2.3

Designing Algorithms and Recursion

ITX2010, CSX3003, IT2230

Data Structures and Algorithms, Information Structures

Learning Objectives

Recursion

Students will be able to:

- Recognize goal and principal of Object-Oriented Programming
- Explain the history of developing a program
- Comprehend the class’s terminologies
- Generate a Python class
- Explain the recursive function concept
- Describe how to solve a problem with the recursion
- Illustrate recursive function workflow
- Synthesize a recursive function

Chapter Outline

Object-Oriented Programming

1. Object-Oriented Concept
   1) Goal and Principle
   2) Program Development
   3) Class's Terminologies
   4) Python Class
2. Recursion
   1) Recursive Function
   2) Recursive Function Workflow
   3) Recursive Solution
   4) Runtime Stack
   5) Multiple Recursion
   6) Recursion Application

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Recursion

Section 1: Object-Oriented Concept

1) Goal and Principle
2) Program Development
3) Class Definitions
4) Python Class

Goal and Principle

Object-Oriented Concept: Goal

- Robustness
  - Capability of handling unexpected input or tolerating with the input which is not explicitly specified
  - Ex: Although an input is type mismatched, it can run and return error message or find the possible closet outcome without any crash.
- Adaptability (Evaluability)
  - Portable to variety of hardware's and software's specifications
  - **Ex**: It can be run on its previous or new Operating Systems or platforms.
- Reusability
  - Reusable code can be adopted to the new program for optimized development time
  - **Ex**: Importing class to support some basic operations in another class

Goal and Principle

Object-Oriented Concept: Principle

- Modularity:
  - It consists of several different module that must properly work and correctly with others to serve a functional requirement.
- Abstraction:
  - Design and Implementation can be integrated in class concept.
- Encapsulation:
  - it gives one programmer freedom to implement the details of a component, without concern that other programmers will be writing code that intricately depends on those internal decisions. [1]

Goal and Principle

Object-Oriented Concept: Principle

Fig 2-1 General Data Structures [1]

Program Development

Object-Oriented Concept

- Non-structured Linear Programming (Spaghetti Code)
  - Program line of code run in sequence and were not prepared based on their responsibility.
- Modular Programming
  - Programs were organized in functions / method or module for serving a single purpose.
- Object-Oriented Programming
  - Functions and Data are developed within a template called class to define a behavior of class instance.

Class Definitions

Object-Oriented Concept

- Python is an object-oriented programming language.
- Class is a template of generating an object with the following components:
    1. Class name
    2. Class attribute
    3. Constructor
    4. Class method

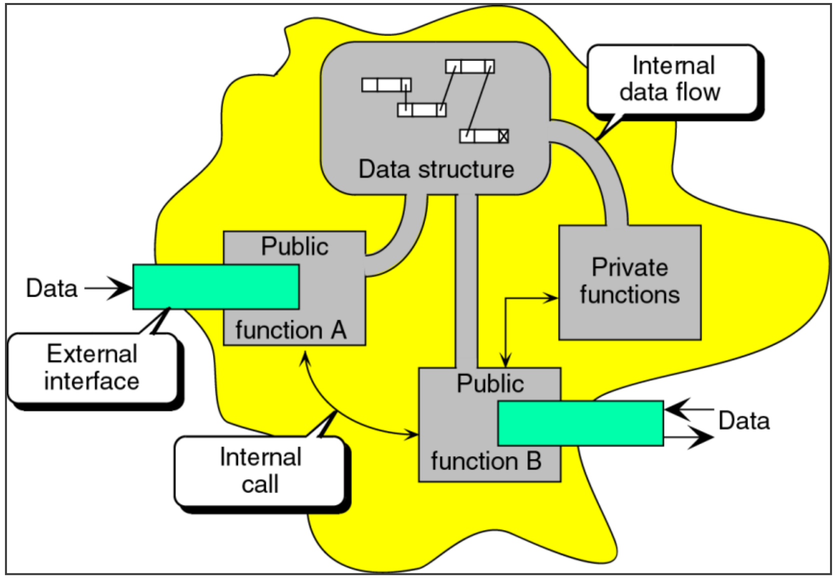
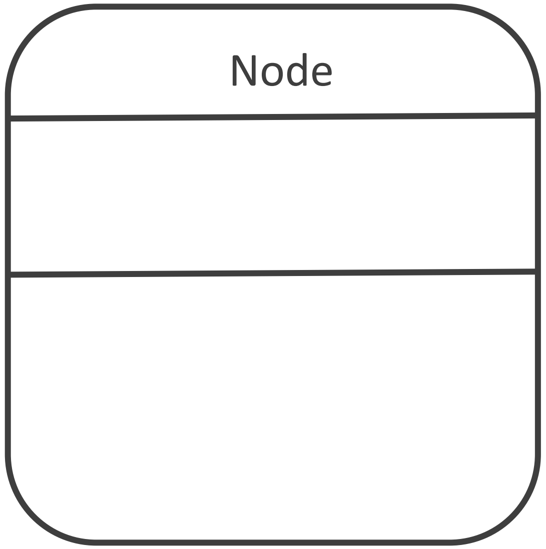


Fig 2-2 General Data Structures [2]

Class Definitions

Object-Oriented Concept

1. Class name
   - Name of class or template name which must be assigned before generating its object or instance.



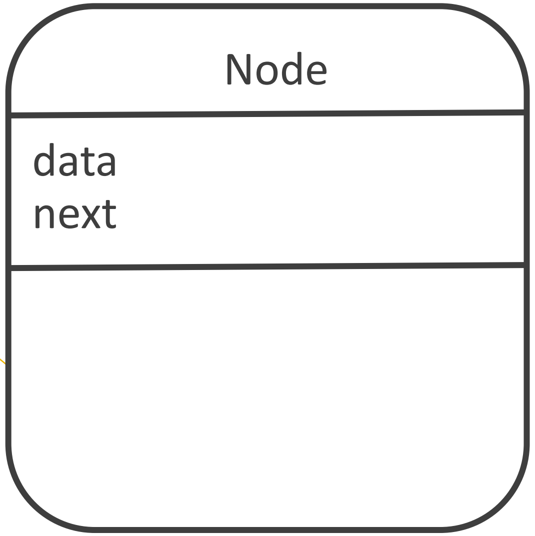

```python
class Node:
  #Constructor

  #Class members
```

