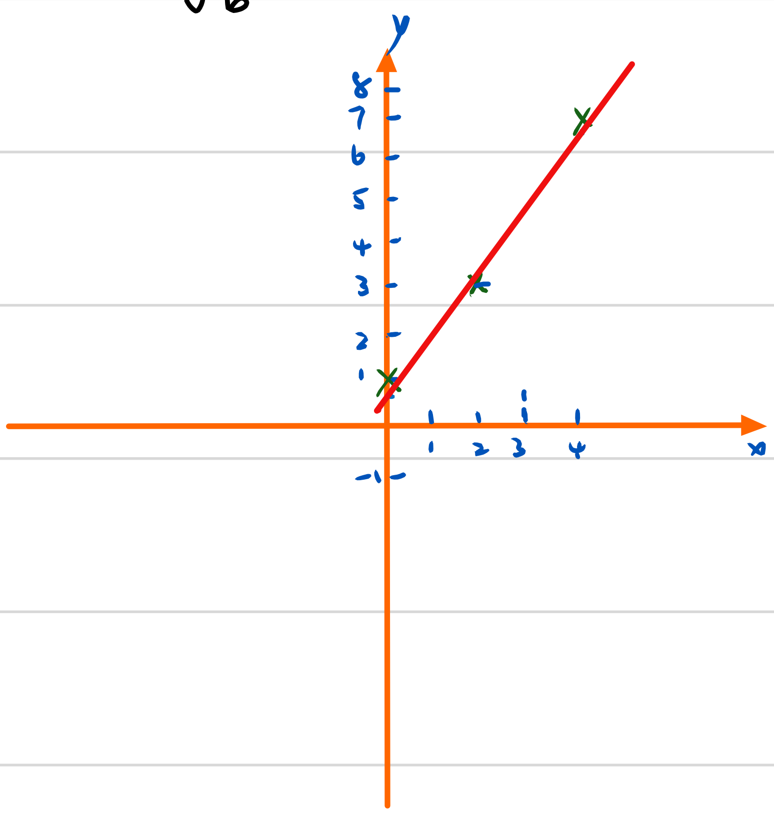
2300012929 尹锦润

一、 $L = \sum_i (wx_i + b - y_i)^2 = (b-1)^2 + (2w+b-3)^2 + (4w+b-7)^2$

$$\frac{\partial L}{\partial w} = 2(2w+b-3)\cdot 2 + 2(4w+b-7)\cdot 4 = 40w + 12b - 68$$

$$\frac{\partial L}{\partial b} = 2(b-1) + 2(2w+b-3) + 2(4w+b-7) = 6b + 12w - 22$$

$$\begin{cases} \frac{\partial L}{\partial w} = 0. \\ \frac{\partial L}{\partial b} = 0. \end{cases} \Rightarrow \begin{cases} w = \frac{3}{2} \\ b = \frac{2}{3} \end{cases}$$



二、

1) $y_i = 1$: $p(y=1 \mid x = x_i) = \sigma(w^T x_i + b)$

$y_i = 0$: $p(y=0 \mid x = x_i) = 1 - \sigma(w^T x_i + b) = \sigma(-w^T x_i - b)$

2)

$$\max_{w,b} \sum_{i\in[n]} \log[P(y = y_i \mid x = x_i)]$$

$$= \max_{w,b} \sum_{i\in[n]} \log[p(y=1 \mid x = x_i)^{y_i} p(y=0 \mid x = x_i)^{1-y_i}]$$

$$= \max_{w,b} \sum_{i\in[n]} y_i \log[\sigma(w^T x_i + b)] + (1-y_i)\log[\sigma(-w^T x_i - b)]$$

$$= \max_{w,b} \left\{ -\sum_{i\in[n]} y_i \log[1 + e^{-w^T x_i - b}] - \sum_{i\in[n]} (1-y_i) \log[1 + e^{w^T x_i + b}] \right\}$$

三、 D为样本

1) $$H(D) = -\sum_{k\in[k]} \frac{|D_k|}{|D|}\log_2\frac{|D_k|}{|D|} = -\left(\frac{4}{9}\log_2\frac{4}{9} + \frac{5}{9}\log_2\frac{5}{9}\right) \doteq 0.991076$$

$$H(D^{A=0}) = -\left(\frac{1}{4}\log_2\frac{1}{4} + \frac{3}{4}\log_2\frac{3}{4}\right) \doteq 0.811278$$

