Radiation, Conduction, and Convection From a Sphere in an Absorbing, Emitting, Gray Medium

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Combined mode heat transfer is solved for an emitting, reflecting sphere in low Peclet number motion through a gray, nonscattering, absorbing, emitting, and conducting infinite medium. The coupled formulation of the energy and radiative transfer equations is solved numerically. The radiative transfer equation is expressed in a unique spatial/directional coordinate system, whose object is to exploit the axisymmetry of the problem. The radiation intensity field is solved using the discrete ordinates method. Results are presented in terms of the Planck and Peclet numbers, and serve as a combined radiation/convection analog to the well-known Nusselt number result for a radiatively nonparticipating medium.

Introduction

Although heat transfer by conduction and convection from a sphere in motion through an infinite medium has been extensively addressed in the literature, the case in which the medium is also radiatively participating has received little attention. In such a medium, the combined modes of heat transfer give rise to temperature profiles distinct from those resulting from either radiation or convection alone. It is necessary to perform a combined mode heat transfer analysis in such cases; it is not accurate simply to add together heat flux correlations taken from independent analyses of either mode.

In nonradiating problems, heat transfer correlations are derived by considering the sphere to be the inner surface of a spherical annulus, where the outer surface is expanded to infinity. In the case of the decoupled momentum and energy equations, and a known velocity field, the energy analysis in this annulus reduces to solution of a second-order differential equation. In the present radiatively participating case an infinite spherical annulus is likewise addressed, although the analysis is considerably more involved. Radiation heat transfer terms in the energy equation are governed by the continuous radiation intensity, which is the solution of the radiative transfer equation. In the present problem it is necessary to solve a coupled formulation for temperature (energy) and radiation intensity in the thermal boundary layer around the sphere. The results of the present analysis, either in graphic or correlation equation form, express heat transfer between a spherical body in motion through a radiatively participating, gray, continuous medium, developed in a manner analogous to the widely used correlations for the special case of a radiatively nonparticipating medium.

The radiation part of the present problem is the most difficult, and hence is the focus of our analysis. Spherically symmetric (no flow) heat transfer by radiation alone in a spherical annulus has been solved by a variety of methods. Ryhming (1966) and Viskanta and Crosbie (1967) presented coupled temperature and intensity solutions by applying an exponential integral solution for the radiative transfer, outlined by Kuznetsov in a 1951 Russian language paper. Viskanta and Merriam (1968) included conduction in the medium in a similar analysis. Bayazitoglu and Suryanarayana (1989) developed closed-form solutions to the pure radiation problem using the spherical harmonics method. Tsai et al. (1989) addressed the radiative transfer for an assumed temperature profile using the discrete ordinates method. These analyses are all one-dimensional in that there is only radial variation in the temperature, leading to spatial radial symmetry. The variation of intensity with direction may be expressed in a single angular coordinate, as these problems are directionally radially symmetric as well. The present problem, flow over a sphere, is spatially two-dimensional and axisymmetric. The resulting radiative transfer expression must involve two angular coordinates. Therefore, none of the previous solutions for radiative transfer in a spherical annulus apply directly to the present problem.

We have applied the discrete ordinates method to solution of the radiative transfer equation. This is a differential method, which may be integrated easily into the computational grid for the conduction/convection side of the problem. Recalling that radiation intensity is a quantity varying spatially (in two dimensions) and directionally (in two dimensions), the discrete ordinates method may be described as a finite difference representation of the problem (in four dimensions), where the spatial grid is chosen to suit the problem, and the directional grid is chosen to satisfy a quadrature. The mathematics of the discrete ordinates method are treated in several texts, such as Lewis and Miller (1984). Application to radiation heat transfer problems is discussed in a series of papers by Fiveland (1984, 1987, 1988) and Truelove (1987, 1988), among others, who have addressed radiation problems in up to three spatial dimensions in Cartesian media. In Cartesian media, the radiative transfer equation contains only spatial derivatives. However, curvilinear media involve directional derivatives as well.

