An approximate model of the Relative Spatial Variation in Response of EMI Instruments

Introduction

The electrical conductivity is commonly used property in precision agriculture to determine the spatial variability of soil characteristics (Corwin and Plant 2005). There are a range of factors that contribute to the electrical conductivity of the soil. These factors are: clay content, moisture ratio, cation exchange capacity, salinity and temperature (Guo et al., 2015; Whelan & Taylor, 2013). By obtaining a measurement of the electrical conductivity, we can better understand the nature of the soil and estimate its properties.

Electrical conductivity surveys are performed using proximal sensors. There are two methods to obtain the electrical conductivity of soil: electrical resistivity (ER)(Samouëlian et al. 2005) and electromagnetic induction (EMI)(Sudduth et al. 2003). Resistivity is an invasive, in-situ method and measures the electrical conductivity directly. The majority of electrical resistivity techniques require the sensor to perform the survey while being stationary. Electromagnetic surveys are non-invasive techniques. They estimate the electrical conductivity indirectly allowing measurements to be measured “on the go” (Adamchuk et al. 2004).

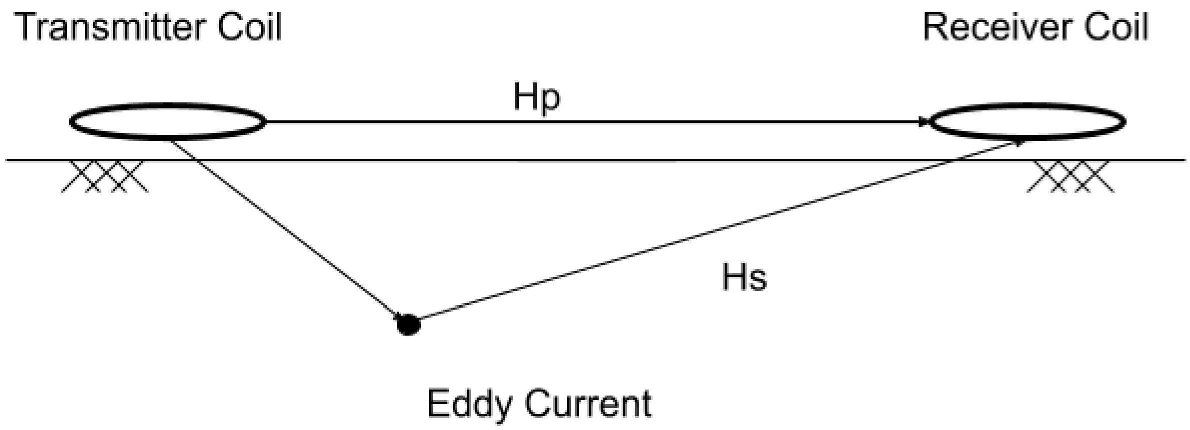
Because both methods are measuring the electrical conductivity, an applied electrical potential in the soil is required to measure its conductance. ER methods supply a current into the soil directly, via electrical contacts, to create the electrical potential; where EMI, as the name suggests, uses electromagnetic induction to do so (Viscarra Rossel & McBratney, 1998).

The purpose of this paper is to develop a method to map the relative response that an electromagnetic induction (EMI) system has on a homogeneous half space. This method will approximate the response using the law of biot-savart, ohm's law, the faraday-lenz law and various assumptions; outlined below.

EMI is a method to estimate the Electrical conductivity in the ground. EMI is an active method therefore consists of a minimum 2 coils - one to transmit an alternating magnetic field (transmitter coil) and the other to receive and record changes in magnetic flux (receiver coil). The complete system is a follows(Callegary et al. 2007):

1. An alternating current is supplied to the transmitter coil
2. The transmitter coil produces an alternating magnetic field (Hp)
3. The changing magnetic field in the ground produces eddy currents
4. The eddy currents then produces a secondary magnetic field (Hs)

