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Answers for COMP2012 A1

Question 1 and 2

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1) Proof by contradiction

Assume ~~It~~ every number in~~a~~ a1, a2, ... an is greater than m.

The sum of all the numbers will be

$S = a1 + a2 + a3 + \dots + an$

Because every number of a1, a2 ... is greater than m.

$S > m + m + \dots + m = n \times m$

Divide both sides by n, we get

$S/n > m$

But in question ~~8~~ $m = (a_1 + a_2 + \dots + an)/~~m~~ n$ which leads to $m = S/n$, so a1, a2, an can't be all greater then m.

Which means there exist some number in a1, a2, .. an is such ~~ge~~ smaller or equal to m to make $m = (a_1 + a_2 + \dots an)/n$

2) a) Truth table ~~b) Truth table~~

p	q	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \leftrightarrow q$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	F	F	F
F	F	F	T	T	T

By ~~et~~ evaluating truth tables of both sides, we could see all possible combination of q and p. And $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $p \leftrightarrow q$ are the same. We could conclude $p \leftrightarrow q = (p \wedge q) \vee (\neg p \wedge \neg q)$ is true

b) Truth table

p	q	r	$p \wedge q \wedge r$	$\neg (p \wedge q \wedge r)$	$\neg p$	$\neg q$	$\neg r$	$\neg p \vee \neg q \vee \neg r$
T	T	T	T	F	F	F	F	F
T	T	F	F	T	F	F	T	T
T	F	T	F	T	F	T	F	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	F	F	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	T	T

By evaluating truth tables of both sides, we could see all possible combination of p, q, r. ~~$(p \wedge q)$~~ $\neg (p \wedge q \wedge r)$ has the same cases with $\neg p \vee \neg q \vee \neg r$. We could conclude $\neg (p \wedge q \wedge r) = \neg p \vee \neg q \vee \neg r$.

Question 3

(Question 3)

a) We set two proposed function: $P(x)$ and $Q(y)$

For $P(x)$: " x is a prime number that is greater than or equal to 3 and less than or equal to 7 "

For ~~P~~ $Q(y)$: " y is a prime number greater then or equal to 10 and less than or equal to 25 "

Proof: ① Statement 1: $\exists \in D\, P(x)$

This is true because there are prime numbers between 3 and 7 in the doman, such as, 3, 5 and 7.

Proof: ② Statement 2: $\exists \in D\, Q(y)$

This is true because there are prime numbers between 10 and 25 in the domain, such as 11, 13, 17, 19, 23

Proof: ③ Statement 3: $\neg(\exists z \in D(P(z) \wedge Q(z)))$

Because $P(x)$ is in range $3 \leq x \leq 7$, $Q(y)$ is in range $10 \leq y \leq 25$, So it is not true there exist a D so $P(x)$, $Q(y)$ both true. But with the "not" sign, not false is true makes it satisfy the condition

With all three conditions true, the proposed $P(x)$, $Q(y)$ satify the above conditions

b) For L_1: $x+y=0$, we can conclude $x=-y$ or $-x=y$

Then we have values from Domain, we get points $(-3,3)$ $(-2,2)$ $(-1,1)$ $(0,0)$ $(1,-1)$ $(2,-2)$ $(3,-3)$ in Set A that ~~contains~~ fullfils requiremane

For L_2: $x-y=0$, we can conclude $x=y$

From values from Domain, we get points $(-3,-3)$ $(-2,-2)$ $(-1,-1)$ $(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$ in Set B

Expressing the findings with set builder notation

$A = \{(x,y) \mid x, y \in D, x+y=0\}$ $B = \{(x,y) \mid x, y \in D, x-y=0\}$

Final result in set form:

$A = \{(-3,3), (-2,2), (-1,1), (0,0), (1,-1), (2,-2), (3,-3)\}$

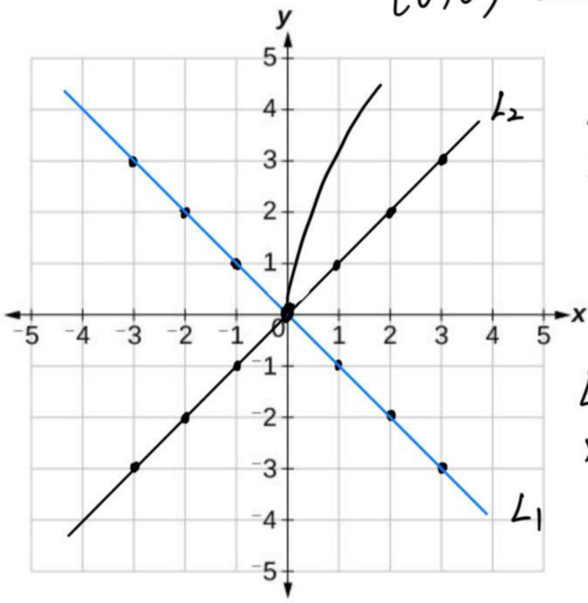
$B = \{(-3,-3), (-2,-2), (-1,-1), (0,0), (1,1), (2,2), (3,3)\}$ $x=0$, $y=0$

The Final answer for L_1, L_2 will be $(0,0)$ ~~x=y~~

Solution guide:

- List out all the points (x, y) of each line in the given domain D;
- Solve the equations diagrammatically with the aid of the *coordinate grid* given below. Explain with the concept of set.
- Express your findings with the aid of set builder representation, and write down the final answer in set form.

$(0,0)$ - solution $x=0$, $y=0$



L_1: Each point's x-coordinate added it's y-coordinate equals 0, x y has opposite value

L_2: Each point's x-coordinate minus it's y-coordinate equals 0 x and y has same value

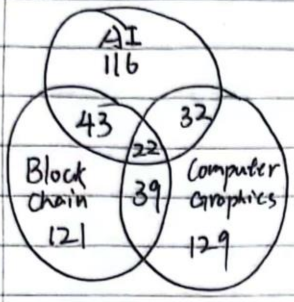
Set $A(L_1)$: $(-3,3)$ $(-2,2)$ $(-1,1)$ $(0,0)$ $(1,-1)$ $(2,-2)$ $(3,-3)$

Set $B(L_2)$: $(-3,-3)$ $(-2,-2)$ $(-1,-1)$ $(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$

Question 4

(Question 4)

1) Venn Diagram



In the Venn Diagram, We set students taking AI as set A, Students taking blockchein as set B, Students taking Computer Graphics as set C

Then we get

$|A| = 116 \quad |B| = 121 \quad |C| = 129$

$|A \cap B| = 43 \quad |B \cap C| = 39 \quad |A \cap C| = 32$

$|A \cap B \cap C| = 22$

To calculate the number of students of Comp, according to the Principle of Inclusion-Exclusion

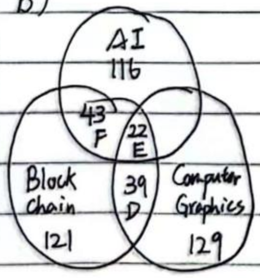
We have $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 116 + 121 + 129 - 43 - 39 - 32 + 22$

$= 274$

Number of COMP Students is 274

b)



From above Venn Diagram, and question, we know a student has to complete at lease 2 cources and one of the course has to be Blockchain to receive FinTech major achievement

The students would be the result of D, E, F

$D = 39 - 22 = 17$

$E = 22$

$F = 43 - 22 = 21$

Students that receive FinTech major achievement = $D + E + F = 17 + 22 + 21 = 60$

60 students will be eligible to receive a degree certificate with a FinTech major

Question 5

Question 5

a) Proof: Truth Table

p	q	$p \to q$	$p \land (p \to q)$	$p \land (p \to q) \to q$
T	T	T	T	T
T	F	F	F	F
F	T	~~F~~ T	~~F~~ T	T
F	F	~~F~~ T	~~F~~ T	T

Therefore, Modus ponens is proved

b) Proof:

Simplication: $p \land q = (p \land q) \to p$

Then, addition: $p = p \to (\cancel{p \lor q}) \; p \lor s)$

Since $(p \lor s) \to \neg r$ and $\neg r$ can be implied

~~Bu~~ Also, $r \lor t$

then Disjunctive syllogism $((r \lor t) \land \neg r \to t$

We get t at last

c) Set p as a student is known to be checting

q as a student is good

r as a student will pass

then $\neg p$ = not cheat $\neg r$ = student fail $\neg q$ = not good

i) This statement if using our set statements, will be if p then q but will be unvalid since $(p \land (p \to \neg r)) = \neg r$ and $\neg r \land (q \to r = \neg q$ the result will bem a student isn't good and will not pass

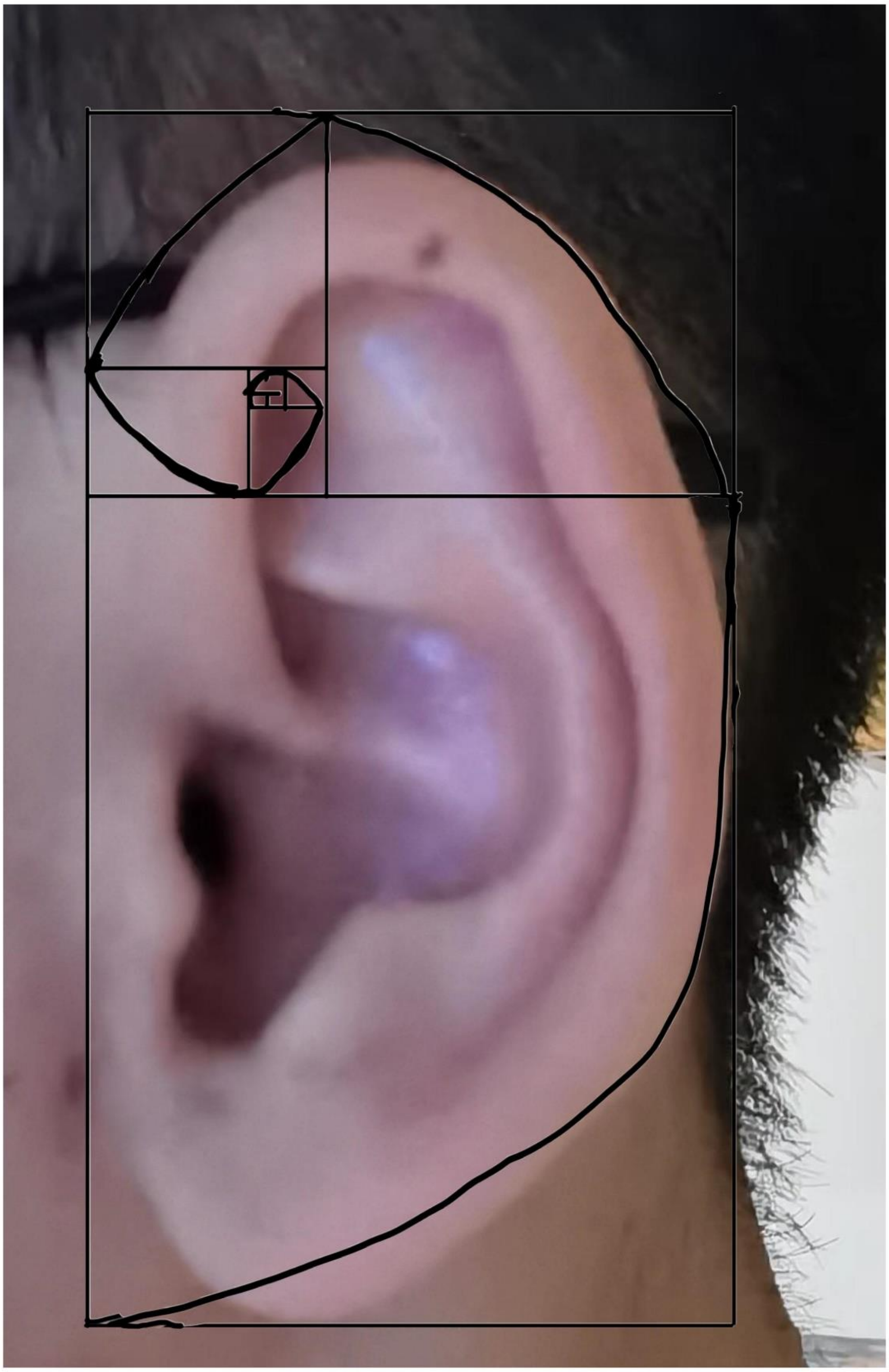
ii/ This statement can be represent as if $\neg p$ then $\neg q$. However, there are no rules to prove and then it would be an unvalid statement

