Comp2012

Assignment 2

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Question 1

a)

Determine the maximum flow of the network G in Figure 1-1,

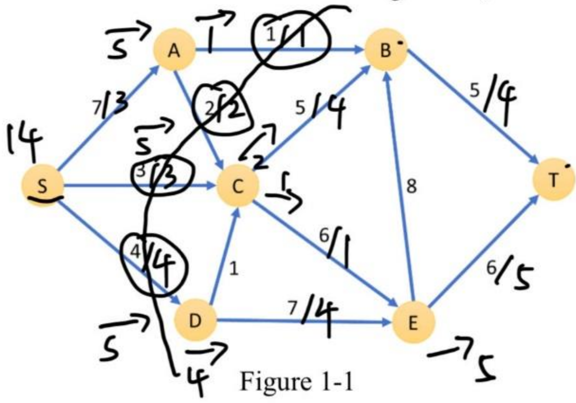


Figure 1-1

Maximum flow

$= 1+2+3+4$

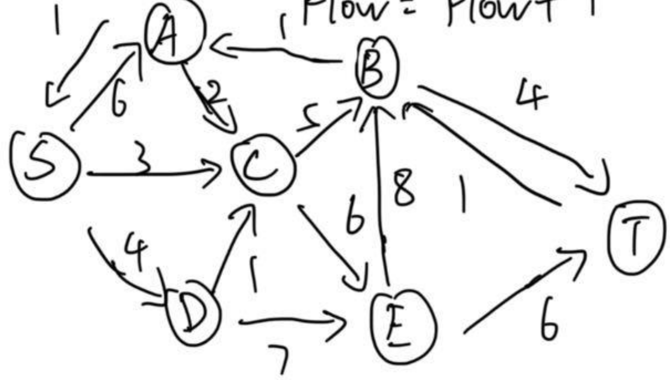
$= 10$

$\therefore$ Maximum flow is 10

1(a) Using the *max-flow min-cut* theorem. (3 marks) b)

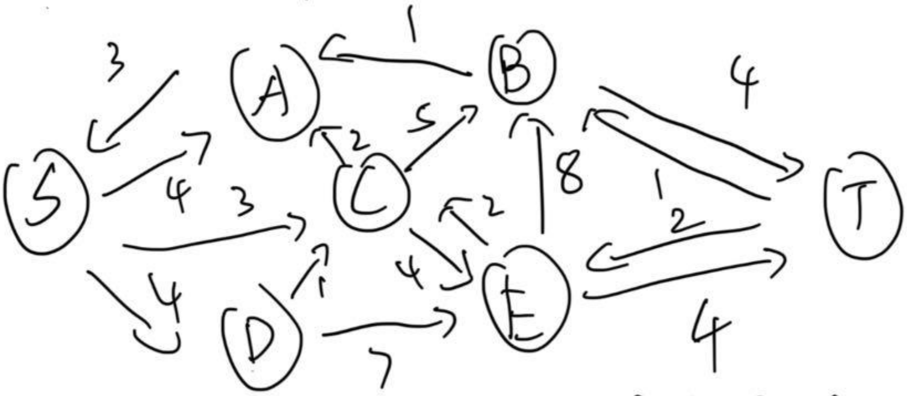
b) Iterction 1: Augmented Path 1 S-A-B-T

