THE HONG KONG POLYTECHNIC UNIVERSITY

DEPARTMENT OF APPLIED MATHEMATICS

Subject Code: AMA1501/ AMA1602

Subject Title: Introduction to Statistics for Business/ Introduction to Statistics

Session: Semester 1, 2022/2023

Date: 3 Dec 2022 Time: 15:15 – 18:15

Time Allowed: THREE Hours

This question paper has 15 pages (attachments included).

Instructions to Candidates: This question paper has 6 questions.

Attempt any **FIVE** questions.

Each question carries equal marks.

Attachments: Formula Sheets, Standard Normal Distribution Table, Student's t-distribution Table, χ^2 Distribution Table and F-distribution Table

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DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

Attempt any **FIVE** questions.

1. In a football tournament in 2018, 103 goals were scored from open play. The distribution of goals scored per 15-min period is given in the following table.

Time of goals (in minutes)	Number of goals scored
0 – 15	11
15 – 30	10
30 – 45	16
45 – 60	29
60 – 75	16
75 – 90	21

Remark: The periods '30 – 45' and '75 – 90' also include added time.

(a) Calculate the mean, median, standard deviation and interquartile range of the distribution. [10 marks]

(b) Calculate the coefficient of skewness using results in (a) and interpret your result briefly. [2 marks]

(c) From the above table, estimate the proportion of goals scored in the last 20 minutes. [3 marks]

(d) Test, at the 5% level of significance, whether 40% of goals were scored in the first half of a match (i.e. first 45 minutes). Assume that the chance of a goal scores in the first half remains the same for all goals. [5 marks]

2. (a) A problem is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the probability that the problem will be solved? [3 marks]

   (b) A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then another ball is drawn at random. What is the probability of second ball being blue? [3 marks]

   (c) How many 3 letter words (may not be meaningful) can be formed with the letters of the word **SIMULATE** each with at least one vowel (A, E, I, O, U)? [3 marks]

   (d) If a six sided die is rolled three times, what is the probability of getting at least one even number and at least one odd number? [3 marks]

   (e) An insurance company insured 2000 motorcycle drivers, 4000 car drivers, and 6000 truck drivers. The probability of an accident involving a motorcycle driver, car driver, and a truck is 0.01, 0.03, and 0.015 respectively. One of the insured persons meets with an accident. What is the probability that he is a motorcycle driver? [8 marks]

3. (a) The weight, X grams (g), of soup put in a tin by machine A is normally distributed with a mean of 120 g and a standard deviation of 5 g.

      i. A tin is selected at random. Find the probability that this tin contains more than 123 g. [3 marks]

      ii. Six tins are selected at random. Find the probability that at least three tins contain less than 123 g. [4 marks]

      iii. The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams. Given that $P(Y < 120) = 0.99$ and $P(Y > 112) = 0.90$, find the value of μ and the value of σ. [4 marks]

   (b) A fair coin is tossed 100 times. With suitable approximation, find the probability of getting 49, 50, or 51 heads. [5 marks]

   (c) A garage uses a particular spare part at an average rate of 3 per week. Assuming that usage of this spare part follows Poisson distribution, find the probability that at least three units are used in a 2-week period. [4 marks]

4. (a) In a massive attempt to compete with its competitors, the Ace Light Bulb Company issued a new line of bulb. Ace took 100 bulbs from their new line which had an established standard deviation of 140 hours. The mean measured lifetime was 1280 hours. Construct a 95% confidence interval for the mean lifetime of Ace's bulbs. [5 marks]

   (b) In the production of size D cells for use as flashlight batteries, the distribution of the operating life for all batteries is approximately normal. Seven batteries were tested and the following operating life (in hours) were recorded:

   21.75 16.23 19.87 15.96 20.25 22.92 21.06

   Construct a 90% confidence interval for the mean life of all the batteries. [7 marks]

