RISK EVALUATION AND MANAGEMENT IN THE SECURITIES MARKET

Master Thesis

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Kirill Degtyarev (113217)

Supervisor
Prof. Dr. Karl Ludwig Keiber, Dipl.-Wi.-Ing.
Europa-Universität Viadrina
Lehrstuhl für Betriebswirtschaftslehre, insb.

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Introduction

One of the main tasks on the financial market is risk management, i.e. to create a portfolio that would satisfy the investor's view of risk, i.e. what amount the investor would be ready to lose at a given possible return. The possibility of risk is due to the uncertainty of trading on the stock exchange due to many factors: microeconomic, macroeconomic, political and investor sentiment. Due to the impossibility of modelling all the factors, in modern science, it is common to model financial assets as random values, for which there are specific risk measures.

To understand the risk on the financial market, it is necessary to understand the definition of risk and methods of risk assessment, which would correctly assess the risk of return on securities. Approaches play not the last role in risk management to risk modelling. Also, for research completeness, it is necessary to consider the basic models for risk management, which would be useful not only in a theoretical aspect but also would be suitable for practical application.

Thus, the relevance of the chosen topic of work does not cause any doubts and arouses interest.

The purpose of the review is to study the theoretical foundations of understanding risk on the financial market. In accordance with the objective, the following tasks are set:

- Defining the concept of risk and approaches to its modelling
- Identifying key risk measures
- Review and comparison of the main risk management models
- The object of consideration is the risk in the financial market.

The subject matter is scientific papers on risk in the financial market and its management, as well as reports of private companies and state institutions showing the practice of risk management.

The work is presented in three chapters. The first chapter deals with theoretical and practical aspects of risk definition approach to risk modelling and risk measures. The second chapter deals with one of the classic methods of risk diversification - the Markovitz portfolio. One of its generalizations - dynamic portfolio with transaction costs - is considered. Then various forecasting methods on financial markets and methods of scenario generation are presented and compared. Arguments are given to the scenario approach to investment. The last chapter demonstrates step by step the implementation of methods in practice with consideration of an example on real data.

1. Randomness on financial markets.

1.1. Approaches to an understanding of randomness in the security markets.

Valuation of the securities is affected by many factors. Some of them are obvious, while others are hidden and required efforts to be determined. Determination of import risk factors and evaluation of their effect is an essential and difficult task in investment [1, p. 741]. That is why it is common to unite risk factors in groups.

There is no unified system of classification of risk factors, and each author tends to determine the most important type of risks. Aswath Damodaran, in his classic book "Investment Valuation" [2, p. 88] identifies such factors:

- Firm-specific risk – a risk that affects only a single firm
- Project risk – a firm has miscalculated product demand
- Competitive risk – the influence of competing firms, existing or potential
- Sector risk – risks affecting the whole sector
- Market risk – uncertainty due to changings in economics.

Nobel laureate in economics William Sharpe considers two types of risks [1, p. 240]:

- Market risk
- Non-market risk (unique risk)

In turn, another Nobel laureate Harry Markowitz in his article [3] understands by the risk only one measure – dispersion of the returns of securities.

Another approach is a probabilistic approach, which became possible thanks to empirical observations on the logarithm of stock prices in papers Samuelson and his predecessors (L. Bachelier, M. Kendall) [4, p. 36]. This approach assumes that stationary increments (e.g. rate of return) or another characteristic of a financial asset could be described by stochastic differential equation (SDE), that includes Brownian motion process [5, p. 111]. Brownian motion is the process W_t for $t \geq 0$ if:

- $W_0 = 0$.

- W has independent increments: for $\forall\ t \geq 0$, future increments $W_{t+u} - W_t$ when $u \geq 0$, independent from past values $W_s, s < t$
- Increments of W normally distributed with mathematical expectation 0 and dispersion $\sigma^2(t - s)$
- All trajectories of the random process W_t lie on the whole interval $[t;\ +\infty]$.

Another key theory that supports the idea to analyze risk as nondeterministic uncertainty was another Nobel work about the market efficiency, that the market reacts on new income information rationally:

- Instantly corrected prices, which are set in such a way that markets stay in equilibrium, leaving no room for arbitration.
- Market players interpret the incoming information homogeneously, while instantly adjusting their decisions when updating the information.
- Market participants are homogeneous in their target-setting, and their actions are "rational".

1.2. Methods of risk assessment

Despite that, in theory, there is no unambiguous understanding, how to divide risk into the groups, in practice, the following division of major risk groups is adopted:

- Market risk
- Credit risk
- Operational risk

In this paper, I consider only market risk, as one of the most significant risks in an investment decision. Figure 1.1. shows how the value of risk spreads between groups in one of the biggest banks in Germany "Deutsche Bank" in 2017 [6].

in € m. (unless stated otherwise)	Dec 31, 2017	Dec 31, 2016	2017 increase (decrease) from 2016 in € m.	in %
Credit risk	10,769	13,105	(2,336)	(18)
Market risk	10,428	14,593	(4,165)	(29)
Trading market risk	3,800	4,229	(429)	(10)
Nontrading market risk	6,628	10,364	(3,736)	(36)
Operational risk	7,329	10,488	(3,159)	(30)
Business risk	5,677	5,098	579	11
Diversification benefit[1]	(7,074)	(7,846)	772	(10)
Total economic capital usage	27,129	35,438	(8,309)	(23)

[1] Diversification benefit across credit, market, operational and strategic risk (largest part of business risk).

Figure 1.1. Table of risk measures between different groups in «Deutsche Bank» in 2017.

For decision making and different evaluation scenarios, it is a common practice to assess risk as a measure of uncertainty using different risk measures. Risk measures are evaluated for each investment strategy separately concerning their development, and further decision about investment is made. After passing an investment horizon, an investor evaluates the strategy, that was constructed in the past and correlates what happened with how he predicted uncertainty by risk measures. Such a comparative analysis between forecast in future (ex-ante) and understanding risk from the past (ex-post) allows for an investor to understand the quality of the forecast and correct it for the future strategies. Consider the main risk measures.

1.2.1. Value at risk и Expected Shortfall

Value at Risk (VaR) was firstly used in the J.P. Morgan company [7, p. 195] and became an alternative to risk evaluation using standard risk measures (dispersion) and Greeks. It allows representing the changing of portfolio value as distribution and determine intervals of losses in respect to a confidence level. The biggest interest for analysts is the consideration using this possible measure level of losses.

The formal definition of VaR (1.1.):

$$VaR_{\alpha} = inf(I: P(L > I) \leq 1 - \alpha) = inf(I: F_L(I) \geq \alpha) \quad (1.1.)$$

, where I – set of losses (or incomes), α– a confidence level. It is common to take into account only two levels of confidence 0.05 and 0.01 (in case of incomes and losses together) or 0.95 and 0.99 (in case of losses only).

