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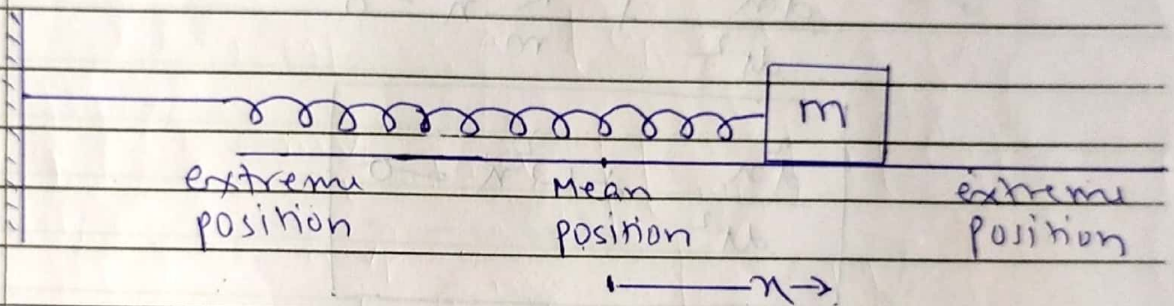
Oscillation OR simple harmonic motion.

Q) What is simple harmonic motion? find expression for it's differential equation. Hence find expression for acceleration, displacement and velocity of a particle performing S.H.M.



Consider a body of mass 'm' is connected with a spring of force constant 'K'. Let x be the displacement of the body from it's mean position and F be the internal restoring force then it is given by

$\therefore F \propto -x$

$\therefore \boxed{F = -Kx}$ .... (1)

Where -ve sign indicates that force and displacement are opposite in direction.

By definition of force.

$\because F = ma$

Since $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

$\therefore \boxed{F = \frac{md^2x}{dt^2}}$ ... (ii)

from eqns (i) and (ii)

$\therefore \frac{md^2x}{dt^2} = -kx$

$\therefore \frac{md^2x}{dt^2} + kx = 0$

$\therefore \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

$\therefore \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$

It differential equation of linear simple harmonic motion.

where $\omega = \sqrt{\frac{k}{m}}$

→ Expression for acceleration.

$\because \frac{d^2x}{dt^2} + \omega^2 x = 0$

$\therefore \frac{d^2x}{dt^2} = -\omega^2 x$

But $\frac{d^2x}{dt^2}$ is acceleration

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$\therefore \boxed{a = -\omega^2 x}$

-ve sign indicates that accelⁿ and displacment are opposite in direction.

→ Expression for velocity

$\therefore a = -\omega^2 x$

Since, $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$

$\therefore v\frac{dv}{dx} = -\omega^2 x$

$\therefore v\,dv = -\omega^2 x\,dx$

$\therefore \int v\,dv = -\omega^2 \int x\,dx$

$\therefore \boxed{\frac{v^2}{2} = \frac{-\omega^2 x^2}{2} + c}$ ---- (iii)

where c is constant of integration

When particle is at extreme then $x = A$ and $v = 0$. Now substitute above conditions in eqn (iii)

$0 = \frac{-\omega^2 A^2}{2} + c$

$\therefore \boxed{c = \frac{\omega^2 A^2}{2}}$

Now substitute value of c in equation (iii)

$\therefore \frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$

$\therefore v^2 = -\omega^2 x^2 + \omega^2 A^2$

$\therefore v^2 = \omega^2 (A^2 - x^2)$

$\therefore \boxed{v = \pm \omega \sqrt{(A^2 - x^2)}}$

→ Expression for displacement

$\because v = \omega \sqrt{(A^2 - x^2)}$

Since $v = dx/dt$

$\therefore \frac{dx}{dt} = \omega \sqrt{(A^2 - x^2)}$

$\therefore \frac{dx}{\sqrt{A^2 - x^2}} = \omega dt$

$\therefore \int \frac{1}{\sqrt{A^2 - x^2}} = \omega \int dt$

$\therefore \sin^{-1}\left(\frac{x}{A}\right) = \omega t + \alpha$

$\therefore \frac{x}{A} = \sin(\omega t + \alpha)$

$\therefore \boxed{x = A \sin(\omega t + \alpha)}$

Where α is constant of integration called intial phase or epoch.

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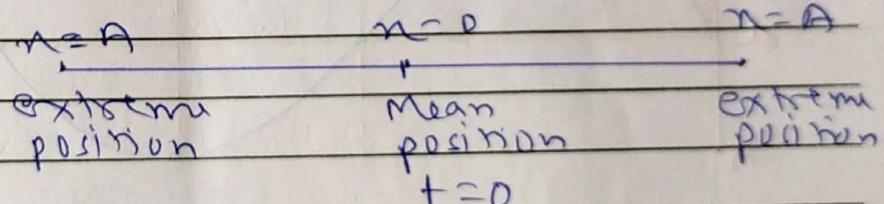
Draw graph of displacement, velocity and acceleration of a particle performing SHM when it starts it's motion from (i) mean position
(ii) extreme position

Case 1) When particle starts it's motion from mean position.

$\because x = A\sin(\omega t + \alpha)$

Since particle starts from mean position, when $t = 0$ then $x = 0$.



