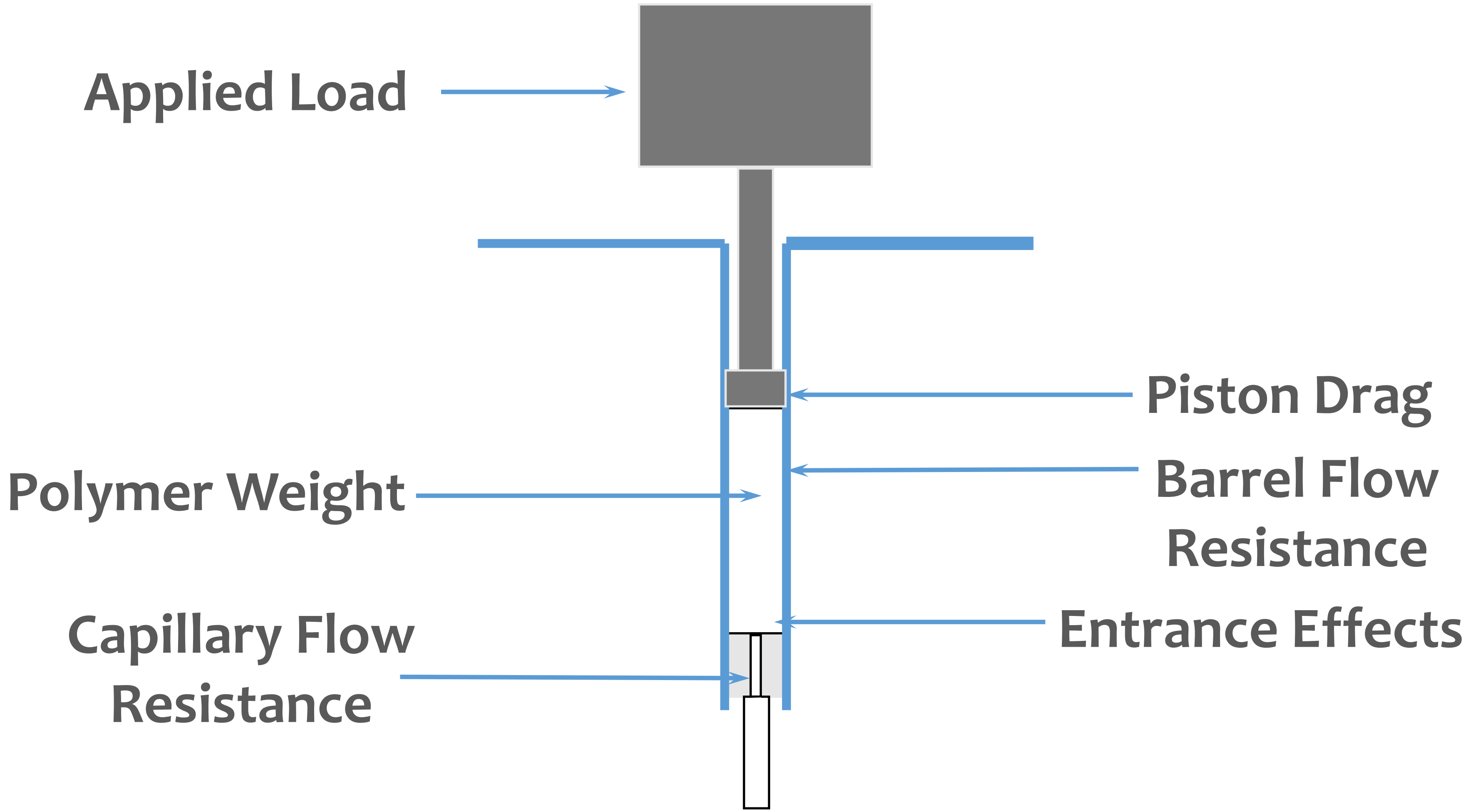
Tecnológico de Monterrey

# Capillary rheometry

# Instron Force Balance

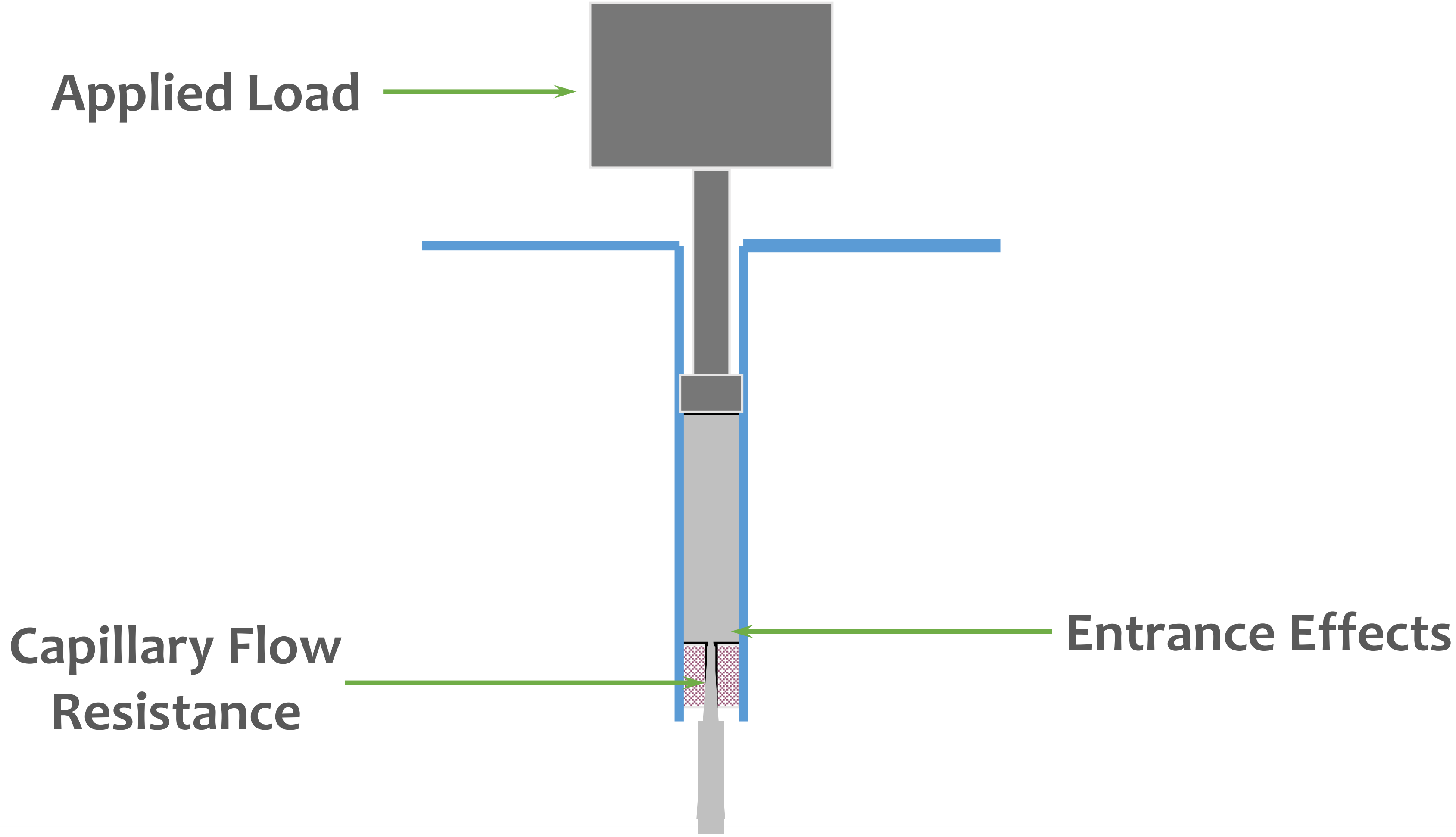


$$F_{applied} = \cancelto{0}{F_{drag}} + \cancelto{0}{F_{barrel}} - \cancelto{0}{F_{resin}} + F_{entrance} + F_{capillary}$$

*after dividing by the appropriate areas:*

$$\Delta P_{total} = \cancelto{0}{\Delta P_{drag}} - \cancelto{0}{\Delta P_{resin}} + \cancelto{0}{\Delta P_{barrel}} + \Delta P_{entrance} + \Delta P_{capillary}$$

# Force Balance



# Note:

The Capillary rheometer is a velocity controlled instrument and the things you get from such type of equipment are:

**piston velocity (in/min) ($P_V$),**
**load (lbf).**

The volumetric flow (Q) is obtained as follows:

$$Q=P_V\ (in/min)*Piston\ Cross\ Section\ Área\ (in^2)=in^3/sec$$

Then, the computer program uses the DIE data (goemetry) to calculate the viscosity of the samples and the procedure is based on a force balance on the DIE.

The load (used for the force balance) is measured either at the top of the plunger or just above the die (all depends on the type of rheometer).

# Note:

Such load is converted by the computer program to a pressure (F/A).