Graphically VaR at X% level of confidence looks like [Figure 1.2.]:

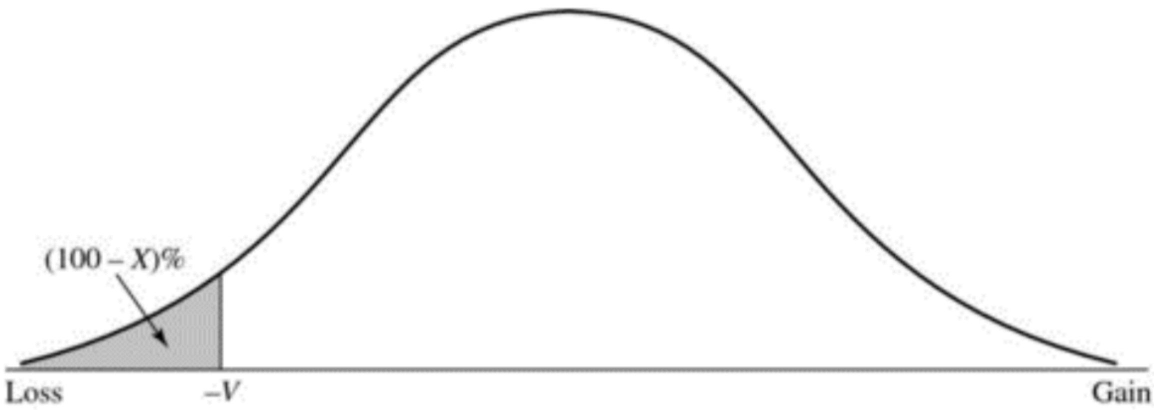


Figure 1.2. The density of distribution of portfolio values changings

There are exist three ways to calculate VaR:

- Parametric
- Simulation based on historical data
- Monte-Carlo method

The parametric method based on the assumption that the distribution of portfolio values based on some theoretical distribution (usually assumed the log-normal distribution or normal distribution of assets' marginal distributions) and therefore VaR could be calculated as (1.2.):

$$VaR_{\alpha} = \mu + \sigma N^{-1}(\alpha)$$

N – a normal distribution with parameters $(\omega\hat{\mu}, \omega^T\Sigma\omega)$ (1.2.)

, where $\hat{\mu}$ – the mathematical expectation of assets' return of assets included in portfolio

ω – weights of securities

Σ – covariation matrix of financial assets.

$N^{-1}(\alpha)$ – a quantile that cuts $(1-\alpha)$% of losses from the density of distribution.

To the drawbacks of this method could be related assumption about the theoretical distribution. For example, consider daily returns of the S&P 500 index for the period (03.01.1950 – 15.06.2018) and compare it visually empirical density of distribution with standard normal distribution [Figure 1.3.]:

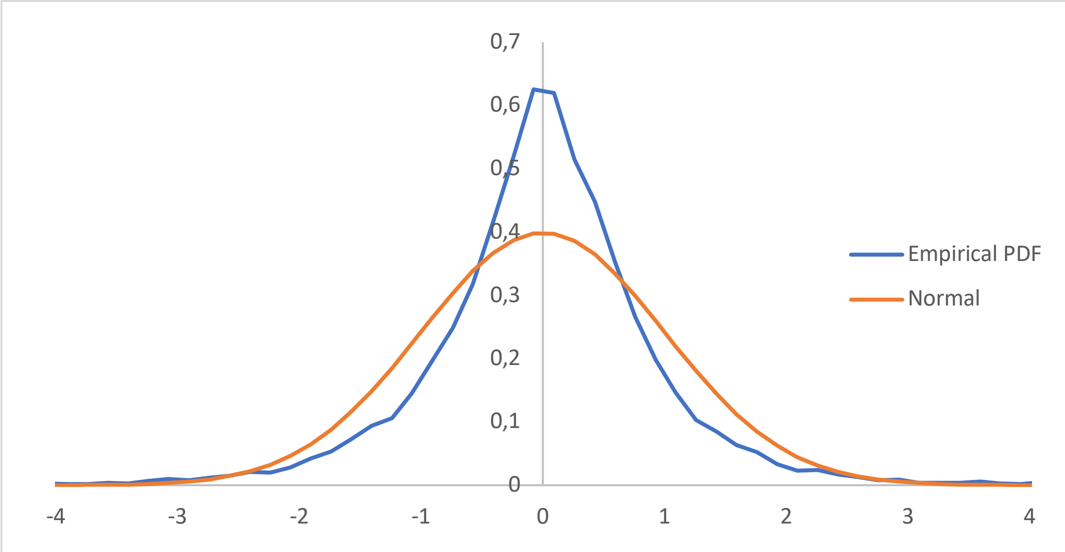


Figure 1.3. Graph of comparison between the density of empirical distribution of S&P 500 returns and standard normal distribution.

As we can see from simple overlay empirical distribution is not similar to the standard normal distribution and tails of the empirical distribution has heavier tails then empirical one has. It might cause significant inaccuracy in VaR evaluation.

Using parametric estimation of VaR under the assumption of non-normal distribution is complicated by the fact, that the distribution of losses could not be represented in the general case with convolution as simple as with the stable distributions (to which is related the normal distribution) [8, p. 291]. The article [9] presents the case of convolution of several t-distributions.

A simulation based on historical data approach assumes the selection of an interval of time for which statistics about portfolio value changings is taken and the estimation of empirical density. After that, the VaR value is estimated as a quantile of some level of losses. In the general case, Kernel density could be used for the estimation of complex empirical distributions [10].

An advantage of such an approach of estimation is that it does not require any assumption about a type of theoretical distribution. To the contrary, it causes full dependency of a forecast of VaR from a choice of an interval of historical data. Despite that, this method is recommended to use for VaR estimation, according to Basel III, with a confidence level equals 99% [11].

Monte–Carlo method for VaR valuation is implemented using the following algorithm [12, p. 169]:

1. Identification of risk factors, their stochastic processes and calculation of the required parameters.

2. Modelling price trajectories with a generator of random numbers and numerical solution for stochastic processes for the determined period.

3. Evaluation of portfolio characteristics for each trajectory.

This process is repeated several times, and then a quantile is calculated.

Alternatively, to VaR value Expected Shortfall (ES) could be calculated, that shows a mathematical expectation of all possible losses [13]:

$$ES_{\alpha} = E[L|L \geq VaR_{\alpha}] \tag{1.3.}$$

The problem, in this case, is high sensitivity to the shape of the heavy left-hand tail.

An important aspect of correct use of VaR is the use of "backtesting", a method that evaluates the correctness of modelling on historical data. Concerning VaR, there are two methods:

- To evaluate VaR on historical data with a certain period and shift it until the present time, checking whether forecasted density differs seriously from real density. In case of significant difference using some metrics, it needs to change the methodology of forecasting VaR in future (to change the length of the historical interval, a level of confidence).