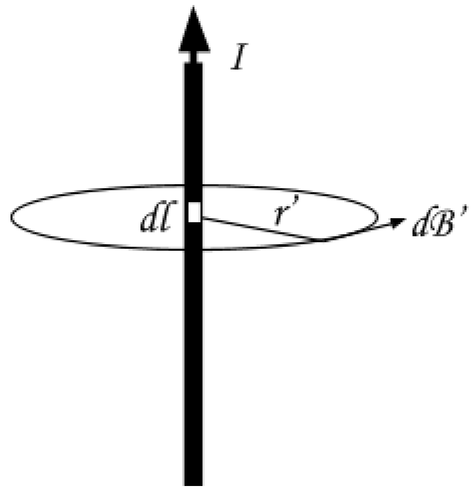
5. The resulting magnetic field (Hs + Hp) induces an Emf in the receiver coil and, after extrapolating Hp and Hs, the apparent electrical conductivity is estimated using: $\sigma_a = \frac{4}{\omega\mu_0 s^2}\Gamma\left(\frac{Hs}{Hp}\right)$ (McNeill 1980)



Based on this system, we will set out to establish a method to describe the magnetic field produced by the transmitter coil for a given current (I) and the response caused by the eddy currents in the ground.

Theory

As the response varies spatially, we must compare the secondary field with the primary field at the receiver Rx. In order to obtain an expression of the ratio between the secondary magnetic field (Hs) and the primary magnetic field (Hp), we must first derive an expression of these magnetic fields. The transmitter, that produces the alternating magnetic field, is usually a multi coiled loop of wire. In principle, when a section of wire is supplied with a current, a magnetic field is created such that it acts orthogonally to the direction of the current and the vector between the section of wire and the location at which the field is produced.



The magnitude of the magnetic flux density, measured in teslas (T), may be determined according to law of Biot-Savart (Jackson 1999; Hauser and Grandy 1973) which states:

$$dB' = \frac{\mu_0}{4\pi} \frac{I dl \times r'}{r'^3}$$

Where r' is the vector (magnitude measured in meters) from the small section of wire (dl), I is the current in Amps, B is the magnetic flux density and μ_0 is the permeability of free space(H/m). The direction of the dB is determined by the "right hand rule" and is a resultant of the cross product of $dl \times r'$. $\times$ is the symbol for the cross product. By summing the contribution from all current carrying elements, the total magnetic flux density is given as:

$$B(r') = \frac{\mu_0}{4\pi} \int_C \frac{I dl' \times r'}{|r'|^3}$$

Simplified:

$$B(r') = \frac{\mu_0 I}{4\pi} \int_C \frac{dl' \times \bar{r}'}{|r'|^2}$$

Where $\bar{r}'$ is the unit vector of r' and I is a constant.

Primary Magnetic Field H_p

Before considering Hs, we must apply a time varying current to Tx so the change in magnetic flux can produce an electromotive force in Rx according to the faraday-lenz law. For simplification in calculations, let's consider this time varying current at a sign wave. Thus the equation for the primary magnetic field is:

$$B(r') = \frac{\mu_0 I sin(\omega t)}{4\pi} \int_C \frac{dl' \times \overline{r'}}{|r'|^2}$$

The primary magnetic field (Hp) is the field produced by the transmitter(s), Tx, at the point where the receiver is; Rx. let s' be the the vector from Tx to Rx and its magnitude is the intercoil spacing that does not vary. By considering the magnetic field is proportional to the magnetic flux density, $B = \mu_0 H$, an expression of the primary magnetic file may be produced:

$$H_p = \frac{I sin(\omega t)}{4\pi} \int_C \frac{dl' \times \overline{(s' - R')}}{|s' - R'|^2}$$

Where H is denotes the magnetic field measured in A/m, t is time in seconds, ω is the angular frequency equal to 2πf and f is the frequency in Hz. It is important to note that s' is the vector from the centre of the transmitter coil to the centre of the receiver coil and does not vary. R' is the vector extending from the center of the transmitter coil to each element dl. The magnitude of R' is equal to the radius of the transmitter coil and is perpendicular to dl in the same plane as the Tx loop. $\overline{s' - R'}$ is the unit vector of the vector resulting from $s' - R'$.

We will finally manipulate this expression to consider the intercoil spacing; equal to the magnitude of s'. Where s is the intercoil spacing and $\overline{s'}$ is the unit vector pointing toward the center of Rx.

$$H_p = \frac{I sin(\omega t)}{4\pi s^2} \int_C \frac{dl' \times \overline{(s' - R')}}{\left|\overline{s'} - R' \bullet \frac{1}{s}\right|^2}$$

Note • is the symbol denoting the dot product.

