Università degli studi di Milano-Bicocca

Financial Markets Analytics

Final Project

S&P 500 Index

Tilted Portfolios Analysis and Comparison

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Introduction

In this group work, we want to better understand the structural characteristics that risk brings to real investment portfolios. In order to understand this empirically, we need to build real portfolios that are concentrated/tilted with respect to a specific level and kind of risk. Our reference will be the definition of "Total," "Systematic," and "Specific" risk as defined and introduced through the CAPM we studied. We will then rely on the SML to investigate the risk level of individual securities and then proceed to the subsequent groupings.

$$E(r_i) = r_i + \beta_i(E(R_M) - r_f)$$

In pursuit of the analysis the above SML equation, must be reconsidered in the form of a regression equation as in the Market Model, also known as single index model. We can interpret such an equation as the "ex-post" version of the SML.

$$r_i = \alpha + \beta_i(R_M) + e_i$$ $\rightarrow$ we excluded the expectations

It is usually applied, for equivalence, in excess returns form:

$$r_i - r_f = \alpha_i + \beta_i(R_M - r_f) + e_i \ [2]$$

In this ex-post framework, we see two new parameters α and e_i.

Given that even in the "ex-ante" version of the SML, the Beta is derived from time series, then the $\beta = COV(R_M, R_i)$ and the β as SML(i) σ^2 the regression slope will match, but only if the time series are of the same length.

In the setting of the Market Model we get two other coefficients, the α_i that should be zero if the CAPM holds and ei proxying the specific risk.

In addition, the equation [2], since it is a regression, yields an R^2 informing on the goodness of fit. This way there are many possible profiles on which to do stock groupings. Firstly, the profile of fitting is based on R^2. Then going in more detail the relations between total risk decomposed in specific risk and systematic risk:

$Total\ Risk\ =\ Systematic\ Risk\ +\ Specific\ Risk$

$$\sigma^2_i\ =\ \beta_i^2\sigma^2_M + \sigma^2 e_i \ [3]$$

The ratio of (Systematic Risk)/(Specific Risk) is the R^2, but such a separation could lead to different characteristics in case of stocks with high or low levels of total risk.
The last profile interesting to investigate it is the one of excess returns

represented by the α_i. Such a return could be positive, null, or negative, but also significant or not as expressed by the specific t-test. Also, the ratio between this potential excess return and the total stock return could be of interest.

Finally, the return/risk profile, varying the adopted measure of return and risk should be of interest.

Theory

A portfolio is a collection of financial investments, in an efficient portfolio, investable assets are combined in a way that produces the best possible expected level of return for their level of risk — or the lowest risk for a target return.

A diversified portfolio contains a mix of distinct asset types and investment vehicles in an attempt at limiting exposure to any single asset or risk.
The Market Model, also known as the single index model and developed by William Sharpe in 1963, is a simple asset pricing model to measure both the risk and the return of a stock.

To simplify analysis, the single-index model assumes that there is only one macroeconomic factor that causes the systematic risk affecting all stock returns and this factor can be represented by the rate of return on a market index, such as the S&P 500, or the Euro Stoxx 50, used in this project.

According to this model, the return of any stock can be decomposed into the expected excess return of the individual stock due to firm- specific factors, commonly denoted by its alpha coefficient (α), the return due to macroeconomic events that affect the market, and the unexpected microeconomic events that affect only the firm.

Below, the equation of the Market Model:

$r_i = \alpha_i + \beta_i \cdot R_M + e_i$

where:
- r_i : expected return on security i
- α : alpha coefficient, excess return
- β_i : beta coefficient, measure of volatility
- R_M : return of the market
- e_i : error term

As we saw above, the relations between total risk can be decomposed in specific risk and systematic risk:

Total Risk = Systematic Risk + Specific Risk: $\sigma_i^2 = \beta_i^2 \cdot \sigma_M^2 + \sigma^2 \cdot e_i$ [3]

Unsystematic risk (or diversifiable, specific risk) can be mitigated through diversification while systematic or market risk is generally unavoidable. The unsystematic risk is specific to a company, industry, market, economy, or country. The most common sources of unsystematic risk are business risk and financial risk. Because it is diversifiable, investors can reduce their exposure through diversification.

