Project 1

Circular Restricted Body Problem

(CRTBP)

AERSP 304

Final Report

We have neither given nor received any assistance on this project.

Team Member	PSU ID	Contributions	Percentages
Thomas Waltz	tiw5261	Assist with code, Final Report	35%
Ankit Gupta	apg5667	Code D and E, Final Report	35%
Kameron Metcalf	kmm8076	Assist with part B	30%

Methodology:

The following report covers the process of simulating orbits in the Earth-Moon system around Lagrange point 2, and Earth-Sun around Lagrange point 4. The equations of motion that were derived in the preliminary report are used once again. The equations can be seen in equations one and two below.

$$x'' - 2y' = \partial U/\partial x = Ux \tag{1}$$

$$y'' + 2x' = \partial U/\partial y = Uy \tag{2}$$

These equations were integrated numerically. The purpose was to find a spacecraft's trajectory in a Lyapunov orbit. The equations were rearranged to solve for x'' and y''. This was done in order to have equations that matched the initial conditions.

$$\mathbf{X}(0) = \{x(0),\ y(0),\ x'(0),\ y'(0)\}^T \tag{3}$$

The unknown variables like mu and T were provided to us. These values varied from the Earth-Moon and Earth-Sun system. The nominal orbit solution was solved using Matlab. ode45 was able to do a majority of the work for us. Once the nominal orbit was calculated, it was plotted in order to visualize the orbit of the spacecraft. Figures 1 and 2 show the plots of part D, and E respectively. The derivatives Uxx, Uyy, and Uxy were calculated using Matlab (derivatives.m). The toolbox "Symbolic Math Toolbox" calculated these values. These values were then inputted into the A matrix mentioned in part C.

Body Frame Orbit Graphs for parts D and E:

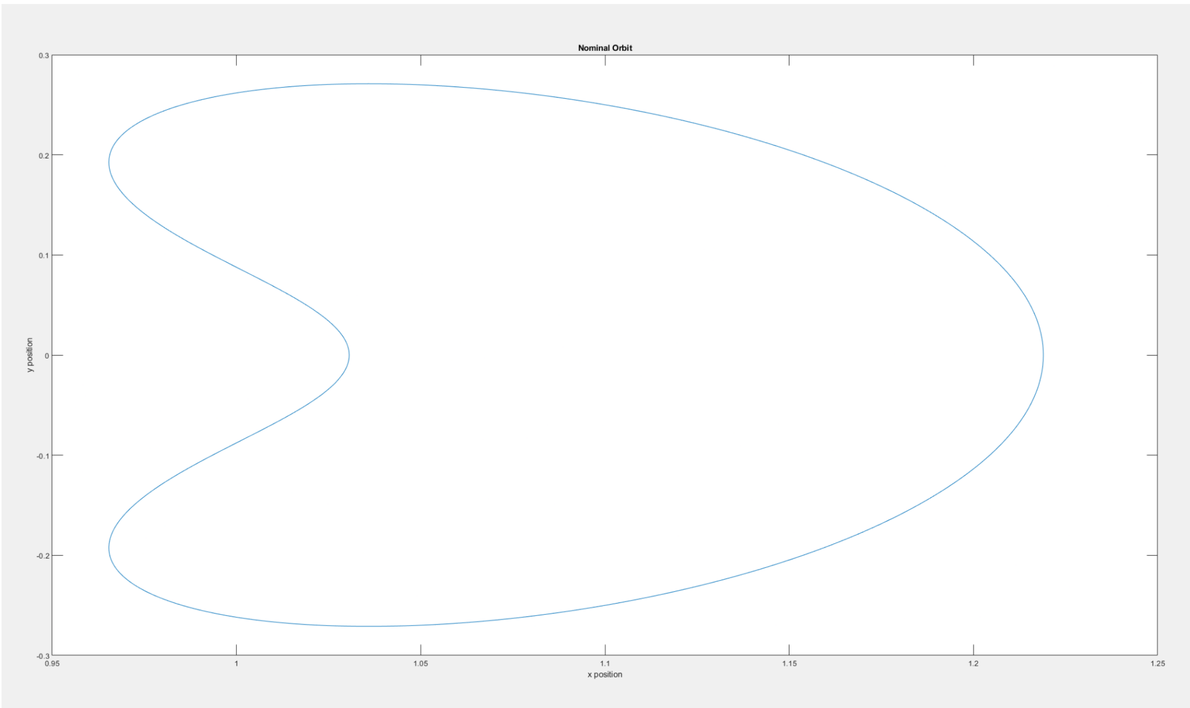

Figure 1. Earth-Moon L2 Lyapunov Body Frame Orbit

The nominal orbit shape appears to be beanlike. At the start of the orbit the trajectory is similar to a large ellipse. The ellipse shape changes as it progresses. The reason for the change in the graph is due to the Lagrange point 2 for Earth-Moon being unstable. The instability for the Lagrange point 2 was shown to be true in part C of the project.

