AMA1602

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Assignment 1

Q1)

Frequency Table:

I	Class Age	Class Boundary	Class Mark Xi	Freq fi	Cumulative Freq
1	0-17	-0.5-17.5	8.5	24	24
2	18-34	17.5-34.5	26	36	60
3	35-44	34.5-44.5	39.5	13	73
4	45-54	44.5-54.5	49.5	12	85
5	55-64	54.5-64.5	59.5	9	94
6	65-80	64.5-80.5	72.5	6	100

a)

Mean:

=fiXi/fi

= [(8.5 * 24) + (26 * 36) + (39.5 * 13) + (49.5 * 12) + (59.5 * 9) + (72.5 * 6)]/100

=(204+936+513.5+594+535.5+435)/100

=3218/100

=32.18

So the mean is 32.18

Mode:

36>24>13>12>9>6

So the mode is within range 18 to 34.

Mode

=17.5+(36-24)/[(36-24)+36-13]*(34-17)

=17.5+(12/35)*17

=23.3286

Standard Deviation of the age distribution:

=sqrt{[((8.5 - 32.18)^2 * 24) + ((26 - 32.18)^2 * 36) + ((39.5 - 32.18)^2 * 13) + ((49.5 - 32.18)^2 * 12) + ((59.5 - 32.18)^2 * 9) + ((72.5 - 32.18)^2 * 6)]/100}

=sqrt(35600.76/100)

=sqrt(356.0076)

=18.8682

Mean is 32.18. Mode is 23.3286. Standard deviation is 18.8682.

b)

Coefficient of skewness

=(mean-mode)/standard deviation

=(32.18-23.3286)/18.8682

=0.4691

The skewness of the age distribution is positively skewed.

c)

If the same 100 video gamers are interviewed two years later, the standard deviation and mode will all remain not change as everyone's age added 2 . The mean will increase 2 into 34.18.

d)

D8

=44.5+(80-73)/12 *10

=44.5+5.83

=50.3

From the Cumulative section of the frequency table, the minimum age of the oldest 20% of video gamers is 50.3.

e)

35 to 44=13 people

30 to 34= [(34.5-30)/(34.5 -17.5)]*36=9.529 people

45 to 50= [(50-44.5)/(54.5-44.5)]*12= 6.667 people

(13+6.667+9.529)/100=29.196%

So 29.196% of the video gamers are aged between 30-50 years old.

Q2)

a)

4-4-3-1 line up so

For the forwards:

There are 6 forwards, and we need to select 3.

C(6, 3) = 6! / (3! * (6-3)!) = 6! / (3! * 3!) = (6 * 5 * 4) / (3 * 2 * 1) = 20

For the midfielders:

There are 8 midfielders, and we need to select 3.

C(8, 3) = 8! / (3! * (8-3)!) = 8! / (3! * 5!) = (8 * 7 * 6) / (3 * 2 * 1) = 56

For the defenders:

There are 7 defenders, and we need to select 4.

C(7, 4) = 7! / (4! * (7-4)!) = 7! / (4! * 3!) = (7 * 6 * 5) / (3 * 2 * 1) = 35

For the goalkeeper:

There are 3 goalkeepers, and we need to select 1.

C(3, 1) = 3! / (1! * (3-1)!) = 3! / (1! * 2!) = (3) / (1) = 3

Total number of different starting lineups=

= 20 * 56 * 35 * 3

= 117,600

Therefore, there are 117,600 different starting lineups that can be formed.

b)

Scenario 1: The additional card is put into one of the empty envelopes.

In this case, there are two empty envelopes out of the seven. The probability of selecting an empty envelope is 2/7. Then probability of putting into an empty envelope is 3/7.

Scenario 2: The additional card is put into one of the envelopes that already contain a card. The probability of selecting an empty envelope is 3/7. Then probability of putting into a card envelope is 4/7.