The discrete ordinates method has been used to solve the radiation part of coupled energy and radiative transfer equation formulations by several authors in Cartesian as well as curvilinear media. Kumar et al. (1988) showed a formulation for combined radiation and convection in flow between infinite parallel plates. Yucel and Williams (1987) and Jamaluddin and Smith (1988) studied combined radiation and conduction in cylindrical media. Jones and Bayazitoglu (1990a) addressed the problem of a sphere in an infinite medium with no convection. In the present problem, coupled energy/radiation analysis using the discrete ordinates method is extended to spherical media with axial (directionally two-dimensional) rather than radial (one-dimensional) symmetry.

Analysis

The geometry of the problem is illustrated in Fig. 1 as an axisymmetric slice of a spherical annulus. Assuming steady

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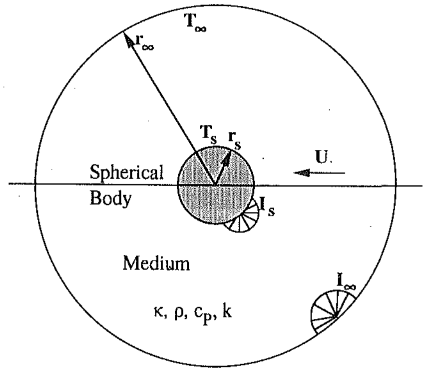

Fig. 1 Spherical body in motion through a gray, absorbing, emitting, conducting medium

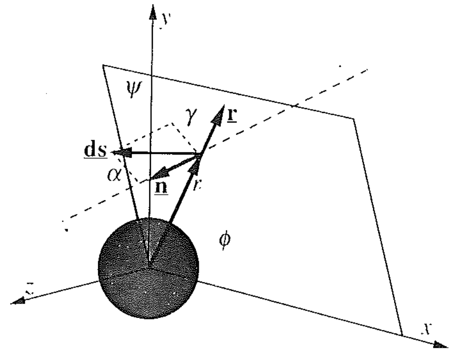

Fig. 2 Spatial/directional coordinate system for use in spatially spherical, axisymmetric geometries

state, constant properties, an index of refraction of unity, and Kirchoff's law, the energy equation may be written:

$$\rho c_P \mathbf{V}\cdot\nabla T - k\nabla^2 T + 4\kappa\sigma_b T^4 - \kappa\int_{4\pi} I\,d\Omega = 0 \tag{1}$$

where the radiation intensity I is integrated over the directional element of solid angle, $d\Omega$. We assume that the medium is gray, so that κ is a spectrally averaged absorption coefficient. The velocity field is assumed to be known. Equation (1) may be written in scalar form in r and ϕ in the usual way.

We assume that the medium is nonscattering. A principal application for this analysis is to express the interphase heat transfer term in gas/particle flows. In such flows, the principal scattering mode is scattering from the particles or droplets themselves. The present problem represents only a single particle, surrounded by the continuous medium in pure (non-particle-laden) form. Although it may be possible for very small particles nearby to effect significant scattering, we neglect this for the sake of clarity. The present analysis could be extended to include scattering in a straightforward manner, as, for instance, in Kim and Lee (1988). The present analysis might be regarded as an inner, single-particle problem, where the outer, multiparticle problem would involve both interparticle scattering and the inner problem heat transfer results. The radiative transfer equation may thus be written:

$$\frac{dI}{ds} + \kappa I = \kappa\frac{\sigma_b}{\pi}T^4 \tag{2}$$

where the radiation intensity I and the differential path length ds are functions of both spatial variables and the two angular variables necessary to define the direction.

The unique spatial/directional coordinate system used in this problem is illustrated in Fig. 2. The spatial point is defined by r and ϕ, where ϕ lies in an axisymmetric plane defined by the polar axis (the azimuthal angle). In spherically symmetric problems, the commonly used coordinate system is an efficient coordinate system as spherical symmetric implies independence from the azimuthal angle.