Afterwards, the rheometer software might give you the following information

Apparent shear rate,

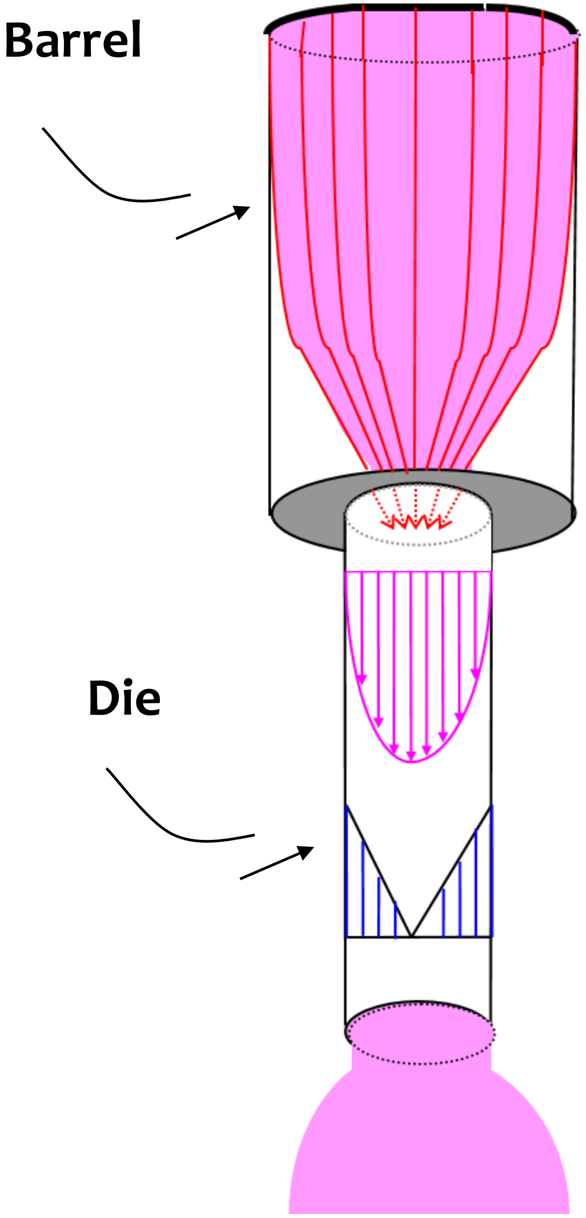
Apparent shear viscosity,

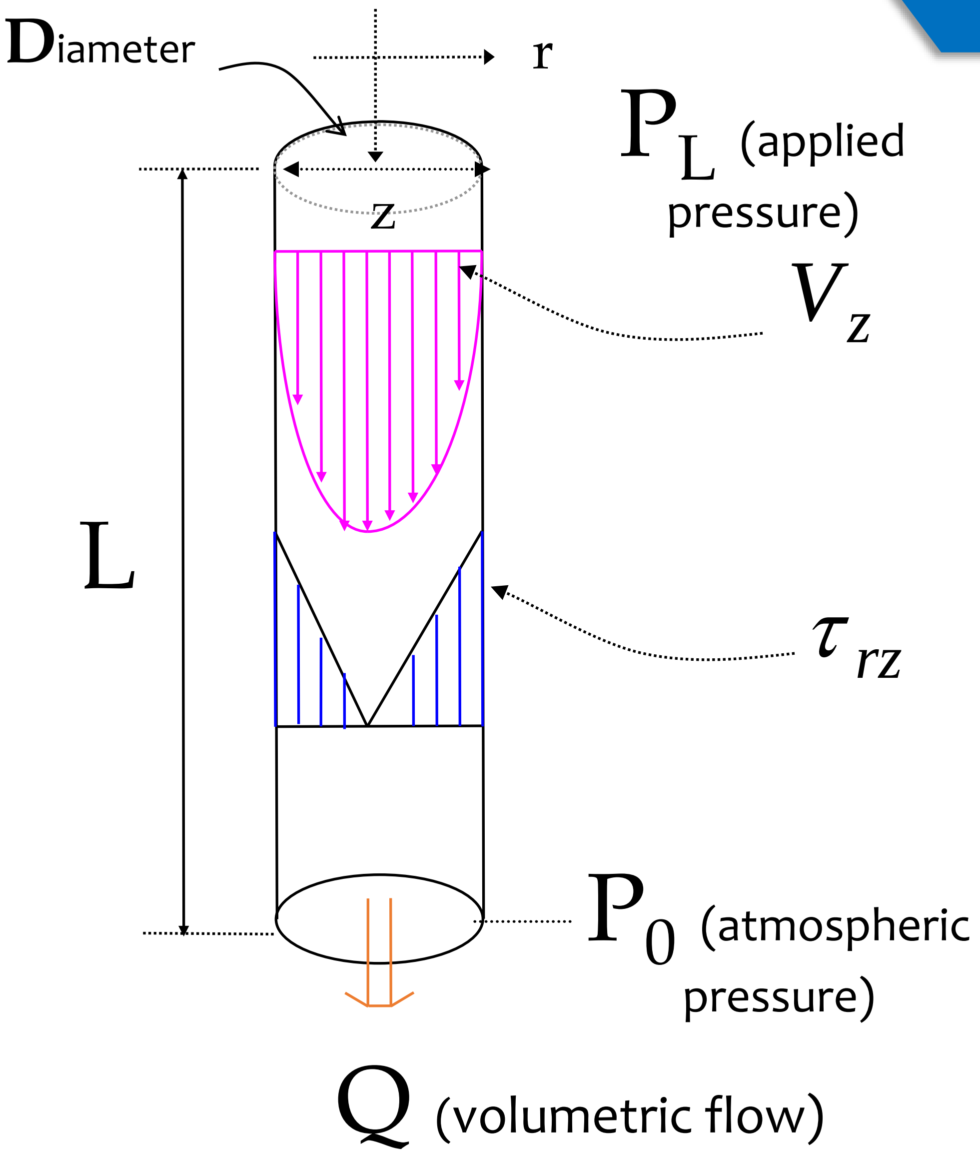
Corrected shear rate,

Corrected viscosity.

( ?????)

To better understand this let's take a closer look of polymer flow in the barrel and in the die…


Barrel
Die


Diameter
r
z
$P_L$ (applied pressure)
$V_z$
L
$\tau_{rz}$
$P_0$ (atmospheric pressure)
Q (volumetric flow)

# Remember the meaning of shear stress?

**Well, that shear stress at the wall is related to the force exerted by the fluid on the barrel's wall:**

(1) $$\frac{F_z}{A} = \tau_w$$

where $A_{barrel} = 2\pi R L$ (2)

and $Fz = \Delta P \text{ x } A_{\text{perpendicular to Fz}}$ (3)

$$F_z = 2\pi R L(\tau_w) = \pi R^2 \Delta P \quad (4)$$

**The stress at the wall** is related to the force exerted by the fluid on the barrel's wall . Such force is the same as that exerted by the plunger which is related to the pressure head.

