Homework 03 - Matrix Inversion

Bruno González Soria (A01169284)
Antonio Osamu Katagiri Tanaka (A01212611)
José Ivan Aviles Castrillo (A01749804)
Jesús Alberto Martínez Espinosa (A01750270)
Katya Michelle Aguilar Pérez (A01750272)

Instructor: Ph.D Daniel López Aguayo

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ITESM Campus Monterrey
Mathematical Physical Modelling F4005
HW3: Matrix inversion
Due Date: February 3-2019, 23:59 hrs.
Professor: Ph.D Daniel López Aguayo

Full names of team members: ______________________________

Instructions: Please write neatly on each page of your homework and send it in pdf format to dlopez.aguayo@tec.mx. Typed solutions in LaTeX (only) will be given extra credit; no late homework will be accepted. Each team should consist (of at most) 5 students.

I. Use Gaussian-Jordan elimination to compute (**by hand**) the inverse of each of the following matrices. In case the matrix is diagonal or triangular, you are allowed to use the results given in class. If the inverse does not exist state clearly why.

(a)
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)
$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

(c)
$$C = \begin{bmatrix} -\frac{4}{\sqrt{3}} & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & e^2 \end{bmatrix}$$

(d)
$$D = \begin{bmatrix} \pi & \pi & \pi \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

II. Is the following matrix invertible?
$$E = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 7 & 8 \\ 0 & 0 & 9 \end{bmatrix}$$
Justify your answer carefully.

III. Verify part **I** with Mathematica and please include your input and output.

IV. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be an arbitrary 2×2 matrix. It can be shown that A is invertible if and only if $ad - bc \neq 0$, and in this case
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
Use the above result to compute the inverse of each of the following matrices

(a)
$$A = \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix}$$

(b)
$$B = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$$

(c)
$$C = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix}$$

(d)
$$D = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

V. Verify with Mathematica all the above answers (from part **IV**); please include your input and output.

VI. Give specific examples of square matrices A, B (not necessarily different), both of size 3, such that:

(a) $A + B$ is not invertible although A and B are invertible.

(b) $A + B$ is invertible although A and B are not invertible.

VII. Determine, **by inspection** (i.e no computations are allowed!), whether the given system has infinitely many solutions or only the trivial solution. *Hint*: use the theorems we saw in class.

(a)

$$\begin{cases} 2x_1 + 3x_2 - x_3 = 0 \\ x_1 - x_2 + 2x_3 = 0 \end{cases}$$

(b)

$$\begin{cases} x_1 - x_3 + 2x_2 = 0 \\ x_2 + 2x_3 = 0 \\ \pi x_3 = 0 \end{cases}$$

VIII. Solve, **by hand**, the following system by matrix inversion (i.e first compute A^{-1} and then $A^{-1}B$.)

$$\begin{cases} x_1 - x_2 - 2x_3 = -5 \\ 2x_1 + 3x_2 + x_3 = 5 \\ 2x_2 + 3x_3 = 8 \end{cases}$$

IX. Consider the following system

$$\begin{cases} x_1 - x_2 - x_3 + 2x_4 = 1 \\ 2x_1 - 2x_2 - x_3 + 3x_4 = 3 \\ -x_1 + x_2 - x_3 = -3 \end{cases}$$

(a) Find the reduced row echelon form of $[A|B]$; then compute $\text{rank}(A)$ and $\text{rank}([A|B])$. Finally, use Mathematica to verify all your answers.

(b) Use the theorem given in class to prove that the system has infinitely many solutions.

(c) Compute the general form of the solution vector (the answer must be two-parametric).

(d) What are the free variables?

(e) Why we **cannot** use matrix inversion for this problem?

X. A Brazilian man, currently living in Canada, made phone calls within Canada, to the United States, and to Brazil. The rates per minute for these calls vary for the different countries. Use the information in the following table to determine the rates.

Month	**Time within Canada (min)**	**Time to the U.S. (min)**	**Time to Brazil (min)**	**Charges (U.S dollar)**
September	90	120	180	252
October	70	100	120	184
November	50	110	150	206

(a) Write down the corresponding linear system and state clearly what does each variable represent in this context.

(b) Compute the inverse of A using Mathematica (**not by hand!**).

(c) Use (b) to solve the system (recall you can use Mathematica to compute the product of matrices) and **interpret the answer** in terms of the context.

