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Subject: Determining the stiffness of three I-beams provided

Aerospace 305W Structures Laboratory

Beam Bending Experiment

Abstract

In this study, three distinct I-beam designs were evaluated to identify the most optimal option. The three I-beams under investigation were all made of aluminium and two of them are carbon composite reinforced, and aluminium reinforced. Prior to the experiment, it was hypothesized that the carbon composite reinforced beam would be the most suitable due to its higher stiffness compared to the aluminium I-beam and lighter weight than the aluminium reinforced I-beam. Three approaches were utilized to determine the stiffness of the I-beams.

The first method involved calculating stiffness physically, while the second and third methods employed the Euler-Bernoulli Beam Equations. After deriving these equations, the slopes of the collected data were analysed to assess stiffness. Two graphs, Load vs Deflection and Load vs Strain, were used in this process. The stiffness values obtained were then used to find the most ideal beam.

Upon concluding the experiment, it was determined that the carbon composite reinforced aluminium beam was indeed the most optimal I-beam among the three tested.

Introduction

The aim of this experiment is to assess whether incorporating composite material to the top and bottom of an aluminium beam will enhance the beam's stiffness more effectively than adding aluminium to these sections. It is anticipated that the carbon composite material in the carbon composite reinforced I-beam will contribute to increased stiffness without significantly increasing the overall weight of the beam.

Stress and strain are two crucial concepts in the context of beam bending. Stress (σ) is defined as:

$$\sigma = \frac{F}{A}$$

Where F is force and A is area. Strain (ε) refers to the deformation or displacement of a material resulting from the applied stress.

$$\varepsilon = \frac{\Delta L}{L}$$

Where ΔL is change in length and L is length. Hooke's Law is used to make a relationship between stress and strain.

$$\sigma = E\varepsilon$$

E, or Young's Modulus of Elasticity, indicates how readily a material can stretch and deform. To determine the stiffness, it is necessary to calculate the Moment of Inertia (I), which is given by:

$$I = \frac{wh^3}{12}$$

Here, 'w' represents the width, and 'h' is the height of the cross section.

To predict the deflection, strain, and stiffness of each beam based on their geometry, the following steps were taken.

The beam was broken into sections A, B, C, D, E as shown in Figure 1.

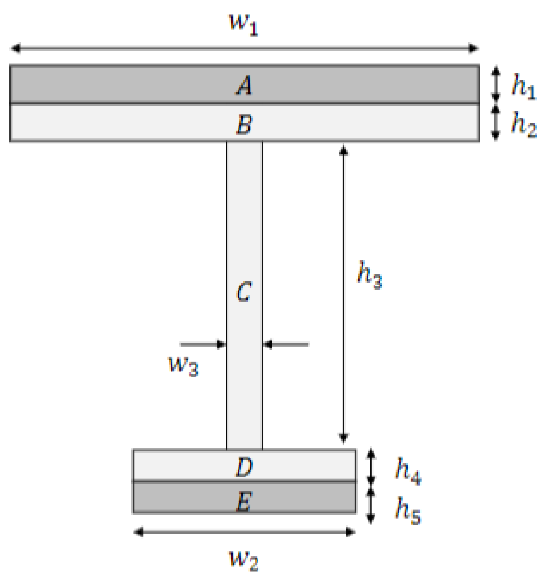

Figure 1. Diagram of I-beam section

For each section, Area (A), Modulus of Elasticity I, Moment of Inertia (I), and Centroid (z) was calculated using Table 1.

Table 1. Calculating E, A, I, and z for a five-section beam

Section	A_i	E_i	I_i	z_i
A	w_1h_1	E_A	$\frac{w_1h_1^3}{12}$	$h_5+h_4+h_3+h_2+\frac{h_1}{2}$
B	w_1h_2	E_B	$\frac{w_1h_2^3}{12}$	$h_5+h_4+h_3+\frac{h_2}{2}$
C	w_2h_3	E_c	$\frac{w_2h_3^3}{12}$	$h_5+h_4+\frac{h_3}{2}$
D	w_3h_4	E_D	$\frac{w_3h_4^3}{12}$	$h_5+\frac{h_4}{2}$
E	w_3h_5	E_E	$\frac{w_3h_5^3}{12}$	$\frac{h_5}{2}$

The Modulus of Elasticity for Aluminium (Al) and Carbon Composite (CC) are:

$E_{Al} = 1\text{x}10^7$ psi $\qquad$ $E_{CC} = 87\text{x}10^4$ psi

For neutral axis of the beam (z*):

$$z^* = \frac{\sum_i E_i A_i z_i}{\sum_i E_i A_i}$$

For stiffness (EI) for each section about z*:

$$(EI)_i^* = E_i(I_i + A_i(z_i - z^*)^2)$$

For Total Stiffness:

$$EI = \sum_i (EI)_i^*$$

To estimate the mass per unit length of a beam, it is necessary to calculate the density and volume of the beam. Density (D) is defined as

$$D = \frac{m}{V}$$

where 'm' represents mass and 'V' denotes volume. Volume (V) is expressed as

$$V = AL$$

where 'A' stands for the cross-sectional area and 'L' corresponds to the length of the beam. The density of aluminium is 2710 kg/m^3 and carbon composite is 1750 kg/m^3.

