Business-IT Alignment Dynamics: A Chaotic Systems Approach

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2.1 Target Audience

This interactive notebook is designed for anyone interested in Business-IT alignment dynamics. It also serves as template for modeling other complex dynamics that use discrete time equations in the form of $x_{t+1} = x_t + \Delta(t, *\text{args})$. The user just needs to apply a few minor customization to the code.

2.2 Getting Started

2.2.1 Installation

1. Clone the repository:

   ```
   git clone https://github.com/Kinshale/pii.git
   cd pii
   ```

2. Install required packages:

   ```
   pip install numpy matplotlib ipywidgets
   ```

2.2.2 Running the Notebook

Choose your preferred environment:

- **Local Jupyter**: Launch Jupyter Notebook and open `pii.ipynb`
- **VS Code**: Open the notebook with Jupyter extension
- **Google Colab**:
  1. Visit https://colab.research.google.com
  2. Select "GitHub" tab and paste the notebook URL
  3. Or open the cloned one

For optimal experience:

- Start with default parameters to observe baseline behavior
- Modify one parameter at a time to understand its effect
- Use the bifurcation tool to identify chaotic parameter regions
- Read the markdown cells

3 Model Description

3.1 Core Equation

The alignment dynamics are governed by:

$$x_{t+1} = x_t + A(x_t) - B(x_t)C(x_t) \tag{1}$$

Where:

- x_t: Percentage of dissatisfied users (misalignment proxy)
- $A(x_t)$: Environmental pressure effect
- $B(x_t)$: IT department efficacy
- $C(x_t)$: Organizational adaptability

3.2 Component Functions

$$A(x_t) = d(1 - x_t) \tag{2}$$

$$B(x_t) = \frac{ax_t(1 - x_t)^g}{1 + ahx_t} \tag{3}$$

$$C(x_t) = \frac{1}{1 + z^s} \quad \text{where} \quad z = \frac{r(1 - x_t)}{x_t(1 - r)} \tag{4}$$

3.3 Parameter Definitions

Parameter	Description	Range	Default
x_0	Initial misalignment	[0.01, 0.99]	0.3
d	Environmental dynamicity	[0.01, 5]	0.5
a	IT department efficacy	[0.1, 10]	2
h	IT system rigidity	[0.1, 5]	1
g	IT investment propensity	[0.1, 5]	1
r	Action threshold	[0.01, 0.99]	0.3
s	Organizational flexibility	[1, 10]	3

Table 1: Model Parameters and Ranges

3.4 Interpretation

Let's take a deeper look at our equation:

$$x_{t+1} = x_t + \underbrace{A(x_t)}_{\text{Environmental Pressure}} - \underbrace{B(x_t)C(x_t)}_{\text{Recovery Mechanism}} \tag{5}$$

x_t – **Alignment** Measures the percentage of dissatisfied users at time t:

- $0 \rightarrow$ Complete satisfaction (perfect alignment)
- $1 \rightarrow$ Utter dissatisfaction (total misalignment)

$A(x_t)$ – **Environmental Pressure:** Increases misalignment due to external factors, representing how competitive environments and technological changes increase dissatisfaction.

$B(x_t) \cdot C(x_t)$ – **Recovery Mechanism:** Reduces misalignment through:

- $B(x_t)$: IT department's effectiveness.
- $C(x_t)$: Organization's adaptability.

3.5 Parameter Analysis

Full interactive simulation available at: https://www.desmos.com/calculator/qdendm1pfg

3.5.1 Environmental Pressure Function

$$A(x_t) = d(1 - x_t) \tag{6}$$

Forms a line passing through $(1, 0)$ with slope $-d$.

- d **(dynamicity)**: Fast changing industries (e.g., a tech startup) have a competition/innovation that rapidly renders old IT systems obsolete.
- $1 - x_t$: As misalignment grows, environmental pressure has less "room" to worsen things.

3.5.2 IT Department Efficacy

$$B(x_t) = \frac{a x_t (1 - x_t)^g}{1 + a h x_t} \tag{7}$$

This looks like a function that peaks at some x_a and then tapers off.

- **a (IT efficacy)**: an higher a can more effectively reduce misalignment.
- **x (current misalignment)**: the more misalignment exists, the more opportunity/pressure there is for IT to act.
- $(1 - x_t)^g$ **(Diminishing Returns)**: as satisfaction improves, the IT department's impact diminishes.
  - I haven't understood g.
- $1 + a h x_t$ **(Saturation)**: even if IT is highly capable (a » 1), inflexible systems (h » 1) limit its efficacy.

