Vector Integral Calculus. Integral Theorems [Chapters 10.4 - 10.9]

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Modelación física matemática (Ene 19 Gpo 1)
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$$\iint_S (\operatorname{curl} F)\cdot n\, dA = \oint_C F\cdot r'(s)\, ds$$

Stokes's Theorem

Line Integrals ⟷ (transforms) Stokes's Theorem

Stokes's Theorem ⟷ (transforms) Surface/Double Integrals

Stokes's Theorem ↕ Green's Theorem

$$\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy = \oint_C (F_1\,dx + F_2\,dy)$$

Green's Theorem — has — Applications:

- Area of a Plane
  - in polar coordinates
    $$A = \frac{1}{2}\oint_C r^2\, d\theta$$
  - region as a line integral over a boundary
    $$A = \frac{1}{2}\oint_C (x\,dy - y\,dx)$$
- Transformation of a double integral of the Laplacian of a function into a line integral of its normal derivative
  $$\iint_R \nabla^2 w\, dx\,dy = \oint_C \frac{\partial w}{\partial n}\, ds$$

Parametric representation of a surface

$$r(u,v) = x(u,v)\,i + y(u,v)\,j + z(u,v)\,k$$

Surface/Double Integrals — defines — Parametric representation of a surface

Surface/Double Integrals ⟷ (transforms) Divergence theorem of Gauss

Surface/Double Integrals — has:

- Tangent vector
  $$r'(t) = \frac{\partial r}{\partial u}u' + \frac{\partial r}{\partial v}v'$$
- Normal vector
  $$N = r_u \times r_v \neq 0$$
- Unit vector
  $$n = \frac{1}{|N|}N$$
  or
  $$n = \frac{1}{|\operatorname{grad} g|}\operatorname{grad} g$$
  if $S = g(x,y,z)$

Surface/Double Integrals — defines:

- without regard to orientation
  $$\iint_S G(r)\, dA = \iint_R G(r(u,v))\,|N(u,v)|\, du\, dv$$
  where
  $$dA = |N|\, du\, dv = |r_u \times r_v|$$
- with regard to orientation
  $$\iint_S F\cdot n\, dA = \iint_R F(r(u,v))\cdot N(u,v)\, du\, dv$$
  where $dA = |N|\, du\, dv = |r_u \times r_v|$

Divergence theorem of Gauss

$$\iiint_T \operatorname{div} F\, dV = \iint_S F\cdot n\, dA$$

Divergence theorem of Gauss ⟷ (transforms) Triple Integrals

Divergence theorem of Gauss — has — Applications:

- Fluid Flow
- Harmonic Functions
- Heat Flow
  - Heat/Diffusion Equation

Kreyszig, E. (2011). Advanced Engineering Mathematics. (M. E. Shannon Corliss, Barbara Russiello, Ed.) (10th ed.). New York: JOHN WILEY & SONS, INC. Retrieved from www.ieee.org.