- To compare forecasted VaR with real value changings using "Traffic lights" approach [14]. Green light if less then 5 cases when real value exceeds the VaR values, Orange light in case of 5 – 10 such cases and red if more then 10 such cases.

Below is listed backtesting of VaR for Deutsche Bank for 2017 [15] [Figure 1.4.]:

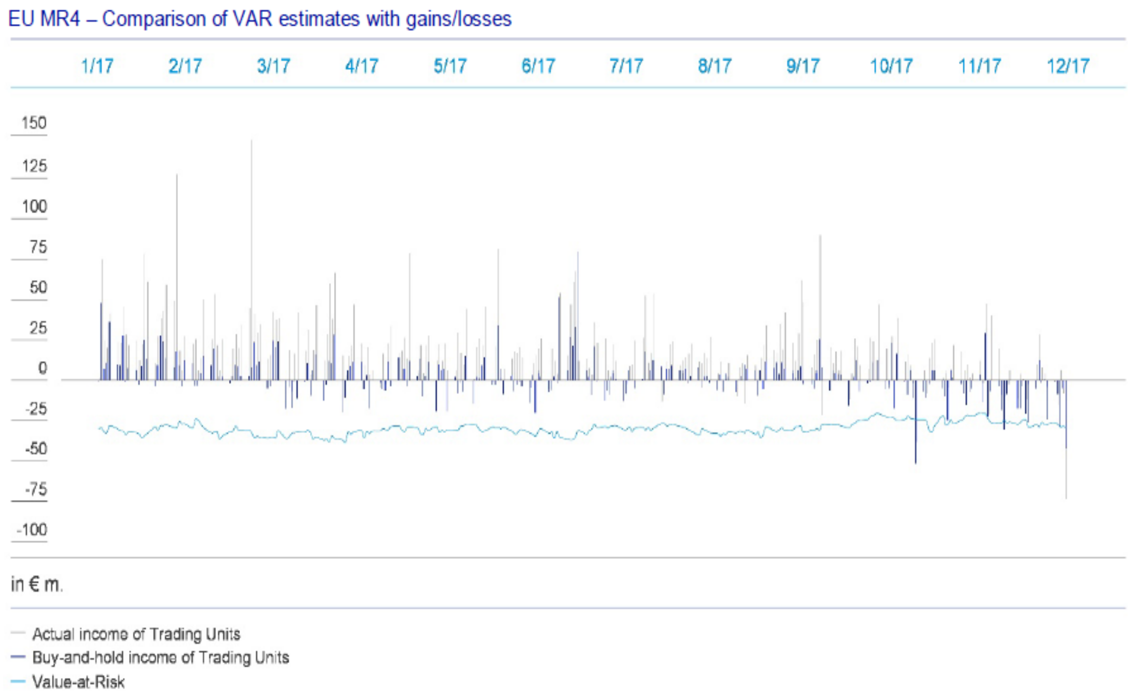
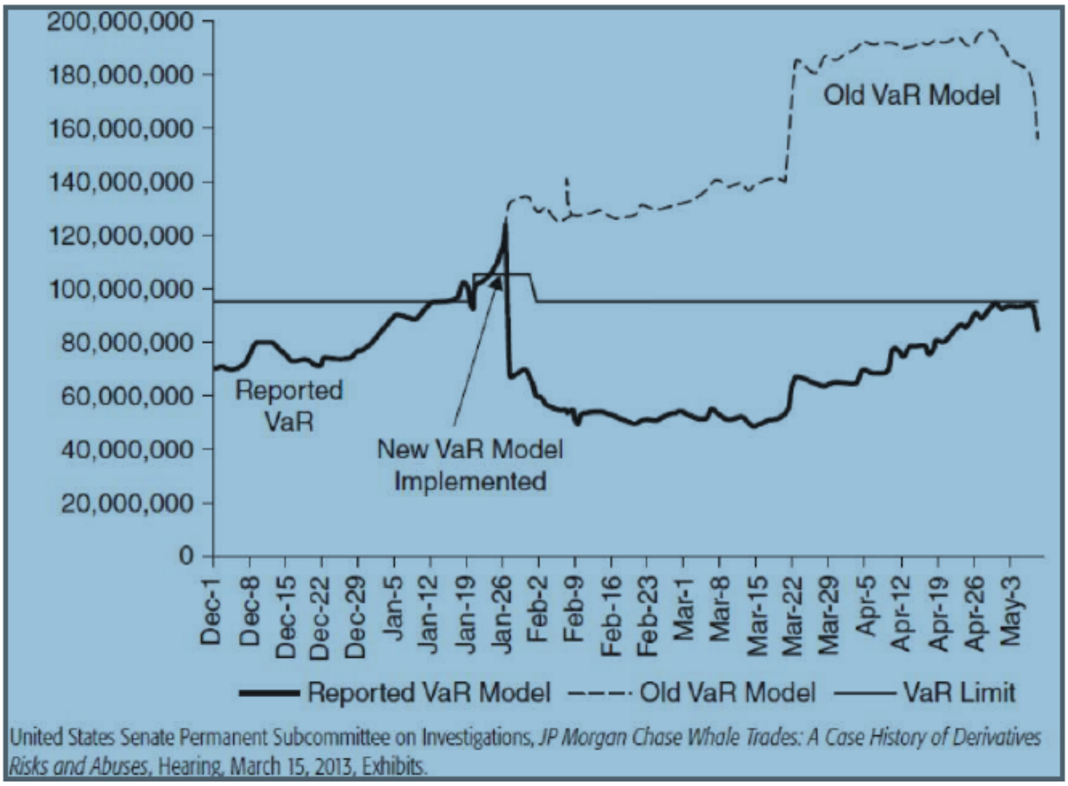


Figure 1.4. Backtesting of VaR for Trading Market Risk of Deutsche Bank for 2017.

As can be seen from the above chart, Deutsche Bank correctly chose the modelling and forecasting methodologies for 2017, as for the whole year there were only 3 cases when the values estimated with VaR were lower than real, which corresponds to the green signal and can be written off as a statistical error.

It is essential to pay close attention to the VaR prediction procedure, and it is incorrect to change procedure within a short period. It can be seen in the practice in the case of London department of JPMorgan Chase, where in April and May 2012 the total loss due to incorrect risk assessment model VaR the total loss of the investment bank was $2 billion. This event was later called London Whale by financiers [16]. Because of the resonance of the event, a commission of inquiry was organized in the U.S. Senate, where it was found that the main reason was the incorrect use of the VaR model [Figure 1.5.]:



United States Senate Permanent Subcommittee on Investigations, *JP Morgan Chase Whale Trades: A Case History of Derivatives Risks and Abuses*, Hearing, March 15, 2013, Exhibits.