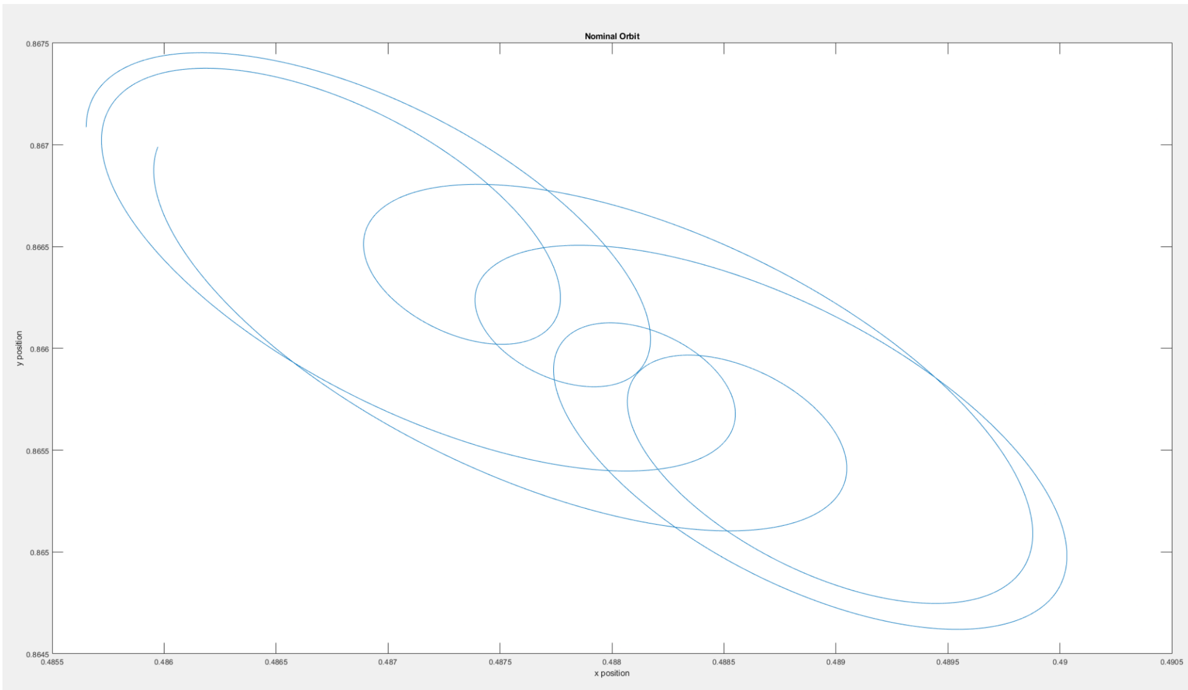

Figure 2. Earth-Sun L4 Lyapunov Body Frame Orbit

The nominal orbit of Earth-Sun orbit in the body frame is seen in figure 2. This orbit contains a skewed elliptical shape. The Lagrange point 4 value for the Earth-Sun orbit is stable. This cannot be determined from the body frame orbit alone. The following graphs in this report will be able to verify the stability of this point.

Inertial Frame Orbit Graphs for parts D and E:

Once the body frame orbits were graphed, they needed to be converted to the inertial frame. This was a simple rotation about the 3 axis. The converted DCM was multiplied by the body frame points. This conversion was done to visualize the orbits in a way that corresponds to the stability of the points.

