Project 2

Quadcopter Mechanics

AERSP 304

We have neither given nor received any assistance with this project.

Team Member	PSU ID	Contributions	Percentages
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Methodology:

The following report is an analysis of a quadcopter's mechanics during flight. Parameters and equations were given for the flight of a quadcopter. The equations were derived from applying rigid body dynamics and Lagrangian mechanics. The position of the quadcopter with respect to the inertial frame is shown below.

$$r = \{x, y, z\}^T$$

The Lagrangian method covered in lecture is used to derive the following equations of motion:

$$\ddot{r} = -g\{0\ 0\ 1\}^T + \frac{T}{m}\begin{Bmatrix}\cos\psi\sin\theta\ cos\phi + \sin\psi\sin\phi \\ \sin\psi\sin\theta\ cos\phi - \cos\psi\sin\phi \\ cos\theta\ cos\phi\end{Bmatrix} \quad (1)$$

$$\{p, q, r\}^T = A\{\phi, \Theta, \psi\}^T, A = \begin{bmatrix}1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\sin\phi\end{bmatrix} \quad (2)$$

$$Iw = -w \times (Iw) + \tau \quad (3)$$

where (ψ, Θ, ϕ) denote the 3-2-1 Euler angle sequence and $w = (p, q, r)^T$ denotes the body angular rates of the quadcopter. Because the quadcopter is symmetric, the moment of inertia matrix, I, is a diagonal matrix. Equation 4 is the combined forces of rotors, in the direction of z-axis:

$$T = \sum_{i=1}^{4} T_i = 4k\Omega_i^2 \qquad (4)$$

Equation 5 consists of roll, pitch, and yaw moments in that order in the matrix. For roll, the second motor has to be subtracted from the fourth motor's angular velocity. Pitch varies from roll, as the first motor decreases while the third motor increases angular velocity. Lastly, yaw is generated by increasing two opposite motors while decreasing the other two motors' angular velocity.

$$\tau = \begin{Bmatrix} L \\ M \\ N \end{Bmatrix} = \begin{Bmatrix} kl(-\Omega_2^2 + \Omega_4^2) \\ kl\left(-\Omega_1^2 + \Omega_3^2\right) \\ b(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{Bmatrix} \qquad (5)$$

Various parameters were provided when coding. These initial conditions and parameters can be seen in the code.

PART 1:

The control input and angular velocity of the four rotors were provided. The flight started by increasing each rotor to a set angular velocity. Equation 8 shows the initial angular velocity during the ascension phase of 1 second.

$$\Omega_i = \Omega_{ihover} + 70\sin\left(\frac{2\pi t}{4}\right) \qquad t < 1,\ i = 1, 2, 3, 4 \qquad (6)$$

After the ascension, the quadcopters' rotors slowed down to achieve a hover state. The decrease in rotor speed lasted for 1 second after the initial ascension. Equation 7 shows the decrease in the angular velocity for each rotor.

$$\Omega_i = \Omega_{ihover} - 70\sin\left(\frac{2\pi t}{4}\right) \qquad 1 \le t \le 2 \qquad (7)$$

The quadcopter has four propellers, so the angular velocity is multiplied by four to obtain the thrust. The thrust profile allowed for the quadcopter to hover at a constant altitude.

$$T = \sum_{i=1}^{4} T_i = 4k\Omega_{i\,hover}^2 = mg \qquad (8)$$

The second step of the quadcopter's motion is a roll. The roll started with an increase in angular velocity of the fourth rotor, and a decrease of angular velocity in the second rotor. These rotors are located diagonally from each other. Equation 9 shows the angular velocities during the initial roll phase. The time this roll occurred can be seen under the equation.

$$\Omega_2^2 = \Omega_{2\,hover}^2 - 70^2 sin\left(\frac{2\pi(t-2)}{4}\right), \quad \Omega_4^2 = \Omega_{4\,hover}^2 + 70^2 sin\left(\frac{2\pi(t-2)}{4}\right), 2 \leq \mathrm{t} \leq 3 \quad (9)$$

After the roll is started the quadcopter needs to counteract the angular velocities in equation 10. The canceling of angular velocity allows the quadcopter to remain at a constant roll angle. The quadcopter is rotating about an axis of rotation.

$$\Omega_2^2 = \Omega_{2\,hover}^2 + 70^2 \sin\left(\frac{2\pi(t-2)}{4}\right), \Omega_4^2 = \Omega_{4\,hover}^2 - 70^2 sin\left(\frac{2\pi(t-2)}{4}\right), 3 \leq \mathrm{t} \leq 4 \quad (10)$$