In the present problem, flow over the sphere violates spherical symmetry, leading instead to axisymmetry. We choose a spatial/directional coordinate system for this axisymmetric problem (Fig. 2), which allows us to represent directions that lie in the plane of axisymmetry (and in parallel planes) using one scalar variable, as opposed to a combination of the polar and azimuthal angles necessary with the more commonly used coordinate system. The present coordinate system is explained as follows: The polar axis is normal to the plane of axisymmetry (**n** in Fig. 2), and the polar angle, α, is the angle away from the polar axis ($\alpha = \pi/2$ lies in the plane of axisymmetry). The azimuthal angle, γ, represents rotation about the polar axis (parallel to the plane of symmetry), using the radial direction as a starting point.

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The intensity pathlength derivative for the spatial/directional coordinate system is derived by Jones (1990) for the coordinate system of Fig. 2:

$$\frac{dI}{ds} = \frac{1}{r^2}\sin\alpha\cos\gamma\frac{\partial}{\partial r}(Ir^2) + \frac{\sin\alpha\sin\gamma}{r\sin\phi}\frac{\partial}{\partial\phi}(I\sin\phi) + \frac{1}{r}\left(\cos\gamma + \frac{\sin\gamma}{\tan\phi}\right)\frac{\partial}{\partial\alpha}(I\cos\alpha) - \frac{\sin\alpha}{r}\frac{\partial}{\partial\gamma}(I\sin\gamma) \tag{3}$$

Energy equation temperature boundary conditions include the temperatures on the sphere's surface and at the far field boundary:

$$T(r_s, \phi) = T_s \tag{4a}$$

$$T(r_\infty, \phi) = T_\infty \tag{4b}$$

as well as leading and trailing edge conditions:

$$\frac{\partial T}{\partial\phi}(r, 0) = 0 \tag{5a}$$

$$\frac{\partial T}{\partial\phi}(r, \pi) = 0 \tag{5b}$$

The radiative transfer equation is first order, requiring only one intensity boundary condition in each variable. However, the known boundary conditions apply to divided ranges of direction. Thus, for diffuse emission and reflection from an opaque, gray sphere, we write:

$$I(r_s, \phi, \alpha, \gamma_{\text{out}}) = \epsilon_s\frac{\sigma_b}{\pi}T_s^4 + (1-\epsilon_s)\frac{1}{\pi}\left|\int_{\pi/2}^{3\pi/2}\int_0^\pi I(r_s, \phi, \alpha, \gamma_{\text{in}})\sin^2\alpha\cos\gamma\,d\alpha\,d\gamma\right| \tag{6a}$$

where γ_{out} on the left side denotes that the boundary condition is given only for those directions facing away from the sphere's surface, and the integration range for γ on the right side includes only those directions facing toward the surface. For directions toward the sphere, we assume a black, diffuse, far field boundary condition:

$$I(r_\infty, \phi, \alpha, \gamma_{\text{in}}) = \frac{\sigma_b}{\pi}T_\infty^4 \tag{6b}$$

Leading and trailing edge boundary conditions also apply to the radiation intensity:

$$\frac{\partial I}{\partial \phi}(r, 0, \alpha, \gamma)=0;\ 0\le\gamma\le\pi \tag{7a}$$

$$\frac{\partial I}{\partial \phi}(r, \pi, \alpha, \gamma)=0;\ \pi<\gamma\le 2\pi \tag{7b}$$

where the spatial location of the boundary condition is again divided by directional range. The problem is symmetric across the plane of axisymmetry, so for the directional polar angle we have:

$$\frac{\partial I}{\partial \alpha}\left(r, \phi, \frac{\pi}{2}, \gamma\right)=0 \tag{8}$$