Flow = Flow + 1



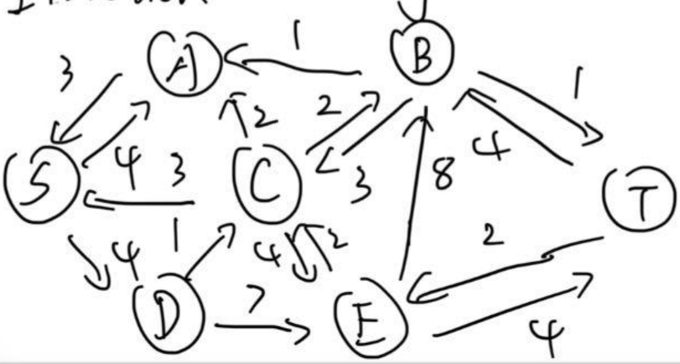
Iterction 2: Augmented Path 2 S-A-C-E-T

Flow = Flow + 2

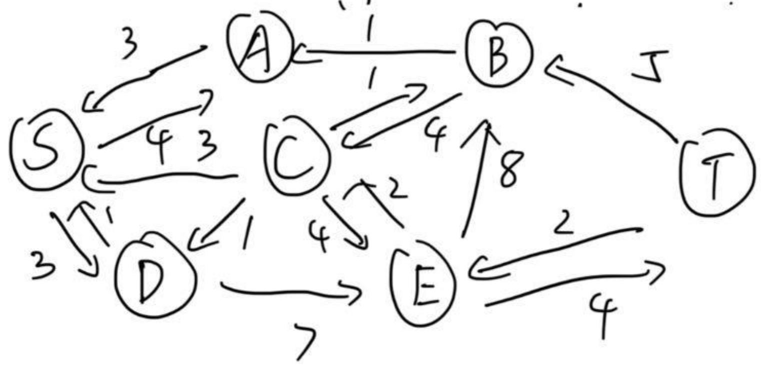


Iterction 3: Augmented Path 3 S C B T

Flow = Flow + 3

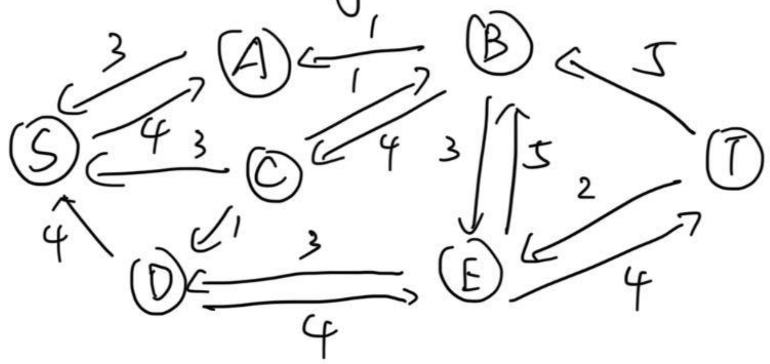


Iteration 4: Augumented Path 4 SDCBT



Flow = Flow + 1

Iteration 5: Augumented Path 5 SDEBT



Flow = Flow + 3

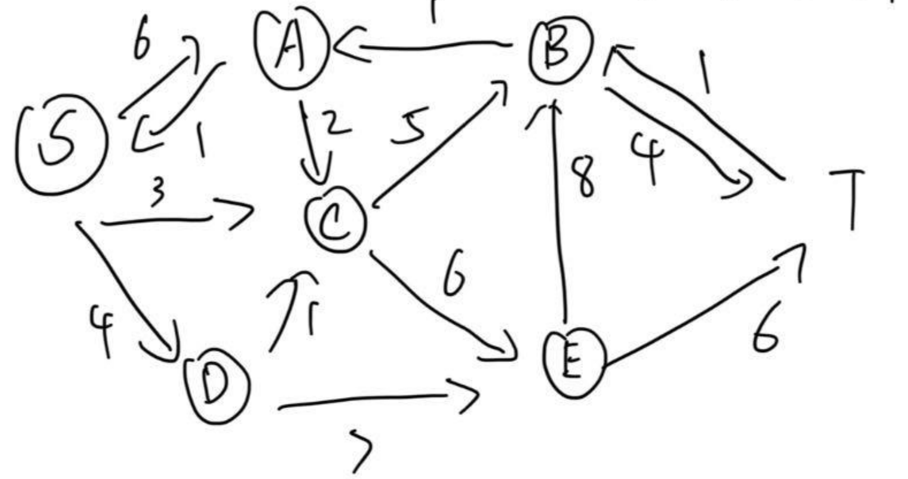
Maximum Flow = 1 + 2 + 3 + 1 + 3 = 10

c)

