Date: April 25, 2023

To: Dr. Beck

From: Ankit Gupta

Subject: Vibration measurement of cantilevered beam

Introduction

The objective of this laboratory experiment was to estimate and evaluate the initial four natural frequencies and the loss factor for a beam, both with and without the inclusion of a tip mass accelerometer. This was done to verify the accuracy of the laser Doppler vibrometer (LDV). Natural frequency refers to the inherent oscillation rate of an object when it is disrupted and can be used to pinpoint the locations of nodes. Nodes are distinct points on a graph where the amplitude of the wave is zero. The equations below can be utilized to compute critical damping ratio(ζ):

$$w_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}}, \zeta = \frac{1}{w_n} \frac{\ln\left(\frac{A_o}{A_n}\right)}{(t_n - t_o)} \quad , \qquad \beta_n L = 1.88, 4.69, 7.85, 11.0 \qquad (1)$$

These equations are derived from the Euler-Bernoulli Beam Theory Equation. In order to determine w_n, E, I, A and ρ must be calculated. The moment of Inertia (I) and area (A) is computed using following equation:

$$I = \frac{bh^3}{12}, A = bh \qquad (2)$$

In this experiment, a cantilever beam was configured with one end secured and the other end left unattached. This arrangement was selected to facilitate the attachment of the tip mass accelerometer to the beam's free end.

Procedure

Figure 1 shows the setup for this experiment.

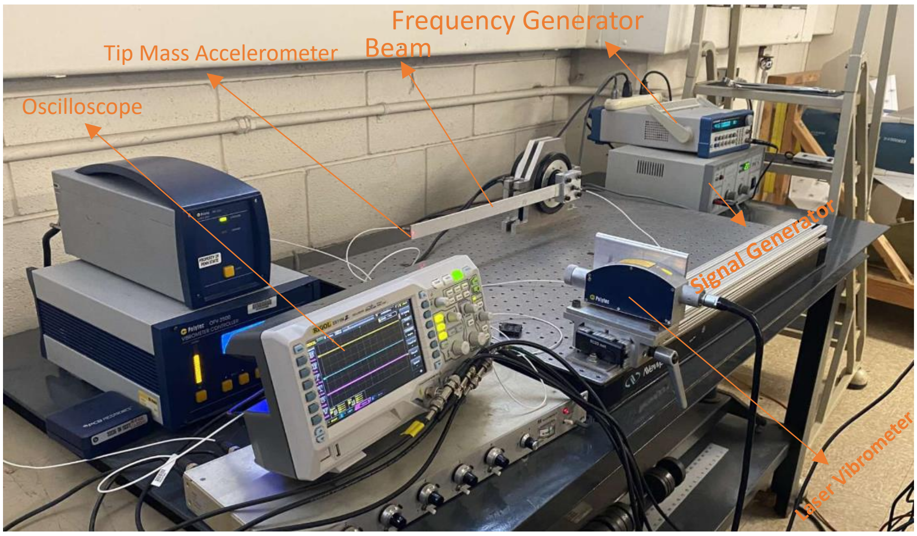


Figure 1. Experiment Setup

The Oscilloscope, is capable of testing and displaying voltage signals as waveforms, typically illustrating voltage changes over time. These waveforms assist in identifying node locations at various frequencies. Generators are used to modify the oscillating frequency of the beam. The aluminium beam was oscillated to determine node locations. The tip mass accelerometer was positioned at the free end of the beam during the measurement of natural frequencies to assess whether significant differences arise when altering the beam's boundary conditions. LabView software was used to collect data for the log decay plot and the white noise Fast Fourier Transform (FFT) graph.

Results and Discussion

The dimension of beam is:

- Length (L) – 17.125 inch
- Width (b) – 0.998 inch
- Thickness (h) – 0.128 inch
- Young's modulus (E) – 10^7 psi
- Density (ρ) – 0.0975 lb/in^2

The following are two tables displaying the location and natural frequency measured for each mode number. Table 1 presents the data when the tip mass is not attached to the beam, while Table 2 demonstrates the data when the tip mass is attached to the beam. The nodes (n) is determined using the equation $n = m - 1$, where m represents the mode number. The Predicted Natural Frequency was computed from the white noise FFT data showed in Figure 2.

Table 1. Natural Frequency without Tip Mass

Mode, m	Number of nodes	Predicted Natural Frequency (Hz)	Measured Natural Frequency (Hz)	Error (%)
1	0	12.6	12.3	2.38
2	1	74.59	72.7	2.54
3	2	226.87	226.2	0.29
4	3	434.5	434.8	0.07

Table 2. Natural Frequency with Tip Mass

Mode, m	Number of nodes	Predicted Natural Frequency (Hz)	Measured Natural Frequency (Hz)	Error (%)
1	0	11.78	11.7	0.68
2	1	71.1	69.6	2.11
3	2	217.78	223.5	0.03
4	3	414.1	416.7	0.62

The reason the frequency with the tip mass is lower than the frequency without the tip mass is due to the change in boundary conditions when the tip mass is added. This causes the moment equation in the Euler-Bernoulli beam theory to increase, leading to a decrease in frequency.

