Student name: Zhu Jin Shun

Student ID: 22101071d

Assignment Q3

a)

Binomial distribution when get r successes out of X trials:P(r;n,p)=Pr(X=r)=(nCr)*p^r*(1-p)^n-r

X is counting r successes, given n trials

Negative Binomial distribution would be X is counting n trials, given r successes:

Probability Mass Function would then be

P(X=k;r,p)=Pr(X=k)=((k-1)C(r-1)) * p^r * (1−p)^k-r

Let $A_1, A_2 \ldots$ be independent and $P(A_i)=p$, $i=1,2,3$

Then $k=r, r+1, r+2$

$$P(Y_r=k)=\binom{k-1}{r-1}p^r(1-p)^{k-r}$$

If $r=1$ $\quad P(Y_1=k)=P(\{X_{k-1}=0\}\cap A_k)$

$$=P(\{X_{k-1}=0\})\cdot P(A_k)=(1-p)^{k-1}p$$

In general $\quad P(Y_r=k)=P(\{X_{k-1}=r-1\}\cap A_k)$

$$=P(\{X_{k-1}=r-1\})\cdot P(A_k)$$

Binomial $(k-1,p)$

$$=\binom{k-1}{r-1}p^{r-1}(1-p)^{k-r}p$$

pmf of negative binomial

Derive for Mean and Variance:

$$E(x)=\sum_{x=r}^{\infty} x\,C\binom{x-1}{r-1}{}^{x-r}=\frac{r}{p}\sum_{x=r}^{\infty}\frac{x\cdot(x-1)!}{r\cdot(r-1)!\,(x-r)!}\,p^{r+1}\,(1-p)^{x-r}$$

$$=\frac{r}{p}\sum_{x=r}^{\infty} C\binom{(x+1)-1}{(r+1)-1}\,p^{r+1}\,(1-p)^{(x+1)-(r+1)}$$

$$=\frac{r}{p}\sum_{y=r+1}^{\infty} C\binom{y-1}{(r+1)-1}\,p^{r+1}\,(1-p)^{y-(r+1)}$$

$$=\frac{r}{p}$$

$$E[X(X+1)]=E(X^2+X)=E(X^2)+E(X)$$

$$=\sum_{X=r}^{\infty} x(x+1)\,C\binom{x-1}{r-1}\,p^{r}\,(1-p)^{x-r}$$

$$=\frac{r(r+1)}{p^2}\sum_{X=r}^{\infty}\frac{(x+1)\,x\cdot(x-1)!}{(r+1)\,r\cdot(r-1)!\,(x-r)!}\,p^{r+2}\,(1-p)^{(x+2)-(r+2)}$$

$$=\frac{r(r+1)}{p^2}\sum_{X=r}^{\infty} C\binom{(x+2)-1}{(r+2)-1}\,(1-p)^{(x+2)-(r+2)}\,p^{r+2}$$

$$=\frac{r(r+1)}{p^2}\sum_{y=r+2}^{\infty} C\binom{y-1}{(r+2)-1}\,p^{r+2}\,(1-p)^{y-(r+2)}$$

$$=\cancel{r(r+1)/p^2}$$

$$=\frac{r(r+1)}{p^2}$$

$$Var(X)=E(X^2)-[E(X)]^2$$

$$=[E(X^2)+E(X)]-E(X)-[E(X)]^2$$

$$=\frac{r(r+1)}{p^2}-\frac{r}{p}-\frac{r^2}{p^2}$$

$$=\frac{r(1-p)}{p^2}$$

Formula:

E(X)=r/p

Var(X)=(r(1-p))/p^2

c)

p=0.05

r=9

E(x)=r/p=9/0.05=180 weeks

Var(x)= r(1-p)/p^2=(9*0.95)/0.0025=3420

Std(x)=sqrt(Var(x))=sqrt(3420)=58.48077

d)(non-coding part)

1 Life:

X=104

r=1

p=0.05

1-p=0.95

P(Survive for 104 weeks (1 live))=(Probability of survive 103 trials)*(Death on 104th trial)

=(1-p)^(X-r)*p^(r)

=(0.95)^103*0.05

=0.0002538

9 Lives:

X=104

r=9

$p=0.05$

$1-p=0.95$

P(Survive for 104 weeks (9 lives))

=(X-1Cr-1)(p)^r(1-p)^X-r

=(103C8)(0.95)^95*(0.05)^9

=0.00355316

=0.003553