

Fundamental review questions:

1) What are the components of a graph?

- vertex (node) → the points or objects in the graph,
- edges (arcs/links) → connect pairs of vertex and represent the relationships between them

2) Give a real-life example of a graph that is directed.

road network with one-way streets. In this graph:

- **Intersections (junctions) are represented as nodes (vertices).**
- **Roads are represented as directed edges**, where an arrow indicates the allowed direction of travel (e.g., one-way streets).

For example, in a city with one-way streets, if you can drive from **Point A to Point B** but not the other way around, the edge would have a direction from A to B, not vice versa.

3) Give a real-life example of a graph that is undirected.

a friendship network on a social media platform.

- **People (users) are represented as nodes (vertices).**
- **Friendships are represented as edges**, where an edge between two people means they are friends.

Since friendships on many platforms (like Facebook) are **mutual**, the edges are **undirected**—if **Person A is friends with Person B**, then **Person B is also friends with Person A**.

4) What does it mean when a graph is called “connected”?

A graph is called **"connected"** if there is a **path between every pair of vertices**. This means that:

- In an **undirected graph**, you can travel from any node to any other node by following edges.
- In a **directed graph**, it is usually specified as **strongly connected** (if there is a directed path between every pair of vertices in both directions) or **weakly connected** (if ignoring edge directions makes the graph connected).

5) What does it mean when a graph is called “fully connected”?

"fully connected" when every pair of vertices is directly connected by an edge.

- In an **undirected graph**, this means there is an edge between every two nodes, forming a **complete graph** (denoted as K_n , where n is the number of vertices).
- In a **directed graph**, a fully connected graph (also called a **complete digraph**) has a **directed edge in both directions** between every pair of nodes.

6) Is a tree connected graph?

yes, a **tree is a connected graph** by definition.

A **tree** is a special type of graph that:

- Is **connected** (there is a path between every pair of nodes).
- Has **no cycles** (it does not contain any loops).

Since a tree is always connected (as long as it has at least one node), removing any edge from a tree would make it **disconnected**.

7) A tree is a specific type of graph. What makes a tree distinguished from graph?

A **tree** is a specific type of graph that is distinguished by the following properties:

- **Connected** – There is a path between any two nodes.
- **Acyclic** – It contains no cycles (no closed loops).
- **Has $n-1$ edges** – A tree with n nodes always has exactly $n-1$ edges.
- **One unique path** – There is exactly **one unique path** between any two nodes.

Difference

- **General graphs** can have **cycles, multiple paths between nodes, and any number of edges**.
- **A tree is always a minimally connected graph**, meaning removing any edge will make it disconnected.

8) What is rooted tree?

A **rooted tree** is a tree in which one specific node is designated as the **root**, and all edges are directed **away from the root**(or toward it, depending on the convention).

- Each node has a **unique parent**, except for the root, which has no parent.
- Nodes are organized into **levels**, with the root at the top.
- Examples: **File systems, organizational charts, family trees**.

9) What is a leaf?

A **leaf** (or **leaf node**) in a tree is a node that **has no children**.

- It is a terminal node at the **end of a branch**.
- Example: In a family tree, a person with no children is a **leaf node**.

10) How many edges are there in a tree having n nodes?

A tree with n nodes always has $n-1$ edges.

- Since a tree is **connected** and **acyclic**, there must be exactly **one less edge than the number of nodes**.
- Example: A tree with **5 nodes** has **4 edges**.

11) How many simple path(s) is(are) there between a pair of tree nodes?

A tree has **exactly one unique simple path** between any two nodes.

- Since a tree has **no cycles**, there is only **one way** to travel between two nodes.
- Example: In a family tree, there is **only one direct path** from a child to their ancestor.

12) What is a “weighted” graph?

A **weighted graph** is a graph where **each edge has a numerical value (weight)** associated with it.

- These weights often represent **distances, costs, or capacities**.
- Example: In a **road network**, edges can be weighted based on the **distance (km)** between cities.