Mathematical Modelling-2022-2023

Assignment-1

Note: Each question has four choices with only one choice as the correct answer.

1. Which of the following options is correct? [1M]
   (A) $\frac{dx}{dt} = xt$; is a continuous time autonomous system.
   (B) $\frac{dx}{dt} = xt$; is a discrete time non autonomous system.
   (C) $\frac{dx}{dt} = x$; is a discrete time autonomous system.
   (D) $\frac{dx}{dt} = x$; is a continuous time autonomous system.

2. For a given matrix $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ with the initial vector $[1\ 1]^T$, how many iterations will be required to compute the largest eigen value by using the power method? [1M]
   (A) Infinite.
   (B) 1.
   (C) 2.
   (D) More than 2 but finite.

3. Given a difference equation $f(n) = nf(n-1) + 2f(n-2)$ with initial conditions f(1) = 0; f(2) = 1. The appropriate statement based on this difference equations is: [2M]
   (A) This difference equation is linear in nature.
   (B) Order of difference equation is not defined.
   (C) $f(n+2) = (n+2)f(n+1) + 2f(n)$ for all $n \in N$.
   (D) $f(4) = 15$.

4. For a given matrix $A = \begin{bmatrix} 3 & -1 \\ 3 & 4 \end{bmatrix}$, [2M]
   (A) trace of matrix is 2.
   (B) determinant of matrix is 9.
   (C) matrix is singular.
   (D) characteristic equation of matrix is $\lambda^2 - 7\lambda + 15 = 0$.

5. The linear function is (which follow principle of super-position): [3M]
   (A) $f_1(x) = rx(1-x)$; r is a constant.
   (B) $f_2(x) = a\sin x$; a is a constant.
   (C) $f_3(x) = ax + b$; a, b are constants.
   (D) $f_4(x) = ax$; a is a constant.

6. For $x_{n+1} = f(x_n)$, which of the following statement is correct? [3M]
   (A) In general, we can not draw cobweb graphs for this difference equation.
   (B) If $f(x)$ is linear function then there will always be only one steady state, (0, 0) and vice-versa is also true.
   (C) The cobwebs for $f(x) = x - 2$ are neither monotonically increasing nor decreasing and hence system is stable but not asymptotically stable.
   (D) $x_{n+1} = 0.5x_n$ and $x_{n+1} = 0.5$ are both asymptotically stable.

7. Consider a linear cell division model with growth rate of 2 per second and migration function is n. The same model is represented as $C_{n+1} = 2C_n + 4n$ with $C_0 = 99$. Mark the most appropriate statement: [3M]
   (A) The equation is of order 1 and homogeneous.
   (B) $C_n = k2^n - n$ is a general solution where k is any constant.
   (C) $C_n = 1002^n + 2n + 1$ is a particular solution.
   (D) $C_n = 1002^n - 2n - 1$ is a particular solution.

8. The statement which violates about Fibonacci's rabbit model $f(n) = f(n-1) + f(n-2)$ with $f(1) = 1$ and $f(2) = 1$ is, [3M]
   (A) The equation is of order 1 and homogeneous.
   (B) $f(4) = 3$.
   (C) $f(n) = \left(\frac{1+\sqrt{5}}{2}\right)^n$ will satisfy the model equation.
   (D) $f(n) = \frac{(1+\sqrt{5})^n}{2}$ will satisfy the model equation.

9. A third degree polynomial $f(x) = x^3 + bx^2 + bx + 1$ can be treated as a characteristic polynomial for some matrix. The same matrix is checked for stability using Jury's stability test. Which of the following statement is most appropriate: [3M]
   (A) Jury's condition can not be applied on this equation because 3rd row is entirely zero.
   (B) This equation has at-least one root with absolute value 1 and hence one of the entries in first column at odd numbered rows will be 0.
   (C) This system may be stable for some value(s) of b.
   (D) In Jury's test, if the third row has all 0 entries, it does not always imply that equation has unit magnitude root.

10. For a given difference equation

$$af(n) + bf(n-1) + cf(n-2) = h(n)$$

   where a, b, c are constants. Which of the following statement is correct? [4M]
   (A) It is linear difference equation of order 2 for any value of constants a, b and c.
   (B) For $h(n) \equiv 0$ and $a = c = 1$; $b = 2$, the general solution is asymptotically stable.
   (C) For $a = 1$, $b = -0.5$ and $c = 0$; $h(n) = 2^n$, the general solution is stable but not asymptotically stable.
   (D) For $a = 1$, $b = -0.5$ and $c = 0$; $h(n) = 2^n$, the general solution is always unstable.

Answers: 1.(D), 2.(B), 3.(C), 4.(D), 5(D), 6.(C), 7.(D), 8.(C), 9.(B), 10.(D)