1 Answer to Problem I

I.a

$$A=\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow R_2 - 2R_1 \rightarrow R_2 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow R_2 - 3R_3 \rightarrow R_2 \Rightarrow \boxed{\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)}$$

I.b

$$B=\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} R_2 - R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} R_1 - R_2 \rightarrow R_1 \\ R_3 - R_2 \rightarrow R_3 \end{array} \Rightarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right) \Rightarrow R_2 - R_3 \rightarrow R_2 \Rightarrow \boxed{\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)}$$

I.c

$$C = \begin{pmatrix} -\frac{4}{\sqrt{3}} & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & e^2 \end{pmatrix} \Rightarrow C^{-1} = \operatorname{diag}\left(\frac{1}{C_1}, \frac{1}{C_2}, \ldots, \frac{1}{C_n}\right) \Rightarrow \boxed{\begin{pmatrix} -\frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{e^2} \end{pmatrix}}$$

I.d

$$D = \left(\begin{array}{ccc|ccc} \pi & \pi & \pi & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} \frac{1}{\pi}R_1 \rightarrow R_1 \\ \frac{1}{2}R_2 \rightarrow R_2 \end{array} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & \frac{1}{\pi} & 0 & 0 \\ 1 & 1 & 1 & 0 & \frac{1}{2} & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} R_2 - R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array} \Rightarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & \frac{1}{\pi} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\pi} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{\pi} & 0 & 1 \end{array}\right) \Rightarrow$$

D is not invertible since: a) the reduced row echelon of D contains at least one zero-row; and b) the matrix on the left-hand side is not precisely the identity matrix.

2 Answer to Problem II

E is invertible; the proof is below.

$$\mathbf{varE} = \begin{pmatrix} \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \mathbf{0} & \mathbf{7} & \mathbf{8} \\ \mathbf{0} & \mathbf{0} & \mathbf{9} \end{pmatrix};$$

Inverse[varE];

MatrixForm[%]

$$\begin{pmatrix} \frac{1}{4} & -\frac{5}{28} & -\frac{1}{126} \\ 0 & \frac{1}{7} & -\frac{8}{63} \\ 0 & 0 & \frac{1}{9} \end{pmatrix}$$

3 Answer to Problem III

III.a

$$\mathbf{varA} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix};$$

Inverse[varA];

MatrixForm[%]

$$\boxed{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}}$$

III.b

$$\mathbf{varB} = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} \end{pmatrix};$$

Inverse[varB];

MatrixForm[%]

$$\boxed{\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}}$$

III.c

$$\mathbf{varC} = \begin{pmatrix} -\frac{4}{\sqrt{3}} & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & e^2 \end{pmatrix};$$

Inverse[varC];

MatrixForm[%]

$$\begin{pmatrix} -\frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{e^2} \end{pmatrix}$$

III.d

Matrix is singular.

$$\mathbf{varD} = \begin{pmatrix} \pi & \pi & \pi \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix};$$

Inverse[varD];

4 Answer to Problem IV

IV.a

$$A^{-1} = \frac{1}{1(8)-2(2)} \begin{pmatrix} 8 & -2 \\ -2 & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 8 & -2 \\ -2 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix}}$$

IV.b

$$B^{-1} = \frac{1}{0(0)-2(3)} \begin{pmatrix} 0 & -2 \\ -3 & 0 \end{pmatrix} = -\frac{1}{6} \begin{pmatrix} 0 & -2 \\ -3 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{pmatrix}}$$

IV.c

$$C^{-1} = \frac{1}{2(0)-0(4)} \begin{pmatrix} 0 & 0 \\ -4 & 2 \end{pmatrix} \Rightarrow ad - bc\ is\ equal\ to\ 0,\ therefore\ \boxed{C^{-1}\ does\ not\ exist.}$$

IV.d

$$D^{-1} = \frac{1}{\mathrm{Cos}[\theta](\mathrm{Cos}[\theta])-(-\mathrm{Sin}[\theta])(\mathrm{Sin}[\theta])} \begin{pmatrix} \mathrm{Cos}[\theta] & \mathrm{Sin}[\theta] \\ -\mathrm{Sin}[\theta] & \mathrm{Cos}[\theta] \end{pmatrix} = \frac{1}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} \begin{pmatrix} \mathrm{Cos}[\theta] & \mathrm{Sin}[\theta] \\ -\mathrm{Sin}[\theta] & \mathrm{Cos}[\theta] \end{pmatrix} =$$

$$\boxed{\begin{pmatrix} \frac{\mathrm{Cos}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} & \frac{\mathrm{Sin}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} \\ -\frac{\mathrm{Sin}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} & \frac{\mathrm{Cos}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} \end{pmatrix}}$$