Fig 2-3 Class name

Class Definitions

Object-Oriented Concept

2. Class attribute
   - It describes the characteristics of class and can be implemented in term of variables.




```
class Node:
  #Constructor
    def __init__(self,initData):
      #Class attibute
      self.data = initData
      self.next = None
```

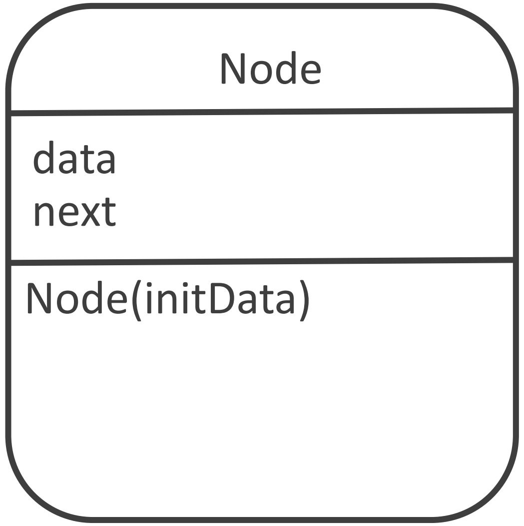
Fig 2-4 Class attribute

Class Definitions

Object-Oriented Concept

3. Constructor

- Class method responses for initializing values of class attributes.




```python
class Node:
  #Constructor
    def __init__(self,initData):
      #Class attibute
      self.data = initData
      self.next = None
```

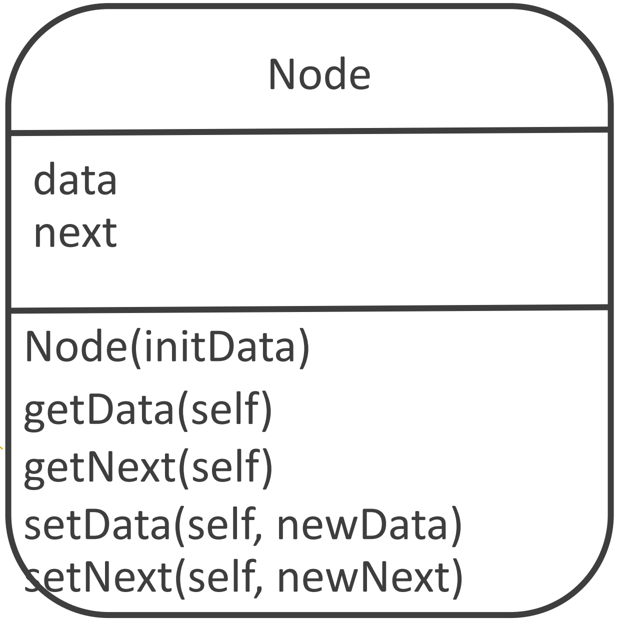
Fig 2-5 Class name

Class Definitions

Object-Oriented Concept

4. Class method

- Class member or function define behavior or operation of class instance (object).




```python
class Node:
  #Constructor
  #Class members
    def getData(self):
      return self.data
    def getNext(self):
      return self.next
    def setData(self,newdata):
      self.data = newdata
    def setNext(self,newnext):
      self.next = newnext
```

Fig 2-6 Class method

Tutorial 1: Python Class Implementation

1.4

```python
#Create Node Class

class Node:
  #Constructor
    def __init__(self,initData):
      #Class attibute
      self.data = initData
      self.next = None
  #Class members
    def getData(self):
      return self.data
    def getNext(self):
      return self.next
    def setData(self,newdata):
      self.data = newdata
    def setNext(self,newnext):
      self.next = newnext
#Create an object of class Node
myNode = Node(10)
print("\n")
#Call the class's method getData
print("Print node data : ", myNode.getData())
```

```
Print node data :  10
```

2

Recursion

Section 2:
Recursion

1) Recursive Function
2) Recursive Function Workflow
3) Recursive Solution
4) Runtime Stack
5) Multiple Recursion
6) Recursion Application

Recursion Function

[7]

What is Recursion?

- **Divide-and-conquer** method:
  - Break the problem into several subproblems that are similar to the original problem
  - Solve the subproblems recursively
  - Combine these solutions to create a solution to the original problem
- Perform three characteristic steps:
  1. **Divide** the problem into one or more subproblems
  2. **Conquer** the subproblems by solving them recursively
  3. **Combine** the problem solutions to form a final solution

Recursion Function

What is Recursion?

- **Recursion** is a process for solving problems by subdividing a larger problem into smaller cases of the problem itself and then solving the smaller, more trivial parts. [3]
- It recurse (call itself) one or more times to handle closely related problems. [7]
- This algorithm follow the **“divide-and-conquer”** method. [7]

Recursion Function

What is Recursion?

- **Recursive Function** is a function that calls itself, directly or indirectly. [4]
- **Ex:** A mathematic example of Fibonacci
- The recursive function consists of two parts:
  1. Termination condition
  2. Function body

$$F(n) = \begin{cases} 1, n = 1,2 \\ F(n-1) + F(n-2), others \end{cases}$$

Fig 2-7 Fibonacci function [3]

Recursion Function

Components

1. **Termination condition (Base case)**
   - A recursive function always contains one or more terminating condition.
   - A condition which recursive function is processing a simple case and do not call itself.
2. **Function body (Recursive case)** – including recursive expansion
   - The main logic of the function contains in the body of the function.
   - It contains the recursion expansion statement that in turn calls the function itself.

Recursion Function

Properties

1. It must have a terminate condition (base case).
   - Without the terminate condition, the recursive function will forever run and consume all stack memory.
2. It must call itself which contain a recursive case.
3. It must change its state until toward to the terminate condition (base case).

- **Note that**:
  - Property 1 and 3 guarantee that the recursive function can stop.
  - Property 2 divides the problem into smaller pieces (Divide and Conquer).

