Electrostatics

↓

study of charges at rest — It always deals with +ve charge (+q)

What is electric flux ($d\phi_E$)

It is the number of lines of force passing through given area (ds) is known as electric flux

Electric flux $d\phi = \vec{E} \cdot \vec{ds}$

$$d\phi = E\,ds\cos\theta$$

To electric flux $\phi = \int d\phi$

$$\phi = \int E\,ds\cos\theta$$

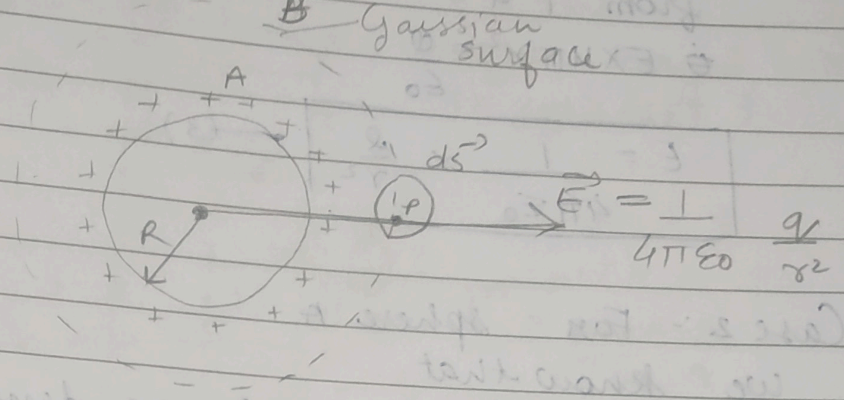
What is Gauss Law

The net electric flux through a closed surface is equal to net charge(Q) enclosed by the surface divided by ε_0

$$\phi = \frac{Q}{\varepsilon_0} \text{ or } \frac{q}{\varepsilon_0}$$

Applications:-

1) Obtain an expression for electric field intensity (E) at a point due to uniformly charged spherical shell or Hollow sphere



1) Consider a +ve charge sphere with radius R

2) Let P be the point on the Gaussian surface where we have to find electric field intensity ($\vec{E}$) of radius r with small surface area ds

Derivation :- Case I :- for Gussian surface. (B)
The total electric flux on the sphere is

$$\phi = \int \vec{E} \cdot \vec{ds}$$
$$\phi = \int E\, ds \cos\theta.$$

Angle betn E and ds is zero.

$$\theta = 0^\circ \qquad \cos 0^\circ = 1.$$
$$\phi = \int E\, ds.$$

But $\int ds = 4\pi r^2$ (Area of spnre

$$\phi = E 4\pi r^2 \quad \text{———} \boxed{(1)}$$

By Gauss Law

$$\phi = \frac{Q}{\varepsilon_0} \quad \text{——— (2)}$$

Electric field inside the hollow/ spherical sphere is zero.

from 1 & 2

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}} \quad \text{——— (3)}$$

Case 2 :- For sphere A

we know that

$$\sigma = \frac{Q}{A} \quad \text{(Surface charge density)}$$

$$Q = \sigma A$$

$$Q = \sigma 4\pi R^2$$

Put in eq^n(3)

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

$$E = \frac{1}{\cancel{4\pi}\varepsilon_0} \frac{\sigma \cancel{4\pi} R^2}{r^2}$$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