But, how can you relate that to the viscosity of the material and to the volumetric flow ?… Some equations will do it for you…

# A momentum balance

gives:

$$\frac{d(r\tau_{rz})}{dr} = \left(\frac{P_o - P_L + \rho g_z}{L}\right) r \qquad (5)$$

$$\int d(r\tau_{rz}) = \left(\frac{P_o - P_L + \rho g_z}{L}\right) \int r\,dr \qquad (6)$$

$$\tau_{rz} = \frac{\Delta P_g}{2L} r + \frac{C_1}{r} \qquad (7)$$

Since $\tau_{rz}$ can not be infinite, the value of C1 must be zero and then...

$$\tau_{rz} = \frac{\Delta P_g}{2L} r \qquad (8)$$

## For a Newtonian Fluid

$$\tau_{rz} = -\eta\,\dot{\gamma} \qquad (9)$$

and since:

$$\dot{\gamma} = \frac{dVz}{dr} \qquad (10)$$

and by substitution of equation 10 in 9

$$\tau_{rz} = -\eta \frac{dVz}{dr} = \frac{\Delta P_g}{2L} r \qquad (11)$$

then the stress at the wall can be calculated at r= ?

$$\tau_w = \frac{\Delta P_g}{2L} R \qquad (12)$$

## We can manipulate equation 11 to look for an expression for the flow rate (Q=Average Velocity x Cross sectional area)

$$-\eta \frac{dVz}{dr} = \frac{\Delta P_g}{2L} r \qquad (13)$$

$$dVz = -\frac{\Delta P_g}{2L\eta} r\, dr \qquad (14)$$

$$\int dVz = \int \frac{\Delta P_g}{2L\eta} r\, dr \qquad (15)$$

**And by integrating we get equation 16:**

$$Vz = -\frac{\Delta P_{Lg}}{4\eta L} r^2 + C_2 \qquad (16)$$

## And the integration constant can be obtained by evaluating the border conditions: BC1: Vz=0, when r=R

BC2: Vz=max, when r=0

$$Vz = -\frac{\Delta P_g}{4\eta L} R^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \qquad (17)$$

$$Vz_{max} = -\frac{\Delta P_g}{4\eta L} R^2 \qquad (18)$$

However, the maximum velocity is a very crude way to calculate the flow rate, since the flow pattern is parabolic for a newtonian fluid, therefore we need to look for an weighted average velocity using the following equation:

$$\langle Vz \rangle = \frac{\int_0^{2\pi} \int_0^{R} V_z \, r \, dr \, d\theta}{\int_0^{2\pi} \int_0^{R} r \, dr \, d\theta} = -\frac{\Delta P_g}{8\eta L} R^2 \qquad (19)$$

The average velocity can be multiplied by the cross section area and

$$Q = \langle Vz \rangle \pi R^2 \qquad (20)$$

$$Q = \pi \frac{\Delta P_g}{8\eta L} R^4 \qquad (21)$$

By rearranging equation 21,

$$\frac{4Q}{\pi R^3} = \frac{\Delta P_g}{2\eta L} R \qquad (22)$$

Observe that the RHS of equation 22 except by eta, is the same as equation 12, and also that the units in this equation are equal to 1/s

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**therefore:**

$$\frac{4Q}{\pi R^3} = \frac{\tau_w}{\eta} \qquad (23)$$

$$\tau_w = \frac{4Q}{\pi R^3}\eta \qquad (24)$$

**let´s:**

$$\Gamma = \frac{4Q}{\pi R^3} \qquad (25)$$

Where $\Gamma$ is called the apparent shear rate

**then:**

$$\tau_w = \eta\,\Gamma \qquad (26)$$

Where $t_w$ the shear stress at the wall (which is $\tau_{rz}$ at $r=R$)

$$\eta_A = \frac{\tau_w}{\Gamma} \qquad (27)$$

Where $\eta_A$ is the apparent shear viscosity

Remember that this treatment is for a Newtonian fluid but at the shear rates at which we measured the polymers in the capillary rheometer, the flow is non-Newtonian (as it is in many polymer processing conditions)

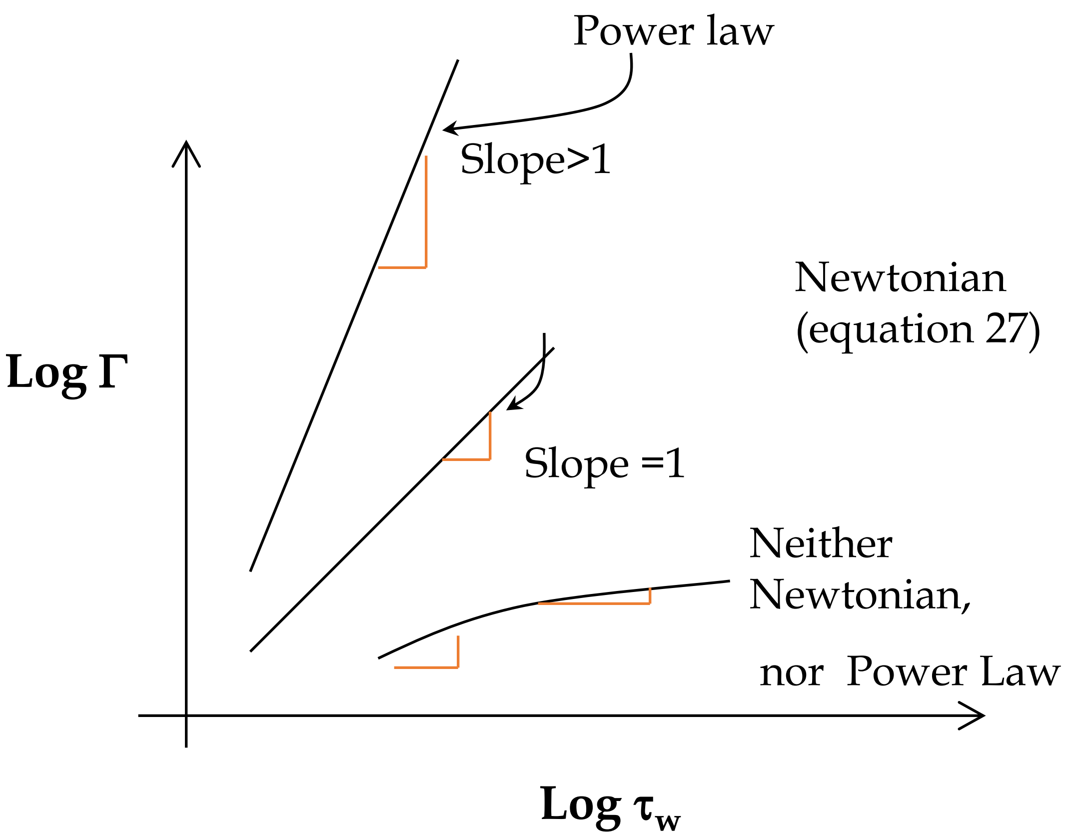
THEN

**what can be done to get the data corrected?**

**we need to use**

# The Rabinowitch correction

Log of $\Gamma$ is plotted against the log of $\Delta P\,R/\,2L$ then:



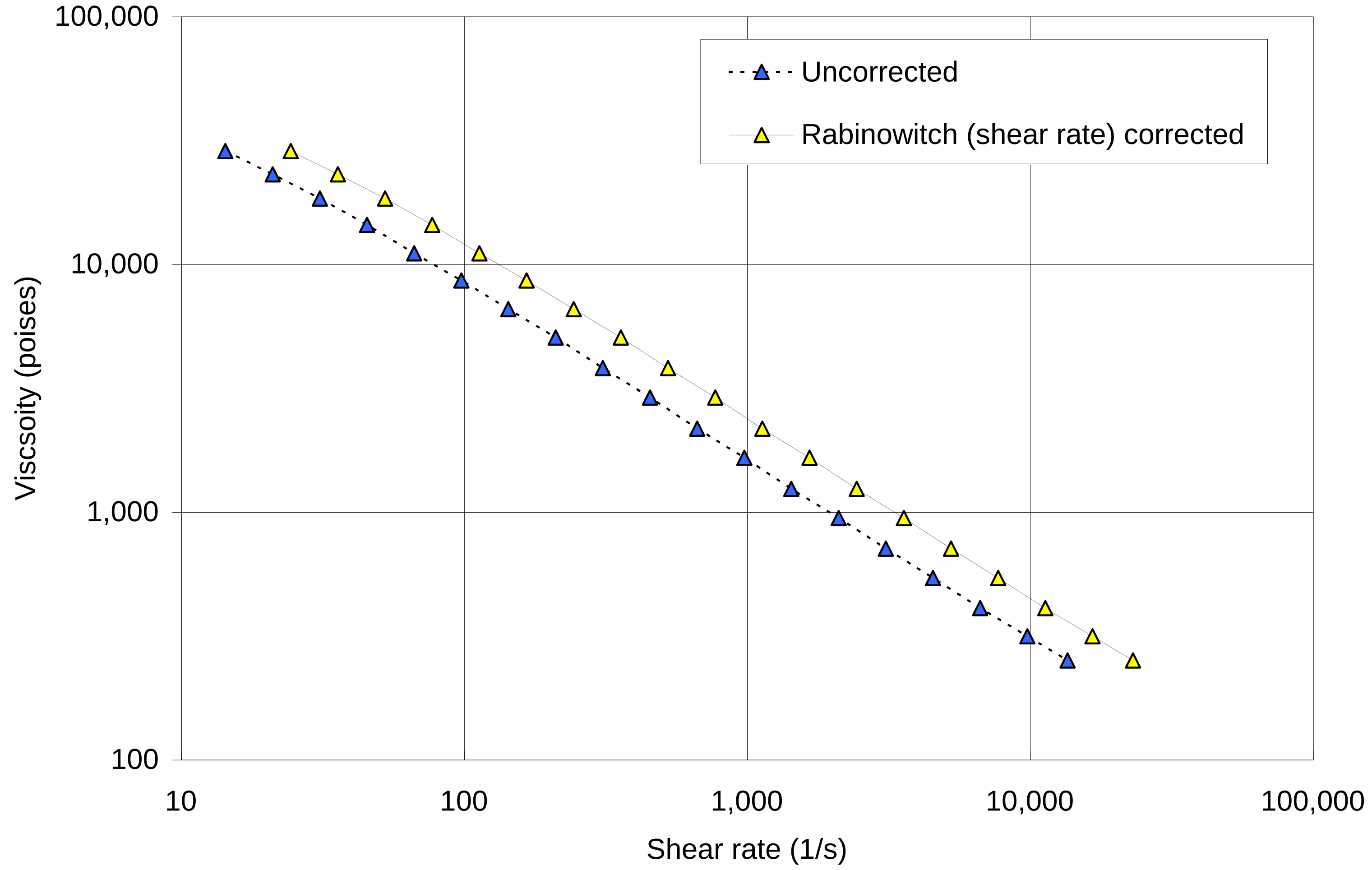

The shear rate at the wall is

$$\left.\frac{dVz}{dr}\right|_{r=R} = \dot{\gamma}_w$$

and is related to the apparent shear rate, $\Gamma$, by the following equation:

$$\dot{\gamma}_w = \left(\frac{3+b}{4}\right)\Gamma$$

# Rabinowitch  for a PS 585 at 2040C

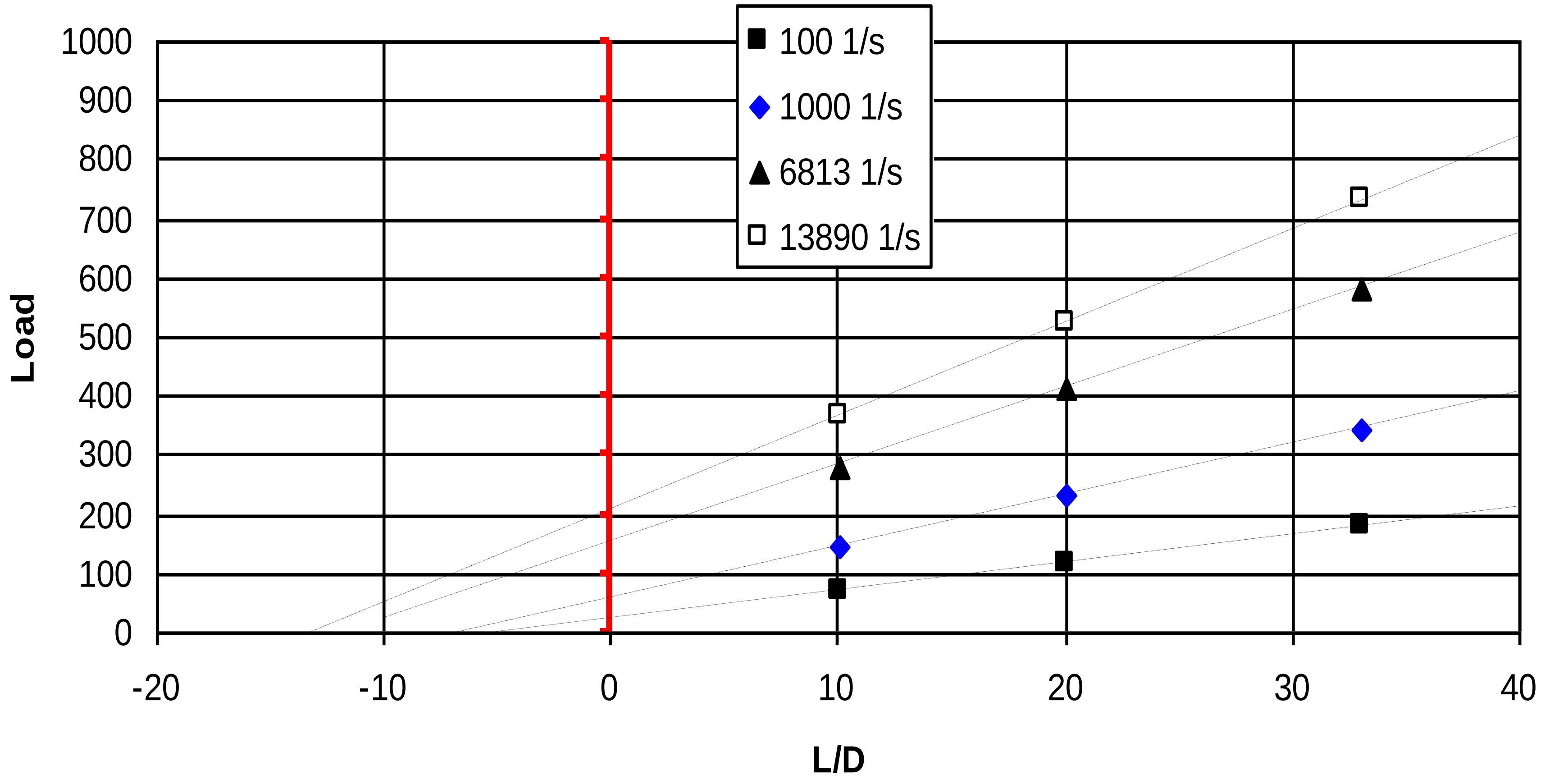


# But…

This is just a correction for the shear rate.  Another correction is needed because the pressure used in the math presented in the previous pages, assumes that the pressure is that due just to viscous drag while the TOTAL PRESSURE **REGISTERED** BY THE INSTRUMENT IS FOR BOTH the viscous and the ___________________ of the polymer.

