Mathematics Paper 1 Questions and Answers - Form 3 End Term 3 2022 Exams

QUESTIONS

SECTION 1 (50 Marks)
Answer ALL questions from this section

1. Evaluate: (2 Marks)

$$\frac{\sqrt{\frac{1}{4}} \; of \; 3\frac{1}{2} + \frac{3}{2}\left(\frac{5}{2} - \frac{2}{3}\right)}{\frac{3}{4} of \; 2\frac{1}{2} \div \frac{1}{4}}$$

2. Solve the equation. (4 Marks)

$$\frac{x+1}{2} - \frac{3}{x} = \frac{x-2}{5}$$

3. The first, third and seventh terms of an increasing arithmetic progression are the three consecutive terms of a geometric progression. If the first term of the arithmetic progression is 10, find the common difference of the arithmetic progression. (3 Marks)

4. Find y if $\log_2 y - 2 = \log_2 92$ (3 Marks)

5. Rationalize the denominator (3 Marks)

$$\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$$

6. A contractor employs 40 men to do a piece of work in 60 days each man working 9 hours a day. The contractor is then required to do the same job in 48 days. How many more men working 10 hours a day does he need to employ. (3 Marks)

7. A student's results in six Mathematics tests were: 24, 28, 32, x, 48 and 50. If the median is 36, find the mean mark. (3 Marks)

8. Given that the dimensions of a rectangle are 20.0cm and 25.0cm. Find the percentage error in calculating the area. (3 Marks)

9. The co-ordinates of the points P and Q are (1,-2) and (4,10) respectively. A point T divides the line PQ in the ratio 2:1
   Determine the co-ordinates of T (2 Marks)

10. A student at a certain college has a 60% chance of passing an examination at the first attempt. Each time a student fails and repeats the examination his chance of passing is increased by 15%.
    Calculate the probability that a student in the college passes an examination at the second or at the third attempt. (4 Marks)

11. What is the square value of

$$\frac{2w(x-2)^2}{y+1} \; \text{if} \; x = 3, \; y = x + 3 \; \text{and} \; w = 2x + y$$

    (2 Marks)

12. A salesman earns a basic wage of KSh. 1500 per week in addition, he is paid commission per week as follows:-

	Commission
For sales upto KSh. 50,000	0%
For sales above KSh. 50,000	

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     i. For the first KSh. 25,000   2%

     ii. For the next KSh. 25000   2½%

     iii. For any amount above KSh.100,000   5%

     During that week, he sold goods worth KSh. 115,000. What was his total pay for that week. (4 Marks)

13. Two grades of tea A and B costs KSh. 25 and KSh. 28 respectively per kg. They are mixed and the mixtures sold at KSh. 31.20 making a profit of 20%. Find the ratio of A:B in the mixture. (4 Marks)

14. The surface area of two similar bottles are $12cm^2$ and $108cm^2$ respectively. If the bigger one has a volume of $810cm^3$. Find the volume of the smaller one. (3 Marks)

15. If tan x = $^1/_{\sqrt{3}}$, Find without tables or calculator,
Sin(90-x) + Cos (90 –x). Leave your answer in surd form in simplest form. (3 Marks)

16. A regular polygon has the sum of all its interior angles as $1260º$. Find the size of each exterior angles in the polygon. (3 Marks)

SECTION II

17. A group of people planned to contribute equally towards a water project which needed KSh. 2,000,000 to complete. However, 40 members of the group withdrew from the project. As a result each of the remaining members were to contribute KSh. 2500 more.
   a. Find the original number of members in the group. (5 Marks)
   b. 45% of the value of the project was funded by Consistuency Development Fund (CDF). Calculate the amount that would be made by each of the remaining members of the group. (3 Marks)
   c. Members contribution were in terms of labour provided and money contributed. The ratio of the value of labour to the money contributed was 6:19, calculate the total amount of money contributed by members. (2 Marks)

18. If $\underset{\sim}{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 14 \\ 1 \end{pmatrix}$ are vectors, find
   a. 2a + 3b (2 Marks)
   b. ½a – b (2 Marks)
   c. If x and y are scalars in the following equation.