Let pt P lies on the surface of sphere

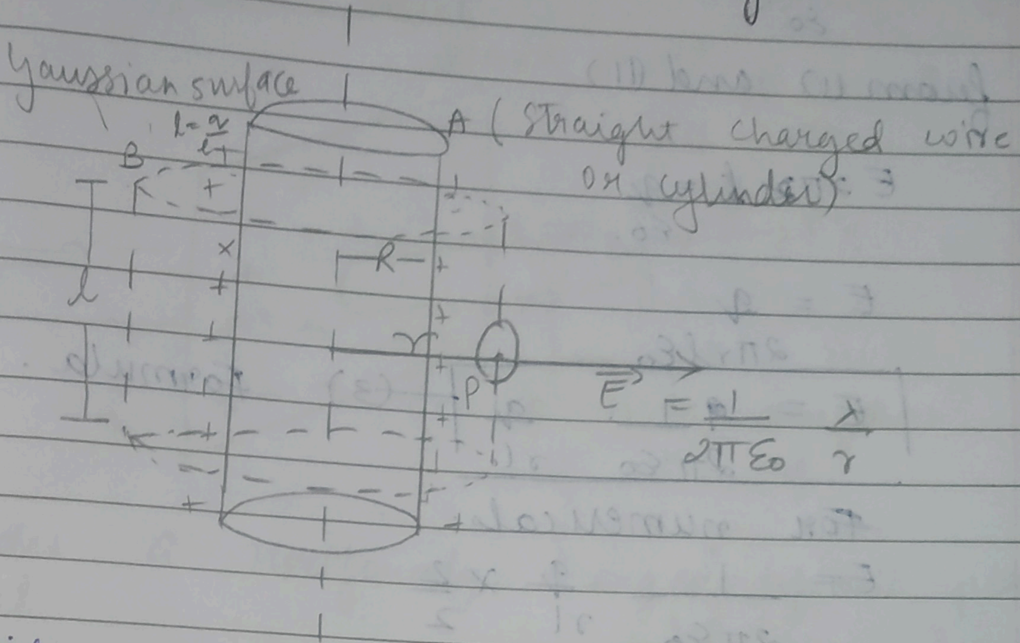
$$r = R.$$

$$E = \frac{\sigma \cancel{R^2}}{\varepsilon_0 \cancel{R^2}}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Obtain an expression for:-

2\. Electric field intensity at a point due to an infinitely long straight charged wire or cylinder



1\) Consider a uniformly charged wire of infinite length

2\) Let P be the point with radius r where we have to find electric field intensity $(\vec{E})$ with small area $d\vec{s}$.

Case I :- For Gaussian surface (B)

Total electric flux is

$$\phi = \int \vec{E} \cdot d\vec{s}$$

$$\phi = \int E\, ds \cos\theta$$

Angle betn E and ds is zero

$$\therefore \theta = 0^\circ \qquad \cos 0^\circ = 1$$

$$\therefore \phi = \int E\, ds.$$

$$\phi = E \int ds$$

But $\int ds = 2\pi r l.$

$$\boxed{\phi = E\, 2\pi r l.} \quad \text{——(1)}$$

By Gauss Law.

$$\phi = \frac{q}{\varepsilon_0} \quad \text{——— (2)}$$

from (1) and (2)

$$E\, 2\pi x\, l = \frac{q}{\varepsilon_0}.$$

$$E = \frac{q}{2\pi r l \varepsilon_0}$$

$$\boxed{E = \frac{1}{2\pi \varepsilon_0} \cdot \frac{q}{r l}} \text{ —— (3)} \quad \text{Formula.}$$

For numericals.

$$E = \frac{1}{2\pi\varepsilon_0} \frac{q}{rl} \times \frac{2}{2}$$

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{rl} \qquad \text{As } \frac{1}{4\pi\varepsilon_0} = 9\times10^9$$

Case 2 :- For charged wire A
we know that

$$\lambda = \frac{q}{l} \quad \text{(linear charge density)}$$

$$q = \lambda l.$$

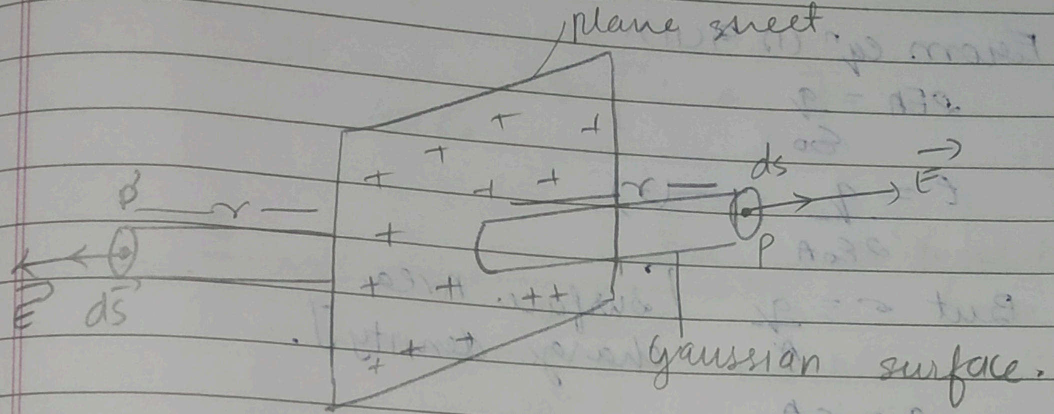
put in eq^n (3)

$$E = \frac{1}{2\pi\varepsilon_0} \frac{q}{rl}$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda \cancel{l}}{r \cancel{l}}$$

$$\boxed{E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}}$$

3 Obtain an expression for electric field intensity $\vec{E}$ at a point due to charged infinite plane sheet.



1) Consider a uniformly charged infinite plane sheet

2) let P be the point at a distance R from the sheet

The total electric flux is

$$\phi = \phi_P + \phi_{P'}$$

$$\phi = \left[\int E\,ds\cos\theta\right]_P + \left[\int E\,ds\cos\theta\right]_{P'}$$

~~$\phi = E\int ds$~~ Angle betn E & ds is zero.