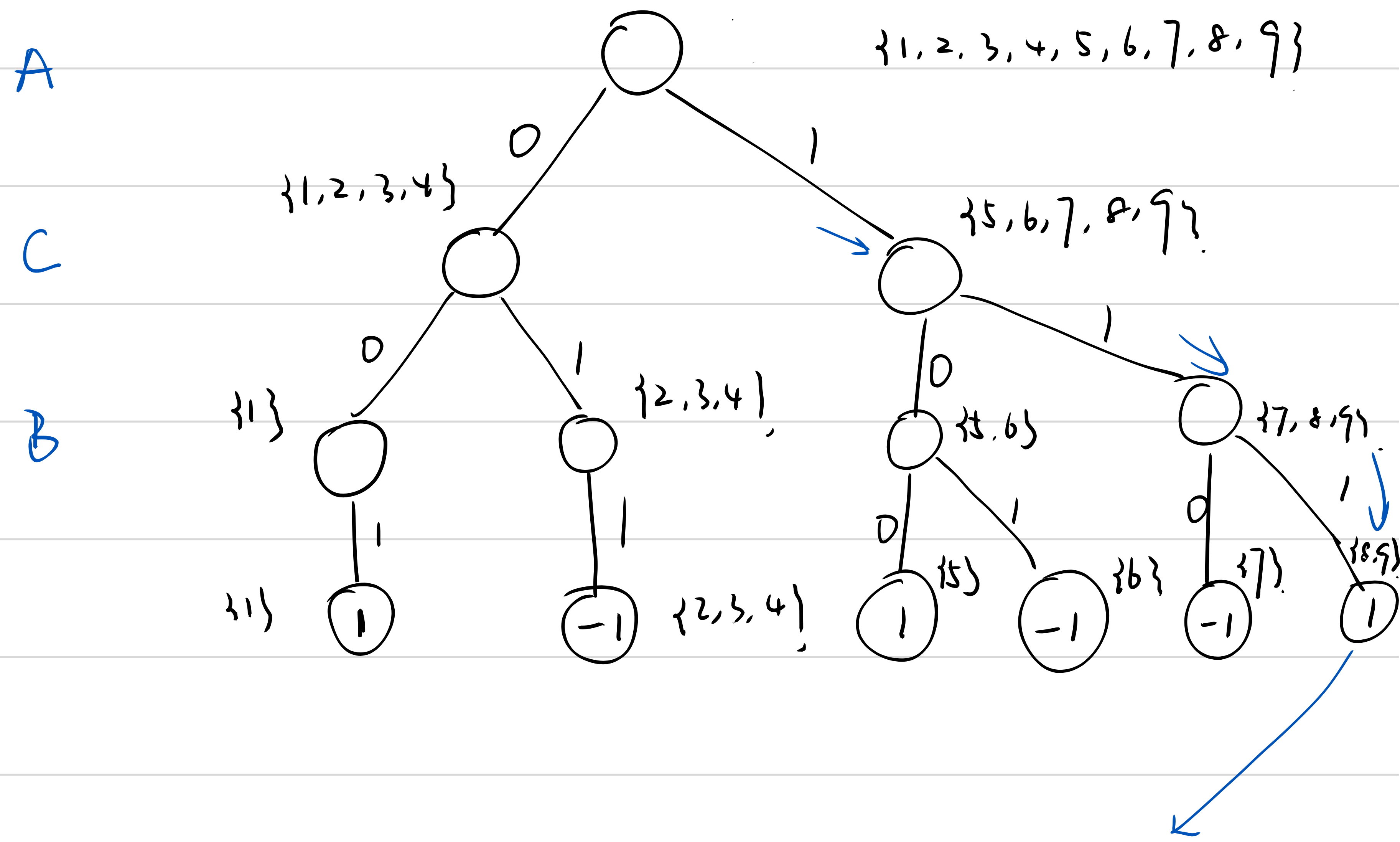
$$H(D^{A=1}) = -\left(\frac{3}{5}\log_2\frac{3}{5} + \frac{2}{5}\log_2\frac{2}{5}\right) \doteq 0.970951$$

$$H(A) = -\left(\frac{4}{9}\log_2\frac{4}{9} + \frac{5}{9}\log_2\frac{5}{9}\right) \doteq 0.991076$$

$$g_R(D,A) = \frac{g(D,A)}{H(A)} = \frac{H(D) - \frac{4}{9}H(D^{A=0}) - \frac{5}{9}H(D^{A=1})}{H(A)}$$

$$\doteq 0.091910 = 9.2\%$$

2)



$x_* = [1,1,1]$ 标签为 1。

四、1) $\frac{\partial a_j}{\partial z_i} = \frac{-e^{z_j}\cdot e^{z_i}}{(\sum_k e^{z_k})^2} = -a_j \cdot a_i \quad (j \neq i)$

$$\frac{\partial a_i}{\partial z_i} = \frac{e^{z_i}(\sum_k e^{z_k}) - e^{z_i} e^{z_i}}{(\sum_k e^{z_k})^2} = \frac{e^{z_i}}{\sum_k e^{z_k}}\left(1 - \frac{e^{z_i}}{\sum_k e^{z_k}}\right) = a_i(1-a_i)$$

$$\frac{\partial L}{\partial z_i} = \sum_j \frac{\partial L}{\partial a_j}\frac{\partial a_j}{\partial z_i} = \left(-\sum_{j\neq i}\frac{\partial L}{\partial a_j} a_j \cdot a_i\right) + \frac{\partial L}{\partial a_i} a_i(1-a_i)$$

$$= \left(-a_i \sum_j \frac{\partial L}{\partial a_j} a_j\right) + a_i \frac{\partial L}{\partial a_i}$$

2) $\frac{\partial a_j}{\partial z_i} = \frac{(\sum_k e^{z_k})}{e^{z_j}} \cdot \frac{-e^{z_j}\cdot e^{z_i}}{(\sum_k e^{z_k})^2} = \frac{-e^{z_i}}{\sum_k e^{z_k}} = -e^{a_i} \quad (i \neq j)$

$$\frac{\partial a_i}{\partial z_i} = \frac{(\sum_k e^{z_k})}{e^{z_i}} \cdot \frac{e^{z_i}}{\sum_k e^{z_k}}\left(1 - \frac{e^{z_i}}{\sum_k e^{z_k}}\right) = 1 - e^{a_i}$$

$$\frac{\partial L}{\partial z_i} = \sum_j \frac{\partial L}{\partial a_j}\frac{\partial a_j}{\partial z_i} = \left(-e^{a_i}\sum_{j\neq i}\frac{\partial L}{\partial a_j}\right) + (1-e^{a_i})\frac{\partial L}{\partial a_i}$$

$$= \left(-e^{a_i}\sum_j \frac{\partial L}{\partial a_j}\right) + \frac{\partial L}{\partial a_i}$$