Intro to Statics

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March 12, 2020

Capacity must exceed demand

Statics is a subsection of mechanics that focuses on bodies that are not moving. Hence, it is the study of bodies at rest. Statics is largely derived from Newton's second and third laws that state:

- The relation between an object's mass, its acceleration, and the applied force, is: $F = ma$
- Forces of action and reaction are equal in magnitude and opposite in direction between two interacting rigid bodies.

1 Defining a Force

A force is "the action of one body upon another". Forces are vectors, they have both a magnitude and a direction. Alternatively, it could be described as "an interaction that, when unopposed, will cause an object to move". From Newton's second law $F = ma$, we can see that a force is a product of a vector (a) and a scalar (m) hence the direction of F is dependent on the direction of the acceleration. The units are as follows:

Acceleration	m/s^2
Mass	kg
Force	N
Force	$kg \cdot m/s^2$

2 Equilibrium

Newton's first law states that a rigid body will remain at rest or continue to move in a straight line if there are no unbalanced forces acting on it. This is when the object is said to be "at equilibrium". The forces on the object "sum to zero".

3 Moments

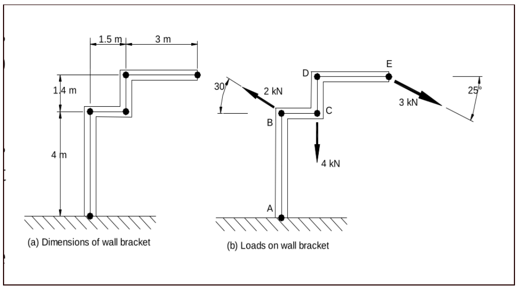
Moments are defined by: $M = Fd$. That is they are *force* $\times$ *perpendicular distance* Moments are the static equivalents of torque and have the same units Nm. Moments are the result of forces that turn a body, these are sometimes called *actions*.

3.1 Solving Moment Questions

Solving Moment Questions can be as easy as three (general) steps:

1. Decompose the force vectors into their x and y components
2. Find the distance from the reference point for each force vector
3. For each reference point, multiply the force components by their perpendicular distance and sum them

3.1.1 Example



Finding the moment around A

$$F_{2x} = 2\cos 30$$
$$F_{2y} = 2\sin 30$$
$$F_{4x} = 0$$
$$F_{4y} = -4$$
$$F_{3x} = 3\cos 25$$
$$F_{3y} = -3\sin 25$$
$$d_{2y} = 4m$$
$$d_{2x} = 0m$$
$$d_{4x} = 1.5m$$
$$d_{4y} = 4m$$
$$d_{3y} = 5.4m$$
$$d_{3x} = 4.5m$$
$$M_a = F_{2x} \cdot d_{2y} + F_{2y} \cdot d_{2x} + F_{4x} \cdot d_{4y} + F_{4y} \cdot d_{4x} + F_{3x} \cdot d_{3y} + F_{3y} \cdot d_{3x}$$
$$M_a = F_{2x} \cdot d_{2y} + F_{4y} \cdot d_{4x} + F_{3x} \cdot d_{3y} + F_{3y} \cdot d_{3x}$$
$$M_a = 2\cos 30 \cdot 4 + -4 \cdot 1.5 + 3\cos 25 \cdot 5.4 - 3\sin 25 \cdot 4.5$$
$$M_a = 19.46\,\mathrm{kNm}$$

3.2 Couples

A couple is two forces that have equal magnitude but opposite direction, but that do not have the same line of action. These forces sum to zero, but generate a moment. The value of a moment is simply the moments of the two forces subtracted from one another. Or, alternatively, The magnitude of the two forces times by the distance between them. IMPORTANT NOTE: Couples have never been in a 121 exam before.

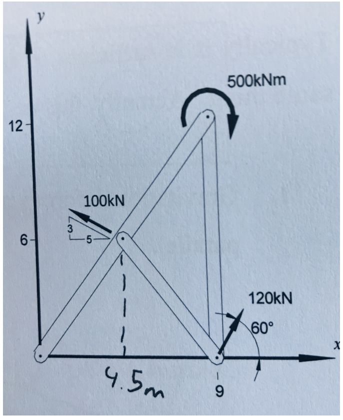
4 Resultants

In engineering it is more common to represent a set of forces acting on an object as its *resultant* instead of individual forces. Hence you have a single vector with a magnitude and angle that could represent multiple forces acting on an object. The resultant is simply the sum of all the forces acting on an object (adding the x and y components of course). The difficulty with these problems comes from finding where to place the resultant on an object.

4.1 Where to place the resultant

When beginning the problem, we chose a point of reference. Sensible points of reference will be the origin or a point where a force acts. Anywhere that you have data for will do but technically you can use any point. You can calculate x and y distances with $X = \frac{M_r}{R_y}$,$Y = \frac{M_r}{R_x}$. This is simply rearranging the moments formula but let's do an example.

4.2 Example



$$\theta = \arctan\frac{3}{5} = 30.96$$
$$R_x = 120\cos 60 - 100\cos 39.6$$
$$R_y = 100\sin 30.96 + 120\sin 60$$
$$|R| = \sqrt{R_x^2 + R_y^2} = 157.49\,\mathrm{kN}$$
$$\theta_r = \arctan\frac{R_y}{R_x} = 99.4$$
$$M_o = -500 + (120\sin 60 \cdot 9) + (100\sin 59.04 \cdot 4.5) + (100\cos 58.04 \cdot 6) = 1129\,\mathrm{kNm}$$
$$X = \frac{M_0}{R_y} = 7.27\,\mathrm{m}$$
$$Y = \frac{M_0}{R_x} = 43.88\,\mathrm{m}$$