$\therefore 0 = A\sin(\omega \times 0 + \alpha)$

$\therefore 0 = A\sin\alpha$

$\therefore \sin\alpha = 0$

$\therefore \alpha = \sin^{-1}(0)$

$\therefore \boxed{\alpha = 0}$

$\therefore \boxed{x = A\sin\omega t}$

$$\because \quad v = \frac{dx}{dt}$$

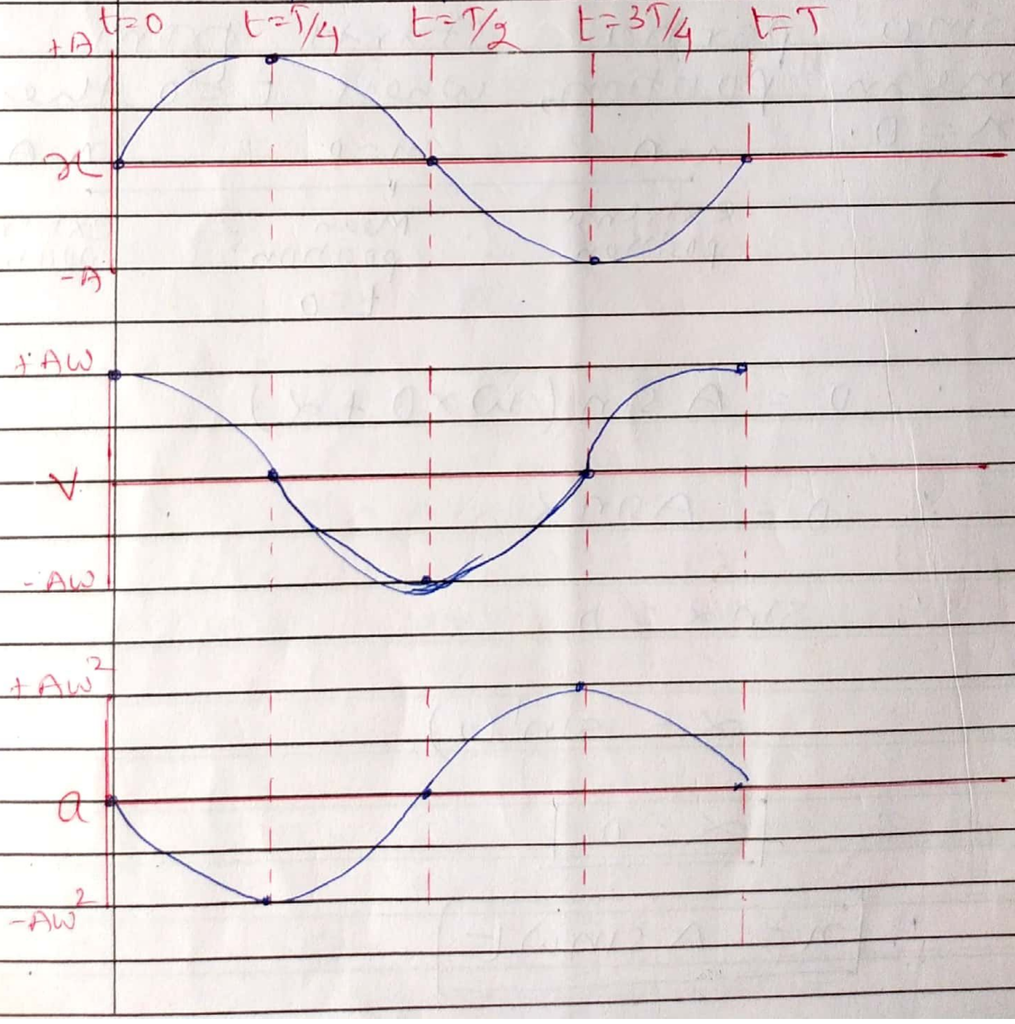
$$\therefore \quad v = \frac{d}{dt}(A \sin \omega t)$$

$$\therefore \quad \boxed{v = A\omega \cos \omega t}$$

$$\because \quad a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t)$$

$$\therefore \quad \boxed{a = -A\omega^2 \sin \omega t}$$

t	0	$T/4$	$T/2$	$3T/4$	T
$\theta = \omega t$	0	$\pi/2$	π	$2\pi/3$	2π
x	0	A	0	$-A$	0
v	$A\omega$	0	$-A\omega$	0	$A\omega$
a	0	$-A\omega^2$	0	$A\omega^2$	0



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Case-2) When particle starts it's motion from extreme position

$\because x = A\sin(\omega t + \alpha) \ldots\ldots$

When particle starts it's motion from extreme position, when $t=0$ then $x = A$

Now substitute above conditions in above eqn

$\therefore A = A\sin(\omega \times 0 + \alpha)$

$\therefore 1 = \sin\alpha$

$\therefore \alpha = \sin^{-1} 1$

$\therefore \boxed{\alpha = 90}$

$\therefore x = A\sin(\omega t + 90)$

$\therefore \boxed{x = A\cos\omega t}$

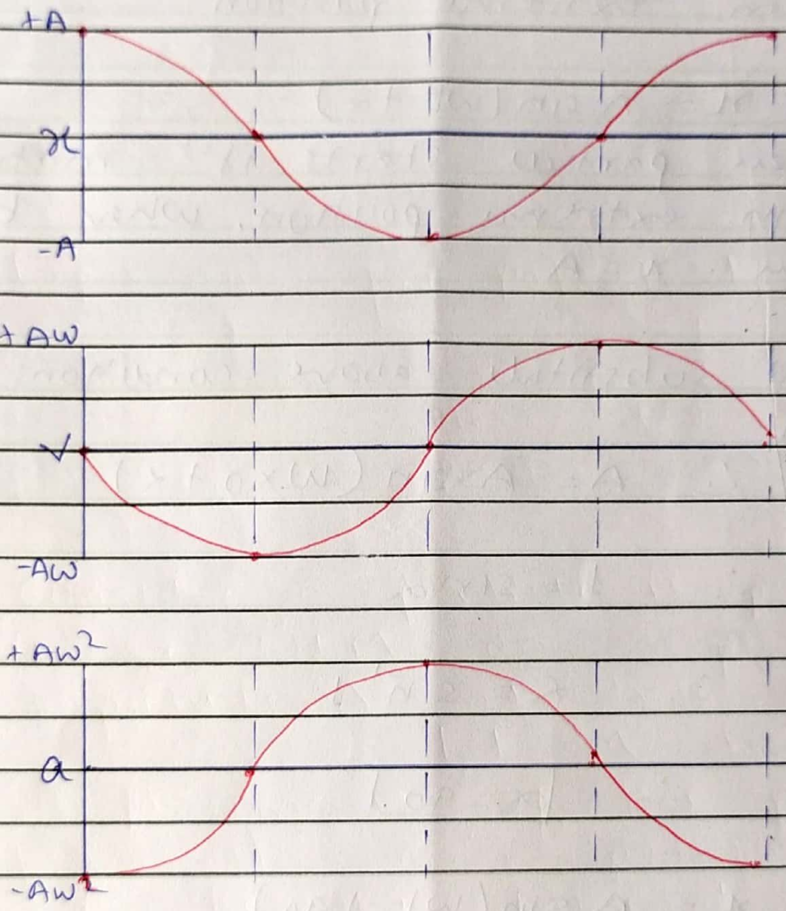
$\because v = \frac{dx}{dt} = \frac{d}{dt}(A\cos\omega t)$

$\therefore \boxed{v = -A\omega\sin\omega t}$

$\because a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega\sin\omega t)$

$\therefore \boxed{a = -A\omega^2\cos\omega t}$

t	0	$T/4$	$T/2$	$3T/4$	T
$\theta = \omega t$	0	$\pi/2$	π	$3\pi/2$	2π
x	A	0	$-A$	0	A
v	0	$-A\omega$	0	$A\omega$	0
a	$-A\omega^2$	0	$A\omega^2$	0	$-A\omega^2$



Show that linear S.H.M is periodic in time. Hence find expression for time period of a particle performing linear S.H.M.