The quadcopter now undergoes pitch motion. The pitch motion is accomplished by increasing the angular velocity of the third rotor and decreasing the velocity of the first rotor. The angular velocity during this step is shown below in equations 11 and 12. The pitch rate is canceled out once the opposite maneuver is applied and assumes steady flight.

$$\Omega_1^2 = \Omega_{1\,hover}^2 - 70^2 \sin\left(\frac{2\pi(t-2)}{4}\right), \Omega_3^2 = \Omega_{3\,hover}^2 + 70^2 \sin\left(\frac{2\pi(t-2)}{4}\right) \qquad , 4 \leq \mathrm{t} \leq 5 \qquad (11)$$

$$\Omega_1^2 = \Omega_{1\,hover}^2 + 70^2 \sin\left(\frac{2\pi(t-2)}{4}\right), \Omega_3^2 = \Omega_{3\,hover}^2 - 70^2 \sin\left(\frac{2\pi(t-2)}{4}\right) \qquad , 5 \leq \mathrm{t} \leq 6 \qquad (12)$$

Results and Discussion:

` To further evaluate the quadcopters motion the following graphs needed to be made, Position in inertial frame, velocity in body frame, Euler angles and angular velocities versus time. The following figures show each of the required relations.

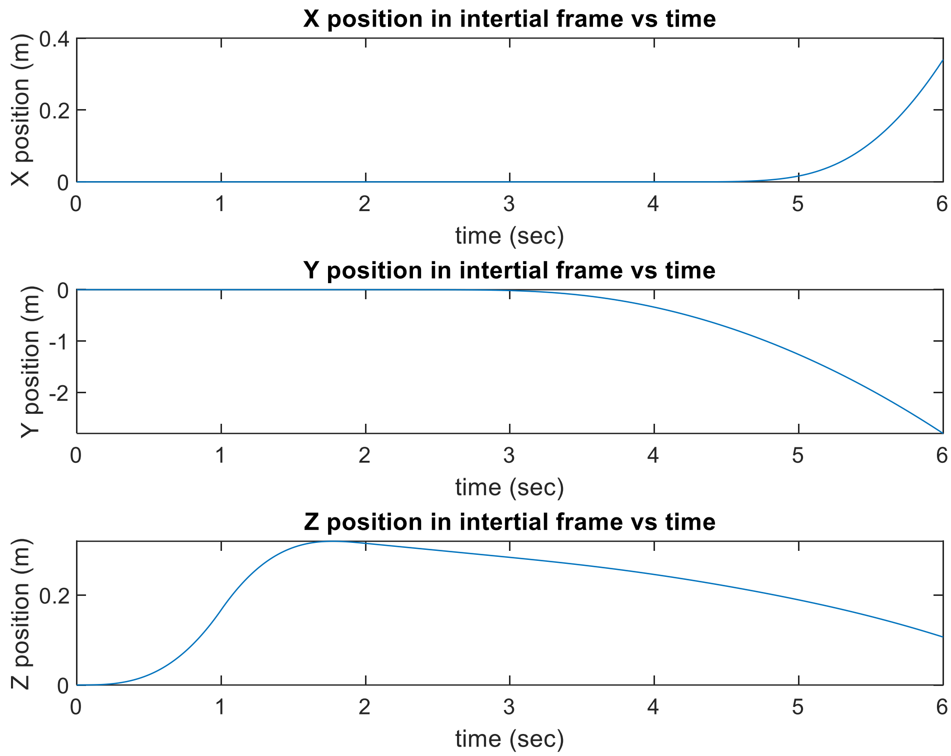

Figure 1. Position in Inertial Frame vs Time

Figure 1 shows the quadcopters inertial position with respect to time. The x position of the drone does not move until the pitch and roll movement at five seconds. The y position remains at a constant zero, until three seconds. 0.5 seconds is when the quadcopter starts to experience a constant roll angle. The z position rises as the quadcopter ascends. When the quadcopter hovers, the z position remains constant. 0.3 meters is when the quadcopter hovers. After the quadcopter begins to roll, the z position decreases. As the quadcopter rolls, we can conclude that it loses altitude.