Figure 1.5. Graphic explanation of incorrectly selected VaR model of JPMorgan Chase

Thus, it can be established that all methods of risk assessment are not without drawbacks and should be used based on forecasting purposes and the requirements of the regulator.

2. Theoretical aspects of portfolio methods of investments

To one of the methods of diversification of risk is related portfolio method of investments, such as Markovitz portfolio and other modification of it. For this methods is relevant such properties [5]:

- One-hold one-period model that the model does not provide the possibility to reassemble its portfolio during the selected investment horizon T. Thus, these models are called still static.
- Distribution of financial assets' returns should be elliptical. This family of distributions includes multinomial normal distribution and other types of distributions [17].
- A portfolio of investments is constructed from all available securities to achieve some optimality.

Consider the modification of a Markovitz portfolio task, which consider reassembling during an investment period and transactional costs.

2.1. Dynamic mean-variance portfolio with considering of transactional costs.

There are n assets in a portfolio. Let $w = (w_1, \dots, w_n)$ is a vector of weights in this portfolio and $w_i = \frac{market \text{ capitalization of investment in asset } i}{\text{Total investments}}$. Expected yield of security i at the moment of time t shall be designated as $\mu_{i,t}$. The standard portfolio deviation is $\sigma_{i,t}$. The measure of the correlation between asset i and j is $\rho_{ij,t}$ in time t. The mean-variance portfolio includes riskiness and risk-free assets. Returns of securities are linear and are calculated as (2.1.):

$$\mu_{i,t} = E\left[r_{i,t}\right] \quad (2.1.)$$

, where $r_{i,t}$ equals to

$$r_i = \frac{p_{i,t} - p_{i,t-1}}{p_{i,t}} \quad (2.2.)$$

$p_{i,t}$ in (2.2.) is a market price on an asset i in time t. The risk of a portfolio is characterized by portfolio variance (2.3.):

$$\sigma_{\Pi}^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1,j>i}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j Cov(r_i, r_j) \tag{2.3.}$$

In a matrix form (2.3.) could be written as (2.4.):

$$\sigma_{\Pi}^2 = w^T \sum w \tag{2.4.}$$

The mean-variance portfolio in one period of time that takes into account not only the investor's interest in maximizing the return of a portfolio but also in minimizing risks of a portfolio in matrix form could be written as (2.5.) [18, p. 92]:

$$Minimize_w \ \frac{c}{2} w^T \Sigma \mathrm{w} - \mathrm{w}^{\mathrm{T}} \mu - w_0 R$$

Subject to: (2.5.)

$$\sum_{i=1}^{N} w_i = 1$$

In the task (2.5.), the parameter c characterizes risk aversion. The higher its value, the less valuable the profitability becomes in comparison with the lower risk (portfolio dispersion).

Restrictions that are taking into account transactional costs.

Commissions from brokers play an important role in stock market trading. A size of commission affects on an optimal plan of investments since different classes of financial assets are subject to commissions differently. Consider the model of accounting for transaction costs in the optimal portfolio selection task as in the paper [19].

The total amount of transactions, related to trading, could be written as (2.6.):

$$I^T x + \phi(x) = 0 \tag{2.6.}$$

, where x is a vector consisting of transactions for each of portfolio's assets. $\phi(x)$ is a function, that characterizes costs associated with purchase and sale. Specification of the function $\phi(x)$ is following (2.7.):

$$\phi_i(x_i) = \begin{cases} \beta_i^- - \alpha_i^- x_i, & x_i < 0 \\ 0, & x_i = 0 \\ \beta_i^+ + \alpha_i^+ x_i, & x_i > 0 \end{cases} \tag{2.7.}$$

Here transactional costs divided on the fixing part and variable cost part. The optimization task (2.5.) should converge under the constraints (2.7.), then the parameter β should be equal to zero.

Dynamic construction of a portfolio.

In order to develop a trading strategy based on an optimal portfolio in the long term, it is necessary to consider changing the composition of the portfolio dynamically over time. One of the most common strategies of dynamic investing is a self-financing strategy, which assumes that an investor invests in trading some initial amount and overtime uses all the money received as a result of positive returns, or conversely reduces the amount of funding due to negative returns for the previous period.

If we change the constraint (2.6.), then we can consider a self-financing strategy [20]:

$$I^T(w_n - w_{n-1}) + \phi(w_n - w_{n-1}) \leq 0 \tag{2.8.}$$

The final optimization task of building an optimal dynamic Mean-variance portfolio with transaction costs (2.9.):

$$Minimize_w \ \frac{c}{2} w^T \Sigma \mathrm{w} - \mathrm{w}^{\mathrm{T}} \mu - w_0 R + \phi(w_n - w_{n-1})$$

Subject to:

$$I^T(w_n - w_{n-1}) + \phi(w_n - w_{n-1}) \leq 0 \tag{2.20.}$$

$$\sum_{i=1}^{N} w_i = 1$$

Portfolio's approach is a flexible tool for determining optimal asset allocation corresponding to an investor's attitude to return and risk. However, there are several disputes about the stability of the obtained w_i weight and some researchers argue that at small displacements of the expected values of the arguments (yield and covariance matrix) from the present values in the future, the obtained weights may be very different from each other [21] and thus this portfolio method maximizes the error. On the other hand, other researchers explain such high variability by the fact that financial assets can be similar in parameters and can be replaced by each other, while the returns and risk measures will not differ much [22].

2.2.Methods of forecast and generating scenarios for portfolios.

The key point in building an investment strategy based on an optimal portfolio is the simulation of expected key indicators (expected returns, volatility). It depends on the accuracy of forecasts to what extent the built portfolio will approach the current optimal value.

2.2.1. Multivariate DCC–GARCH.

Multivariate GARCH

The multi-dimensional generalized autoregressive model of conditional heteroscedasticity (mGARCH) [23] can be considered as a universal model for forecasting portfolio profitability and volatility. The mGARCH model is written as (2.21):

$$r_t = \mu_t + a_t \tag{2.21.}$$

, where a_t is equal to (2.22.)

$$a_t = H_t^{1/2} z_t \tag{2.22.}$$

In the equation (2.21.) r_t is a vector of dimension n of returns at the moment of time t.

$$z_t \sim N(0, I).$$

μ_t is a mean value of returns, which could be modelled as a vector autoregressive–moving-average model VARMA(p,q) [24, p. 424] (2.23.):

$$\Phi(B)\mu_t = \phi_0 + \Theta(B)a_t \tag{2.23.}$$

, where $\Phi(B) = I - \Phi_1 B - \ldots - \Phi_p B^p$ and $\Theta(B) = I - \Theta_1 B - \ldots - \Theta_q B^q$

