COMPSCI 120 Assignment One

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Question 1

a.

b.

Find $152615278636986567767^{12345678} \bmod 5$

$$
\begin{aligned}
152615278636986567767^{12345678} \bmod 10 &= 7^{12345678} \\
(7^2) \bmod 10 = 49 \bmod 10 &= 9 \bmod 10 \\
(7^4) \bmod 10 = (7^2 \cdot 7^2) \bmod 10 = (9 \cdot 9) \bmod 10 &= 1 \bmod 10 \\
12345678 &= 308669 \cdot 4 + 2 \\
(7^{4k} \cdot 7^2) \bmod 10 = (1 \cdot 9) \bmod 10 &= 9 \\
9 \bmod 5 &= 4 \\
152615278636986567767^{12345678} \bmod 5 &= 4
\end{aligned}
$$

c.

Suppose $a \bmod 2 = 1$, *find all possible values of* $(4a + 1) \bmod 6$

$$
\begin{aligned}
a \bmod 2 &= 1 \\
a &= 2k + 1 \\
(4a + 1) \bmod 6 & \\
(8k + 5) \bmod 6 & \\
(A + B) \bmod C = (A \bmod C + B \bmod C) \bmod C & \\
(8k + 5) \bmod 6 = (2k + 5) \bmod 6 & \\
2k + 5 \bmod 6 = ? &
\end{aligned}
$$

$k = -1$	3	
$k = 0$	5	
$k = 1$	1	
$k = 2$	3	Solutions are $\{1, 3, 5\}$
$k = 3$	5	
$k = 4$	1	
$k = 5$	3	

Question 2

a.

b.

c.

d.

Question 3

a.

Check whenever 1928467 is a UPC

There is only one condition for n to be a UPC.

$$c = (3M + N)\%10$$

Where:

- The sum of the odd position digits (not including the last) is M
- The sum of the even position digits (not including the last) is N

So for 1928467:

$$\begin{aligned} M &= 1 + 2 + 4 = 7 \\ N &= 9 + 8 + 6 = 23 \\ \therefore C &= (3 \cdot 7 + 23)\%10 = 4 \\ C &\neq 0 \\ 7 &\neq 10 - 4 \end{aligned}$$

$\therefore$ 1928467 is not a UPC

b.

c.

Question 4

a.

b.