Dynamics Summary

Alexander Bailey

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Kinematics of Particles

THE SEVEN HOLY STEPS

Here are Hazim's 7 holy steps for kinematics of particles:

1. Identify provided information and what is required
2. Sketch the motion and identify points of interest on the path
3. Choose coordinate system
4. Construct the kinematic conditions table
5. For each interval, identify the type of acceleration and choose the appropriate equations to solve
6. Solve equations for required information
7. Check that your answers make sense

Ensure that when you integrate it is definite integration (with limits) because we will almost always know the limits and it's preferred over indefinite integration. Additionally, ALWAYS draw a FBD for a question (or maybe a sketch of the motion) and note that you might want to draw a kinematics condition table for every part of a question as the conditions change.

Rectilinear Motion

Also called linear motion, rectilinear motion is motion in 1 dimension i.e a straight line. These kinematic equations form the basis of rectilinear motion:

$$v = \frac{ds}{dt},\ a = \frac{dv}{dt},\ s = v\frac{ds}{dt}$$

If acceleration is constant, we can derive these equations by integration:

$$v - v_0 = a(t - t_0) \tag{1}$$
$$v^2 - v_0^2 = 2a(s - s_0) \tag{2}$$
$$s - s_0 = v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \tag{3}$$

The general approach for a rectilinear motion question is detailed in the 7 holy steps but for the solution steps we will use any of the equations above (and maybe others, depending on the question). Acceleration (a) will be one of four things: constant, a function of time, a function of velocity or a function of displacement. Each of these have different methods.

1. Const.: use the above equations for constant velocity to solve
2. a(t): solve $a = \frac{dv}{dt}$
3. a(v): solve $a = \frac{dv}{dt}$ or $vdv = ads$ depending on what you are solving for
4. a(s): solve $vdv = ads$

Plane Curvilinear Motion

Simple Projectile Motion

Simple Projectile Motion is for objects with no thrust or other forces. It is 'simple'. The primary assumptions are: $a_x = 0$ and $a_y = -9.81$ (a_y might changed based on where you put the positive y etc). The holy steps are the same as for rectilinear motion with just a few additional considerations. We now must handle 2 dimensions. This means we consider for v (for example) both v_x and v_y. This will be the same for s, for a etc.

Circular Motion

Circular Motion is a special case of curvilinear motion involving motion about a fixed point with constant radius. These rotational motion equations will be important:

$$v = r\omega \tag{4}$$

$$a_n = r\omega^2 = \frac{v^2}{r} \tag{5}$$

$$a_t = r\alpha \tag{6}$$

Normal Tangential Co-ordinates
It is almost always more convenient to use Normal Tangential (n-t) co-ordinates in circular motion problems. n-t co-ordinates place the origin at the particle, this means that we can't talk about displacement / position but we can more easily keep track of a particles velocity and acceleration when considering a_t and a_n. The normal component is responsible for changing the direction of the vector. It will always act towards the center and will always be positive (just look at the equation). The tangential component is responsible for changing the magnitude of the vector. It will always act 'tangentially' i.e touching the circle at the particle and in the *direction of motion* If the velocity of the particle is constant then the tangential acceleration is 0.

Kinetics of Particles

Relative motion of particles

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

The way to remember this is that the first letter is what you are looking for and the second is what you are relative to. $v_{A/B}$ is read as "The velocity of A relative to B".

THE SIX HOLY STEPS

1. Identify provided vector quantities and what is required
2. State the relative motion to use
3. Construct the kinematic conditions table

4. Draw the velocity or acceleration diagram
5. Solve equations (using sine/cosine or by vector methods)
6. Check that your answers make sense

Drawing velocity/acceleration diagrams

General conventions are on page 51 of the coursebook. Start from O (for origin, which is just some point), $v_{A/B}$ starts at b and ends at a, try and draw to scale.

Kinematics of Rigid Bodies

There are three types of motion for R.B. (Rigid Bodies).

1. Pure Translation $\implies \alpha = \omega = 0$
2. Fixed-axis Rotation (F.A.R.)
3. General Plane Motion (combination of the above)

For a rigid body we can only have one ω, α but different linear velocities at points. In this course, we only look at velocities as relative accelerations of R.B. is hard. Additionally, these equations could be useful:

$$|\underline{v}_{A/B}| = |\omega \underline{r}_{A/B}| \tag{7}$$

$$\underline{v}_{A/B} = \omega \times \underline{r}_{A/B} \tag{8}$$

The direction of $\underline{v}_{A/B}$ will be normal to $\underline{r}_{A/B}$ in the direction of ω. The seven holy steps of kinematics of particles still apply. The words "Rolling without Slip" imply: if point 'A' is on the wheel for any instant, $\underline{v}_{A/Sr}$ is 0. Generally, $\underline{v}_{Sr}$ will be 0.

Kinetics of Rigid Bodies

The equation of motion that we have covered so far ($\sum F = ma$) relates forces to acceleration. In rotational motion, we can form the new equation:

$$\sum M_G = I_G \alpha \tag{9}$$

Some important things to note: General Plane Motion is not considered in this course.

THE SEVEN HOLY STEPS

1. Identify provided information and what is required
2. Draw a FBD, Note: Forces, Moments and Lines of Action
3. Choose coordinate system (including positive rotation)
4. State any kinematic constraints
5. Write equations of motions ($\sum F = ma$, $\sum M_G$)
6. Solve equations for required motions
7. Check that your answers make sense

Example conditions include:

1. Rolling without slip, velocity of the point touching the ground is zero
2. Pure Translational Motion, rotational acceleration is zero
3. Linking translational motion and rotational motion, some equation

The general addition in kinetics of rigid bodies is the use of moments to solve problems. Look at the statics notes for more on moments. Otherwise, look to kinetics of particles for more.