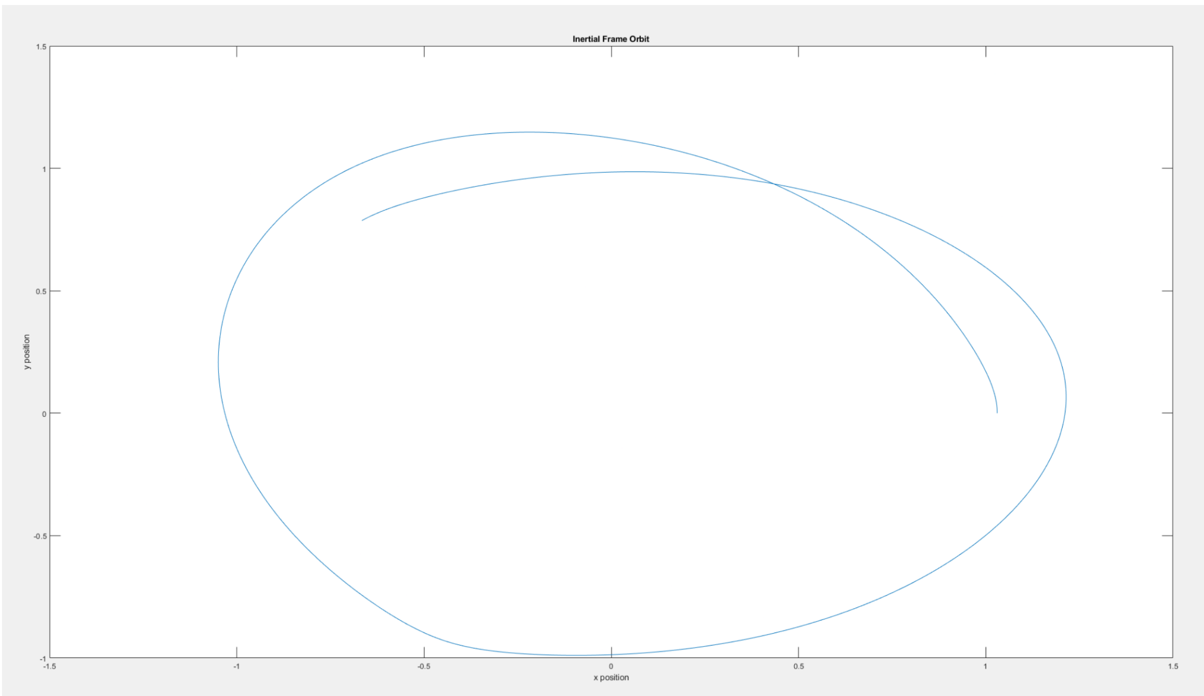

Figure 3. Earth-Moon L2 Lyapunov Inertial Frame Orbit

Figure 3 shows that the inertial frame orbit of Earth-Moon for Lagrange point 2 shows a more circular orbit. Although the orbit is more circular, it still remains an unstable point. The instabilities are shown by the "bumps" in the orbit. This misshaped circular orbit was to be expected in the inertial frame as the Lagrange point 2 for Earth-Moon is unstable.

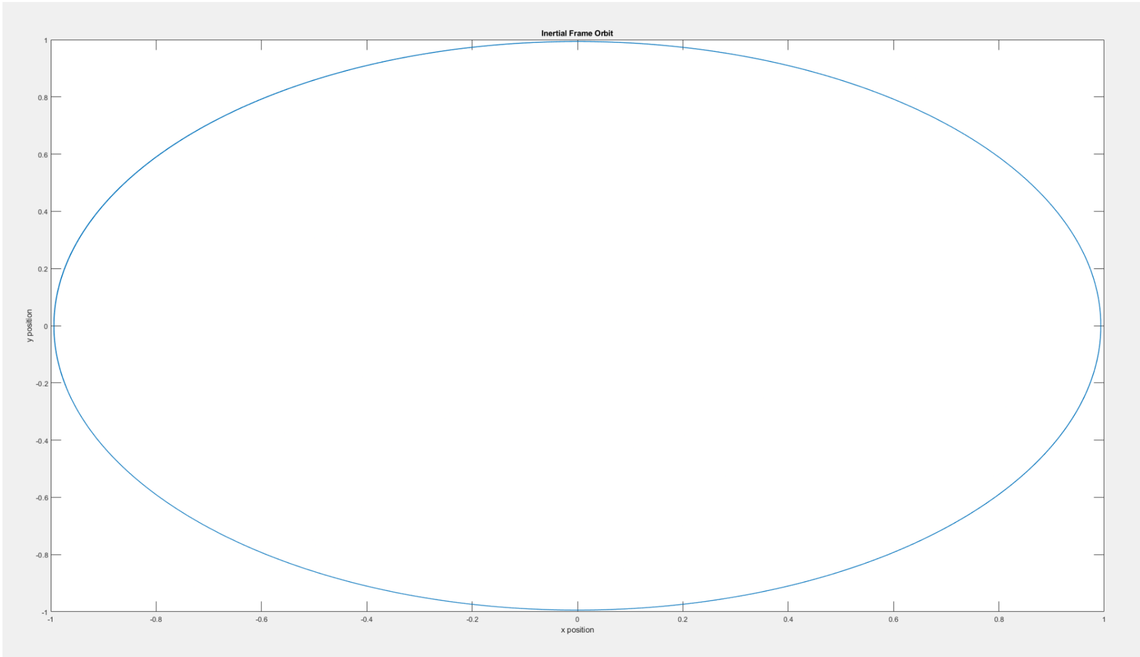

Figure 4. Earth-Sun L4 Lyapunov Inertial Frame Orbit

Figure 4 shows the inertial frame orbit of Earth-Sun for Lagrange point 4. This orbit is a perfect ellipse. This is because Lagrange point 4 in the Earth-Sun orbit is stable which we got from part C.

Non-Linear Position and Velocity versus Time for parts D and E:

The initial conditions listed in equation 3 needed to be perturbed. The perturbation initial conditions were provided in "EM_l2-304P1.mat" and "EM_l4-304P1.mat". This was done by changing the original values by adding the perturbation values. Equation 5 shows this step. The matlab ode45 function was used once again to obtain the perturbed values. ode45 integrated equations 1 and 2, similarly to the first few steps in this part. The initial perturbation vector can be seen in equation 4.

$$\partial \mathbf{X}(0) = \{\partial x(0),\ \partial y(0),\ \partial x'(0),\ \partial y'(0)\}^T \tag{4}$$

$$\mathbf{X}(t) = \mathbf{X}(0) + \partial\mathbf{X}(0) \tag{5}$$

Once the perturbation values were calculated, the departure motion at each time was computed. The equation used can be seen below, in equation 6. The values for the departure motion were plotted.

