作业一. 2300012929 尹锦润

1. 解 (1) $\lambda = Ax$. $E=\int dE = \int_0^L \frac{\lambda dx}{(L+b-x)^2} \cdot \frac{1}{4\pi\varepsilon_0}$

$$E = \frac{A}{4\pi\varepsilon_0}\left(\frac{x}{L+b-x} + \ln(L+b-x)\right)\Big|_0^L$$

$$E = \frac{A}{4\pi\varepsilon_0}\left(\frac{L}{b} + \ln\frac{b}{L+b}\right)$$

$$\therefore \vec{E} = \frac{A}{4\pi\varepsilon_0}\left(\frac{L}{b} + \ln\frac{b}{L+b}\right)\hat{e}_x$$

(2) $\lambda = A(L+b-x)^2$

$$E = \int dE = \int_0^L \frac{\lambda dx}{(L+b-x)^2} \cdot \frac{1}{4\pi\varepsilon_0}$$

$$E = \frac{AL}{4\pi\varepsilon_0}$$

$$\therefore \vec{E} = \frac{AL}{4\pi\varepsilon_0}\hat{e}_x$$

2. 解: 取距离球心 x 的球面为高斯面

$$\Phi_e = \frac{q_内}{\varepsilon_0} = \left(\frac{1}{\varepsilon_0}\int_a^x \frac{A}{r} 4\pi r^2 dr\right) + \frac{Q}{\varepsilon_0}$$

$$\Phi_e = \oint\!\!\oint_{面} \vec{E}\cdot d\vec{S} = E\cdot 4\pi x^2$$

$$\therefore E = \frac{A}{2\varepsilon_0}\left(1-\frac{a^2}{x^2}\right) + \frac{Q}{4\pi\varepsilon_0}$$

$\because$ E处处相等 $\therefore$ $A = \frac{Q}{2\pi a^2}$

3. 解 取距离圆筒中心轴为 r 处 长为 l 的圆柱面为高斯面.

(1)

$$\Phi_e = \frac{q_内}{\varepsilon_0} = 0 \quad \therefore E = 0.$$

(2)

$$\Phi_e = \frac{q_内}{\varepsilon_0} \quad \Phi_e = \frac{2\pi R_1 l \sigma}{\varepsilon_0}$$

$$\Phi_e = \oint\!\!\oint_{侧面} \vec{E}\cdot d\vec{S} + \oint\!\!\oint_{上底面} \vec{E}\cdot d\vec{S} + \oint\!\!\oint_{下底面} \vec{E}\cdot d\vec{S}$$

$$\Phi_e = 2\pi r l E$$

$$\therefore E = \frac{R_1 \sigma}{\varepsilon_0 r} \qquad \vec{E} = \frac{R_1 \sigma}{\varepsilon_0 r}\hat{e}_r$$

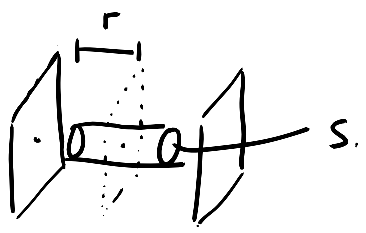
(3) $\Phi_e = \dfrac{q_{内}}{\varepsilon_0} \qquad \Phi_e = \dfrac{2\pi(R_1 - R_2) l \sigma}{\varepsilon_0}$

$$\Phi_e = \oint_{侧} \vec{E}\cdot d\vec{S} + \oint_{上底} \vec{E}\cdot d\vec{S} + \oint_{下底} \vec{E}\cdot d\vec{S}$$

$$\Phi_e = 2\pi r l E$$

$$\therefore E = \frac{(R_1 - R_2)\sigma}{\varepsilon_0 r} \qquad \vec{E} = \frac{(R_1 - R_2)\sigma}{\varepsilon_0 r}\hat{e}_r$$

4.解 取距离平板中心 r，底面面积 S 的圆柱面为高斯面



内：$\Phi_e = \dfrac{q_{内}}{\varepsilon_0} \qquad \Phi_e = \dfrac{S\cdot 2r\rho}{\varepsilon_0}$

$$\Phi_e = 2ES$$

$$\therefore E = \frac{\rho r}{\varepsilon_0}, \quad \vec{E} = \frac{\rho r}{\varepsilon_0}\hat{e}_r$$

外：$\Phi_e = \dfrac{q_{内}}{S} \qquad \Phi_e = \dfrac{S\cdot d\cdot \rho}{\varepsilon_0}$

$$\Phi_e = 2ES$$

$$\therefore E = \frac{\rho d}{2\varepsilon_0}, \quad \vec{E} = \frac{\rho d}{2\varepsilon_0}\hat{e}_r$$

5.解：取距筒中心为零电势点，对于距离中心 r 的位置

① $r < a$： $E = 0 \quad U = 0$

② $a \le r \le b$： 取长为 l 圆柱为高斯面
$$\begin{cases} \Phi_e = \dfrac{q_{内}}{\varepsilon_0} = \dfrac{\pi(r^2 - a^2) l \rho}{\varepsilon_0} \\ \Phi_e = E_r \cdot 2\pi r \cdot l \end{cases} \Rightarrow E_r = \frac{\rho}{2\varepsilon_0}\left(r - \frac{a^2}{r}\right)$$

$$U_r = \int_r^a \frac{\rho}{2\varepsilon_0}\left(x - \frac{a^2}{x}\right)dx \quad \therefore U_r = -\frac{\rho}{2\varepsilon_0}\left(\frac{1}{2}(r^2 - a^2) - a^2 \ln\frac{r}{a}\right)$$

③ $r > b$：
$$\begin{cases} \Phi_e = \dfrac{q_{内}}{\varepsilon_0} \quad \Phi_e = \dfrac{\pi(b^2 - a^2) l \rho}{\varepsilon_0} \\ \Phi_e = E_r \cdot 2\pi r \cdot l \end{cases} \Rightarrow E_r = \frac{\rho}{2\varepsilon_0}\frac{(b^2 - a^2)}{r}$$

$$U_r = U_b + \int_r^b E_r dx \quad \therefore U_r = \frac{\rho}{2\varepsilon_0}\left[-\frac{b^2 - a^2}{2} - a^2 \ln\frac{b}{a} + (b^2 - a^2)\ln\frac{b}{r}\right]$$

6、解：对于 (0, y) 处点电势：(y>0)

$$u = \int_{-l}^{l} \frac{q\,dx}{2l} \frac{1}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2+y^2}}$$

$$u = \frac{q}{8\pi\varepsilon_0 l} \ln \frac{l+\sqrt{l^2+y^2}}{-l+\sqrt{l^2+y^2}}$$

$$E_y = -\frac{du}{dy} \qquad E_y = -\frac{q}{8\pi\varepsilon_0 l} \frac{d}{dy}\left(\ln(l+\sqrt{l^2+y^2}) - \ln(-l+\sqrt{l^2+y^2})\right)$$

$$E_y = -\frac{q}{8\pi\varepsilon_0 l}\left[\frac{1}{l+\sqrt{l^2+y^2}} - \frac{1}{-l+\sqrt{l^2+y^2}}\right]\frac{y}{\sqrt{l^2+y^2}}$$

$$= -\frac{q}{8\pi\varepsilon_0 l} \cdot \frac{y}{\sqrt{l^2+y^2}} \cdot \frac{-2l}{y^2}$$

$$= \frac{q}{4\pi\varepsilon_0 y\sqrt{l^2+y^2}}$$