

CS 527/ECE 599 Error Correcting Codes  
Assignment No. 1, Due: Thursday January 14, 2021

1. In a transmission system a 0 is encoded as 00000 and 1 as 11111 and these bits are sent through the binary symmetric channel where the bit error probability is  $p$ . At the receiving end the decoding is done by majority voting. What is the probability of error  $P_E$  assuming  $p = 0.1$ ? When 0 is encoded as 0000000 and 1 as 1111111 and the decoding is done again by majority voting, what is the value of  $P_E$  for  $p = 0.1$ ?
2. A random number is selected uniformly from 0; 1; 2; 3; 4; 5; 6; 7 without replacement until 3 is chosen. Let  $X$  denote the number of selection. Find the entropy  $H(X)$  in bits.
3. A box of 50 semiconductor chips includes 3 defective ones. 5 chips are randomly chosen from this box. Let  $X$  denote the number of defective chips. Find the entropy of  $H(X)$  in bits.
4. World Series. The world series is a seven game-series that terminates as soon as either team wins four games. Let  $X$  be the random variable that represents the outcome of a World Series between teams  $A$  and  $B$ ; some possible values of  $X$  are  $AAAA$ ,  $ABABBB$ , and  $BABABAA$ . Let  $Y$  be the number of games played, which ranges from 4 to 7. Assuming that  $A$  and  $B$  are equally matched and the games are independent, calculate  $H(X)$  and  $H(Y)$ .
5. Erasure Channel. Consider the discrete memoryless channel as shown in Figure 1. Assuming  $P(X = 0) = \frac{2}{3}$  and  $P(X = 1) = \frac{1}{3}$ , and  $p = \frac{1}{4}$ , find
  - (a)  $H(X), H(Y)$
  - (b)  $H(Y/X), H(X/Y)$
  - (c)  $H(X, Y)$
  - (d)  $I(X, Y)$
6. A fair coin is flipped until the first head occurs. Let  $X$  denote the number of flips required. Find  $H(X)$ .

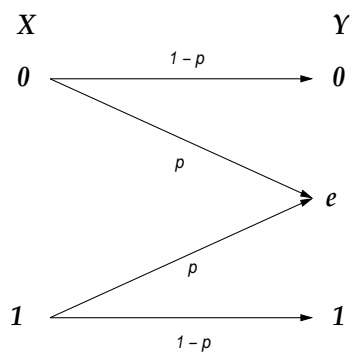


Figure 1: Erasure Channel