The Hong Kong Polytechnic University

COMP2012 Discrete Mathematics

Assignment 1

(Due: 23:59, 22nd October, 2023)

Guideline:

- This is an individual assignment.
- Please submit the soft copy of your answer to Blackboard (as a doc/docx/pdf file).
- You just need to write your answer. There is no need to copy questions.

Questions:

Question 1 **[10 marks]**

You are given n integers $a_1, a_2, \ldots, a_n$.

Let $m = (a_1 + a_2 + \ldots + a_n)/n$.

Prove that there exists some number in $a_1, a_2, \ldots, a_n$ such that it is smaller than or equal to m.

Question 2 **[10 marks]**

Prove the following logic using a truth table.

(a) $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$ (5 marks)

(b) $\neg(p \land q \land r) \equiv \neg p \lor \neg q \lor \neg r$ (5 marks)

(Hint: $p \leftrightarrow q$ means p and q are logically equivalent)

Question 3 **[10 marks]**

(a) Let the domain D contain all prime numbers ≥ 3 and ≤ 25.

Suggest two propositional functions $P(x)$ and $Q(y)$ so that all the following statements are true at the same time:

- $\exists\, x \in D\ P(x)$
- $\exists\, y \in D\ Q(y)$
- $\neg\, (\exists\, z \in D\ P(z) \land Q(z))$

Prove that your proposed functions P and Q satisfy the above conditions. (5 marks)

(b) **Solve** the following two straight lines L_1 and L_2, where:

$$\begin{cases} L_1: x + y = 0 \\ L_2: x - y = 0 \end{cases}$$

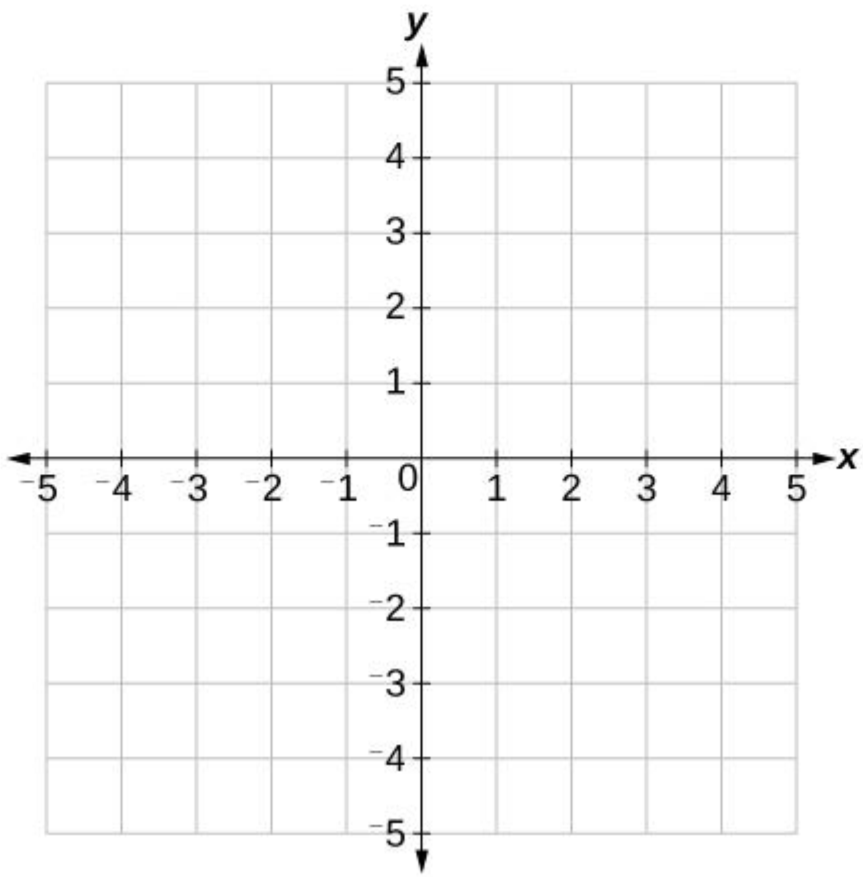
(5 marks)

Assume that:

- $x, y \in \mathrm{D}$, where D is a domain {-3, -2, -1, 0, 1, 2, 3}
- Sets A and B contain all the points of L_1 and L_2, respectively.