   xa – yb = $\begin{pmatrix} -13 \\ -18 \end{pmatrix}$, form two equations simultaneously hence solve for x and y. (6 Marks)

19. The figure below shows two intersecting circles of centres C and D radii 16cm and 20cm respectively. The two circles substend angles θ_1 and θ_2 at their centres respectively and intersect at P and Q as shown.
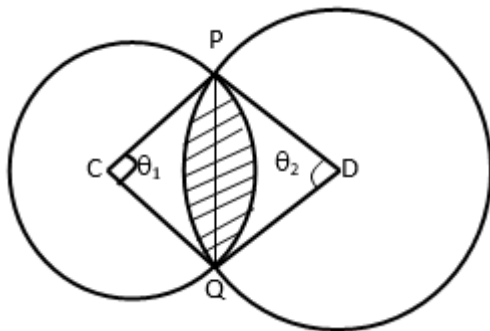
Given that the area of triangle PCQ = $80.14cm^2$,
Calculate the size of
   i. The angle marked θ_1 (2 Marks)
   ii. The angle marked θ_2 (3 Marks)
   iii. The area of the shaded region (5 Marks)

20. The diagram below represents a solid frustrum consisting of a hemispherical bottom and a conical frustrum at the top.
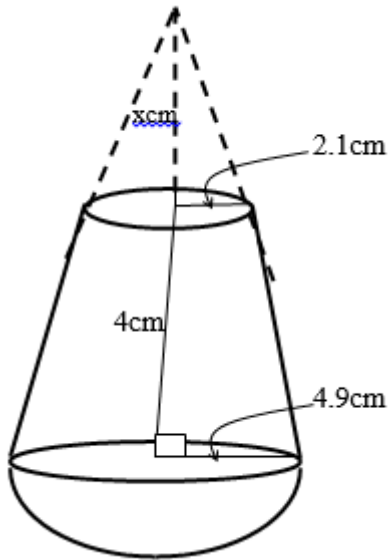
a. Calculate the value of x(height of the smaller one) (2 Marks)
b. Calculate:
   i. Surface area of the solid (4 Marks)
   ii. Volume of the solid (4 Marks)

21. Town B is 102km on a bearing of 112º from town A. Town C is 94 km on a bearing of 062º from town B. Town D is 073º from town A and 336º from town C.
    a. Using a scale of 1cm rep. 20km, draw a diagram to show the positions of towns A, B, C and D.
    b. Using the diagram in (a) above, determine
       i. Bearing of town B from town D. (1 Mark)
       ii. Bearing of town A from town C (1 Mark)
    c. The distance AC and BD (2 Marks)

22. A surveyor recorded the measurements of a field in a field book using lines AB 260m as shown below.

	B	
	130	R 40
	70	Q 10
	50	P 20
S50	10	
	A	

a. Sketch the map of the field (4 Marks)
b. Find the area of the field in hectares (6 Marks)

