The Hong Kong Polytechnic University Department of Applied Mathematics

AMA1501 Introduction to Statistics for Business/ ${\rm AMA1602\ Introduction\ to\ Statistics}$ ${\rm 2023/24\ Semester\ Two}$

Assignment

Due date: 20 Mar 2024, 23:59

1. In order to study the age distribution of video gamers in the United States, 100 video gamers are randomly selected and interviewed. The results are given in the following table.

Age	Number of video gamers
0 to 17	24
18 to 34	36
35 to 44	13
45 to 54	12
55 to 64	9
65 to 80	6

Source: https://www.statista.com/statistics/189582/age-of-us-video-game-players/

(a) Calculate the mean, mode and standard deviation of the age distribution.

(9 marks)

- (b) Calculate the coefficient of skewness using the results in (a). Comment the skewness of the age distribution. (4 marks)
- (c) Describe the changes of the results in (a) if the same 100 video gamers are interviewed two years later. (3 marks)
- (d) Estimate, from the frequency distribution table, the minimum age of the oldest 20% of video gamers. (4 marks)
- (e) Estimate, from the frequency distribution table, the proportion of video gamers aged between 30 to 50 years. (5 marks)

- 2. (a) In a football team, there are 6 forwards, 8 midfielders, 7 defenders and 3 goal-keepers. The manager is selecting 11 players as a starting lineup for a match and adopting a 4-3-3 formation (3 forwards, 3 midfielders, 4 defenders and a goalkeeper). How many different starting lineups can be formed? (4 marks)
 - (b) Suppose that there are 7 envelopes, in which 4 of them contain a card and the remaining 3 are empty. An additional card is now put into one of these envelopes at random. After that the envelopes are shuffled and an envelope is selected. Find the probability that this envelope is empty. (4 marks)
 - (c) An electric circuit consists of two components, A and B. From the past testing result, the probability that only A fails is 0.3, the probability that B fails is 0.4, and the probability that both A and B fail is 0.2. Find the probability of B fails, given that A fails.

 (4 marks)
 - (d) A student is taking a test which consists of 3 questions. The probabilities that he answers each of the questions correctly are 0.3, 0.4 and 0.5 respectively. Find the probability that he answers at least 1 question correctly. (4 marks)
 - (e) The six faces of an unbiased die are labelled with "A, A, A, B, B, C" respectively. Meanwhile, there are three bags where bag A contains 3 white balls and 2 black balls, bag B contains 3 white balls and 4 black balls and bag C contains 4 white balls and 5 black balls. The die is rolled and one of the three bags is chosen according to the result of the roll. A ball is drawn from the chosen bag and it is a white ball. Find the probability that this ball is drawn from bag B. (9 marks)
- 3. (a) It is found that 5% of the distilled water bottles are contaminated. The distilled water bottles are packed into boxes of 5 each. A store owner has purchased 25 boxes of distilled water bottles.
 - i. Find the probability that less than 3 contaminated distilled water bottles are found in a box. (3 marks)
 - ii. Use suitable approximation to find the probability that more than 3 contaminated distilled water bottles are found in these 25 boxes. (5 marks)
 - iii. Use suitable approximation to find the probability that more than 15 boxes of distilled water bottles are having no contaminated distilled water bottle.

(5 marks)

- (b) A hospital uses an equipment 1.8 times per week on average. Assume that the usage of the equipment per week follows a Poisson distribution. Find the probabilities of the following events happening.
 - i. The equipment is used less than 3 times in a week. (4 marks)
 - ii. The equipment is used less than 3 times in each of three successive weeks.

(4 marks)

- iii. The equipment is used less than 3 times in a three-week period. (4 marks)
- 4. The weight of each bag of coffee beans sold by a shop follows a normal distribution with a mean of 510 g and a standard deviation of 18 g. A bag of coffee beans with weight less than 500 g is considered as below standard, and that with weight less than 485 g is considered as underweight. Dan buys 3 bags of coffee beans from the shop. He will lodge a complaint if at least 2 of them are below standard, and he will return a bag of coffee beans if it is underweight.
 - (a) Find the probability that a bag of coffee beans selected randomly is below standard but not underweight. (4 marks)
 - (b) It takes 20 g of coffee beans for a cup of coffee. Dan randomly selected one bag of coffee beans. Determine the maximum number of cups of coffee that he can made in 90% of time. (4 marks)
 - (c) Find the probability that the average weight of the 3 bags of coffee beans bought by Dan is less than 490 g. (5 marks)
 - (d) Find the probability that Dan will not lodge a complaint. (5 marks)
 - (e) Given that Dan will lodge a complaint, find the probability that he will return exactly 2 bags of coffee beans. (7 marks)