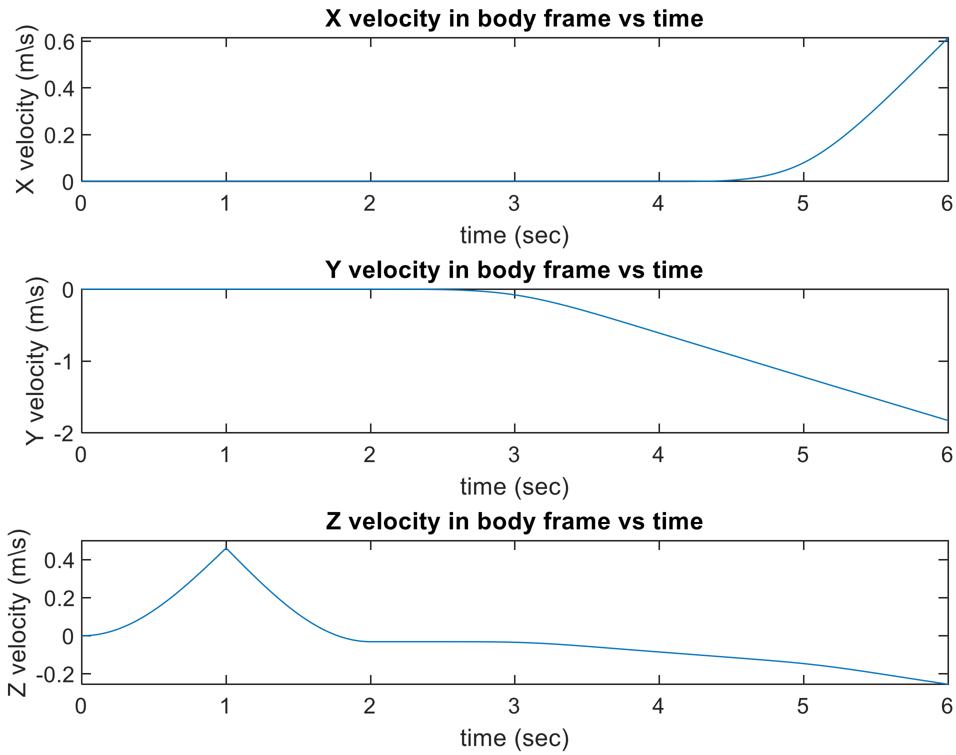

Figure 2. Velocity in Body Frame vs Time

Figure 2 shows the velocities in each direction in the body frame. The x and y velocity in the body frame looks similar to the x and y position in the inertial frame shown in figure 1. The x velocity remains zero until about 4.5 seconds. Between four and five seconds is when the pitch movement occurred, which is detailed in Part 1: Step C. The Y velocity remains zero until just before the three second mark. The second three denotes the beginning of the constant roll angle. The z velocity experiences exponential growth during the ascension phase. It also experiences exponential decay as the quadcopter hovers. During the roll phase it remains at zero, until the pitching movement causes the quadcopter to increase in the z velocity.