Ans:- Since displacement of a particle performing linear S.H.M at any time instant is given by

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$$\boxed{x = A\sin(\omega t + \alpha)} \quad \cdots (1)$$

By adding time equals to time period new displacement is given by

$$x = A\sin[\omega(t+T)+\alpha]$$

since $T = \frac{2\pi}{\omega}$

$$\therefore x = A\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right)+\alpha\right]$$

$$\therefore x = A\sin[\omega t + 2\pi + \alpha]$$

$$\therefore x = A\sin[2\pi + (\omega t + \alpha)]$$

$$\therefore \boxed{x = A\sin(\omega t + \alpha)} \quad \cdots (11)$$

Hence linear S.H.M is periodic time.

Let T be the time period then it is given by

$$T = \frac{2\pi}{\omega}$$

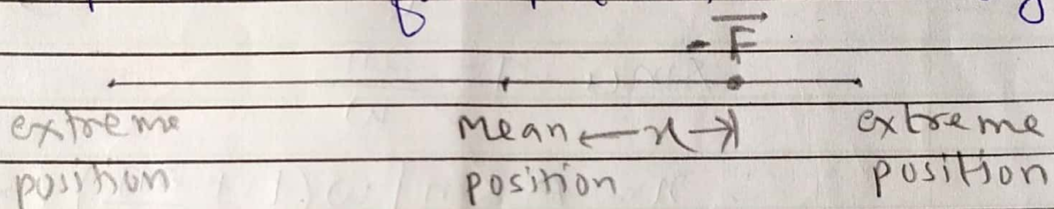
since $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{a}{x}}$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{K}{m}}} = \frac{2\pi}{\sqrt{a/x}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{x}{a}}}$$

Expression for kinetic energy, potential energy and total energy of a particle performing linear S.H.M. Draw graph for it.

(1) Expression for potential energy



Consider a particle is performing linear S.H.M on a path of amplitude 'A' and time period 'T'. Let x be the displacement of the particle from it's mean position and 'f' be the internal restoring force then it is given by

$$\boxed{F = -Kx}$$

where K is force constant. -ve sign indicates that force and displacement are opposite in direction.

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In order to displace the particle further work has to be done against above internal restoring force. Let dw be the small work done for small displacement dx then it is given by

$$dW = -F dx$$

-ve sign indicates that force and displacement are opposite in direction

$$\therefore dW = -(-Kx)dx$$

$$\therefore dW = Kx dx$$

Let W be the total work done for total displacement 'x' then it is given by

$$\therefore \int_0^W dW = \int_0^x Kx dx$$

$$\therefore [W]_0^W = K \int_0^x x dx$$

$$\therefore [W - 0] = K \left[\frac{x^2}{2}\right]_0^x$$

$$\therefore W = K \left(\frac{x^2}{2} - 0\right)$$

$$\therefore \boxed{W = \frac{1}{2} K x^2}$$

Since work done stores as an energy

$$\therefore \boxed{PE = \frac{1}{2}Kx^2}$$

At mean position, $x = 0$

$$\therefore \boxed{(PE)_{mean} = 0}$$

At extreme position, $x = A$

$$\therefore \boxed{(PE)_{ext} = \frac{1}{2}KA^2}$$

(ii) Expression for kinetic energy

$$\because KE = \frac{1}{2}mv^2$$

since $v = \omega\sqrt{A^2 - x^2}$

$$\therefore KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

since $\omega = \sqrt{\frac{K}{m}}$

$$\therefore \boxed{KE = \frac{1}{2}K(A^2 - x^2)}$$

At mean position, $x = 0$

$$\boxed{(KE)_{mean} = \frac{1}{2}KA^2}$$

At extreme position, $x = A$

$$\boxed{(KE)_{ext} = 0}$$

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(iii) Expression for total energy