Recursion Function

Properties

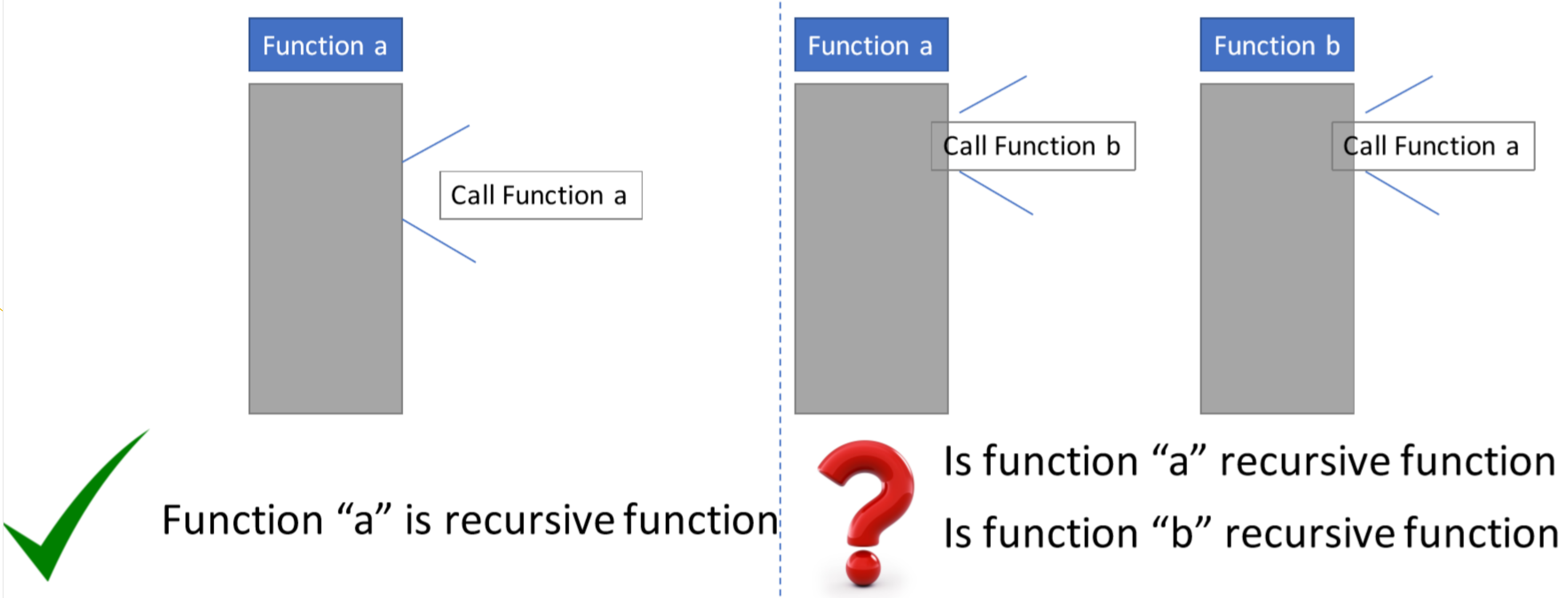


Fig 2-8 Recursive functions [3]

Recursion Function Workflow

- Given a print function:

```
def printRev( n ):
  if n > 0 :
    print( n )
    printRev( n-1 )
```

➡

```
def printInc( n ):
  if n > 0 :
    printInc( n-1 )
    print( n )
```

Fig 2-9 Example of print function [3]

Recursion Function Work Flow

- What is the output of each function?

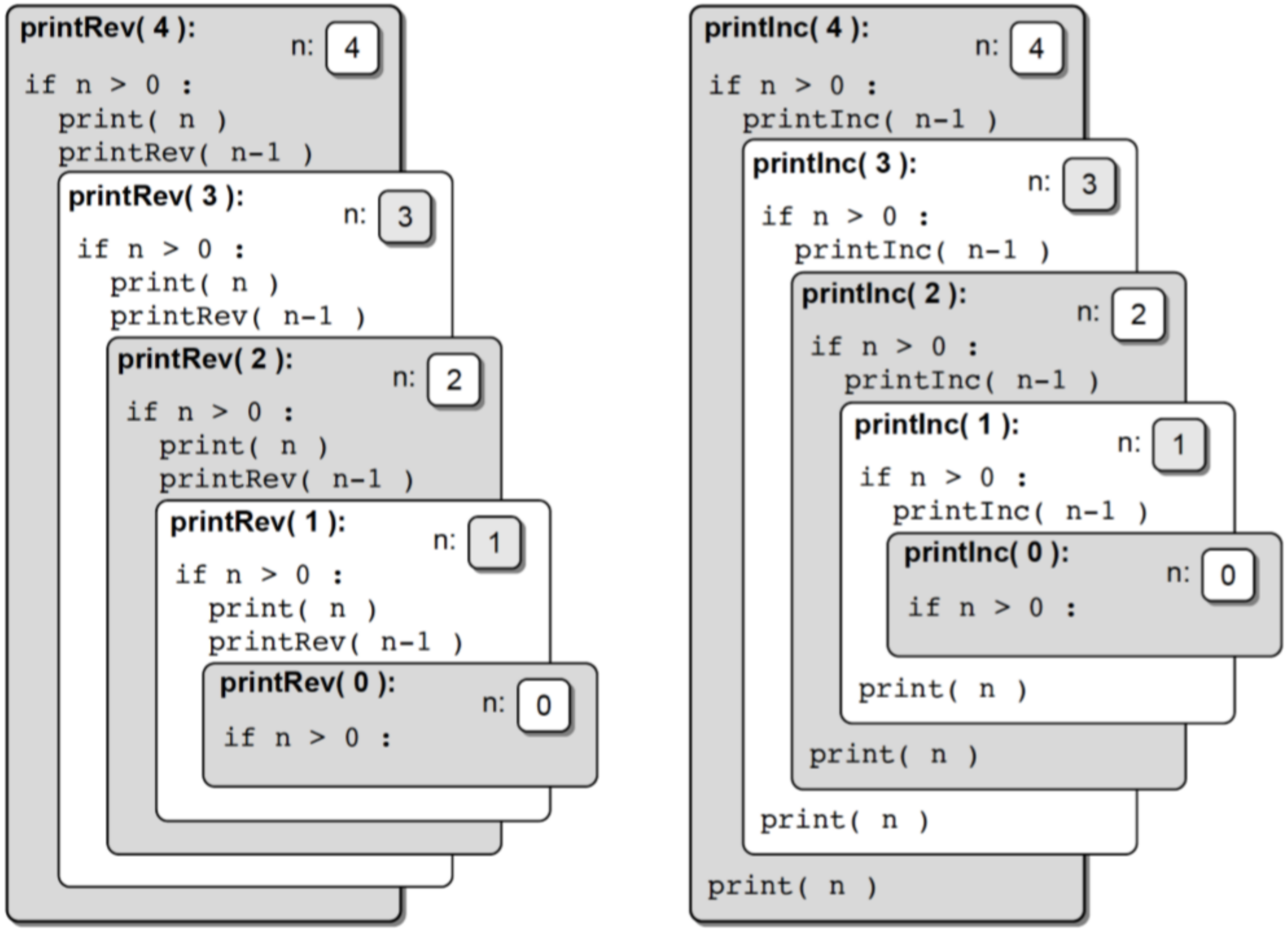


Fig 2-10 Print function work flow [3]