$\cos 0 = 1$.

$$\phi = \left[\int E\,ds\right]_P + \left[\int E\,ds\right]_{P'}$$

$$\phi = \left[E\int ds\right]_P + \left[E\int ds\right]_{P'}$$

But $\int ds = A$.

$$\phi = EA + EA$$

$$\phi = 2EA. \quad \text{——— (1)}$$

By Gauss law.

$$\phi = \frac{q}{\varepsilon_0} \quad —(2).$$

From eq^n (I) & (II)

$$2EA = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\varepsilon_0 A} \quad —(3)$$

But $\sigma = \frac{q}{A}$ [surface Area charge density].

$$q = \sigma A.$$

Put in (3)

$$E = \frac{q}{2\varepsilon_0 A}$$

$$E = \frac{\sigma \cancel{A}}{2\varepsilon_0 \cancel{A}}$$

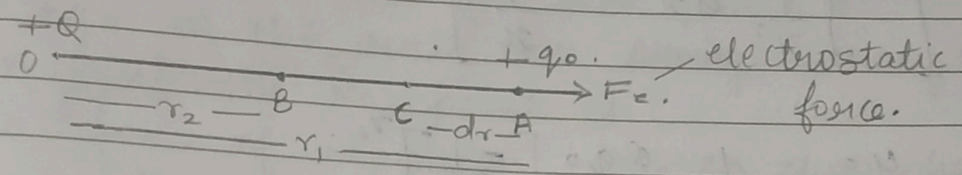
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Page No.
Date

Episode II

Derive an expression for Electrostatic Potential Energy

$$\Delta U = \frac{1}{4\pi\varepsilon_0} Qq_0 \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$



1) Let +Q be the source charge at pt O and +q_0 be the small charge at pt A
2) The force of repulsion between two charg is $F_e = \frac{1}{4\pi\varepsilon_0} \frac{Qq_0}{r^2}$ —— (1)

3) let dw be the small work done to move a charge which increase potential energy (d

$$dw = du = \text{Force} \times \text{displacement}$$
$$du = -F_e \times dr.$$

Total work done or potential energy is obta by integrating du

$$u = \int_{r_1}^{r_2} du$$

$$u = \int_{r_1}^{r_2} -F_e \times dr$$

$$U = \int_{r_1}^{r_2} -\frac{1}{4\pi\varepsilon_0} \frac{Q q_0}{r^2} \, dr.$$

$$U = -\frac{1}{4\pi\varepsilon_0} Q q_0 \int_{r_1}^{r_2} \frac{1}{r^2} \, dr.$$

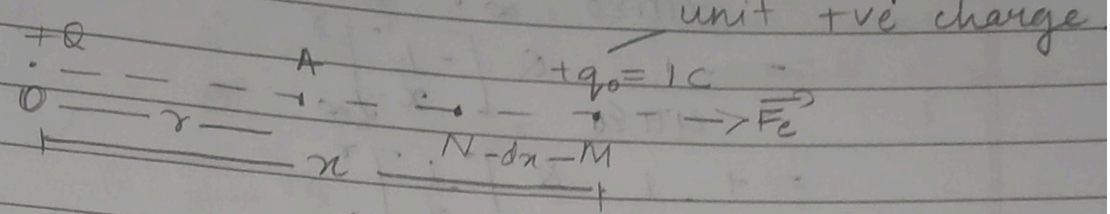
$$\left[\int \frac{1}{x^2} \, dx = \frac{-1}{x} \right].$$

$$U = \frac{-1}{4\pi\varepsilon_0} Q q_0 \left[\frac{-1}{r} \right]_{r_1}^{r_2}.$$

$$= +\frac{1}{4\pi\varepsilon_0} Q q_0 \left[\frac{1}{r_2} - \frac{1}{r_1} \right].$$

$$U = \frac{1}{4\pi\varepsilon_0} Q q_0 \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$$

Electrostatic potential energy due to point charge.



1) Let +Q be the charge at point O
2) Let $+q_0$ be the unit +ve charge brought from infinity to point M.
3. The force of repulsion between two charges

$$F_e = \frac{1}{4\pi\varepsilon_0} \frac{Q q_0}{x^2}$$

But $q_0 = 1C$.

$$\boxed{F_e = \frac{1}{4\pi\varepsilon_0} \frac{Q}{x^2}}$$

4) let dw be the small work done for a small displacement dx.

dw = Force × displacement.