The S&P 500 Index

The S&P 500 Index, established by Standard & Poor's in 1957, is a prominent market-capitalization-weighted index that includes 500 of the largest publicly traded companies in the United States. It serves as a critical benchmark for the U.S. stock market, encompassing a wide range of industries and sectors. The companies in the S&P 500 are selected based on criteria such as market size, liquidity, and industry representation, providing a comprehensive snapshot of the overall market and economic health. The index's market-cap weighting methodology gives more significance to larger companies, ensuring that changes in the stock prices of the biggest corporations have a proportionate impact on the index's performance.

Investors and analysts rely on the S&P 500 as a gauge of market trends and investor sentiment. Its broad and diverse composition makes it a reliable indicator of the U.S. economy. The index's performance is closely watched, influencing investment strategies and economic policies. Many financial products, such as mutual funds and exchange-traded funds "ETFs", are designed to track the S&P 500, making it a central component of both professional and retail investment portfolios. This widespread use and acceptance underscore its significance in the financial markets, reflecting the health and direction of the broader economy.

Data

The data was gathered by the author using a snippet of code available at this link: Code Snippet. The script makes use of the Investor's Exchange API in order to gather the data of all stocks between 2013-02-08 and 2018-02-07. Through a data cleaning process, the data has been stripped of any empty or null values and is ready to be used for the project as is.

The dataset exposes the following columns of data:

- Date - in format: yy-mm-dd

- Open - price of the stock at market open (USD)
- High - Highest price reached in the day (USD)
- Low - Lowest price reached in the day (USD)
- Close - Close price of the day (USD)
- Volume - Number of shares traded
- Name - the stock's ticker name

The chart shows the index price on a time series chart to better appreciate how it moved as a whole.

Figure 1 - S&P 500 Index Price Chart

As an initial exploratory analysis the following table shows the 5 best performing and 5 worst performing stocks of the index during the five years period.

5 Best Performing Stocks				5 Worst Performing Stocks			
Stock	Close 2013	Close 2018	Increase	Stock	Close 2013	Close 2018	Loss
NVDA	12.37 $	228.80 $	+1,749 %	RRC	20.23 $	2.89 $	-85 %
NFLX	25.85 $	264.86 $	+923 %	CHR	72.15 $	13.12 $	-81 %
ALGN	32.73 $	234.33 $	+615 %	DISCA	71.47 $	23.12 $	-67 %
EA	17.37 $	123.05 $	+608 %	DISCK	64.60 $	21.95 $	-66 %
STZ	31.85 $	114.15 $	+572 %	MOS	61.40 $	25.25 $	-58 %

Data - Processing

The data was already suited to perform the rolling regression. However, an easy preprocessing phase was conducted to make the rolling regression a little bit easier. First, it was necessary to compute the log returns of both the S&P 500 index and each stock and select the data with the same days of both dataset. Then the two datasets are merged together to work more efficiently.

Rolling Regression

A rolling regression was performed on the data to estimate the coefficients of the excessive returns, the beta, the specific and systematic risk for each stock. Out of the five years of data, the dataset is divided in two so that the training set has the data of the first four years and the test set contains the data of the fifth year. The actual regression was performed with a moving window of 180 days on the training set by applying the ordinary least square function between each stock and the value of the index. The coefficients are then obtained by averaging the multiple coefficients for each stock. The test set was then used later for comparing the set of possible portfolios.

Portfolios building

After selecting 100 stocks out of the 500, different tilted portfolios with different filters have been created to be compared.
The alpha was not considered in building the portfolios because none of them was statistically significant.