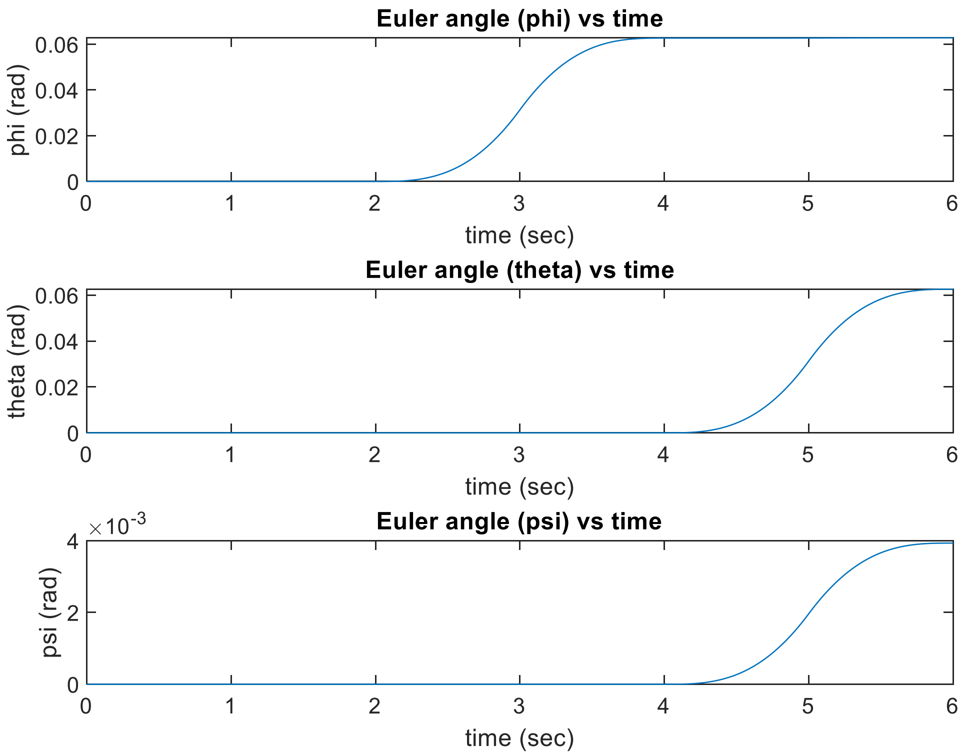

Figure 3. Euler Angles vs Time

Figure 3 shows the Euler angles in radians vs time. Phi remains at zero until two seconds, and then phi increases for the next two seconds. Phi rotates up to 0.06 radians during the roll phase. After four seconds of flight, the quadcopter remains at a constant roll angle of 0.06 radians. Theta and psi remain zero until about the 4.5 second mark. Then their thetas both start to increase. The increase in theta and psi's angle is due to the pitching movement. Theta evens out at 0.06 radians, and psi evens out around 0.004 radians. Their peak angles are reached at 6 seconds each.

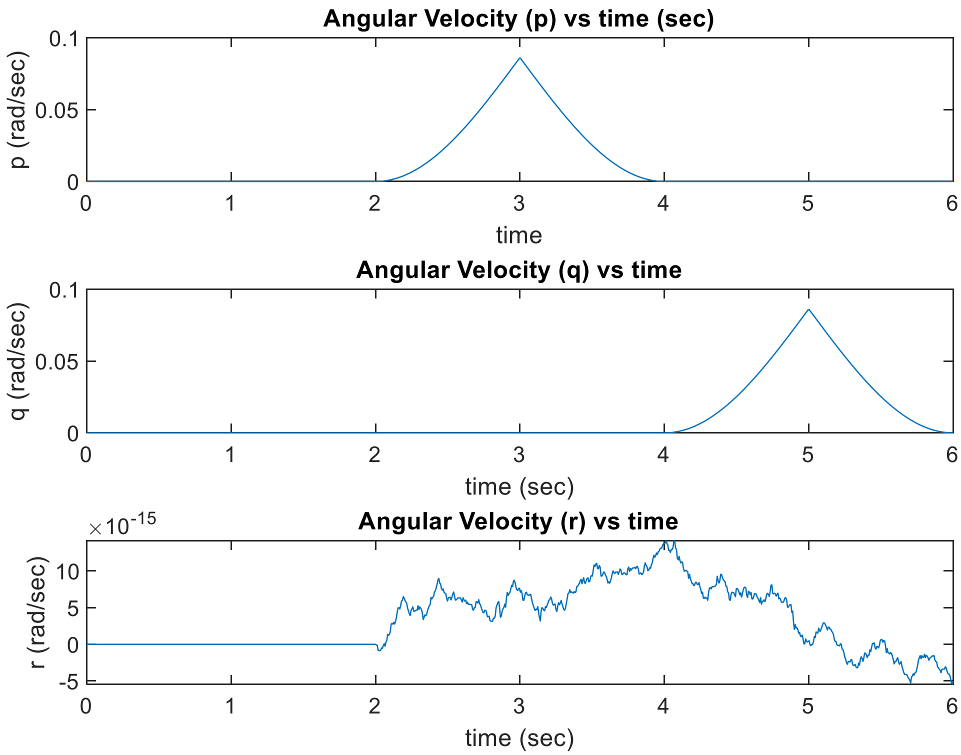

Figure 4. Angular Velocities vs Time

Figure 4 shows the angular velocity in radians per second versus time. The angular velocity, p, is zero for 2 seconds, then zero again after 4 seconds. P exponentially increases between seconds two and three, then exponentially decreases between 3 and 4. It reaches a peak of about 0.08 radians per second in three seconds. During this time the quadcopter is in the rolling movement. It rolls one way, then it counteracts and rolls back. Angular velocity, q, is zero for four seconds. The same behavior is followed as the p, but at a later time. Growth is experienced when the quadcopter reaches its highest pitch angle. Angular velocity, r, is zero for two seconds. At two seconds r begins to increase sporadically. It reaches a peak value of $10 * 10^{-15}$ radians per second in four seconds. Four seconds is when the quadcopter begins its pitching movement. After four seconds the angular velocity decreases to a minimum of $-5 * 10^{-15}$ at about 5.8 seconds into flight.