To determine the stiffness by using Euler-Bernoulli Beam Equations Theory, the equations for displacement (w) and strain (ε) are derived using following formulas:

$$p(x) = \frac{dV}{dx} \quad V(x) = -\int_c \frac{d\sigma}{dz} z dA \quad \sigma(x,z) = E\epsilon(x,z) \quad \epsilon(x,z) = -\frac{z}{\rho(x)} \quad \rho(x) = \frac{dx^2}{d^2w}$$

$$p(x) = \frac{d^2}{dx^2}\left[\frac{d^2w}{dx^2}\int_c E(x,z)z^2 dA\right]$$

After establishing this equation, several key assumptions were made:

- Hooke's Law is applicable, indicating that the material exhibits linear elasticity.
- The beam's length is much greater than its cross-section and deflections.
- Plane sections remain plane and perpendicular to the neutral axis.

So, for a constant cross section:

$$p(x) = EI\frac{d^4w}{dx^4}$$

Based on the diagram (Figure 2), and an equation can be derived in order to calculate the stiffness of the beams for this experiment.

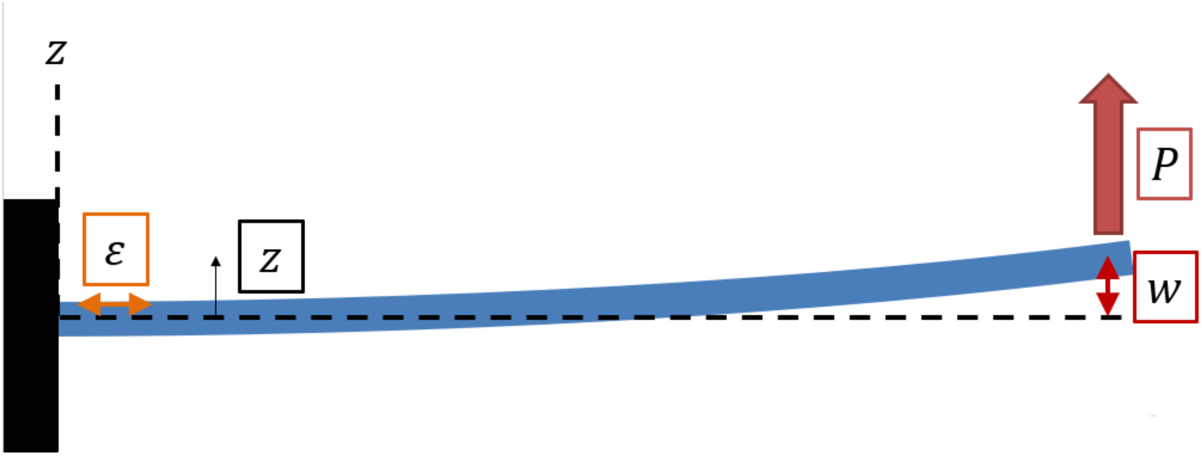

Figure 2. Beam Bending Diagram

To determine the stiffness equation, we integrate to get w(x). There is no distributed load acting on the beam, so:

$$\frac{d^4w}{dx^4} = 0$$

$$\frac{d^3w}{dx^3} = c_1$$

$$\frac{d^2w}{dx^2} = c_1x + c_2$$

$$\frac{dw}{dx} = \frac{c_1x^2}{2} + c_2x + c_3$$

$$w(x) = \frac{c_1x^3}{6} + \frac{c_2x^2}{2} + c_3x + c_4$$

Using boundary conditions to find constants of integration:

$$x = 0 \quad w(0) = 0 \quad \frac{dw}{dx}(0) = 0$$

$$x = L \quad M(L) = EI\frac{d^2w}{dx^2}(L) = 0 \quad V(L) = EI\frac{d^3w}{dx^3}(L) = -P$$

$$c_1 = \frac{-P}{EI} \quad c_2 = \frac{PL}{EI} \quad c_3 = 0 \quad c_4 = 0$$

$$w(x) = \frac{P}{EI}\left(-\frac{x^3}{6} + \frac{Lx^2}{2}\right)$$

$$\varepsilon(x,z) = -z\frac{d^2w}{dx^2} \quad \frac{d^2w}{dx^2} = \frac{P}{EI}(-x + L)$$