Single filter

Max specific risk

Specific risk, is the risk inherent to an individual asset that is not correlated with the index risk, this portfolio tries to exploit the unique characteristics of these individual stocks.
Although high specific risk also means that the returns of these stocks are less predictable and more volatile, this approach could lead to higher returns if the specific risks of the chosen stocks are rewarded by the market.

Ticker	alpha	alpha_pvalue	beta	beta_pvalue	r_squared	specific_risk	systematic_risk
CHK	-0.002256	0.407070	1.769304	3.439421e-03	0.127972	0.001659	0.000215
AMD	0.000462	0.452205	1.611138	3.041992e-04	0.098376	0.001267	0.000168
NI	-0.000391	0.428794	0.679797	3.328530e-02	0.160094	0.001013	0.000029
FCX	-0.001448	0.481385	1.946223	6.066662e-07	0.224964	0.000988	0.000241
LNT	0.000213	0.621854	0.637445	6.278175e-02	0.174287	0.000978	0.000027
...	...	...	...	...	...	...	...
CRM	0.000142	0.610578	1.372058	5.506960e-08	0.286740	0.000269	0.000113
MGM	0.000080	0.552340	1.497360	3.178322e-08	0.320510	0.000268	0.000131
YUM	-0.000339	0.488024	1.065282	5.860023e-05	0.262721	0.000268	0.000068
ATVI	0.000782	0.543113	1.187489	5.985418e-06	0.223022	0.000267	0.000086
COP	-0.000729	0.505391	1.247013	1.879335e-03	0.309123	0.000267	0.000096

100 rows × 7 columns

Figure 2 - max specific risk tilted portfolio

Min tot risk

By focusing on stocks with the lowest total risk, the portfolio aims to achieve more stable and predictable returns.
This approach prioritizes capital preservation and steady growth over the potential for higher, but more volatile, returns.

	alpha	alpha_pvalue	beta	beta_pvalue	r_squared	specific_risk	systematic_risk	tot_risk
Ticker								
PEP	0.000107	0.633178	0.591406	3.443843e-04	0.304761	0.000047	0.000021	0.000068
PG	-0.000088	0.559668	0.569423	1.849204e-03	0.267813	0.000054	0.000020	0.000074
RSG	0.000338	0.379069	0.668540	2.077454e-09	0.333353	0.000048	0.000027	0.000075
KO	-0.000095	0.492759	0.546744	6.025716e-03	0.253213	0.000057	0.000019	0.000076
WM	0.000343	0.448501	0.646297	8.529866e-08	0.307181	0.000051	0.000025	0.000076
...	...	...	...	...	...	...	...	...
SPG	-0.000278	0.500061	0.681102	2.325957e-02	0.233606	0.000104	0.000029	0.000133
MAA	0.000184	0.552925	0.565190	8.870724e-03	0.156496	0.000114	0.000019	0.000133
XL	-0.000125	0.455039	0.836943	1.118164e-03	0.336708	0.000089	0.000044	0.000133
ROP	0.000158	0.504475	1.083031	9.024310e-17	0.493176	0.000065	0.000068	0.000134
SJM	-0.000173	0.451220	0.675731	1.605620e-03	0.221283	0.000107	0.000028	0.000134

100 rows × 8 columns

Figure 3 - min tot risk tilted portfolio

max β

This tilted portfolio tries to maximize the portfolio's exposure to market movements. The portfolio aims to achieve higher returns during market upswings, as these stocks tend to outperform the market when it rises.
The approach takes a higher risk ,because of the increased volatility and risk of substantial losses during market downturns, for the potential of higher returns.