The radiation intensity is continuous through a 2π rotation of γ. Parallel to the plane of axisymmetry plane we have:

$$I(r, \phi, \alpha, 0)=I(r, \phi, \alpha, 2\pi) \tag{9}$$

While the spatial grid for numerical solution is chosen to suit the problem, the directional grid is chosen as a specific set of directions (ordinates) and their associated integration weights. In the present two-dimensional quadrature, each ordinate is specified by a pair (α_l, γ_m), where $1\le l\le L$ and $1\le m\le M$, and we follow the suggestion of Abu-Shumays (1977) by letting $w_{l,m}=w_l w_m$. Fiveland (1987) noted that numerical stability is enhanced if the weights for each coordinate are equal. Thus, we allow $w_l=w_L$ for all l, and likewise $w_m=w_M$. In the present problem, we choose an unconstrained distribution of γ(rotation parallel to the plane of axisymmetry), $\gamma_m=(m=1/2)\Delta\gamma$, where $\Delta\gamma=2\pi/M$. The weight w_m is then solved from the γ component of the first (flux) moment of area, integrated over an octant of the directional unit sphere, $\Omega=4\pi$:

$$\int_0^{\pi/2}\cos\gamma\, d\gamma=\sum_{m=1}^{M/4}\cos\gamma_m\, w_m \tag{10a}$$

The weight w_l is solved from the zeroth moment over an octant:

$$\int_0^{\pi/2}\int_0^{\pi/2}\sin\alpha\, d\alpha\, d\gamma=\frac{\pi}{2}=\sum_{m=1}^{M/4}\sum_{l=1}^{L} w_l w_m \tag{10b}$$

We choose an even cosine distribution for the α_l's, thus favoring directions near the plane of axisymmetry. Thus $\alpha_l=(L+1/2-l)[\Delta(\cos\alpha)]$, and the complete first moment of area integrated over an octant:

$$\int_0^{\pi/2}\int_0^{\pi/2}\sin^2\alpha\cos\gamma\, d\alpha\, d\gamma=\frac{\pi}{4}$$
$$=\sum_{m=1}^{M/4}\sum_{l=1}^{L}\sin\alpha_l\cos\gamma_m\, w_l w_m \tag{10c}$$

serves to define $\Delta(\cos\alpha)$. Table 1 shows the α ordinates corresponding to a quadrature of fourth order in α (note that for the axisymmetric problem, only the half range $0\le\alpha\le\pi/2$ need be considered).

For ultimate solution, the overall numerical scheme is to estimate temperature, solve the radiative transfer equation for the temperature estimate, and use the resulting intensity to solve the energy equation with the T^4 term linearized about the temperature estimate. The temperature solution is then compared to the temperature estimate, a new estimate formed by overrelaxing the estimation error, and the loop is repeated until convergence is achieved. The energy equation is multiplied by an element of volume, $2\pi r^2\sin\phi\, d\phi\, dr$, and integrated between $r_{j-1/2}$ and $r_{j+1/2}$, and between $\phi_{k-1/2}$ and $\phi_{k+1/2}$, to reach the final form of the finite difference equation. A variable grid is used in r to provide very fine spacing near the sphere's surface and expand to very large values for the outer boundary. A uniform grid is used in the ϕ direction. The energy equation is block tridiagonal and could be solved with a block matrix version of the Thomas algorithm; however, since iteration is already required for the linearized terms, the more rapid, iterative solution method of alternating direction implicit (ADI, see Anderson et al., 1984) is used to solve the energy equation in the r and ϕ directions. For convergence to within 0.1 percent, about 25 iterations are required for combined mode (neither radiation or convection dominated) or convection-dominated cases. Radiation-dominated cases require many more iterations, but no instance of solution divergence is found.

The radiative transfer equation is multiplied by an element of volume and solid angle, $2\pi r^2\sin\phi\, d\phi\, dr\sin\alpha\, d\alpha\, d\gamma$, and integrated over Δr, $\Delta\phi$, and the solid angle element. Partial derivatives are formulated over the computational cells by assuming linear forms by the intensity in each cell over each variable.