$dw = -F_e \, dx$. -ve sing indicates force and displacement ar

Total work done (W) to move a charge from infinity (∞) to r distance. is obtained by integrating dw.

$$W = \int_{\infty}^{r} -F_e \, dx.$$

$$W = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0} \frac{Q}{x^2} \, dx.$$

$$w = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0} \frac{Q}{x^2} \, dx$$

$$W = -\frac{1}{4\pi\varepsilon_0} Q \int_{\infty}^{r} \frac{1}{x^2}\, dx.$$

$$= \frac{-1}{4\pi\varepsilon_0} Q \left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= +\frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{x}\right]_{\infty}^{r}$$

$$= +\frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{r} - \underbrace{\frac{1}{\infty}}_{0}\right]$$

$$= +\frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

$$\boxed{W = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}}$$

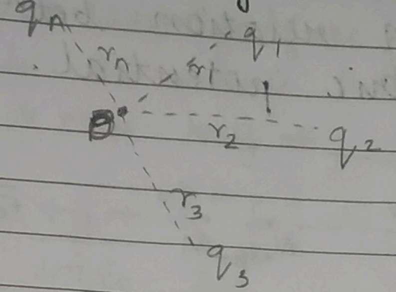
But $v = \frac{W}{q_0}$.

$$v = \frac{\frac{1}{4\pi\varepsilon_0} \frac{Q}{r}}{q_0}.$$

$q_0 - 1C$.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}.}$$

Derive an expression for electrostatic potential energy due to system of charges.



[Jaha charge waha potential eg q_1 pe V_1] 5

Consider a system of charge $q_1, q_2, q_3 \cdots q_n$ are at distance $r_1, r_2, r_3 \cdots r_n$ from point O

The potential V_1 at point O due to q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

The potential V_2 at pt O due to q_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}.$$

The potential V_3 at pt O due to q_3 is

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}.$$

The potential V_n at pt O due to q_n is

$$V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

Acc. to principle of superposition

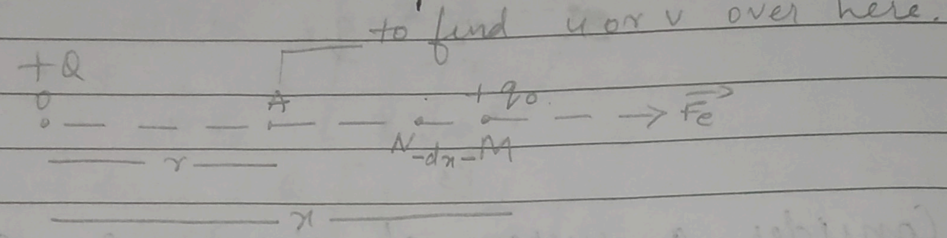
$$V = V_1 + V_2 + V_3 \cdots V_n$$

$$V = \left[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \cdots \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \cdots \frac{q_n}{r_n}\right]$$

Episode III

Derive ~~an~~ the ~~exp~~ relation beth electric field and electric potential.



Consider a charge +Q at point O
Let $+q_0$ be the charge displaced by small displacement dx.
work done is given by.

$$dw = -F\,dx.$$

Dividing both side by q_0.