5 Answer to Problem V

V.a

Custom2by2Inverse[varM_]:=Module[{vM = varM, a, b, c, d, Res},

(* Let's check if the matrix is of size 2x2 *)

If[Dimensions[vM][[1]] == 2&&Dimensions[vM][[2]] == 2,

a = vM[[1, 1]];

b = vM[[1, 2]];

c = vM[[2, 1]];

d = vM[[2, 2]];

(* The matrix is invertible if and only if ad − bc ≠ 0 ... *)

If[$a * d - b * c$ == 0,

Print["[ERROR] The matrix is not invertible."]

];

(* Use the provided equation to calculate the inverse of the 2x2 matrix *)

Res = $\frac{1}{a*d-b*c}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$;

(* Use Mathematica's function Inverse[] to verify the calculation *)

If[Res == Inverse[varM],

Print["[PASS] The calculation is EQUAL to Mathematica's function Inverse[",

MatrixForm[varM], "]."],

Print["[FAIL] The calculation is NOT EQUAL to Mathematica's function Inverse[",

MatrixForm[varM], "]."]

];

(*Return the calculated inverse of vM*)

Res,

(*Display an error message if the matrix size is not 2x2*)

Print("[ERROR] Unfortunately, this function only works with 2x2 matrices.")

];

];

$$\mathbf{Custom2by2Inverse}\left[\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{8} \end{pmatrix}\right];$$

MatrixForm[%]

[PASS] The calculation is EQUAL to Mathematica's function Inverse[$\begin{pmatrix} 1 & 2 \\ 2 & 8 \end{pmatrix}$].

$$\begin{pmatrix} 2 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

V.b

$$\mathbf{Custom2by2Inverse}\left[\begin{pmatrix} \mathbf{0} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} \end{pmatrix}\right];$$

MatrixForm[%]

[PASS] The calculation is EQUAL to Mathematica's function Inverse[$\begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$].

$$\begin{pmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{pmatrix}$$

V.c

Infinite expression encountered.

Matrix is singular.

$$\mathbf{Custom2by2Inverse}\left[\begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{4} & \mathbf{0} \end{pmatrix}\right];$$

MatrixForm[%]

[ERROR] The matrix is not invertible.

V.d

$$\mathbf{Custom2by2Inverse}\left[\begin{pmatrix} \mathrm{Cos}[\theta] & -\mathrm{Sin}[\theta] \\ \mathrm{Sin}[\theta] & \mathrm{Cos}[\theta] \end{pmatrix}\right];$$

MatrixForm[%]

[PASS] The calculation is EQUAL to Mathematica's function Inverse[$\begin{pmatrix} \mathrm{Cos}[\theta] & -\mathrm{Sin}[\theta] \\ \mathrm{Sin}[\theta] & \mathrm{Cos}[\theta] \end{pmatrix}$].

$$\begin{pmatrix} \frac{\mathrm{Cos}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} & \frac{\mathrm{Sin}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} \\ -\frac{\mathrm{Sin}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} & \frac{\mathrm{Cos}[\theta]}{\mathrm{Cos}[\theta]^2+\mathrm{Sin}[\theta]^2} \end{pmatrix}$$

6 Answer to Problem VI

(a) A + B is not invertible although A and B are invertible.
(b) A + B is invertible although A and B are not invertible.