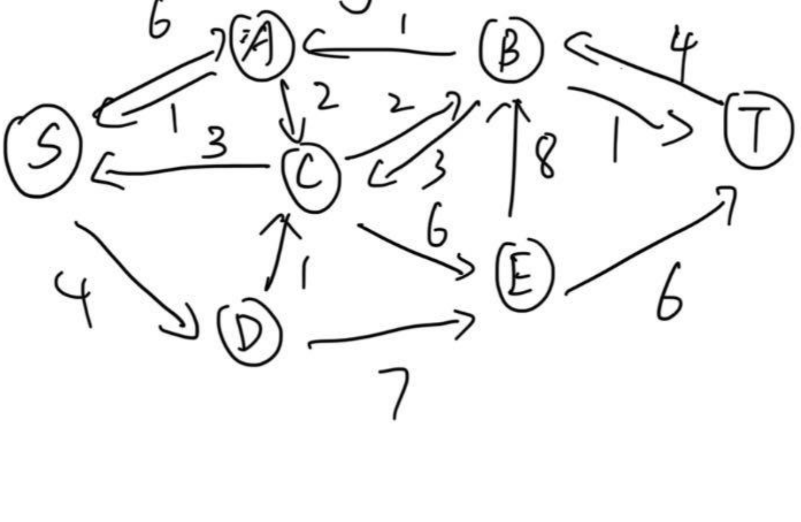
c) Flow = 0

Iteration 1: Augumented Path 1: S-A-B-T

Flow = 0 + 1

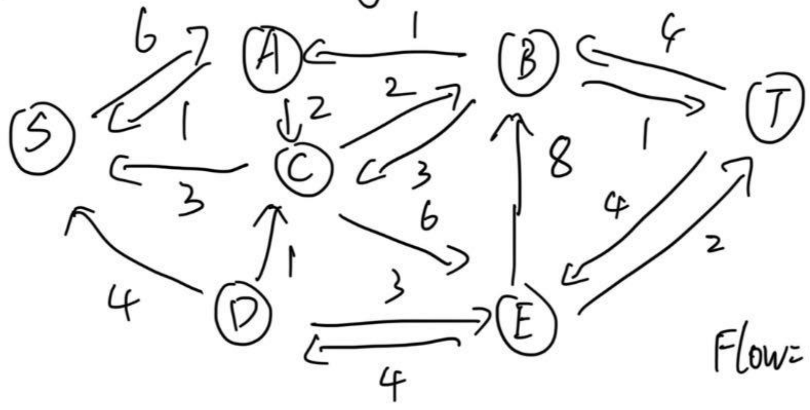


Iteration 2: Augumented Path 2: S-C-B-T



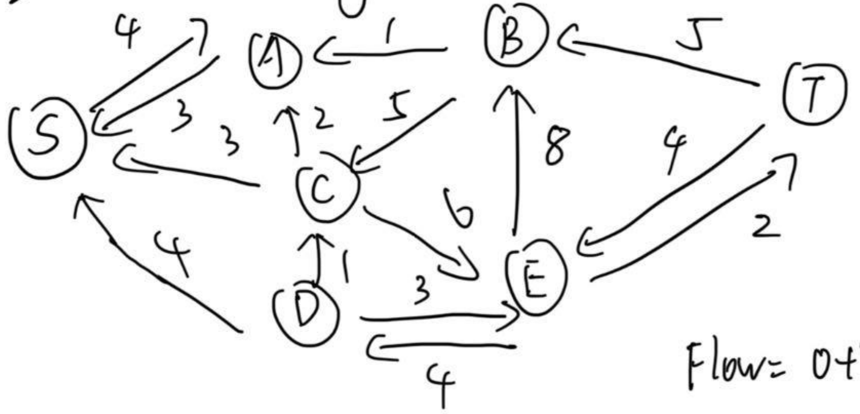
Flow = 0+1 +3

Iteration 3: Augmented Path 3: S-D-E-T



Flow=0+1+3+4

Iteration 4: Augmented Path 4: S-A-C-B-T



Flow= 0+1+3+4+2

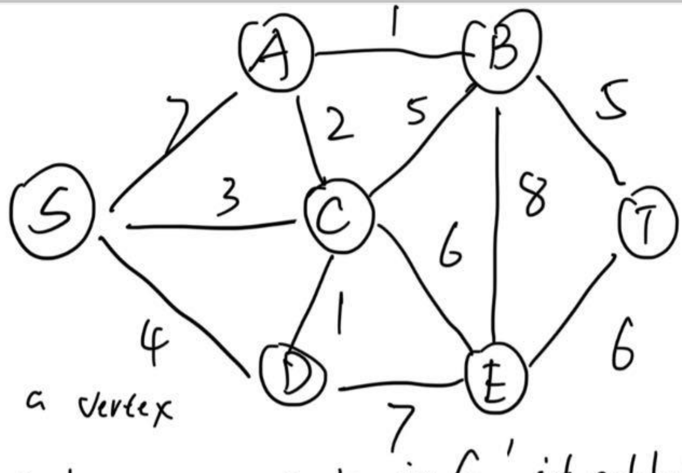
No more augmented path can be drawn. Hence maximum flow= 0+1+3+2+4= 10 is reached

d)

