AMA2104 Random Class Quiz 2

NAME: **ID:**

Problem 1. The joint probability mass function of X and Y is given by

$$p(x,y) = \frac{x+y}{30}, \text{ for } x = 0,1,2,3 \text{ and } y = 0,1,2.$$

Find $P(X+Y=3)$.

Solution:
$P(X+Y=3) = P(X=1,Y=2) + P(X=2,Y=1) + P(X=3,Y=0) = \frac{3}{10}$

Problem 2. Let X and Y be continuous random variables having the joint density function

$$f(x,y) = \begin{cases} 4xy, & 0 < x < 1, \quad 0 < y < 1 \\ 0 & \quad elsewhere \end{cases}$$

Find $P(X > 2Y)$.

Solution:

$$P(X > 2Y) = \int_0^1 \int_0^{\frac{x}{2}} 4xydydx = \int_0^1 2x \cdot (\frac{x}{2})^2 dx = \frac{1}{8}$$

Problem 3. Let X and Y be continuous random variables having the joint density function

$$f(x,y) = \begin{cases} Cxy, & -1 < x < 0, \quad x < y < 0 \\ 0 & \quad elsewhere \end{cases}$$

where C is an unknown constant to be determined. Find C.

$$\iint_\Omega f(x,y)dxdy = \int_{-1}^0 \int_x^0 Cxydydx = C\int_{-1}^0 x \cdot \left(-\frac{x^2}{2}\right) dx = \frac{C}{8} = 1 \Rightarrow C = 8$$

Problem 4. A car dealer sells 0, 1, 2 or 3 luxury cars on any day with a uniform distribution. After selling a car, the dealer also tries to persuade the customers to buy an extended warranty for the car. Each customer has 50-50 chance of whether to buy the warranty or not. Let X denote the number of luxury cars sold on a given day, and let Y denote the number of extended warranties sold. Write down the joint probability mass function $p(x,y)$ as a table.

Solution:

(a) The joint probability mass function $p(x,y)$ is given by

		y			
		0	1	2	3
	0	$\frac{1}{4}$	0	0	0
x	1	$\frac{1}{8}$	$\frac{1}{8}$	0	0
	2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	0
	3	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{1}{32}$

Problem 5. Given the joint probability density functions of X, Y as follows, (i)$f(x,y) = xe^{-x(y+1)}$, $x > 0$, $y > 0$; (ii) $f(x,y) = xe^{-(x+y)}$.
Please determine whether X, Y are independent ? You may NOT compute their marginal density functions directly, instead, you may apply more convenient method such as separation method.

By factorization theorem, if $f(x,y) = h(x) \cdot g(y) \Rightarrow X \perp\!\!\!\perp Y$ otherwise, $X \not\!\perp\!\!\!\perp Y$.

$$\begin{aligned} f(x,y) &= xe^{-(x+y)} \\ &= xe^{-x} \cdot e^{-y} \\ &= h(x) \cdot g(y)) \Rightarrow X \perp\!\!\!\perp Y \end{aligned}$$

$$\begin{aligned} f(x,y) &= xe^{-x(y+1)}, x > 0, y > 0 \\ f_X(x) &= \int_0^{+\infty} f(x,y)dy \\ &= \int_0^{+\infty} xe^{-x(y+1)}dy \\ &= e^{-x(y+1)}\Big|_{y=0}^{y=+\infty} \\ &= e^{-x} \\ f_Y(y) &= \int_0^{+\infty} f(x,y)dx \\ &= \int_0^{+\infty} xe^{-x(y+1)}dx \\ &\overset{\text{IBP}}{=} \frac{xe^{-x(y+1)}}{-(y+1)}\Bigg|_{x=0}^{x=+\infty} - \int_0^{+\infty} \frac{e^{-x(y+1)}}{-(y+1)}dx \\ &= \cdots = \frac{1}{(y+1)^2} \\ f(x,y) &\neq f_X(x) \cdot f_Y(y) \Rightarrow X \not\!\perp\!\!\!\perp Y \end{aligned}$$