3.5.3 Organizational Adaptability

$$C(x_t) = \frac{1}{1 + z^s} \quad \text{where} \quad z = \frac{r(1 - x_t)}{x_t(1 - r)} \tag{8}$$

This is a **sigmoid** function in disguise. Sigmoids are exploited for modeling "threshold behaviors".

- **r (activation threshold)**: below r, the organization resists to change ($C(x) \to 0$). But when misalignment crosses a certain threshold adaptability kicks in.
- **s (flexibility)**: higher s make the sigmoid steeper (sharper transition from resistance to adaptation).

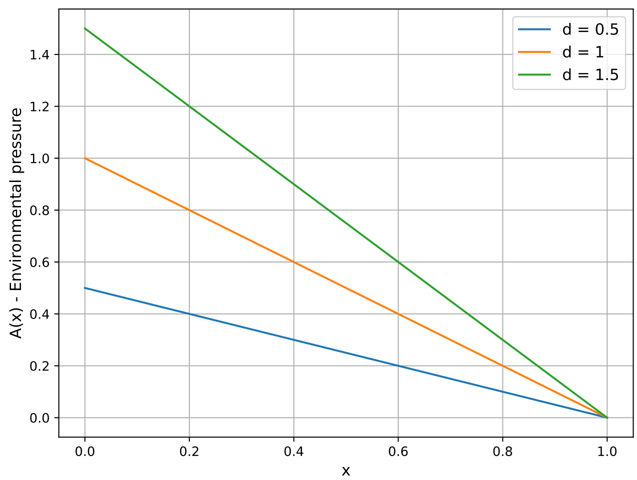

Figure 1: Environmental pressure function for varying d values

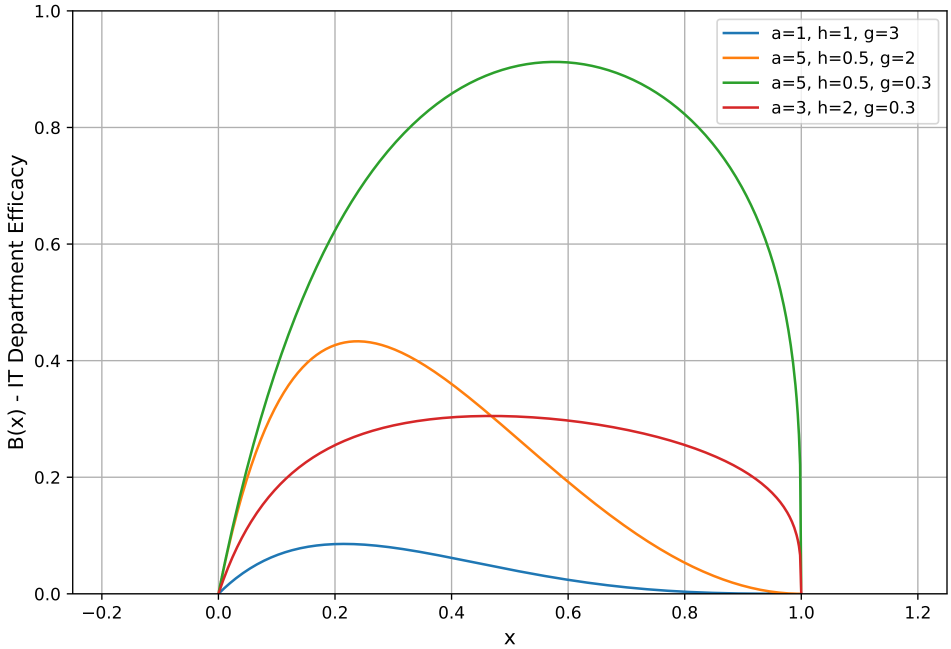

Figure 2: IT efficacy function showing the effect of parameters a, h, and g

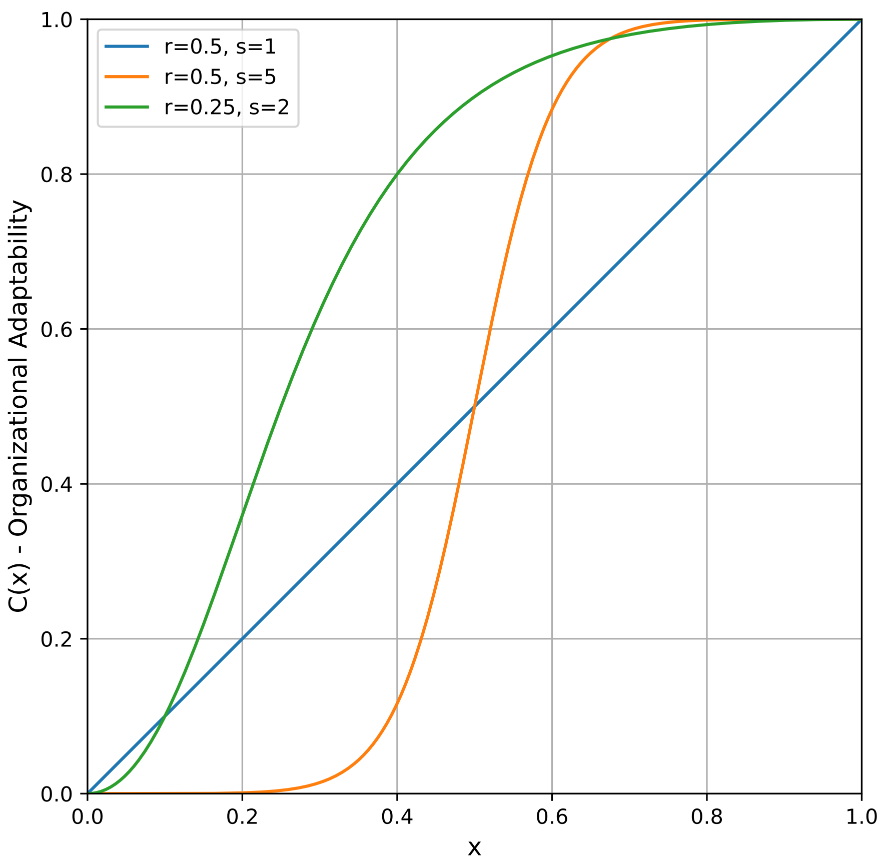

Figure 3: Organizational adaptability function demonstrating threshold behavior

4 Implementation

4.1 Technological Stack

- Python 3.10.12 (managed via Conda)
- Jupyter Notebook for interactive exploration, with markdown explanations attached
- Core dependencies (Python Libraries)
  - NumPy for numerical computations
  - Matplotlib for visualization graphs
  - ipywidgets for parameter sliders

4.2 Key lines of code

4.2.1 Simulating the equation

The equation is hardcoded:

```
def A(x, d):
    return d * (1 - x)

def B(x, a, h, g):
    return (a * x * (1 - x) ** g) / (1 + a * h * x)

def C(x, r, s):
    if x == 0:  # Avoid division by zero
        return 0
    z = (r * (1 - x)) / (x * (1 - r))
