AMA2104 Random Class Quiz 1

NAME:

ID:

Problem 1. Let A and B be two events. Assume $P(A) = 0.5$ and $P(B) = 0.6$.

(1) If A and B are independent, what is $P(A \cap B)$?
(2) If $P(A \cap B) = 0.1$, what is $P(A \mid B)$?

Solution:
Note that $P(A) = 0.5$ and $P(B) = 0.6$.
(1) If A and B are independent, $P(A \cap B) = P(A)P(B) = 0.5 \times 0.6 = 0.3$
(2) If $P(A \cap B) = 0.1, P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.6} = \frac{1}{6}$

Problem 2. If $P(A) = \frac{2}{3}, P(B) = \frac{3}{5}$, and $P(A \mid B) + P(B \mid A) = \frac{19}{25}$, find $P(A \cap B)$.

Solution: Let $P(A \cap B) = p$.
$P(A \mid B) + P(B \mid A) = \frac{5}{3}p + \frac{3}{2}p = \frac{19}{25} \Rightarrow P(A \cap B) = p = \frac{6}{25}$.

Problem 3. Urn I contains 3 white and 3 black balls and Urn II contains 3 white and 4 black balls. An urn is chosen at random with equal probability, and a ball is randomly selected from that urn. Find the probability that the ball chosen is black.

Solution:

$$P(\text{black}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{4}{7} = \frac{15}{28}$$

Problem 4. If $P(A) = \frac{2}{5}, P(B) = \frac{4}{5}$, and $P(A \bigcup B) = 1$, find $P(A \mid B)$ and $P(B \mid A)$.

Solution:

$$P(A) + P(B) - P(A \bigcap B) = P(A \bigcup B),$$

$$P(A \bigcap B) = \frac{1}{5}.$$

Then

$$P(A \mid B) = \frac{1}{4}, \qquad P(B \mid A) = \frac{1}{2}.$$