   (c) The weights of 15 Hong Kong students had a sample mean of 107 lbs and a sample standard deviation of 10 lbs. Twelve Macau students had a mean weight of 112 lbs and a standard deviation of 8 lbs. Construct a 90% confidence interval to estimate the difference of the mean weights between the two student populations. State any assumption(s)/approximation(s) used. [8 marks]

5. (a) An electrical repair service claims that 10% of the service calls made result solely from appliances not having been plugged properly into the receptacle. A random sample of 200 work invoices produced 15 in which the only "repairs" were the plugging in of the appliance. Do the results indicate that the repair service's claim is justified? Test at the 1% level of significance. [6 marks]

(b) In an marketing survey for a new product, there was some question as to whether or not the potential buyers under 30 years of age view the product differently from those over 30. Two thousand and five hundred individuals were interviewed. The results were as follows:

	Interested	Neutral	Not interested
Under 30	400	100	500
Over 30	600	400	500

What conclusion can we draw about whether age is related to the preference on the new product at the 1% level of significance? [7 marks]

(c) Past records had shown that the scores of students who take a certain mathematics test are normally distributed with mean 75. The mathematics teachers would like to know whether a group of current year students is typical. They decide to test the hypothesis that current year students are typical versus the alternative that they are not typical. When a group of 16 students take the test, the average score is 82 and the variance is 36. What conclusion should be drawn? Use the 10% level of significance. State any assumption(s)/approximation(s) used. [7 marks]

6. (a) A manufacturer of kitchen clocks claims that a certain model will last at least 5 years. A random sample of the lifespan of 100 clocks are showed as follows.

Lifespan of the clock (in years)	5 – 6	6 – 7	7 – 8	8 – 9	9 – 10
Frequency	15	25	22	18	20

Conduct a hypothesis test to test whether the random sample follows uniform distribution at the 5% level of significance. [5 marks]

(b) The abilities of a group of 10 officers are ranked independently by their manager and their department head. Rank 1 is given to the best officer, Rank 2 to the second best,. . ., and so on, until Rank 10 to the officer with the worst performance. The two sets of ranks are shown in the following table:

Officer	Manager's ranking	Head's ranking
A	5	6
B	4	4
C	3	5
D	1	2
E	2	3
F	6	1
G	7	9
H	10	8
I	9	7
J	8	10

Calculate the coefficient of rank correlation as a measure of the consistency of the two rankings. [3 marks]

(c) An investigator has data on 1,000 individuals who have been in psychotherapy for five years. Variable x tells the mean number of hours per week the individual received psychotherapy over the five years. Variable y tells the score of the individual on a personality test after five years. Here are the data:

$$\sum xy = 30,000, \quad \sum x = 3,000, \quad \sum x^2 = 14,000,$$
$$\sum y = 5,000, \quad \sum y^2 = 80,000$$

i. Find the least squares regression equation. [4 marks]

ii. Interpret the coefficient estimate of the independent variable x. [2 marks]

iii. Predict the score of an individual with $x = 4$. [2 marks]

iv. What is the proportion of variation in y that cannot be explained by the fitted equation in i.? [4 marks]

End

Formula sheet

1. Sample Statistics:

	Ungrouped data	Grouped data
Arithmetic Mean	$\dfrac{\Sigma x}{n}$	$\dfrac{\Sigma fx}{\Sigma f}$
Standard Deviation	$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\dfrac{\Sigma x^2 - (\Sigma x)^2/n}{n-1}}$	$\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f - 1}} = \sqrt{\dfrac{\Sigma fx^2 - \frac{(\Sigma fx)^2}{\Sigma f}}{\Sigma f - 1}}$

2. Probability Distributions:

   (a) Binomial $P(r) = {}_nC_r p^r (1-p)^{n-r}$

   (b) Poisson $P(r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$

3. Standard Errors:

   (a) Mean $\dfrac{\sigma}{\sqrt{n}}$

   (b) Proportion $\sqrt{\dfrac{p(1-p)}{n}}$

   (c) Difference between means $\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

