 Due: 6 pm on Wednesday, May 28

Assignment 12

1 Revisit 3-Dimensional Matching

Recall that in the last assignment we proved that 3-dimensional matching problem is NP-complete. Now let's consider the following maximization version: Given disjoint sets X, Y, and Z, and a set $T \subseteq X \times Y \times Z$ of ordered triples, we want to find out the maximum size $|M|$ of a 3-dimensional matching $M \subseteq T$.

Give a 3-approximation algorithm. Prove its approximation ratio and give its running time.

2 Bounded Subset Sum

Suppose you are given a list of N positive integers $L = [a_1, a_2, \ldots, a_N]$, and a positive integer C. The problem is to find a subset $S \subseteq \{1, 2, \ldots, N\}$ such that

$$T(S) = \sum_{i \in S} a_i \leq C$$

and $T(S)$ is as large as possible.

(a) Prof. Luo proposes the following greedy algorithm for obtaining an approximate solution to this maximization problem:

Initialize $S \leftarrow \{\}, T = 0$
for $i = 1, 2, \ldots, N$ **do**
  if $T + a_i \leq C$ **then**
    $S \leftarrow S \cup \{i\}$
    $T \leftarrow T + a_i$
  end if
end for
return S

Show that Prof. Luo's algorithm is not a ρ-approximation algorithm for any fixed value ρ. (Use the convention that $\rho > 1$.)

(b) Describe a 2-approximation algorithm for this maximization problem that runs in $O(N)$ time. Prove its approximation ratio.

3 Hitting Set

We are given a set $A = \{a_1, \ldots, a_n\}$ and a collection $B_1, \ldots, B_m$ of subsets of A. Also, each element $a_i \in A$ has a weight $w_i \geq 0$. We call $H \subseteq A$ is a *hitting set* if $H \cap B_i$ is not empty for each i. Now the problem is to find a hitting set H that minimizes the total weight of the elements in H, $\sum_{a_i \in H} w_i$.

Let $b = \max_i |B_i|$. Give a b-approximation algorithm that runs in polynomial time. Prove the approximation ratio. (*Hint: consider LP rounding.*)