$$\varepsilon(x,z) = \frac{Pz}{EI}(x - L)$$

After deriving w(x) and $\varepsilon(x, z)$ equations, they can be modified by replacing 'x' with L_d in the displacement equation and L_s in the strain equation, where 'z' represents the midpoint of the I-beam. The Euler-Bernoulli Beam Equations are employed to assess the stiffness of the aluminium, carbon composite, and aluminium reinforced beams. The data gathered from the experiment will be illustrated in two graphs: load vs deflection and load vs strain (as shown in the Results section). Once the data points are plotted, a best-fit line will be drawn. For the load vs deflection graph, the slope will be equal to m_d, which can then be used in the Euler-Bernoulli Beam Equation to determine stiffness. The same principle applies to the load vs strain graph; since the slope is equal to m_e, the Euler-Bernoulli Beam Equation for strain will be used to ascertain the stiffness.

Procedure

The equipment utilized for this experiment included a Linear Potentiometer, S-beam load cell, strain gauge, data acquisition system, and software. A linear potentiometer (Figure 4) is a position sensor employed to measure displacement along a specific axis. It functions as a resistor with a resistance value proportional to displacement. In this experiment, it was used to gauge the deflection of the three beams during loading.

Beam load cells convert force into an electrical signal, which is then interpreted using an S-load cell (Figure 3), which allows for the measurement of both tension and compression forces.

A strain gauge (Figure 5) is a sensor that measures electrical resistance changes corresponding to strain. It operates as a resistor with a resistance value directly proportional to strain and inversely proportional to the area. Given the small resistance value, a strain gauge is essentially a thin wire looped back and forth. In this experiment, the strain gauge was positioned at the bottom of the beams to determine the external forces (Tension) acting on each of the three beams.

A data acquisition system is used to convert, store, transmit, and process collected data. In this experiment, the data acquisition system converted the data gathered by the linear potentiometer and strain gauge into load, deflection, and strain values. Once the data was collected, it was transmitted to the computer software. This software then plotted the data on two graphs, one displaying strain and the other deflection based on the applied load.

Figure 3. S-load Cell

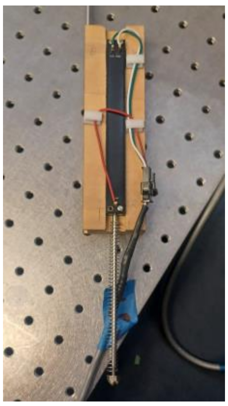

Figure 4. Potentiometer

Figure 5. Strain Gauge

The test articles used for this experiment were three I-beams: aluminum, carbon composite reinforced, aluminum reinforced, and aluminum. These beams are shown in order in Figure 6. The aluminum beam (right) serves as our standard design. The carbon composite reinforced beam (left) is an aluminum beam with a layer of carbon fiber composite on both the top and bottom surfaces. The aluminum reinforced beam (middle) consists of an aluminum beam with an additional layer of aluminum on the top and bottom.

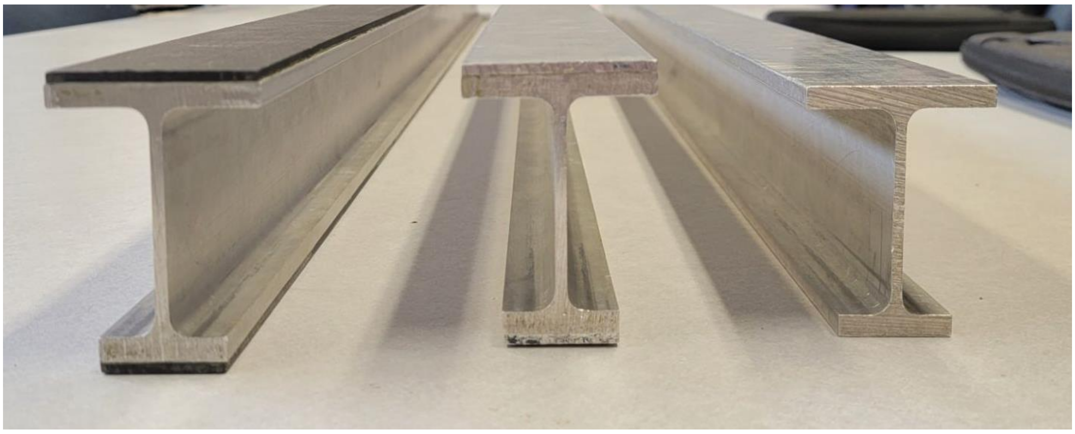

Figure 6. Three beams side by side

The experimental setup involved a cantilever beam configuration (Figure 7). The beams were positioned horizontally, with one end clamped and the other end free. All three beams measured in length (L). A strain gauge was placed Ls from the root of the beam (the clamped end), and the linear potentiometer was situated L_d away from the beam's root. A load was applied to the free end of the beam, and the resulting response was used to determine the stiffness of the three beams.

