

$$S_1(x) = \frac{-31(x-2)^3 + 128(x-1) + 29(x-2)}{2}, 1 \leq x \leq 2$$

3 Spline exercise

3.1 a

L(x) has to be continuous, So

$$L_i(x) = y(t_i) + \frac{y(t_{i+1}) - y(t_i)}{t_{i+1} - t_i}(x - t_i)$$

And to compute L(1.8)

$$L_2(x) = -4.6218 + 8.6284, 1.6 \leq x \leq 2.0$$

$$L_2(1.8) = L(1.8) = -0.05075$$

3.2 b

For C(x) to be a natural cubic spline- C(x),C'(x),C"(x) has to be continuous and

$$C''(x_0) = C''(x_n) = 0$$

we create a table with values of i, t_i,y_i,h_i,b_i and z_i then the system looks like

$$H\vec{z} = \vec{b}$$

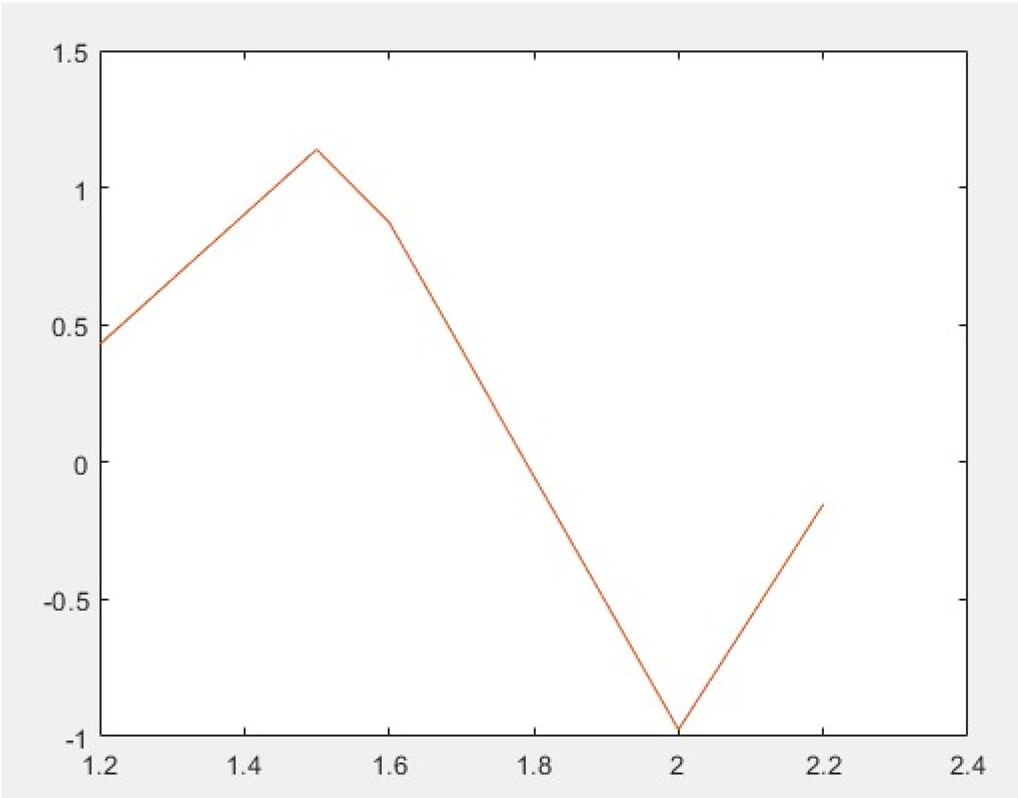
we use Matlab to solve this to get values of z

We need to solve for C(1.8), So

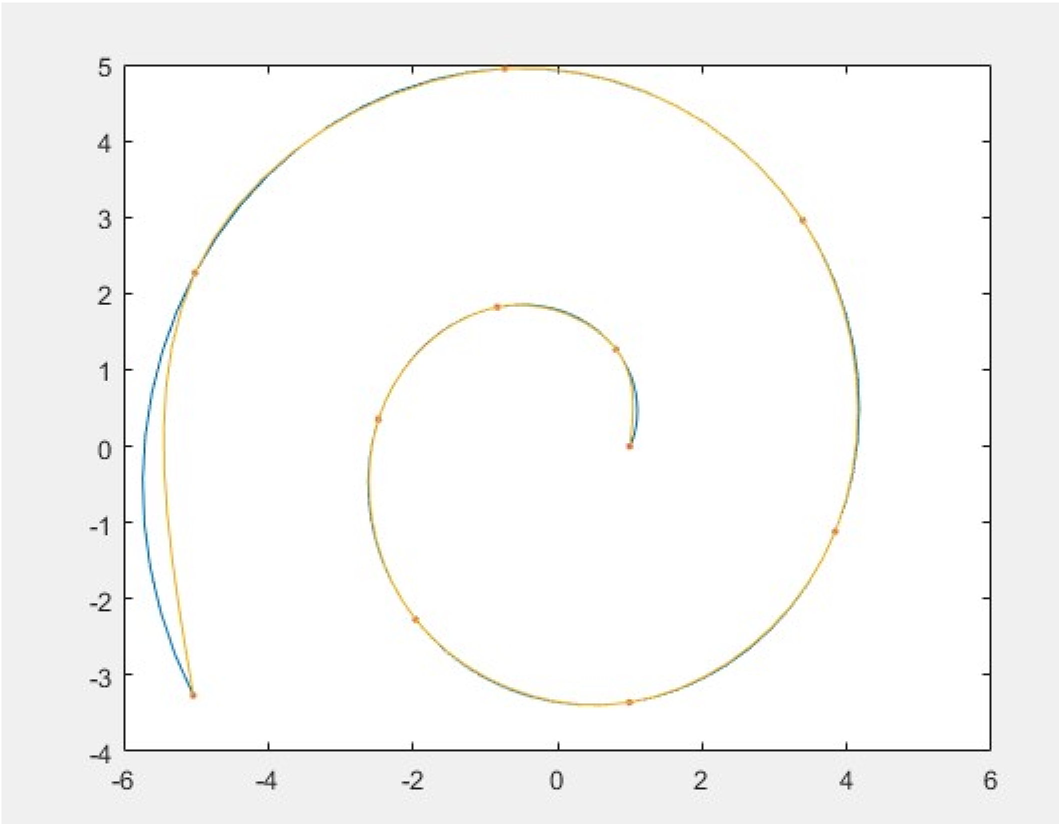
$$C_2(x) = 22.3(x-1.6)^3 - 12.4(2-x)^3 - 10.18x + 17.9, 1.6 \leq x \leq 2$$

$$C_2(1.8) = C(1.8) = -0.29$$

4 Linear Spline in Matlab



5 Interpolating a Spiral with Natural Spline



The spline is the same at the central portion of the spiral. the errors become pronounced near the spiral's end. This occurs due to the boundary condition S"=0.