The Hong Kong Polytechnic University
Department of Applied Mathematics

AMA1501 Introduction to Statistics for Business/
AMA1602 Introduction to Statistics

2023/24 Semester Two

Assignment Solution Outline

This is part of the solutions only. You need to write more to get full marks in test/exam.

1. (a) mean $= \dfrac{8.5 \times 24 + 26 \times 36 + 39.5 \times 13 + 49.5 \times 12 + 59.5 \times 9 + 72.5 \times 6}{100} = 32.18$

   $\sum fx = 3218$
   $\sum fx^2 = 139156$
   s.d. $= 18.9632$
   mode $= 17.5 + \dfrac{12}{12+23}(34.5 - 17.5) = 23.3286$

   (b) $SK = \dfrac{\text{mean} - \text{mode}}{\text{s.d.}} = 0.4668$

   (c) mean increases by 2
   mode increases by 2
   s.d. no change

   (d) $D_8 = 44.5 + \dfrac{7}{12}(54.5 - 44.5) = 50.333$

   (e) $\dfrac{\dfrac{34.5-30}{34.5-17.5} \times 36 + 13 + \dfrac{50-44.5}{54.5-44.5} \times 12}{100} = 29.13\%$

2. (a) ${}_6C_3 \times {}_8C_3 \times {}_7C_4 \times {}_3C_1 = 20 \times 56 \times 35 \times 3 = 117600$

   (b) If the additional card is put into an empty envelope then there will be 2 empty envelopes remaining, otherwise there will be 3 empty envelopes.
   The required probability $= \dfrac{3}{7} \times \dfrac{2}{7} + \dfrac{4}{7} \times \dfrac{3}{7} = \dfrac{18}{49}$

   (c) Let A and B be the events that components A and B fail, respectively.
   $P(A \cap B') = 0.3$
   $P(B) = 0.4$
   $P(A \cap B) = 0.2$
   $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(A \cap B)}{P(A \cap B) + P(A \cap B')} = \dfrac{0.2}{0.2 + 0.3} = 0.4$

   (d) $P(\text{at least 1 correct}) = 1 - P(\text{all wrong})$
   $= 1 - (1 - 0.3)(1 - 0.4)(1 - 0.5) = 0.79$

   (e) W – the ball drawn is white
   A, B, C – bags A, B, C are chosen, respectively

   $P(W|A) = \dfrac{3}{5}, \quad P(W|B) = \dfrac{3}{7}, \quad P(W|C) = \dfrac{4}{9}$

   $P(A) = \dfrac{1}{2}, \quad P(B) = \dfrac{1}{3}, \quad P(C) = \dfrac{1}{6}$

   $$P(B|W) = \frac{\frac{1}{3} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{3} \times \frac{3}{7} + \frac{1}{6} \times \frac{4}{9}} = 0.2763$$

3. (a) i. X = no. of contaminated distilled water bottles in a box of 5 bottles
$X \sim b(5, 0.05)$
$P(X < 3) = (1 - 0.05)^5 + 5(0.05)(1 - 0.05)^4 + 10(0.05)^2(1 - 0.05)^3$
$= 0.7738 + 0.2036 + 0.02143 = 0.9988$

ii. Y = no. of contaminated distilled water bottles in 25 boxes (= 125 bottles)
$Y \sim b(125, 0.05)$

Since $n > 100$, $p < 0.1$, use Poisson approximation.
$Y \sim Po(6.25)$
$$P(Y > 3) = 1 - P(Y \leq 3) = 1 - e^{-6.25}\left(1 + \frac{6.25}{1} + \frac{6.25^2}{2!} + \frac{6.25^3}{3!}\right) = 0.8697$$

iii. W = no. of boxes having no contaminated distilled water bottle
p = probability that no contaminated distilled water bottle in a box
$W \sim b(25, p)$, where $p = P(X = 0) = 0.7738$

Since $np, nq > 5$, use normal approximation.
$np = 19.345$, $npq = 4.3758$
$W \sim N(19.345, 4.3758)$
$$P(W > 15) = P\left(Z > \frac{15.5 - 19.345}{\sqrt{4.3758}}\right) = P(Z > -1.84) = 1 - 0.0329 = 0.9671$$

(b) i. X = usage of the equipment in a week
$X \sim Po(1.8)$
$P(X < 3) = 0.7306$

ii. $(0.7306)^3 = 0.3900$

iii. Y = usage of the equipment in three weeks
$Y \sim Po(1.8 \times 3 = 5.4)$
$P(Y < 3) = 0.09476$

4. W = weight of a bag of coffee beans
$W \sim N(510, 18^2)$

(a) below standard $\Rightarrow W < 500$
not underweight $\Rightarrow W \geq 485$
$$P(485 \leq W < 500) = P\left(\frac{485 - 510}{18} \leq Z < \frac{500 - 510}{18}\right) = P(-1.39 \leq Z < -0.56)$$
$= 0.2877 - 0.0823 = 0.2054$

(b) c = minimum weight of a bag of coffee beans in 90% of time
$$P(W > c) = 0.9 \quad \Rightarrow \quad \frac{c - 510}{18} = -1.28 \quad \Rightarrow \quad c = 486.96$$
$486.96/20 = 24.348 \quad \Rightarrow \quad$ 24 cups of coffee can be made

(c) $\overline{W}$ = average weight of 3 bags of coffee beans
$\overline{W} \sim N(510, 18^2/3)$
$$P(\overline{W} < 490) = P\left(Z < \frac{490 - 510}{18/\sqrt{3}}\right) = P(Z < -1.92) = 0.0274$$

(d) $P(\text{below standard}) = P(W < 500) = 0.2877$
X = no. of below standard bags in 3 bags of coffee beans
$X \sim b(3, 0.2877)$
lodge a complaint $\quad \Rightarrow \quad X \geq 2$
$P(\text{no complaint}) = P(X < 2) = (1 - 0.2877)^3 + 3(0.2877)(1 - 0.2877)^2 = 0.7993$

(e) $P(\text{under weight}) = P(W < 485) = 0.0823$

Y = no. of underweight bags in 3 bags of coffee beans = no. of bags returned in 3 bags of coffee beans

$Y \sim b(3, 0.0823)$

Since "return 2 bags of coffee beans" means "return 2 bags of coffee beans and lodge a complaint" $\Rightarrow$ $Y = 2$

$$P(\text{return 2 bags of coffee beans} \,|\, \text{lodge a complaint})$$
$$= \frac{P(\text{return 2 bags of coffee beans} \cap \text{lodge a complaint})}{P(\text{lodge a complaint})}$$
$$= \frac{P(Y = 2)}{1 - P(\text{no complaint})}$$
$$= \frac{3(0.0823)^2(1 - 0.0823)}{1 - 0.7993} = 0.0929$$