$$\frac{dw}{q_0} = \frac{-F\,dx}{q_0}$$

But $dv = \dfrac{dw}{q_0}$

$$E = \frac{F}{q_0}$$

$$dv = -E\,dx.$$

$$E = \frac{-dv}{dx}.$$

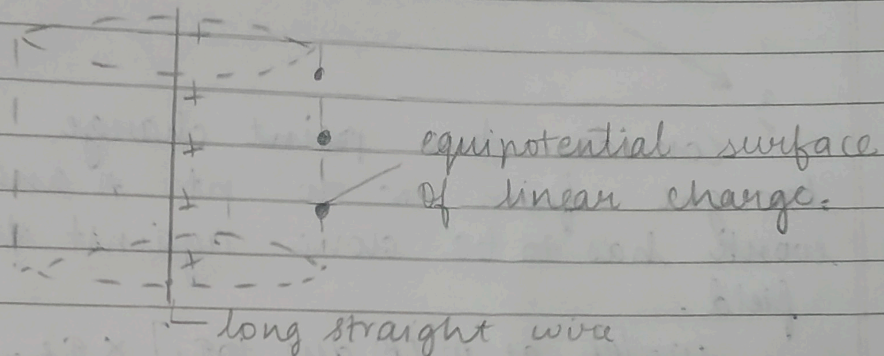
$$\boxed{E = \frac{-dv}{dx}}$$

Page No.
Date

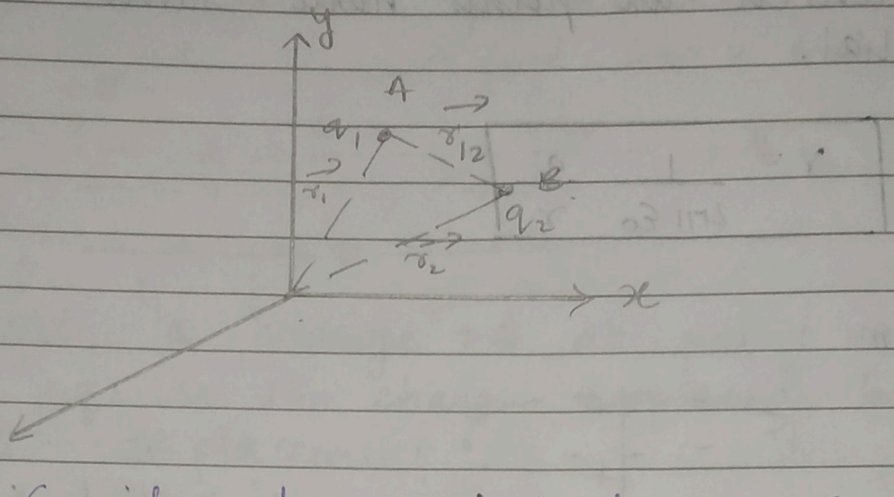
Explain equipotential surface.
An equipotential surface is the surface in which all points have same electric potential.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

Diagram



Obtain an expression for potential energy of a system of two point charge.



Consider two ~~point~~ charge q_1 and q_2 brought from ∞ to pt A and B, so work has to be done against the electric field.

$$W = [\text{PE at pt (B) due to } q_1] \times q_2.$$

$$W = V_1 \times q_2$$

$$W = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{12}} \times q_2.$$

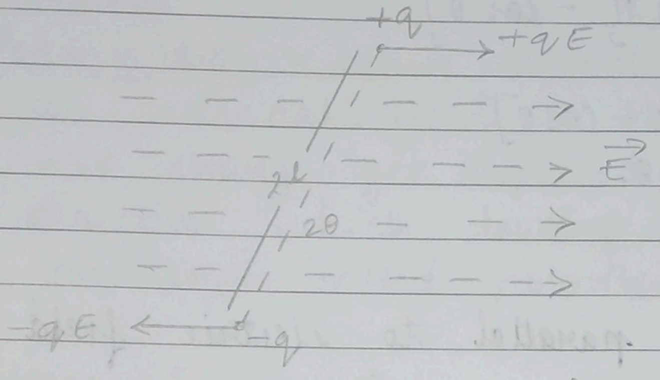
Work done (W) = PE (U)

$$\therefore U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}.$$

Obtain an expression for P.E of an electric dipole in an electric field. What will be the P.E if

1] dipole is perpendicular to electric field

2] dipole is parallel to electric field.



(-q +q)

Consider a dipole is placed in uniform electric field.

This dipole experience a torque.

$$\tau = PE\sin\theta \quad \text{——(1)}$$

Let W be the work done to rotate a dipole from angle θ_0 to θ

$$W = \int_{\theta_0}^{\theta} \tau \, d\theta$$

$$W = \int_{\theta_0}^{\theta} PE\sin\theta . d\theta$$

$$W = PE\int_{\theta_0}^{\theta} \sin\theta \, d\theta.$$

$$W = -PE\left[\cos\theta\right]_{\theta_0}^{\theta}$$

Work done(w) = PE(U)

$U = +PE\left[\cos\theta_0 - \cos\theta\right]$

$$W = -PE\left[\cos\theta - \cos\theta_0\right].$$

$$W = +PE\left[\cos\theta_0 - \cos\theta\right]$$

$$W = +PE\left[\cos\theta_0 - \cos\theta\right]$$

Case I

If dipole is perpendicular to electric field.

$\theta_0 = \frac{\pi}{2}$.

$U = PE\,[\cos\theta_0 - \cos\theta]$

$U = PE\left[\cos\frac{\pi}{2} - \cos\theta\right]$

$U = PE\,[0 - \cos\theta]$

$U = -PE\cos\theta$.

Case II

If dipole is parallel to electric field

$\theta_0 = 0$

$U = PE\,[\cos\theta_0 - \cos\theta]$

$U = PE\,[\cos 0 - \cos\theta]$

$U = PE\,[1 - \cos\theta]$

Episode IV

Some important defination.

1. Dielectric :- It is an insulator which is used to store electrical energy

2. Polarization :- when dielectric are placed in an external field, their +ve and -ve charge get displaced in opposite direction which develop dipole moment this process is called polarization of material.

Types of dielectric.

1. Polar dielectric
2. Non-Polar dielectric.

1) Polar dielectric.

fig 1

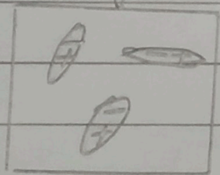
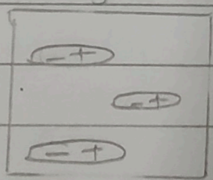


fig 2

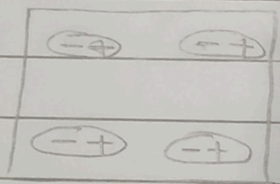


The molecule of polar dielectric have permanent tiny dipole moment.
when this dipole are not applied electric field then it is randomly oriented and net dipole moment is zero as shown in fig 1