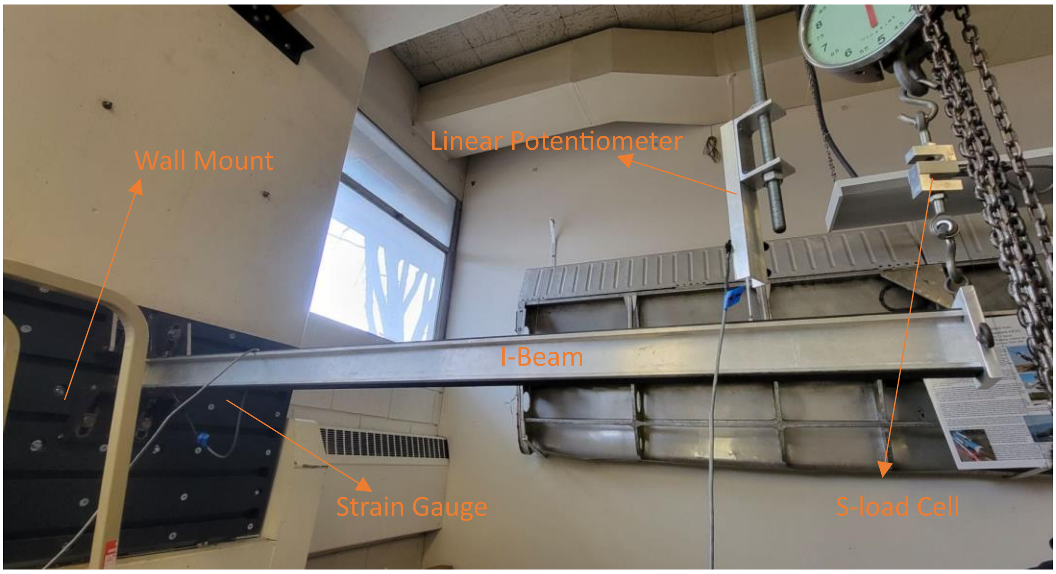

Figure 7. Beam Bending Setup

To ensure proper alignment of the beams before applying the load, the root end was securely clamped to the wall, as depicted in Figure 8. An I-shaped frame was positioned at the free end of the beam (Figure 9), if the beam was aligned correctly, it would fit precisely inside the frame. The I-shaped frame was used to apply the load to the beam (shown in Figure 10). A chain connected the I-shaped frame to the bottom of the S-beam load cell. Another chain, attached to the top of the S-beam load cell, was linked to a pulley. The pulley system facilitated the addition of extra load to the beam. In this setup, as one of the chains was pulled downward, the load would lift the beam from the free end, creating deflection. This chain system is illustrated in Figure 10.

Figure 8. Clamped End

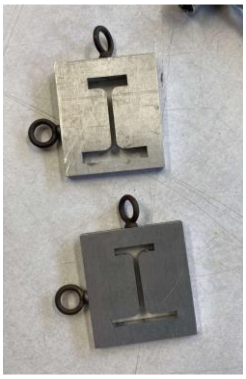

Figure 9. Frame at Free End

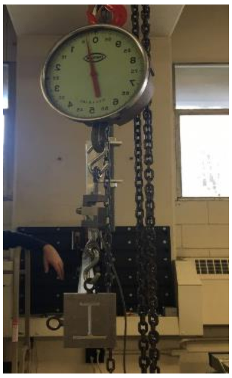

Figure 10. Loading chain

To prevent the beams from breaking under the applied load, they were unloaded after approximately 80 pounds of load had been applied. This setup was designed to ensure the beams remained intact while determining their stiffness.

