Due: 6 pm on Wednesday, April 2

Assignment 6

1 Aggregate Analysis and Accounting Method

Suppose we have a data structure where the cost $T(i)$ of the i-th operation is:

$$T(i) = \begin{cases} 2i & \text{if } i \text{ is a power of } 2, \\ 1 & \text{otherwise.} \end{cases}$$

What is the amortized cost of the operation? Use both the **aggregate method** and the **accounting method** to analyze the amortized running time.

2 Potential Method

We want to design a data structure S with real numbers, with the following two operations:

1) $\texttt{insert}(S, x)$ inserts the object x in S.

2) $\texttt{delete}(S)$ removes the $\lceil |S|/2 \rceil$ largest elements of S.

Propose your implementation of the data structure and analyze the amortized costs of the two operations with the **potential method**, such that the amortized runtime of $\texttt{insert}(S, x)$ is $O(1)$, and that of $\texttt{delete}(S)$ is 0.

3 LP Formulation

Formulate the following problems as LPs:

(a) minimize $||Ax - b||_1$ subject to $||x||_\infty \leqslant 1$.

(b) minimize $||x||_1$ subject to $||Ax - b||_\infty \leqslant 1$.

(c) minimize $||Ax - b||_1 + ||x||_\infty$.

In each problem, $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ are given, and $x \in \mathbb{R}^n$ is the optimization variable.

($||X||_1 = \sum_i |X_i|, ||X||_\infty = \max_i |X_i|$)

4 Construct a Hyperplane

Describe a method for constructing a hyperplane that separates two given polyhedra:

$$\mathcal{P}_1 = \{x \in \mathbb{R}^n | Ax \leqslant b\}, \mathcal{P}_2 = \{x \in \mathbb{R}^n | Cx \leqslant d\}.$$

Your method must return a vector $a \in \mathbb{R}^n$ and a scalar γ such that

$$a^T x > \gamma, \forall x \in \mathcal{P}_1;\ a^T x < \gamma, \forall x \in \mathcal{P}_2.$$

You can assume that $\mathcal{P}_1$ and $\mathcal{P}_2$ do not intersect. If you know several methods, you should give the most efficient one.