Conditional dispersion of the value a_t is characterized by matrix H_t. In turn, $H_t^{1/2}$ can be obtained by decomposition of the H_t Holetsky matrix. The conditioned dispersion of H_t is presented in the form (2.24.):

$$\Phi(B)\mu_t = \phi_0 + \Theta(B)a_t \tag{2.24.}$$

DCC-GARCH

Among the conditional dispersion models, Dynamic Conditional Correlation Model (DCC) allows to model changing the interrelationships between securities in a portfolio through the modelling of the conditional correlation matrix ρ_t and a conditional dispersion D_t (2.25.):

$$\mathrm{H_t} = \mathrm{D_t}\rho_t D_t \tag{2.25.}$$

, where $D_t = diag\left(\sqrt{\mathrm{h}_{11,t}}, \ldots, \sqrt{\mathrm{h}_{nn,t}}\right)$

$$\mathrm{h_{it}} = \alpha_{i0} + \sum_{q=1}^{Q} \alpha_{iq}\alpha_{i,t-q}^2 + \sum_{p=1}^{P} \beta_{ip}\mathrm{h}_{i,t-p} \tag{2.26.}$$

An important condition is that the equation matrix (2.25.) is positively determined.

ρ_t is a symmetric correlation matrix of standardized deviations ϵ_t:

$$\epsilon_t = \mathrm{D}_t^{-1}\mathrm{a}_t \tag{2.27.}$$

The matrix ρ_t is decomposed in a general case into (2.28.) and (2.29.):

$$\rho_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{2.28.}$$

$$Q_t = \left(1-\sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n\right)\overline{Q_t} + \sum_{m=1}^{M} a_m \alpha_{t-1}\alpha_{t-1}^T + \sum_{n=1}^{N} b_n Q_{t-1} \tag{2.29.}$$

The order of the DCC model (M, N) is set by parameters in equation (2.29.). More details on the DCC-GARCH model can be read in the papers [25,26].

Thus, the choice of a forecasting model depends on a system of preconditions that the researcher sets in his analysis. In this paper, risk management in the financial market in the long term is considered, and therefore it would be logical to choose DCC-GARCH model, as it takes into account the variability in the structure of interrelationships of securities in the investment portfolio. DCC-GARCH model is used for forecasting and generating scenarios in Chapter 3 of this paper.

3. Practical implementation of dynamic portfolio methods for risk management

For practical implementation of portfolio methods, I took six American companies with good indicators of reliability and profitability. For this purpose, I used conclusions of analysts on a website of New York stock exchange NASDAQ and chose stocks of the companies with a recommendation "Buy" and above, and as a risk-free asset annual returns to repayment on three-month treasury bills (3-Month Treasury Bill) which have been taken from a website [27]. Other data have been taken from the service "Yahoo finance" [28].

Such companies were Amazon (AMZ), Marathon Oil Corp. (MPC), Diamondback Energy (FANG), UnitedHealth Group (UNH), Equinix (EQIX), Alphabet (GOOGL). Historic data is taken daily from 31.12.2013 to 02.07.2020. The forecast horizon is approximately 1 year (328 days ahead).

3.1. Forecasting and generating scenarios for the portfolio using the DCC-GARCH method.

Using results of modelling for each security separately with ARIMA – GARCH models, it is possible to start the general modelling of a portfolio of investments and its indicators necessary for portfolio methods of optimization (a vector of covariance matrixes Σ and a vector of expected returns μ_t). Details of the implementation of the DCC-GARCH model are listed in Table 3.1.

Table 3.1.

Details of the implementation of the DCC-GARCH model for forecasting and generation of investment portfolio scenarios.

Input:	Historical data of 6 company equities.
Start date: **End date:**	31.12.2013 15.03.2019
Output:	1. Forecast values of the vector of expected yield vectorsμ_t and the vector of expected covariance matrices Σ_t. 2. The vector with scenarios s at each period of time t.
Technical details:	Modelling is done using the R programming language with rmgarch package [29] with functions dccroll and dccsim. A rolling forecast approach is used with the following parameters refit.every = 30, n.ahead = 1, window.size = 300. The result is 13 models. Scenario generation based on built models with moving forecast. The number of simulations at each moment of time S = 200.
Model:	13 VARMA (0, 0) models with drift - DCC (1, 1) - GARCH (1, 1) with the distribution of residuals as a multi-dimensional Student distribution.

As a result of model evaluation on a training and forecasting sample, forecasts with the following measures of deviation from the real values of the test sample were obtained [Table 3.2]:

Table 3.2.

Values of deviation measures for each security between forecast values and test values.

	ME	RMSE	MAE	MPE	MAPE
AMZN	0.0000603310	0.01871251	0.01302376	108.83015	127.0409
EQIX	0.0006032670	0.02239820	0.01451018	111.81318	123.3104
FANG	-0.0013342400	0.05205301	0.03073403	99.24808	100.6958
GOOGL	-0.0004431890	0.02167310	0.01366882	101.08717	135.4935
MPC	-0.0002876029	0.04399883	0.02739920	Inf	Inf
UNH	0.0001408508	0.02736994	0.01675791	97.11054	107.1306

Besides considering the deviations of the predicted values of expected returns from the test ones, one can also consider the ability of the model to predict the conditional variance (2.25.) [Figure 3.1.]:

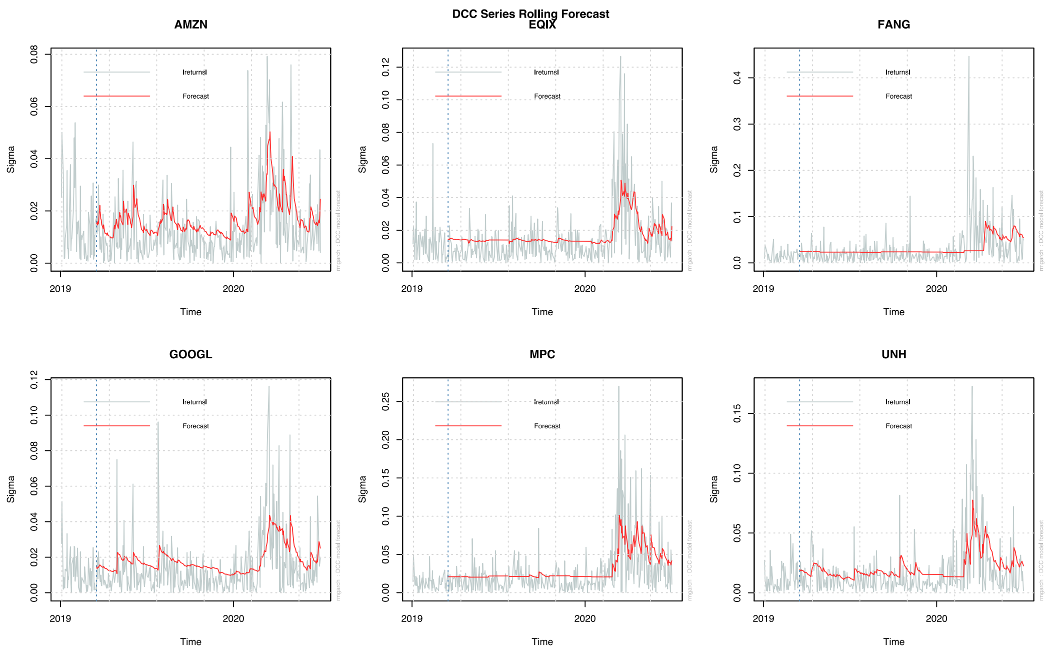


Figure 3.1. Graph of forecasted conditional dispersion and real dispersion from the test data.

