Answer all questions: (Full marks=100)

Section A (short questions)

1. Evaluate the following limits.

(a) $\lim_{x \to 32} \sqrt[5]{x}$ (b) $\lim_{x \to -2}(x^3 + 5x^2 - 3x + 1)$

[6 marks]

2. Evaluate the following limits:

(a) $\lim_{x \to 0} \frac{\sin 2x}{\tan 3x}$ (b) Evaluate $\lim_{x \to -\infty} \frac{4x-3}{x^2+2x-1}$

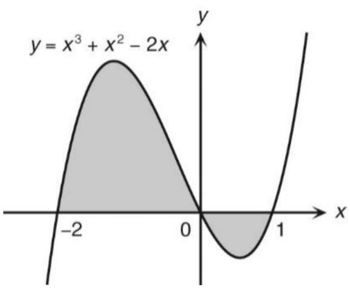
[7 marks]

3.

(a) Find $\int \left(3x^2 - \frac{2}{x}\right) dx$. (b) Find $\int (3x + e^{-2x})\, dx$.

[7 marks]

4. Find the area of the region bounded by the curve $y = x^3 + x^2 - 2x$ and the x-axis.



[8 marks]

5. Let $f(x) = 5x^2$. Find $f'(1)$ from first principles. [8 marks]

6. Find $\frac{dy}{dx}$ of the following functions.

(a) $y=(2-\frac{5}{x})(5+x^{\frac{2}{3}})$ **(b)** $y=\frac{4x^2-3x+3}{2x-3}$

[10 marks]

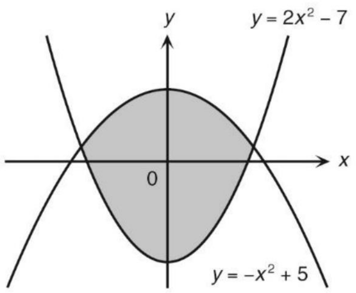
7. Find $\frac{dy}{dx}$ of the following functions and simplify the answers.

(a) $y=\cos(\sin\sqrt{x})$ **(b)** $y=\tan(x\cos 2x)$

[8 marks]

8. Find $\int \frac{\tan x-\sin^2 x}{\cot x-\cos^2 x}\,dx$. [8 marks]

9. Find the area of the region bounded by the curves $y=2x^2-7$ and $y=-x^2+5$.



[8 marks]

Section B (Long Questions)

10. It is given that $g(x)=\frac{x^2-3x-10}{1-x}$.

(a) Find the domain of $g(x)$.

(b) **(i)** Find the x- and y-intercepts of the curve $y=g(x)$.

(ii) Find the turning points of the curve.

(c) Find the range of values of x for which the curve is

(i) concave upwards,

(ii) concave downwards.

(d) Find the asymptotes of the curve.

(e) Using the results of (a) – (d), sketch the curve $y=g(x)$. [15 marks]

11. **(a)** Using a suitable substitution, find $\int \tan^{n-2} x \sec^2 x \, dx$, where *n* is an integer and $n \geq 2$.

(b) **(i)** Using (a), show that $\int_0^{\frac{\pi}{4}} \tan^n x \, dx = \frac{1}{n-1} - \int_0^{\frac{\pi}{4}} \tan^{n-2} x \, dx$, where *n* is an integer and $n \geq 2$.

(ii) Hence, evaluate $\int_0^{\frac{\pi}{4}} \tan^3 x \, dx$.

(c) Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^3 x \, dx$.

[15 marks]

End of paper

The table of derivatives

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$
$\operatorname{cosec} x = \frac{1}{\sin x}$	$-\operatorname{cosec} x \cot x$
$\sec x = \frac{1}{\cos x}$	$\sec x \tan x$
$\cot x = \frac{\cos x}{\sin x}$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$
$\coth x$	$-\operatorname{cosech}^2 x$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

Table of Standard Integrals

1. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

2. $\displaystyle\int \frac{dx}{x} = \ln|x| + C$

3. $\displaystyle\int e^x\,dx = e^x + C$

4. $\displaystyle\int \sin x\,dx = -\cos x + C$

5. $\displaystyle\int \cos x\,dx = \sin x + C$

6. $\displaystyle\int \tan x\,dx = -\ln\left|\cos x\right| + C$

7. $\displaystyle\int \cot x\,dx = \ln\left|\sin x\right| + C$

8. $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

9. $\displaystyle\int \sec^2 x\,dx = \tan x + C$

10. $\displaystyle\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

11. $\displaystyle\int \sec x\,dx = \ln\left|\sec x + \tan x\right| + C$

12. $\displaystyle\int \operatorname{cosec} x\,dx = \ln\left|\operatorname{cosec} x - \cot x\right| + C$

13. $\displaystyle\int \sinh x\,dx = \cosh x + C$

14. $\displaystyle\int \cosh x\,dx = \sinh x + C$

15. $\displaystyle\int \tanh x\,dx = \ln\cosh x + C$

16. $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad (|x| < a)$

17. $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2+a^2}\right) + C'$

18. $\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2-a^2}\right) + C' \quad (x > a)$

Linearity: $\displaystyle\int (\lambda f(x) + \mu g(x))\,dx = \lambda \int f(x)\,dx + \mu \int g(x)\,dx$

Integration by substitution: $\displaystyle\int f(u(x))\frac{du}{dx}\,dx = \int f(u)\,du$

Integration by parts: $\displaystyle\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$