When dipole is placed in external electric field it develop a dipole moment in the direction of applied electric field

Def^n:- A molecule in which centre of mass of +ve charge does not coinside with centre of mass of -ve charge is called polar dipole
eg:- HCl, H_2O

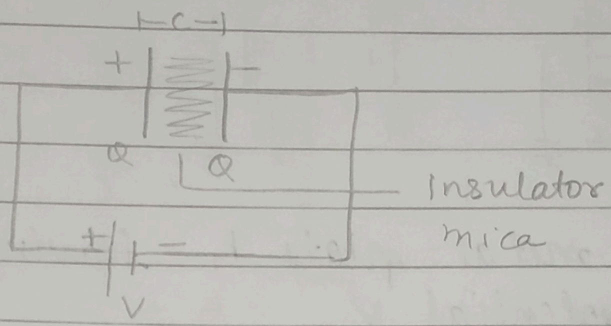
2 Non - Polar Dielectric
In this dielectric centre of mass are seperated when it is placed in ~~el~~ external field E.
In this net dipole moment is present



Def^n:- A molecule in which centre of mass of +ve charge coincide with centre of mass of -ve charge is called as non-polar dielectric.

Capacitor

What is capacitor? Explain capacitance



It is a two parallel plate condenser seperated by insulator to store a charge is called as capacitor.

Formula :- $V \propto Q$

$$\boxed{V = \frac{Q}{C}}$$

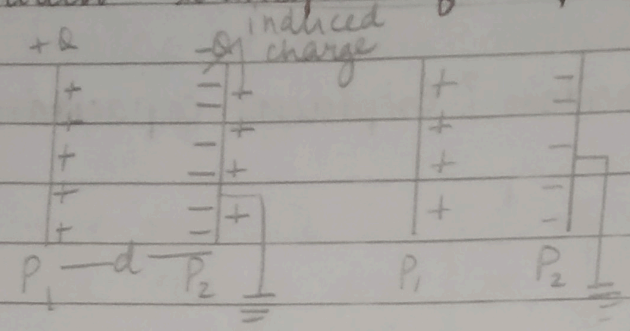
C is called capacitance.

$$\boxed{C = \frac{Q}{V}}$$

It unit is Farad (F) or ~~colum~~ coulomb/volt

To induce charge

Explain Principle of capacitor.



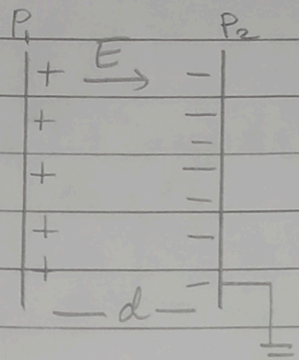
1. Consider a metal plate P_1 having +ve charge +Q and potential V
2. It's capacity is given by

$$C_1 = \frac{Q}{V}$$

3. If another plate P_2 is brought near to P_1 it is seen that -ve charge is produced inner ~~pth~~ plate of P_2 and equal +ve charge is developed on the outer plate of P_2
4. If plate P_2 is grounded the +ve charge on P_2 flows to earth and potential at P_2 i.e V_1 decreases

$$\therefore \quad C_2 = \frac{Q}{V - V_1}$$

Obtain an expression for capacitance 'C' of a parallel plate capacitor <u>without</u> dielectric



1) Consider a parallel plate capacitor P_1 and P_2 seperated by distance 'd' and area 'A'
2) The electric field in the inner region of the plate add up

$$E = E_1 + E_2$$

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\cancel{2}\sigma}{\cancel{2}\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

But

$$\sigma = \frac{Q}{A}$$

$$\boxed{E = \frac{Q}{A\varepsilon_0}} \longrightarrow (I)$$

We know that

$$\boxed{E = \frac{V}{d}} \longleftarrow (II)$$

From eq^n (1) & (2)

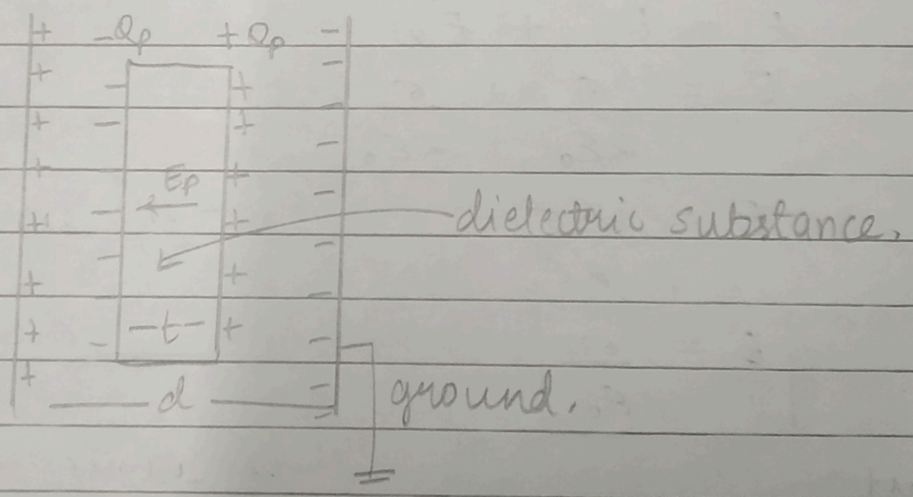
$$\frac{Q}{A\varepsilon_0} = \frac{V}{d}$$