iii) This statement can be represent as if q then $\neg p$. Using (modus ponens) $q \land (q \to r) = r$. We can conclude this statement is valid

iv) This statement can be represent as if $\neg p$ then $\neg q$. But there are no rules to prove so we consider it as unvalid.

Only iii statement ~~is~~ follows logic of statement

Question 6



Picture of my ear with golden spiral(a beautiful gift from my parents)

Question 7

Question 7

a) ~~$D^T =$~~ $D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $D^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ $E = \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$

$$D^T \times E = \begin{bmatrix} 1\times7+3\times9+5\times11 & 1\times8+3\times10+5\times12 \\ 2\times7+4\times9+6\times11 & 2\times8+4\times10+6\times12 \end{bmatrix} = \begin{bmatrix} 7+27+55 & 8+30+60 \\ 14+36+66 & 16+40+72 \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 83 & 84 \\ 113 & 121 \end{bmatrix}} = \begin{bmatrix} 89 & 98 \\ 116 & 128 \end{bmatrix}$$

$$= \begin{bmatrix} 89 & 98 \\ 116 & 128 \end{bmatrix}$$

b) Proof: $A^2 = A \cdot A$ $B^2 = B \cdot B$

Left Hand side

$A^2 = A \cdot A$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times2 & 1\times0+0\times-1 \\ 2\times1+-1\times2 & 2\times0+-1\times-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$B^2 = B \cdot B$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+-1\times1 & 2\times-1+-1\times3 \\ 1\times2+3\times1 & 1\times-1+3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ -5 & -7 \end{bmatrix}$$

Right Hand Side

$A - B$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 0-(-1) \\ 2-1 & -1-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}$$

$A + B$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 0+(-1) \\ 2+1 & -1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ 3 & 2 \end{bmatrix}$$

$(A-B) \times (A+B)$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times3+1\times3 & -1\times(-1)+1\times2 \\ 1\times3+(-4)\times3 & 1\times(-1)+(-4)\times2 \end{bmatrix}$$

$$= \begin{bmatrix} -3+3 & 1+2 \\ 3+(-12) & -1+(-8) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ -9 & -9 \end{bmatrix}$$

$LHS = \begin{bmatrix} -2 & 5 \\ -5 & -7 \end{bmatrix}$ $RHS = \begin{bmatrix} 0 & 3 \\ -9 & -9 \end{bmatrix}$

We can conclude $LHS \neq RHS$, which means $A^2 - B^2 = (A-B)(A+B)$ is false. $A^2 - B^2$ doesn't equal $(A-B)(A+B)$

c) $C^3 = C\cdot C\cdot C$

$$= \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\cdot\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\cdot\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$= \begin{bmatrix}1\times1+0\times0+2\times2 & 1\times0+0\times2+2\times0 & 1\times2+0\times1+2\times3\\ 0\times1+2\times0+1\times2 & 0\times0+2\times2+1\times0 & 0\times2+2\times1+1\times3\\ 2\times1+0\times0+3\times2 & 2\times0+0\times2+3\times0 & 2\times2+0\times1+3\times3\end{bmatrix}\cdot\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$= \begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}\cdot\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$= \begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\cdot\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$\cancel{=\begin{bmatrix}5\times1+\end{bmatrix}}$$

$$= \begin{bmatrix}5\times1+0\times0+8\times2 & 5\times0+0\times2+8\times0 & 5\times2+0\times1+8\times3\\ 2\times1+4\times0+5\times2 & 2\times0+4\times2+5\times0 & 2\times2+4\times1+5\times3\\ 8\times1+0\times0+13\times2 & 8\times0+0\times2+13\times0 & 8\times2+0\times1+13\times3\end{bmatrix}$$

$$= \begin{bmatrix}5+0+16 & 0+0+0 & 10+0+24\\ 2+0+10 & 0+8+0 & 4+4+15\\ 8+0+26 & 0+0+0 & 16+0+39\end{bmatrix} = \begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$$