23. Construct the parallelogram ABCD where AB = 8cm, BC = 6cm angle ABC = 1200. Using a ruler and a pair of compass only. (3 Marks)
    a. Draw in the diagram diagonals and construct the circumcircle ABD (2 Marks)
    b. Drop a perpendicular from D to meet AB. Let the perpendicular cut diagonal AC at x. (2 Marks)
    c. Drop a perpendicular from B to meet DC and cut diagonal AC at Y. (2 Marks)
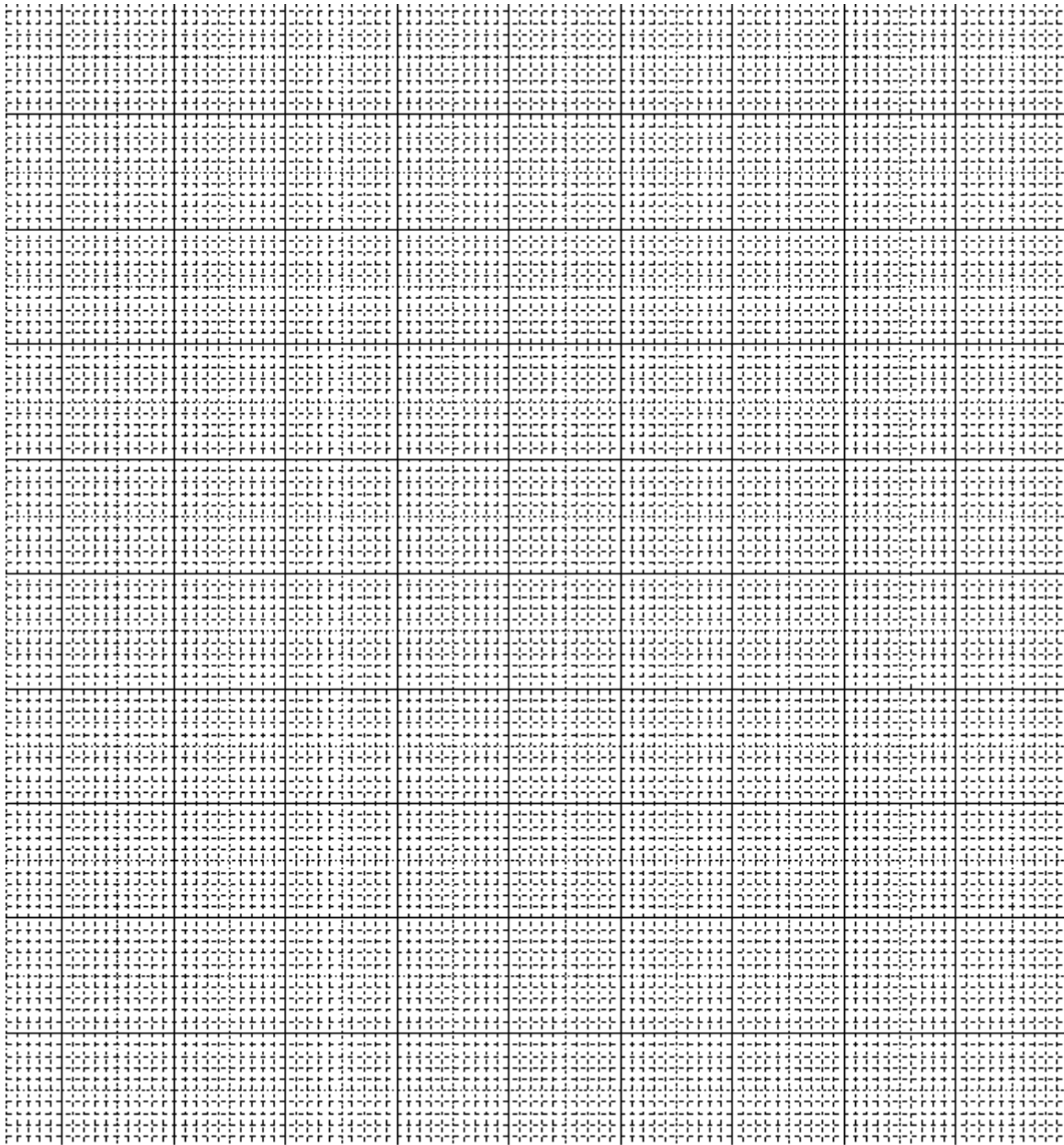   d. Measure XY (2 Marks)

24.

a. Complete the table below for $y = 2x^3 + x^2 - 5x + 2$.
   For the interval $-3 \leq x \leq 3$. (2 marks)

x	-3	-2	-1	0	0.5	1	2	3
$2x^3$	-54		-2		0.25		16	
x^2	9	4			0.25	1		
-5x			5	0	-2.5	-5	-10	
+2	2	2	2	2	2	2	2	2
y			6					50

b. Draw the graph of $y = 2x^3 + x^2 - 5x + 2$ for the interval $-3 \leq x \leq 3$ (3 Marks)
c. Use your graph to solve equation $y = 2x^3 + x^2 - 5x + 2$ (1 Mark)
d. Use your graph to solve equation $y = 2x^3 + x^2 - 11x - 10$ (2 Marks)
e. Find the gradient of the curve at $x = 2$ (2 Marks)

MARKING SCHEME

No.	