$$\frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

But $C = \frac{Q}{V}$

$$\therefore \boxed{C = \frac{A\varepsilon_0}{d}}$$

Obtain an expression for capacitance of a parallel plate capacitor with dielectric slab between the plate



1) Consider a parallel plate capacitor P_1 and P_2 seperated by distance 'd' having area 'A'

2) let E_0 be the electric field intensity without dielectric

$$\boxed{E_0 = \frac{Q}{A\varepsilon_0}} \quad \text{—— (1)}$$

Date
Page

Let E_P be the electric field intensity when dielectric is introduced into the plate

$$E_p = \frac{Q}{\varepsilon_0 A}$$

Net field (E) inside the dielectric

$$E = E_0 - E_P$$

$$\boxed{E = \frac{E_0}{K}} \quad \text{--- (3)}$$

K — dielectric constant (direct likhna hai)

We know that

$$E = \frac{V}{d}$$

$$V = E \times d$$

Potential difference (V) with dielectric

$$V = \underset{\text{बाहर}}{E_0 \times (d-t)} + \underset{\text{अन्दर}}{E \times t}$$

$$V = E_0 \times (d-t) + \frac{E_0}{K} \times t \quad \text{— from(3)}$$

$$V = E_0 \left[(d-t) + \frac{t}{K} \right]$$

$$V = \frac{Q}{\varepsilon_0 A} \left[(d-t) + \frac{t}{K} \right] \quad \text{— from(1)}$$

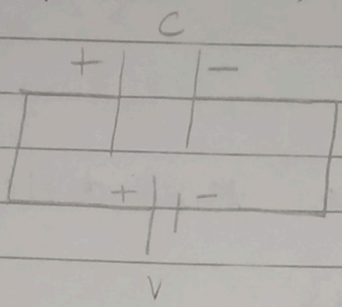
But $C = \dfrac{Q}{V}$

$\cancel{\dfrac{Q}{V}}$ $\cancel{\varepsilon}$

$$C = \frac{\cancel{Q}}{\dfrac{\cancel{Q}}{\varepsilon_0 A} \left[(d-t) + \dfrac{t}{K} \right]}$$

$$C = \frac{\varepsilon_0 A}{(d-t) + \dfrac{t}{K}}.$$

Derive an expression for energy stored in a capacitor (U)



1) Consider a capacitor 'c' charged by a battery V

2) Let 'q' be the charge ~~on~~ in the capacitor during the process of charging

$$C = \frac{q}{V}$$

$$\therefore V = \frac{q}{C}$$

3) Let dW be the work done to transfer a charge

$$dW = V \times dq$$

$$dW = \frac{q}{C} \times dq$$

$$\left[\because V = \frac{W}{q} \qquad W = V \times q \right]$$

Total work done is

$$W = \int_0^Q dW.$$

$$= \int_0^Q \frac{q}{C} \times dq.$$

$$= \frac{1}{C} \int_0^Q q \, dq$$

$$W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$W = \frac{1}{C}\left[\frac{Q^2}{2} - \frac{0^2}{2}\right]$$

$$W = \frac{1}{C}\left[\frac{Q^2}{2}\right]$$

$$W = \frac{Q^2}{2C}$$

But $C = \frac{Q}{V}$

$$Q = CV.$$

$$W = \frac{(CV)^2}{2C}$$

$$W = \frac{C^{\cancel{2}}V^2}{2\cancel{C}}$$

$$W = \frac{CV^2}{2}$$

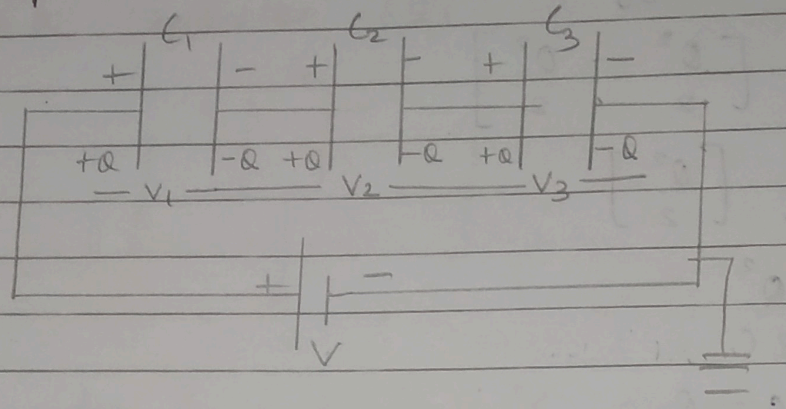
$$\boxed{W = \frac{1}{2}CV^2}$$

$$\boxed{W = \frac{1}{2}QV}$$

Work done (W) = Energy stored (U)