$$\partial\mathbf{X}(t) = \mathbf{X}(t) - \mathbf{X}n(t) \tag{6}$$

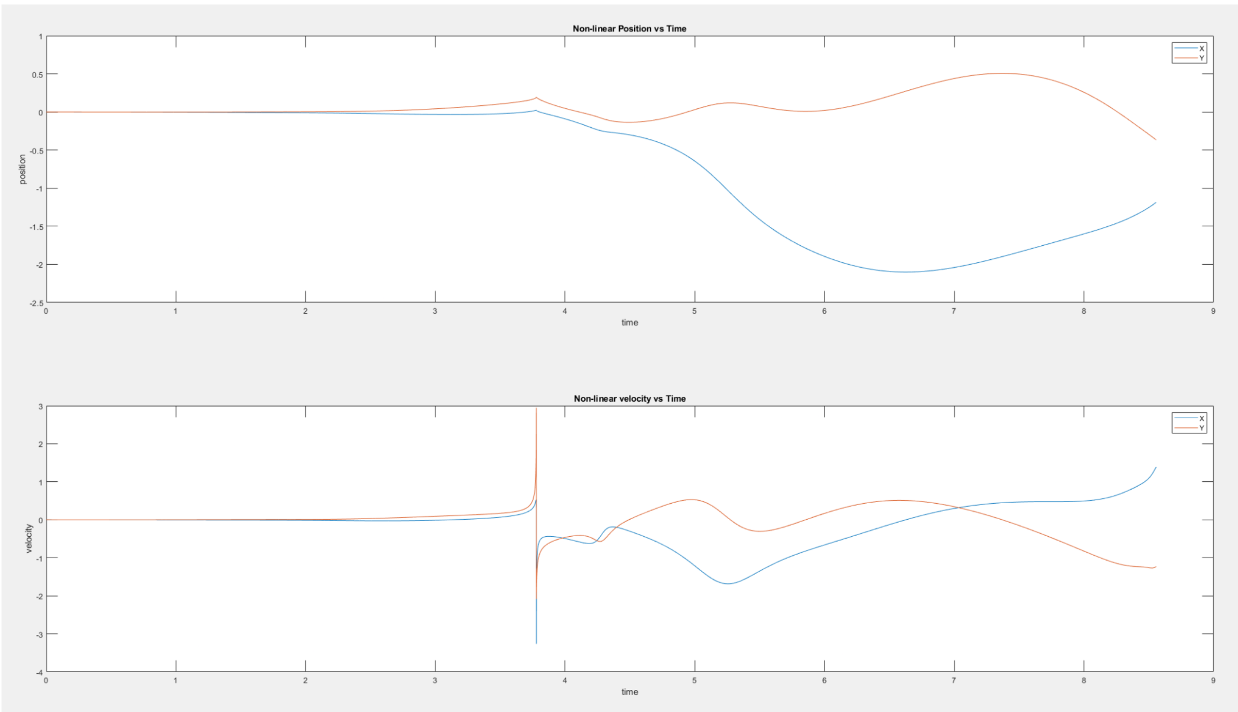

Figure 5. Non-linear Earth-Moon Position and Velocity versus time

The top graph has x-axis being time, and y-axis being position. The bottom graph has x-axis being time, and y-axis being velocity. The relationships in these graphs are non-linear. Both x and y motion start off with straight lines. After time progresses, the motion of x and y become non-linear. Figure 5 shows this relation.

