7.

解: $E_r \cdot 2\pi r = \dfrac{\sigma_0 2\pi R_1}{\varepsilon_0}$ $\therefore E_r = \dfrac{\sigma_0 R_1}{r\varepsilon_0}$

$$u_1 - u_2 = \int_{R_1}^{R_2} E_r \, dr$$

$$= \frac{\sigma_0 R_1}{\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r}\, dr$$

$$= \frac{\sigma_0 R_1}{\varepsilon_0} \ln \frac{R_2}{R_1}$$

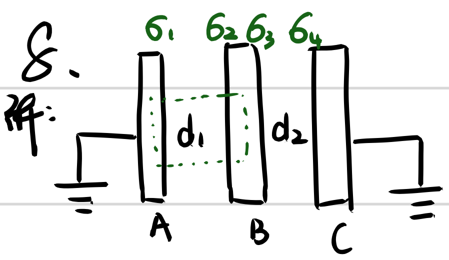
$$u_1 - u_r = \int_{R_1}^{r} E_r \, dt$$

$$= \frac{\sigma_0 R_1}{\varepsilon_0} \ln \frac{r}{R_1}$$

$$\therefore u_r = u_1 - (u_1 - u_r) = u_1 - \frac{(u_1-u_2)\ln\frac{r}{R_1}}{\ln\frac{R_2}{R_1}} = u_1 \frac{-\ln r + \ln R_2}{\ln R_2 - \ln R_1} + u_2 \frac{\ln r - \ln R_1}{\ln R_2 - \ln R_1}$$

8.

解:




$$\begin{cases} \sigma_1 + \sigma_2 = 0 \\ \sigma_3 + \sigma_4 = 0 \\ (\sigma_2 + \sigma_3) S = Q \\ \sigma_2 \cdot d_1 = \sigma_3 \cdot d_2 \end{cases} \Rightarrow \begin{cases} \sigma_2 = \dfrac{d_2}{d_1+d_2}\dfrac{Q}{S} \\ \sigma_3 = \dfrac{d_1}{d_1+d_2}\dfrac{Q}{S} \\ \sigma_1 = -\dfrac{d_2}{d_1+d_2}\dfrac{Q}{S} \\ \sigma_4 = -\dfrac{d_1}{d_1+d_2}\dfrac{Q}{S} \end{cases}$$

$$U_B = \frac{\sigma_2 \cdot d_1 \cdot S}{\varepsilon_0 \cdot S} = \frac{\sigma_2 \cdot d_1}{\varepsilon_0} = \frac{d_1 d_2}{d_1+d_2}\frac{Q}{\varepsilon_0 S}$$

9.

(1) $\oint_S \vec{D_1} \cdot d\vec{s} = D_1 \cdot 4\pi r^2 = \begin{cases} Q, & r \geq R \\ 0, & r < R \end{cases}$ $\therefore \vec{D_1} = \begin{cases} \dfrac{Q}{4\pi r^2}\hat{e_r} & R \leq r \\ 0, & r < R \end{cases}$

$r < R$: $\vec{E} = 0$.

$R < r < a$: $\varepsilon_0 \vec{E} = \vec{D}$ $\therefore \vec{E} = \dfrac{Q}{4\pi r^2}\dfrac{\hat{e_r}}{\varepsilon_0}$

$a < r < b$: $\varepsilon_0 \varepsilon_r \vec{E} = \vec{D}$ $\therefore \vec{E} = \dfrac{Q}{4\pi r^2}\dfrac{\hat{e_r}}{\varepsilon_0 \varepsilon_r}$

$r > b$: $\varepsilon_0 \vec{E} = \vec{D}$ $\therefore \vec{E} = \dfrac{Q}{4\pi r^2}\dfrac{\hat{e_r}}{\varepsilon_0}$.