The obtained forecasted values of returns could be converted to the price for each stock with the transformation (3.1.):

$$P_{t+1} = P_t \times (1 + \mu_t) \tag{3.1.}$$

Also, it is possible to plot the "vertical difference" chart between the forecasted price and the price in the test sample for each security [Figure 3.2.]:

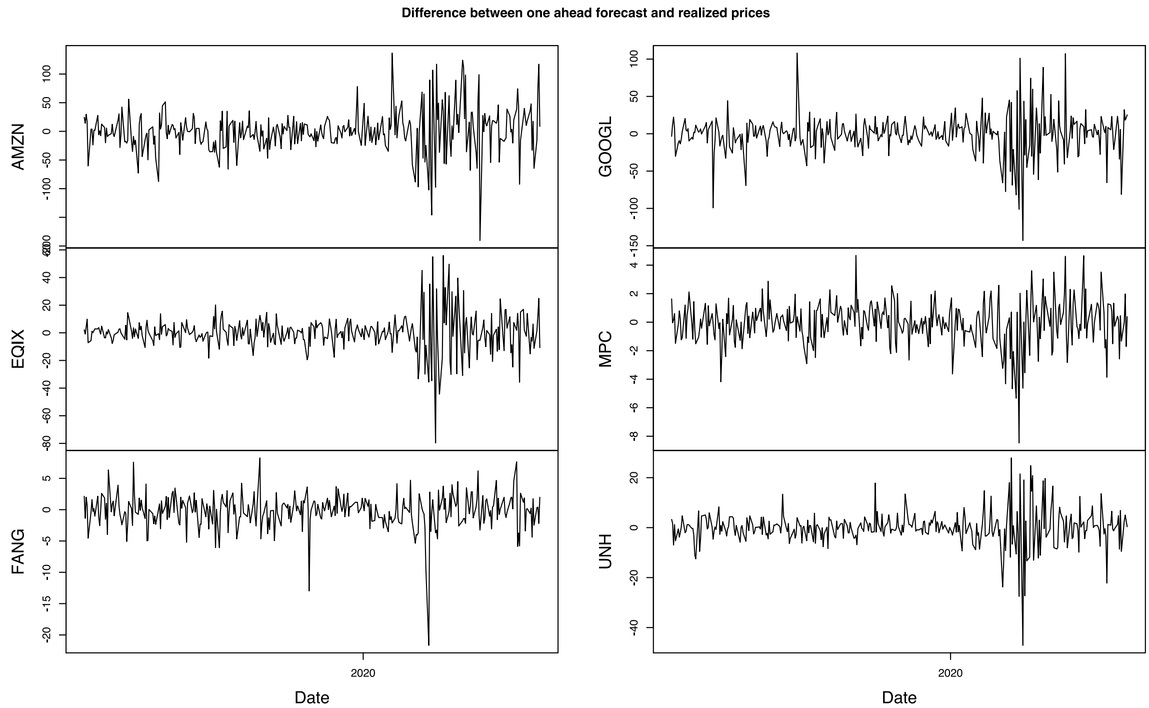


Figure 3.2. Graphs of the difference between the prices predicted and prices in the test sample

Thus, using the rolling forecast method, the difference between real and forecast values is reduced in comparison with a one-step forecast. The ability of the model to adjust to changing conditions on financial markets is essential for an optimal portfolio. And the dynamic portfolio strategy is suitable for using the rolling forecast method.

3.2.Development of dynamic Mean-variance portfolios

In this part of the thesis, two dynamic Mean-variance portfolios are considered, that was mentioned in paragraph 2.1.: one that includes a risk-free asset and one that consists only of risky financial assets. The most interesting is their consideration and comparison at different values of risk aversion coefficient c in equation (2.5.). Two cases are considered: c = 1 (investor has a moderate attitude towards risk), c = 5 (investor has a strong risk aversion). The case when c < 0 is unrealistic, i.e. when the investor prefers the risk of return and therefore, is not considered.

Let's first consider a case with moderate risk attitude. In Fig. 3.3. the daily returns of two portfolios are compared:

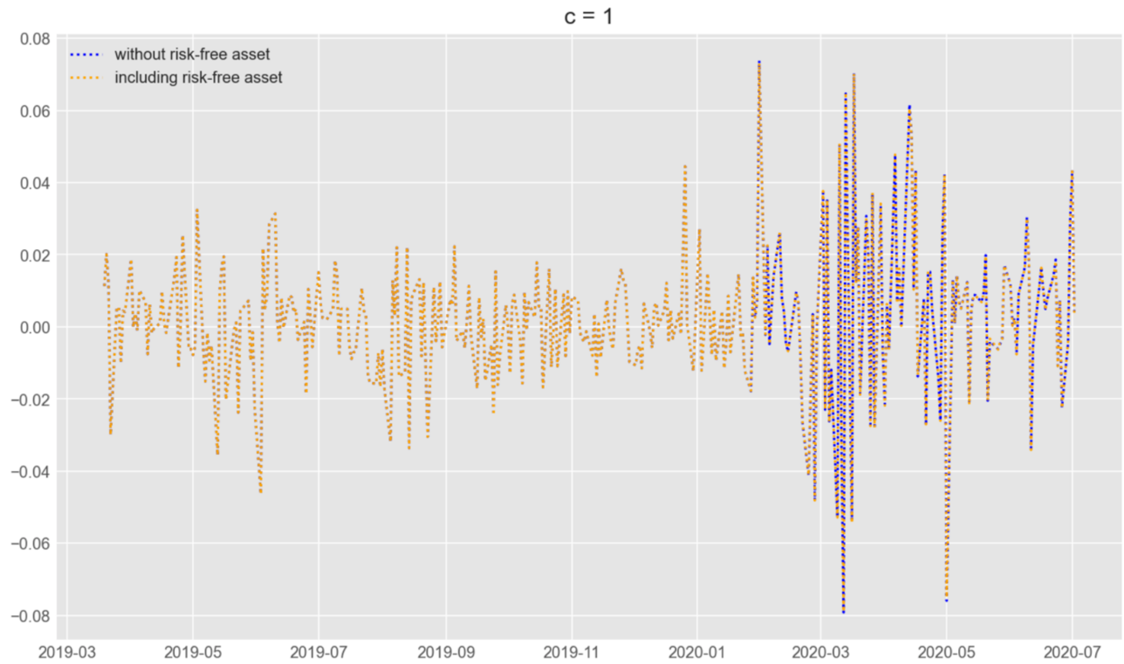


Figure 3.3. Comparison of daily returns for both portfolios when c = 1.

