Compsci 120	The University of Auckland
Assignment 1	
Due: Friday, March 20th, by 23.59pm	*Semester 1, 2020*

This assignment covers the material in the first chapter of your coursebook. There are **four** problems listed below. To get full credit on this assignment, pick any **three** of them to complete! If you submit more than three problems, markers will grade the **first** three problems written problems and ignore the others.

All problems are worth the same number of points; complete and accurate answers to any three problems below will grant you full credit on this assignment. If you are stuck or confused by any of the problems, feel free to post on Piazza! While you cannot post questions like

- "hey guys what's the answer to 1(a)," or
- "did anyone else think that the answer to 2(b) was 24,"

you **can** ask questions like

- "I don't think I fully get how to use the % operator, can you give some examples", or
- "what's a check digit," or even "I'm totally stuck on 3(b), where do I even begin?"

Show all working; problems that do not show their work will receive reduced marks. Once you're done with it, upload your completed assignment as a single PDF to Canvas before the due date (go to Assignments/Assignment 1 and press "Submit Assignment" button).

Please note that late assignments cannot be marked under any circumstances. (With that said: if you find yourself unable to complete this assignment due to illness, injury, or personal/familial misfortune, please email us as soon as possible. We can come up with solutions to help you succeed in CS120!)

Best of luck, and enjoy the problems!

1. *(Integers: primes, divisibility, parity.)*
   (a) Let n be a positive integer. Prove that two numbers $n^2 + 3n + 6$ and $n^2 + 2n + 7$ cannot be prime at the same time.
   (b) Find $15261527863698656776 7^{12345678}\%5$ without using a calculator.
   (c) Let a be an integer number. Suppose $a\%2 = 1$. Find all possible values of $(4a + 1)\%6$.

2. *(Integers: $\%, \equiv$)*
   (a) Suppose a, b, n are integer numbers and $n > 0$. Prove that
   $$(a + b)\%n = (a\%n + b\%n)\%n.$$
   (b) Let a, b, n be integer numbers and $n > 0$. Suppose $a\%n \geq b\%n$. Prove that
   $$a\%n - b\%n = (a - b)\%n.$$
   (c) Consider the following claim:

   Claim 1. *Suppose a, b, n are integer numbers and $n > 0$. Then*
   $$a\%n - b\%n = (a - b)\%n.$$
   Show that this claim is not correct. (Hint: To prove that this claim is not correct you need to find an example of integer values a, b, n, where $n > 0$, but $a\%n - b\%n \neq (a - b)\%n$.)

(d) Let a, b, n be integer numbers and $n > 0$. Suppose $a\%n = b\%n$. Prove that

$$a\%n - b\%n = (a - b)\%n = (b - a)\%n.$$

3. *Integers: %*

   A number is a **Universal Product Code** (UPC) if its last digit agrees with the following computations:

   - The sum of the odd position digits (not including the last) is M. That is we add the first digit to the third digit to the fifth digit etc.
   - The sum of the even position digits (not including the last) is N.
   - $c = (3M + N)\%10$.
   - If $c = 0$ then the check digit is 0.
   - If $c \neq 0$, then the check digit is $10 - c$.

   (a) Check whenever 1928467 is a UPC.

   (b) Suppose you are given a 7 digit number which is a UPC. Prove that if a mistake is made when scanning the number, causing one digit to be read incorrectly, then you will be able to tell that an error has been made. **Hint:** use results from question 2.

   (c) Find an example of two 7-digit UPCs that have equal last digits, and disagree with each other at exactly two of the other digits numbers.

4. *Rational, irrationals*

   (a) Prove that $\log_3(2^7) - 5$ is irrational.

   (b) Let x and y be two positive integer numbers. Prove that $\frac{1}{x} + y$ can never be equal to $\frac{y}{x}$.