Results

Aluminium Beam

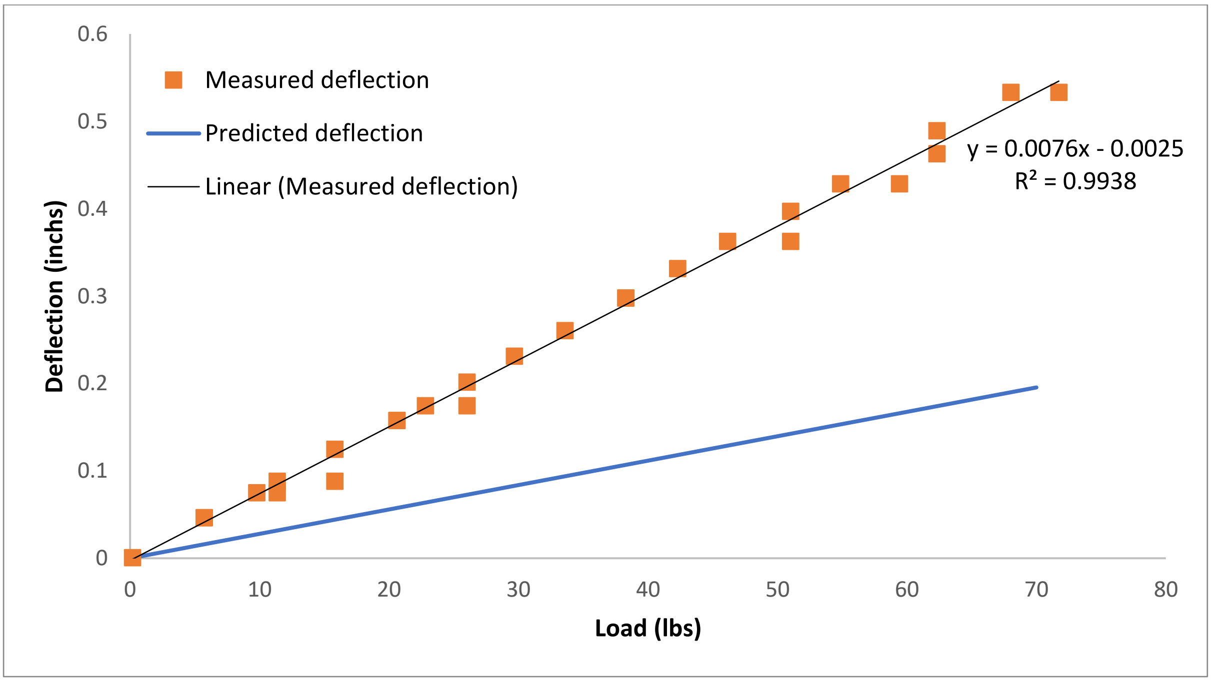

Figure 11. Graph of Load versus Deflection for Aluminium Beam

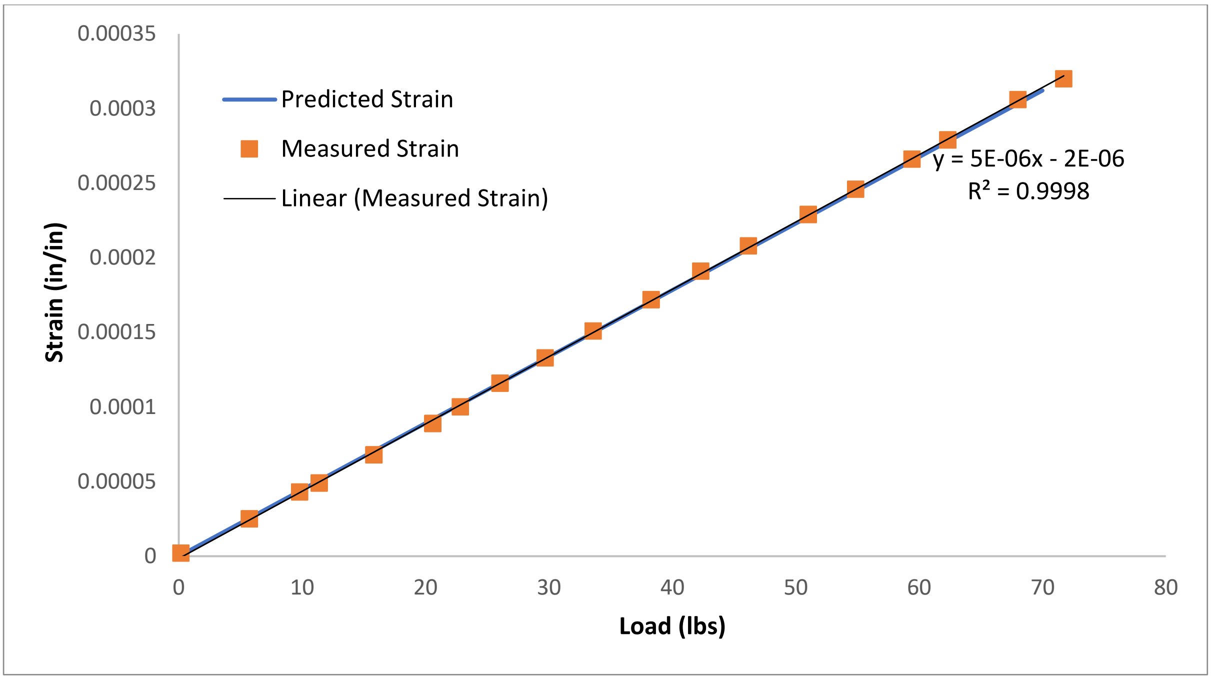