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Capacitor connected in Series.



when the capacitor are connected one after another in a single path then such a connection is called as series connection

In this connection charge remaine same.

$$Q_1 = Q_2 = Q_3 = Q \quad \text{——} (I)$$

In this connection potential difference is different

$$V = V_1 + V_2 + V_3 \quad \text{——} (II)$$

But,

$$C = \frac{Q}{V}$$

$$V = \frac{Q}{C}$$

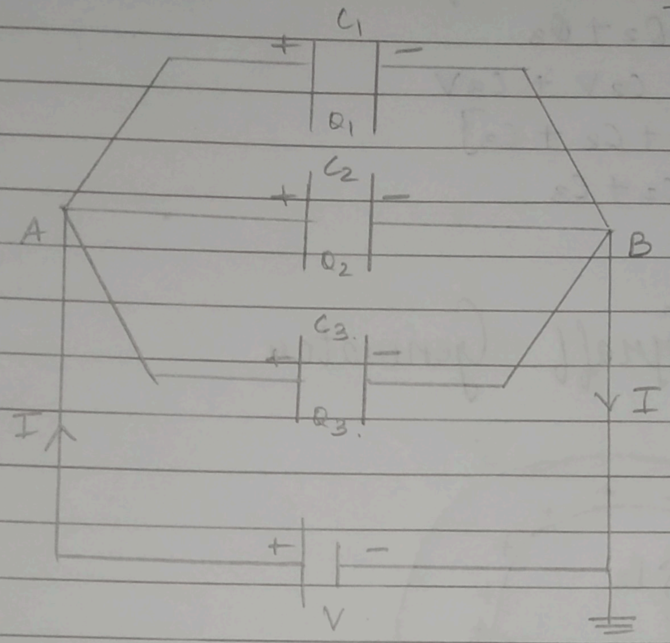
Put in eq (II)

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

$$\frac{\cancel{Q}}{C} = \cancel{Q}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitor connected in Parallel



When the capacitor are connected one below another between two point A and B than such a connection is know as parallel connection

In this connection potential difference is same

$$V_1 = V_2 = V_3 = V \quad \text{—— (I)}$$

In this connection charge on each capacitor is different

$$Q = Q_1 + Q_2 + Q_3 \quad \text{—— (II)}$$

But $C = \frac{Q}{V}$

$$Q = CV$$

Put $C = QV$ in eq^n (ii)

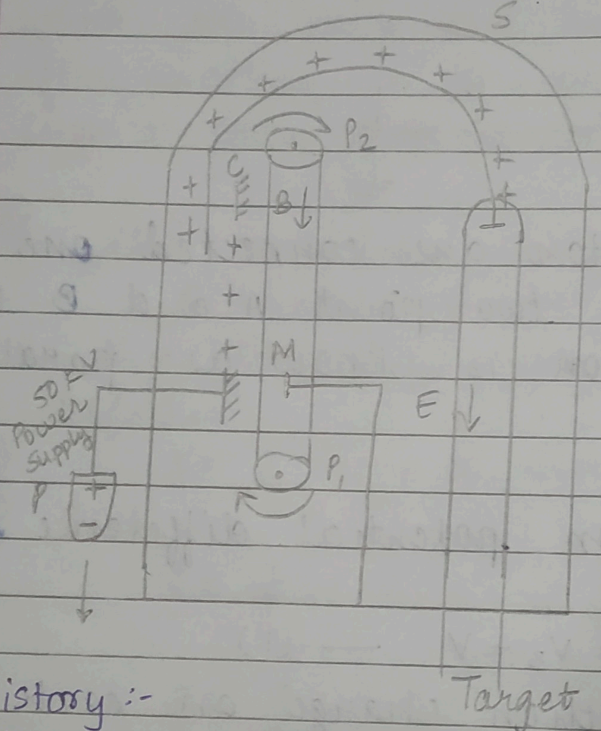
$$Q = Q_1 + Q_2 + Q_3$$

$$CV = C_1V + C_2V + C_3V$$

$$C\cancel{V} = \cancel{V}[C_1 + C_2 + C_3]$$

$$C = C_1 + C_2 + C_3$$

Van de Graff Generator



History :-

It is a device used to develop high potential of 10^7 V.

It works on the principle of corona discharge

Uses:-

To produce radioactive isotopes.

To study nuclear structure.