Asset Pricing. Factor models.

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ABOUT THIS TEMPLATE

This a short template about risk-factor models, which were discussed in the class. You can find more expanded information in the **lecture**.

Standard multi-factor model

Everybody knows about Capital Asset Pricing Model (CAPM). It is a fundamental model for asset pricing theory. However, it is quite unrealistic on real data. There are much more factors, that can be used to describe performance of an asset.

$$R_i = \alpha_i + R_f + \beta_1 * Factor_1 + \beta_2 * Factor_2 + ... + \beta_n * Factor_n + \epsilon_i$$

- α_i - constant
- R_i - expected rate of return
- R_f - risk-free rate
- $Factor_i$ - factor's **excess** returns
- β - factor's coefficient (sensitivity\loading)
- ϵ_i - idiosyncratic shock

Fama-French 3 factors model, 1992

Standard Fama-French 3 factors model.

$$R_i = \alpha_i + R_f + \beta_1 * R_{MKT} + \beta_2 * R_{SMB} + \beta_3 * R_{SML} + \epsilon_i$$

- α_i - constant
- R_i - expected rate of return
- R_f - risk-free rate
- R_{MKT} - market **excess** return
- R_{SMB} - **excess** returns of small-cap companies over large-cap companies (Small Minus Big)
- R_{HML} - **excess** returns of value stocks (high book-to-price ratio) over growth stocks (low book-to-price ratio) (High Minus Low)
- β - factor's coefficient (sensitivity\loading)
- ϵ_i - idiosyncratic shock

Fama-French-Carhart (F-F factors plus momentum factor, 1997)

Momentum - another factor. It is not F-F factor. It cannot be explained by F-F three factors (bad fit, negative correlation). It is possible to built 4 factors model, using momentum and Fama-French factors.

$$R_i = \alpha_i + R_f + \beta_1 * R_{MKT} + \beta_2 * R_{SMB} + \beta_3 * R_{SML} + \beta_4 * R_{MOM} + \epsilon_i$$

- α_i - constant
- R_i - expected rate of return
- R_f - risk-free rate
- R_{SMB} - **excess** returns of small-cap companies over large-cap companies (Small Minus Big)
- R_{HML} - **excess** returns of value stocks (high book-to-price ratio) over growth stocks (low book-to-price ratio) (High Minus Low)
- R_{MOM} - **excess** return of purchasing assets with strong performance, and selling assets with poor recent performance
- β - factor's coefficient (sensitivity\loading)
- ϵ_i - idiosyncratic shock

Fama-French 5 factors model, 2015

Momentum - another factor. It is not F-F factor. It cannot be explained by F-F three factors (bad fit, negative correlation). It is possible to built 4 factors model, using momentum and Fama-French factors.

$$R_i = \alpha_i + R_f + \beta_1 * R_{MKT} + \beta_2 * R_{SMB} + \beta_3 * R_{SML} + \beta_4 * R_{RMW} + \beta_5 * R_{CMA} + \epsilon_i$$

- α_i - constant
- R_i - expected rate of return
- R_f - risk-free rate
- R_{SMB} - **excess** returns of small-cap companies over large-cap companies (Small Minus Big)
- R_{HML} - **excess** returns of value stocks (high book-to-price ratio) over growth stocks (low book-to-price ratio) (High Minus Low)
- R_{RMW} - **excess** return on the two robust operating profitability portfolios minus **excess** return on the two weak operating profitability portfolios (Robust Minus Weak)
- R_{CMA} - **excess** return on the two conservative investment portfolios minus **excess** return on the two aggressive investment portfolios (Conservative Minus Aggressive)
- β - factor's coefficient (sensitivity\loading)
- ϵ_i - idiosyncratic shock

Betting against beta

- Purchase stocks with low beta and sell stocks with high beta. We construct zero-investment portfolio (as always).
- Arithmetically we subtract short position from long position. **This procedure is identical for SMB, HML and other factors**.
- Find more information here

Methodologies

- Maio and Santa-Clara, 2011
- Fama and MacBeth, 1973

Maio and Santa-Clara

- **Step 1**: Estimate betas for individual portfolios from time-series regressions (monthly data, excess return)
- Take betas from this regression
- **Step 2**: Cross-sectional regressions
- $\overline{R_i - R_f}$ is the average excess return for a given portfolio over all the years
- Obtain individual explanation errors α_i for each of the 25 portfolios
- Main part — that all factors are priced in the cross-section (meaning their prices of risks λ are statistically and economically significant numbers)
- It is possible to check: R^2, **all or almost all alphas are close to zero** and **what are the worst portfolios (that have too high absolute alphas)**

Necessary to look at:

- $R^2_{OLS} = 1 - \frac{Var_N(\hat{\alpha}_i)}{Var_N(\overline{R_i - R_f})}$
- $MAE = |C| + \frac{1}{N}\sum_{i=1}^{n} |\hat{\alpha}_i|$

Fama and MacBeth

The main difference between these two methods is to **use monthly returns to estimate parameters λat monthly horizon, and then combining them**

- **Step 1**: Estimate betas for individual portfolios from time-series regressions (monthly data, excess return)
- Take betas from this regression
- **Step 2**: Cross-sectional regression for each month
- Take the average excess return for a given portfolio over all the years
- Take lambdas from this regression for each time period, so that we have a time-series of λ_{0t} and all other lambdas
- **Step 3**: Take lambdas and do standard tests for H_0: **lambda is taken from a distribution with zero mean** $t - stat(\lambda_{0t}) = mean(\lambda_{0t}) * \frac{\sqrt{periods}}{volatility(\lambda_{0t})}$
- If this t-stat is above 2 or below -2, we may conclude that the factor is important and significant in pricing

Important features

- When you use factor models, you have to get insignificant alpha and significant factors
- You also have to look at R^2, but be ready to have very low positive value - it is okay for financial data
- Look at the sign before the coefficients - it helps to understand the "direction" of factor
- Only significant coefficients have practical and theoretical sense
- More information about factor models you can find on Fama-French website