Recursion Solution

- A recursive solution can:
  1. Subdivides a problem into smaller version of itself.
  2. Find a based case
  3. Find a recursion case

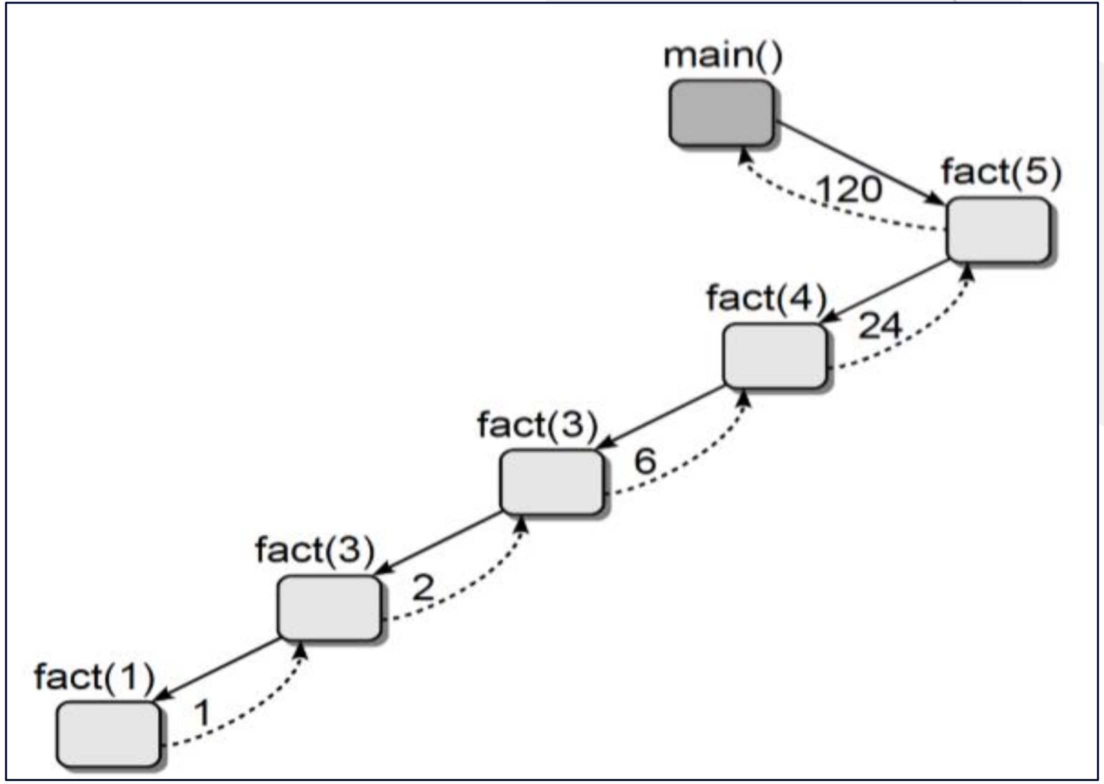
Tutorial 2: Recursive Function of Factorial [3]

- **<u>Ex</u>**: Factorial of n (n!)
- A recursive solution can:
  1. Subdivides a problem into smaller version of itself.

     5 x 4 x 3 x 2 x 1
  2. Find a based case

     n =0, n! = 1
  3. Find a recursion case

     n! = n(n-1)!




```
# Compute n!
def fact( n ):
  assert n >= 0, "Factorial not defined for negative values."
  if n < 2 :
    return 1
  else :
    return n * fact(n - 1)
```

Runtime Stack

- The base case will be called last but must be firstly computed.

```
def main():
  y = fact( 2 )
main()
```

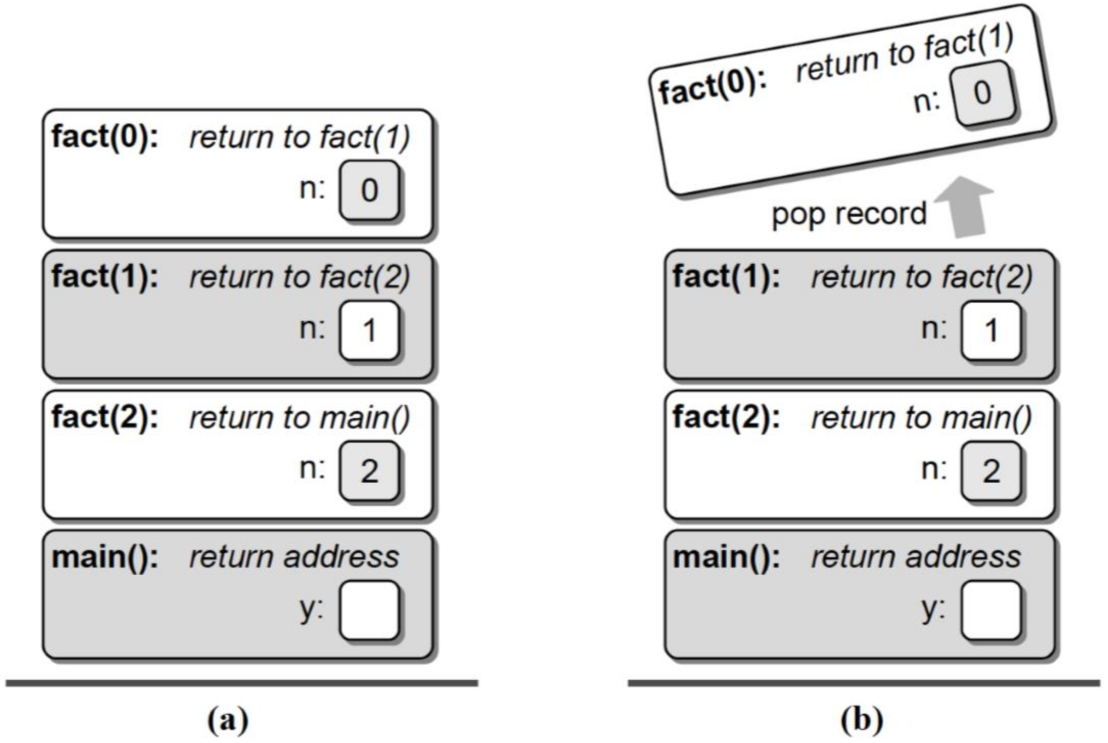


Fig 2-11 Stack runtime [3]