Figure 12. Graph of Load versus Strain for Al Beam

Carbon Composite Reinforced Aluminium Beam

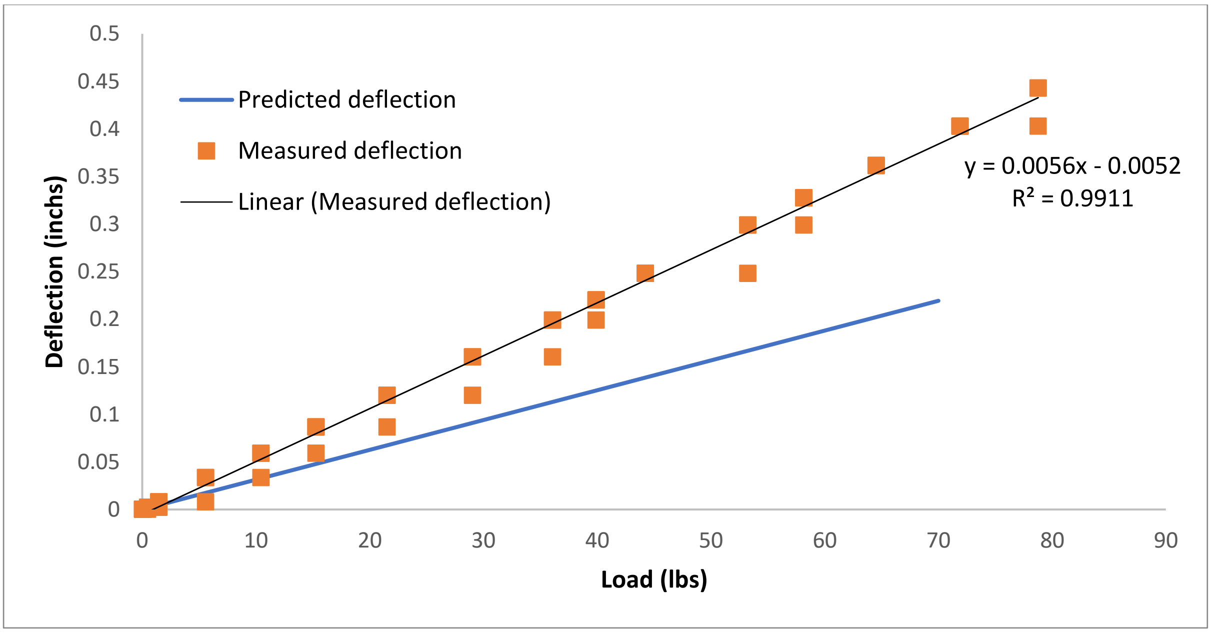

Figure 13. Graph of Load versus Deflection for Carbon Composite Reinforced Al Beam