Secondary Magnetic Field H_s

We now make some assumptions about our system to formulate an equation for the secondary field (Hs) produced from a point r' away from Tx in a homogeneous half-space. Firstly, as we only need to consider relative change of response of each point in the conductor. We may simplify the system by observing the effects of the changing flux density on a loop orthogonal to the primary magnetic field at r' as if it were the path of part of the eddy current caused by B(r). Each theoretical loop is required to have consistent radius (R_e) and conductance (σ). Secondly, for purposes of analysis, we are to consider our half-space is homogeneous with a consistent conductance and assume a permeability equal to μ_0. Lastly, we assume that we have low induction numbers. Low induction numbers are defined by [McNeill 1980](#) as:

The magnetic field produced by the eddy current, at r’, at the receiver may be expressed as:

$$H_s = \frac{I_e}{4\pi} \int_C \frac{dl'_e \times \overline{((s'-r')-R'_e)}}{((s'-r')-R'_e)^2}$$

Where (s’ - r’) describes the vector from the focus point at r’ to the receiver position at s’ and $\overline{(s'-r')}$ is its unit vector, I_e is the theoretical loop current with radius R'_e and dl'_e is the vector representing the small element of the loop contributing to the magnetic field. Because we are using low induction numbers, we may use ohm’s law to calculate I_e:

$$I_e = V\sigma$$

Where V is voltage in volts and is equal to the electromotive force (reference). We now apply the faraday-Lenz law (Jackson 1999):

$$I_e = -\frac{d\Phi}{dt}\sigma$$

Where $\Phi = A\mu 0 Hp$, $A = \pi R_e^2$ and R_e is significantly small.

$$I_e = -A\sigma\mu_0 \frac{d\|H(r')\|}{dt}$$

By deriving and substituting equation (equation number), the current may finally be expressed as:

$$I_e = -A\sigma\mu_0\omega \frac{\mu_0 I cos(\omega t)}{4\pi} \int_C \frac{dl' \times \overline{(r'-R')}}{|r'-R'|^2}$$

From this, an expression of the secondary magnetic field is obtained.

$$H(r')_s = -A\sigma\mu_0\omega \frac{I cos(\omega t)}{16\pi^2} \left\| \int_C \frac{dl' \times \overline{(r'-R')}}{|r'-R'|^2} \right\| \left(\int_C \frac{dl'_e \times \overline{((s'-r')-R'_e)}}{((s'-r')-R'_e)^2} \right)$$

Where $\int_C dl'_e = 2\pi R_e$.

Comparing Hs to Hp

By comparing the time variables of the Hp and Hs equations(ie $sin(\omega t)$ for Hp and $-cos(\omega t)$ for Hs) we see that the primary and secondary magnetic fields are in quadrature as Hs proceeds Hp by $\frac{\pi}{2}$. Using the quadrature component $\Gamma(\cdot)$ (please explain):

$$\Gamma\left(\frac{H(r')s}{Hp}\right) = \sigma \frac{\mu_0\omega s^2}{4} \left\| \int_C \frac{dl' \times \overline{(s'-R')}}{\left|\overline{s}'-R' \cdot \frac{1}{s}\right|^2} \right\|^{-1} \left\| \int_C \frac{dl' \times \overline{(r'-R')}}{|r'-R'|^2} \right\| \left\| \int_C \frac{dl'_e \times \overline{((s'-r')-R'_e)}}{((s'-r')-R'_e)^2} \right\| R_e^2$$

Now all components that are dictated by the spatial variation of r’ may be substituted by the function G(r’):

$$\Gamma\left(\frac{H(r')s}{Hp}\right) = \sigma \frac{\mu_0\omega s^2}{4} G(r') \qquad \text{(change quadrature symbol)}$$

Where:

$$G(r') = \left\| \int_C \frac{dl' \times \overline{(s'-R')}}{\left|\overline{s'-R'} \cdot \frac{1}{s}\right|^2} \right\|^{-1} \left\| \int_C \frac{dl' \times \overline{(r'-R')}}{|r'-R'|^2} \right\| \left\| \int_C \frac{dl'_e \times \overline{((s'-r')-R'_e)}}{((s'-r')-R'_e)^2} \right\| R_e^2$$

The function G(r') may be used to understand the spatial variation of the response.

Numerical Solution

Because the geometry, dimensions and coil orientations are consistent, the sum of all the responses may be considered constant.

