AMA2104 Probability and Engineering Statistics

2024-25 Semester 2 Assignment 2

Due Date: April 13, Sunday 11:00 pm.

- Put the following information on the top right corner of the front page of your assignment.
  - Your name and student number
  - Subject code: AMA2104
  - Subject lecturer: Dr. James Huang
- **In all the computations, if you need to round a number, please keep four decimal places.**
- Highlight the final answers by boxing them.
- Photograph your solutions onto a **PDF** file named YourName-StuID, otherwise the marker (not James) cannot write on your solution, then you cannot see the marking but only the score.
- You may using the app "CamScanner" or other softwares. Make sure that the file is complete, legible, in correct order and orientation.
- Upload/attach your assignment solution pdf file at the same place you've downloaded this assignment by pressing the "Browse My Computer", then choose your pdf file, and then press Submit. You may re-submit the assignment again, to a maximum of **twice**, before the due time. After submitting, check and make sure your submission is successful. Your submission is via **polyu BB**.
- **No late submission is allowed. Email submission will not be marked.**

1. In a study conducted in the Forestry and Wildlife Department at Virginia Tech, J. A. Wesson examined the influence of the drug succinylcholine on the circulation levels of androgens in the blood. Blood samples were taken from wild, free-ranging deer immediately after they had received an intramuscular injection of succinylcholine administered using darts and a capture gun. A second blood sample was obtained from each deer 30 minutes after the first sample, after which the deer was released. The levels of androgens at time of capture and 30 minutes later, measured in nanograms per milliliter (ng/mL), for 15 deer are given in the table below.

Deer	Androgen (ng/mL)		d_i
	At Time of Injection	30 Minutes after Injection	
1	2.76	7.02	4.26
2	5.18	3.10	-2.08
3	2.68	5.44	2.76
4	3.05	3.99	0.94
5	4.10	5.21	1.11
6	7.05	10.26	3.21
7	6.60	13.91	7.31
8	4.79	18.53	13.74
9	7.39	7.91	0.52
10	7.30	4.85	-2.45
11	11.78	11.10	-0.68
12	3.90	3.74	-0.16
13	26.00	94.03	68.03
14	67.48	94.03	26.55
15	17.04	41.70	24.66

Assume that the populations of androgen levels at time of injection has a normal distribution $\mathrm{N}(\mu_1, \sigma_1^2)$ with unit ng/mL, and the androgen levels 30 minutes later has another normal distribution $\mathrm{N}(\mu_2, \sigma_2^2)$ with unit ng/mL. Draw a 95% confidence interval for $\mu_2 - \mu_1$.

2. Analysis of a random sample consisting of $m = 20$ specimens of cold-rolled steel to determine yield strengths resulted in a sample average strength of $\bar{x} = 29.8$ ksi. A second random sample of $n = 25$ two-sided galvanized steel specimens gave a sample average strength of $\bar{y} = 34.7$ ksi. Assuming that the two yield-strength distributions are normal with $\sigma_1 = 4.0$ and $\sigma_2 = 5.0$.

   (a) Find the 99% confidence interval of the difference $\mu_1 - \mu_2$ between the population means of the two normal distributions.

   (b) Does the data indicate that the corresponding true average yield strengths μ_1 and μ_2 are different? Carry out a test at significance level $\alpha = 0.01$. Write down the test statistic, its value, the rejection region, and the scientific conclusion as your answer.

3. Quantitative noninvasive techniques are needed for routinely assessing symptoms of peripheral neuropathies, such as carpal tunnel syndrome (CTS). The article "A Gap Detection Tactility Test for Sensory Deficits Associated with Carpal Tunnels Syndrome" (Ergonomics, 195: 2588 – 2601) reported on a test that involved sensing a tiny gap in an otherwise smooth surface by probing with a finger; this functionally resembles many work-related tactile activities, such as detecting scratches or surface defects. When finger probing was not allowed, the sample average gap detection threshold for $m = 8$ normal subjects was 1.71mm, and the sample standard deviation was 0.53mm; for $n = 10$ CTS subjects, the sample mean and sample standard deviation were 2.53mm and 0.87mm, respectively.

   (a) Does this data suggest that the true average gap detection threshold for CTS subjects exceeds that for normal subjects? Use significant level $\alpha = 0.01$.

   (b) Build a 99% confidence interval for $\mu_1 - \mu_2$, where μ_1 is the true average gap detection threshold for normal subjects, and μ_2 is this value for CTS subjects.

4. The firmness of a piece of fruit is an important indicator of fruit ripeness. The Magness-Taylor firmness was determined for one sample of 20 golden apples with a shelf life of zero days, resulting in a sample mean of 8.74 and sample standard deviation of 0.66, and another sample of 20 apples with a shelf life of 20 days, with a sample mean and sample standard deviation of 4.96 and 0.39, respectively.

   (a) Calculate a confidence interval for the differences between the true average firmness of zero-day apples and true average firmness of 20-day apples using a confidence level of 95%.

   (b) Test with significance level 5% whether there is a difference between the true average firmness of zero-day apples and true average firmness of 20-day apples.

5. Persons having Reynaud's syndrome are apt to suffer a sudden impairment of blood circulation in fingers and toes. In an experiment to study the extent of this impairment, each subject immersed a forefinger in water and the resulting heat output (cal/cm^2/min) was measured. For $m = 10$ subjects with the syndrome, the average heat output was $\bar{x} = 0.64$ with sample standard deviation 0.2, and for $n = 10$ nonsufferers, the average output was 2.05 , with sample standard deviation 0.4. Let μ_1 and μ_2 denote the true average heat outputs for the two types of subjects. Assume that the two distributions of heat output are normal with equal variance.

   (a) Build a 95% confidence interval for $\mu_1 - \mu_2$.

   (b) Test, under significance level 5%, the null hypothesis that $\mu_1 = \mu_2$, against the alternative hypothesis that $\mu_1 \neq \mu_2$.

   (c) Test, under significance level 5%, the null hypothesis that $\sigma_1^2 = \sigma_2^2$, against the alternative hypothesis that $\sigma_1^2 \neq \sigma_2^2$.

6. A soft-drink dispensing machine is said to be out of order if the variance of the contents exceeds 1.15 deciliters. If a random sample of 25 drinks from this machine has sample variance of 2.03 deciliters, does this indicate at the 0.05 level of significance that the machine is out of order?Assume that the contents are approximately normally distributed.

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