1.	**Numerator** $$\sqrt{\frac{1}{4}} \text{ of } \frac{7}{2} + \frac{3}{2}\left(\frac{15-4}{6}\right)$$ $$\frac{1}{2} \times \frac{7}{2} \times \frac{3}{2} \times \frac{11}{6}$$ $$\frac{7}{4} + \frac{33}{12} = \frac{21+33}{12} = \frac{54}{12}$$ **Denominator** $$\frac{3}{4} \text{ of } \frac{5}{2} \div \frac{1}{4}$$ $$\frac{15}{8} \times \frac{4}{1} = \frac{5}{12}$$ $$\frac{54}{12} \div \frac{15}{2}$$ $$\frac{54}{6} \ x \ \frac{2}{15} = \frac{3}{5}$$ $$= \frac{3}{\underline{5}}$$
2.	L.C.M. 10x $\frac{x + 1}{2} \times 10x - \frac{3}{x} 10x = \frac{x - 2}{5} \times 10x$ $5x(x + 1) - 30 = 2x(x - 2)$ $5x^2 + 5x - 30 = 2x^2 - 4x$ $3x^2 + x - 30 = 0$ $3x^2 + 10x - 9x - 30 = 0 \ (10,-9)$ $x(3x + 10) - 3(3x + 10) = 0$ $(x - 3)(3x + 10) = 0$ $x - 3 = 0 = 3$ $3x + 10 = 0 = \frac{-10}{3}$
3.	$T_1 = 10 + 0d = 10$ $T_3 = 10 + 2d$ $T_T = 10 + 6d$ $\frac{10 + 2d}{10} = \frac{10 + 6d}{10 + 2d}$ $(10 + 2d)(10 + 2d) = 10(10 + 6d)$ $100 + 40d + 4d^2 = 100 + 60d$ $60d = 40d + d^2$ $4d^2 = 20d$ $d = 5$

4	$Log_2 y\ 2 = Log_2 92$ $Log_2 y - Log_2 92 = 2$ $Log_2\ (y/92) = 2$ $y/92 = 2^2$ $y = 4 \times 92$ $y = 368$
5	$\dfrac{2\sqrt{3}}{(\sqrt{3}+\sqrt{2})} \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$ $= \dfrac{2\sqrt{3}x\sqrt{3}-2\sqrt{3}x\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$ $= \dfrac{2x3-2\sqrt{6}}{3-2}$ $= \dfrac{6-2\sqrt{6}}{1}$ $= 6 - 2\sqrt{6}$
6	Men Days Hours 40 60 9 ? 48 10 No. of men to work 48 days 10 hrs $= \dfrac{40 \times 60 \times 9}{48 \times 10}$ $= 45$ $= 45 - 40 = 5$ Men
7	Mean – of six $\underline{n}$, $\underline{n}$ + 1 2 2 32, x Median 36 $= \dfrac{32 + x}{2}$ $72 = 32 + x$ $72 - 32 = x$ $40 = x$ Mean mark $= \dfrac{24 + 28 + 32 + 40 + 48 + 50}{6}$ $x = 37$

