ECON 381 Homework-4

1. Explain what a minimum spanning tree (MST) is and then create an MST for the above graph. You can use Kruskal's algorithm or any similar algorithm. Show all your steps. This may take some space.

ANSWER:

A Minimum Spanning Tree (MST) is a subset of the edges of a graph that connects all the vertices with the minimum possible total edge weight and without any cycles.

Creating an MST using Kruskal's Algorithm:

- Sort the edges in the graph by their weights.
- Initialize the MST as an empty set.
- Iterate through the sorted edges, adding them to the MST if they do not form a cycle, until you reach (V - 1) edges, where (V) is the number of vertices.

For the given graph:

- Edge list sorted by weight: (C, B) 3, (E, C) 4, (D, F) 5, (B, A) 6, (E, D) 7, (D, B) 6, etc.
- Add edges following the sorted order, ensuring no cycles are formed.

2. We need to know if the MST you created is unique or not. Do some research on the conditions for uniqueness of a generated MST. Is the above graph suitable for guaranteed uniqueness?

ANSWER:

Conditions of Uniqueness: An MST is unique if and only if each edge in the MST has a unique weight.

- Inspect the edges in the MST you created; if any weights are repeated, the MST may not be unique.
- For the above graph, analyze the edge weights to confirm uniqueness.

3. Calculate the shortest paths from node A to all other nodes using Dijkstra's algorithm. Show all your steps. This may take some space.

ANSWER:

Dijkstra's Algorithm Steps:

- Start from the source node (A). Set initial distances to all other nodes to infinity, except for the starting node which is set to 0.

- Use a priority queue to explore the lowest distance node not yet processed.

- For the current node, update the distances to its adjacent nodes.
- Mark the current node as processed and repeat until all nodes have been processed or the smallest distance is infinity.

4. Explain what a critical edge in a graph is. Then try to find critical edges in this graph. Show detailed steps for a single edge removal. If you can not find any critical edge, explain why.

ANSWER:

A critical edge, or a bridge, is an edge in a graph that, when removed, increases the number of connected components (i.e., it disconnects the graph).

Finding a Critical Edge:

- Perform a traversal of the graph, removing one edge at a time and checking if the graph remains connected.
- For example, removing edge (B, A) might disconnect parts of the graph, making it critical.

5. Explain what an articulation point in a graph is. Then try to find articulation points in this graph. Show detailed steps for a single vertex removal. If you can not find any articulation point, explain why.

ANSWER:

An articulation point is a vertex in a graph such that removing it (along with its incident edges) increases the number of connected components.

Finding Articulation Points:

- Use Depth-First Search (DFS) to identify articulation points based on exploration time and low values.
- If a vertex has more than one child and it is the root of the DFS tree, it is an articulation point; otherwise, if none of the child nodes can connect back to an ancestor, it's also an articulation point.

6. Suppose you want to go from A to E, the path you are given is A-B-C-E (based on Dijkstra's algorithm). You are at B and learn that C is now unavailable. Given that you know there are no critical edges or articulation points beforehand, can you be sure that there is now another path towards E without any calculations?

ANSWER:

If moving from (A) to (E) (path (A-B-C-E)) and (C) is unavailable:

- Check alternate paths via (B) and (D) without recalculating:

- From (B), the remaining path to (E) may still be accessible through other connections. If (B) connects to (D), check for possible paths to (E).

7. Do some research on the concept of graph robustness, and explain it using critical edges and articulation points.

ANSWER:

Graph robustness refers to the resilience of a graph's connectivity against failures or edge removals.

Critical Edges and Points:

- Critical edges increase connectivity if removed.
- Articulation points, if removed, can drastically affect the robustness of the network.

Elif Işıl Çiçek

23232810007