Solution guide:

- List out all the points (x, y) of each line in the given domain D;
- Solve the equations diagrammatically with the aid of the *coordinate grid* given below. Explain with the concept of set.
- Express your findings with the aid of set builder representation, and write down the final answer in set form.



Question 4 **[10 marks]**

COMP department provides 3 elective courses (AI, Blockchain, and Computer graphics), and all the Year 4 COMP students need to take at least ONE elective course. According to the elective enrollment report of this Academic Year, 116 students have chosen an AI course, 121 students have chosen a Blockchain course, and 129 students have chosen a Computer graphics course. If 43 students take both AI and Blockchain courses; 32 students take both AI and Computer graphics courses; 39 students take both Blockchain and Computer graphics courses, and 22 take all three elective courses.

With the aid of Venn Diagram, show the steps to calculate:

(a) The total number of Year 4 COMP students of this Academic Year. (5 marks)
(b) Assuming a student completes any two elective courses, including Blockchain, he or she will receive a degree certificate with a FinTech major. How many persons, assuming that every student passes the courses, will be eligible to receive this? (5 marks)

Question 5 **[15 marks]**

Given the table of rules of inferences in Appendix I (at the back page).

(a) Prove the rule "Modus ponens" using a truth table (2 marks)
(b) Given: $p \wedge q$
$(p \vee s) \rightarrow \neg r$
$r \vee t$
Prove: t (5 marks)

(c) Consider the following two statements

S1: If a student is known to be cheating, then he/she will not be passed.

S2: If a student is good, he/she will be passed.

Determine which one of the statements (i) to (iv) follows from S1 and S2 as per sound inference rules of logic. (8 marks)

(i) If a student is known to be cheating, he is good
(ii) If a student is not known to be cheating, he is not good
(iii) If a student is good, he is not known to be cheating
(iv) If a student is not good, he is not known to be cheating

<u>Solution guide:</u>

Starting with writing down premises, i.e. the propositional logic of statements S1, S2, and the given statements (i) to (iv).

Question 6 **[5 marks]**

Take an original photo by yourself of an everyday object or a scene that you feel is beautiful, which can be fit with a golden spiral. Attach the photo and draw a golden spiral on it.

Question 7 **[10 marks]**

Matrix operations

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, E = \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$

I_d = Identity matrix, O_d = zero matrix (all entries are zero)

(where subscript *d* is the dimension of the matrix)

Evaluate:

(a) $D^T E$ (2 marks)
(b) Prove whether $A^2 - B^2 = (A - B)(A + B)$ (3 marks)
(c) If $C^3 - 6C^2 + 7C + kI_3 = O_3$, find the value of *k* (5 marks)

Question 8 **[10 marks]**

(a) Determine the time complexity of the following two algorithms which check whether a input number is prime. (8 marks)

IsPrime-A (*n*)	IsPrime-B (*n*)
1. for integer *i*←2 to *n-1*	1. for integer *i*←2 to sqrt(*n-1*)
2. if *n* mod *i* ==0 then	2. if *n* mod *i* ==0 then
3. return False	3. return False
4. return True	4. return True

Hint: You need to write down the run time frequency of each line, and then estimate the final complexity. In addition, you may try to test the pseudocode with some sample *n*.

(b) Which one is more efficient? (1 mark) Why? (1 mark)