    return 1 / (1 + z ** s)

def delta(x, d, a, h, g, r, s):
    return A(x, d) - B(x, a, h, g) * C(x, r, s)

def simulate(x0, d, a, h, g, r, s, steps=100):
    x = np.zeros(steps)
    x[0] = x0
    for t in range(steps - 1):
        x[t + 1] = np.clip(x[t] + delta(x[t], d, a, h, g, r, s), 0, 1)
```

4.2.2 Long term behavior

Here is the key logic behing classifing the equation.

```
x = simulate(x0, d, a, h, g, r, s, steps)

last_values = x[-10:]

if np.std(last_values) < 0.001:  # Stable state
final_val = np.mean(last_values)
if final_val < 0.1:
    return x, "ALIGNED"
elif final_val > 0.9:
    return x, "MISALIGNED"
else:
    return x, "PARTIAL_ALIGNMENT"
else:  # Dynamic state
if len(np.unique(np.round(last_values, 2))) > 3:
    return x, "CHAOTIC"
else:
    return x, "OSCILLATING"
```

4.2.3 Phase Portrait Analysis

The phase space visualization algorithm:

```
def phase_portrait(d, a, h, g, r, s, n_points=200):
    x = np.linspace(0, 1, n_points)
    dx = np.array([delta(xi, d, a, h, g, r, s) for xi in x])

    # Arrow placement logic
    arrow_indices = np.linspace(0, len(x)-1, 20, dtype=int)
    norm = Normalize(vmin=0, vmax=np.max(np.abs(dx)))

    for xi, dxi in zip(x[arrow_indices], dx[arrow_indices]):
        if dxi > 0:  # Right arrow
            plt.arrow(xi, 0, 0.02, 0, ...)
        elif dxi < 0:  # Left arrow
            plt.arrow(xi, 0, -0.02, 0, ...)

    plt.plot(x, dx, 'k-')  # Main curve
    plt.axhline(0, color='black', linestyle=':')  # Zero line
```

4.2.4 Bifurcation Analysis

The chaotic regime detection algorithm:

```
def bifurcation_analysis(param, p_min, p_max, fixed_params, n_points=500):
    param_values = np.linspace(p_min, p_max, n_points)
    n_transient = 200  # Skip initial transient
    n_samples = 100    # Points to plot per parameter

    for p in param_values:
        params = fixed_params.copy()
        params[param] = p

        x = 0.3  # Initial value
        # Burn-in phase
        for _ in range(n_transient):
            x = np.clip(x + delta(x, **params), 0, 1)

        # Sample stable points
        x_vals = []
        for _ in range(n_samples):
            x = np.clip(x + delta(x, **params), 0, 1)
            x_vals.append(x)

        plt.plot([p]*n_samples, x_vals, 'k.', markersize=0.5)
```

4.3 Numerical Considerations

- State clipping ensures $x_t \in [0, 1]$ remains meaningful
- All floating-point operations use NumPy's float64 precision
- The bifurcation analysis skips 200 transient iterations to focus on long-term behavior
- Phase portrait arrows are normalized to avoid cluttering

5 Results

5.1 Interactive Tools

The notebook provides three powerful ways to explore alignment dynamics. You can adjust the parameters of the equations through **sliders**. The diagram will update in real-time.

Figure 4: Interactive sliders

5.1.1 Time Evolution Simulation

The time evolution simulation displays how alignment changes over successive iterations. You can also tweak the initial condition (x_0) and the number of iterations.

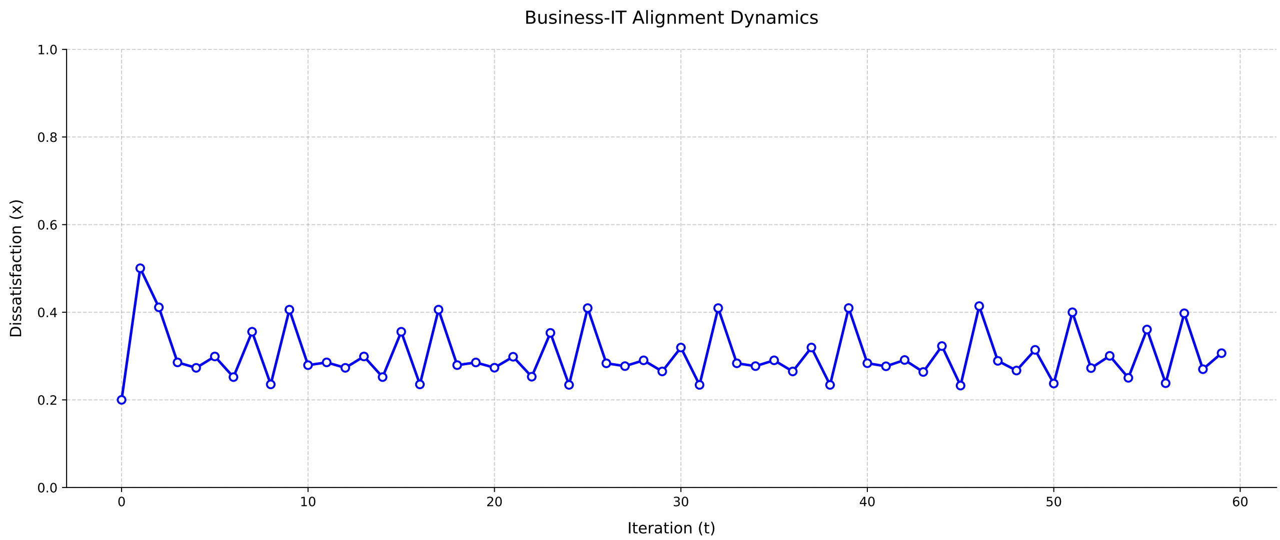

Figure 5: Time evolution showing chaotic behaviour ($d = 0.5$, $a = 6$, $h = 0.4$, $g = 2$, $r = 0.25$, $s = 5$)

5.1.2 Phase Portrait Analysis

The phase portait examines the system's underlying dynamics through multiple visual cues. Arrow directions indicate whether misalignment tends to increase or decrease at each state point, while a color gradient represents the rate of change intensity.

Note: Stable equilibrium points occur where the curve crosses zero with a negative slope - these represent self-correcting alignment levels.

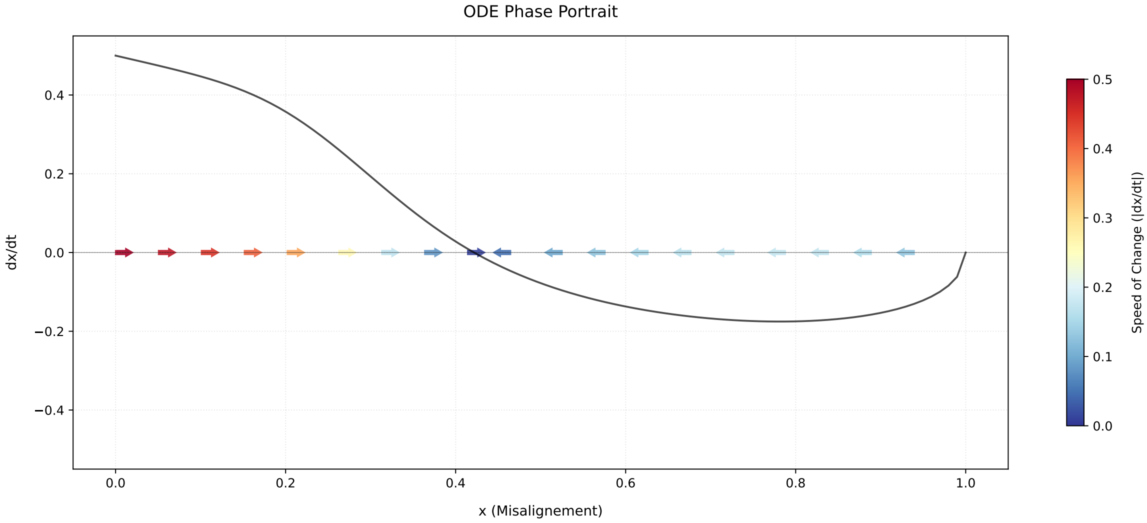

Figure 6: Phase portrait ($d = 0.5$, $a = 2$, $h = 1$, $g = 0.5$, $r = 0.3$, $s = 3$)

5.1.3 Bifurcation Diagram

Users can select any parameter for the x-axis via a dropdown menu and focus on specific ranges of interest. Adjust the sliders, and you may get the chacteristic period-doubling bifurcations and the emergance of chaotic behaviour.

Pro tip: Set h (IT rigidity) to a low value for observing beatiful patterns.