   (d) Difference between proportions $\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$

4. Test Statistics:

   (a) $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (one sample)

   $$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \quad \text{(two samples)}$$

(b) $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ (one sample)

$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ (two samples) where $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

(c) $\chi^2 = \sum \dfrac{(O - E)^2}{E}$

5. Correlation and Regression:

(a) Product moment correlation coefficient

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{\left[n\Sigma x^2 - (\Sigma x)^2\right]\left[n\Sigma y^2 - (\Sigma y)^2\right]}}$$

(b) Spearman's rank correlation coefficient

$$R_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

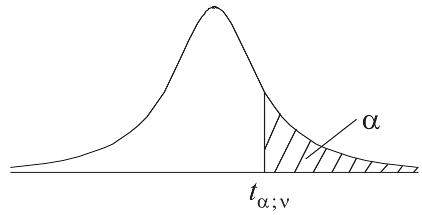
(c) Least squares regression line $y = a + bx$

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \qquad a = \frac{\Sigma y}{n} - \frac{b\Sigma x}{n}$$

Table of the Student's *t*-distribution

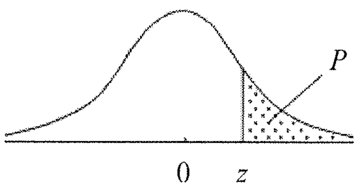
The table gives the values of $t_{\alpha;\nu}$ where

$\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table of the Standardised Normal Distribution



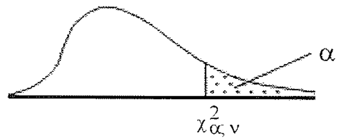
The table gives the probability

$$P = \Pr(Z > z)$$

where $Z \sim N(0,1)$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

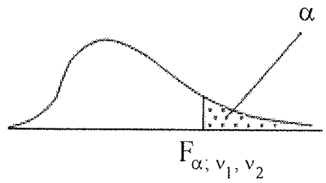
Table of the Chi-square Distribution



$\alpha =$	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	$=\alpha$
$\nu = 1$	0.0000393	0.000157	0.000628	0.000982	0.00393	0.0158	0.0642	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	$\nu = 1$
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	90.405	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

Table of F-Distribution (i)

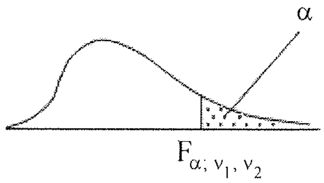
For each pair of values ν_1 and ν_2, the table gives the values of $F_{\alpha;\,\nu_1,\,\nu_2}$ with $\alpha = 0.05,\ 0.025,\ 0.01,$ and 0.001.