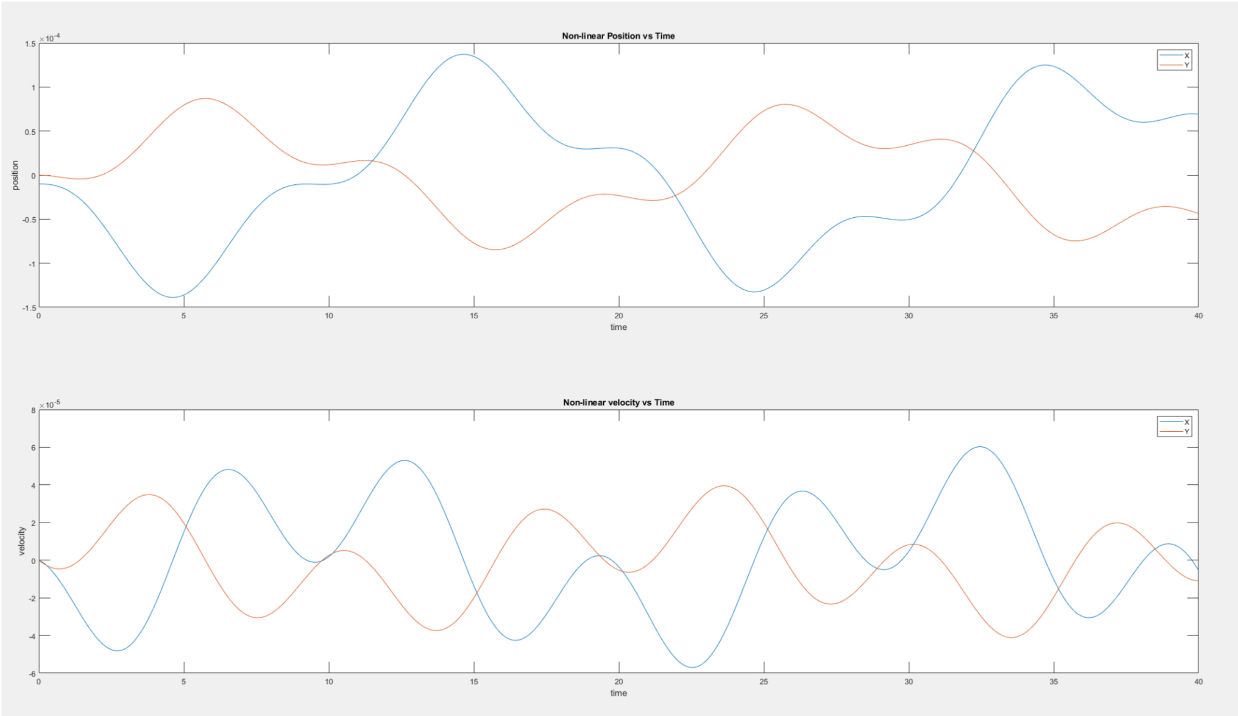

Figure 6. Non-Linear Earth-Sun Position and Velocity versus time

In figure 6, the top graph has x-axis being time, and y-axis being position. The bottom graph has x-axis being time, and y-axis being velocity. Both x and y motions do not start off small. Instead they display large waves. These waves cancel each other after one time period. This canceling of eachother out is due to Lagrange point 4 of Earth-Sun being stable.

Linearized Position and Velocity versus Time for parts D and E:

The linearized equations of motion were then solved and graphed. These equations can be seen in equation 7. These equations of motion were solved with the initial conditions.

$$\partial \mathbf{X}' = A\partial \mathbf{X},\ \partial \mathbf{X} = \{\partial x,\ \partial y,\ \partial x',\ \partial y'\}^T \tag{7}$$

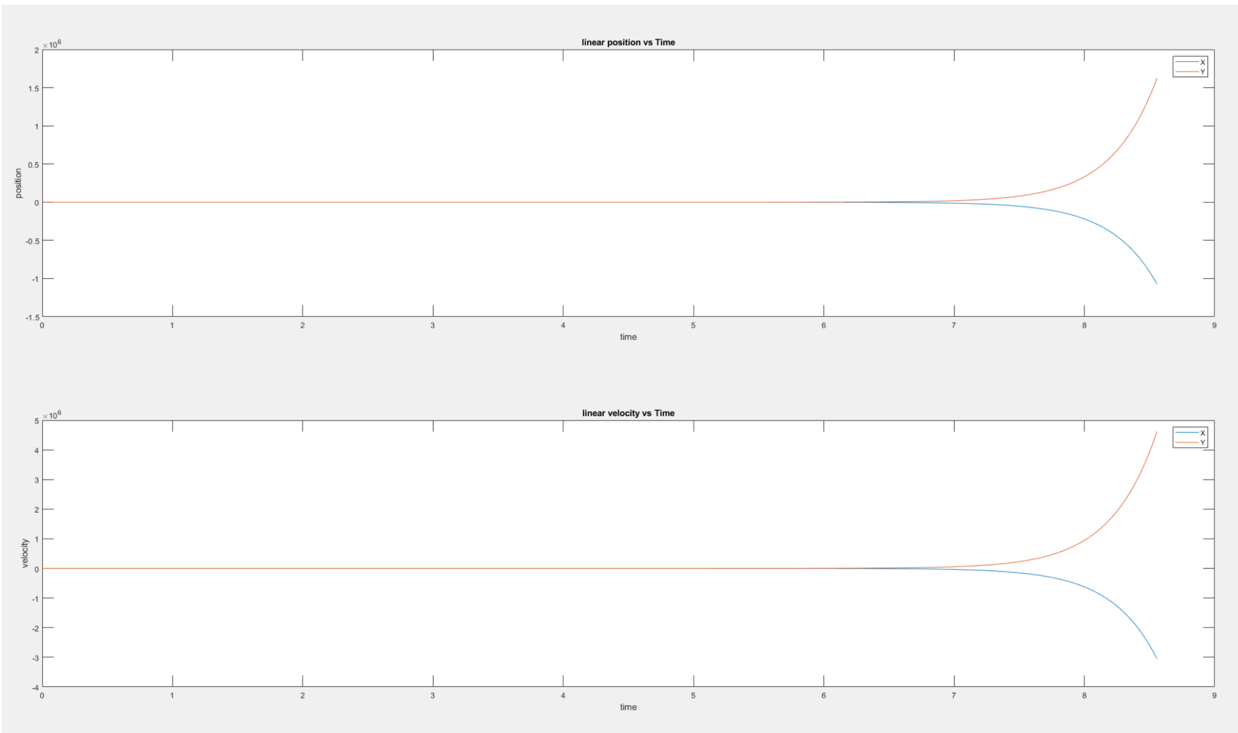

Figure 7. Linear Earth-Moon Position and Velocity versus time

Figure 7 depicts unstable behavior. The x and y motion slowly begin to depart from one another over time. The relationship seen in figure 7 is different from that seen in figure 5.