Multiple Recursion

- Some problem - like Fibonacci, require multiple recursive function.

```
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...
```

$$\text{fib}(n) = \begin{cases} 1, n = 1,2 \\ fib(n-1) + fib(n-2), others \end{cases}$$

```
fib(n): //assuming n >= 1
  if n=1 or n=2:
    return 1
  else:
    return fib(n - 1)+fib(n-2)
```

Fig 2-12 Example of Fibonacci recursive function [3]

Multiple Recursion

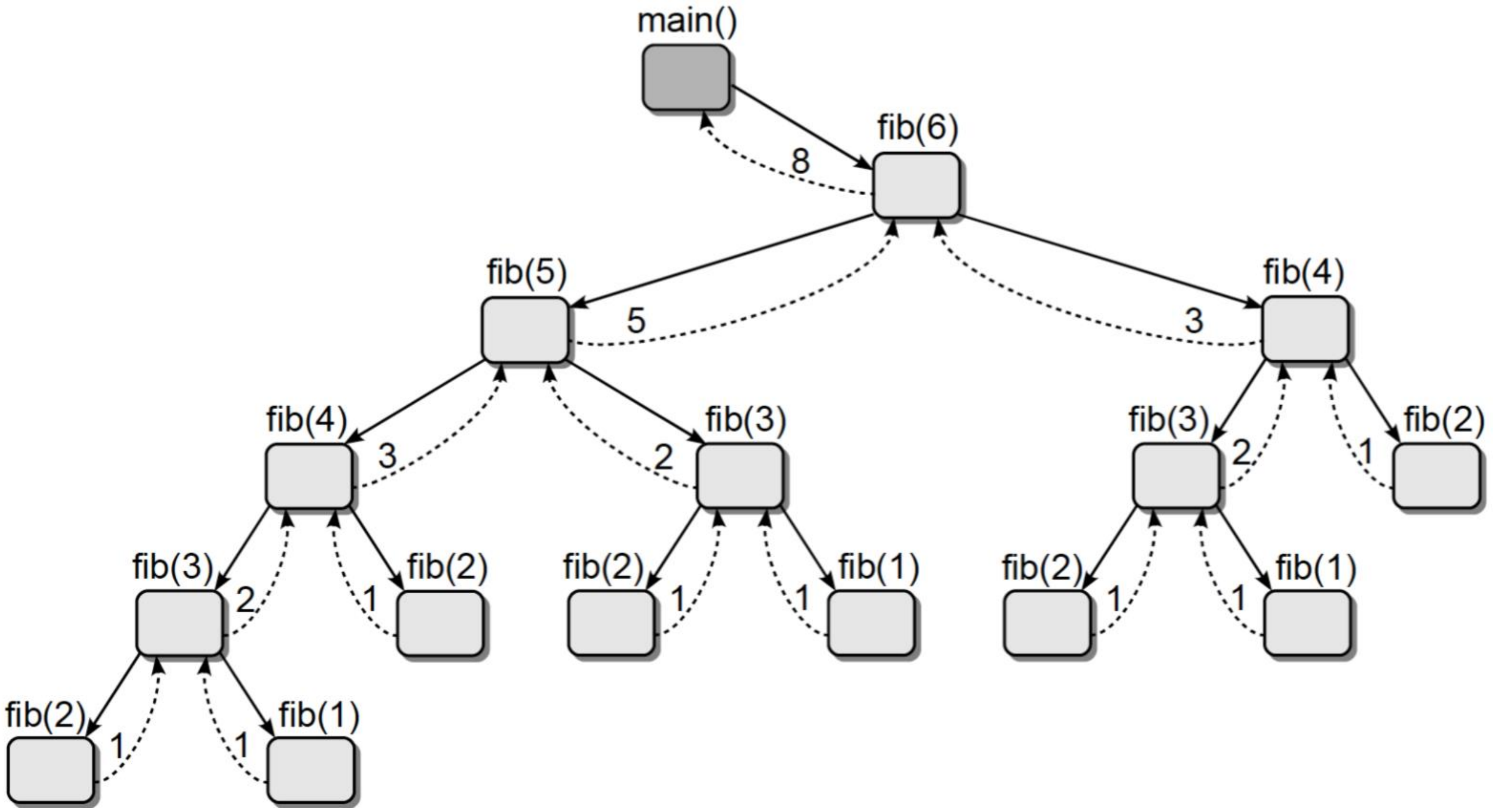


Fig 2-13 Logical view of executing Fibonacci recursive function [3]

Recursion Applications

- Many application can be solved by using recursion
    1. Merge sort algorithm
    2. Binary search
    3. Towers of Hanoi
    4. Tic-Tac-Toe

[7]

Recursion Applications

Merge sort algorithm

```
MERGE(A, p, q, r)
n_L = q − p + 1          // length of A[p : q]
n_R = r − q              // length of A[q + 1 : r]
let L[0 : n_L − 1] and R[0 : n_R − 1] be new arrays
for i = 0 to n_L − 1     // copy A[p : q] into L[0 : n_L − 1]
    L[i] = A[p + i]
for j = 0 to n_R − 1     // copy A[q + 1 : r] into R[0 : n_R − 1]
    R[j] = A[q + j + 1]
i = 0                    // i indexes the smallest remaining element in L
j = 0                    // j indexes the smallest remaining element in R
k = p                    // k indexes the location in A to fill
```

[7]

Recursion Applications

Merge sort algorithm

```
// As long as each of the arrays L and R contains an unmerged element,
//     copy the smallest unmerged element back into A[p : r].
while i < n_L and j < n_R
    if L[i] ≤ R[j]
        A[k] = L[i]
        i = i + 1
    else A[k] = R[j]
        j = j + 1
    k = k + 1
// Having gone through one of L and R entirely, copy the
//     remainder of the other to the end of A[p : r].
```

