Modelling Process Notes

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Modelling Process

Factors

Anything that will have some effect on the your calculations e.g. drag, friction, mass, buoyancy, area. Note that separate things qualities will count separately e.g. a triangle's area and a square's.

Assumptions

A statement that makes the problem simpler - cancels factors. e.g. 'Total length of wood is not reduced when it is cut' or 'There are no significant currents'

Precise Problem Statement

"Given [KEY FACTORS AND ASSUMPTIONS], Find [THE VALUE YOU'RE ASKED TO FIND]"

Formulating a Model

$x \propto y, 1/z \implies x = \frac{ky}{z}$

Modelling Forces

Use Newton's 2nd law, subtract negative forces and add positive ones. Use the ones from the list.

1 Ordinary Differential Equations

Ordinary differential equations are equations containing one or more functions of one independent variable. You can recognise an ODE from a PDE (partial differential equation) because a PDE will contain ∂ (pronounced 'del') and ODEs have standard 'd'. $\frac{dy}{dx}$ means y is the dependent variable and x is the independent variable. $\frac{dx}{dt}$ x is dependent, t is independent.

Properties

Order

Highest derivative (also equal to number of values needed to find a particular solution) e.g.

$$\frac{dy}{dx} = 5x \text{ 1st Order}$$
$$\frac{d^4y}{dx^4} = \frac{dy}{dx} + 2 \text{ 4th Order}$$

Linear

Involves only derivatives of y and terms of y to the 1st power e.g. ONLY $\frac{dy}{dx}$, y etc.

$$\frac{d^4y}{dx^4} + \frac{dy}{dx} = 2 \text{ is linear}$$
$$\frac{dy}{dx} = 2y + 3 \text{ is linear}$$

Homogeneity

If all (non-zero) terms involve the dependent variable then the equation is homogeneous

$$\frac{dx}{dt} = x \text{ is homogeneous}$$
$$\frac{dy}{dx} = 2y + 3 \text{ is not homogeneous (3 does involve x)}$$

Forming Differential Equations

In typical exam questions there are few points at which you will form a differential equation: modelling a set of forces in the typical modelling questions, using proportionality or previous knowledge. Typically the modelling questions will use Newton's 2nd law which states $\sum F = ma$ and then you can sum the forces and use it to find mass/acceleration (or their derivatives).

Solving Differential Equations

1. Direct Integration
2. Separation of Variables
3. Euler's Method
4. Integrating Factor

2 Probability