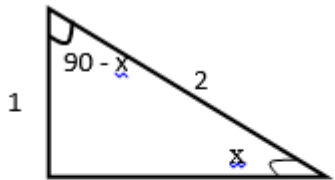|	
8	Absolute error $= \dfrac{\text{max - working + working - min}}{2}$ $= \dfrac{20.05 \times 25.05 - 19.95 \times 24.95}{2}$ $= \dfrac{4.5}{2}$ $= 2.25$ % error $= \dfrac{\text{absolute}}{\text{actual}} \times 100$ $= \dfrac{2.25}{20.0 \times 25.0} \times 100$ $= 0.45\%$
9	$$\underset{2}{\overset{P(1,-2) \qquad m \qquad T \qquad n \qquad Q(4,10)}{\rule{6cm}{0.4pt}}}$$ $OT = \dfrac{m}{m+n}(OQ) + \dfrac{m}{m+n}(OP)$ $= \dfrac{2}{3}\begin{pmatrix}4\\10\end{pmatrix} + \dfrac{1}{3}\begin{pmatrix}1\\-2\end{pmatrix}$ $= \begin{pmatrix}\frac{8}{3}\\\frac{20}{3}\end{pmatrix} + \begin{pmatrix}\frac{1}{3}\\\frac{-2}{3}\end{pmatrix}$ $= \begin{pmatrix}\frac{9}{3}\\\frac{18}{3}\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$ $T(3,6)$
10	 P (Second or third) $= P(FP)$ or $P(FFP)$ $= 0.4 \times 0.69 + 0.4 \times 0.31 \times 0.7935$ $= 0.276 + 0.098394$ $= 0.374394$
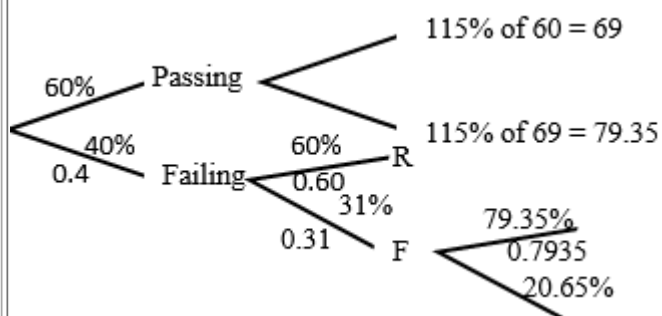
11	$\dfrac{2w(x-2)^2}{y+1}$ $X = 3$ $Y = 3 + 3 = 6$ $W = 2 \times 3 + 6$ $= 6 + 6 = 12$ $\therefore \dfrac{2 \times 12(3-2)^2}{6+1}$ $\dfrac{24 \times 12}{7} = \dfrac{24}{7}$ $x = 3,\ y = x + 3$ and $w = 2x + y$
12	The 1st 25000 = $2500 \times \dfrac{2}{100}$ = 500/= Next 25000 = $25000 \times \dfrac{2.5}{100}$ = 625/= Reamaining 15,000 = $15000 \times \dfrac{5}{100}$ = 750/= Total commission = 500 + 625 + 750 = 1875/= Weekly pay 1500 + 1875 = 3375/=
13	Cost of mixture $\Rightarrow \dfrac{31.20 \times 100}{120} = Sh.\ 26$ $\left(\dfrac{A}{A+B}\right)25 + \left(\dfrac{A}{A+B}\right)28 = 26$ $25A + 28B = 26A + 26B$ $\dfrac{2B}{B} = \dfrac{A}{B}$ $\dfrac{A}{B} = \dfrac{2}{1}$ $\therefore A : B = 2:1$
14	A.S.F = 108:12 = 9:1 L:S:F = $\sqrt{A.S.F}$ or A.S.F = $(L.S.F)^2$ L:S:F = 3 :1 (L:S:F)3 = V.S.F V.S.F = 27:1 Volume of smaller = $\dfrac{1}{27} \times 810$ = 30cm^3

15	$\text{Sin}\,(90-x\,)+\text{Cos}\,(90-x)$ $=\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}$ $=\dfrac{\sqrt{3}+1}{2}$
16	Sum = (2n – 4)90 = 1260 2n – 4 = 14 $=\dfrac{2n}{2}=\dfrac{18}{2}$ n = 9 (No. of sides) Size of exteruior $=\dfrac{360}{n}$ $=40^{0}$

| 17 | a. Let the group members be x

Originally, each member was to contribute

$$\frac{2,000,000}{x}$$

After withdrawal of 40 members, each member had to contribute.

$$\frac{2,000,000}{x-40}$$

But, the same amount was to be raised despite the number of members.

