1 $\bar{d} = 9.848,\ s_d = 18.474$

$$CI = \bar{d} \pm t_{0.025,14} \frac{S_d}{\sqrt{15}} = 9.848 \pm 2.145 \times \frac{18.474}{\sqrt{15}}$$

$$= 9.848 \pm 10.2316$$

$$(\text{or, } [-0.3836, 20.0796])$$

2. (a). $Z_{\alpha/2} = Z_{0.005} = 2.576$. The confidence interval is

$$CI = \bar{x} - \bar{y} \pm Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}} = 29.8 - 34.7 \pm 2.576 \sqrt{\frac{4^2}{20} + \frac{5^2}{25}}$$

$$= \boxed{-4.9 \pm 3.4561} \text{ or equivalently, } \boxed{[-8.3561, -1.4439]}.$$

(b) Step 1: H_0: $\mu_1 - \mu_2 = d_0 = 0$, H_1: $\mu_1 - \mu_2 \neq 0$

$\alpha = 0.01$

Step 2: Test statistic and its value:

$$Z = \boxed{\frac{\bar{X} - \bar{Y} - d_0}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}} = \frac{-4.9}{\sqrt{\frac{4^2}{20} + \frac{5^2}{25}}} = \boxed{-3.6522}$$

Rejection region: $R_\alpha = \{x : |x| > Z_{\alpha/2} = 2.576\} \ni Z$.

Step 3: Reject H_0, so we draw the conclusion that the data indicates that the true yield strengths μ_1 and μ_2 are significantly different.

3 (a) Step 1: $H_0: \mu_1 - \mu_0 = 0$, $H_1: \mu_1 - \mu_0 < 0$. $\alpha = 0.01$.

Step 2. Test statistic and its value:

$$T = \frac{\bar{X} - \bar{Y} - 0}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}} = \frac{1.71 - 2.53}{\sqrt{\frac{0.53^2}{8} + \frac{0.87^2}{10}}} = \boxed{-2.4634}$$

$$\nu = \frac{\left(\frac{S_1^2}{m} + \frac{S_2^2}{n}\right)^2}{\frac{(S_1^2/m)^2}{m-1} + \frac{(S_2^2/n)^2}{n-1}} = 15.1071 \approx 15.$$

Rejection region $R_\alpha = (-\infty, -t_{\alpha,\nu}) = \boxed{(-\infty, -2.602)} \not\ni T$.

Step 3. We do not reject H_0. The conclusion is that the data does not suggest that the true average gap detection threshold for CTS subjects exceeds that for normal subjects significantly.

(b). $CI = \bar{X} - \bar{Y} \pm t_{\alpha/2,\nu} \sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}$

$$= 1.71 - 2.53 \pm 2.947 \sqrt{\frac{0.53^2}{8} + \frac{0.87^2}{10}}$$

$$= \boxed{-0.82 \pm 0.9810}$$ or equivalently,

$$\boxed{[-1.8010, 0.1610]}$$

4 (a). degree of freedom

$$\nu = \frac{\left(\frac{S_1^2}{m}+\frac{S_2^2}{n}\right)^2}{\frac{(S_1^2/m)^2}{m-1}+\frac{(S_2^2/n)^2}{n-1}} = 30.8267 \approx 31.$$

However, we still use 30 since it is the closest degree of freedom provided by the table.

The Confidence interval is

$$CI = \bar{X} - \bar{Y} \pm t_{0.025,30}\sqrt{\frac{S_1^2}{m}+\frac{S_2^2}{n}}$$

$$= 8.74 - 4.96 \pm 2.042\sqrt{\frac{0.66^2}{20}+\frac{0.39^2}{20}}$$

$$= \boxed{3.78 \pm 0.3500} \quad \text{or equivalently, } \boxed{[3.4300, 4.1300]}$$

(b). Let μ_1 be the true average firmness of zero-day apples, and μ_2 be this value for 20-day apples.

step 1: $H_0: \mu_1 - \mu_2 = 0$. $\quad H_1: \mu_1 - \mu_2 \neq 0$.

$\alpha = 0.05$

step 2: Test statistic and its value:

$$T = \frac{\bar{X} - \bar{Y} - 0}{\sqrt{\frac{S_1^2}{m}+\frac{S_2^2}{n}}} = \frac{8.74 - 4.96}{\sqrt{\frac{0.66^2}{20}+\frac{0.39^2}{20}}} = 22.0510$$

Rejection region: $R_\alpha = \{x : |x| > t_{\frac{\alpha}{2},30} = 2.042\} \ni T$.

Step 3. Reject H_0. We draw the conclusion that there is a significant difference between the true average firmness of zero-day apples and the true average firmness of 20-day apples.

5 (a). $S_{pooled}^2 = \dfrac{9 \times 0.2^2 + 9 \times 0.4^2}{18} = 0.1$

$$CI = \bar{X} - \bar{Y} \pm t_{\alpha/2, 18} \, S_{pooled} \sqrt{\frac{1}{m} + \frac{1}{n}}$$

$$= 0.64 - 2.05 \pm 2.101 \times \sqrt{0.1} \times \sqrt{0.2}$$

$$= \boxed{-1.41 \pm 0.2971}, \text{ or equivalently, } \boxed{[-1.7071, -1.1129]}$$

(b) $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \neq 0$. $\alpha = 0.05$.

Test statistic and its value:

$$T = \frac{\bar{X} - \bar{Y} - 0}{S_{pooled}\sqrt{\frac{1}{m} + \frac{1}{n}}} = \frac{-1.41}{\sqrt{0.1 \times 0.2}} = \boxed{-9.9702}$$

Rejection region: $R_\alpha = \{x : |x| > t_{\frac{\alpha}{2}, 18} = 2.101\} \ni T$.

So, we reject H_0.

(c) $H_0: \sigma_1^2 = \sigma_2^2$, $H_1 = \sigma_1^2 \neq \sigma_2^2$, Let $S_1 = 0.2$, $S_2 = 0.4$

$$F = \frac{S_1^2}{S_2^2} = \frac{0.2^2}{0.4^2} = 0.25$$

$$\frac{\alpha}{2} = 0.025, \; R_\alpha = \left(0, \frac{1}{f_{0.025, 10-1, 10-1}}\right) \cup \left(f_{0.025, 10-1, 10-1}, \infty\right)$$

$$= \left(0, \frac{1}{4.03}\right) \cup (4.03, \infty)$$

$$= (0, 0.248) \cup (4.03, \infty) \not\ni F$$

$\therefore$ cannot reject H_0

6. Step 1: $H_0: \sigma^2 = \sigma_0^2 = 1.15$, $H_1: \sigma^2 > 1.15$.

$\alpha = 0.05$.

Step 2: Test statistic:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{24 \times 2.03}{1.15} = 42.3652$$

Rejection region: $R_\alpha = (\chi^2_{0.05,24} = 36.415, \infty) \ni \chi^2$

Step 3: Reject H_0. So we can draw the conclusion that the machine is out of control.