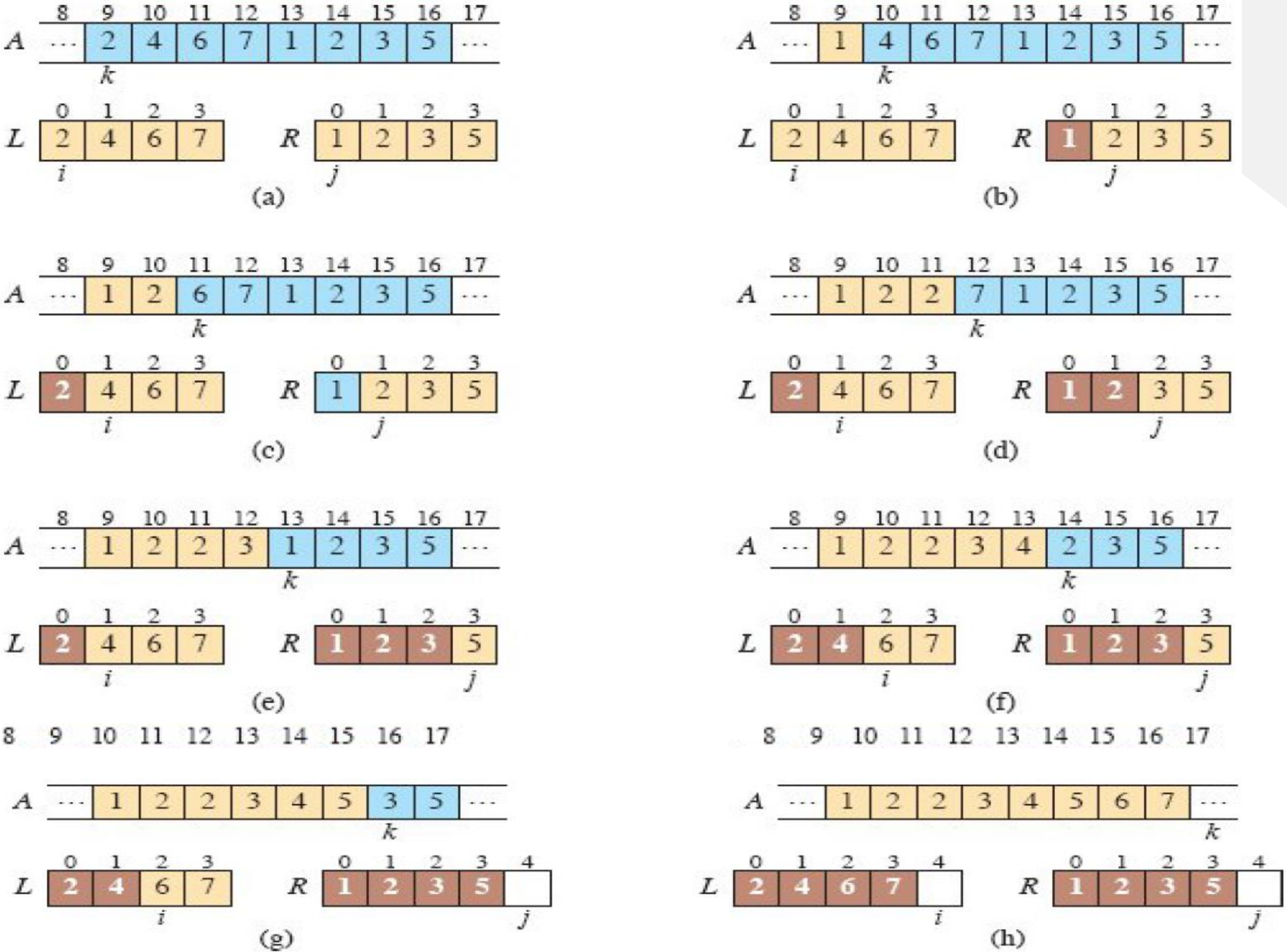
[7]

Recursion Applications

Merge sort algorithm

// Having gone through one of L and R entirely, copy the
// remainder of the other to the end of $A[p:r]$.
while $i < n_L$
    $A[k] = L[i]$
    $i = i + 1$
    $k = k + 1$
while $j < n_R$
    $A[k] = R[j]$
    $j = j + 1$
    $k = k + 1$

2.2.6 Merge sort algorithm [7]



[7]

Recursion Applications

Merge sort algorithm

$$T(n) = \begin{cases} \Theta(1) & \text{if } n < n_0 \,, \\ D(n) + aT(n/b) + C(n) & \text{otherwise} \,. \end{cases}$$

```
MERGE-SORT(A, p, r)
if p ≥ r                                  // zero or one element?
    return
q = ⌊(p + r)/2⌋                           // midpoint of A[p : r]
MERGE-SORT(A, p, q)                       // recursively sort A[p : q]
MERGE-SORT(A, q + 1, r)                   // recursively sort A[q + 1 : r]
// Merge A[p : q] and A[q + 1 : r] into A[p : r].
MERGE(A, p, q, r)
```