The conservative form of the intensity pathlength derivative, Eq. (3), is used to promote numerical stability. However, in this form, it arises that $dI/ds\neq 0$ for constant I in a direct discretization (Lewis and Miller, 1984). The discretized equation is brought back into balance ($dI/ds=0$ for constant I) by altering the difference coefficients for the angular terms, as described in Jones and Bayazitoglu (1990b).

The radiative transfer equation is solved for the current temperature estimate using nested marching algorithms in the explicitly bound variables r, ϕ, and α. In γ, continuity provides only an implicit boundary condition. Hence, the r, ϕ, and α dependencies are solved for an assumed γ distribution, followed by iterations to solve the correct γ dependence. Due to the large number of intensity values over the four-dimensional range, convergence is judged by convergence of the radiation heat flux in two dimensions. Generally three to four cycles are sufficient, using the intensity values from the previous temperature iteration to start the process.

The steep temperature gradients near the sphere surface dictate a fairly fine radial grid for accurate temperature profile representation. A non-uniform radial grid of 72 points produces the results reported here. Coarser grids usually result in underprediction of heat flux from the sphere surface, although if heat flux alone is of interest (as opposed to temperature profile), smaller radial grids are often found to give acceptable results. This general criterion applied to judge grid fineness is agreement within 1 percent with the heat flux computed in the special cases reported by Viskanta and Crosbie (1967) and by Tsai et al. (1989) (no flow, finite spherical annulus, radiation only). A polar angular quadrature of four points (in the half range $0\le\alpha\le\pi/2$) and a uniform azimuthal grid of 16 points (in the range $0\le\gamma\le 2\pi$; hence four by four or 16 ordinates in each octant) is found to be the coarsest grid conforming to 1 percent agreement with the test cases. Coarser grids lead both to overprediction of heat flux and numerical instability. Ray effects are mitigated in this problem by temperature field smoothing due to conduction/convection.

Table 1 Ordinates for a quadrature of fourth order in the direction angle out of the axisymmetric plane ($\alpha=\pi/2$ lies in the axisymmetric plane)

l	α_l	w_l
1	0.47139	0.25
2	0.88099	0.25
3	1.17902	0.25
4	1.44317	0.25

Results and Discussion

The controlling parameters of the solution are illustrated by nondimensional forms of the energy and radiative transfer equations. Using the radiative transfer equation to substitute dI/ds for the radiation terms, we write:

$$\frac{1}{\mathrm{Pl}}\int_0^{2\pi}\int_0^{\pi}\frac{1}{\zeta^2}\sin\alpha\cos\gamma\frac{\partial}{\partial\zeta}(\zeta^2\Phi)\sin\alpha\,d\alpha\,d\gamma$$
$$+\frac{1}{\mathrm{Pl}}\int_0^{2\pi}\int_0^{\pi}\frac{\sin\alpha\sin\gamma}{\zeta\sin\phi}\frac{\partial}{\partial\phi}(\sin\phi\,\Phi)\sin\alpha\,d\alpha\,d\gamma$$
$$+\frac{1}{\mathrm{Pl}}\int_0^{2\pi}\int_0^{\pi}\frac{1}{\zeta}\left(\cos\gamma+\frac{\sin\gamma}{\tan\phi}\right)\frac{\partial}{\partial\alpha}(\cos\alpha\,\Phi)\sin\alpha\,d\alpha\,d\gamma$$
$$-\frac{1}{\mathrm{Pl}}\int_0^{2\pi}\int_0^{\pi}\frac{\sin\alpha}{\zeta}\frac{\partial}{\partial\gamma}(\sin\gamma\Phi)\sin\alpha\,d\alpha\,d\gamma$$
$$-\left[\frac{1}{\zeta^2}\frac{\partial}{\partial\zeta}\left(\zeta^2\frac{\partial\Theta}{\partial\zeta}\right)+\frac{1}{\zeta^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial\Theta}{\partial\phi}\right)\right]$$
$$+\frac{\mathrm{Pe}}{2}\left[\left(\frac{v_r}{U}\right)\frac{\partial\Theta}{\partial\zeta}+\frac{1}{\zeta}\left(\frac{v_\phi}{U}\right)\frac{\partial\Theta}{\partial\phi}\right]=0 \quad (11)$$