$\because TE = KE + PE$

$= \frac{1}{2}K(A^2 - x^2) + \frac{1}{2}Kx^2$

$= \frac{1}{2}KA^2 - \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2$

$= \frac{1}{2}KA^2$

$\because (TE)_{mean} = (KE)_{mean} + (PE)_{mean}$

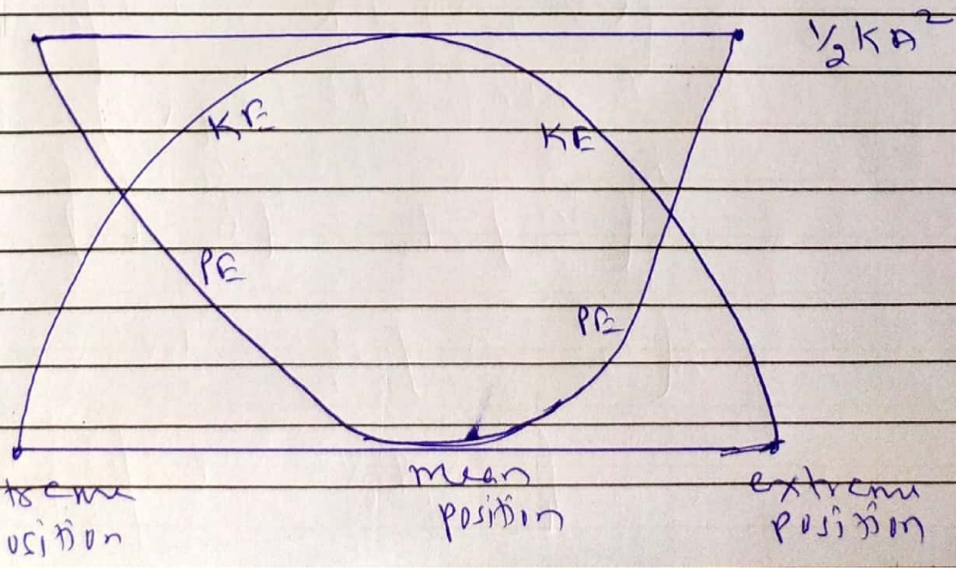
$= \frac{1}{2}KA^2 + 0$

$= \frac{1}{2}KA^2$

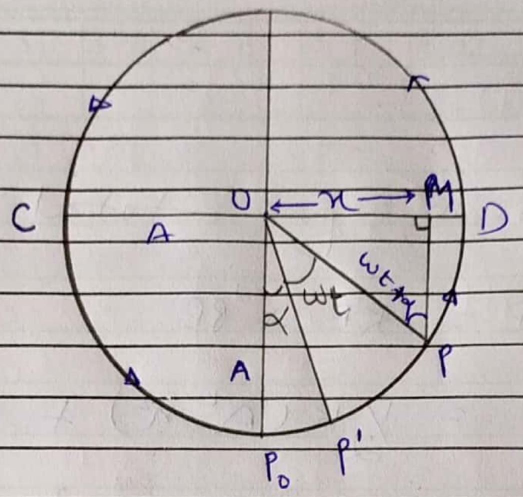
$(TE)_{ext} = (KE)_{ext} + (PE)_{ext}$

$= 0 + \frac{1}{2}KA^2$

$= \frac{1}{2}KA^2$



Show that linear S.H.M is a projection of uniform circular motion along one of the diameter



Consider a particle 'p' is performing uniform circular motion on circular path of radius 'A' in anticlockwise sense. Projection of particle 'p' is point 'M' on diameter CD. As particle 'p' is performing circular motion it's projection 'M' is performing to and fro motion on diameter CD.

In Δ OMP, $\angle M = 90°$, $\angle P = \omega t + \alpha$

$$\therefore \sin(\omega t + \alpha) = \frac{OM}{OP}$$

$$\therefore \sin(\omega t + \alpha) = \frac{x}{A}$$

$$\therefore x = A\sin(\omega t + \alpha)$$

Now diff above eqn w.r.t

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\left\{A\sin(\omega t + \alpha)\right\}$$

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$\therefore v = A\omega \cos(\omega t + \alpha)$

Again diff above eqn w.r.to 't'

$\therefore \frac{dv}{dt} = \frac{d}{dt}\left[A\omega \cos(\omega t + \alpha)\right]$

$\therefore a = -A\omega^2 \sin(\omega t + \alpha)$

$\therefore a = -\omega^2 \left[A \sin(\omega t + \alpha)\right]$

$\therefore \boxed{a = -\omega^2 x}$

Hence linear S.H.M is a projection of uniform circular motion along one of the diameter.

Explain composition of two SHM's having same time period along the same path.

Ans) Consider two SHM's of the same time period along the same path. Let A_1 and A_2 be the amplitudes, α_1 and α_2 be the initial phase of two S.H.M's then their displacements at any time instant are given by -

$\therefore x_1 = A_1 \sin(\omega t + \alpha_1)$

$\therefore x_2 = A_2 \sin(\omega t + \alpha_2)$

Let x be the resultant amplitude then it is given by

$\therefore x = x_1 + x_2$

$\therefore x = A_1 \sin(\omega t + \alpha_1) + A_2 \sin(\omega t + \alpha_2)$

$\therefore x = A_1 \{\sin \omega t \cos\alpha_1 + \cos \omega t \sin\alpha_1\}$
$+ A_2 [\sin \omega t \cos\alpha_2 + \cos \omega t \sin\alpha_2]$

$\therefore x = A_1 \sin \omega t \cos\alpha_1 + A_1 \cos \omega t \sin\alpha_1$
$+ A_2 \sin \omega t \cos\alpha_2 + A_2 \cos \omega t \sin\alpha_2$

$\therefore x = \sin \omega t (A_1 \cos\alpha_1 + A_2 \cos\alpha_2]$
$+ \cos \omega t \{A_1 \sin\alpha_1 + A_2 \sin\alpha_2]$

$\therefore$ Let $A_1 \cos\alpha_1 + A_2 \cos\alpha_2 = R \cos\delta$ ---(1)

$A_1 \sin\alpha_1 + A_2 \sin\alpha_2 = R \sin\delta$ --(2)

$\therefore x = \sin \omega t \, R\cos\delta + \cos \omega t \, R\sin\delta$

$\therefore x = R[\sin \omega t \cos\delta + \cos \omega t \sin\delta)$

$\therefore \boxed{x = R \sin(\omega t + \delta)}$

Hence composition of two S.H.M's is also a S.H.M. where R is resultant amplitude and δ is resultant initial phase.

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To find 'R'

By squaring and adding eqns (I) and (II)

$\therefore R^2\sin^2\alpha + R^2\cos^2\alpha = (A_1\sin\alpha_1 + A_2\sin\alpha_2)^2 + (A_1\cos\alpha_1 + A_2\cos\alpha_2)^2$

$\therefore R^2 = A_1^2\sin^2\alpha_1 + A_2^2\sin^2\alpha_2 + 2A_1A_2\sin\alpha_1\sin\alpha_2 + A_1^2\cos^2\alpha_1 + A_2^2\cos^2\alpha_2 + 2A_1A_2\cos\alpha_1\cos\alpha_2$

$\therefore R^2 = A_1^2 + A_2^2 + 2A_1A_2(\sin\alpha_1\sin\alpha_2 + \cos\alpha_1\cos\alpha_2)$

$\therefore R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_1 - \alpha_2)$

$$\therefore \boxed{R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_1 - \alpha_2)}}$$

Case 1) If two S.H.M's are in the same phase i.e. $\alpha_1 - \alpha_2 = 0$ and $\cos 0 = 1$

$\therefore R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$

$= \sqrt{(A_1 + A_2)^2}$

$$\boxed{R = A_1 + A_2}$$

Case-2) If two S.H.M's are opposite in phase i.e. $\alpha_1 - \alpha_2 = 180°$ and $\cos 180 = -1$

$$\therefore R = \sqrt{A_1^2 + A_2^2 - 2A_1A_2}$$

$$= \sqrt{(A_1 - A_2)^2}$$

$$\boxed{R = A_1 - A_2}$$

Case-3) If two S.H.M's are having phase diff of $\pi/2$ and $\alpha_1 - \alpha_2 = \pi/2$

$\cos \pi/2 = 0$

$$\therefore \boxed{R = \sqrt{A_1^2 + A_2^2}}$$

To find 'δ'

Divide eqn (II) by eqn (I)

$$\therefore \frac{R \sin \delta}{R \cos \delta} = \frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2}$$

$$\therefore \tan \delta = \frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2}$$

$$\therefore \boxed{\delta = \tan^{-1}\left[\frac{A_1 \sin \alpha_1 + A_2 \sin \alpha_2}{A_1 \cos \alpha_1 + A_2 \cos \alpha_2}\right]}$$

