KEY

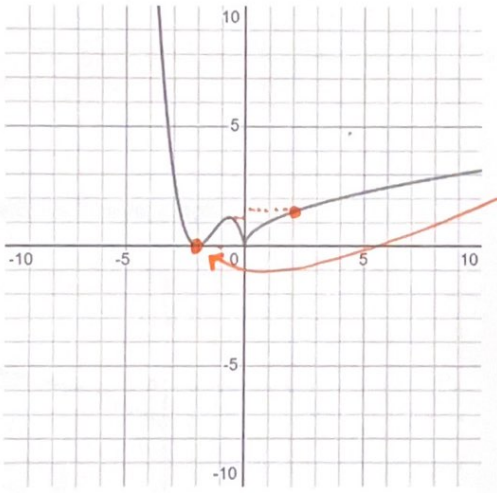
Math 141 Final Review

1. For both functions below, Find $f(-2)$

   a. $f(x) = 2x^2 - x + 3$

$f(-2) = 2(-2)^2 - (-2) + 3$
$= 2(4) + 2 + 3$
$= 13$

   b.



$f(-2) = 0$

2. Where is the graph from 1 (b)

   a. How can you tell it is a function?

It passes the vertical line test.

   b. What is it's domain? Range?

Domain: $(-\infty, \infty)$ left, right
Range: $[0, \infty)$ down, up

   c. What are its intercepts?

x intercepts: $(-2, 0)$, $(0, 0)$
↑ also y intercept

   d. Does it have an inverse function? How can you tell?

No it does not. It is not one to one (doesn't pass the horizontal line test)

   e. On what open intervals is it increasing?

$(-2, -0.7)$, $(0, \infty)$ remember, use x values only to describe where

   f. Does it have any local maximums? If so, what are they?

Yes, a local max of 1.2 at $x = -0.7$

Even: $f(x) = f(-x)$

Odd: $f(-x) = -f(x)$

3. Is the function even, odd, or neither? $f(x) = \frac{5}{2x^2-3}$

$f(-x) = \frac{5}{2(-x)^2-3} = \frac{5}{2x^2-3}$ ← match

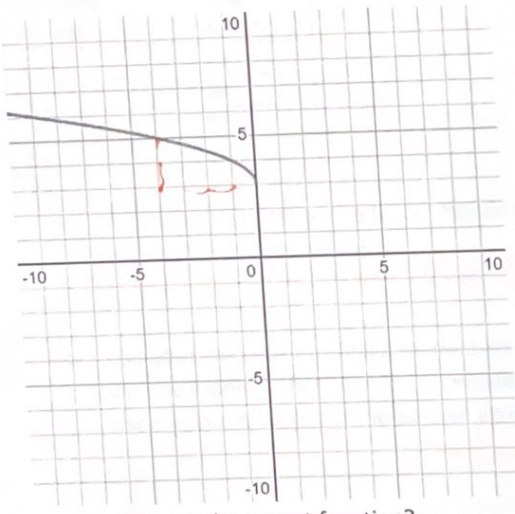
It's even!

$-f(x) = -\frac{5}{2x^2-3} = \frac{-5}{2x^2-3} = \frac{5}{-2x^2+3}$

4. What does the graph of an even function look like? The graph of an odd function?

symmetry about y axis. Symmetry about origin.

5. Consider the following graph.



a. What is the parent function?

$f(x) = \sqrt{x}$ The square root function

b. What transformations have been done to it?

reflected over y axis and shifted up 3

c. What is the equation of this graph?

$y = \sqrt{-x} + 3$

6. $f(x) = 2\log_2(x-4)+1$

a. What is the parent function?

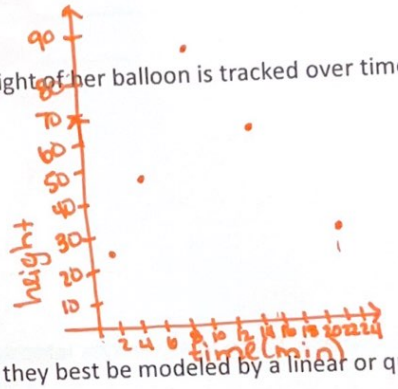
$f(x) = \log_2(x)$

b. What transformations of it give you this function?

Shift right 4, vertical stretch by a factor of 2, and shift up 1.

7. Elliott accidentally lets go of her balloon. If the height of her balloon is tracked over time, we see the following:

Minutes Passed	Height of Balloon
2	25
5	48
10	87
15	67
22	32



a. Plot the points next to the table. Would they best be modeled by a linear or quadratic function? How do you know?

quadratic because the data goes up then back down

b. Use your calculator to find a model that estimates the height of the balloon best. Then use the model to find how long it would take for the balloon to hit the ground. What is the maximum height of the balloon according to the model (not the table)?

STAT → EDIT STAT → CALC → QUADREG

8. $f(x) = 5x^4 + 3x^3 - 2x^2 + 1$

a. What is the end behavior?

LC: 5
degree: 4
rise on the left and rise on the right

b. What are the possible rational zeros?

$\frac{\pm 1}{\pm 1, \pm 5} = \pm 1, \pm \frac{1}{5}$

9. $f(x) = 3x(x-4)^3(x-1)^2$

a. What is the end behavior?

LC: 3

degree: 6

rise on the left and rise on the right

b. What are the zeros? What is the multiplicity of each zero? Does the graph cross or touch there?

Zero	mult.	cross/touch
0	1	cross
4	3	cross
1	2	touch

c. Sketch the graph.