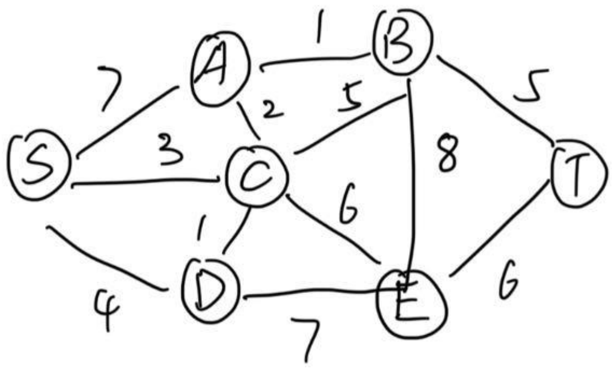
Yes, because for the same network G, to determine the maximum flow, the Ford-Fulkerson method used 5 iterations to find the maximum flow, while Edmonds Karp's method just required 4 iterations. So 1(c)'s algorithm outperforms 1(b)'s algorithm.

e)

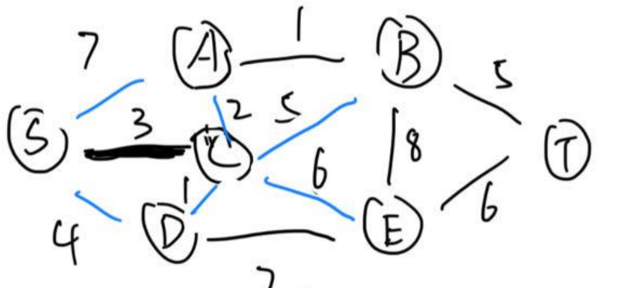
e) network G'
Prim algorithm
Let E' be set of edges that such edge is incident to a vertex in G' and it does not form any cycle in G' if added to G'



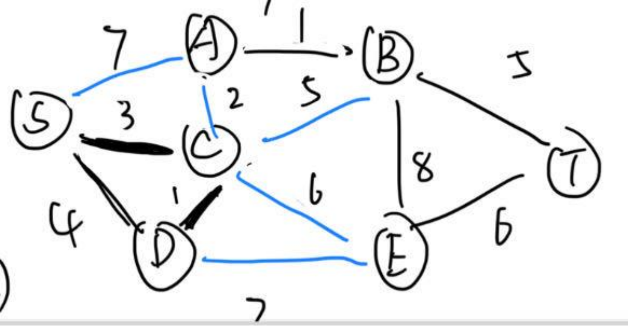
Initialization:
Pick (S, C)



Iteration 1:
E' shown in blue color
pick (C, D)



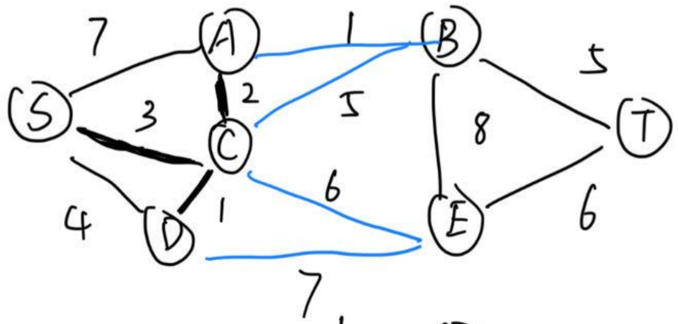
Iteration 2:
E' shown in blue color
pick (A, C)



Iteration 3:

E' show in blue color

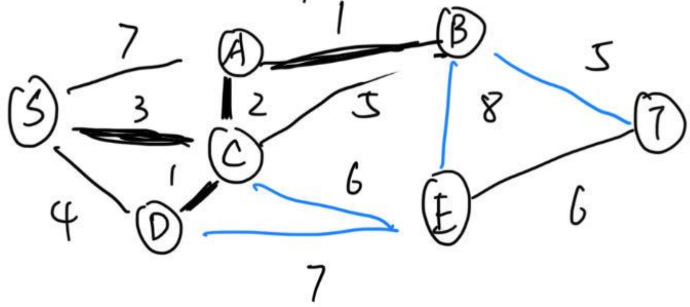
pick (A,B)



Iteration 4:

E' show in blue color

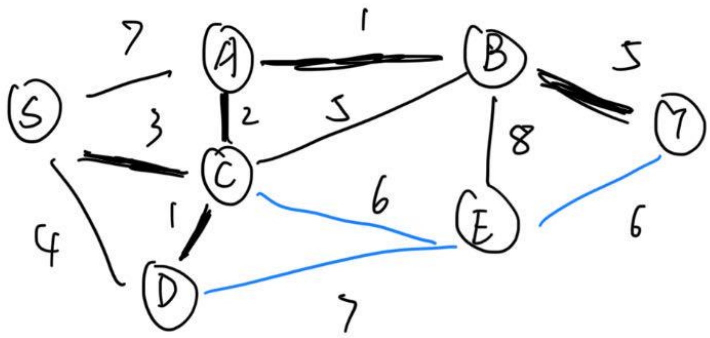
pick (B,T)



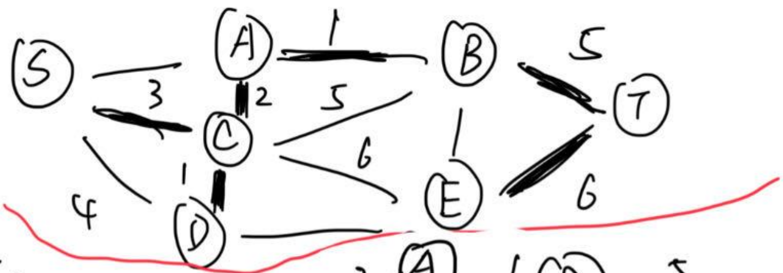
Iteration 5:

E' show in blue color

pick (T,E)



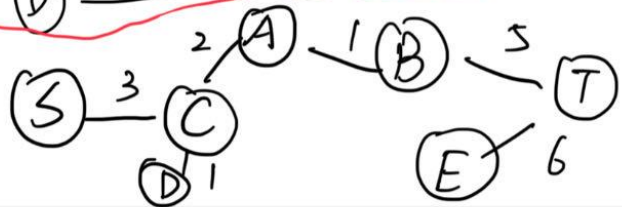
Iteration 6:



Final graph

Sum of weight = 3+2+1+1+5+6

= 20



Question 2

a)

Question 2)

a) Step 1: Unit property $x+\bar{x}=1$

$(A+\bar{A})=1 \quad (A+\bar{A})(AB+AB\bar{C})$

$= AB+AB\bar{C}$.

Step 2: Absurption laws

$x+x\cdot y=x$ assume $AB=x$

$AB+AB\bar{C} = x+x\cdot\bar{C}$

so $AB+AB\bar{C} = AB$

Therefore, Simplifying logic of $(A+\bar{A})\cdot(AB+AB\bar{C})$

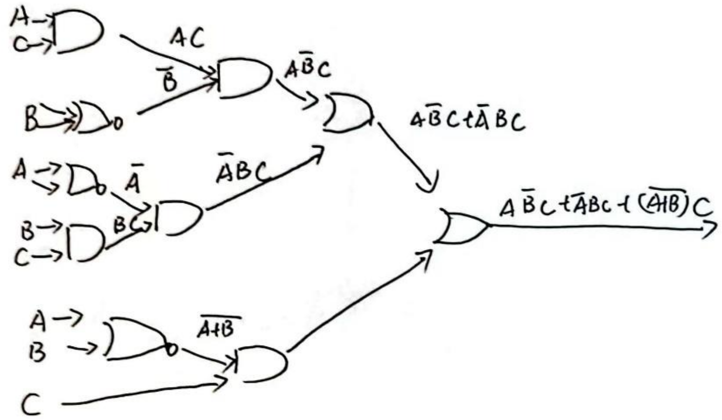
is: AB

b) and c)