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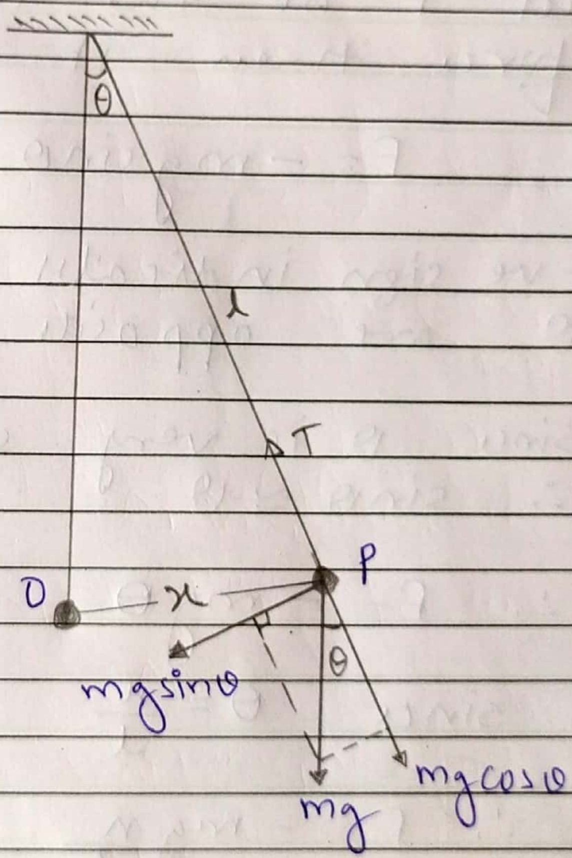
What is simple pendulum? Show that for small displacement simple pendulum is performing linear S.H.M. Hence find expression for time period of simple pendulum.



Consider a simple pendulum of length 'l' having bob of mass 'm'. At point 'p' pendulum is making an angle 'θ' with it's mean posion. At this position two forces are acting on the bob.

i) Weight of the bob which is acting vertically down words

(ii) Tension in the string which is acting towards rigid support.

Weight of the bob (mg) can be resolved into two components

(i) $mg\cos\theta$ which balances tension in the string

(ii) $mg\sin\theta$ which is acting towards the mean position.

Let 'F' be the internal restoring force then it is given by

$$F = -mg\sin\theta$$

-ve sign indicates that force and θ are opposite in direction.

Since θ is very small.

$\therefore \sin\theta \approx \theta$

$\therefore f = -mg\theta$

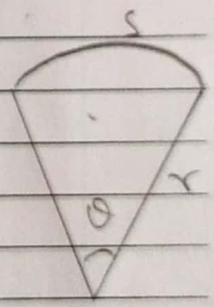
since $\theta = \frac{x}{l}$

$\therefore F = -mg\frac{x}{l}$

$\therefore F = -\left(\frac{mg}{l}\right)x$

since $\frac{mg}{l}$ is constant

$\therefore \boxed{F \propto -x}$



$s = r\theta$

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Hence for small displacement simple pendulum is performing linear S.H.M.

* Expression for time period.

$\because F = -\frac{mgx}{l}$

since $f = ma$

$\therefore ma = -\frac{mgx}{l}$

$\therefore \boxed{a = -\frac{gx}{l}}$ ---- (I)

for linear S.H.M,

$\boxed{a = -\omega^2 x}$ ---- (II)

from eqns (I) and (II)

$\omega^2 = g/l$

$\therefore \boxed{\omega = \sqrt{g/l}}$ --- (III)

Let 'T' be the time period of simple pendulum then it is given by

$$\because T = \frac{2\pi}{\omega}$$

$$\therefore T = \frac{2\pi}{\sqrt{g/l}}$$

$$\therefore \boxed{T = 2\pi\sqrt{l/g}}$$

Q) What is seconds pendulum? find expression for it's length.

Ans) A pendulum whose time period is two seconds is called seconds pendulum.

$$\because T = 2\pi\sqrt{l/g}$$

since $T = 2$ sec

$$\therefore 2 = 2\pi\sqrt{l/g}$$

$$\therefore 1 = \pi\sqrt{l/g}$$

$$\therefore 1 = \frac{\pi^2 l}{g}$$

$$\therefore \boxed{l = g/\pi^2}$$

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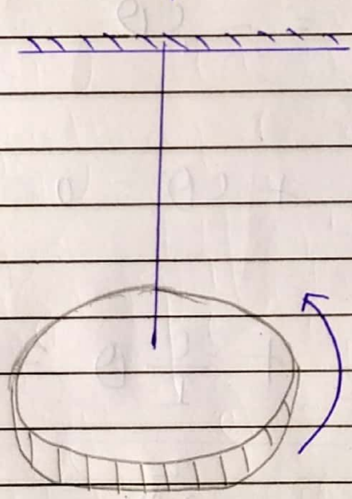
Q) What is angular S.H.M? find expression for it's differential equation. Hence find expression for time period of angular S.H.M.

Ans) A motion in which restoring torque acting on rotating body is directly proportional to angular displacement of the body and opposite in direction is called angular simple harmonic motion.



Consider a metallic disc attached ~~carefully~~ centrally to a thin wire (preferably nylon or metallic wire) hanging from a rigid support and performing angular S.H.M.

Let 'τ' be the restoring torque acting and θ be the angular displacement of the disc then it is given by

$\because \tau \propto -\theta$

$\therefore \boxed{\tau = -c\theta}$ --- (I)

Where c is torsional constant and defined as restoring torque per unit angular displacement. –ve sign indicates that τ and θ are opposite in direction.