	alpha	alpha_pvalue	beta	beta_pvalue	r_squared	specific_risk	systematic_risk	tot_risk
Ticker								
FCX	-0.001448	0.481385	1.946223	6.066662e-07	0.224964	0.000988	0.000241	0.001229
URI	-0.000043	0.473107	1.931314	3.613741e-11	0.345788	0.000409	0.000222	0.000630
MU	0.000247	0.286127	1.799892	8.073737e-08	0.242899	0.000565	0.000193	0.000758
CHK	-0.002256	0.407070	1.769304	3.439421e-03	0.127972	0.001659	0.000215	0.001874
QRVO	-0.000614	0.638561	1.759877	2.243522e-08	0.242892	0.000575	0.000196	0.000771
...	...	...	...	...	...	...	...	...
MLM	0.000143	0.536437	1.303077	1.285353e-09	0.269968	0.000248	0.000101	0.000349
ADS	-0.000334	0.524366	1.301408	1.252290e-09	0.325635	0.000213	0.000102	0.000314
ROK	0.000023	0.561876	1.299003	7.214749e-15	0.454872	0.000104	0.000100	0.000204
LUK	-0.000666	0.299134	1.295310	4.004226e-13	0.424488	0.000122	0.000101	0.000223
MCHP	0.000209	0.591206	1.291668	4.112280e-12	0.387502	0.000140	0.000099	0.000239

100 rows × 8 columns

Figure 4 - max beta tilted portfolio

Sequential screening

max β -> max R^2 -> min tot risk

This tilted portfolio aims to maximize exposure to market movements, maintain a strong relationship with the index, and exhibit lower overall volatility, thereby targeting high returns while mitigating risk.

Ticker	alpha	alpha_pvalue	beta	beta_pvalue	r_squared	specific_risk	systematic_risk	tot_risk
ITW	0.000271	0.495926	1.070079	2.369949e-22	0.596980	0.000040	0.000067	0.000107
HON	0.000113	0.589950	1.065472	5.008669e-16	0.577885	0.000044	0.000066	0.000110
USB	-0.000047	0.614887	1.051847	1.991313e-13	0.521029	0.000052	0.000066	0.000118
APH	0.000288	0.553015	1.038665	3.937541e-16	0.477303	0.000067	0.000063	0.000130
ROP	0.000158	0.504475	1.083031	9.024310e-17	0.493176	0.000065	0.000068	0.000134
...	...	...	...	...	...	...	...	...
NSC	-0.000039	0.555112	1.198704	1.588334e-07	0.351213	0.000161	0.000085	0.000246
IR	-0.000199	0.524165	1.290619	3.642414e-10	0.434872	0.000147	0.000099	0.000247
SEE	0.000029	0.532292	1.169262	1.309658e-06	0.310528	0.000165	0.000082	0.000247
HBAN	-0.000103	0.541342	1.381368	2.395143e-12	0.440420	0.000132	0.000116	0.000248
KEY	-0.000112	0.471049	1.424780	5.766422e-10	0.441754	0.000136	0.000120	0.000256

100 rows × 8 columns

Figure 4 - max beta, max R, min tot risk tilted portfolio

min β -> max R^2 -> min tot risk

This tilted portfolio aims to minimize market exposure, maintain a strong relationship with the index, and exhibit lower overall volatility.
By combining these criteria, the portfolio seeks to achieve stable returns while effectively mitigating risk.

Ticker	alpha	alpha_pvalue	beta	beta_pvalue	r_squared	specific_risk	systematic_risk	tot_risk
PEP	0.000107	0.633178	0.591406	3.443843e-04	0.304761	0.000047	0.000021	0.000068
PG	-0.000088	0.559668	0.569423	1.849204e-03	0.267813	0.000054	0.000020	0.000074
RSG	0.000338	0.379069	0.668540	2.077454e-09	0.333353	0.000048	0.000027	0.000075
KO	-0.000095	0.492759	0.546744	6.025716e-03	0.253213	0.000057	0.000019	0.000076
WM	0.000343	0.448501	0.646297	8.529866e-08	0.307181	0.000051	0.000025	0.000076
...	...	...	...	...	...	...	...	...
COL	0.000213	0.568037	0.910439	3.824273e-03	0.384806	0.000112	0.000049	0.000161
AVY	0.000354	0.444196	1.020524	6.177785e-07	0.370424	0.000098	0.000062	0.000161
BLL	0.000156	0.614580	0.944951	1.751213e-07	0.313287	0.000109	0.000052	0.000161
CVX	-0.000407	0.439978	0.975715	2.297870e-04	0.357775	0.000105	0.000058	0.000163
DHR	-0.000092	0.533314	0.967685	1.929734e-07	0.399708	0.000108	0.000055	0.000163