Part 2:

The following equations represent the quadcopter in autopilot:

$$T = (g + ({z_r}' - z') + (z_r - z))\frac{m}{cos\phi cos\Theta} \quad (13)$$

$$L = I_{xx}(({\phi_r}' - \phi') + (\phi_r - \phi)) \quad (14)$$

$$M = I_{yy}(({\Theta_r}' - \Theta') + (\Theta_r - \Theta)) \quad (15)$$

$$N = I_{zz}(({\psi_r}' - \psi') + (\psi_r - \psi)) \quad (16)$$

subscript ‘r’ for equations 13-16 represents the reference signal for that particular variable. From these autopilot equations, the following can be derived:

$$\Omega_1^2 = \frac{T}{4k} - \frac{M}{2kl} + \frac{N}{4b} \quad (17)$$

$$\Omega_2^2 = \frac{T}{4k} - \frac{L}{2kl} - \frac{N}{4b} \quad (18)$$

$$\Omega_3^2 = \frac{T}{4k} + \frac{M}{2kl} + \frac{N}{4b} \quad (19)$$

$$\Omega_4^2 = \frac{T}{4k} + \frac{L}{2kl} - \frac{N}{4b} \quad (20)$$

with these equations in mind, the motion of the quadcopter will be simulated for the 120 seconds with the following initial conditions:

$$x = y = x' = y' = z' = p = q = r = 0, z = 1m, \Theta = \phi = \psi = 10°$$

For the reference position, the desired value is $z_r = 10m$. Unlike the reference position, all other reference variables are set to zero.

Results and Discussion:

Proving the angular velocity satisfies the autopilot equation.

$$L = kl(-\Omega_2^2 + \Omega_4^2)$$

$$L = kl\left[-(\frac{T}{4k} - \frac{L}{2kl} - \frac{N}{4b}) + (\frac{T}{4k} + \frac{L}{2kl} - \frac{N}{4b})\right]$$

$$L = kl\left[\frac{L}{2kl} + \frac{L}{2kl}\right]$$

$$L = kl\left[\frac{L}{kl}\right]$$

$$L = L$$

$$M = kl(-\Omega_1^2 + \Omega_3^2)$$

$$M = kl\left[-(\frac{T}{4k} - \frac{M}{2kl} + \frac{N}{4b}) + (\frac{T}{4k} + \frac{M}{2kl} + \frac{N}{4b})\right]$$

$$M = kl\left[\frac{M}{2kl} + \frac{M}{2kl}\right]$$

$$M = kl\left[\frac{M}{kl}\right]$$

$$M = M$$

$$N = b(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2)$$

$$N = b\left[(\frac{T}{4k} - \frac{M}{2kl} + \frac{N}{4b}) - (\frac{T}{4k} - \frac{L}{2kl} - \frac{N}{4b}) + (\frac{T}{4k} + \frac{M}{2kl} + \frac{N}{4b}) - (\frac{T}{4k} + \frac{L}{2kl} - \frac{N}{4b})\right]$$