$$\therefore \vec{E} = \begin{cases} 0 & r < R \\ \dfrac{Q}{4\pi r^2}\dfrac{\hat{e_r}}{\varepsilon_0} & R < r < a \text{ 或 } r > b \\ \dfrac{Q}{4\pi r^2}\dfrac{\hat{e_r}}{\varepsilon_0 \varepsilon_r} & a < r < b \end{cases}$$

(2) $\vec{p} = \chi_e \varepsilon_0 \vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \frac{Q}{4\pi r^2}\frac{\varepsilon_r - 1}{\varepsilon_r}\hat{e}_r$

$\sigma' = \vec{p}\cdot\vec{n}$. 内表. $\sigma' = -\frac{Q}{4\pi a^2}\frac{\varepsilon_r - 1}{\varepsilon_r}$

外: $\sigma' = \frac{Q}{4\pi b^2}\frac{\varepsilon_r - 1}{\varepsilon_r}$

(3) 介质体内无自由电荷且均匀极化 $\rho' = 0$

10. (1)

$E_1 = E_2 = \frac{U_0}{d}$

$D_1 = \varepsilon_0\varepsilon_r E_1 = \frac{U_0}{d}\varepsilon_0\varepsilon_r$

$D_2 = \varepsilon_0 E_2 = \frac{U_0}{d}\varepsilon_0$

方向从上到下.

$\sigma \cdot \frac{S}{2} = \oiint D\,ds = D\frac{S}{2}$

∴ $\sigma_1 = \frac{U_0}{d}\varepsilon_0\varepsilon_r$. $\sigma_2 = \frac{U_0}{d}\varepsilon_0$

(2) $\sigma_0 = \sigma_2 = \frac{U_0}{d}\varepsilon_0$

∴ $\Delta W = \frac{1}{2}\Delta Q U_0$

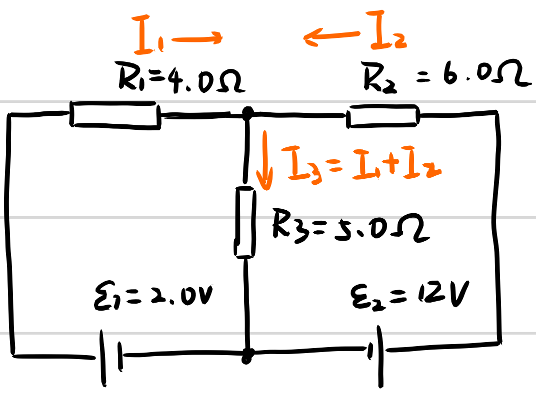
$= \frac{1}{2}U_0 \; \frac{1}{2}(\sigma_1 - \sigma_0)S$

$= \frac{1}{4}U_0\frac{U_0}{d}\varepsilon_0(\varepsilon_r - 1)S$

$= \frac{1}{4}\frac{U_0^2}{d}\varepsilon_0(\varepsilon_r - 1)S$ (增加量)

11.

(1)




$\begin{cases} -\varepsilon_1 + R_1 I_1 + R_3 I_3 = 0 \\ -\varepsilon_2 + R_2 I_2 + R_3 I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_1 = -\frac{19}{37}\,A \\ I_2 = \frac{49}{37}\,A \\ I_3 = \frac{30}{37}\,A \end{cases}$

∴ 通过 $R_3$ 电流 $I_3 = \frac{30}{37}$ A 从上到下.

(2) $\begin{cases} -\varepsilon_1 + R_3 I_2 = 0 \\ -\varepsilon_2 + R_2 I_2 + R_3 I_2 = 0 \end{cases} \Rightarrow \begin{cases} R_2 = 25\Omega \\ I_2 = 0.4\,A \end{cases}$

12、

$$U_d = \int_0^d \frac{I}{\gamma 2\pi r^2}\,dr = \frac{I}{2\pi\gamma d}$$

$$\Delta U_1 = U_1 - U_{1.6} = \frac{3}{8\pi} \times 10^4 \doteq 1193.7\,V$$

$$\Delta U_2 = U_{10} - U_{10.6} = \frac{3}{530\pi} \times 10^4 \doteq 18.0\,V$$