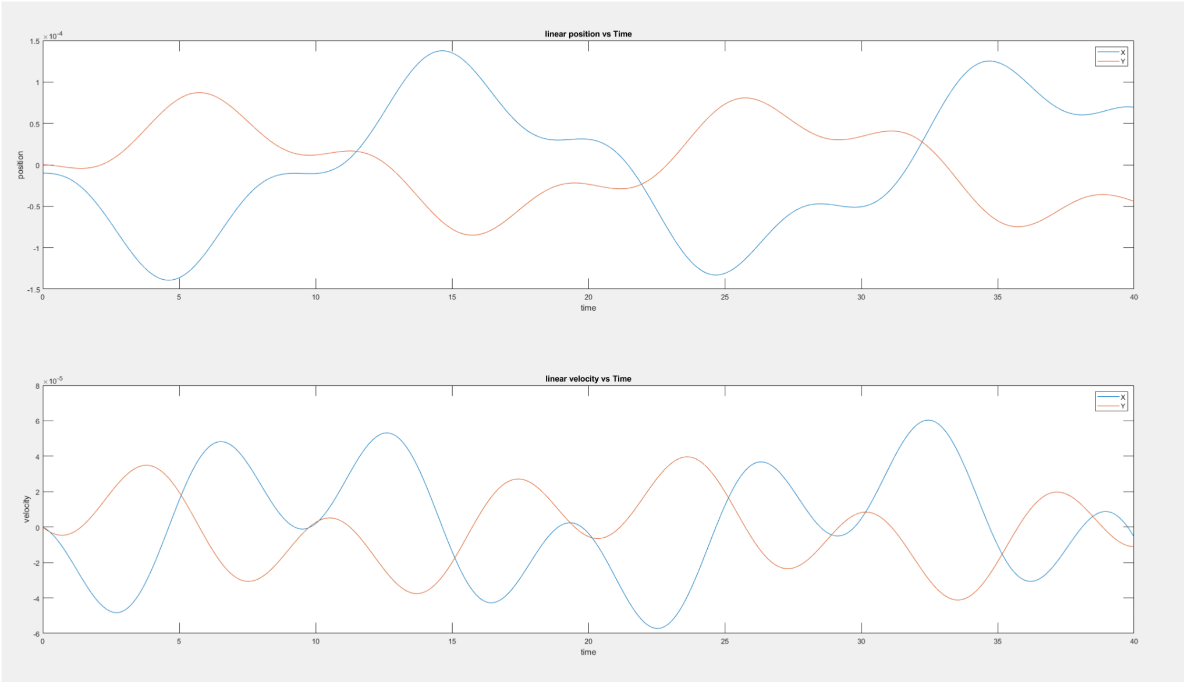

Figure 8. Linear Earth-Sun Position and Velocity versus time

Figure 8 and figure 6 show exactly the same graphs, since the Lagrange point 4 for Earth-Sun is stable. If the graphs do not vary from each other, they are stable. The motion in the graphs cancel eachother out.

Non-Linearized and Linearized Position/Velocity versus Time for Parts D:

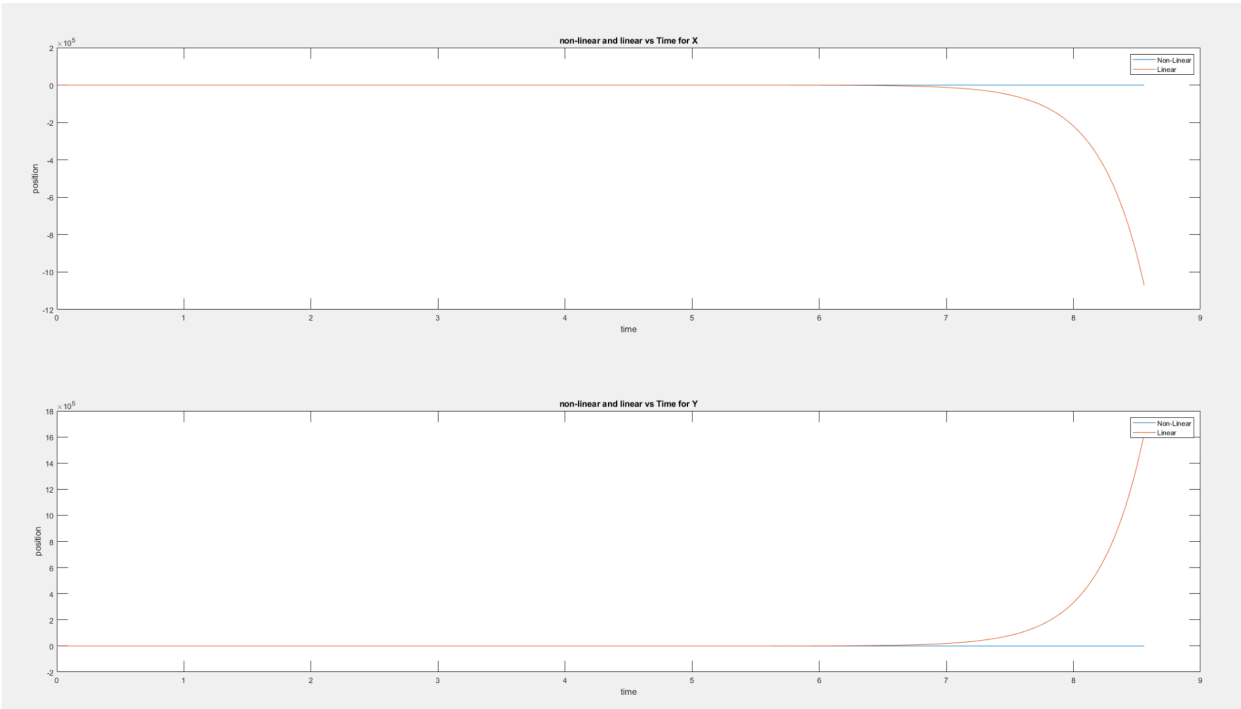

Figure 9. Earth-Moon Non-Linear and Linear Position vs Time Departure

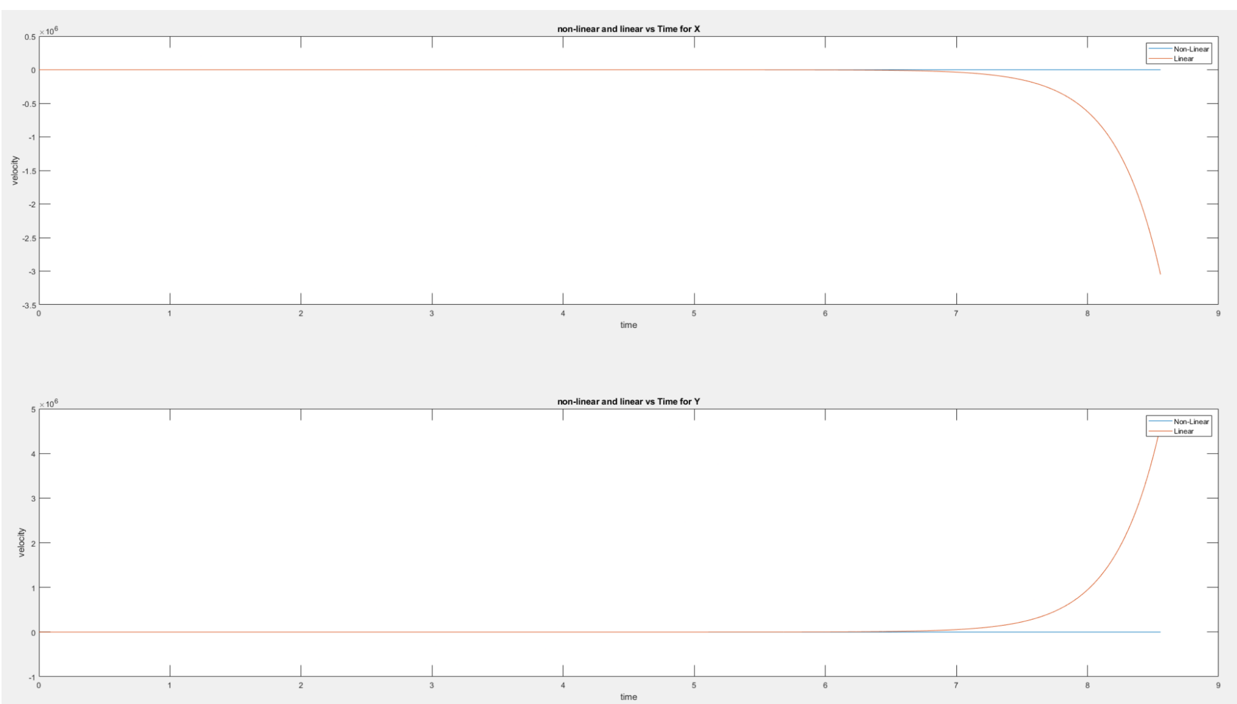

Figure 10. Earth-Moon Non-Linear and Linear Velocity vs Time Departure

The Non-Linear and Linear position and velocity versus time graphs shown in figures 9 and 10 reinforce the results that Lagrange point 2 is unstable. The lines do not match, and instead they separate. Figures 1, 3, 5, and 7 all show the instability of Lagrange Point 2 for Earth-Moon. The linear and non-linear lines being separate makes sense for figures 9 and 10. The trend of random-like motion seen in the body and inertial frame orbit carries through each series of graphs detailed above.

Non-Linearized and Linearized Position/Velocity versus Time for Parts E:

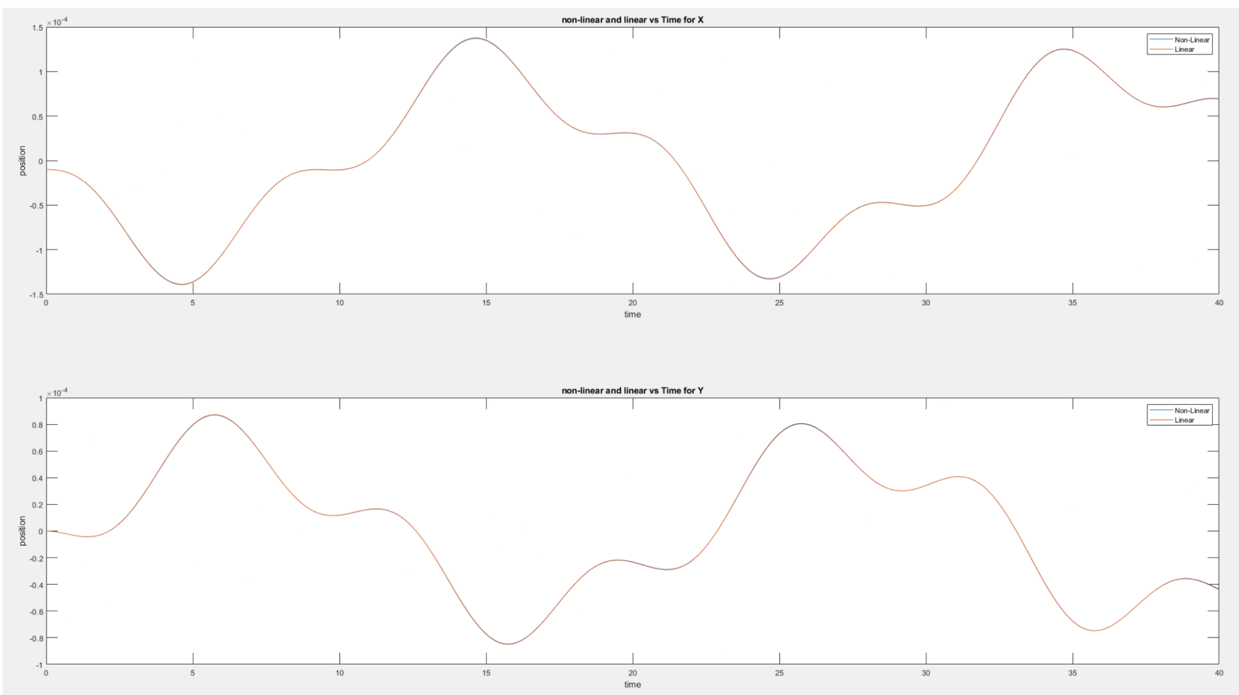

Figure 11. Earth-Sun Non-Linear and Linear Position vs Time Departure

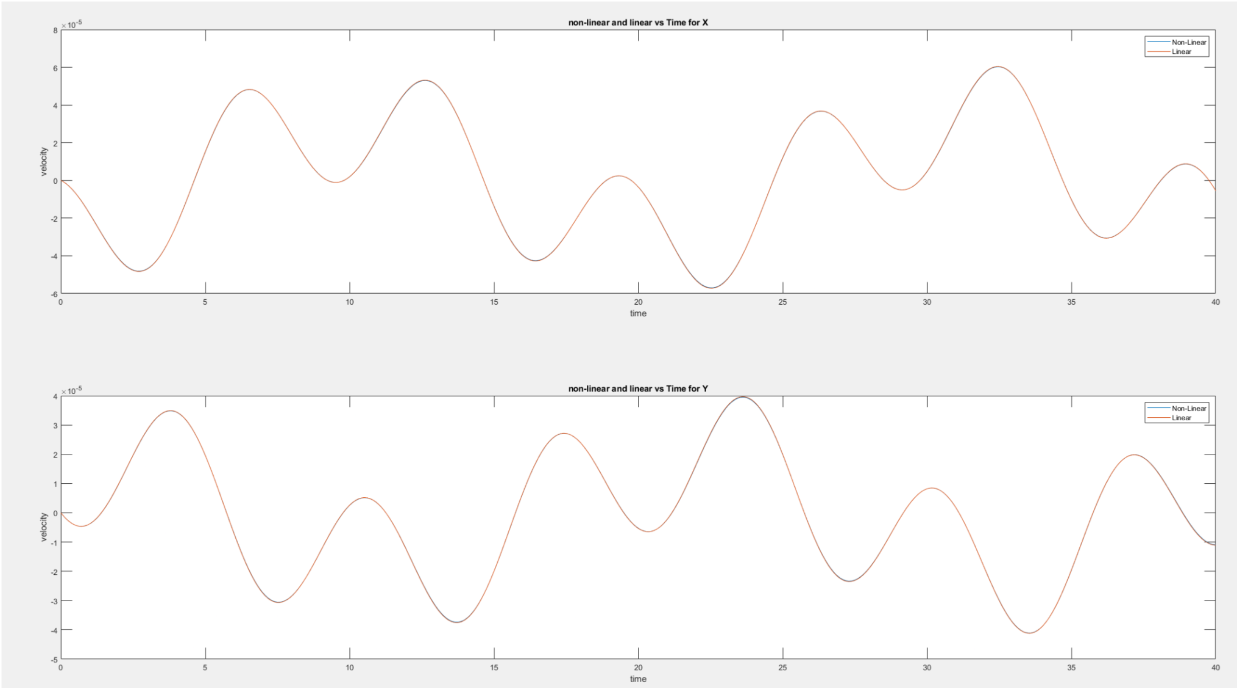

Figure 12. Earth-Sun Non-Linear and Linear Velocity vs Time Departure

Figures 11 and 12 are different from figures 9 and 10. Where two lines can be seen in figures 9 and 10, only one can be seen for Figure 11 and Figure 12. This is because the Lagrange point 4 for Earth-Sun is stable, so the lines are overlapping one another. Figures 2, 4, 6, and 8 also show trends of the Lagrange point 4 for Earth-Sun being stable. The body and inertial frame orbits for Earth-Sun also show signs of stability, and it is proved to be true in the rest of the graphs.