Let Selecting empty card be Event A, then putting an empty card into envelope without card is event B, putting an empty card into envelope with card is event B'

$P(A) = P(A|B) \cdot P(B) + P(A|B') \cdot P(B')$

$=2/7 * 3/7 + 3/7 * 4/7$

=0.3673

Total Probability(envelope is empty) =0.3673

c)

P(A)=0.3

P(B)=0.4

P(A and B)=0.2

P(B/A)=P(A and B)/P(A)=0.2/0.3=2/3=0.667

The probability of B fails, given that A fails is 0.667.

d)

P(Q1)=0.3

P(Q2)=0.4

P(Q3)=0.5

P(no question answered correctly)

=(1-0.3)*(1-0.5)*(1-0.6)

=0.7*0.6*0.5

=0.21

P(answers at least one question correctly)

=1- P(no question answered correctly)

=1-0.21

=0.79

The probability that he answers at least 1 question correctly is 0.79.

e)

Probability of rolling result of the die:

P(R = A) = 3/6 = 1/2

P(R = B) = 2/6 = 1/3

P(R = C) = 1/6

The probabilities of drawing a white ball from each bag:

P(W | A) = 3/5

P(W | B) = 3/7

P(W | C) = 4/9

P (W)

= 3/7*1/3+1/6*4/9+3/5*1/2

= 977/1890

P(B/W)

=P(W/B)*P(B)/P(W)

=(3/7*1/3)/(977/1890)

=270/977

=0.2764

The probability that this ball is drawn from bag B is 0.2764.

Q3)

a)

P (contaminated bottle)= 0.05

P (non-contaminated bottle) = 1 - 0.05 = 0.95

i)

The probability that less than 3 contaminated distilled water bottles are found in a box equals to the probabilities of finding 0, 1, or 2 contaminated bottles. Set X as the number of contaminated bottles in a box.

$P(X < 3)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$=C(5, 0) * 0.05^0 * 0.95^5 + C(5, 1) * 0.05^1 * 0.95^4 + C(5, 2) * 0.05^2 * 0.95^3$

$=0.9988$

The probability that less than 3 contaminated distilled water bottles are found in a box is 0.9988.

ii)

$P(X > 3)$

$= 1 - P(X \leq 3)$

$=1-0.9988$

=0.0012

The probability that more than 3 contaminated distilled water bottles are found in these 25 boxes is 0.0012.

iii)

μ = 25 * 0.05 = 1.25

σ = sqrt(25 * 0.05 * 0.95) =1.085

P(X > 15)

= P(Z > (15 + 0.5 - μ) / σ)

=P(Z > (15 + 0.5 - 1.25) / 1.085)

=P(Z > 0.4018)

=0.3446

The probability that more than 15 boxes of distilled water bottles are having no contaminated distilled water bottle is 0.3446.

b)

i)

The probability the equipment is used less than 3 times in a week equals to the probabilities of the equipment is used 0,1,2 times a week. Set X as the number of usages of the equipment per week.

$\lambda = 1.8$

$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$P(X = 0) = (e^{\wedge}(-1.8) * 1.8^{\wedge}0) / 0! = e^{\wedge}(-1.8) = 0.1653$

$P(X = 1) = (e^{\wedge}(-1.8) * 1.8^{\wedge}1) / 1! = 1.8 * e^{\wedge}(-1.8) =0.2976$

$P(X = 2) = (e^{\wedge}(-1.8) * 1.8^{\wedge}2) / 2! = (1.8^{\wedge}2 * e^{\wedge}(-1.8)) / 2 = 0.2678$

$P(X < 3) = 0.1653 + 0.2976 + 0.2678 = 0.7307$

The probability that the equipment is used less than 3 times in a week is 0.7307.

ii)

$P(X < 3 \text{ in each of three weeks}) = (P(X < 3))^{\wedge}3 = 0.7307^{\wedge}3 = 0.3821$

The probability that the equipment is used less than 3 times in each of three successive weeks is 0.3821.

iii)

The probability the equipment is used less than 3 times in a three-week period equals to the probabilities of the equipment is used 0,1,2 times in a three-week period. Set Y as the number of usages of the equipment in a three-week period.

$\lambda(\text{Three week period}) = 1.8*3=5.4$

$P(X = 0) = (e^{(-5.4)} * 5.4^0) / 0! = e^{(-5.4)} = 0.0045$

$P(X = 1) = (e^{(-5.4)} * 5.4^1) / 1! = 5.4 * e^{(-5.4)} = 0.0243$

$P(X = 2) = (e^{(-5.4)} * 5.4^2) / 2! = (5.4^2 * e^{(-5.4)}) / 2 =0.0657$

P(Y < 3 in a three-week period)

$= P(Y = 0) + P(Y = 1) + P(Y = 2)$

$=0.0045 + 0.0243 + 0.0657$

$= 0.0945$

The probability that the equipment is used less than 3 times in a three-week period is 0.0945.