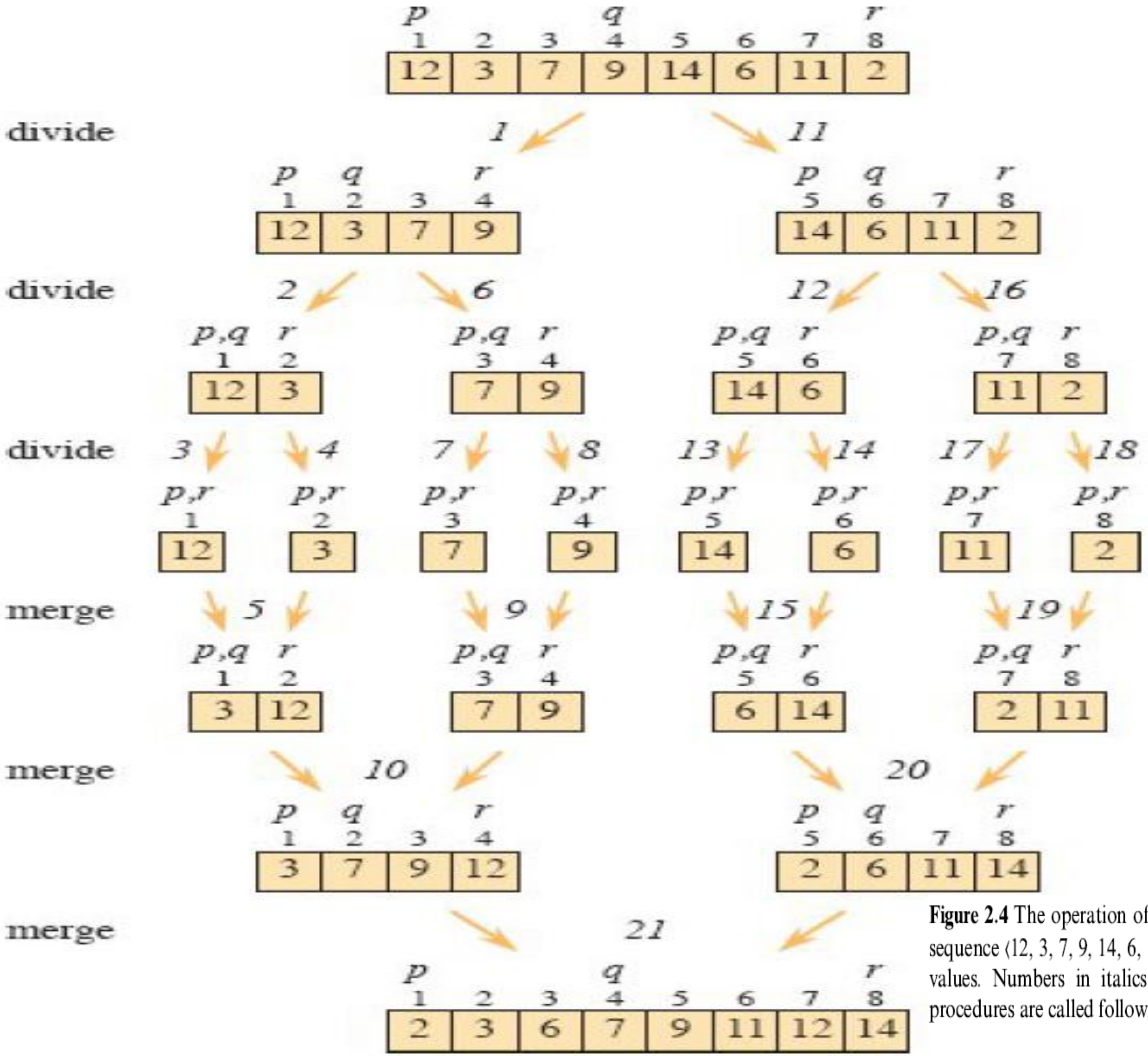


Figure 2.4 The operation of merge sort on the array *A* with length 8 that initially contains the sequence ⟨12, 3, 7, 9, 14, 6, 11, 2⟩. The indices *p*, *q*, and *r* into each subarray appear above their values. Numbers in italics indicate the order in which the MERGE-SORT and MERGE procedures are called following the initial call of MERGE-SORT(*A*, 1, 8).

Analysis of merge sort

Here's how to set up the recurrence for $T(n)$, the worst-case running time of merge sort on n numbers.

Divide: The divide step just computes the middle of the subarray, which takes constant time. Thus, $D(n) = \Theta(1)$.

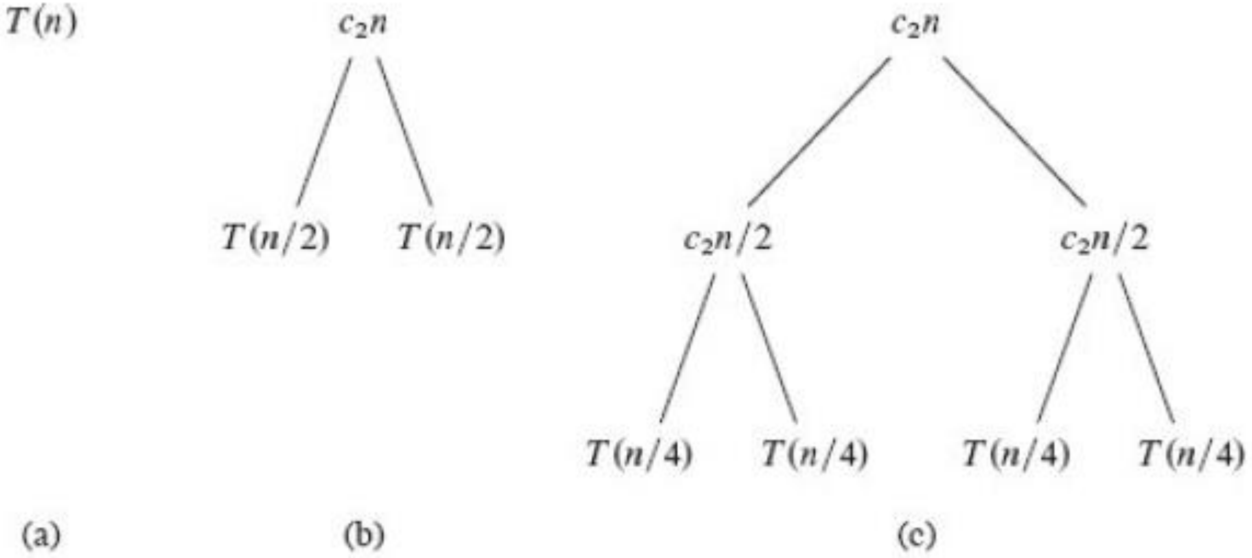
Conquer: Recursively solving two subproblems, each of size $n/2$, contributes $2T(n/2)$ to the running time (ignoring the floors and ceilings, as we discussed).

Combine: Since the MERGE procedure on an n-element subarray takes $\Theta(n)$ time, we have $C(n) = \Theta(n)$.