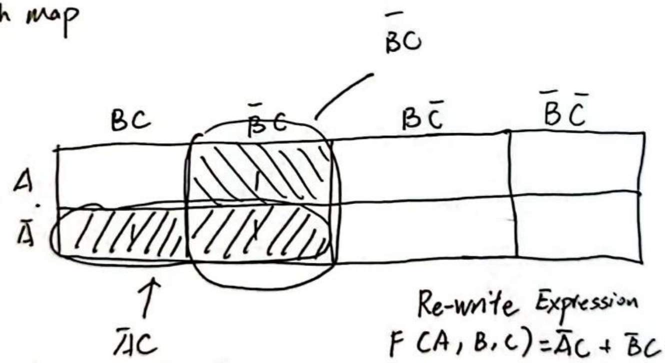
Q2

b) According to De Morgon's low $\bar{A}\bar{B} = \overline{A+B}$

$F(A,B,C) : A\bar{B}C + \bar{A}BC + \overline{(A+B)}C$



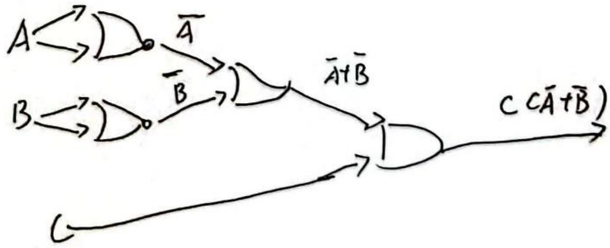
c) Karnough map



Re-write Expression

$F(A,B,C) = \bar{A}C + \bar{B}C = C(\bar{A}+\bar{B})$

Combinational circuit

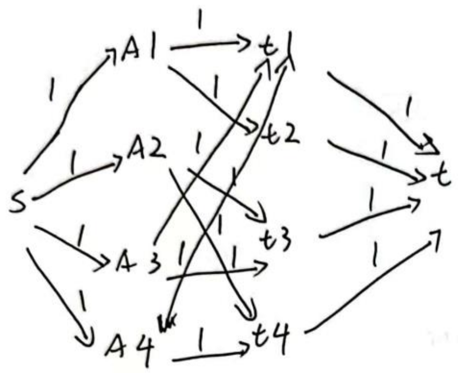


Question 3

(Q3) Assume Paul, Mary, Peter, Susie as A1, A2, A3, A4

Assume Tuesday Morning, Tuesday afternoon, Thursday morning, Friday afternoon as t1, t2, t3, t4

Graph (Bipartite) Initial



G1

$s \to A1 \to t1 \to t$

$\downarrow$

$s \xleftarrow{1} A1 \xleftarrow{1} t1 \xleftarrow{1} t$

G2

$s \xrightarrow{1} A2 \xrightarrow{1} t3 \xrightarrow{1} t$

$\downarrow$

$s \xleftarrow{1} A2 \xleftarrow{1} t3 \xleftarrow{1} t$

G3 $s \xrightarrow{1} A4 \xrightarrow{1} t4 \xrightarrow{1} t$

$\downarrow$

$s \xleftarrow{1} A4 \xleftarrow{1} t4 \xleftarrow{1} t$

G4 $s \xrightarrow{1} A3 \xrightarrow{1} t1 \xrightarrow{1} A1 \xrightarrow{1} t2 \xrightarrow{1} t$

$\downarrow$

$s \xleftarrow{1} A3 \xleftarrow{1} t1 \xleftarrow{1} A1 \xleftarrow{1} t2 \xleftarrow{1} t$

Use Edmonds - Karp Algorithm

Iteration 1 G1

Fewest edge < s, u1, t1, t >

Residual capacity : 1

Iteration 2 G2

Fewestest edge < s, u2, t3, t >

Residual capacity : 1

Iteration 3 G3

fewest edge < s, u4, t4, t >

Residual capacity : 1

Iteration 4 G4

Fewest edge < s, u3, t1, u1, t2, t >

Residual capacity : 1

Final result G5

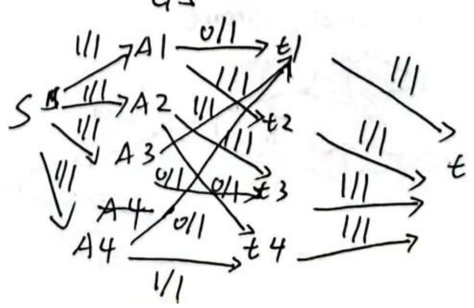
Paul: Tuesday afternoon

Mary: Thursday morning

Peter: Tuesday morning

Susie: Friday afternoon

G5

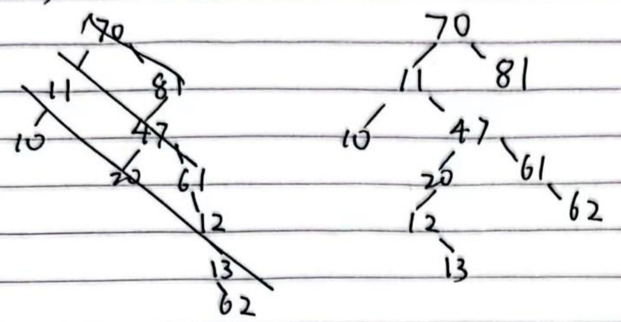


Question 4

2012 A2

Q 4)