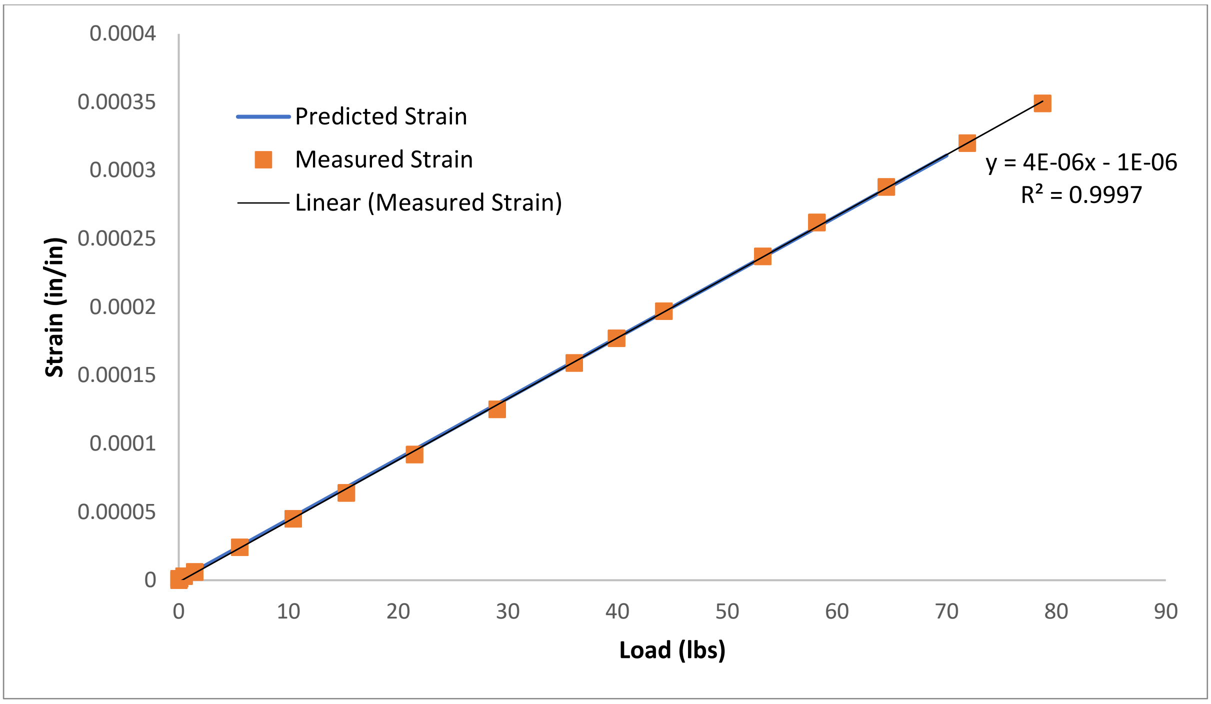

Figure 14. Graph of Load versus Strain for Carbon Composite Reinforced Al Beam

Aluminium Reinforced Aluminium Beam

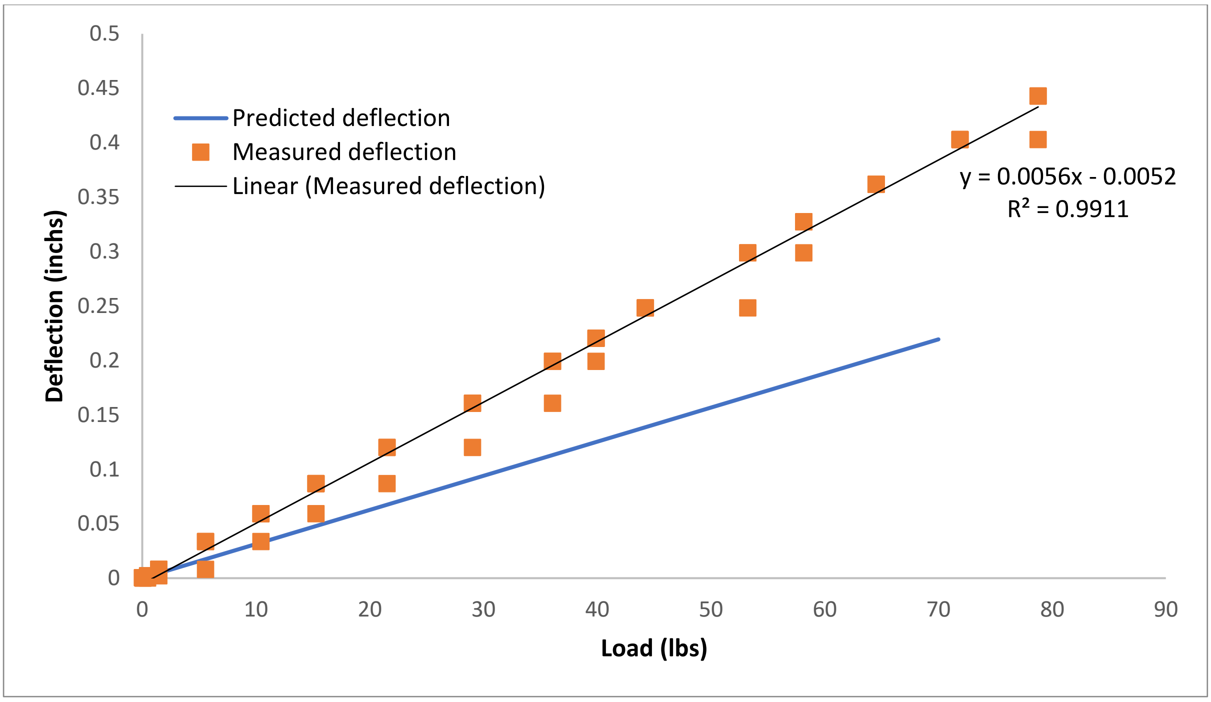

Figure 15. Graph of Load versus Deflection for Aluminium Reinforced Al Beam

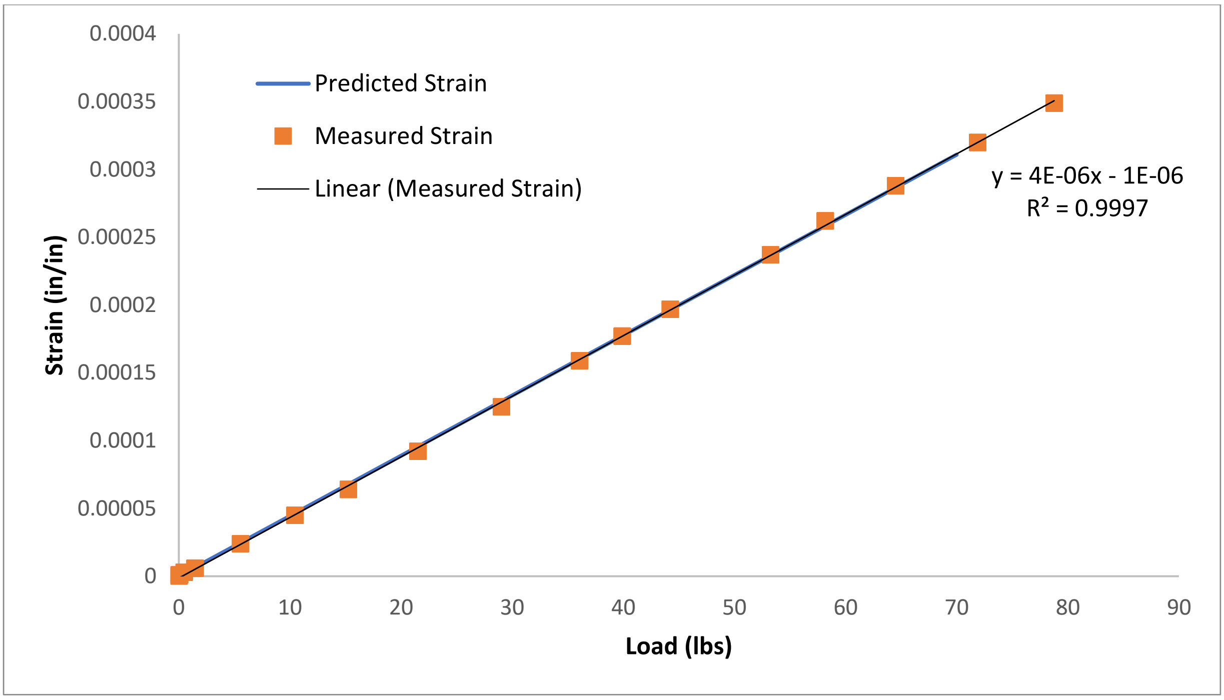

Figure 16. Graph of Load versus Strain for Aluminium Reinforced Al Beam

With the slopes established for the Load vs Deflection and Load vs Strain graphs, the Euler-Bernoulli Beam Equations can be used to evaluate the stiffness of the aluminium, carbon composite reinforced, and aluminium reinforced I-beams. The predicted and calculated stiffness values from deflection (EI_w) and strain (EI_e) are presented in Table 2 below.

Table 2. EI values and Masses for Beams

Beam	EI_w (lbs in^2)	EI_e (lbs in^2)	$EI_{predicted}$ (lbs in^2)	Mass (lb)
Aluminium	11.1E+06	26.9E+06	30.1E+06	12.4
Carbon Composite Reinforced	15.1E+06	29.9E+06	26.9E+06	12.7
Aluminium Reinforced	14.6E+06	42.7E+06	33.0E+06	13.0

The results somewhat align with the predictions, as the line of best fit and the predicted line are similar for some of the graphs. For the aluminium beam, the stiffness values for deflection, strain and predicted are in ascending order. With the carbon composite reinforced beam, the stiffness values of deflection, followed by predicted, and then strain are in ascending order. For the aluminium reinforced beam, the stiffness values of deflection, followed by predicted, and then strain are in ascending order. In all the beams, the deflection stiffness value was the smallest, while the strain value was the largest in carbon composite reinforced beam and aluminium reinforced beam. Differences in the values can be attributed to the accuracy of the instruments used for data collection. For instance, while collecting data for all the beams, the trial had to be repeated several times due to the linear potentiometer shifting during loading of the beam.