100 rows × 8 columns

Figure 5 - min beta, max R, min tot risk tilted portfolio

Max specific risk -> min tot risk

This portfolio among the high-specific-risk stocks, further screens for those with the lowest total risk, ensuring that the overall portfolio volatility is minimized.
By combining these criteria, the portfolio seeks to capitalize on unique stock characteristics while maintaining overall stability, aiming for potentially higher returns from specific risks while effectively mitigating overall portfolio risk.

Ticker	alpha	alpha_pvalue	beta	beta_pvalue	r_squared	specific_risk	systematic_risk	tot_risk
FE	-0.000401	0.542957	0.519166	7.636893e-02	0.108845	0.000163	0.000018	0.000181
VTR	-0.000266	0.494283	0.508003	6.357508e-02	0.092231	0.000171	0.000019	0.000190
TGT	-0.000405	0.410457	0.694968	7.138765e-03	0.172387	0.000170	0.000028	0.000198
DRI	0.000181	0.538168	0.785937	1.057693e-03	0.178308	0.000167	0.000037	0.000204
ORLY	0.000275	0.411365	0.846426	1.189958e-02	0.257157	0.000170	0.000043	0.000213
...	...	...	...	...	...	...	...	...
DHI	0.000021	0.532591	1.309044	1.157170e-08	0.295357	0.000247	0.000102	0.000349
FLR	-0.000918	0.410438	1.386357	4.062200e-03	0.381220	0.000237	0.000114	0.000351
CTXS	-0.000289	0.565609	1.185680	6.312877e-04	0.273504	0.000266	0.000087	0.000353
ATVI	0.000782	0.543113	1.187489	5.985418e-06	0.223022	0.000267	0.000086	0.000353
LB	-0.000572	0.360583	1.037653	2.753428e-04	0.202065	0.000287	0.000067	0.000354

100 rows × 8 columns

Figure 6 - max specific risk, min tot risk tilted portfolio

Min tot risk-> max log returns

This tilted portfolio aims to minimize overall volatility by first selecting stocks with the lowest total risk, ensuring that the portfolio is stable and less exposed to market fluctuations. Among these low-risk stocks, it further screens for those with the highest logarithmic returns, targeting stocks that have demonstrated strong growth potential.

Ticker	close
PBCT	15382.5589
IPG	19819.1110
DRE	20033.2600
CNP	22555.1000
KIM	24819.6300
...	...
K	68196.0000
DGX	68395.6910
LLY	68628.7800
MDT	69000.4500
D	69605.9200

100 rows × 1 columns

Figure 7 - min tot risk, max log return tilted portfolio

Portfolios Analysis

The portfolios are designed to be equally weighted, they are weekly rebalanced to always have the same weights. By fixing the initial portfolio values at 100 at the start of the sample, it's possible to track and visualize the changes in values over time.

From the graph below it's possible to notice that the portfolio of stocks with the max specific risk, or the one with the red line, outperforms the others by a great margin from the beginning to mid 2015. However, from that point it started to underperform and only a year later started to perform like the others.

The green line is the portfolio of stocks with max specific risk and min total risk. It's the one that performs quite well: it only underperforms for a short period of time but in the end it's the second best performing portfolio.