Table 3 shows the node location for each Measured Natural Frequency, which matches with our mode shapes predicted in Figure 5.

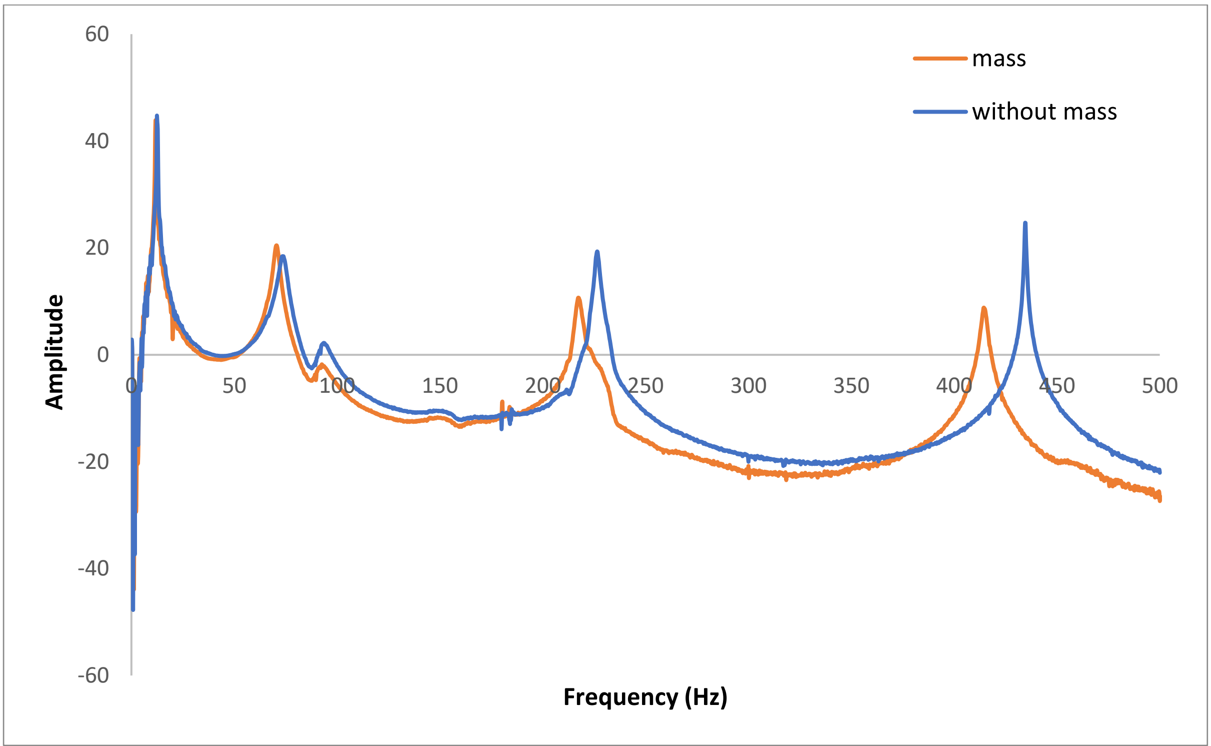


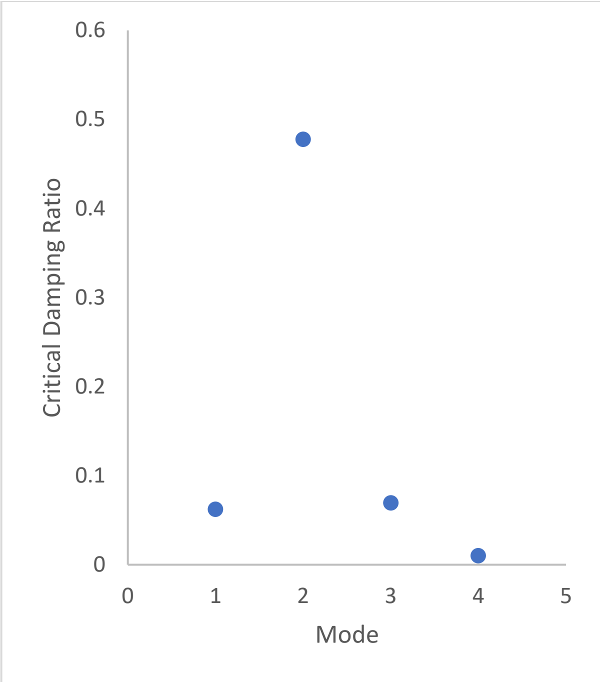
Figure 2. White Noise FFT of Laser Doppler Vibrometer

Table 3. Node location

Measured Natural Frequency (Hz)	1st Node (in)	2nd Node (in)	3rd Node (in)
11.7	N/A	N/A	N/A
12.3	N/A	N/A	N/A
69.6	13.625	N/A	N/A
72.7	13.250	N/A	N/A
223.5	15.375	9.000	N/A
226.2	14.875	5.625	N/A
416.7	15.875	11.25	6.125
434.8	15.500	11.00	5.875

To calculate the critical damping ratio, the equation 1 and Log decrement method is used. The first mode shapes values, mentioned in equation 1, were also used to calculate the critical damping ratio.

