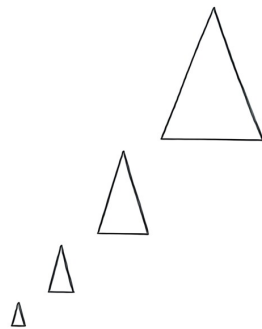


例如

$$u_n(x) = \begin{cases} 2(n+1)(x - \frac{1}{n+1}), & x \in [\frac{1}{n+1}, \frac{1}{2}(\frac{1}{n+1} + \frac{1}{n})] \\ -2(n+1)(x - \frac{1}{n}), & x \in [\frac{1}{2}(\frac{1}{n+1} + \frac{1}{n}), \frac{1}{n}] \\ 0, & x \text{ 于 } [0, 1] \text{ 的其他地方} \end{cases}$$



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这里 $u_n(x) \in C[0, 1]$, $\max_{x \in [0, 1]} u_n(x) = \frac{1}{n} = u_n(x_n)$, $x_n = \frac{1}{2}(\frac{1}{n+1} + \frac{1}{n})$, $\forall n \in \mathbb{N}$

所以 $\sum_{n=1}^{\infty} u_n(x_n)$ 发散. 但是, 由于 $\sum_{k=n+1}^{\infty} u_k(x) \leq \frac{1}{n+1} \rightarrow 0 \quad (n \rightarrow \infty)$, $\forall x \in [0, 1]$.

所以 $\sum_{n=1}^{\infty} u_n(x)$ 于 $[0, 1]$ 一致收敛.

所以, 存在 $x_n \in I$ s.t. $\sum_{n=1}^{\infty} u_n(x_n)$ 发散并不能导出 $\sum_{n=1}^{\infty} u_n(x)$ 于 I 不一致收敛.