Homework 2

Ankit Gupta

28 May 2023

1 Practice on Newton's Divided Differences

Answer

x	$f[]$	$f[,]$	$f[,,]$	$f[,,,]$
-1	2			
		-3		
1	-4		2	
		5		$-\frac{11}{24}$
3	6		$-\frac{3}{4}$	
		2		
5	10			

$$f_3(x) = 2 - 3(x+1) + 2(x+1)(x-1) - \frac{11}{24}(x+1)(x-1)(x-3)$$

2 Practice on Polynomial Interpolation

2.1 a

x	0	2	3	4
y	7	11	28	63

Answer

$$x_0 = 0, x_1 = 2, x_2 = 3, x_3 = 4, y_0 = 7, y_1 = 11, y_2 = 28, y_3 = 63$$

Using the Lagrange Interpolation formula (calculator)

$$y(x) = x^3 - 2x + 7$$

2.2 b

Answer

x	$y[]$	$y[,]$	$y[,,]$	$y[,,,]$
0	7			
		2		
2	11		5	
		17		1
3	28		9	
		35		
4	63			

$$y(x) = x^3 - 2x + 7$$

2.3 c

$$p(x) = x^4 - x^3 + x^2 - x + 1$$

x	-2	-1	0	1	2	3
$p(x)$	-31	5	1	1	11	61

x	-3	-2	-1	0	1	2	3
$h(x)$	193	-31	5	1	1	11	61

x	$f[]$	$f[,]$	$f[,,]$	$f[,,,]$	$f[,,,,]$	f[, , , , ,]	f[, , , , , ,]
-3	193						
		-162					
-2	31		136				
		-26		-114			
-1	5		22		96		
		-4		-18		-72	
0	1		4		24		72
		0		6		0	
1	1		10		24		
		10		30			
2	11		40				
		50					
3	61						

$$h(x) - p(x) = \frac{193 * (x+3)}{.....}$$

$$h(x) = \frac{x^6 - 3x^5 + 5x^4 + 5x^3 + 14x^2 - 22x + 10}{10}$$

2.4 d

$$p(x) = x^4 - x^3 + x^2 - x + 1$$

x	-2	-1	0	1	2	3
$p(x)$	-31	5	1	1	11	61

x	-2	-1	0	1	2	3
$q(x)$	-31	5	1	1	11	30

Answer

$$p(x) - q(x) = (61 - 30)\frac{(x+2)(x+1)(x-0)(x-1)(x-2)}{(3+2)(3+1)(3-0)(3-1)(3-2)}$$

$$p(x) = x^4 - x^3 + x^2 - x + 1$$

$$q(x) = -\frac{31x^5}{120} + x^4 + \frac{7x^3}{24} + x^2 - \frac{61x}{30} + 1$$

3 Theory of Polynomial Interpolation

$$p(x) = 5x^3 - 27z^2 + 45x - 21$$

$$q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$$

x	1	2	3	4
y	2	1	6	47

Answer

$$x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, y_0 = 2, y_1 = 1, y_2 = 7, y_3 = 47$$

x	$f[]$	$f[,]$	$f[,,]$	$f[,,,]$
1	2			
		-1		
2	1		3	
		5		5
3	6		18	
		41		
4	47			

x	1	2	3	4
y	2	1	6	47
$p(x)$	2	1	6	47
$q(x)$	2	1	6	47

4 a question to ponder

Answer

x	$f[]$	$f[,]$	$f[,,]$	$f[,,,]$	$f[,,,,]$
-2	1				
		3			
-1	4		2		
		7		-1	
0	11		-1		0
		5		-1	
1	16		-4		0
		-3		-1	
2	13		-7		
		-17			
3	-4				

The interpolation of the data comes to be cubic but it is not sure that the data came from cubic function.