Q4)

a)

Let X be the weight of a randomly selected bag of coffee beans.

mean= 510g

standard deviation =18g

$P(485 \le X < 500)$

$= P((485 - 510) / 18 \le Z < (500 - 510) / 18)$

$=P(-1.3889 \le Z < -0.5556)$

$= P(Z < -0.5556) - P(Z < -1.3889)$

$= 0.2889 - 0.0823$

$= 0.2066$

The probability that a bag of coffee beans selected randomly is below standard but not underweight is 0.2066.

b)

Let Y be the weight of a randomly selected bag of coffee beans.

$P(Y \leq y) = 0.9$

$Z = 1.2816$

$Z = (y - 510) / 18$

$(y - 510) / 18 = 1.2816$

$y - 510 = 1.2816 * 18$

$y - 510 = 23.069$

$y = 533.069$

533 g / 20 g = 26.65 =26 cups

The maximum number of cups he can made in 90% of time is 26 cups.

c)

Let Z be the standardized random variable for the average weight of the 3 bags

$Z = (X - 510) / (18 / \text{sqrt}(3))$

$P(Z < (490 - 510) / (18 / \text{sqrt}(3)))$

$=P(Z<(20)/6sqrt(3))$

$=P(Z < -1.7321)$

$=0.0416$

The probability that the average weight of the 3 bags of coffee beans bought by Dan is less than 490 g is 0.0416.

d)

As Dan will lodge a complaint if at least 2 of them are below standard, so the probability of him not complainting is when there are only 0 or 1 bag below standard.

Let B1, B2, B3 be the weights of the three bags of coffee beans.

Probability of a bag to be on or above standard:

$Z = (500 - 510) / 18 = -0.5556$

$P(Z >= -0.5556) =0.7111$

P(0 bags below standard)

$= P(B1 > 500 g) * P(B2 > 500 g) * P(B3 > 500 g)$

$= 0.7111 * 0.7111 * 0.7111$

$= 0.3596$

Probability of a bag to be below standard:

$P=1- P(Z >= -0.5556)=0.2889$

P(1 bag below standard)

= P(X1 < 500 g) * P(X2 > 500 g) * P(X3 > 500 g) + P(X1 > 500 g) * P(X2 < 500 g) * P(X3 > 500 g) + P(X1 > 500 g) * P(X2 > 500 g) * P(X3 < 500 g)

=(0.2889 * 0.7111 * 0.7111) + (0.7111 * 0.2889 * 0.7111) + (0.7111 * 0.7111 * 0.2889)

=0.4383

P(Dan will not lodge a complaint)

= P(0 bags below standard) + P(1 bag below standard)

= 0.3596 + 0.4383

=0.7979

The probability that Dan will not lodge a complaint is approximately 0.7979.

e)

P(Underweight)

= P(weight < 485g)

= P(Z < (485 - 510) / 18)

= P(Z < -1.39)

=0.0823

P(lodge a complaint)=1-P(not lodge a complaint)=1-0.7979=0.2021

Dan will return exactly 2 bags of coffee beans means there are two situations

1.Exactly two bag are both underweight and below standard

2. Two bags are underweight, and the third bag is below standard

Situation 1:

P(2 bags underweight+1 bag above standard

= C(3, 2) * P(underweight)^2 * (1 - P(Below Standard))^(3 - 2)

= 3 * 0.0823^2 * (1 - 0.2889)

= 0.0144

Situation 2:

P(Below Standard but not Underweight)

= 0.2889 - 0.0823

=0.2066

P(3 bags below standard+2 bags underweight)

= C(3, 2) * P(underweight)^2 * (P(Below Standard but not Underweight))^(3 - 2)

= 3 * 0.0823^2 * 0.2066

= 0.0042

P(return exactly 2 bags of coffee beans)

=P(Situation 1)+P(Situation 2)

=0.0042+0.0144

=0.0186

P((return exactly 2 bags of coffee beans | lodge a complaint)

= P(return exactly 2 bags of coffee beans) / P(lodge a complaint)

= 0.0186/0.2021

= 0.0920

Given that Dan will lodge a complaint, the probability that he will return exactly 2 bags of coffee beans is 0.0920.