Therefore:

$$\frac{2,000,000}{x} = \frac{2,000,000}{x-40} - 2500$$

$$(x-4)\frac{2,000,000}{x} = \frac{2,000,000}{x-40} \times (x-4) - x(x-4)(2500)$$

$$2,000,000 x - 8,000,000 = 2,000,000x - 2500x^2 + 100000x$$

$$25x^2 - 1000x - 800,000 = 0$$

$$x^2 - 40x - 32,000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{40 \pm \sqrt{1600 + 4 \times 32,000}}{2}$$

X = 200 or – 160

Original no. of members = 200

b. Fund from CDF $= \frac{45}{100} \times 2,000,000 = 900,000$

Amount remaining to be contributed

2,000,000 – 900,000 = 1,100,000

Amount contributed by each member =

$= \frac{1,100,000}{160} = 6875/=$ |

$$a = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad b = \begin{pmatrix} 14 \\ 1 \end{pmatrix}$$

(a) $2a + 3b =$

$$2\begin{pmatrix} 3 \\ -4 \end{pmatrix} + 3\begin{pmatrix} 14 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ -8 \end{pmatrix} + \begin{pmatrix} 42 \\ 3 \end{pmatrix} = \begin{pmatrix} 48 \\ -5 \end{pmatrix}$$

(b) $\frac{1}{2}\begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 14 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -1 \end{pmatrix} - \begin{pmatrix} 14 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} -12\frac{1}{2} \\ -1 \end{pmatrix}$$

c). Given xa – yb = (-13)

(-22)

Form a simultaneous equation hence find x and y (6 Marks)

Using substitution and elimination.

$$x\begin{pmatrix} 3 \\ -4 \end{pmatrix} - y\begin{pmatrix} 14 \\ 1 \end{pmatrix} = \begin{pmatrix} -13 \\ -18 \end{pmatrix}$$

i. 3x – 14y = -13

ii. -4x – y = -22

from (i) 3x = -13 + 14y

x = $\frac{13 + 14y}{3}$

-4$\left(\frac{-13 + 14y}{3}\right)$ - y = -22

52-56y – 3y = -66

-59y = -118

y = 2

3x – 14y = -13
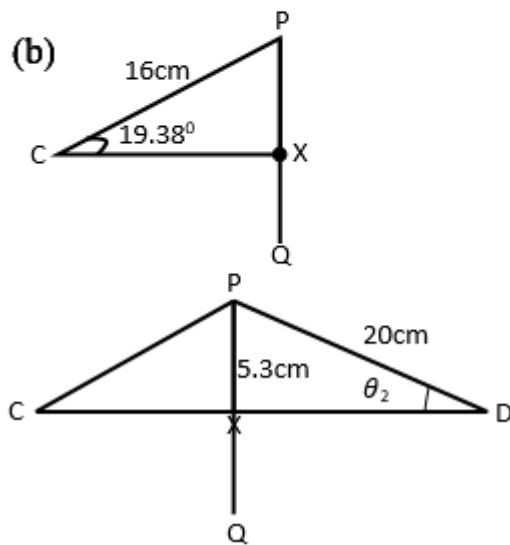
3x – 14 x 2 = -13

3x = -13 + 28

3x = 15

x = 5

$$80.14\text{cm}^2 = \frac{1}{2}(16 \times 16)\,\text{Sin}\,\theta$$

$$\text{Sin}\theta = \frac{80.14}{128}$$

$$\text{Sin}\theta = 0.6261$$

$$\theta = \text{Sin}^{-1}0.6261$$

$$= 38.76^0$$

(b)

19

Sin 19.38 = $\dfrac{PX}{16\ cm}$

PX = 16sin 19.38^0

= 5.3cm^3

(c) Area of shaded region

$$\left[\frac{22}{7}x16^2 \ x \ \frac{38.76}{360} - \frac{1}{2}x16^2 Sin38.76\right]$$
$$+ \left[\frac{22}{7} \ x \ 20^2 \ x \ \frac{30.73}{360} - \frac{1}{2} \ x \ 20^2 Sin \ 30.73^0\right]$$

=(86.626 − 80.136) + (107.311 − 102.199)

6.49 + 5.112 = 11.602cm^2

Sin θ_2 = $\dfrac{5.3cm}{20cm}$

θ_2 = 15.37 x 2

= 30.73^0

	a. $\dfrac{x}{x + 4} = \dfrac{2.1}{4.9}$	

$4.9x = 2.1x + 8.4$

$x = 3cm$

Height = 3cm +4cm = 7cm

b. A = $\pi r2$ = ($\dfrac{22}{7}$ x 4.9 x 4.9) x 2

$= 150.92cm^2$

Area of frustrum

Slant $\sqrt{7^2 + 4.9^2}$ = 8.545cm (Big cone)

