

Q1:

```
>> syms x k
```

```
>> syms f(x)
```

```
>> f(x)=symsum(x^k, k, 0, inf) % 符号函数
```

```
f(x)= piecewise(1 <= x, Inf, abs(x) < 1, -1/(x - 1))
```

```
>> y=symsum(x^k, k, 0, inf) % 符号表达式
```

```
y = piecewise(1 <= x, Inf, abs(x) < 1, -1/(x - 1))
```

```
% 用符号函数求解
```

```
>> f(-1/3), f(1/sym(pi)), f(3)
```

```
ans = 3/4
```

```
ans = -1/(1/pi - 1)
```

```
ans = Inf
```

```
% 用符号表达式求解
```

```
>>subs(y, x, -1/3), subs(y, x, 1/sym(pi)), subs(y, x, 3)
```

```
ans = 3/4
```

```
ans = -1/(1/pi - 1)
```

```
ans = Inf
```

Q2:

```
1)>> syms t y
```

```
>> y=abs(sin(t))
```

```
>> dydt=diff(y, t)
```

```
dydt = sign(sin(t))*cos(t)
```

2) % 求 0 处的左导数

```
>> syms d positive
```

```
>> dydt0 = limit((subs(-sin(t), t, 0)-subs(-sin(t), t, -d))/d,
```

```
d, 0)
```

```
dydt0 = -1
```

% 求 pi/2 处的导数

```
>> dydt1=subs(dydt, t, pi/2)
```

```
dydt1 = 0
```

Q3:

% 逐一验证各表达式是否与 e 相等

```
>> syms n
```

```
>> logical(limit((1+1/n)^n, n, inf)==exp(sym(1)))
```

```
ans =1
```

```
>> logical(limit(n/(factorial(n)^(1/n)) , n,
```

```
inf)==exp(sym(1)))
```

```
ans =1
```

```
>> logical(symsum(1/factorial(n), 0, inf)==exp(sym(1)))
```

```
ans =1
```

Q4(选做题):

1)

求证:  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}} = e$

证明: 只需证明  $\lim_{n \rightarrow \infty} \ln \frac{n}{\sqrt[n]{n!}} = 1$

$$\begin{aligned} \ln \frac{n}{\sqrt[n]{n!}} &= \frac{1}{n} (\ln n + \ln \frac{n}{2} + \dots + \ln \frac{n}{n-1} + \ln \frac{n}{n}) \\ &= -\frac{1}{n} (\ln \frac{1}{n} + \ln \frac{2}{n} + \dots + \ln \frac{n-1}{n} + \ln 1) \end{aligned}$$

事实上,  $\sim$  所述部分可视为对  $[0, 1]$   $n$  等分后求矩形面积和  
即对  $n \rightarrow \infty$  后,  $\sim$  部分为  $\ln x$  在  $[0, 1]$  上定积分

$$\therefore \lim_{n \rightarrow \infty} \ln \frac{n}{\sqrt[n]{n!}} = - \int_0^1 \ln x dx = - (x \ln x - x) \Big|_0^1 = 1$$

$\therefore$  结论得证

2)

```
>> syms n S(n) T(n) u(n)
```

```
>> S(n)=(1/n + 1)^n;
```

```
>> T(n)=n/(factorial(n)^(1/n));
```

```
>> u(n)=exp(sym(1));
```

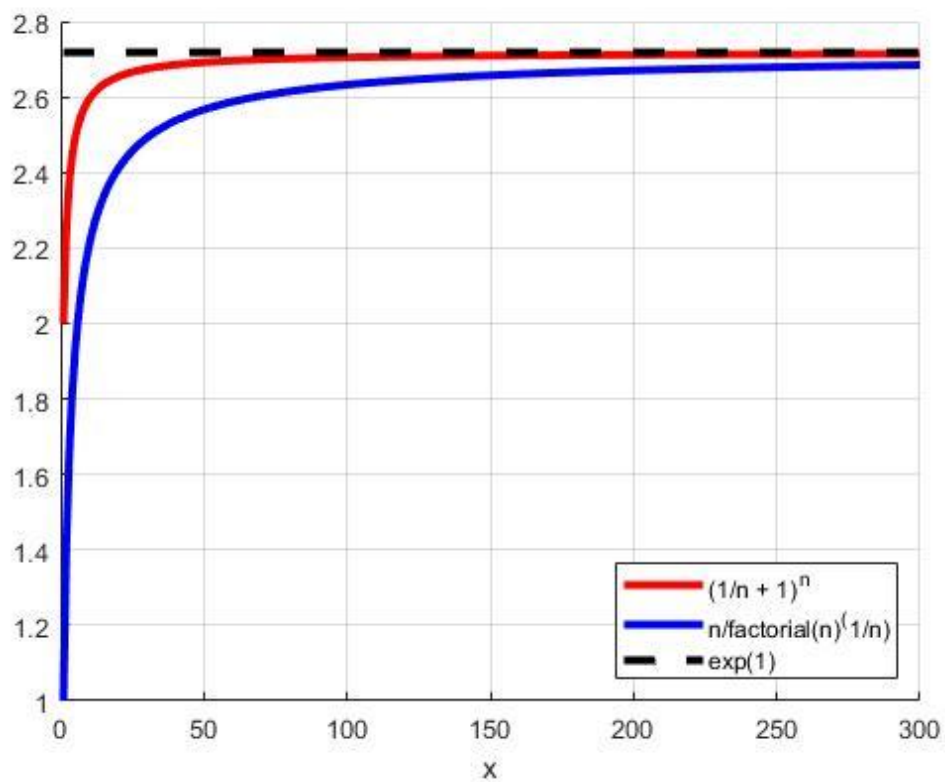
```
>> hold on
```

```
x=1:300
```

```

plot(x, subs(S,n,x), 'r', 'LineWidth', 3)
plot(x, subs(T,n,x), 'b', 'LineWidth', 3)
plot(x, subs(u,n,x), '--k', 'LineWidth', 3)
legend(char(S),char(T),char(u),'Location','SouthEast')
grid on
xlabel('x')
hold off
% 结果如下图

```



% 所以  $(1/n+1)^n$  收敛速度更快