To calculate the critical damping ratios, data was obtained from the accelerometer readings. The critical damping ratio for each mode can be observed in Figure 3 and Figure 4, representing the scenarios without and with the tip mass, respectively.



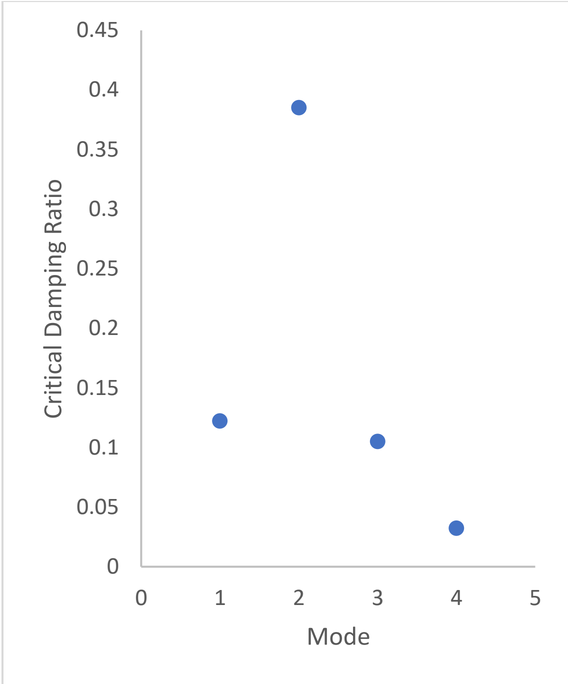


Figure 3. Critical Damping Ratio without Tip Mass

Figure 4. Critical Damping Ratio with Tip Mass

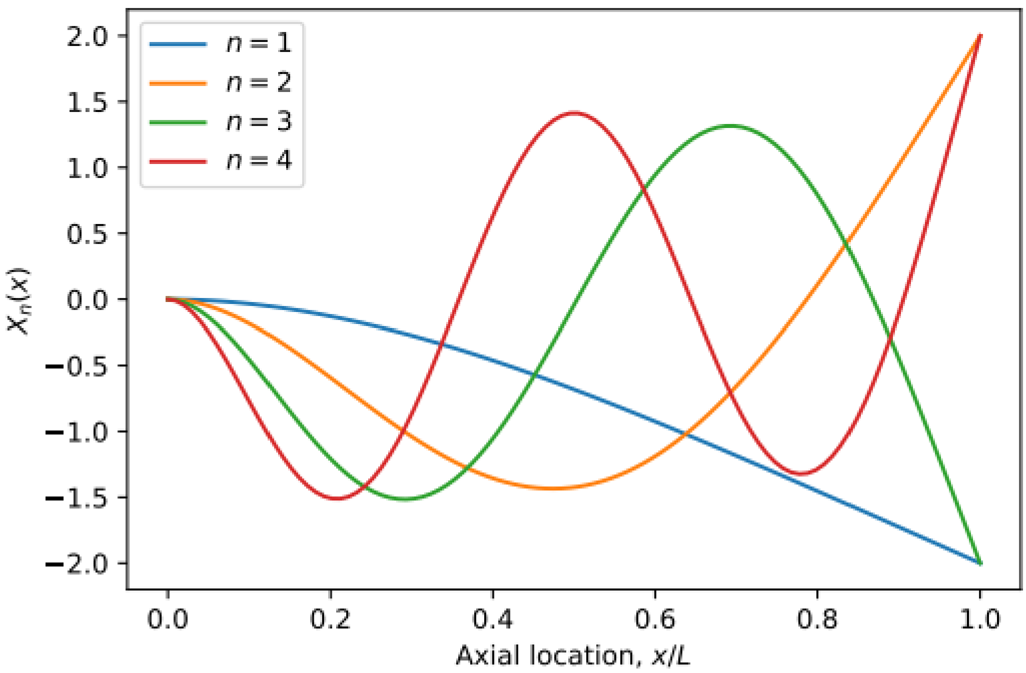


Figure 5. First four predicted mode shapes of cantilevered beam

Challenges faced during this lab included determining the accuracy of measured natural frequency values. Numerous factors can affect the natural frequency, such as mass, stiffness, temperature, and boundary conditions. One way to mitigate this issue is by conducting multiple tests, which can help determine the accuracy and precision of the data. Inaccurate natural frequency values can cause variations in node locations, potentially impacting the experiment's objectives.

Conclusion

In conclusion, the lab's objective, which was to assess the validity of the Laser Doppler Vibrometer (LDV), was found to be reasonably close to the predicted values. The percent error between the measured values and the white noise FFT (predicted) values ranged from approximately 0% to 3%, indicating satisfactory results. Thus, the objective of this experiment was met. In future studies, using different types or sizes of aluminium beams could help further determine the accuracy of the Laser Doppler Vibrometer (LDV). Another approach would involve altering some of the factors that influence the natural frequency. The accelerometer was attached using wax, which could potentially loosen over time and affect the results. In future experiments, a more consistent and reliable wax or alternative adhesive should be used to ensure that the accelerometer remains securely attached throughout the testing process.