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	24	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	249.0	251.1	252.2	253.3	254.3
	648	800	864	900	922	937	948	957	963	969	977	985	997	1006	1010	1014	1018
	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6106	6157	6235	6287	6313	6339	6366
	4053 ×100	5000 ×100	5404 ×100	5625 ×100	5764 ×100	5859 ×100	5929 ×100	5981 ×100	6023 ×100	6056 ×100	6107 ×100	6158 ×100	6235 ×100	6287 ×100	6313 ×100	6340 ×100	6366 ×100
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5
	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5
	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.64	8.59	8.57	8.55	8.53
	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.3	14.2	14.1	14.0	14.0	13.95	13.9
	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.6	26.4	26.3	26.2	26.1
	167.0	148.5	141.1	137.1	134.6	132.8	131.5	130.6	129.9	129.2	128.3	127.4	125.9	125.0	124.5	124.0	123.5
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.77	5.72	5.69	5.66	5.63
	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.51	8.41	8.36	8.31	8.26
	21.20	18.00	16.69	15.98	15.52	15.20	15.0	14.80	14.66	14.55	14.37	14.20	13.93	13.75	13.65	13.56	13.46
	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.76	45.77	45.09	44.75	44.40	44.05
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.53	4.46	4.43	4.40	4.36
	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.28	6.18	6.12	6.07	6.02
	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.47	9.29	9.20	9.11	9.02
	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24	26.92	26.42	25.91	25.14	24.60	24.33	24.06	23.79
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.84	3.77	3.74	3.70	3.67
	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.12	5.01	4.96	4.90	4.85
	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.31	7.14	7.06	6.97	6.88
	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41	17.99	17.56	16.90	16.44	16.21	15.99	15.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.41	3.34	3.30	3.27	3.23
	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.42	4.31	4.25	4.20	4.14
	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.07	5.91	5.82	5.74	5.65
	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.32	12.73	12.33	12.12	11.91	11.70
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.12	3.04	3.01	2.97	2.93
	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	3.95	3.84	3.78	3.73	3.67
	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.28	5.12	5.03	4.95	4.86
	25.42	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	11.54	11.19	10.84	10.30	9.92	9.73	9.53	9.34
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.90	2.83	2.79	2.75	2.71
	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.61	3.51	3.45	3.39	3.33
	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.73	4.57	4.48	4.40	4.31
	22.86	16.39	13.90	12.56	11.71	11.13	10.69	10.37	10.11	9.87	9.57	9.24	8.72	8.37	8.19	8.00	7.81
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.74	2.66	2.62	2.58	2.54
	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.37	3.26	3.20	3.14	3.08
	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.33	4.17	4.08	4.00	3.91
	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.74	8.44	8.13	7.64	7.30	7.12	6.94	6.76
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.61	2.53	2.49	2.45	2.40
	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.17	3.06	3.00	2.94	2.88
	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.02	3.86	3.78	3.69	3.60
	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.32	6.85	6.52	6.35	6.17	6.00
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.51	2.43	2.38	2.34	2.30
	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.02	2.91	2.85	2.79	2.72
	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.78	3.62	3.54	3.45	3.36
	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.71	6.25	5.93	5.76	5.59	5.42

Table of F-Distribution (ii)

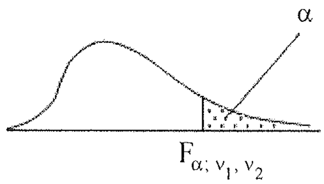
For each pair of values ν_1 and ν_2, the table gives the values of $F_{\alpha;\,\nu_1,\,\nu_2}$ with $\alpha = 0.05,\ 0.025,\ 0.01,$ and 0.001.



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	24	40	60	120	∞
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.42	2.34	2.30	2.25	2.21
	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.89	2.78	2.72	2.66	2.60
	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.59	3.43	3.34	3.25	3.17
	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.23	5.78	5.47	5.30	5.14	4.97
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.35	2.27	2.22	2.18	2.13
	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.79	2.67	2.61	2.55	2.49
	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.43	3.27	3.18	3.09	3.00
	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40	6.13	5.85	5.41	5.10	4.94	4.77	4.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.29	2.20	2.16	2.11	2.07
	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.70	2.59	2.52	2.46	2.40
	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.29	3.13	3.05	2.96	2.87
	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.54	5.10	4.80	4.64	4.47	4.31
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.24	2.15	2.11	2.06	2.01
	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.63	2.51	2.45	2.38	2.32
	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.18	3.02	2.93	2.84	2.75
	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81	5.55	5.27	4.85	4.54	4.39	4.23	4.06
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.19	2.10	2.06	2.01	1.96
	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.56	2.44	2.38	2.32	2.25
	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.08	2.92	2.83	2.75	2.65
	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.05	4.63	4.33	4.18	4.02	3.85
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.15	2.06	2.02	1.97	1.92
	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.50	2.38	2.32	2.26	2.19
	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.00	2.84	2.75	2.66	2.57
	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.87	4.45	4.15	4.00	3.84	3.67
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.11	2.03	1.98	1.93	1.88
	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.45	2.33	2.27	2.20	2.13
	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	2.92	2.76	2.67	2.58	2.49
	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.70	4.29	3.99	3.84	3.68	3.51
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.08	1.99	1.95	1.90	1.84
	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.41	2.29	2.22	2.16	2.09
	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.86	2.69	2.61	2.52	2.42
	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.56	4.15	3.86	3.70	3.54	3.38
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.05	1.96	1.92	1.87	1.81
	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.37	2.25	2.18	2.11	2.04
	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.80	2.64	2.55	2.46	2.36
	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.44	4.03	3.74	3.58	3.42	3.26
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.03	1.94	1.89	1.84	1.78
	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.33	2.21	2.14	2.08	2.00
	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.75	2.58	2.50	2.40	2.31
	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.33	3.92	3.63	3.48	3.32	3.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.00	1.91	1.86	1.81	1.76
	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.30	2.18	2.11	2.04	1.97
	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.70	2.54	2.45	2.35	2.26
	14.19	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.23	3.82	3.53	3.38	3.22	3.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	1.98	1.89	1.84	1.79	1.73
	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.27	2.15	2.08	2.01	1.94
	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.66	2.49	2.40	2.31	2.21
	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.14	3.74	3.45	3.29	3.14	2.97