where $\Theta=T/T_s$, $\Phi=I/4\sigma_b T_s^4$, and $\zeta=r/r_s$. The first group of four terms in Eq. (11) is the radiation part of the energy equation, the second group is the conduction part, and the third group is the advection part. The radiation part is governed by the Planck number, $\mathrm{Pl}=k/(4r_s\sigma_b T_s^3)$, and the advection part by the Peclet number, $\mathrm{Pe}=(2Ur_s\rho c_P)/k$. The Planck number results instead of the more familiar Stark number ($N=\mathrm{Pl}\ \kappa r_s$) because the medium radial coordinate, r, is nondimensionalized by the sphere radius, r_s, rather than by the absorption coefficient, κ. The radiative transfer equation is written:

$$\frac{1}{\zeta^2}\sin\alpha\cos\gamma\frac{\partial}{\partial\zeta}(\zeta^2\Phi)+\frac{\sin\alpha\sin\gamma}{\zeta\sin\phi}\frac{\partial}{\partial\phi}(\sin\phi\Phi)$$
$$+\frac{1}{\zeta}\left(\cos\gamma+\frac{\sin\gamma}{\tan\phi}\right)\frac{\partial}{\partial\alpha}(\cos\alpha\,\Phi)-\frac{\sin\alpha}{\zeta}\frac{\partial}{\partial\gamma}(\sin\gamma\Phi)$$
$$+\Phi(\kappa r_s)=\frac{1}{4\pi}(\kappa r_s)\Theta^4 \quad (12)$$

to illustrate the governing parameter κr_s; the importance of ϵ_s is indicated by the boundary conditions. Note that κr_s is the nondimensionalized radius of the sphere rather than any physically meaningful optical thickness, as the region inside r_s is not part of the medium. Further, due to the nonlinear influence of Θ upon Φ, the temperature ratio T_s/T_∞ is also a governing parameter. Summarizing, the important parameters in this problem are Pl, Pe, κr_s, ϵ_s, and T_s/T_∞. The rather large number of controlling parameters, as compared to conduction/convection without radiation, is a result of the combined mode nature of the problem. In pure radiation, for instance, it may be possible to nondimensionalize heat flux in such a way as to remove κr_s from the parameter list. However, with combined modes, this is not justifiable.

Figure 3 shows the nondimensionalized heat flux leaving the surface of a black sphere hotter than the surrounding medium for a variety of Pl and T_s/T_∞, as a function of κr_s, for $\mathrm{Pe}=0$. The results are nondimensionalized by the factor $[\sigma_b(T_s^4-T_\infty^4)+k(T_s-T_\infty)/2r_s]$ in order to be bounded over the entire range from radiation-dominated to conduction-dominated cases (In viewing Fig. 3 from the point of determining the increase in heat flux calculated by considering a conduction-dominated case to which radiation effects have been added, the nondimensionalizing basis must be kept in mind.) Note that in the $\mathrm{Pe}=0$ case, the problem is spatially one dimensional. For high Pl, the result is dominated by conduction, and approximates the familiar result $\mathrm{Nu}=2$. As κr_s increases, radiation begins to have an effect. At low Pl, a radiation-dominated case, the pure radiation result of $q/\sigma_b(T_s^4-T_\infty^4)$ versus κr_s is nearly recovered. At $\mathrm{Pl}=1$, clearly a combined mode case, variation with T_s/T_∞ is more apparent due to dependence of each heat transfer mode on a different order of T. It is interesting to note the finite outer-annular radius, which must be employed computationally to approach infinite medium heat flux results. Figure 3 was computed using $r_\infty/r_s=200$, which gives results within 0.5 percent of the infinite medium results in conduction-dominated cases. In combined mode and radiation dominated cases it was possible to achieve similar accuracy with much smaller computational domains. For $\mathrm{Pl}=1$, $r_\infty/r_s=100$ was sufficient, while for $\mathrm{Pl}=0.1$, only $r_\infty/r_s=50$ was necessary.

