1 **[7 marks]**Let

$$f(x) = \begin{cases} e^{-\frac{1}{x}}, & \text{if } x > 0, \\ ax + b, & \text{if } x \leq 0. \end{cases}$$

Find a and b such that $f(x)$ is differentiable everywhere and compute $f'(x)$.

2 **[12 marks, 4 marks each]** Evaluate the following limits:

(a) $\lim\limits_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

(b) $\lim\limits_{x \to 0} \dfrac{x^2 \sec^{1131}(1130x)}{\arcsin(x)}$

(c) $\lim\limits_{x \to -\infty} \dfrac{2x+1}{\sqrt{x^2+5}}$

3 **[7 marks]** Find all the local maxima and minima of f, hence find its global maximum and minimum, where $f(x) = x^3 - 3x^2 + 1$ on interval $[-\frac{1}{2}, 4]$.

4 **[7 marks]** Show that the equation $e^x = \frac{1}{2} + \cos(2x) - 2\sin x$ has a UNIQUE solution in the interval $\left(0, \dfrac{\pi}{4}\right)$.

5 **[7 marks]** Let $f(x) = 2x + \arcsin(x)$.
(a) Show that $f(x)$ is one-to-one on the closed interval [0,1].
(b) Use the linear approximation of f^{-1} around 0 to estimate $f^{-1}(0.05)$.

6 **[15 marks, 5 marks each]** Evaluate the following definite integrals:
(a)

$$\int_2^4 \frac{x+2}{x^2+3x-4}\, dx$$

(b)

$$\int x \tan^2(x)\, dx$$

(c)

$$\int_1^{\sqrt{3}} \tan^{-1}\left(\frac{1}{x}\right)\, dx$$

7 **[5 marks]** Evaluate the definite integral

$$\int_0^1 \ln\left(x^2+1\right)\, dx$$

8 **[10 marks]** Evaluate the definite integral

$$\int_1^{\sqrt{2}} \frac{1}{x\sqrt{4x^2+1}}\, dx$$

9 [**5 marks**] Find the definite integral

$$\int_0^{\frac{\pi}{2}} x^2 \sin(x)\, dx$$

10 [**5 marks**] Find the definite integral

$$\int_{-1}^{1} \frac{x^2}{\sqrt{4-x^2}}\, dx$$

11 [**10 marks**] Find the length of the arc of the parabola $y^2 = x$, $0 \leq x \leq 1$.

12 [**10 marks**] Find the values of c that satisfy the mean value theorem for integrals for the function $f(x) = \cos(2x - \pi)$ on the interval $[\frac{3\pi}{4}, \pi]$.

***** End *****

Table of Trigonometric Formulas

Basic formulas (n is an integer)

$\cos(2n\pi + A) = \cos A,$ $\quad \sin(2n\pi + A) = \sin A,$ $\quad \tan(n\pi + A) = \tan A$

$\cos(-A) = \cos A,$ $\quad \sin(-A) = -\sin A,$ $\quad \tan(-A) = -\tan A$

$\cos(\pi - A) = -\cos A,$ $\quad \sin(\pi - A) = \sin A,$ $\quad \tan(\pi - A) = -\tan A$

$\cos\left(\frac{\pi}{2} - A\right) = \sin A,$ $\quad \sin\left(\frac{\pi}{2} - A\right) = \cos A,$ $\quad \tan\left(\frac{\pi}{2} - A\right) = \cot A$

$\sin^2 A + \cos^2 A = 1,$ $\quad 1 + \tan^2 A = \sec^2 A,$ $\quad 1 + \cot^2 A = \csc^2 A$

$\cos 0 = 1,$ $\quad \cos(\pi/3) = 1/2,$ $\quad \cos(\pi/2) = 0$

$\sin 0 = 0,$ $\quad \sin(\pi/3) = \sqrt{3}/2,$ $\quad \sin(\pi/2) = 1$

Compound angle formulas

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\quad \sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\quad \cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Double angle formulas

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\cos^2 A = \frac{1 + \cos 2A}{2}, \quad \sin^2 A = \frac{1 - \cos 2A}{2}$

Conversion formulas

$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$ $\quad \sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$ $\quad \sin(x + y) - \sin(x - y) = 2 \cos x \sin y$

$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$ $\quad \cos(x + y) + \cos(x - y) = 2 \cos x \cos y$

$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$ $\quad \cos(x + y) - \cos(x - y) = -2 \sin x \sin y$

Table of Differentiation Formulas

$f(x)$	$f'(x)$	Remark
c	0	c is a constant
x^n	nx^{n-1}	n is a real number
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$\cot x$	$-\csc^2 x$	
$\sec x$	$\sec x \tan x$	
$\csc x$	$-\csc x \cot x$	
e^x	e^x	
a^x	$a^x \ln a$	$a > 0$, real constant
$\ln x$	$1/x$	$x > 0$
$\log_a x$	$(\log_a e)/x$	$a > 0$, real constant
$\sin^{-1} x$	$1/\sqrt{1-x^2}$	$-1 < x < 1$
$\cos^{-1} x$	$-1/\sqrt{1-x^2}$	$-1 < x < 1$
$\tan^{-1} x$	$1/(1+x^2)$	

Table of Integrals

	$f(x)$	$\int f(x)\,\mathrm{d}x$
1	$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
2	$\dfrac{1}{x} \quad (x \neq 0)$	$\ln\lvert x\rvert + C$
3	e^x	$\mathrm{e}^x + C$
4	$\sin x$	$-\cos x + C$
5	$\cos x$	$\sin x + C$
6	$\tan x$	$-\ln\lvert\cos x\rvert = \ln\lvert\sec x\rvert + C$
7	$\cot x$	$\ln\lvert\sin x\rvert + C$
8	$\sec x$	$\ln\lvert\sec x + \tan x\rvert + C$
9	$\csc x$	$-\ln\lvert\csc x + \cot x\rvert + C$
10	$\sec x \tan x$	$\sec x + C$
11	$\csc x \cot x$	$-\csc x + C$

	$f(x)$	$\int f(x)\,\mathrm{d}x$
12	$\dfrac{1}{a^2 + x^2} \quad (a \neq 0)$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$
13	$\dfrac{1}{a^2 - x^2} \quad (a \neq 0)$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert + C$
14	$\dfrac{1}{\sqrt{a^2 - x^2}} \quad (a > 0)$	$\sin^{-1}\left(\dfrac{x}{a}\right) + C$
15	$\dfrac{1}{\sqrt{x^2 + a^2}} \quad (a \neq 0)$	$\ln\lvert x + \sqrt{x^2 + a^2}\rvert + C$
16	$\dfrac{1}{\sqrt{x^2 - a^2}} \quad (a \neq 0)$	$\ln\lvert x + \sqrt{x^2 - a^2}\rvert + C$
17	$\sqrt{x^2 + a^2}$	$\dfrac{x}{2}\sqrt{x^2 + a^2} + \dfrac{a^2}{2}\ln\lvert x + \sqrt{x^2 + a^2}\rvert + C$
18	$\sqrt{x^2 - a^2}$	$\dfrac{x}{2}\sqrt{x^2 - a^2} - \dfrac{a^2}{2}\ln\lvert x + \sqrt{x^2 - a^2}\rvert + C$
19	$\sqrt{a^2 - x^2} \quad (a > 0)$	$\dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x}{a}\right) + C$