Table of F-Distribution (iii)

For each pair of values ν_1 and ν_2, the table gives the values of $F_{\alpha;\,\nu_1,\,\nu_2}$ with $\alpha = 0.05,\ 0.025,\ 0.01,\ \text{and}\ 0.001$.



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	24	40	60	120	∞
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	1.96	1.87	1.82	1.77	1.71
	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.24	2.12	2.05	1.98	1.91
	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.62	2.45	2.36	2.27	2.17
	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56	4.31	4.06	3.66	3.37	3.22	3.06	2.89
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.95	1.85	1.80	1.75	1.69
	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.22	2.09	2.03	1.95	1.88
	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.58	2.42	2.33	2.23	2.13
	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	3.99	3.59	3.30	3.15	2.99	2.82
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.93	1.84	1.79	1.73	1.67
	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.19	2.07	2.00	1.93	1.85
	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.55	2.38	2.29	2.20	2.10
	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.92	3.52	3.23	3.08	2.92	2.75
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.91	1.82	1.77	1.71	1.65
	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.17	2.05	1.98	1.91	1.83
	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.52	2.35	2.26	2.17	2.06
	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.86	3.46	3.18	3.02	2.86	2.69
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.90	1.81	1.75	1.70	1.64
	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.15	2.03	1.96	1.89	1.81
	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.49	2.33	2.23	2.14	2.03
	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.80	3.41	3.12	2.97	2.81	2.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.89	1.79	1.74	1.68	1.62
	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.14	2.01	1.94	1.87	1.79
	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.47	2.30	2.21	2.11	2.01
	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.75	3.36	3.07	2.92	2.76	2.59
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.79	1.69	1.64	1.58	1.51
	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.01	1.88	1.80	1.72	1.64
	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.29	2.11	2.02	1.92	1.80
	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.40	3.01	2.73	2.57	2.41	2.23
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.70	1.59	1.53	1.47	1.39
	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.88	1.74	1.67	1.58	1.48
	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.12	1.94	1.84	1.73	1.60
	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54	3.32	3.08	2.69	2.41	2.25	2.08	1.89
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.61	1.50	1.43	1.35	1.25
	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.76	1.61	1.53	1.43	1.31
	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	1.95	1.76	1.66	1.53	1.38
	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.78	2.40	2.11	1.95	1.76	1.54
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.52	1.39	1.32	1.22	1.00
	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.64	1.48	1.39	1.27	1.00
	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.79	1.59	1.47	1.32	1.00
	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96	2.74	2.51	2.13	1.84	1.66	1.45	1.00