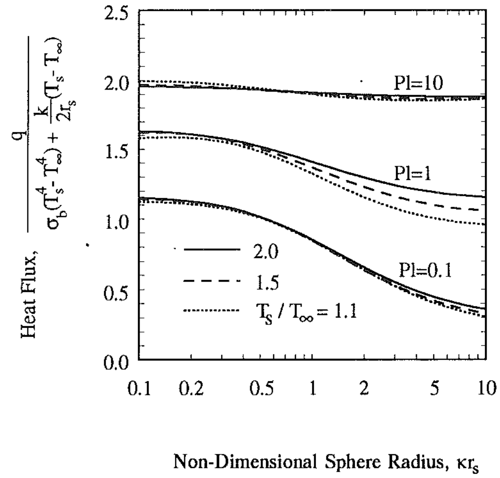

Fig. 3 Combined radiation and conduction heat flux from a hot, black sphere in an infinite medium

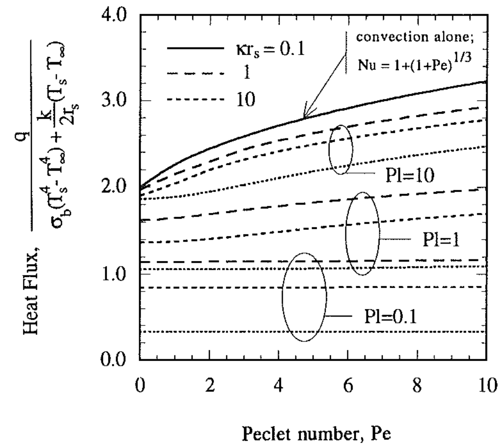

Fig. 4 Combined radiation and convection heat flux from a hot, black sphere in an infinite medium at low Peclet numbers, for $T_s/T_\infty=1.5$

The energy equation is decoupled from the momentum equation in this problem, and so, in $\mathrm{Pe}\neq 0$ cases, any descriptive velocity field may be used. We have used the velocity field of Stokes flow in the following results (see Panton, 1984):

$$v_r=-\frac{U}{2}\cos\phi\left[\left(\frac{r_s}{r}\right)^3-3\left(\frac{r_s}{r}\right)+2\right] \quad (13a)$$

$$v_\phi=\frac{U}{4}\sin\phi\left[-\left(\frac{r_s}{r}\right)^3-3\left(\frac{r_s}{r}\right)+4\right] \quad (13b)$$

This velocity field is valid for Reynolds numbers less than one. However, for slightly higher Reynolds numbers, the deviation of the velocity field from Eqs. (13) due to separation in the wake has only a minor effect on heat transfer.

Figure 4 shows the effect of low Peclet numbers on combined radiation and convection heat flux from the surface of a hot, black sphere in a gray, nonscattering infinite medium. The indicated heat flux is a mean, integrated over the sphere and normalized by the surface area. The results in Fig. 4 were computed using a spatial angular grid of nine points, which was found to give results within 1 percent of results for a 19-point grid. Also shown is the convection-only result $Nu = 1 + (1 + Pe)^{1/3}$, which is valid up to a Peclet number of about 10. Note that as with the radiation/conduction results, inclusion of radiation effects results in a higher overall heat flux, but a lower nondimensional heat flux due to the effect of the nondimensionalizing factor. At high Planck numbers the heat flux is dominated by conduction and convection, and variation with Pe is similar to variation for convection alone. At low Pl, the radiation terms in Eq. (11) dominate the conduction and convection, and Pe does not have an effect. Presumably, at higher Pe, outside the range of Stokes flow, low Pl cases will show greater variation with Pe. Although the objective of this study has focused on low Pe flow, the formulation given is general, and results for higher Pe could easily be developed.

Acknowledgments

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