Tiling 3 x N Problem

Course: Algorithm Design
Tags: LeetCode

Part 1: Draw the "End Pieces" (The Building Blocks)

When you reach the right side of the grid, the puzzle *must* end in one of two ways. Draw these now:

Type A: The "Short Block" (Width 2)

These are blocks that take up exactly 2 columns.

Draw a 3×2 rectangle. There are 3 ways to fill it with dominoes so it doesn't stick out to the left.

1. **Three horizontal bars** stacked on top of each other.
2. **"U" shape:** One horizontal on top, two verticals below.
3. **"Upside-down U":** Two verticals on top, one horizontal below.

- **Memorize this number: 3**.
- (Whenever you use one of these, you use up **2** columns).

Type B: The "Long Bricks" (Width 4, 6, 8...)

These are weird blocks that lock together so tightly you cannot cut them in the middle.

Draw a 3×4 rectangle.

- Try to fill it so you **cannot** draw a straight vertical line through the middle.
- You will find exactly **2 ways** to do this (they look like a braid or zipper).
- This is true for **any** even width ($4, 6, 8 \ldots$). There are always exactly **2** special "braided" patterns that fit that specific length perfectly without breaking.
- **Memorize this number: 2**.
- (This applies to width 4, width 6, width 8, etc.)

Part 2: The "Timeline" Trace (How to Calculate)

Do not think about formulas. Think about **adding up scenarios**.

Let's calculate the answers for Width 0, 2, 4, 6.

1. Width 0 (Empty)

- Answer: **1** way (Do nothing).

2. Width 2

- Look at your "Type A" drawing. How many are there?
- Answer: **3**.

3. Width 4

- **Scenario 1:** Ends with a "Short Block" (Width 2).
  - Cost: **3** variations.
  - Remaining Space: $4 - 2 = 2$. (We solved Width 2 in the previous step: answer was **3**).

- Math: $3 \times 3 = \mathbf{9}$.

- **Scenario 2:** Ends with a "Long Brick" (Width 4).
  - Cost: **2** variations.
  - Remaining Space: $4 - 4 = 0$. (Answer was **1**).
  - Math: $2 \times 1 = \mathbf{2}$.
- **Total:** $9 + 2 = \mathbf{11}$.

4. Width 6

- **Scenario 1:** Ends with "Short Block" (Width 2).
  - Cost: **3**.
  - Remaining: Width 4 (Answer was **11**).
  - Math: $3 \times 11 = \mathbf{33}$.
- **Scenario 2:** Ends with "Long Brick" (Width 4).
  - Cost: **2**.
  - Remaining: Width 2 (Answer was **3**).
  - Math: $2 \times 3 = \mathbf{6}$.
- **Scenario 3:** Ends with "Long Brick" (Width 6).
  - Cost: **2**.
  - Remaining: Width 0 (Answer was **1**).
  - Math: $2 \times 1 = \mathbf{2}$.
- **Total:** $33 + 6 + 2 = \mathbf{41}$.

Part 3: What to write on your Cheat Sheet

Don't memorize the algebra. Memorize this sentence:

> ***"To find the answer for N:"***
>
> 1. *Take the previous answer ($N - 2$) and multiply by **3**.*
> 2. *Then, take **every** earlier answer ($N - 4, N - 6, \ldots$ all the way to 0) and multiply each by **2**.*
> 3. *Add them all up.*

Example Check ($N = 6$):

- Previous ($N = 4$) is 11 $\rightarrow 11 \times 3 = 33$.
- Earlier ($N = 2$) is 3 $\rightarrow 3 \times 2 = 6$.
- Earlier ($N = 0$) is 1 $\rightarrow 1 \times 2 = 2$.
- **Sum:** $33 + 6 + 2 = 41$.