AMA1131 Calculus
Assignment 1

Due Date: Friday, 11 Oct 2024, before 11:59pm.

- Put the following information on the top right corner of the front page of your homework.
  - Your name and student number
  - Subject code: AMA1131
  - Subject lecturer: Dr Bob He
- You should finish all questions.
- Photograph your solutions onto a PDF file named YourName_StuID, otherwise the marker (not the lecturer) cannot write on your solution, then you cannot see the marking but only the score.
- You may use the app "CamScanner" or other softwares. Make sure that the file is complete, legible, in correct order and orientation.
- Upload/attach your homework solution pdf file at the same place you've downloaded this homework by pressing the "Browse My Computer", then choose your pdf file, and then press Submit. You may re-submit the homework again, to a maximum of twice, before the due time. After submitting, check and make sure your submission is successful.
- No late submission is allowed. It may not be marked.

1. Consider the function $f(x) = \frac{\sqrt{9-x^2}}{x}$ and $g(x) = \arccos(\frac{x}{4})$.
   (a) Find the largest possible domain of f and g.
   (b) Show that f is one-to-one on $(0, 2)$. Then find the inverse function of f.
   (c) Find expression for the function fg.
   (d) Find expression for the function $\frac{g}{f}$.
   (e) Find explicit expressions for the functions $f \circ g$ and $g \circ f$.

   [10 marks]

2. Without using any calculator to evaluate $\sin(2\arctan(3/5))$. [10 marks]

3. Determine whether or not the function $y = f(x) = \frac{3x-2}{x+1}$ is one-to-one. If so, find $f^{-1}(y)$. [10 marks]

4. Suppose $f(x) = \sin(x^2)$, prove
$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = 2x\cos(x^2)$$
without the aid of differentiation. [10 marks]

5. Evaluate the following limits (L'Hopital's rule is prohibited)

   (a) $\lim\limits_{x\to 4} \frac{x^2+5x+4}{x^2+3x-4}$;

   (b) $\lim\limits_{x\to 0} \frac{x}{\sqrt{1-x}-1}$;

   (c) $\lim\limits_{t\to 1}(\frac{1}{(t-1)\sqrt{t}} - \frac{1}{t-1})$;

   (d) $\lim\limits_{x\to 1} \frac{x^4-1}{\sin(x-1)}$.

[10 marks]

6. Let

$$f(x) = \begin{cases} 3x & \text{if } x \le 0, \\ \frac{2x^3-2}{x^2-1} & \text{if } 0 < x \le 1, \\ \frac{x^2+2ax+b}{x-1} & \text{if } x > 1, \end{cases}$$

   where a and b are real numbers.

   (a) Find $f(0)$.

   (b) Find $\lim\limits_{x\to 0} f(x)$ if it exists, otherwise explain your result.

   (c) Find a and b so that $\lim\limits_{x\to 1} f(x)$ exists.

[10 marks]

7. Solve equations for x

   (a) $\log_2(x-2) = 3 - \log_2(x-1)$ [5 marks]

   (b) $\log_{16}(2x+3) + \log_{16}(x+5) = \log_4 x$. [5 marks]

8. Use the intermediate value theorem (IVT) to show that the equation

$$3x = 1 + \tan^{-1} x + \sin x$$

   has a solution in the interval $(0, 1)$. Justify your steps and answer. [10 marks]

9. Evaluate the following limits (L'Hopital's rule is prohibited)

   (a) $\lim\limits_{x\to 2} \tan(x^2 - x + 1)\dfrac{\sin(x-2)}{2(x-2)}$;

   (b) $\lim\limits_{x\to\infty} \dfrac{x\cos x}{(x^2+1)\tan^{-1} x}$;

   (c) $\lim\limits_{x\to 0^+} \left(\sqrt{\dfrac{1}{x}} - \sqrt{\dfrac{1}{x} + \dfrac{1}{1130}}\right)$;

   (d) $\lim\limits_{x\to -\infty} \left(\sqrt{4x^2 - x + 1} + 2x\right)$.

[10 marks]

10. Let

$$f(x) = \begin{cases} \dfrac{1}{4}, & x \le 0, \\ \dfrac{1}{2} - \dfrac{\sin 4x}{16x}, & 0 < x < 1, \\ \dfrac{1}{\sqrt{16 - x^2}}, & 1 \le x \le 3, \\ \dfrac{3\sqrt{7}+1}{21} - \dfrac{\sin(2x-6)}{7(x^2-9)}, & x > 3. \end{cases}$$

    Find the values x for which the function is **not** continuous. [10 marks]