As can be seen from the charts, the daily returns of two different portfolios almost completely coincide with each other. This may indicate that the rate of return on a risk-free asset is so low compared to a risky asset that the investor makes similar decisions to invest, which is also the case when there is no risk-free asset. A similar situation exists with the graph of conditional volatility.

Since the only parameter that may differ between these two portfolios is the weight of assets, the investment performance with moderate risk perception is almost the same in both cases. The only indicator that is different for these two portfolios is the Sharp coefficient [Figure 3.4.]:

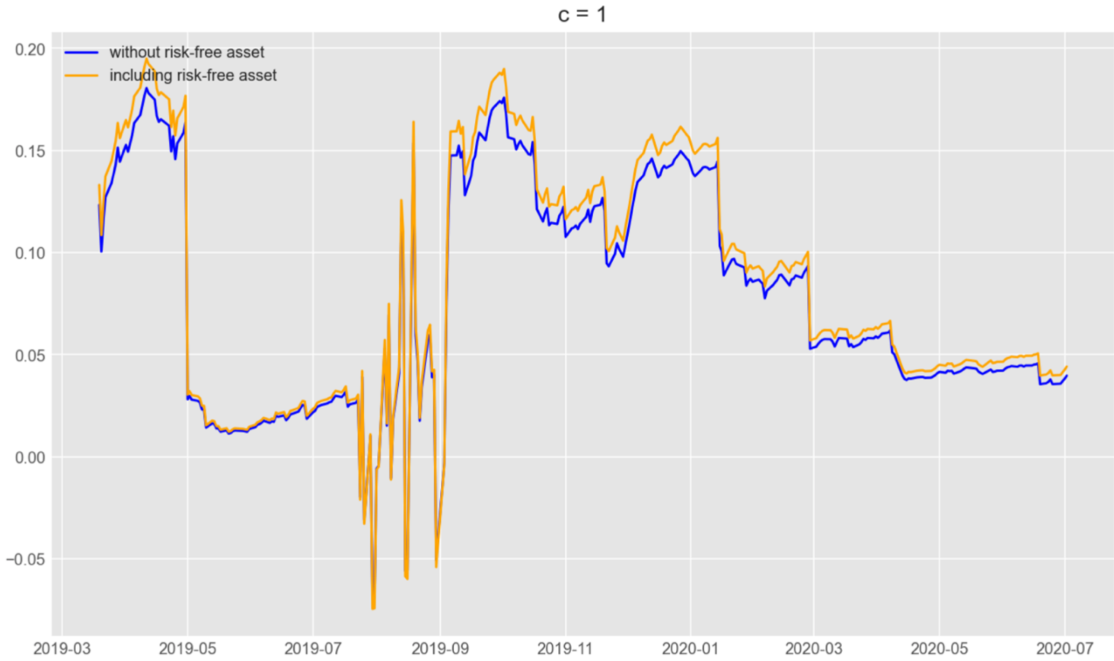


Figure 3.4. Comparison of Sharpe's coefficient between two portfolios

The explanation for the higher rate over the entire investment horizon for a portfolio that includes a risk-free asset can be explained by the fact that a risk-free asset generates a small return on the total volume of the portfolio, but its inclusion in the portfolio does not make it riskier.

Aggregate indicators of both portfolios in case of c = 1 [Figure 3.5.]:

	Average daily return	Last value of cumulative return	Average of volatility	Average Sharpe ratio
without risk-free	0.001723	1.659028	0.000346	0.078144
including risk-free	0.001710	1.652478	0.000342	0.084460

Figure 3.5. Aggregate indicators of both portfolios in case of c = 1

Figure 3.6. is shown the procedure of backtesting of VaR for both portfolios (since their graphs of predicted VaR over time are the same):

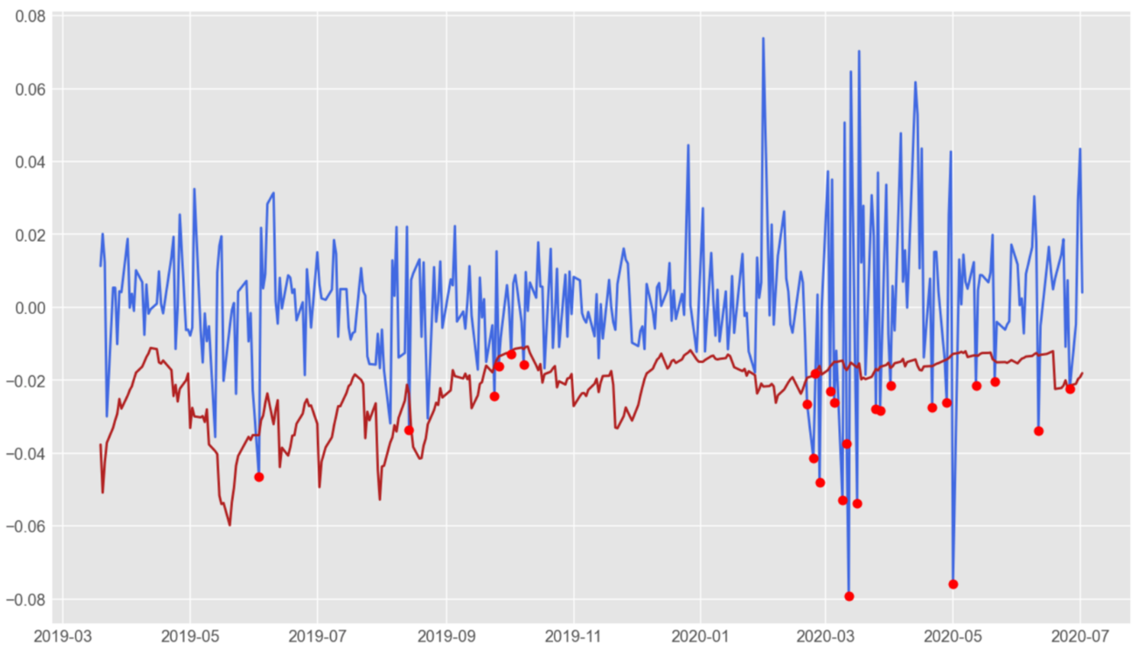


Figure 3.6. Backtesting of VaR 5% for the case c = 1

Applying to this procedure "Traffic lights" approach, it could be seen that this model or trading strategy is not suitable for further use. However, if the entire investment horizon would be divided into two parts: before 11 months 2019 and after, it turns out that the part on the left shows that the actual portfolio returns outside of VaR can be attributed to a statistical error, while in the second part the situation has changed and it is necessary to revise the forecasting model and the investment model.