$$N = b\left[\frac{N}{4b} + \frac{N}{4b} + \frac{N}{4b} + \frac{N}{4b}\right]$$

$$N = b\left[\frac{N}{b}\right]$$

$$N = N$$

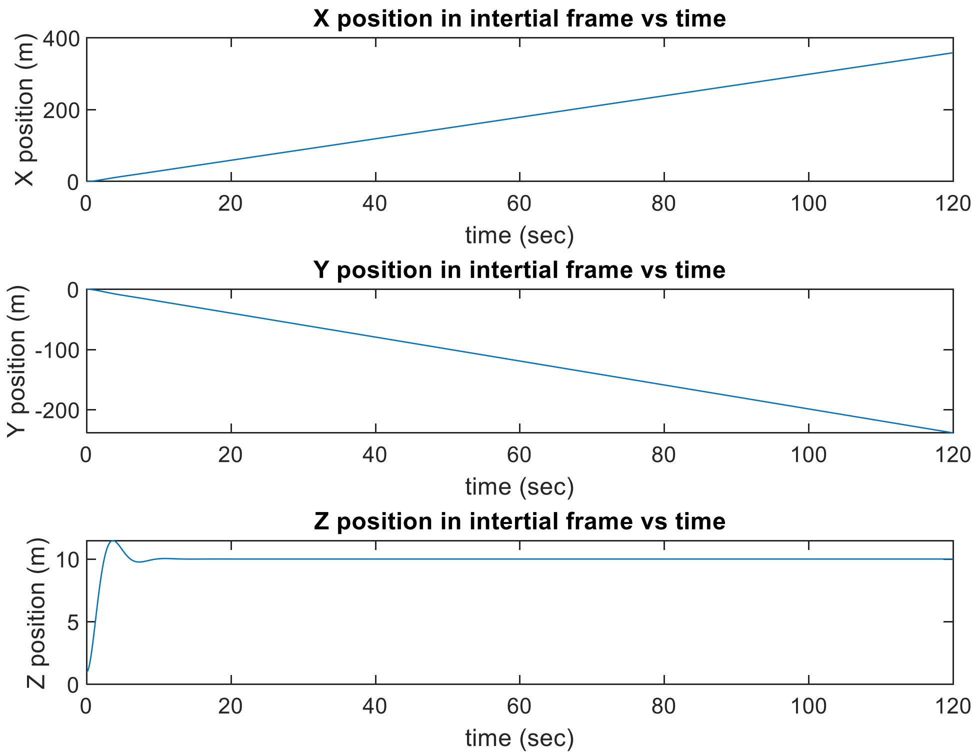

Figure 5: Autopilot Position in Inertial Frame vs Time

For the X position graph, the position increases linearly with time. The position starts at zero meters at zero seconds but increases to about 375 meters around 120 seconds. The opposite can be said about the Y position graph. For that graph the linear relationship stays the same as the X graph, but the relationship is inverted. The Y graph starts in a position of zero but decreases linearly to -225 meters at 120 seconds. The Z position is different from both the X and Y position. It increases to about 13 meters during the first 5 seconds. After about 10 seconds, the Z position decreases and stabilizes at 10 meters for the rest of the flight.

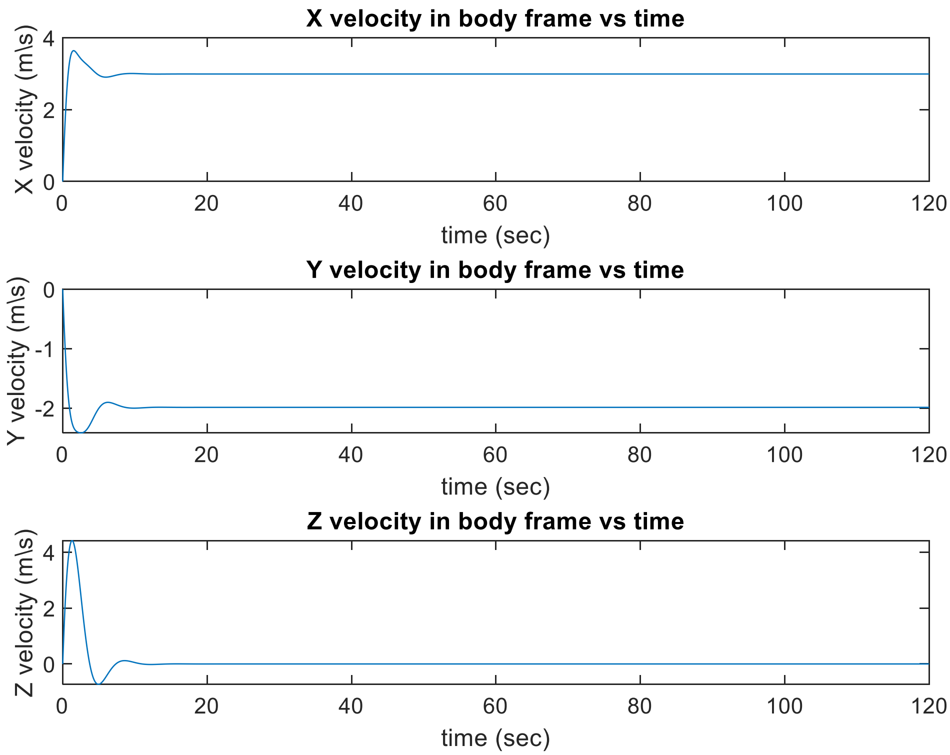

Figure 6: Autopilot Velocity in Body Frame vs Time

In the X velocity graph, the velocity starts at 0 m/s and increases until 3 seconds and approaches a maximum around 3.75 m/s. The velocity then decreases to 3 m/s around 5 seconds and stabilizes at this velocity for the remainder of the flight time. Y velocity undergoes a similar velocity change but in the negative Y direction. As it starts at 0 m/s it changes to -2.5 m/s in 2 seconds. The velocity changes to -2 m/s at 7 seconds and stabilizes at -2 m/s for the remaining time of the flight. Z velocity changes velocity like X velocity but more drastically. Z velocity increases to 4 m/s almost instantaneously in about 1 second. Then the Z velocity decreases to -1 m/s at 5 seconds. It increases again to 0 m/s and stabilizes for the remaining time. What can be learned from these velocity graphs is that the quadcopter will overshoot the preset states by the user but will, in time, stabilize itself and come back to the required state.