$\int_v G(r')$ is constant

Though this method is only intended for observations of the spatial variation of the response magnetic fields $H(r')_s$, a numerical solution for $\Gamma\left(\frac{Hs}{Hp}\right)$ may buy be obtained by considering that the hypothetical eddy currents aren't a true representation of the magnitude of Hs and therefore is only proportional to (equation above). ie

$$\Gamma\left(\frac{H(r')s}{Hp}\right) = \sigma \frac{\mu_0 \omega s^2}{4} k \int_v G(r')$$

Where k is the eddy current assumption adjustment factor that may be numerically obtained via calibration. Note that k needs to be obtained for each variation of the system design.

Finally, this can be rearranged to make the conductivity the subject:

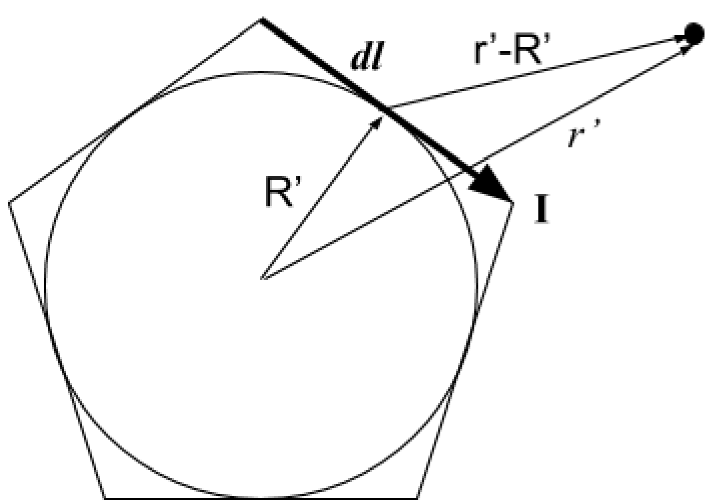
$$\sigma = \Gamma\left(\frac{Hs}{Hp}\right) \frac{4}{\mu_0 \omega s^2} \frac{1}{k \int_v G(r')}$$

This equation displays similarities to the "apparent conductivity" σ_a derived by McNeill 1980, used for specific coil configurations wherein both the transmitter coil and the receiver coil are located within the same plane s' away from each other. With this configuration, the coils are placed on the surface of the half-space; first, parallel to the surface (vertical dipole) then perpendicular to the surface (horizontal dipole).

Methodology

Before creating a function for approximating the relative response of a homogeneous half-space subject to an alternating magnetic field, we must establish a method for determining the orientation of the transmitter Tx and receiver Rx coils. For this distinction, we will use a unit vector that is orthogonal to the plane parallel to each loop Tx and Rx. These vectors will be denoted as T_X' and R_X'.

In order to practically utilize the expressions for Hp, Hs and G(r') computationally, in an approximation model, we must approximate the magnetic flux density B(r') produced by the the transmitter coil, this coil is to be simplified as a single loop of current carrying wire that is a polygon size n. Each side of the polygon may then be treated as a finite element of current carrying wire and may be substituted as dl.



The length of each side of the polygon may be calculated using the following method:

$dl = 2R \cdot tan(\frac{\pi}{n})$

Where R is the radius of the coil and n is the number of sides or the polygon. By substituting this into equation (give number), an approximation of the magnetic flux density may be determined:

$$B'(r') \approx \lim_{n\to\infty} \sum_{n}^{i} \frac{\mu_0 I}{2\pi} \frac{R \cdot tan(\frac{\pi}{2}) \cdot \overline{dl_i} \times \overline{(r'-R'_i)}}{(r'-R'_i)^2}$$

By substituting the s' = r', the vector from Tx to Rx, we have an approximation of the primary magnetic field:

$$H_p \approx \lim_{n\to\infty} \sum_{n}^{i} \frac{I}{2\pi} \frac{R \cdot tan(\frac{\pi}{2}) \cdot \overline{dl_i} \times \overline{(s'-R'_i)}}{(s'-R'_i)^2}$$

The vectors R' and and dl are, by definition perpendicular to each other in the same plane as the Tx loop. Therefore we may define:

$$\overline{dl_i} = \overline{T_x' \times R_i}$$

Where:

$$R_{i+1} = RM(T_x, \frac{2\pi}{n}) \cdot R_i$$

RM is the function that operates as the Euler rotation matrix (reference) that rotates a vector around an axis. R_1 may be any radius vector. Note; the approximation for $B'(r')$ may be used to generate a vector plot of that magnetic flux density resulting from Tx and will be useful in the discussion in conjunction with the depth response plots. We must now consider a variation of the function we derived for describing the relative response:

$$\Gamma\left(\frac{H(r')s}{Hp}\right) = \sigma \frac{\mu_0 \omega}{4} U'(r')^{-1} P'(r') S'(r') R_e^2$$

