Dynamics Summary

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Kinematics of Particles

The 7 Holy Steps

Here are Hazim's 7 holy steps for kinematics of particles:

1. Identify provided information and what is required
2. Sketch the motion and identify points of interest on the path
3. Choose coordinate system
4. Construct the kinematic conditions table
5. For each interval, identify the type of acceleration and choose the appropriate equations to solve
6. Solve equations for required information
7. Check that your answers make sense

Ensure that when you integrate it is definite integration (with limits) because we will almost always know the limits and it's preferred over indefinite integration. Additionally, ALWAYS draw a FBD for a question (or maybe a sketch of the motion) and note that you might want to draw a kinematics condition table for every part of a question as the conditions change.

Rectilinear Motion

Also called linear motion, rectilinear motion is motion in 1 dimension i.e a straight line. These kinematic equations form the basis of rectilinear motion:

$$v = \frac{ds}{dt},\ a = \frac{dv}{dt},\ s = v\frac{ds}{dt}$$

If acceleration is constant, we can derive these equations by integration:

$$v - v_0 = a(t - t_0) \tag{1}$$
$$v^2 - v_0^2 = 2a(s - s_0) \tag{2}$$
$$s - s_0 = v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \tag{3}$$

The general approach for a rectilinear motion question is detailed in the 7 holy steps but for the solution steps we will use any of the equations above (and maybe others, depending on the question). Acceleration (a) will be one of four things: constant, a function of time, a function of velocity or a function of displacement. Each of these have different methods.

1. Const.: use the above equations for constant velocity to solve
2. a(t): solve $a = \frac{dv}{dt}$
3. a(v): solve $a = \frac{dv}{dt}$ or $vdv = ads$ depending on what you are solving for
4. a(s): solve $vdv = ads$

Plane Curvilinear Motion

Simple Projectile Motion

Simple Projectile Motion is for objects with no thrust or other forces. It is 'simple'. The primary assumptions are: $a_x = 0$ and $a_y = -9.81$ (a_y might changed based on where you put the positive y etc). The holy steps are the same as for rectilinear motion with just a few additional considerations. We know must handle 2 dimensions. This means we consider for v (for example) both v_x and v_y. This will be the same for s, for a etc.

Circular Motion

Circular Motion is a special case of curvilinear motion involving motion about a fixed point with constant radius. These rotational motion equations will be important:

$$v = r\omega \tag{4}$$

$$a_n = r\omega^2 = \frac{v^2}{r} \tag{5}$$

$$a_t = r\alpha \tag{6}$$

Normal Tangential Co-ordinates
It is almost always more convenient to use Normal Tangential (n-t) co-ordinates in circular motion problems. n-t co-ordinates place the origin at the particle, this means that we can't talk about displacement / position but we can more easily keep track of a particles velocity and acceleration when considering a_t and a_n. The normal component is responsible for changing the direction of the vector. It will always act towards the center and will always be positive (just look at the equation). The tangential component is responsible for changing the magnitude of the vector. It will always act 'tangentially' i.e touching the circle at the particle and in the *direction of motion* If the velocity of the particle is constant then the tangential acceleration is 0.

1 Kinetics of Particles