(a) x1 x2 x3

$$\begin{pmatrix} 1 & -1 & -2 \\ 3 & 9 & 4 \\ -4 & -6 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 2 \\ -3 & -6 & -4 \\ 2 & 3 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ -2 & -3 & 0 \end{pmatrix}$$

Let's check that x1 and x2 are invertible where as x3 is not invertible.

$$\mathbf{x1} = \begin{pmatrix} 1 & -1 & -2 \\ 3 & 9 & 4 \\ -4 & -6 & 4 \end{pmatrix}$$

Inverse[x1]//MatrixForm

$$\begin{pmatrix} \frac{15}{13} & \frac{4}{13} & \frac{7}{26} \\ -\frac{7}{13} & -\frac{1}{13} & -\frac{5}{26} \\ \frac{9}{26} & \frac{5}{26} & \frac{3}{13} \end{pmatrix}$$

$$\mathbf{x2} = \begin{pmatrix} 1 & 1 & 2 \\ -3 & -6 & -4 \\ 2 & 3 & -4 \end{pmatrix}$$

Inverse[x2]//MatrixFrom

MatrixFrom $\left[\left\{\left\{\frac{18}{11}, \frac{5}{11}, \frac{4}{11}\right\}, \left\{-\frac{10}{11}, -\frac{4}{11}, -\frac{1}{11}\right\}, \left\{\frac{3}{22}, -\frac{1}{22}, -\frac{3}{22}\right\}\right\}\right]$

$$\mathbf{x3} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ -2 & -3 & 0 \end{pmatrix}$$

Inverse[x3]//MatrixForm

Inverse::sing: Matrix$\{\{2,0,0\},\{0,3,0\},\{-2,-3,0\}\}$ is singular.

$\{\{2,0,0\},\{0,3,0\},\{-2,-3,0\}\}$

(b) x4 x5 x6

$$\begin{pmatrix} \mathbf{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} & \mathbf{0} \\ \mathbf{3} & \mathbf{4} & \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{4} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{7} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{4} & \mathbf{0} \\ \mathbf{3} & \mathbf{4} & \mathbf{2} \end{pmatrix}$$

Let' s check that x4 and x4 are not invertible where as x6 is invertible

$$\mathbf{x4} = \begin{pmatrix} \mathbf{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} & \mathbf{0} \\ \mathbf{3} & \mathbf{4} & \mathbf{0} \end{pmatrix}$$

Inverse[x4]

Inverse::sing: Matrix $\{\{3,0,0\},\{0,4,0\},\{3,4,0\}\}$ is singular.

$$\mathbf{x5} = \begin{pmatrix} \mathbf{4} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{2} \end{pmatrix}$$

Inverse[x5]

Inverse::sing: Matrix $\{\{4,0,0\},\{3,0,0\},\{0,0,2\}\}$ is singular.

$$\mathbf{x6} = \begin{pmatrix} \mathbf{7} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{4} & \mathbf{0} \\ \mathbf{3} & \mathbf{4} & \mathbf{2} \end{pmatrix}$$

Inverse[x6]//MatrixForm

$$\begin{pmatrix} \frac{1}{7} & 0 & 0 \\ -\frac{3}{28} & \frac{1}{4} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

7 Answer to Problem VII

a) First we creat a Matrix (A) of the system and the augmented Matrix (B)

$A = \{\{2, 3, -1\}, \{1, -1, 2\}\}$//MatrixForm

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$B = \{\{2, 3, -1, 0\}, \{1, -1, 2, 0\}\}$//MatrixForm

$$\begin{pmatrix} 2 & 3 & -1 & 0 \\ 1 & -1 & 2 & 0 \end{pmatrix}$$

MatrixRank[A]

2

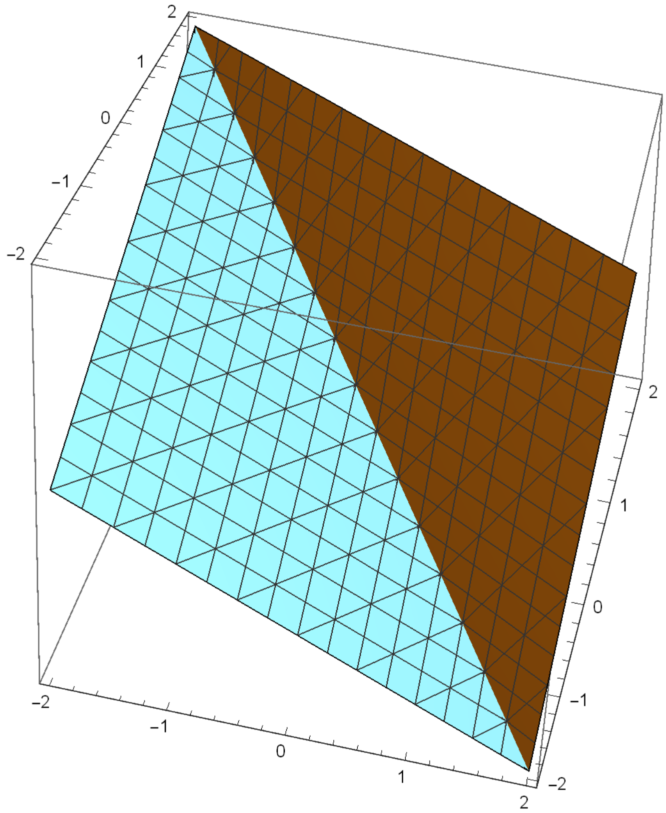
MatrixRank[B]