4(a) 70 11 47 81 20 61 10 12 13 62



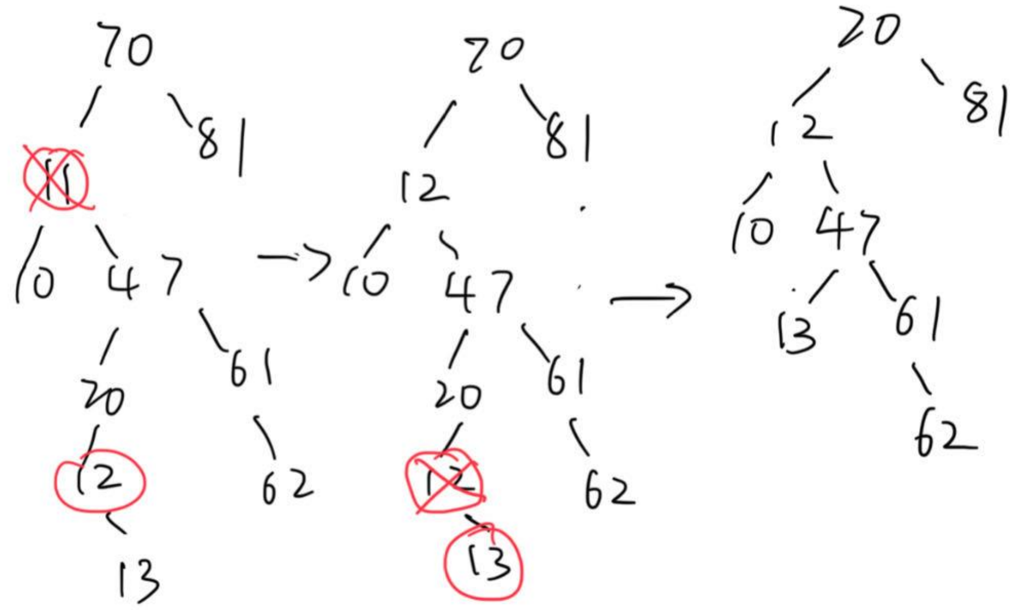
4(b) No, this tree isn't a balanced tree. Because for a balanced binary tree, the height of its left and right subtrees differ at most 1. But in the created tree, the height for right is 1 (81) and left is 4 (62) which doesn't meet condition.

4(c) 70 11 10 47 20 12 13 61 62 81

4(d) 10 13 12 20 62 61 47 11 81 70

4(e) 10 ~~12 1~~ 11 12 13 20 47 61 62 70 81 f)

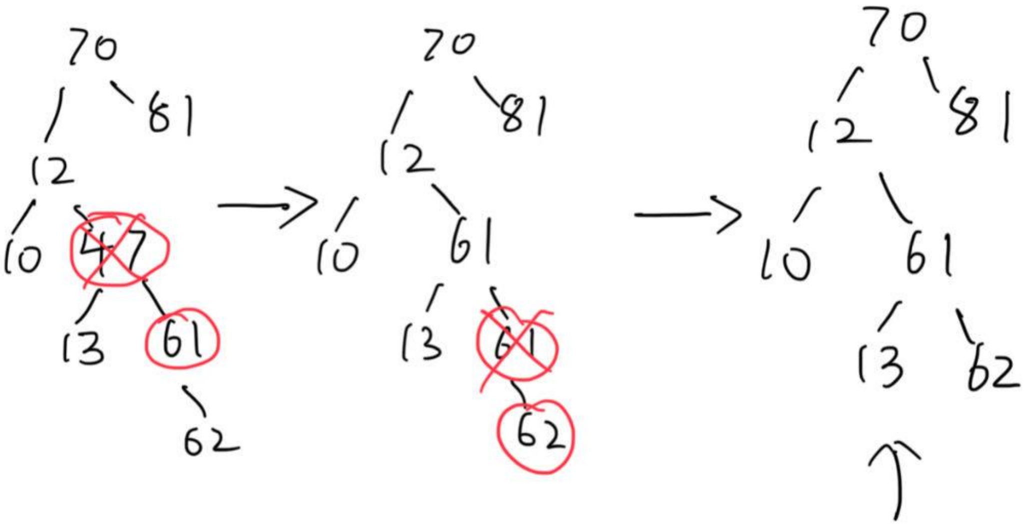
4f) Delete 11



Delete 11 then find 12 as successor to replace 11

12 replaced 11 then find 12's only child 13 to replace 12

Then Delete 47



Final result graph

Delete 47 then find 61 as its successor to replace

61 replace 47, then find it's only Child 62 to replace 61

Question 5

(Q5)