$6C^2$ = calculated above $C\cdot C\cdot 6$

$$= \begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\times 6$$

$$= \begin{bmatrix}5\times6 & 0\times6 & 8\times6\\ 2\times6 & 4\times6 & 5\times6\\ 8\times6 & 0\times6 & 13\times6\end{bmatrix} = \begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}$$

$7C = C\cdot 7$

$$= \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\times 7 = \begin{bmatrix}1\times7 & 0\times7 & 2\times7\\ 0\times7 & 2\times7 & 1\times7\\ 2\times7 & 0\times7 & 3\times7\end{bmatrix} = \begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}$$

$$I_3 = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \quad O_3 = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

C^3-6C^2+7C

$$= \begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix} - \begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix} + \begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}$$

$$\cancel{= \begin{bmatrix}21-30+7 & 0-0+24\\ 12-12+0 & \\ 34-48+14 & \end{bmatrix}} = \begin{bmatrix}21-30 & 0-0 & 34-48\\ 12-12 & 8-24 & 23-30\\ 34-48 & 0-0 & 55-78\end{bmatrix} + \begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}$$

$$= \begin{bmatrix}-9&0&-14\\0&-16&-7\\-14&0&-23\end{bmatrix} + \begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}$$

$$= \begin{bmatrix}-9+7 & 0+0 & -14+14\\ 0+0 & -16+14 & -7+7\\ -14+14 & 0+0 & -23+21\end{bmatrix} = \begin{bmatrix}\cancel{-16}\ -2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{bmatrix}$$

$C^3-6C^2+7C \cancel{m} + kI_3 = O_3$

$$= \begin{bmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{bmatrix} + \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\times k = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} \rightarrow \begin{bmatrix}1k-2&0&0\\0&1k-2&0\\0&0&1k-2\end{bmatrix} = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$1\times(k) + (-2) = 0$

$k = 2$

The value of k is $\underline{2}$.

Question 8

Question 8

a)

Is-Prime-A(n)	Cost	Frequency
for integer i ← 2 to n-1	C_1	At most n-1
if n mod i == 0 then	C_2	At most n-1
return False	C_3	At most 1
return True	C_4	At most 1

Is-Prime - B(n)	Cost	Frequency
for integer i ← 2 to floor(sqrt(n))+1 ~~sqrt(n-1)~~	C_5	At most sqrt ~~(n-2)~~ (n-1)+1
if n mod i == 0 then	C_6	At most sqrt ~~(n-2) (n)+~~ (n-1)+1
return False	C_7	At most 1
return True	C_8	At most 1

O (Final complexity for Is-Prime-A(n) = $C_1 + C_2 + C_3 + C_4 = O($~~sqrt~~$(n-1)) + O($~~sqrt~~$(n-1)) + O(1) + O(1) = O(n)$

O (Final complexity for Is-Prime-B(n) = $C_5 + C_6 + C_7 + C_8 = O(\text{sqrt}(n-1)) + O(\text{sqrt}(n-1)) + O(1) + O(1) = O(\text{sqrt}(n))$

b) Is-Prime ~~(B)~~ B(n) is more efficient because when n gets larger, sqrt(n-1) will be much smaller than (n-1). This makes the range of numbers fewer for Is-Prime B(n) to look for which leads to fewer iterations on finding out the result. So Is-Prime B(n) is more efficient.