$= \sqrt{2.1^2 + 3^2}$ = 3.662cm (Small cone)

=($\dfrac{22}{7}$ x 49 x 8.545)–($\dfrac{22}{7}$ x 2.1 x 3.662)

131.593 – 24.1692

107.4238cm^2

Total surface area

= 150.92 + 107.4238

= 258.34cm^2

volume of hemisphere

$\dfrac{2}{3}$ x $\dfrac{22}{7}$ x 4.93 = 246.5 cm^3

Volume of frustrum:

$\dfrac{1}{3}$ x $\dfrac{22}{7}$ x 4.92 x 7) – ($\dfrac{1}{3}$ x $\dfrac{22}{7}$ x 2.1^2 x 3)

176.07 – 13.86

= 162.21cm^3

20

21
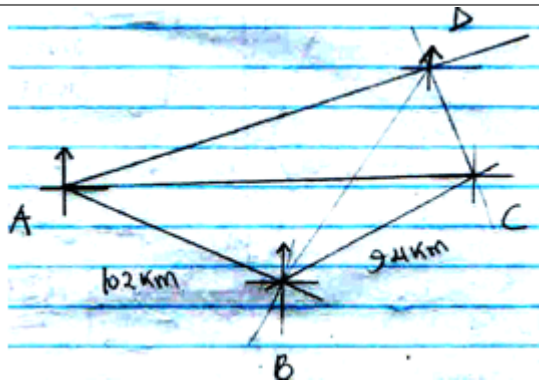
a.

b. Bearing of town B from town D

$= 212^0 \pm 1^0$

Bearing of town A from C
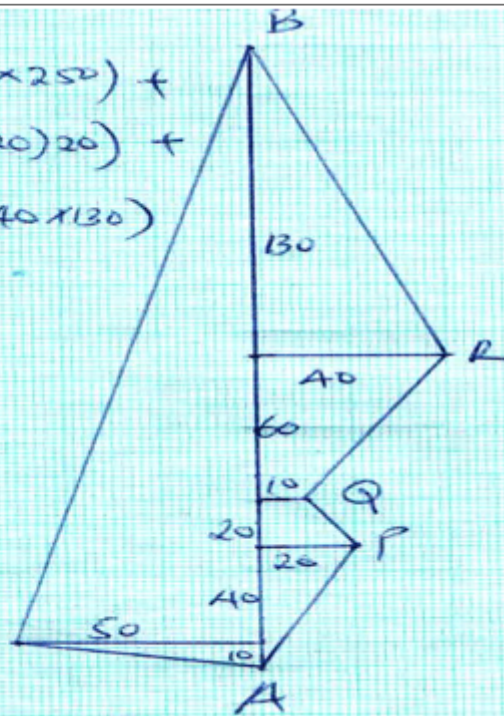
$= 268^0 \pm 1^0$

c. Distance

AC = 9cm x 20km = 180km

BD = 5.4cm x 20km = 108km

22	Area: $(\frac{1}{2} \times 10 \times 50) + (\frac{1}{2} \times 50 \times 250) +$ $(\frac{1}{2} \times 20 \times 50) + \frac{1}{2} (10 + 20) 20) +$ $\frac{1}{2} (40 + 10) 60) + (\frac{1}{2} \times 40 \times 130)$ $= \underline{\underline{5775 m^2}}$ 1 ha $= 10,000 m^2$? $= 5775 m^2$ $\dfrac{5775 \times 1}{10,000}$ $= \underline{\underline{0.5775 \; ha}}$	
23	

24(a)

x	-3	-2	-1	0	0.5	1	2	3
$2x^3$	-54	-16	-2	0	0.25	1	16	54
x2	9	4	1	0	0.25	1	4	9
-5x	15	10	5	0	-2.5	-5	-10	
+2	2	2	2	2	2	2	2	2
y	-28	0	6	2	0	0	12	50