Use Dijkstra algorithm to find shortest Path

Initialization

Q = (Q:0) (T:∞) (R:∞) (S:∞) (M:∞) (VA:∞) (U:∞) (W:∞) (Y:∞)

Iteration 1

Extract vertex Q Update Vertices M,R,T

Q = (T:100) (R:100) (M:185) (S:∞) (VA:∞) (U:∞) (W:∞) (Y:∞)

S = (Q:0)

Iteration 2

Extract T, Update S,W,U

Q = (R:100) (M:185) (S:200) (U:200) (W:215) (VA:∞) (Y:∞)

S: (Q:0) (T:100)

Iteration 3

Extract R, update VA

Q = (M:185 ~~R:100~~) (S:200) (U:200) (W:215) (VA:271) (Y:∞)

S = (Q:0) (T:100) (R:100)

Iteration 4

Extract M ~~update~~

Q = ~~(R:100)~~ (S:200) (U:200) (W:215) (VA:271) (Y:∞)

S = (Q:0) (T:100) (R:100) (M:185)

Iteration 5

Extract S

Q: ~~R:100 S:200~~ (U:200) (W:215) (VA:271) (Y:∞)

S = (Q:0) (T:100) (R:100) (M:185) (S:200)

Iteration 6

Extract U, update Y

Q = (W:215) (VA:271) (Y: 283) S = (Q:0) (T:100) (R:100) (M:185) (S:200) (U:200)

Iteration 7

Extract W update Y

Q = (VA:271) (Y:276) S = (Q:0) (T:100) (R:100) (M:185) (S:200) (U:200) (W:215), ~~W~~

Iteration 8

Extract VA

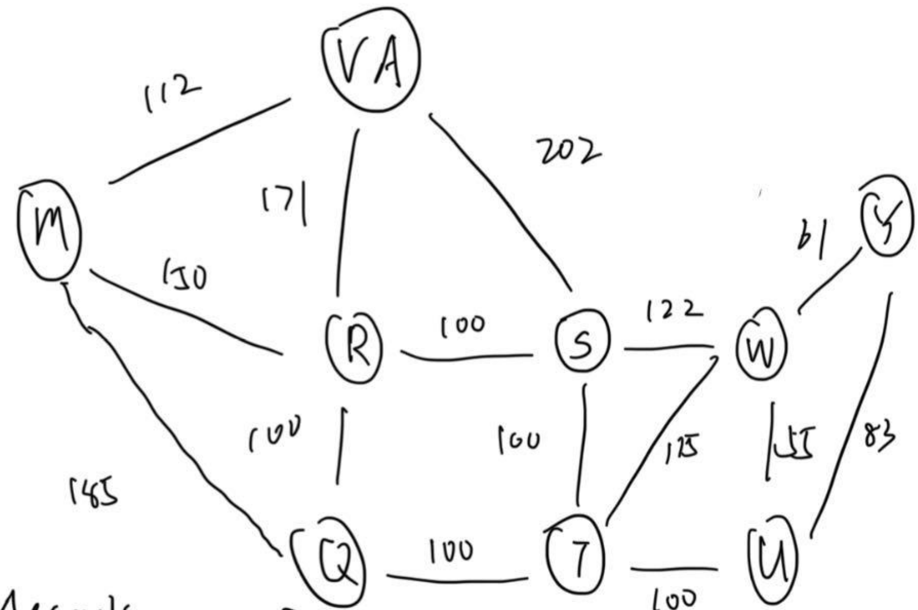
Q = (Y: 276) S = (Q:0) (T:100) (R:100) (M:185) (S:200) (U:200) (W:215) (VA: 245)

Iteration 9

Extract Y

Q = ∅ S = (Q:0) (T:100) (R:100) (M:185) (S:200) (U:200) (W:215) (VA:245) (Y:276)

Diagram for Question 5



5a According to Dijkstra algorithm

i) Q → M   185 meters

ii) Q → T → W → Y : 276 meters

iii) Q → R → VA: 271 meters

5b

Q → T → W → Y

Shortest Path Q → T → W → Y