Question 9

Question 9

Mathemotical Induction

$LHS = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots - \frac{1}{2n}$ $RHS = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n}$

Basis step: When $=1$ $LHS = 1 - \frac{1}{2} = \frac{1}{2}$ $RHS = \frac{1}{1+1} = \frac{1}{2}$ $LHS = RHS$ $\forall 1 \in \mathbb{Z}^+$ true

Inductive step: We assume the statement is true for $k=n$

then equction $LHS = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{(-1)^{k-1}}{2k}$ $RHS = \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k}$

If we set $n = k+1$ equction will beome

$LHS = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{(-1)^{-1}}{2k} + \frac{1}{2k+1} - \frac{1}{2k+2}$

Because $n=k$, $LHS = RHS = \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k}$

Thus when $n=k+1$ $LHS = \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} + \frac{1}{2k+1} - \frac{1}{2k+2}$

Simplify we get $LHS = \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} + \frac{1}{2k+1} + (\frac{1}{k+1} - \frac{1}{2k+2})$

$= \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2k+2}$

Cause $n=k+1$, then first element for right Hand side is $\frac{1}{k+2}$

$RHS = \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2k+2}$

$LHS = RHS$

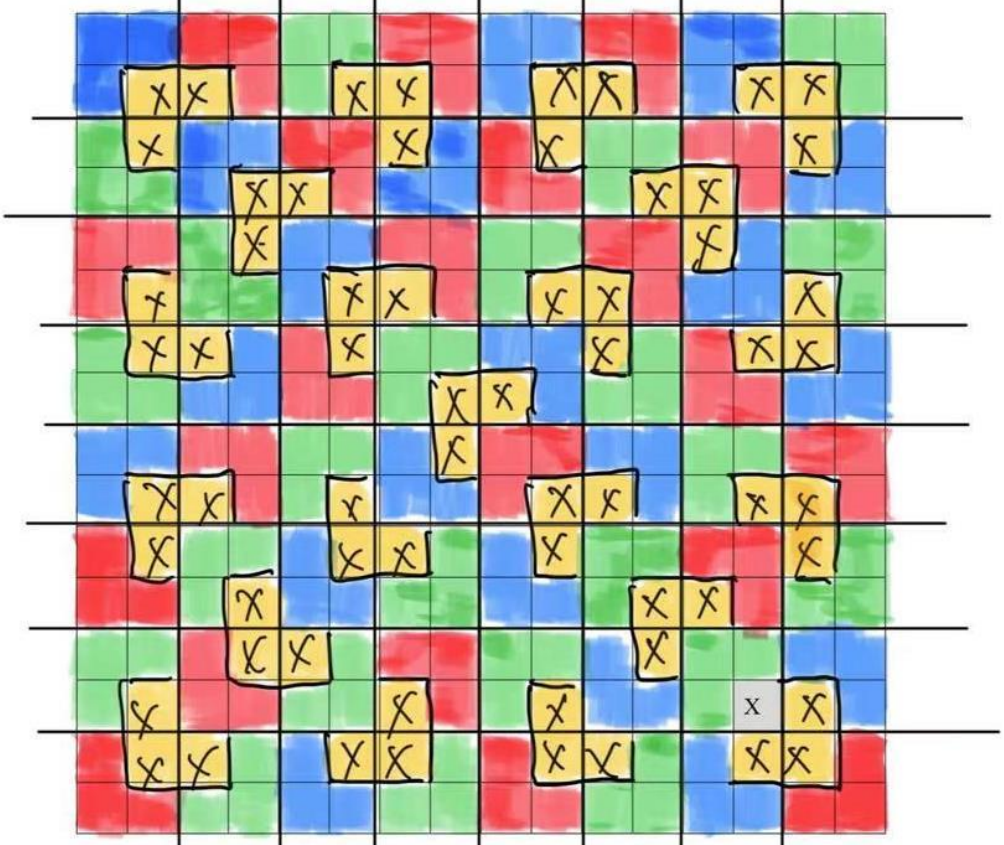
Thus, we con prove thot $\forall n \mathbb{Z}^+$ $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} \dots + \frac{1}{2n}$

Question 10

Question 10 **[10 marks]**

We are referring to the tiling problem in "MI: example 3" in the slides of lecture #5.

Now, we are given the following 16×16 checkboard with a missing square (X).



A triomino looks like one of the followings.