It's interesting to notice the orange portfolio, which is made up of stocks with the highest beta and R^2. From 2013 to mid 2017 it performed like the others, but in the last six months its value rose sharply and ended up being the best one.
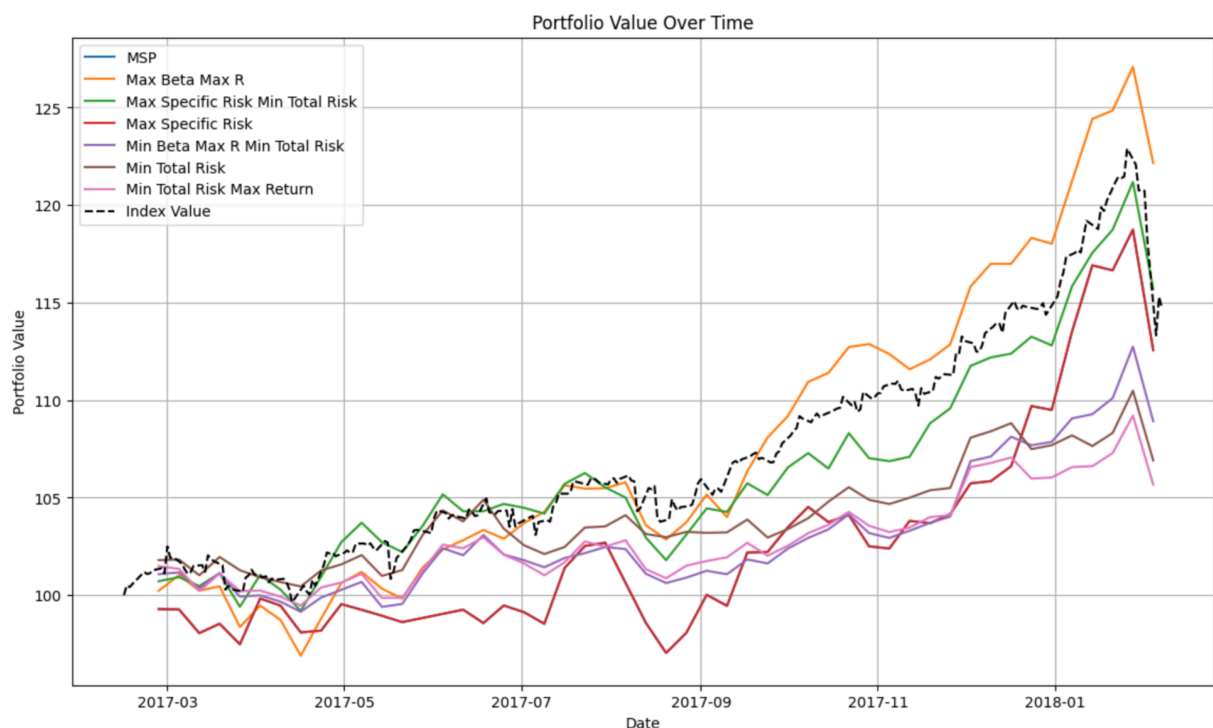

Figure 8 - Portfolios performance time series chart

Conclusions

The table below presents a detailed comparison of several portfolios, highlighting their respective returns, volatilities, and the ratio between the two in 2017. This comparative analysis allows investors to identify which portfolios offer the most favorable balance of risk and return, guiding informed investment decisions. The ratio of returns to volatility, is a metric used to understand the risk-adjusted performance of a portfolio. A higher ratio suggests that the portfolio provides a better return for a given level of risk, making it more attractive to investors.

Portfolio	Return	Volatility	Ratio
Max specific risk	13.37%	10.91%	1.23
Max beta, max R	21.89%	9.20%	2.38
Max specific risk, min total risk	14.87%	9.16%	1.62
Min beta, max R, Min total risk	7.76%	6.73%	1.15
Min total risk	5.02%	6.53%	0.77
Min total risk, max return	4.12%	6.35%	0.65
S&P 500	14.7%	3.82%	3.84

The statistical measurements were applied to the data not used in the rolling regression, that's the year 2017.

The portfolio with the highest ratio ultimately had the maximum beta, the highest R, and also achieved the greatest return.

We observed that even the portfolio designed to minimize total risk on the 4 years, that we analyzed to estimate the coefficients, exhibits greater volatility than the index. Therefore, when evaluating portfolios with new data, the index remains the superior choice in terms of ratio between the return and the volatility.

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