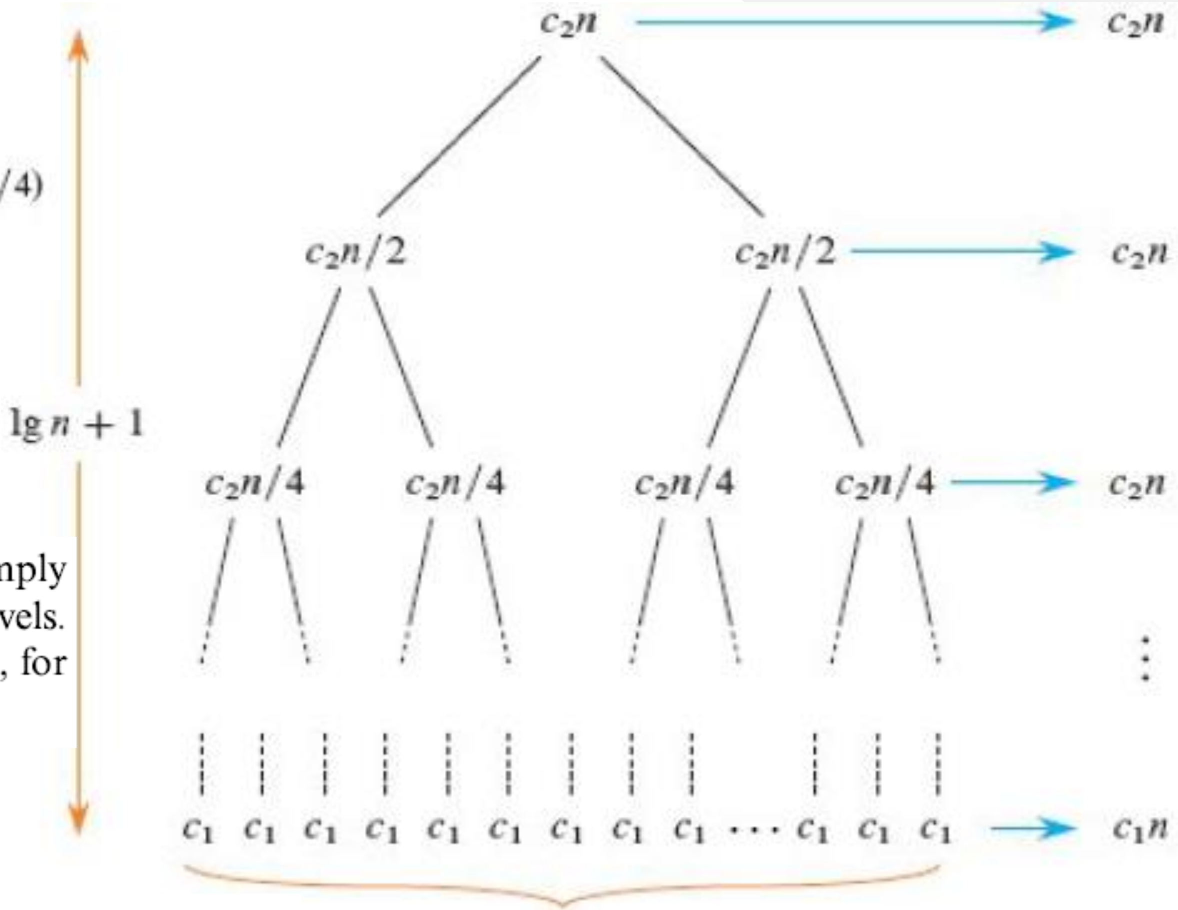
When we add the functions $D(n)$ and $C(n)$ for the merge sort analysis, we are adding a function that is $\Theta(n)$ and a function that is $\Theta(1)$. This sum is a linear function of n. That is, it is roughly proportional to n when n is large, and so merge sort's dividing and combining times together are $\Theta(n)$. Adding $\Theta(n)$ to the $2T(n/2)$ term from the conquer step gives the recurrence for the worst-case running time $T(n)$ of merge sort:

$$T(n) = 2T(n/2) + \Theta(n) . \tag{2.3}$$

Merge sort algorithm [7] 2.2.6



To compute the total cost represented by the recurrence (2.4), simply add up the costs of all the levels. The recursion tree has $\lg n + 1$ levels. The levels above the leaves each cost $c_2 n$, and the leaf level costs $c_1 n$, for a total cost of $c_2 n \lg n + c_1 n = \Theta(n \lg n)$.



n

Total: $c_2 n \lg n + c_1 n$

(d)

Figure 2.5 How to construct a recursion tree for the recurrence (2.4). Part **(a)** shows $T(n)$, which progressively expands in **(b)–(d)** to form the recursion tree. The fully expanded tree in part (d) has $\lg n + 1$ levels. Each level above the leaves contributes a total cost of $c_2 n$, and the leaf level contributes $c_1 n$. The total cost, therefore, is $c_2 n \lg n + c_1 n = \Theta(n \lg n)$.

Recursion Applications

Binary Search

- The problem of searching can be divided into smaller version of itself by:
  - Each splitting, the list will be cut into two parts.
  - This will be repeated until the target key is found or there is no more item to be searched.

Recursion Applications

Binary Search

```
recBinarySearch(target, A, first, last):

  //if the sequence cannot be subdivided further, we are done.
  if first > last:
    return False

  else:
    mid <-- (last + first)/2
    if A[mid] = target:
      return True

    else if target < A[mid]:
      return recBinarySearch(target, A, first, mid-1)

    else:
      return recBinarySearch(target, A, mid+1, last)
```

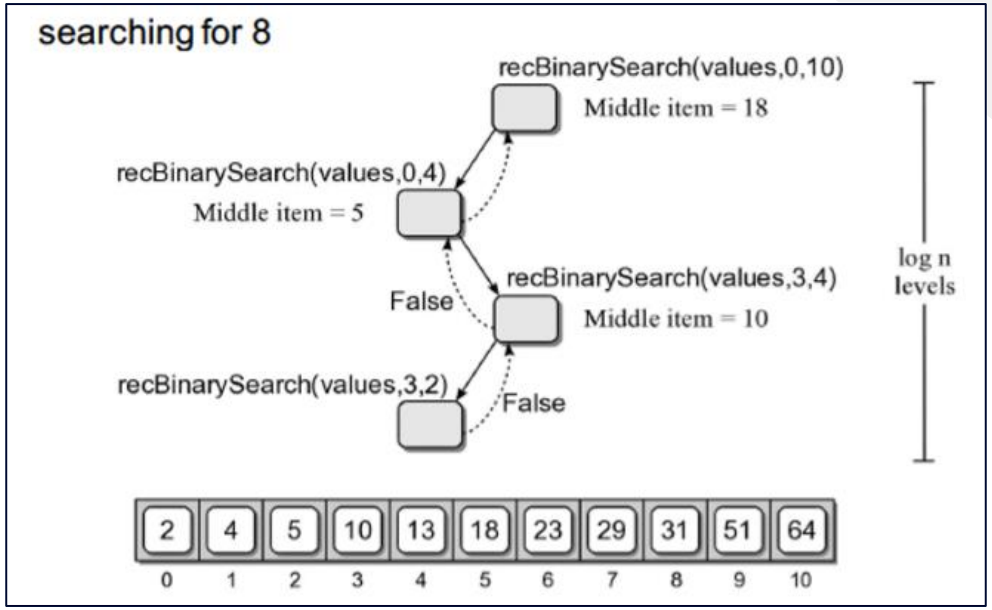


Fig 2-14 Binary search as recursive function [3]

References

Texts | Integrated Development Environment (IDE)

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