Question 9 **[10 marks]**

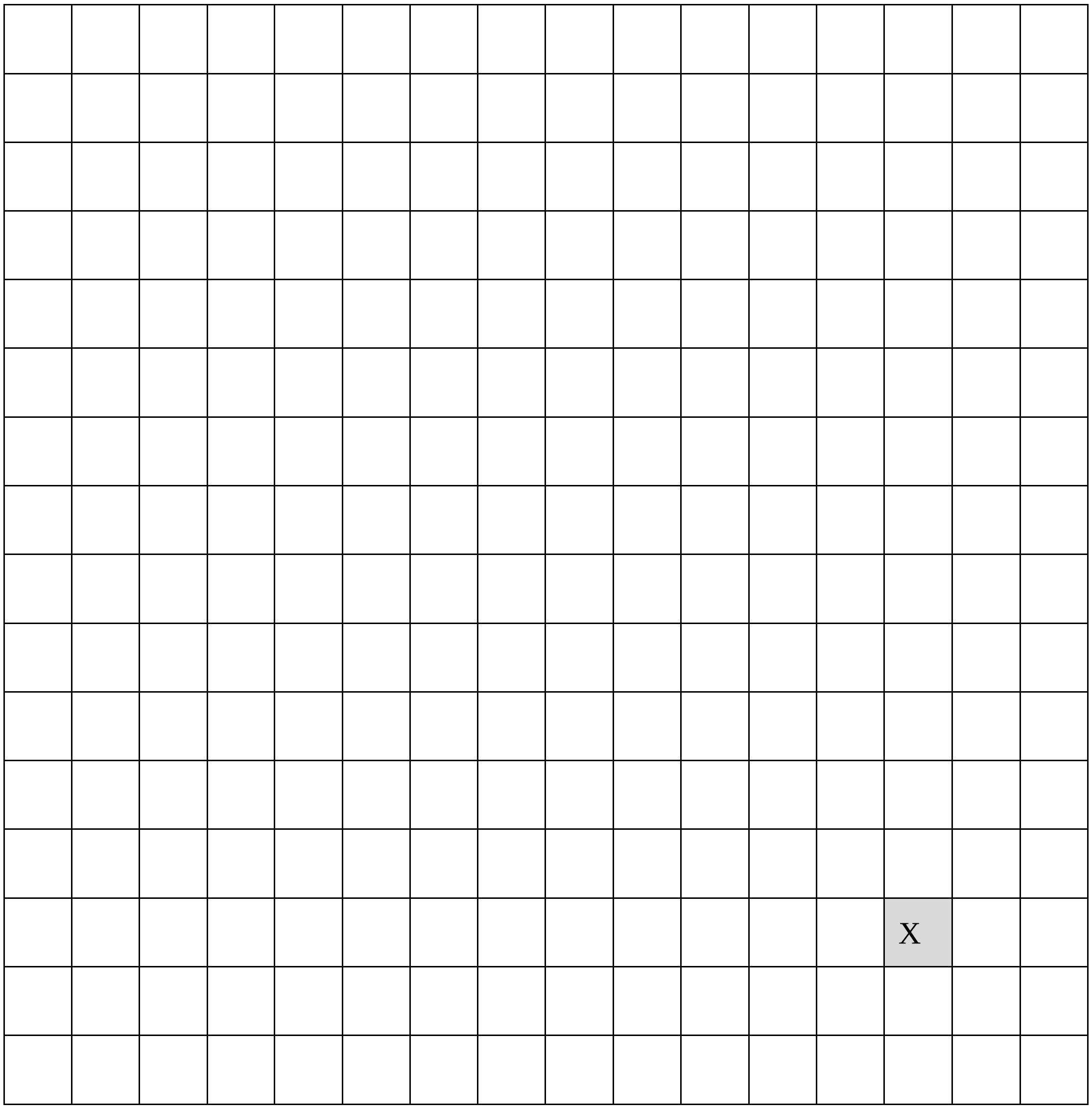
By means of Mathematical Induction, show that $\forall n \in \mathbb{Z}^+$ (i.e. non-zero positive integer),

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}$$

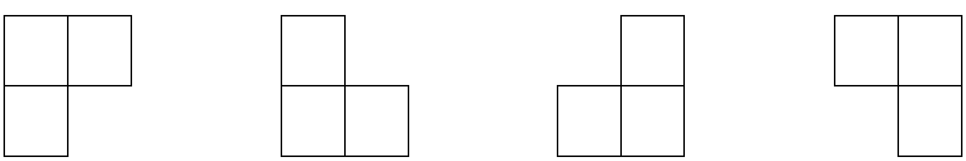
Question 10 **[10 marks]**

We are referring to the tiling problem in "MI: example 3" in the slides of lecture #5.

Now, we are given the following 16×16 checkboard with a missing square (X).



A triomino looks like one of the followings.



Draw the above checkboard after tiling it with triominoes. Adjacent triominoes should be filled in different colours (like in the slides).

Solution guide:

The way of tiling please refer to the solution of lab 5 question 1.

End of Assignment 1

Appendix I

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution