By Bisection Method

a) given f(x) = x^3-3,
finsd out the approximate initial bounds for the function to approximate the root ..
b) find ot the approximated root of given function using false position technique.
c) find out theaccuracy of the method based on the following condition ,
Er<Es, Where Es=0.001

x	f(x)=x^3-3
0	-3
1	-2
2	5
3	24
4	61

(1,2) are the initial roots of the equation.

x	f(x)=X^4 - X-10
0	-10
1	-10
2	4
3	68
4	242

f(X)=X^3-3

Iteration	Xl	Xu	f(Xl)	f(Xu)	Xmid	f(Xmid)	f(Xl)*f(Xmid)	Error
1	1	2	-2	5	1.5	0.375	-0.75	
2	1	1.5	-2	0.375	1.25	-1.046875	2.09375	FALSE
3	1.25	1.5	-2	0.375	1.375	-0.40039063	0.80078125	FALSE
4	1.375	1.5	-2	0.375	1.4375	-0.02954102	0.059082031	FALSE
5	1.4375	1.5	-2	0.375	1.46875	0.16842651	-0.336853027	FALSE
6	1.4375	1.46875	-2	0.1684265	1.453125	0.06837845	-0.136756897	FALSE
7	1.4375	1.453125	-2	0.0683784	1.4453125	0.01915407	-0.038308144	FALSE
8	1.4375	1.445313	-2	0.0191541	1.44140625	-0.00525945	0.010518909	FALSE
9	1.44140625	1.445313	-2	0.0191541	1.443359375	0.00693079	-0.013861582	FALSE
10	1.44140625	1.443359	-2	0.0069308	1.442382813	0.00083154	-0.001663083	TRUE

f(x)=X^4 - X-10

S.no	Xl	Xu	f(Xl)	f(Xu)	Xmid	f(Xmid)	f(Xl)*f(Xmid)	Error
1	1	2	-10	4	1.5	-6.4375	64.375	
2	1.5	2	-10	4	1.75	-2.37109375	23.7109375	FALSE
3	1.75	2	-10	4	1.875	0.48461914	-4.846191406	FALSE
4	1.75	1.875	-10	0.48462	1.8125	-1.02024841	10.20248413	FALSE
5	1.8125	1.875	-10	0.48462	1.84375	-0.28773403	2.877340317	FALSE
6	1.84375	1.875	-10	0.48462	1.859375	0.09337813	-0.933781266	FALSE
7	1.84375	1.85938	-10	0.09338	1.8515625	-0.09843343	0.984334312	FALSE
8	1.85156	1.85938	-10	0.09338	1.85546875	-0.00284285	0.02842847	FALSE
9	1.85547	1.85938	-10	0.09338	1.857421875	0.04518868	-0.451886752	FALSE
10	1.85547	1.85742	-10	0.04519	1.856445313	0.02115319	-0.211531937	TRUE

f(x) = x^2 - sin(x) - 0.5

Iteration	Xl	Xu	f(Xl)	f(Xu)	Xmid	f(Xmid)	f(Xl)*f(Xmid)	Error
1	0	2	-0.5	2.5907026	1	-0.34147098	0.170735492	
2	1	2	-0.5	2.5907026	1.5	0.75250501	-0.376252507	FALSE
3	1	1.5	-0.5	0.752505	1.25	0.11351538	-0.05675769	FALSE
4	1	1.25	-0.5	0.1135154	1.125	-0.13664259	0.068321297	FALSE
5	1.125	1.25	-0.5	0.1135154	1.1875	-0.01728067	0.008640334	FALSE
6	1.1875	1.25	-0.5	0.1135154	1.21875	0.0466825	-0.023341248	FALSE
7	1.1875	1.21875	-0.5	0.0466825	1.203125	0.01434286	-0.007171432	FALSE
8	1.1875	1.203125	-0.5	0.0143429	1.1953125	-0.00155833	0.000779164	TRUE

By False Position Method

a) given f(x) = x^3-3,
find out the approximate initial bounds for the function to approximate the root ..
b) find ot the approximated root of given function using false position technique.
c) find out theaccuracy of the method based on the following condition ,
Er<Es, Where Es=0.001

	f(x)=x^3-3
0	-3
1	-2
2	5
3	24
4	61

(1,2) are the initial roots of the equation.

x	f(x)= X^4 - X-10
0	-10
1	-10
2	4
3	68
4	242

f(X)=X^3-3

s.no	Xl	Xu	f(Xl)	f(Xu)	Xr	f(Xr)	f(Xr)*f(Xl)	Error
1	1	2	-2	5	1.2857143	-0.87464	1.749271137	
2	1.285714	2	-0.8746356	5	1.3920596	-0.30243	0.264512111	FALSE
3	1.39206	2	-0.3024255	5	1.4267336	-0.09579	0.028967971	FALSE
4	1.426734	2	-0.0957855	5	1.4375093	-0.02948	0.002824068	FALSE
5	1.437509	2	-0.0294833	5	1.4408067	-0.00899	0.000265201	FALSE
6	1.440807	2	-0.008995	5	1.4418109	-0.00274	2.46175E-05	TRUE

f(x) = X^4 - X -10

s.no	Xl	Xu	f(Xl)	f(Xu)	Xr	f(Xr)	f(Xr)*f(Xl)	Error
1	1	2	-10	4	1.714286	-3.07788	30.77884215	
2	1.714286	2	-3.07788	4	1.838531	-0.4128	1.270546036	FALSE
3	1.838531	2	-0.4128	4	1.853636	-0.04777	0.019720093	FALSE
4	1.853636	2	-0.04777	4	1.85536	-0.00543	0.000259423	TRUE

f(x) = x^2 - sin(x) -

s.no	Xl	Xu	f(Xl)	f(Xu)	Xr	f(Xr)	f(Xr)*f(Xl)	Error
1	0	2	-0.5	2.590703	0.323551	-0.71325	0.356625032	
2	0.323551	2	-0.7132501	2.590703	0.6854592	-0.66317	0.473009087	FALSE
3	0.685459	2	-0.6631743	2.590703	0.9533764	-0.40645	0.269546062	FALSE
4	0.953376	2	-0.4064483	2.590703	1.0953107	-0.18937	0.076967115	FALSE
5	1.095311	2	-0.1893651	2.590703	1.1569338	-0.07708	0.014595975	FALSE
6	1.156934	2	-0.0770785	2.590703	1.181292	-0.02965	0.002285117	FALSE
7	1.181292	2	-0.0296466	2.590703	1.1905549	-0.01115	0.000330684	TRUE
8	1.190555	2	-0.0111542	2.590703	1.194025	-0.00416	4.64212E-05	TRUE

Secant Methoc

$f(x)$= X^4 - X - 10

Es = 0.001

S.no	xl	f(xl)	xu	f(xu)	Xr	f(xr)	f(xr)*f(xl)	|f(xr)| < 0.001	Eabs < 0.001	Er < Es
1	1	-10	2	4	1.714286	-3.07788	30.77884215	FALSE		
2	1.714286	-3.07788	2	4	1.838531	-0.4128	1.270546036	FALSE	FALSE	FALSE
3	1.838531	-0.4128	2	4	1.853636	-0.04777	0.019720093	FALSE	FALSE	FALSE
4	1.853636	-0.04777	2	4	1.855363	-0.00543	0.000259423	FALSE	FALSE	TRUE
5	1.855363	-0.00543	2	4	1.855559	-0.00062	3.34544E-06	TRUE	TRUE	TRUE

Newton Rphson Method

$f(x) = x^3 - 2x^2 - 5$ | a) X0 = 1 | $f'(x) = 3x^2 - 4x$

Iteration	Xi	f(Xi)	f'(Xi)	Xi+1	f(Xi+1)
1	1	-6	-1	-5	-180
2	-5	-180	95	-3.105263158	-54.22831317
3	-3.105263158	-54.22831317	41.34903047	-1.793785898	-17.20714291
4	-1.793785898	-17.20714291	16.82814714	-0.771264359	-6.648483035
5	-0.771264359	-6.648483035	4.869603574	0.594038367	-5.496137964
6	0.594038367	-5.496137964	-1.317508723	-3.577575731	-76.38766011
7	-3.577575731	-76.38766011	52.70744725	-2.128299317	-23.69978396
8	-2.128299317	-23.69978396	22.10217122	-1.056016249	-8.407978616
9	-1.056016249	-8.407978616	7.569575955	0.054743279	-5.005829597
10	0.054743279	-5.005829597	-0.209982635	-23.78451179	-14591.3757
11	-23.78451179	-14591.3757	1792.24705	-15.64312542	-4322.402873
12	-15.64312542	-4322.402873	796.6946206	-10.21770549	-1280.546847
13	-10.21770549	-1280.546847	354.0753385	-6.601111202	-379.7905746
14	-6.601111202	-379.7905746	157.1284521	-4.184040523	-113.2590199
15	-4.184040523	-113.2590199	69.25474738	-2.548643324	-34.5460893
16	-2.548643324	-34.5460893	29.68132167	-1.384743357	-11.49029333
17	-1.384743357	-11.49029333	11.29151593	-0.367139219	-5.31906955
18	-0.367139219	-5.31906955	1.872930494	2.472832456	-2.108676649
19	2.472832456	-2.108676649	8.453371239	2.722280468	0.352683994
20	2.722280468	0.352683994	11.34331096	2.691188664	0.005931431
21	2.691188664	0.005931431	10.96273462	2.69064761	1.77781E-06
22	2.69064761	1.77781E-06	10.95616325	2.690647448	1.59872E-13
23	2.690647448	1.59872E-13	10.95616128	2.690647448	0

$f(x) = x^3 - 2x^2 - 5$ | b) X0 = 4 | $f'(x) = 3x^2 - 4x$

Iteration	Xi	f(Xi)	f'(Xi)	Xi+1
1	4	27	32	3.15625
2	3.15625	6.518463135	17.26074219	2.778603253
3	2.778603253	1.011312336	12.0474951	2.694659468
4	2.694659468	0.044054142	11.00493108	2.690656341
5	2.690656341	9.74318E-05	10.95626927	2.690647448
6	2.690647448	4.80179E-10	10.95616128	2.690647448

$f(x) = x - \cos(x)$ | a) X0 = 0 | $f'(x) = 1 + \sin(x)$

Iteration	Xi	f(Xi)	f'(Xi)	Xi+1
1	0	-1	1	1
2	1	0.459697694	1.841470985	0.750363868
3	0.7503639	0.018923074	1.681904953	0.739112891
4	0.7391129	4.64559E-05	1.673632544	0.739085133
5	0.7390851	2.84721E-10	1.673612029	0.739085133

$f(x) = x - \cos(x)$ | b) X0 = pi/2 | $f'(x) = 1 + \sin(x)$

Iteration	Xi	f(Xi)	f'(Xi)	Xi+1
1	1.5707963	1.570796327	2	0.785398163
2	0.7853982	0.078291382	1.707106781	0.739536134
3	0.7395361	0.000754875	1.673945288	0.739085178
4	0.7390852	7.51299E-08	1.673612062	0.739085133

Newton Forward Difference

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

x	y	Δy	Δ2y	Δ3y	Δ4y
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

The value of x at you want to find the $f(x): x = 1895$

$$h = x_1 - x_0 = 1901 - 1891 = 10$$

$$p = \frac{x - x_0}{h} = \frac{1895 - 1891}{10} = 0.4$$

Newton's forward difference interpolation formula is

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \cdot \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \cdot \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \cdot \Delta^4 y_0$$

$$y(1895) = 46 + 0.4 \times 20 + \frac{0.4(0.4-1)}{2} \times -5 + \frac{0.4(0.4-1)(0.4-2)}{6} \times 2 + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24} \times -3$$

$$y(1895) = 46 + 8 + 0.6 + 0.128 + 0.1248$$

$$y(1895) = 54.8528$$

Solution of newton's forward interpolation method $y(1895) = 54.8528$

x	1891	1901	1911	1921	1931
Y	46	66	81	93	101

x	y	Δy	Δ2y	Δ3y	Δ4y
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

The value of x at you want to find the $f(x): x = 1925$

$$h = x_1 - x_0 = 1901 - 1891 = 10$$

$$p = \frac{x - x_n}{h} = \frac{1925 - 1931}{10} = -0.6$$

Newton's backward difference interpolation formula is

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\cdot\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\cdot\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\cdot\nabla^4 y_n$$

$$y(1925) = 101 + (-0.6)\times 8 + \frac{-0.6(-0.6+1)}{2}\times -4 + \frac{-0.6(-0.6+1)(-0.6+2)}{6}\times -1 + \frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{24}\times -3$$

$$y(1925) = 101 - 4.8 + 0.48 + 0.056 + 0.1008$$

$$y(1925) = 96.8368$$

Solution of newton's backward interpolation method $y(1925) = 96.8368$

x	300	304	305	307
y	2.4771	2.4829	2.4843	2.4871

x	y	1st order	2nd order
300	2.4771		
		0.00145	
304	2.4829		0
		0.0014	
305	2.4843		0
		0.0014	
307	2.4871		

The value of x at you want to find the $f(x)$: $x = 301$

Newton's divided difference interpolation formula is

$$f(x) = y_0 + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2]$$

$$y(301) = 2.4771 + (301 - 300) \times 0.00145 + (301 - 300)(301 - 304) \times 0$$

$$y(301) = 2.4771 + (1) \times 0.00145 + (1)(-3) \times 0$$

$$y(301) = 2.4771 + 0.00145 + 0$$

$$y(301) = 2.47858$$

...divided difference interpolation method y(301)=2.47858

$x = (-1 + 2y - 3z)/5$ | 0 x

$y = (2 + 3x - z)/9$ | 0 y

$z = (-3+2x-y)/7$ | 0 z

iteration	x	y	z
0	0	0	0
1	-0.2	0.222222222	-0.428571429
2	0.146031746	0.203174603	-0.517460317
3	0.191746032	0.328395062	-0.415873016
4	0.180881834	0.332345679	-0.420700428
5	0.185358529	0.329260659	-0.424368859
6	0.186325579	0.331160494	-0.422649086
7	0.186053649	0.331291758	-0.422644191

$x = (-1 + 2y - 3z)/5$

$y = (2 + 3x - z)/9$

$z = (-3+2x-y)/7$

0	x
0	y
0	z

iteration	x	y	z
0	0	0	0
1	-0.2	0.155555556	-0.507936508
2	0.166984127	0.334320988	-0.428621819
3	0.190901487	0.333480698	-0.421668246
4	0.186393227	0.331205325	-0.422631267
5	0.186060891	0.331201549	-0.422725681

Gauss Seidel Method

x = (-1 + 2y -3z)/5
y = (2 + 3x -z)/9
z = (-3+2x-y)/7

Initial values
(x, y, z) = (0, 0, 0)

S.No.	x	y	z
0	0	0	0
1	-0.2	0.155555556	-0.507936508
2	0.166984127	0.334320988	-0.428621819
3	0.190901487	0.333480698	-0.421668246
4	0.186393227	0.331205325	-0.422631267
5	0.186060891	0.331201549	-0.422725681

true value	0.186	0.331	-0.422
errors	x	y	z
|Er|	0.032726115	0.06085381	0.171667145
accuracy	99.96727389	99.93914619	99.82833285

Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Lagrangian method for linear interpolation.

Table Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

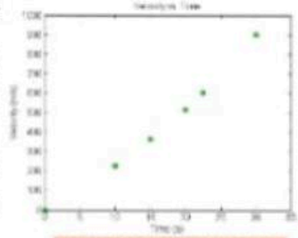

Figure. Velocity vs. time data for the rocket example

2

Linear Interpolation:

```
octave:1> v_t=[0 227.04 362.78 517.35 602.97 901.67]
v_t =

        0   227.0400   362.7800   517.3500   602.9700   901.6700

octave:2> t_s=[0,10,15,20,22.5,30]
t_s =

        0   10.0000   15.0000   20.0000   22.5000   30.0000

octave:3> t_v=[0,227.04,362.78,517.35,602.97,901.67]
t_v =

        0   227.0400   362.7800   517.3500   602.9700   901.6700

octave:4> >> % find t=16second using the Lagrangian method for Linear interpolation.
```

```
octave:4> t=16;
```

```
octave:9> A=interp1(t_s,t_v,t,'linear')
A = 393.69
```

```
octave:10> << % find t=16 for Quadratic interpolation>>
```

```
octave:0> B=interp1(t_s,t_v,t,'spline')
B = 392.07
```

```
octave:13> << % find t=16 for Cubic interpolation>>
```

```
octave:13> c=interp1(t_s ,v_s ,t,'cubic')
error: 'v_s' undefined near line 1, column 1
octave:14> c=interp1(t_s ,v_t ,t,'cubic')
c = 392.13
```