Discussion

The various measurements of stiffness, EI, yielded inconsistent outcomes. The strain stiffness appears to be more dependable, as its values are closer to the predicted ones. Additionally, positioning the linear potentiometer nearer to the beam's midpoint along its cross section could have contributed to higher accuracy for EI_w. Among the three I-beams tested, the aluminium reinforced beam exhibited the highest stiffness but was also the heaviest. As a result, the composite beam, which has comparable stiffness and lower weight, would be a more suitable choice. It is important to consider why Predicted and Measured deflection produce different outcomes, which could be due to the mechanical component of the linear potentiometer creating friction. It's also possible that some I-beams may not have a linear slope. There are four potential explanations for this. First, the issue could be caused by the S loading cell, which should be perpendicular to the beam and free from twisting or rotating with the chain during the experiment. Second, the sensors may be sensitive. Third, the beam's boundary conditions might be a contributing factor. Lastly, the I-beams' repeated use by numerous groups over the years may have led to slight deformation, resulting in a different response due to fatigue in the beam.

Conclusion

In general, reinforcing beams increases their stiffness and the beam's stiffness can be enhanced using carbon composite reinforcement on the I-beam without increasing its weight. The stiffness of each beam can be determined from deflection, strain, and prediction. The mass can then be calculated based on this density, area, and length. One suggestion for future experiments is to try a different setup to compare the stiffness values with those determined in this study. Another recommendation is to use newer beams to avoid potential differences in responses due to material fatigue.