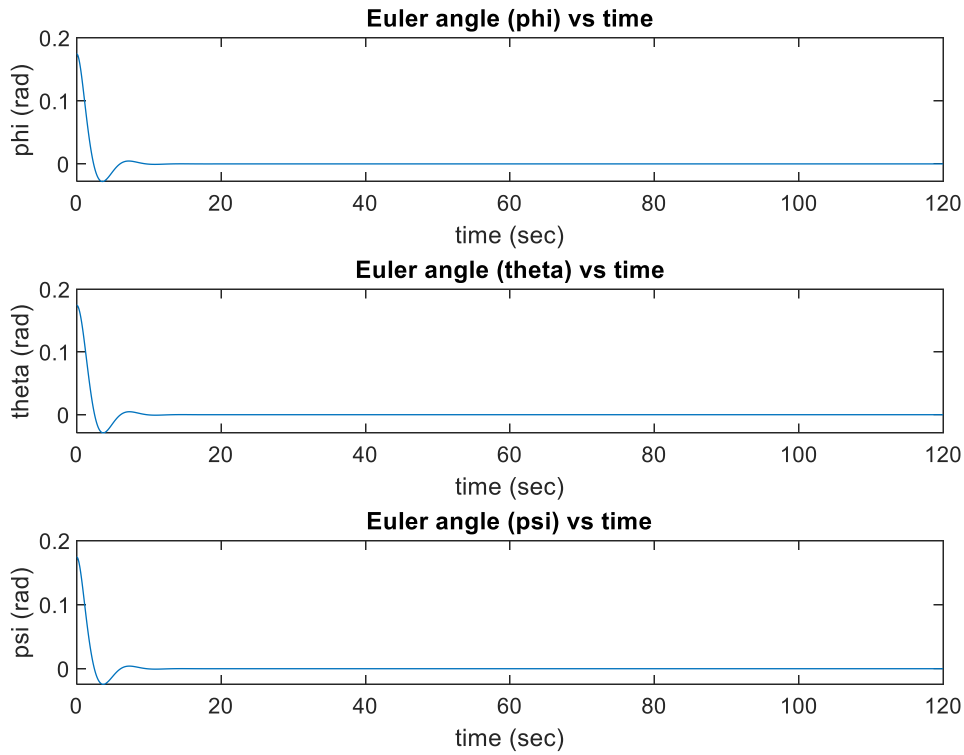

Figure 7: Autopilot Euler Angle vs Time

For Figure 7, all angle types, that being phi, psi, and theta were set to about 0.175 radians. Then in 2 seconds, the angles would decrease to -0.025 radians. Eventually the angles would stabilize at 0 radians around 10 seconds. Like what was learned in Figure 6, the states would tend to overshoot their preset values but then would stabilize to their required state.