2

Like the Rank[A]=Ranl[B] but less than n, the system has **infinitely many solutions** to prove that we could plot it

ContourPlot3D[$\{2x + 3y - z == 0, x - y + 2z == 0\}, \{x, -2, 2\}, \{y, -2, 2\}, \{z, -2, 2\}$]

Out[ ]=



In the plote we can se that it is a intersection of two planes in one line
b) First we creat a Matrix (F) of the system and the augmented Matrix (G)

F = {{1, −1, 2}, {0, 1, 2}, {0, 0, Pi}}//MatrixForm

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & \pi \end{pmatrix}$$

G = {{1, −1, 2, 0}, {0, 1, 2, 0}, {0, 0, Pi, 0}}//MatrixForm

$$\begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & \pi & 0 \end{pmatrix}$$

And calculate the rank of the matrix to evaluate it

MatrixRank[F]

3

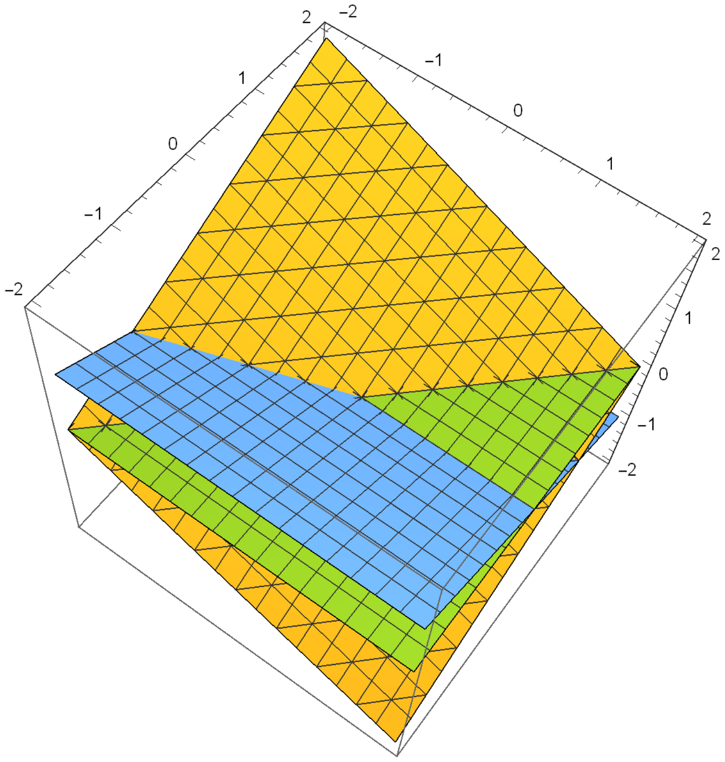
MatrixRank[G]

3

Like the Rank[A]=Ranl[B] = n, the system has a **unique solution** to prove that we could plot it

ContourPlot3D[{$x - y + 2z == 0, y + 2z == 0$, Pi $* z == 0$}, {$x, -2, 2$}, {$y, -2, 2$}, {$z, -2, 2$}]

Out[ ]=



In the plot we can see that the intersection of the planes is in one point, for this has a unique solution

8 Answer to Problem VIII

First we calculate the Inverse Matrix of T

$$T = \begin{pmatrix} 1 & -1 & -2 \\ 2 & 3 & 1 \\ 0 & 2 & 3 \end{pmatrix}$$

Create the augmented matrix [T—I]

$$\begin{pmatrix} 1 & -1 & -2 \\ 2 & 3 & 1 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

And reduce the T matrix

$R_2 - 2R_1$

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 5 & 5 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$1/5(R_2)$