10. Give an example of a rational function with a horizontal asymptote of y=4/3, a vertical asymptote of x=3, and a hole at x=-1.

$$y = \frac{(4x+4)(x-2)}{3(x^2-2x-3)}$$

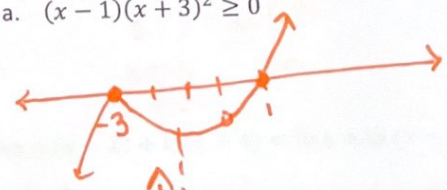
Need: • $(x+1)$ in the top & bottom
• $(x-3)$ in the bottom for V.A.
• same degree on top & bottom & LC of 4 on top, LC of 3 on bottom

11. Solve the inequality

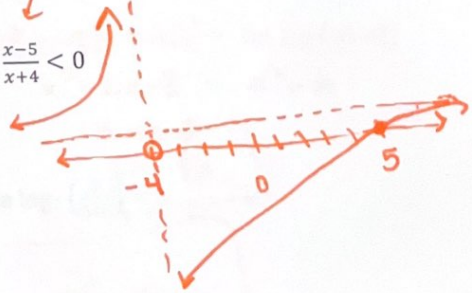
a. $(x-1)(x+3)^2 \geq 0$

cross touch

end behavior ↙ ↗



b. $\frac{x-5}{x+4} < 0$



12. Find the inverse function of $f(x) = \sqrt[3]{x-1} + 5$. Use function notation for the inverse.

$y = \sqrt[3]{x-1} + 5$

$x = \sqrt[3]{y-1} + 5$

$x - 5 = \sqrt[3]{y-1}$

$(x-5)^3 = y - 1$

$y = (x-5)^3 + 1$

$f^{-1}(x) = (x-5)^3 + 1$

13. Find the composition of $f(x) = 2x + 1$ *and* $g(x) = x^2 + 3x + 1$

$(f \circ g)(x) = 2(x^2+3x+1) + 1$

$= 2x^2 + 6x + 2 + 1$

$(f \circ g)(x) = 2x^2 + 6x + 3$

14. Solve $25 = 3 * 5^{x-4} + 4$

$25 = 3 \cdot 5^{x-4} + 4$

$\frac{21}{3} = \frac{3 \cdot 5^{x-4}}{3}$

$7 = 5^{x-4}$

$\ln 7 = (x-4)\ln 5$

$\frac{\ln 7}{\ln 5} = x - 4$

$\frac{\ln 7}{\ln 5} + 4 = x$

15. Solve $\log_2 x + \log_2(x+6) = 4$. Check your work.

$\log_2(x(x+6)) = 4$

$2^4 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$x = -8$ $\quad x = 2$

check: $\log_2(-8) + \log_2(-8+6)$

$\log_2(2) + \log_2(2+6)$

$1 + \log_2(8) = 1 + 3 = 4$

16. The price of one gallon of gas was \$3.04 last month. This month it is \$3.73. If the price rises exponentially, find an equation to model this. t = months since last month

$y = 3.04\, e^{rt}$

$3.73 = 3.04\, e^{r(1)}$

$\frac{3.73}{3.04} = e^r$

$\ln\left(\frac{3.73}{3.04}\right) = r$

$0.20455 = r$

$y = 3.04\, e^{.20455t}$

17. Solve $\ln(x-2) + \ln(x+4) = \ln x + \ln(x-1)$ check.

$\ln((x-2)(x+4)) = \ln(x(x-1))$

$x^2 + 2x - 8 = x^2 - x$

$3x = 8$

$x = 8/3$

check: $\ln(8/3 - 2) + \ln(8/3 + 4) = \ln 8/3 + \ln(8/3 - 1)$

$\ln(2/3) + \ln\frac{20}{3} = \ln\frac{8}{3} + \ln\frac{5}{3}$

$\ln\frac{40}{3} = \ln\frac{40}{3}$ ✓

18. Rewrite $\log_7\left(\frac{1}{343}\right) = y$.

$7^y = \frac{1}{343}$

19. Solve the system either by elimination or substitution. Show your work. $\begin{cases} x+5=y \\ -3x+y=-3 \end{cases}$

$-3x+(x+5)=-3$

$-2x+5=-3$

$-2x=-8$

$x=4$

$y=9$

$(4,9)$

20. The community college theater department sold three kinds of tickets to its latest play production. The adult tickets sold for \$15, the student tickets sold for \$10 and the child tickets for \$8. The theater department was thrilled to have sold 250 tickets and brought in \$2,825 in one night. The number of student tickets sold is twice the number of adult tickets sold. How many of each type did the department sell?

A = adult tix

S = student tix

C = child tix

$A+S+C=250$

$15A+10S+8C=2825$

$S=2A$

$\left[\begin{array}{cccc} 1 & 1 & 1 & 250 \\ 15 & 10 & 8 & 2825 \\ 2 & -1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc} 1 & 0 & 0 & 75 \\ 0 & 1 & 0 & 150 \\ 0 & 0 & 1 & 25 \end{array}\right]$

75 Adult tix sold, 150 student tickets, and 25 child tickets.

21. Use partial fraction decomposition to rewrite $\frac{2x+1}{x^2-7x+12}$

$\frac{2x+1}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$

$2x+1 = A(x-4)+B(x-3)$

$2x+1 = Ax+Bx-4A-3B$

$\begin{cases} A+B=2 \\ -4A-3B=1 \end{cases}$

$A=2-B$

$-4(2-B)-3B=1$

$-8+4B-3B=1$

$B=9$

$A=2-9$

$A=-7$

$\frac{2x+1}{x^2-7x+12} = \frac{-7}{x-3} + \frac{9}{x-4}$

22. Solve the system $\begin{cases} x-y=-1 \\ y=x^2+1 \end{cases}$

$x-(x^2+1)=-1$

$x-x^2-1=-1$

$-x^2+x=0$

$-x(x-1)=0$

$x=0 \quad x=1$

$y=1 \quad y=2$

$(0,1), (1,2)$