B$_2$ If all values
B1 Any 4 values of y

24(b)
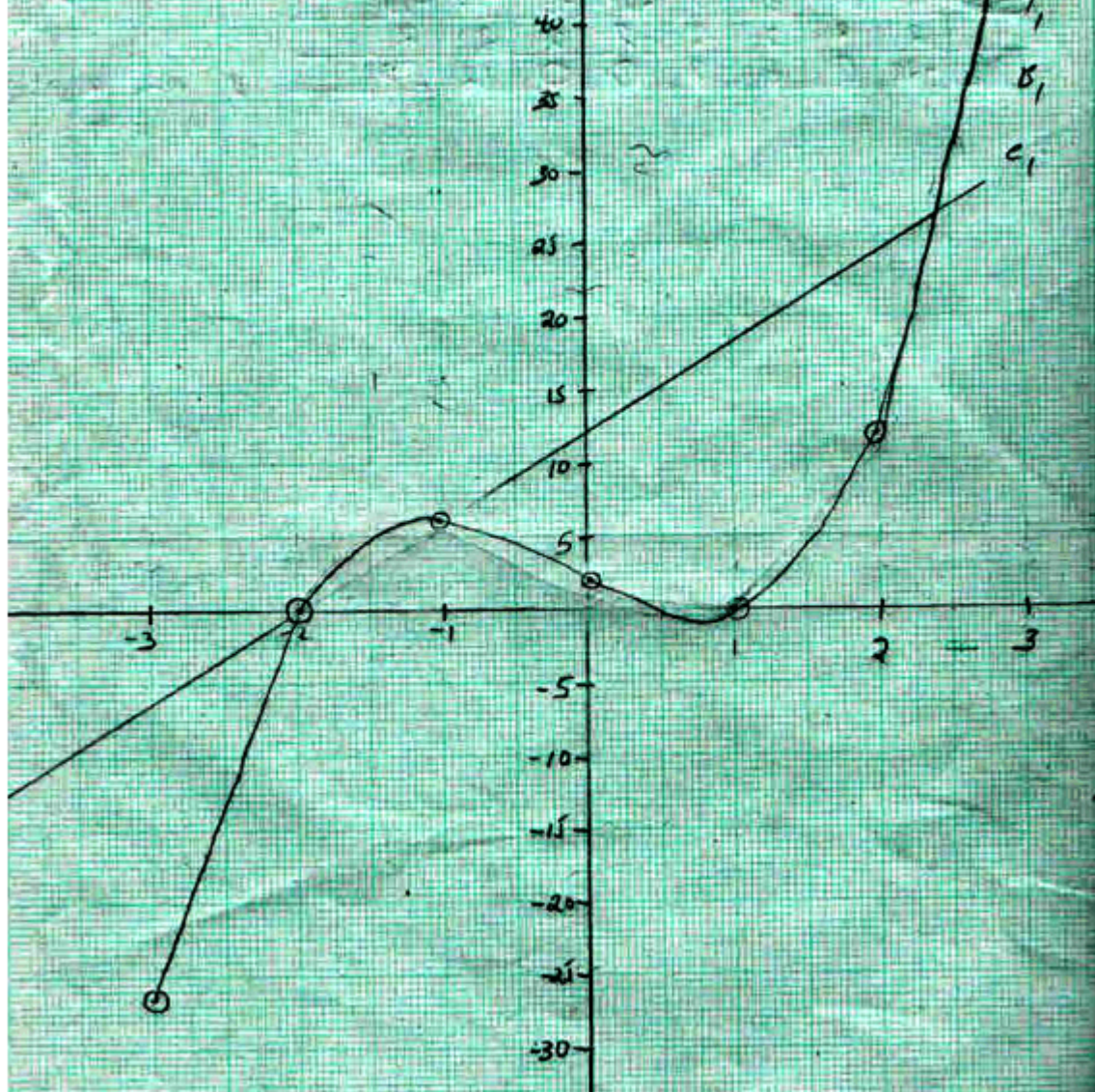
24(c) - (e)

(c) = $(-2, 0.5, 1) \pm 0.1$

(d) Line $y = 6x + 12$

  $x = -2, -1, 2.5 \pm 0.1$

(e) $\dfrac{0 - 2.0}{1.3 - 2.5} = 1.667$