

# Compsci 120 Notes

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## Modulo

Modulo can be considered the 'remainder operator'. It is represented here with ' mod ' but appears (in the text book and elsewhere as '%')

## Congruence

$$a \bmod n \equiv b \implies a \bmod n = b \bmod n$$

$$\begin{aligned} a \bmod n &= b \bmod n \\ \implies a - b &= nk, k \in \mathbb{Z} \end{aligned}$$

This gives us the operations:

$$(a + c) \bmod n = (b + d) \bmod n \tag{1}$$

$$(ac) \bmod n = (bd) \bmod n \tag{2}$$

$$(a^k) \bmod n = (b^k) \bmod n \tag{3}$$

## Finding the last digit

The last digit of any decimal number  $a$  can be found by  $a \bmod 10$  for example,

Finding the last digit of  $213047^{129314}$

$$213047^{129314} \bmod 10$$

Observe,  $213047 \bmod 10 = 7$

$$\implies 213047^{129314} \bmod 10 = 7^{129314} \bmod 10 \text{ (from eqn. 3)}$$

Notice,  $(7^2) \bmod 10 = 49 \bmod 10 = 9$

Notice,  $(7^2 \cdot 7^2) \bmod 10 = 81 \bmod 10 = 1$

So for any  $k$ ,  $(7^{4k}) \bmod 10 = (7^2 \cdot 7^{4k}) \bmod 10 = 1$

Because  $129314 = 129300 + 12 + 2$

We have  $(7^{129314}) \bmod 10 = 9$

So the last character is 9

## Probability

Ordered means that order matters e.g. a pattern of letters. With repetitions means that repeated values will have separate outcomes. Remember that it is, "given  $n$  items choose  $k$ ".

### Ordered Choice WITH Repetitions

$$n^k$$

### Unordered Choice WITH Repetitions

$$\frac{(k-1+n)!}{k!(n+k-1)!} = \binom{n+k-1}{k}$$

### Ordered Choice without Repetitions

$$\frac{n!}{k!}$$

### Unordered Choice without Repetitions

$$\frac{n!}{k!(n-k)!} = \binom{n}{k}$$

## Functions

For a function to exist the domain of  $g$  must equal the co-domain of  $f$

## Graphs

## Proofs

### Direct Proof

A direct proof is when you take simple known facts, axioms and theorems and work them algebraically until you prove the thing you are looking for is true or false. For statements like 'p implies q' or 'if p then q' we can assume p is true and use our known facts/algebra to prove q.

### Proof by Contradiction

Say we have a fact p we want to prove is true. We assume p is false then reach a contradiction so in fact p must be true. Say we want to prove p implies q. Then we can assume p is true, but q is false. Then we reach a contradiction.

### Proof by Cases

Say we want to prove a statement in the form 'for all numbers/graphs/etc' but the 'numbers/graphs/etc' can be thought of as several families so we deal with each separately in different 'cases'. Eg. we could consider odd & even numbers.

### Proof by Construction

Proof by construction is a bit of an odd ball but can be brought down to simply producing an example of what you are trying to prove. If the statement is of the form "there exists x" it is a valid proof to just produce a numerical/physical example otherwise you must work in general i.e "a graph with n vertices".

## Proof by Induction