Assignment (due online on Sunday, 2020/3/8)

Please type up your work as a report, and upload to NTUCOOL as a pdf file. Remember to label all axes in your figures. Each figure should be assigned a figure number (for example, Figure 1), and each figure should be properly referred to in text (for example: "the results are plotted in Figure 1"). Attach the computer code you used to make these plots as appendices (附錄) at the end of your report. Try to explain your code concisely by using comments in your code.

1. What programming language do you plan to use in this course?

2. What platform do you plan to use for coding exercises in class (bring your own laptop/tablet, or use your smartphone)? If you plan to use your smartphone, what app or online computing engine will you use?

3. For a periodic square wave with a wavelength of L_0 and a wave amplitude of A_0, defined as

$$f(x) = \begin{cases} A_0, & \text{for } 0 \leq x < L_0/2 \\ -A_0, & \text{for } L_0/2 \leq x < L_0 \end{cases}, \tag{1}$$

its Fourier series can be expressed as

$$f(x) = \frac{4A_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\left(\frac{(2n-1)2\pi}{L_0} x\right). \tag{2}$$

For $A_0 = 0.5$ m and $L_0 = 1$ m, write a program to compute this Fourier series using the first 3 terms (i.e., sum from $n = 1$ to $n = 3$), the first 10 terms (i.e., sum from $n = 1$ to $n = 10$), and the first 50 terms (i.e., sum from $n = 1$ to $n = 50$). Compare the three resulting approximate square waves in the same plot. Your results should look similar to Figure 1.

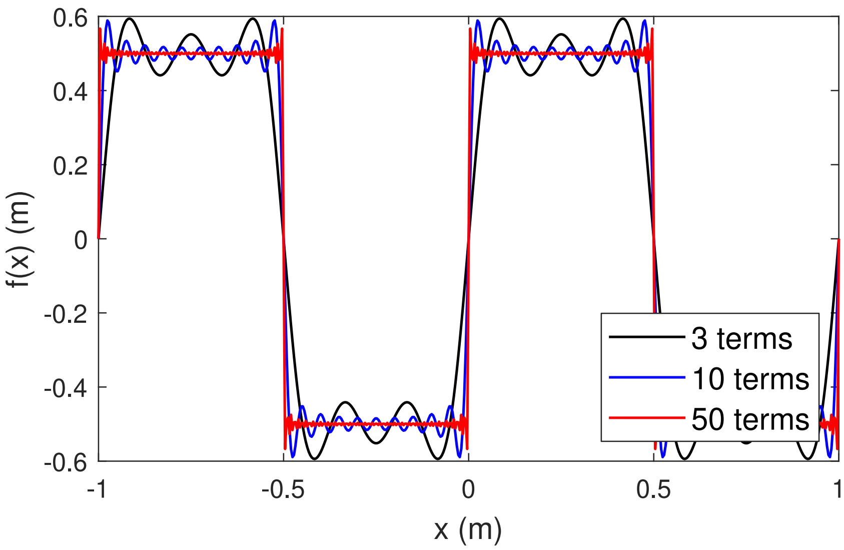

Figure 1: Reference plot for Problem 3.

4. Now, consider the approximate square wave using only the first 3 terms in (2), again with $A_0 = 0.5$ m and $L_0 = 1$ m. We see that this wave essentially consists of 3 sine waves

added together, and we can modify the expression so that it becomes a solution under the small-amplitude wave theory:

$$\eta(x,t) = \frac{4A_0}{\pi} \sum_{n=1}^{3} \frac{1}{2n-1} \sin(k_n x - \omega_n t), \tag{3}$$

where

$$k_n = \frac{(2n-1)2\pi}{L_0}, \tag{4}$$

and

$$\omega_n = \sqrt{g k_n \tanh(k_n h_0)} \tag{5}$$

is required by the dispersion relation, and $h_0 = 0.05$ m is the water depth. Since each of the three sine waves has a different wavelength, each of them propagates at a different speed. Therefore, as time goes by, the wave shape will not remain the same – it shall decompose into a series of waves.

Write a program to compute (3) at $t = 0$ s, $t = 0.25$ s, and $t = 0.5$ s, and show the results in the same plot. Your results should look similar to Figure 2.

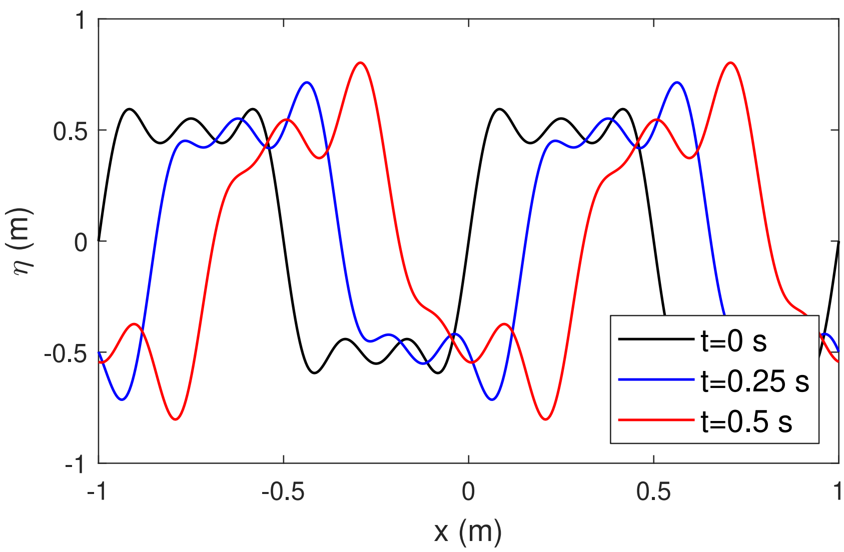

Figure 2: Reference plot for Problem 4.