Consider the second case when the investor has a higher risk aversion, i.e. c = 5 [Figure 3.7.]:

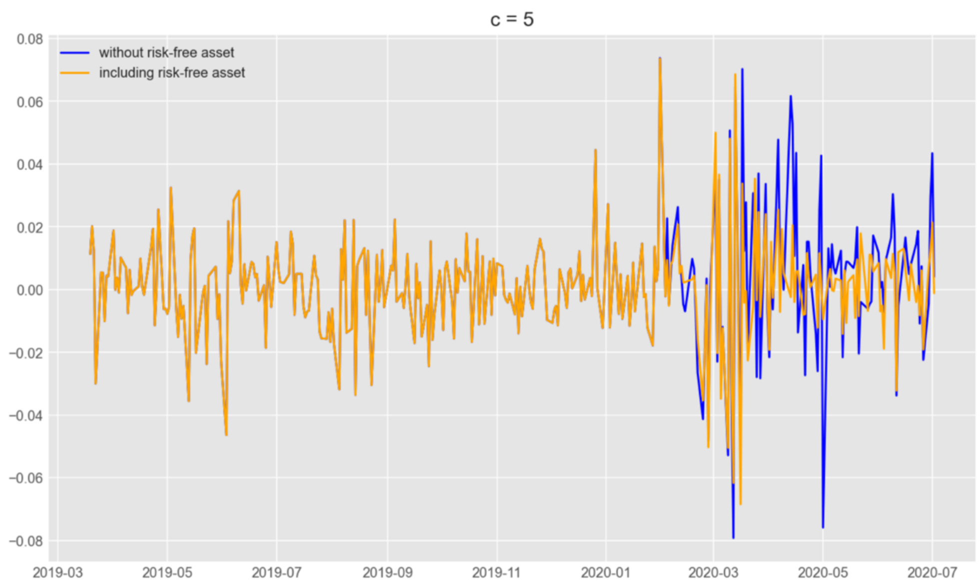


Figure 3.7. Comparison of daily returns for both portfolios in case of c = 5

It can be seen from Fig. 3.7. that an investor prefers less risky securities when the price volatility increases, which is also confirmed by the conditional volatility chart [Fig. 3.8.]:

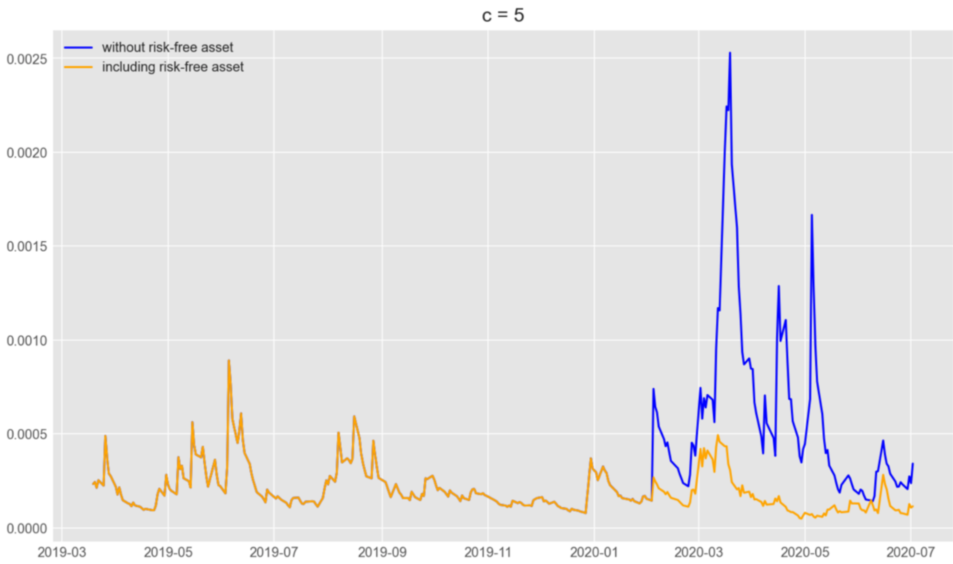


Figure 3.8. Comparison of conditional volatility of two portfolios when c = 5

Aggregate indicators for both portfolios in case of $c = 5$ is presented in Figure 3.9.:

	Average daily return	Last value of cumulative return	Average of volatility	Average Sharpe ratio
without risk-free	0.001722	1.658488	0.000346	0.078140
including risk-free	0.000644	1.185209	0.000202	0.079706

Figure 3.9. Summary investment performance for both portfolios in case of c = 5.

Consequently, in Fig. 3.9. the indicators responsible for efficiency (average daily returns and Sharp's coefficient) are lower at c = 5 than in Fig. 3.5. and, on the other hand, the indicators responsible for risk measures are lower in a situation with higher risk perception.

Thus, building and comparing different portfolios in different situations, under different scenarios, provides the main tool for understanding and managing uncertainty (risk).

Conclusion

The paper aimed to consider why there is uncertainty on the stock exchange, why it is necessary to define the concept of risk, how to model it and how to manage it. The probabilistic approach to securities evaluation was considered, and the transition to the concept of risk was made consistently. Also, the main risk measures were given, the advantages and disadvantages of using each method were considered. In the end, the risk management model was considered - dynamic Mean-variance portfolio taking into account transaction costs. Different sources were analyzed, where the main disadvantages of the model were indicated.

After defining the VaR concept and the methods of its calculation, the problem of mismatching the normal distribution with the empirical distribution was shown on the example of S&P 500 returns distribution and assumptions were made as to how it can harm the risk forecast. At the end of the first chapter, the case of JPMorgan Chase, the most significant investment bank, was discussed, which showed the importance of using a calibrated risk assessment model.

The second chapter presents the mathematical formulation of the optimal Mean-variance portfolio problem and presents the problems that an analyst can face in building this portfolio. For investment on a long-term horizon of investment should be considered a dynamic compilation of the portfolio with the consideration of transaction costs. For forecasting financial indicators important at drawing up of a portfolio of investments, such as expected profitableness and covariance between the financial assets included in a portfolio, it is possible to use the DCC-GARCH model.

In the third Backtesting procedure, leads to better accuracy of the forecast. In the final part of the third chapter, the analysis of two dynamic portfolios is given: including risk-free measure and in which, such chapter, the main analysis was portfolio modelling methodology. As an alternative to conventional forecasting, a rolling forecast method has been proposed, which, with adequately selected parameters based on the class of assets is absent. To better understand risk management, it is necessary to compare competing approaches to modelling and portfolio building and to identify risk measures and performance indicators that are important for the investor.

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