$\because \tau = I\alpha$

since $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$

$\therefore \boxed{\tau = I\frac{d^2\theta}{dt^2}}$ --- (II)

from eqns (I) and (II)

$$I\frac{d^2\theta}{dt^2} = -c\theta$$

$$\therefore I\frac{d^2\theta}{dt^2} + c\theta = 0$$

$$\therefore \frac{d^2\theta}{dt^2} + \frac{c}{I}\theta = 0$$

$$\therefore \boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$$

It is differential equation of angular S.H.M.

where, $\omega = \sqrt{\frac{c}{I}}$

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Expression for time period of angular SHM:

Let T be the time period of angular S.H.M then it is given by

$$T = \frac{2\pi}{\omega}$$

since $\alpha = \frac{d^2\theta}{dt^2} = -\omega^2\theta$

$$\omega = \sqrt{\frac{\alpha}{\theta}}$$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{\text{Angular accel}^n}{\text{per unit angular displacement}}}}$$

Note

$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{\alpha}{\theta}} = \sqrt{\frac{c}{I}}$$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{\alpha}{\theta}}} = \frac{2\pi}{\sqrt{c/I}}$$

$$\boxed{\therefore T = 2\pi\sqrt{\frac{\theta}{\alpha}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{c}}}$$

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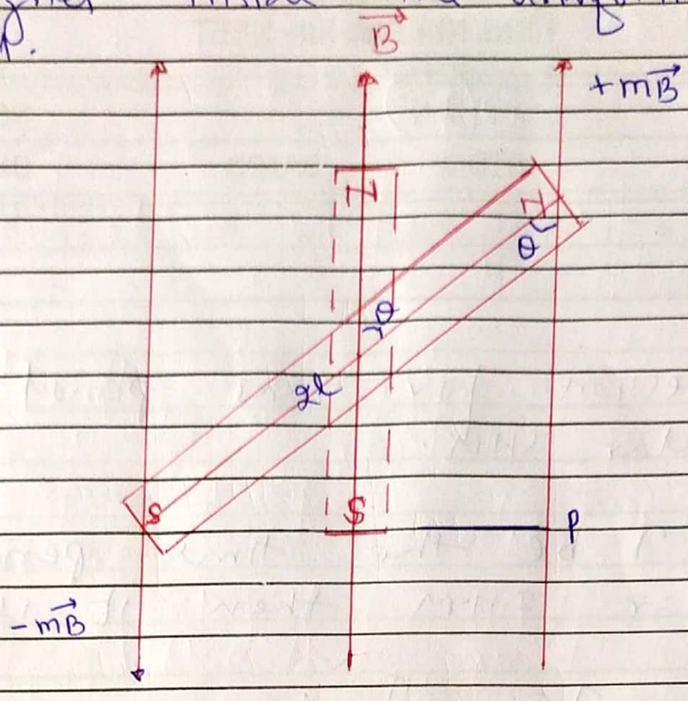
Expression for vibration of a magnet inside the uniform magnetic field.



Consider a magnet of magnetic length $(2l)$, magnetic moment μ and pole strength 'm', is placed inside a magnetic field of induction $\vec{B}$.

If magnet is tilted by an angle 'θ' from it's mean position, then both the poles experience equal force of 'mB' in opposite directions.

Let τ be the torque acting on it then it is given by

$$\tau = (mB)(SP)$$

$$\tau = (mB)(2l\sin\theta)$$

$$\tau = m2lB\sin\theta$$

$$\boxed{\tau = \mu B\sin\theta} \quad \ldots(\mu = m2l)$$

since θ is very small. Hence $\sin\theta$ is approx. θ.

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MARKS									

$\therefore \tau = -\mu B \theta$

Since μB is constant. -ve sign indicates that τ and θ are opposite in direction.

$\boxed{\tau \propto -\theta}$

Hence magnet is performing angular ~~acceleration~~ simple harmonic motion

→ Expression for time period of oscillation of magnet inside the magnetic field.

$\tau = -\mu B \theta$

since $\tau = I\alpha$

$\therefore I\alpha = -\mu B \theta$

$\therefore \boxed{\alpha = -\frac{\mu B}{I}\theta}$ --- (1)

for angular S.H.M,

$\boxed{\alpha = -\omega^2 \theta}$ --- (11)

from eqns (1) and (11)

$\boxed{\omega = \sqrt{\frac{\mu B}{I}}}$

Let T be the time period of magnet oscillation then it is given by

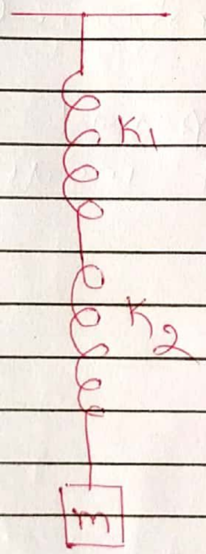
$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{MB}{I}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{MB}}}$$

Consider two springs of force constant K_1 and K_2.

(1)

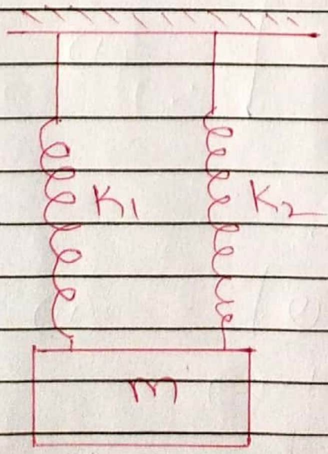


If K be the resultant force constant then it is given by

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\boxed{K = \frac{K_1 K_2}{K_1 + K_2}}$$

(2)



If K be the resultant force constant then it is given by

$$\boxed{K = K_1 + K_2}$$

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5.14 Damped Oscillations:

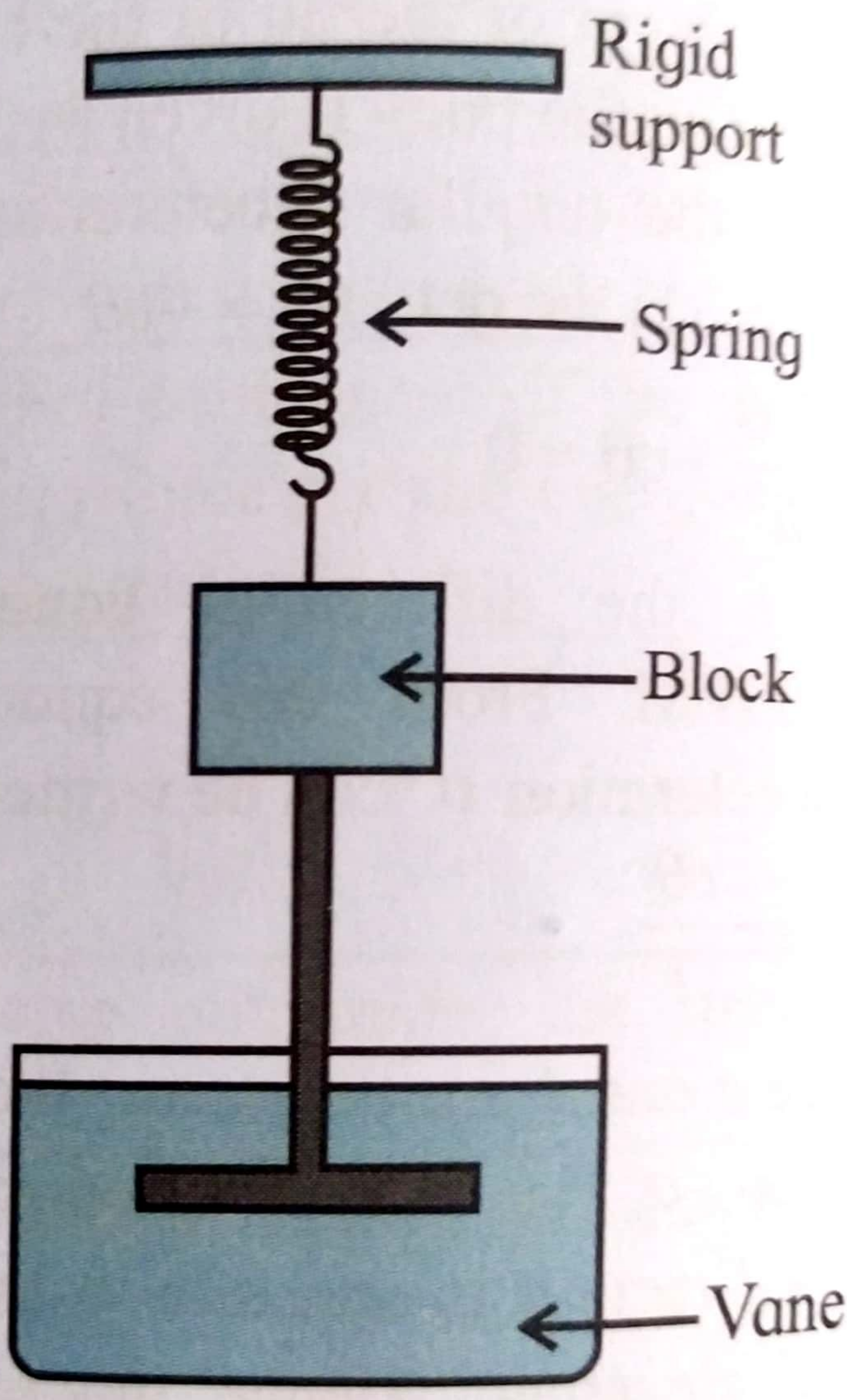


Fig. 5.13: A damped oscillator.

If the amplitude of oscillations of an oscillator is reduced by the application of an external force, the oscillator and its motion are said to be *damped*. *Periodic oscillations of gradually decreasing amplitude are called damped harmonic oscillations and the oscillator is called a damped harmonic oscillator.*

For example, the motion of a simple pendulum, dies eventually as air exerts a viscous force on the pendulum and there may be some friction at the support.

Figure 5.13 shows a block of mass m that can oscillate vertically on a spring. From the block, a rod extends to vane that is submerged on a liquid. As the vane moves up and down, the liquid exerts drag force on it, and thus on the complete oscillating system. The mechanical energy of the block-spring system decreases with time, as energy is transferred to thermal energy of the liquid and vane.

The damping force (F_d) depends on the nature of the surrounding medium and is directly proportional to the speed v of the vane and the block

$$\therefore F_d = -bv$$

Where b is the damping constant and negative sign indicates that F_d opposes the velocity.

For spring constant k, the force on the block from the spring is $F_s = -kx$.

Assuming that the gravitational force on the block is negligible compared to F_d and F_s, the total force acting on the mass at any time t is

$$F = F_d + F_s$$

$$\therefore ma = F_d + F_s$$

$$\therefore ma = -bv - kx$$

$$\therefore ma + bv + kx = 0$$

$$\therefore m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad \text{--- (5.35)}$$

The solution of Eq. (5.35) describes the motion of the block under the influence of a damping force which is proportional to the speed.

The solution is found to be of the form

$$x = Ae^{-bt/2m}\cos(\omega' t + \phi) \qquad \text{--- (5.36)}$$

$\left(Ae^{-bt/2m}\right)$ is the amplitude of the damped harmonic oscillations.

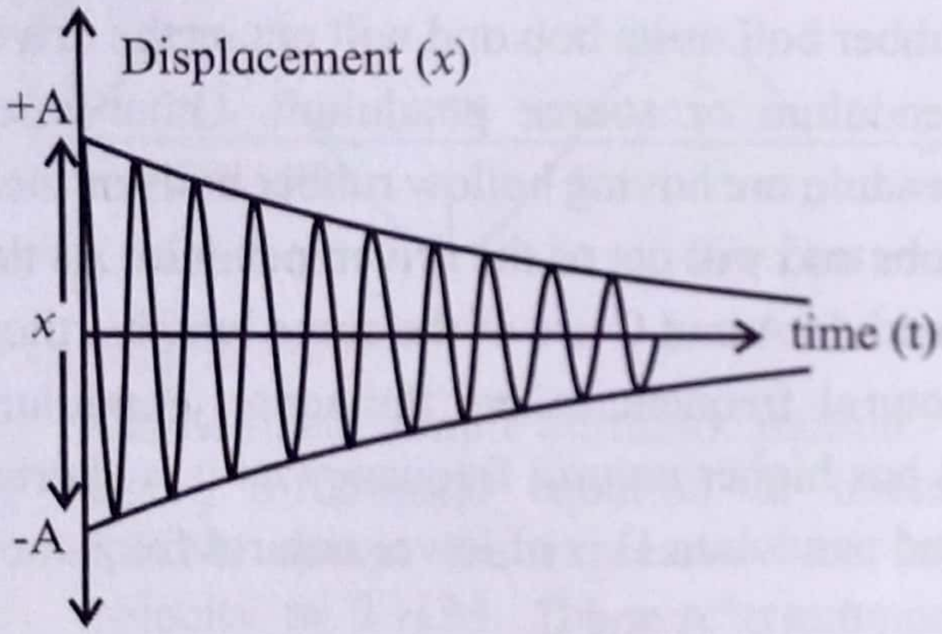


Fig. 5.14: Displacement against time graph.

As shown in the displacement against time graph (Fig 5.14), the amplitude decreases with time exponentially. The term $\cos(\omega' t + \phi)$ shows that the motion is still an S.H.M.

The angular frequency, $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

Period of oscillation, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$

The damping increases the period (slows down the motion) and decreases the amplitude.

5.15 Free Oscillations, Forced Oscillations and Resonance:

sound.