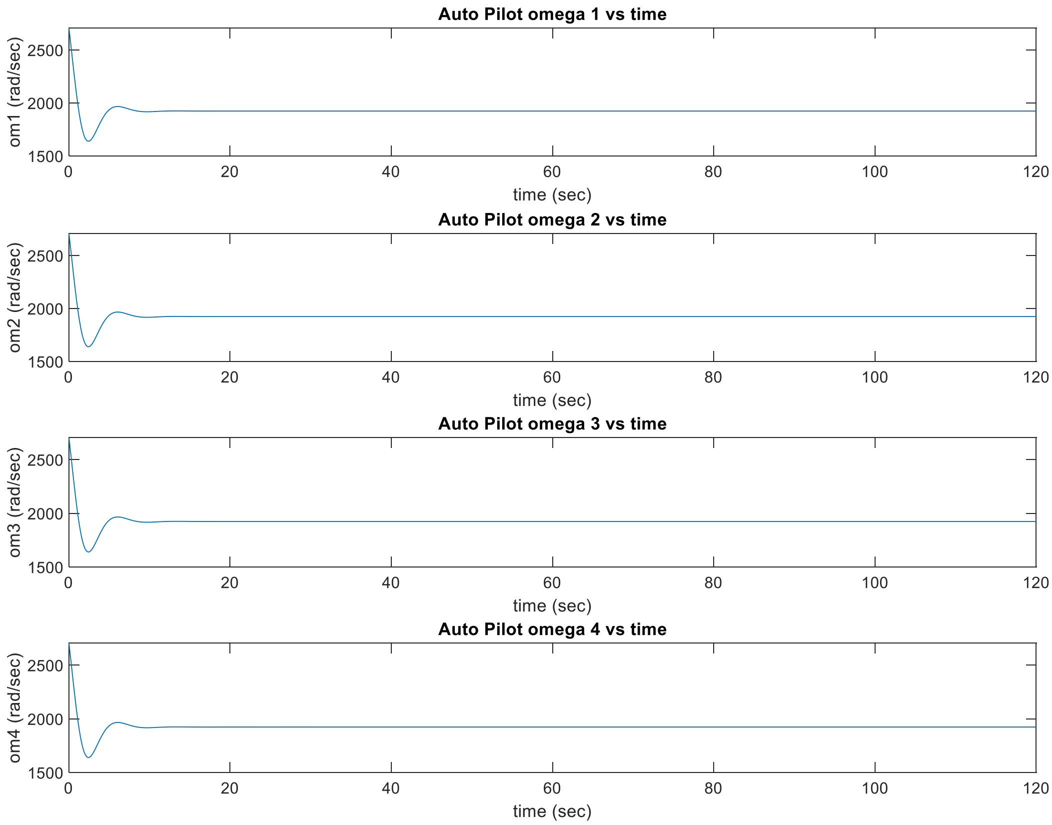

Figure 8: Autopilot Omega vs Time

Figure 8's omegas, like Figure 7's Euler angles, all appear to share the same values with each other. The autopilot omega starts at 2600 rad/s, that value sharply declines in 5 seconds to 1600 rad/s. Recalling the trend that all states are initially overshot, the omega begins to increase to about 1950 rad/s after another 5 seconds. Finally, all omega values reach a value of 1900 rad/s and remain constant for the remainder of the flight time. The constant omega value shows that the autopilot successfully takes control of the quadcopter.

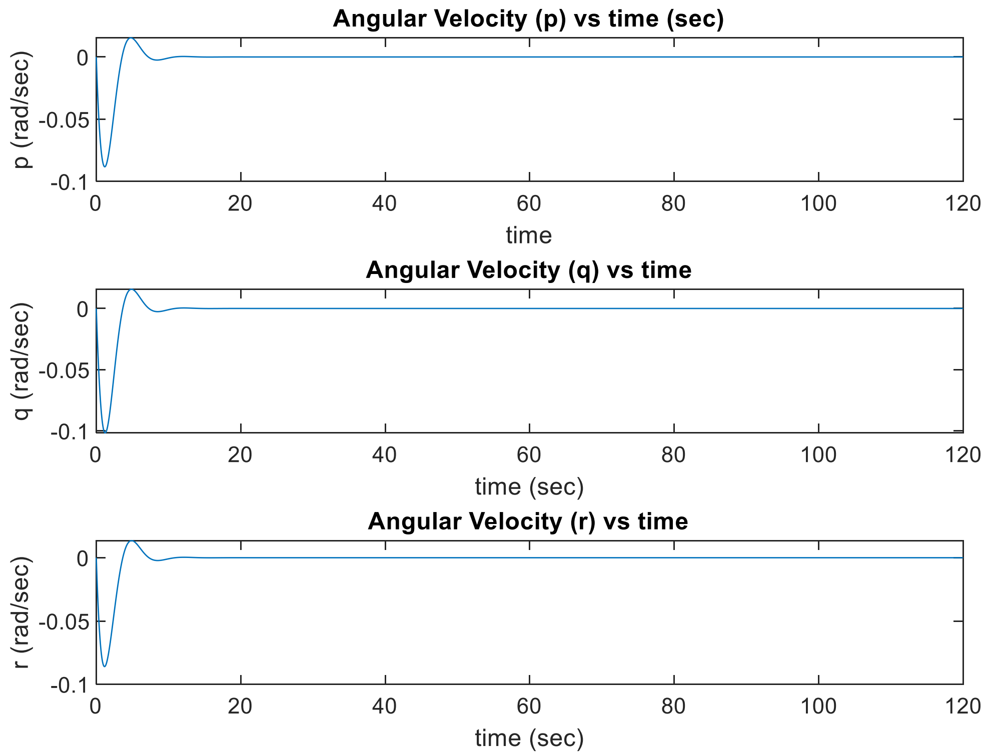

Figure 9: Autopilot Angular Velocity vs Time

Figure 9 represents the autopilot angular velocity of the quadcopter in respect to time. All angular velocity angles p, q, and r are set to 0 radians at 0 seconds. All angular velocities decrease to -0.8 rad/sec in just 1 second. The value of the angular velocity then rises sharply to 0.2 rad/sec in 3 seconds. The quadcopter then begins to approach its stable states as the angular velocities decrease back to 0 rad/sec.