

# A NUMERICAL EXERCISE

2020/3/30

NTU ESOE 5136

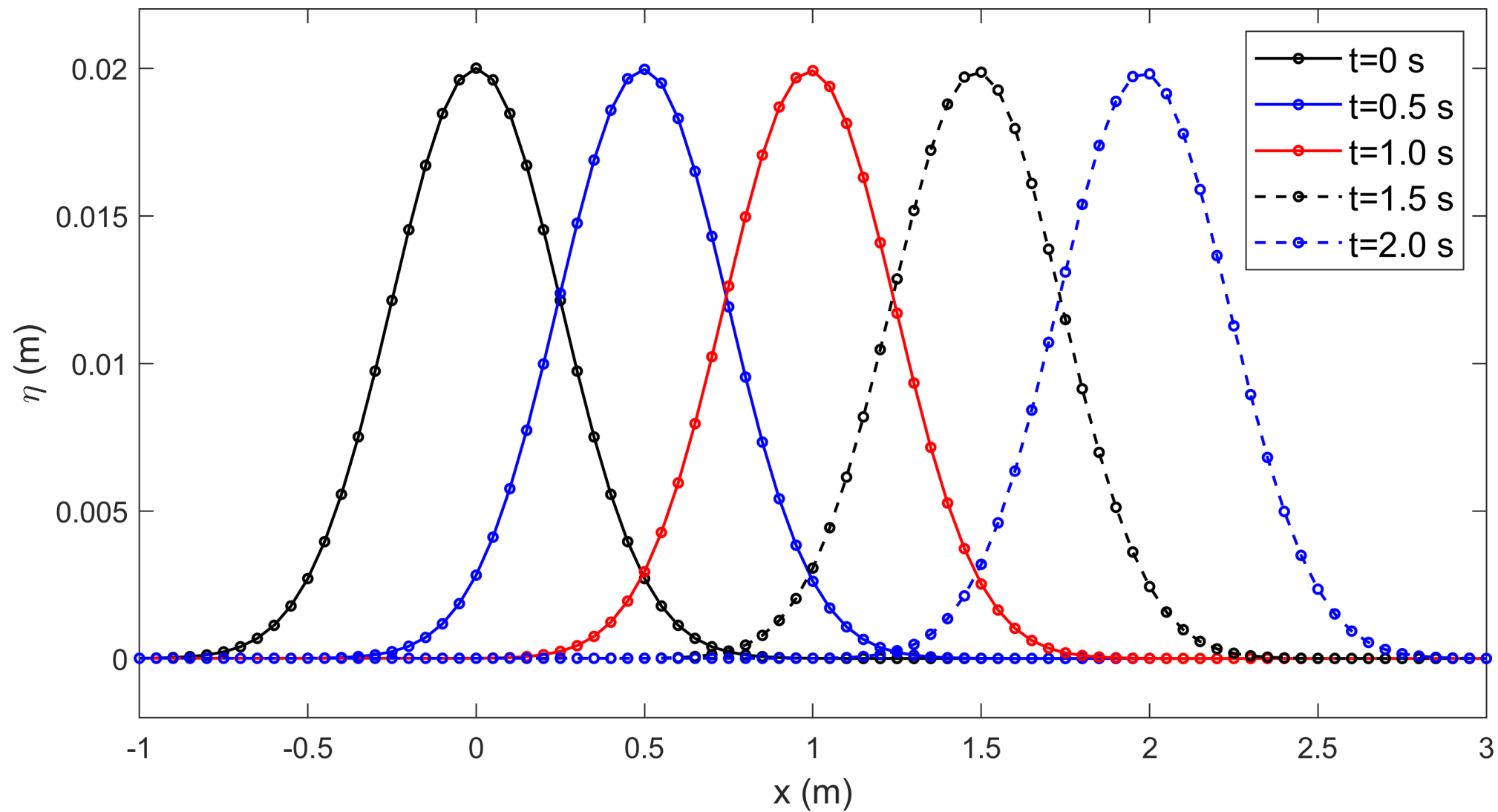
Use the SSP-RK scheme to solve the 1DH LSWE:

$$\left\{ \begin{array}{l}
 \text{First round:} \\
 \eta_i^{(*)} = \eta_i^{(n)} - \frac{\Delta t}{12\Delta x} \left( -U_{i+2}^{(n)} h_{i+2} + 8U_{i+1}^{(n)} h_{i+1} - 8U_{i-1}^{(n)} h_{i-1} + U_{i-2}^{(n)} h_{i-2} \right) \\
 U_i^{(*)} = U_i^{(n)} - \frac{\Delta t}{12\Delta x} g \left( -\eta_{i+2}^{(n)} + 8\eta_{i+1}^{(n)} - 8\eta_{i-1}^{(n)} + \eta_{i-2}^{(n)} \right) \\
 \\
 \text{Second round:} \\
 \eta_i^{(**)} = \frac{3}{4}\eta_i^{(n)} + \frac{1}{4}\eta_i^{(*)} - \frac{1}{4} \frac{\Delta t}{12\Delta x} \left( -U_{i+2}^{(*)} h_{i+2} + 8U_{i+1}^{(*)} h_{i+1} - 8U_{i-1}^{(*)} h_{i-1} + U_{i-2}^{(*)} h_{i-2} \right) \\
 U_i^{(**)} = \frac{3}{4}U_i^{(n)} + \frac{1}{4}U_i^{(*)} - \frac{1}{4} \frac{\Delta t}{12\Delta x} g \left( -\eta_{i+2}^{(*)} + 8\eta_{i+1}^{(*)} - 8\eta_{i-1}^{(*)} + \eta_{i-2}^{(*)} \right) \\
 \\
 \text{Third round:} \\
 \eta_i^{(n+1)} = \frac{1}{3}\eta_i^{(n)} + \frac{2}{3}\eta_i^{(**)} - \frac{2}{3} \frac{\Delta t}{12\Delta x} \left( -U_{i+2}^{(**)} h_{i+2} + 8U_{i+1}^{(**)} h_{i+1} - 8U_{i-1}^{(**)} h_{i-1} + U_{i-2}^{(**)} h_{i-2} \right) \\
 U_i^{(n+1)} = \frac{1}{3}U_i^{(n)} + \frac{2}{3}U_i^{(**)} - \frac{2}{3} \frac{\Delta t}{12\Delta x} g \left( -\eta_{i+2}^{(**)} + 8\eta_{i+1}^{(**)} - 8\eta_{i-1}^{(**)} + \eta_{i-2}^{(**)} \right)
 \end{array} \right.$$

# CONDITIONS

1. constant water depth:  $h=0.1$  m
2.  $x = -1 : 0.05 : 3$  (length = 81;  $\Delta x = 0.05$ )
3.  $t = 0 : 0.05 : 2$  (length = 41;  $\Delta t = 0.05$ )
4.  $g = 9.81$
5. mirror BCs
6. ICs:  $\eta(x, 0) = 0.02e^{-8x^2}, \quad U(x, 0) = \frac{\eta(x, 0)}{h} \sqrt{gh}$

# RESULTS



First round:

$$\eta_i^{(*)} = \eta_i^{(n)} - \frac{\Delta t}{12\Delta x} \left( -U_{i+2}^{(n)}h_{i+2} + 8U_{i+1}^{(n)}h_{i+1} - 8U_{i-1}^{(n)}h_{i-1} + U_{i-2}^{(n)}h_{i-2} \right)$$

$$U_i^{(*)} = U_i^{(n)} - \frac{\Delta t}{12\Delta x} g \left( -\eta_{i+2}^{(n)} + 8\eta_{i+1}^{(n)} - 8\eta_{i-1}^{(n)} + \eta_{i-2}^{(n)} \right)$$

Second round:

$$\eta_i^{(**)} = \frac{3}{4}\eta_i^{(n)} + \frac{1}{4}\eta_i^{(*)} - \frac{1}{4} \frac{\Delta t}{12\Delta x} \left( -U_{i+2}^{(*)}h_{i+2} + 8U_{i+1}^{(*)}h_{i+1} - 8U_{i-1}^{(*)}h_{i-1} + U_{i-2}^{(*)}h_{i-2} \right)$$

$$U_i^{(**)} = \frac{3}{4}U_i^{(n)} + \frac{1}{4}U_i^{(*)} - \frac{1}{4} \frac{\Delta t}{12\Delta x} g \left( -\eta_{i+2}^{(*)} + 8\eta_{i+1}^{(*)} - 8\eta_{i-1}^{(*)} + \eta_{i-2}^{(*)} \right)$$

Third round:

$$\eta_i^{(n+1)} = \frac{1}{3}\eta_i^{(n)} + \frac{2}{3}\eta_i^{(**)} - \frac{2}{3} \frac{\Delta t}{12\Delta x} \left( -U_{i+2}^{(**)}h_{i+2} + 8U_{i+1}^{(**)}h_{i+1} - 8U_{i-1}^{(**)}h_{i-1} + U_{i-2}^{(**)}h_{i-2} \right)$$

$$U_i^{(n+1)} = \frac{1}{3}U_i^{(n)} + \frac{2}{3}U_i^{(**)} - \frac{2}{3} \frac{\Delta t}{12\Delta x} g \left( -\eta_{i+2}^{(**)} + 8\eta_{i+1}^{(**)} - 8\eta_{i-1}^{(**)} + \eta_{i-2}^{(**)} \right)$$

h = 0.1, g = 9.81	x = -1:0.05:3, t = 0:0.05:2
left boundary, i=L $\begin{cases} \eta_{L-1} = \eta_{L+1} \\ \eta_{L-2} = \eta_{L+2} \end{cases} \begin{cases} U_{L-1} = -U_{L+1} \\ U_{L-2} = -U_{L+2} \end{cases}$	right boundary, i=R $\begin{cases} \eta_{R+1} = \eta_{R-1} \\ \eta_{R+2} = \eta_{R-2} \end{cases} \begin{cases} U_{R+1} = -U_{R-1} \\ U_{R+2} = -U_{R-2} \end{cases}$
$\eta_i^{(n=1)} = 0.02e^{-8x_i^2}$	$U_i^{(n=1)} = \frac{\eta_i^{(n=1)}}{h} \sqrt{gh}$

POSSIBLE CODE STRUCTURE

initialization

start t-loop

define temporary variables

set ghost cells / BCs

first SSP-RK x-loop

set ghost cells / BCs

second SSP-RK x-loop

set ghost cells / BCs

third SSP-RK x-loop

end t-loop