Free and forced vibrations

Free vibrations (Oscillation):

A body is said to be executing free oscilaltions, if it vibrates with its natural frequency.

When a body performs free vibrations, it continuously loses energy due to frictional resistance of the surrounding medium. Therefore the amplitude of vibrations goes on decreasing and finally the body comes to rest.

Example:

i. If a stretched string is plucked at some point, it performs free vibrations with its natural frequency.

ii. When the bob of a simple pendulum is displaced from its mean position and released, it perfoms free vibrations with its natural frequency.

iii. Vibrations of a tuning fork.

iv. Oscillations of a spring.

Natural frequency:

The frequency at which the body performs the free vibrations is called its natural frequency.

The natural freqency of a body depends upon the dimensions, mass, elastic properties and the mode of vibrations of the vibrating body.

Forced vibration:

The vibrations of a body in which body vibrates with frequency other than its natural frequency under the action of an external periodic force are called as forced vibrations.

The amplitude of forced vibrations depends upon the difference between the natural frequency of vibrating body and frequency of external periodic force. The amplitude is large, when the difference between frequency of external periodic force and the natural frequency of body is small.

Example:

i. In resonance tube experiment the vibrations of air column inside the tube when vibrating fork held over it, are forced vibrations.

ii. When soldiers are marching on suspension bridge, the bridge performs forced vibrations.

iii. Vibrations of pendulum of clock.

Damping:

Resonance

I. **Definition:** *The phenomenon of setting a body into oscillations of large amplitude by the influence of another vibrating body having the same natural frequency is called resonance.*

II. **Condition:**

i. The natural frequency of the driver oscillator must be equal to that of the driven oscillator.

ii. Oscillations produced by the driver and the driven oscillator must be in same phase.

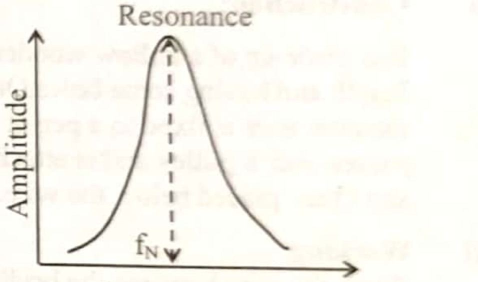
III. **Explanation:**

i. The amplitude of the forced vibrations depends on the difference between the natural frequency and the frequency of the applied periodic force.

ii. When the difference is large, the response of the body is poor and the amplitude of vibrations is small. The resonance is called flat resonance.

iii. As the difference in frequency decreases, the body vibrates more readily and the amplitude of the vibrations increases. The resonance is called sharp resonance.

iv. Fig. show variation of the amplitude of the forced vibrations with the frequency of the applied force. f_N is the natural frequency of the body and curve is called the resonance curve.



Resonance curve

IV. **Example:**

i. If two identical pendulums are suspended from a horizontal string and first is set into oscillations. Then second also starts vibrating with its amplitude increasing to maximum. Phase difference between first and second is $\frac{\pi}{2}$.

ii. If two tuning forks of same frequencies are mounted at two ends of a hollow wooden box. As one fork is set into vibrations then the other fork also starts vibrating with an amplitude which increases to some maximum value.

iii. In sonometer experiment a small piece of paper is placed at center of a wire between two knife edges. A vibrating T.F. is placed on the box with its stem in contact. As the length of the wire is adjusted so that its natural frequency is equal to that of the TF, resonance takes place. The piece of paper is violently thrown off the wire as it vibrates with maximum amplitude at resonance.

iv. In resonance tube experiment, air column of proper height vibrates in resonance with a vibrating TF held near its mount. A note of loud intensity is heard from the tube at resonance.

v. Small bottles give out a loud note when air is blown in them from the mouth at resonance.

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MCQ's of Oscillation

1) $\because v = \frac{v_{max}}{2}$

$\therefore \omega\sqrt{A^2 - x^2} = \frac{A\omega}{2}$

$\therefore A^2 - x^2 = \frac{A^2}{4}$

$\therefore A^2 - \frac{A^2}{4} = x^2$

$\therefore \frac{3}{4}A^2 = x^2$

$\therefore \boxed{x = \frac{\sqrt{3}}{2}A}$

[A]

2) $x = 6\sin(100t + \pi/4) \cdots ①$

$x = A\sin(\omega t + \alpha) \cdots (II)$

from eqns ① and (II)

$A = 6\,cm = 6\times10^{-2}\,m$

$\omega = 100$

$\because (KE)_{max} = \frac{1}{2}m v_{max}^2$

$= \frac{1}{2}\times1\times(A\omega)^2$

$= \frac{1}{2}\times[6\times10^{-2}\times100]^2$

$\boxed{KE = 18\,J}$ (D)

3) $\because l = g/\pi^2$

$\therefore \frac{l_1}{l_2} = \frac{g_1}{g_2}$

$\therefore \frac{1}{l_2} = \frac{g}{(g/6)}$

$\therefore \frac{1}{l_2} = \frac{6}{1}$

$\therefore \boxed{l_2 = \frac{1}{6}\ m}$

(A)

4) $T = 2\pi\sqrt{\frac{m}{K}}$

$\frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}}$

$\because K_1 = \frac{K \cdot K}{K+K}$

$\boxed{K_1 = K/2}$

$K_2 = K + K$

$\boxed{K_2 = 2K}$

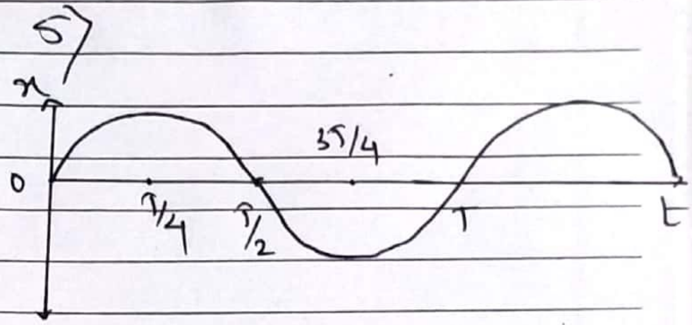
$\frac{T_1}{T_2} = \sqrt{\frac{2K}{(K/2)}}$

$= \sqrt{4/1}$

$\boxed{\frac{T_1}{T_2} = \frac{2}{1}}$

(C)

5)



(B) - The force is maximum at time 3T/4.

Explaination

$\therefore F = ma$

$\therefore F_{max} = m\, a_{max}$

And accelⁿ is maximum when displacement is maximum.