First, we are to choose the hypothetical eddy current radius R_e such that it is at least less than half the increments of x, y and z in r' = [x, y, z]. Ie:

$$2R_e \leq dx = dy = dz$$

This is so that, geometrically, no eddy current loop can overlap, distorting the contribution to the secondary magnetic field field Hp. Second, we shall consider the component independent of r'; that, after being determined may be considered a constant.

$$U'(r') = \int_C \frac{dl' \times \overline{(s'-R')}}{(s'-R'_i)^2} \approx \lim_{n\to\infty} \sum_{n}^{i} \frac{R \cdot tan(\frac{\pi}{2}) \cdot \overline{dl_i} \times \overline{(s'-R'_i)}}{(s'-R'_i)^2}$$

Now we use the same method to estimate the magnetic field vector of G(r') produced from Tx at r':

$$P'(r') = \int_C \frac{dl' \times \overline{(r'-R')}}{|r'-R'|^2} \approx \lim_{n\to\infty} \sum_{n}^{i} \frac{R \cdot tan(\frac{\pi}{2}) \cdot \overline{dl_i} \times \overline{(r'-R'_i)}}{(r'-R'_i)^2}$$

This approximation improves as n heads toward infinity. With this method, the magnetic flux density may be computationally produced for each location at r'. This vector B' is not only important for its magnitude to calculate the hypothetical eddy current I_e, but also for its direction, to determine the orientation of the hypothetical eddy current loop that is to be orthogonal to B'.

$$\overline{dl_{ei}} = \overline{B' \times R_{ei}}$$

Where

$$R_{ei+1} = RM(B', \tfrac{2\pi}{n}) \cdot R_{ei}$$

Finally, the vector that dictates the magnetic field resulting from: the eddy current:

$$S'(r') = \int_C \frac{dl'_e \times \overline{((s'-r')-R'_e)}}{((s'-r')-R'_e)^2} \approx \lim_{n\to\infty} \sum_n^i \frac{R \cdot tan(\frac{\pi}{2}) \cdot \overline{dl_{ei}} \times \overline{((s'-r')-R'_{ei})}}{((s'-r')-R'_{ei})^2}$$

Now we have an approximate methods for obtaining the vector components of G(r'), we may also like to consider the orientation of the receiver coil. This is the case where there is one receiver coil with center s' away from Tx and orientation R'_x. Therefore, we may like to take it into consideration:

$$\Gamma\left(\frac{H(r')s}{Hp}\right) = \sigma \frac{\mu_0 \omega s^2}{4} (R'_x \bullet U'(r'))^{-1} \|P'(r')\| (R'_x \bullet S'(r'))$$

Using this equation, A function

We may also find a numeric solution for $\Gamma\left(\frac{Hs}{Hp}\right)$ by obtaining the constant, k, by using the system on a homogeneous half space with a known electrical conductivity. We can rearrange equation (Number):

$$k = \Gamma\left(\frac{Hs}{Hp}\right) \frac{4}{\sigma \mu_0 \omega s^2} \frac{1}{G}$$

Where:

$$G = \int_V G'(r') dV$$

$$dV = dxdydz$$

Two transmitters

To do

How to extrapolate the data: H = Hp +Hs
Skin effect!
Units!!!

Induction in ground
It is important to consider that, due to the oscillating nature of the eddy current in the ground, we should expect some impedance that should affect the response

Results

Discussion

The ground that the magnetic field acts upon as a homogeneous half space. As it is subject to an alternating magnetic field, eddy currents are produced. Previous studies have assumed that these eddy currents only act in the x-y plane and therefore their calculations adhere to that particular model (McNeill 1980) (Add the others too). On the contrary, we shall assume that eddy currents act in the plane that is orthogonal to the magnetic field lines. Therefore, at each new point, we must consider a new, unique, plane at which the eddy currents flow as per the . In reality, current is not restricted to one particular plane; because as it flows to a new location, the orientation of the magnetic field line may also change. Therefore we will only consider eddy current that flow in a very small circular path; with radius dr. We can idolise these small currents as flowing in a small loop with a resistance inversely proportional to the conductance of the homogeneous half space. It is important to not that by only considering these small eddy current loops, the total response can not be calculated. The response calculated using this method may be compared with the surrounding responses to establish the spatial response variation.