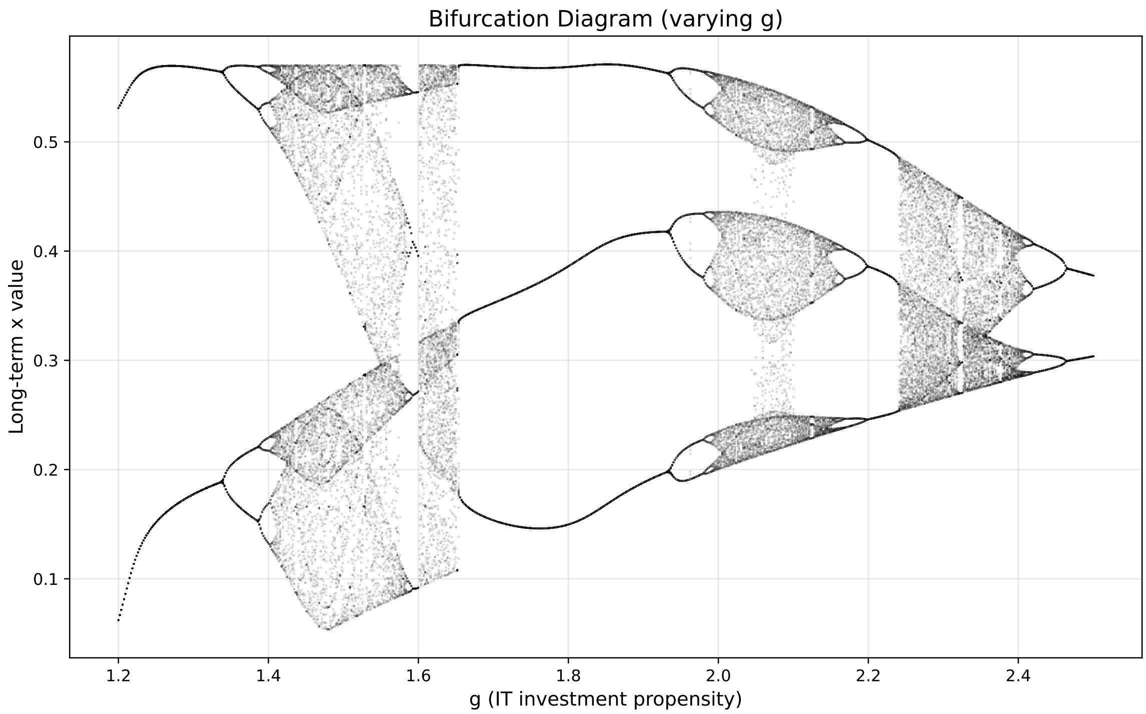

Figure 7: Shouldn't this be exposed at Louvre? ($d = 0.5$, $a = 7$, $h = 0.3$, $r = 0.3$, $s = 5.1$)

5.2 Interesting Cases

5.2.1 Alignment as a function of a and h

How variations in parameters **a** (efficiency) and **h** (rigidity) affect the long term alignment?

In order to find it out I built a countour line plot, where the output is displayed both as as a color gradient and and as countour lines. From that we can infer that improving efficiency (a↑) while reducing rigidity (h↓) decreases dissatisfaction, but with diminishing marginal returns. This is a pattern consistent with real world IT system behavior.

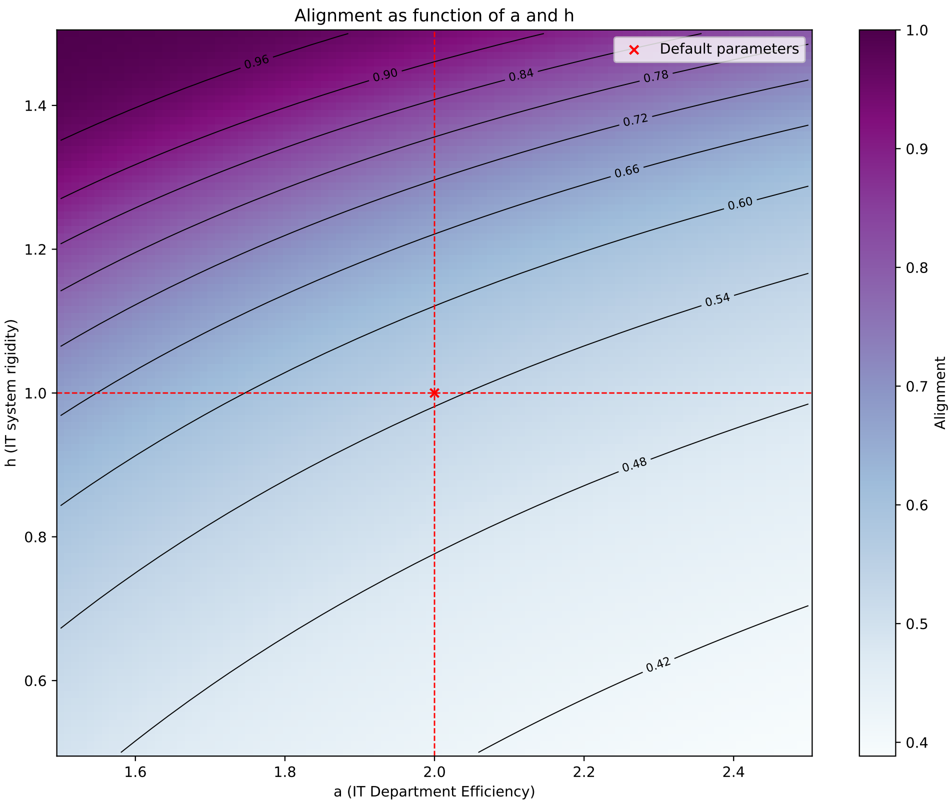

Figure 8: Shouldn't this be exposed at Louvre? ($d = 0.5$, $a = 7$, $h = 0.3$, $r = 0.3$, $s = 5.1$)

6 Personal Reflection

To be written at the end of the project. Some key points: Data Science master degree, I like beatiful graphs, real world application of mathematics, collaborating with a professors, learned a lot (even Latex documents).

References

[1] Veritasium. (2020, January 29). This Equation Will Change How You See the World.

[2] Strogatz, S. H. (2018). *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*. CRC Press.

[3] Luftman, J. (2003). *Assessing IT/business alignment*. Information Systems Management, 20(4), 9-15.

A Appendix: Complete Python Code

The full implementation is available at: `https://github.com/Kinshale/pii`