USEFUL ELEMENTARY FORMULAS

Trigonometric Formulae

❖ **Table of Standard Values:**

$\boldsymbol{\theta \rightarrow}$	**0**	$\boldsymbol{\frac{\pi}{6}}$ ***or*** $\boldsymbol{30^o}$	$\boldsymbol{\frac{\pi}{4}}$ ***or*** $\boldsymbol{45^o}$	$\boldsymbol{\frac{\pi}{3}}$ ***or*** $\boldsymbol{60^o}$	$\boldsymbol{\frac{\pi}{2}}$ ***or*** $\boldsymbol{90^o}$	$\boldsymbol{\pi}$ ***or*** $\boldsymbol{180^o}$
$\boldsymbol{\sin\theta}$	**0**	$\boldsymbol{\frac{1}{2}}$	$\boldsymbol{\frac{1}{\sqrt{2}}}$	$\boldsymbol{\frac{\sqrt{3}}{2}}$	**1**	**0**
$\boldsymbol{\cos\theta}$	**1**	$\boldsymbol{\frac{\sqrt{3}}{2}}$	$\boldsymbol{\frac{1}{\sqrt{2}}}$	$\boldsymbol{\frac{1}{2}}$	**0**	$\boldsymbol{-1}$
$\boldsymbol{\tan\theta}$	**0**	$\boldsymbol{\frac{1}{\sqrt{3}}}$	**1**	$\boldsymbol{\sqrt{3}}$	∞	**0**

❖ **Fundamental Identities:**

1) $\sin^2\theta + \cos^2\theta = 1$
2) $1 + \tan^2\theta = \sec^2\theta$
3) $1 + \cot^2\theta = cosec^2\,\theta$

❖ **Addition / Subtraction Formulae:**

1) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
2) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

❖ **Functions of $(-\boldsymbol{\theta})$**

1) $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$
2) $cosec\,(-\theta) = -cosec\,\theta$ and $\sec(-\theta) = \sec\theta$
3) $\tan(-\theta) = -\tan\theta$ and $\cot(-\theta) = -\cot\theta$

❖ **Functions of $(\boldsymbol{2\theta})$**

1) $\sin 2\theta = 2\sin\theta\cos\theta$
$$= \frac{2\tan\theta}{1+\tan^2\theta}$$
2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$$
3) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
4) $1 - \cos 2\theta = 2\sin^2\theta$ AND 5) $1 + \cos 2\theta = 2\cos^2\theta$
6) $1 - \sin 2\theta = (\cos\theta - \sin\theta)^2$ AND 7) $1 + \sin 2\theta = (\cos\theta + \sin\theta)^2$

❖ Functions of $(\boldsymbol{\theta})$

1) $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

$$= \frac{2\tan\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$$

2) $\cos\theta = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$

$$= 1 - 2\sin^2\left(\frac{\theta}{2}\right)$$

$$= 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$= \frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$$

3) $\tan\theta = \dfrac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}$

4) $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$ AND 5) $1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$

6) $1-\sin\theta = \left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)^2$ AND 7) $1+\sin\theta = \left(\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right)^2$

❖ Functions of $(\boldsymbol{3\theta})$

1) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
2) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
3) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

❖ Factorisation Formulae:

1) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
2) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
3) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
4) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$, If $C > D$

$$= 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right) \text{ , If } D > C$$

❖ De factorisation Formulae:

1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

❖ The formula in a nutshell

1) $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ and $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$
2) $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ and $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$
3) $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ and $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$
4) $\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$ and $\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$
5) $\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ and $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=\sec\theta$
6) $\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$ and $\sec\left(\frac{\pi}{2}+\theta\right)=-\operatorname{cosec}\theta$
7) $\sin(\pi-\theta)=\sin\theta$ and $\sin(\pi+\theta)=-\sin\theta$
8) $\cos(\pi-\theta)=-\cos\theta$ and $\cos(\pi+\theta)=-\cos\theta$

❖ Properties of Inverse trigonometric functions:

1) **Identities**
   - $i)$ $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$
   - $ii)$ $\sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2}$
   - $iii)$ $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$
2) **Functions of $(-x)$**
   - $i)$ $\sin^{-1}(-x)=-\sin^{-1}(x)$
   - $ii)$ $\tan^{-1}(-x)=-\tan^{-1}(x)$
   - $iii)$ $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$
3) **Reciprocal Function:**
   - $i)$ $\sin^{-1}\left(\frac{1}{x}\right)=\operatorname{cosec}^{-1}(x)$ and $\operatorname{cosec}^{-1}\left(\frac{1}{x}\right)=\sin^{-1}(x)$
   - $ii)$ $\cos^{-1}\left(\frac{1}{x}\right)=\sec^{-1}(x)$ and $\sec^{-1}\left(\frac{1}{x}\right)=\cos^{-1}(x)$
   - $iii)$ $\tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}(x)$ and $\cot^{-1}\left(\frac{1}{x}\right)=\tan^{-1}(x)$
4) $i)$ $\tan^{-1}\left[\frac{A+B}{1-AB}\right]=\tan^{-1}A+\tan^{-1}B$

   $ii)$ $\tan^{-1}\left[\frac{A-B}{1+AB}\right]=\tan^{-1}A-\tan^{-1}B$

❖ Factorization:

1) $A^2-B^2=(A-B)(A+B)$
2) $A^3-B^3=(A-B)(A^2+AB+B^2)$
3) $A^3+B^3=(A+B)(A^2-AB+B^2)$
4) $(A+B)^2=(A^2+2AB+B^2)$
5) $(A-B)^2=(A^2-2AB+B^2)$
6) $(A+B)^3=(A^3+3A^2B+3AB^2+B^3)$
7) $(A-B)^3=(A^3-3AB^2+3A^2B-B^3)$