ITESM Campus Monterrey
Mathematical Physical Modelling F4005
HW1: Introduction to matrices
Due Date: January 21-2019, 23:59 hrs.
Professor: Ph.D Daniel López Aguayo

Full names of team members: ____________________

Instructions: Please write neatly on each page of your homework and send it in pdf format to dlopez.aguayo@tec.mx. Typed solutions in LaTeX (only) will be given extra credit; no late homework will be accepted. Each team should consist (of at most) 5 students.

I. In each of Problems 1 to 2, calculate the indicated matrix combination, with A the first matrix listed and B the second.

1. $\begin{bmatrix} 1 & -1 & 3 \\ 2 & -4 & 6 \\ -1 & 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} -4 & 0 & 0 \\ -2 & -1 & 6 \\ 8 & 15 & 4 \end{bmatrix}$; $2A - 3B$

2. $\begin{bmatrix} -22 & 1 & 6 & 4 & 5 \\ -3 & -2 & 14 & 2 & 25 \\ 18 & 1 & 16 & -4 & -6 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 3 & 1 & 14 \\ -8 & 6 & -10 & 4 & 10 \\ 21 & 6 & 17 & 3 & 2 \end{bmatrix}$; $-2A + 6B$

II. Verify the above answers using Mathematica; please attach your input and output.

III. In each of Problems 3 to 6, compute the products AB and BA where possible. Specify any products that are not defined and justify why. **Finally, use Mathematica to verify your answers**.

3. $A = \begin{bmatrix} -4 & 6 & 2 \\ -2 & -2 & 3 \\ 1 & 1 & 8 \end{bmatrix}$; $B = \begin{bmatrix} -2 & 4 & 6 & 12 & 5 \\ -3 & -3 & 1 & 1 & 4 \\ 0 & 0 & 1 & 6 & -9 \end{bmatrix}$

4. $A = \begin{bmatrix} -1 & 6 & 2 & 14 & -22 \end{bmatrix}$; $B = \begin{bmatrix} -3 \\ 2 \\ 6 \\ 0 \\ -4 \end{bmatrix}$

5. $A = \begin{bmatrix} -2 & 1 \\ 2 & 0 \\ 0 & 9 \\ 6 & -5 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 1 & -5 \\ 0 & 4 & 2 \end{bmatrix}$

IV. Use Mathematica to determine the dimension (size) of the matrix AB where A and B are as in problem 3 of III.

V. In each of the following problems, determine whether AB and BA are defined and determine how many rows and columns each product has if it is defined.

6. A is 14×21, B is 21×14.

7. A is 18×4, B is 18×4.

VI. Optional problem (**for those familiar with mathematical induction**). Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Compute A^2, A^3 and A^4. Once done this, conjecture a formula and prove by induction that

$$A^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix} \text{ for all } n \geq 1.$$

VII. A factory manufactures three products (doohickies, gizmos and widgets) and ships them to two warehouses for storage. The number of units of each product shipped to each warehouse is given by the matrix

$$A = \begin{bmatrix} 200 & 75 \\ 150 & 100 \\ 100 & 125 \end{bmatrix}$$

where a_{ij} is the number of units of product i sent to warehouse j and the products are taken in alphabetical order. The cost of shipping one unit of each product by truck is 1.50 per doohickey, 1.00 per gizmo, and 2.00 per widget. The corresponding unit costs to ship by train are 1.75, 1.50 and 1.00.

(a) Organize these costs into a matrix B and then use matrix multiplication to show how the factory can compare the cost of shipping its product to each of the two warehouses by truck and train.

(b) Let C be the product matrix obtained above. Find c_{22} and explain what does c_{22} means in this context.

VIII. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$.

(a) Compute, by hand, $A^2, A^3, \ldots, A^7$. Verify your answer with Mathematica (please include your input/output).

(b) Compute, by hand, A^{2001}. Verify your answer with Mathematica (please include your input/output).

IX. We say that a matrix A is *nilpotent* if there exists a natural number N such that A^N is the zero matrix. The smallest such N is called the **index** of A. Let

$$A = \begin{bmatrix} 0 & 2 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Prove that A is nilpotent. What is the index of A?

X. In each of the following, find the explicit form of the 4×4 matrix $A = [a_{ij}]$ that satisfies the given condition:

1. $a_{ij} = (-1)^{i+j}$ for all $i, j = 1, \ldots 4$.
2. $a_{ij} = j - i$ for all $i, j = 1, \ldots 4$.
3. $a_{ij} = \begin{cases} 1 & \text{if } 6 \le i + j \le 8 \\ 0 & \text{otherwise} \end{cases}$

XI. <u>Optional problem</u> (**for those familiar with mathematical induction**). Let $A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$. Prove, by mathematical induction, that

$$A^n = \begin{bmatrix} cos(n\theta) & -sin(n\theta) \\ sin(n\theta) & cos(n\theta) \end{bmatrix} \text{ for all } n \ge 1.$$

Finally, use Mathematica to compute A^5 and compare the output with the above formula. Do the values match? Now use the *TrigReduce* command!