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R_1 + R_2$

$R_3 - 2R_2$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{3}{5} & \frac{1}{5} & 0 \\ -\frac{2}{5} & \frac{1}{5} & 0 \\ \frac{4}{5} & -\frac{2}{5} & 1 \end{pmatrix}$$

$R_1 + R_3$

$R_2 - R_3$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{7}{5} & -\frac{1}{5} & 1 \\ -\frac{6}{5} & \frac{3}{5} & -1 \\ \frac{4}{5} & -\frac{2}{5} & 1 \end{pmatrix}$$

Here is the Inverse Matrix of T

$$T^{-1} = \begin{pmatrix} \frac{7}{5} & -\frac{1}{5} & 1 \\ -\frac{6}{5} & \frac{3}{5} & -1 \\ \frac{4}{5} & -\frac{2}{5} & 1 \end{pmatrix}$$

Then multiply $T^{-1}S = X$

$$\begin{pmatrix} \frac{7}{5} & -\frac{1}{5} & 1 \\ -\frac{6}{5} & \frac{3}{5} & -1 \\ \frac{4}{5} & -\frac{2}{5} & 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 5 \\ 8 \end{pmatrix} = \begin{pmatrix} -7-1+8 \\ 6+3-8 \\ -4-2+8 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

Therefore $x_1 = 0, x_2 = 1, x_3 = 2$

9 Answer to Problem IX

a) Finding the reduced row echelon form of [A|B]:

$A = \{\{1, -1, -1, 2\}, \{2, -2, -1, 3\}, \{-1, 1, -1, 0\}\};$

$B = \{\{1\}, \{3\}, \{-3\}\};$

$M = \{\{1, -1, -1, 2, 1\}, \{2, -2, -1, 3, 3\}, \{-1, 1, -1, 0, -3\}\};$

RowReduce[M]//MatrixForm

$$\begin{pmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Determining the Rank of matrix A:

MatrixRank[A]

2

Determining the Rank of [A|B]:

MatrixRank[M]

2

b)

According to the Theorem given in class: If rank(A) = rank([A—B]) and both are less than n, then the system has infinitely many solutions. As proven before, the matrix A of size 3 X 4 has a rank of 2 as well as the augmented matrix [A—B], which is less that the number of rows n = 3.

c) The general form of the solution vector is:

Letting $x_2 = s$ and $x_4 = t$: (x_1, x_2, x_3, x_4) = (s+2-t, s, 1+t, t) where s, t ϵ $\Re$.

d)

Since there are only two equations to solve for the 4 variables there will be two free variables that cannot be fixed to a unique value, thus resulting in an infinite amount of possible solutions. In this case, the free variables are X_2 and X_4.

e)

This matrix cannot be inverted since a triangular matrix is only invertible if no element on its main diagonal is 0, and this requirement is not fulfilled by the system.

10 Answer to Problem X

a) The corresponding linear system is as follows:

$$\begin{aligned} &\mathrm{A} \cdot 90 + \mathrm{B} \cdot 120 + \mathrm{C} \cdot 180 = 252 \\ &\mathrm{A} \cdot 70 + \mathrm{B} \cdot 100 + \mathrm{C} \cdot 120 = 184 \\ &\mathrm{A} \cdot 50 + \mathrm{B} \cdot 110 + \mathrm{C} \cdot 150 = 206 \end{aligned}$$

Letting the variables correspond to:
A = Time of calls within Canada
B = Time of calls to the U.S.
C = Time of calls to Brazil

b) Letting:

$\boldsymbol{A} = \{\{90, 120, 180\}, \{70, 100, 120\}, \{50, 110, 150\}\};$

$\boldsymbol{B} = \{\{252\}, \{184\}, \{206\}\};$

The inverse of A is:

Inv = Inverse[*A*]

MatrixForm[Inv]

$$\begin{pmatrix} \frac{1}{60} & \frac{1}{60} & -\frac{1}{30} \\ -\frac{1}{24} & \frac{1}{24} & \frac{1}{60} \\ \frac{1}{40} & -\frac{13}{360} & \frac{1}{180} \end{pmatrix}$$

c) Using the inverse of A to solve the system:

Inv.*B*

$\left\{\left\{\frac{2}{5}\right\}, \left\{\frac{3}{5}\right\}, \left\{\frac{4}{5}\right\}\right\}$

This shows the total rate charged for the calls made to each country through the three months.