

Assignment (due online on Sunday, 2020/5/17)

不用寫報告，線上繳交要求結果圖和程式碼即可，這次的作業成果會在下次的作業中用到。

For this assignment, you **do not** need to write a report. Just make sure your code works, generate the required plots, and attach your code. The two functions you write in this assignment will be used in the next assignment.

1. Implement a function that performs the MUSCL data reconstruction.

The function should take a vector (which represents cell-averaged data calculated by a numerical model) as input, such as $\bar{u}_i, \bar{u}_{i+1}, \dots$, and output the intercell values, such as $u_{i-1/2}^+, u_{i-1/2}^-, u_{i+1/2}^+, u_{i+1/2}^-, \dots$.

As a way to test your MUSCL reconstruction, you can try the following inputs:

$$\vec{u} = [0, 0, 1, 4, -4, 0, 0, 5, 4, 2, -1, 2]. \quad (1)$$

In return, you should get something like

$$\begin{aligned} \vec{u}^+ &= [0, 0, 0.25, 4.00, -4.00, 0, 0, 5.00, 4.67, 3.20, -1.00, 0.50, 0] \\ \vec{u}^- &= [0, 0, 0, 1.75, 4.00, -4.00, 0, 0, 5.00, 3.33, 0.80, -1.00, 3.50] \end{aligned} \quad (2)$$

A plot of the results is shown in Figure 1

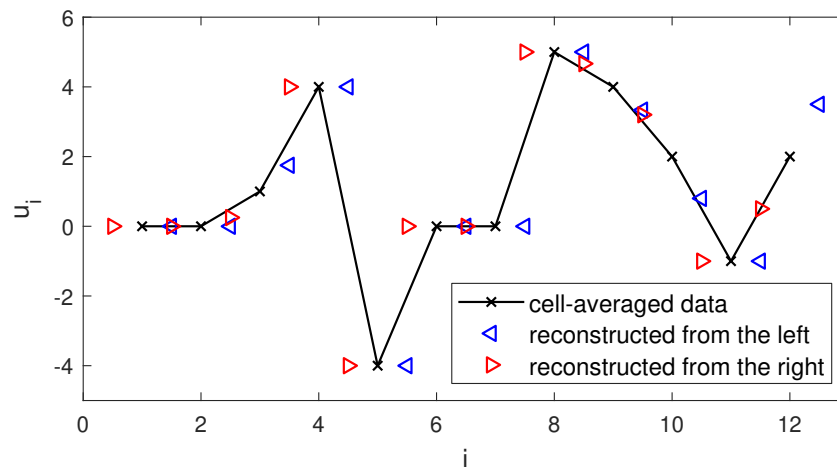


Figure 1: Test results of my MUSCL data reconstruction function.

Question to think about: how do you want to implement the boundary conditions or the ghost cells? You can do anything you want in this assignment. However, keep in mind that in the next assignment when you use this function in your numerical model, you need to deal with the boundary conditions somehow.

2. Run your code for the inputs

$$\vec{u} = [8, 4, 1, 0, 0, 0, 4, 3, 0, -5, -5, 2]. \quad (3)$$

Write down your results for \vec{u}^+ and \vec{u}^- , show a plot like Figure 1, and attach your code.

3. Implement a function that performs as the HLLC approximate Riemann solver (see the handout for 2020/5/11).

For example, the function may take the following as inputs (for $i = 1, 2, 3, \dots$):

$$\left\{ \begin{array}{l} H_{i-1/2}^- \\ H_{i-1/2}^+ \\ U_{i-1/2}^- \\ U_{i-1/2}^+ \end{array} \right. \text{ and } \left\{ \begin{array}{l} [HU]_{i-1/2}^- \\ [HU]_{i-1/2}^+ \\ H_{i-1/2}^-(U_{i-1/2}^-)^2 + \frac{1}{2}g\left((\eta_{i-1/2}^-)^2 + 2\eta_{i-1/2}^-h_{i-1/2}\right) \\ H_{i-1/2}^+(U_{i-1/2}^+)^2 + \frac{1}{2}g\left((\eta_{i-1/2}^+)^2 + 2\eta_{i-1/2}^+h_{i-1/2}\right) \end{array} \right. , \quad (4)$$

and output the intercell numerical fluxes $F_{i-1/2}$ and $G_{i-1/2}$ (for $i = 1, 2, 3, \dots$).

To test your HLLC approximate Riemann solver, you can try the following:

$$\left\{ \begin{array}{l} \vec{\eta}^- = [0.01, 0.01, 0.26, 0.32, -0.23, -0.23, -0.09] \\ \vec{\eta}^+ = [0.01, 0.10, 0.32, -0.12, -0.23, -0.09, -0.09] \\ \vec{U}^- = [0.02, 0.08, 0.35, 0.41, -0.31, -0.32, 0.02] \\ \vec{U}^+ = [-0.02, 0.16, 0.41, -0.21, -0.32, -0.17, -0.02] \\ h = 8 \text{ (constant)} \end{array} \right. . \quad (5)$$

Compute the relevant quantities, feed the quantities into your HLLC solver, and the outputs should be approximately:

$$\left\{ \begin{array}{l} \vec{F} = [0.00, 0.56, 2.87, 2.87, -2.45, -2.51, -0.00] \\ \vec{G} = [2.21, 1.51, 21.93, 31.76, -16.68, -16.89, -5.63] \end{array} \right. . \quad (6)$$

4. Run your code for the inputs

$$\left\{ \begin{array}{l} \vec{\eta}^- = [-0.01, -0.01, -0.33, -0.40, 0.28, 0.29, 0.13] \\ \vec{\eta}^+ = [-0.01, -0.13, -0.40, 0.17, 0.29, 0.13, 0.13] \\ \vec{U}^- = [-0.02, -0.09, -0.38, -0.45, 0.36, 0.36, -0.02] \\ \vec{U}^+ = [0.02, -0.18, -0.45, 0.28, 0.36, 0.20, 0.017] \\ h = 10 \text{ (constant)} \end{array} \right. . \quad (7)$$

Write down your results for \vec{F} and \vec{G} , and attach your code.