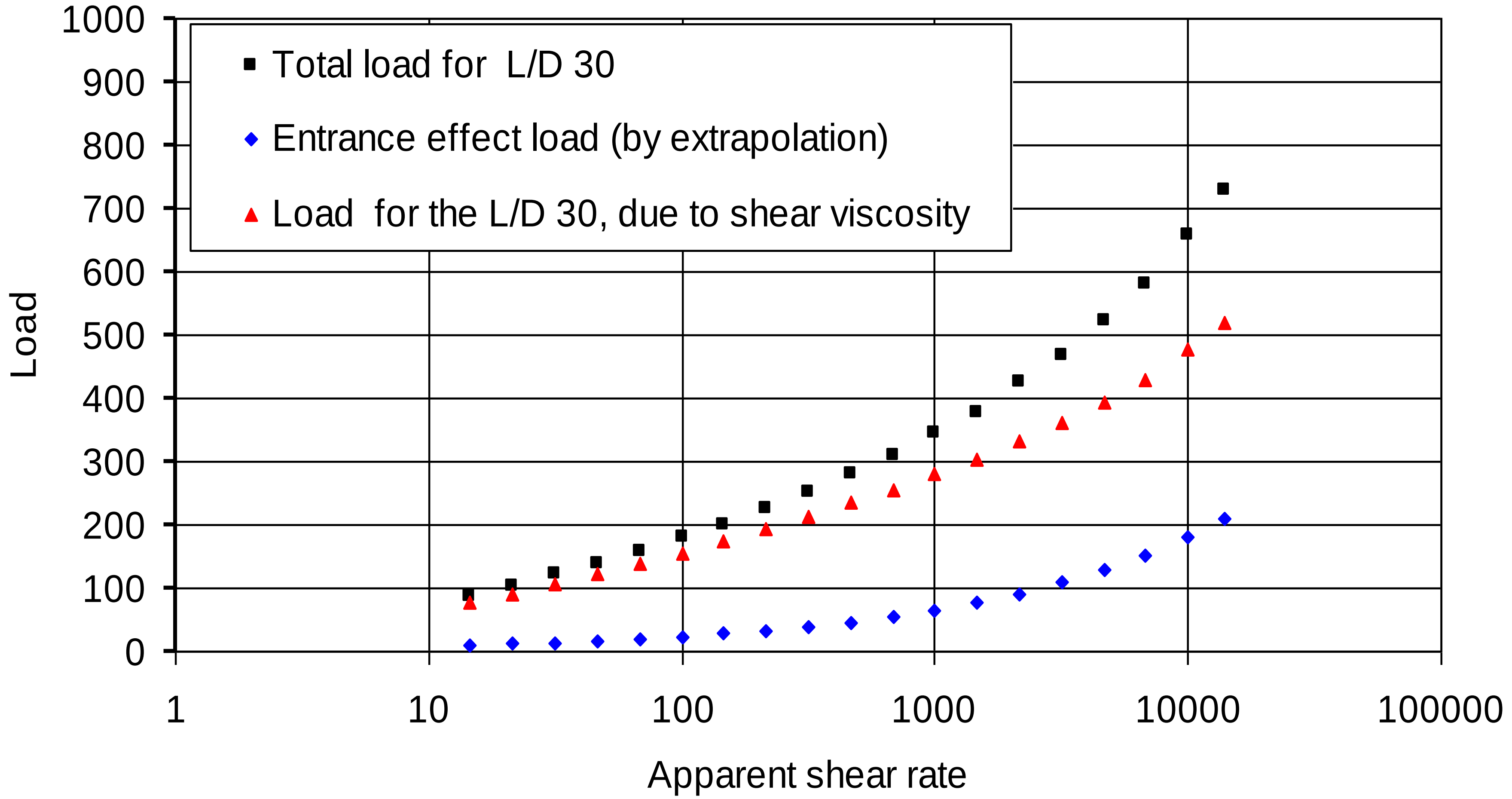
The correction needed is called the…

# Bagley Correction:
## getting pressure entrance effects by extrapolation

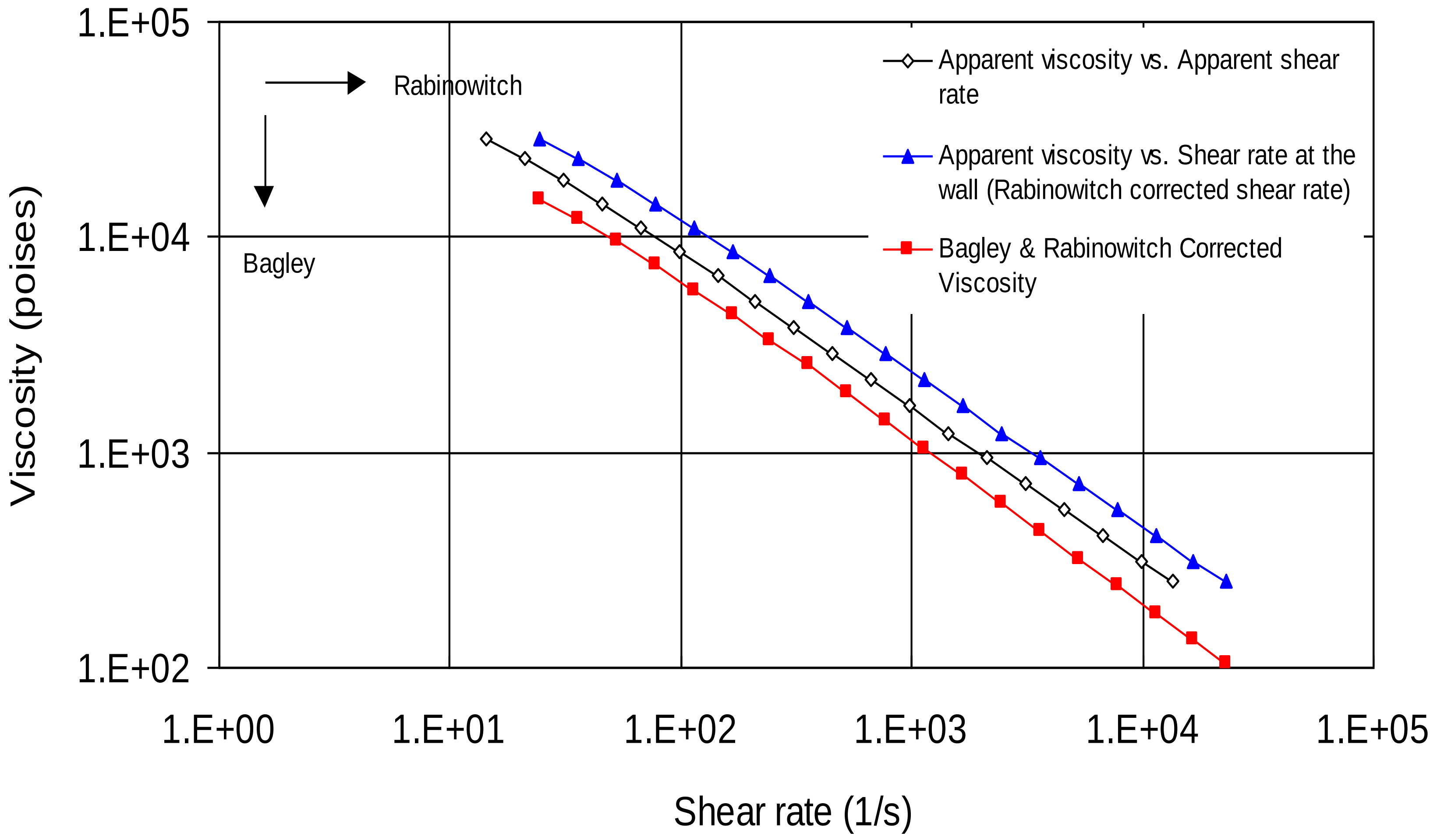


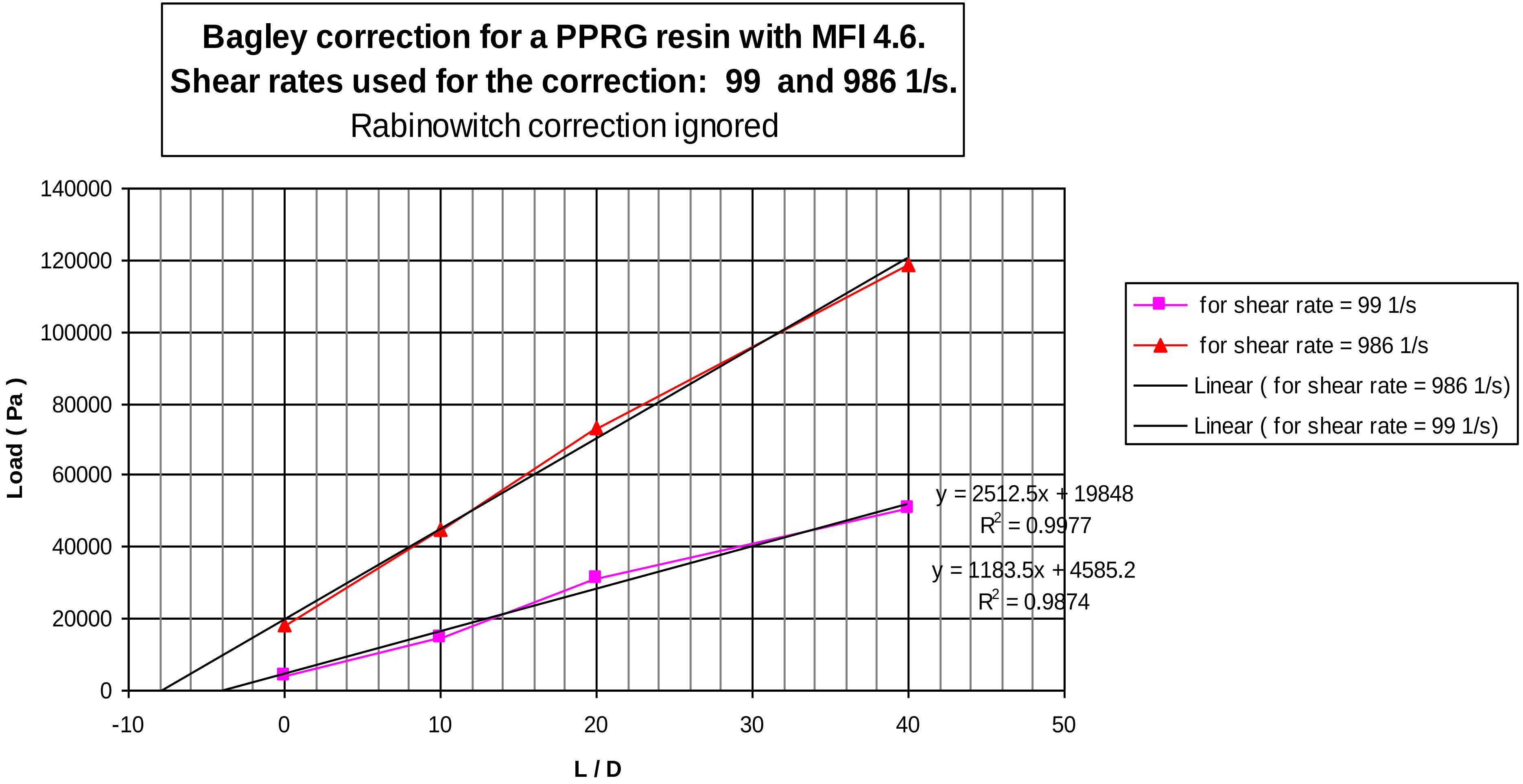

*Load versus L/D for various shear rates for the PS 585 resin at 204°C*

# Load ($lb_f$) versus apparent shear rate (1/s)



# Rabinowitch and Bagley corrected viscosity




Bagley correction for a PPRG resin with MFI 4.6.
Shear rates used for the correction: 99 and 986 1/s.
Rabinowitch correction ignored
Load ( Pa )
140000
120000
100000
80000
60000
40000
20000
0
-10
0
10
20
30
40
50
L / D
y = 2512.5x + 19848
R2 = 0.9977
y = 1183.5x + 4585.2
R2 = 0.9874
for shear rate = 99 1/s
for shear rate = 986 1/s
Linear ( for shear rate = 986 1/s)
Linear ( for shear rate = 99 1/s)

# Please, answer the following questions:

The apparent viscosity ($\Gamma$) is obtained by multiplying the volumetric flow Q by:
Suppose you work all day doing some viscosity tests in the Instron and then you realized that while the instrument is set up for a L/D =40 with a 0.05 in diameter, unadvertedly you use a L/D =40 with a 0.03 in diameter.

**How would that affect your results?**

**What would you do?**

**Would you throw the results away and start all over again?**

**What other use you can think of for the entrance pressure data?**

# Review

Suppose the software in the capillary rheometer computer crashes and the only thing you can read is load and piston speed velocity.

At that moment  a customer has a problem and it requires capillary viscosity data. What would you do to report the viscosity with the information you get from the Instrument?

Summarize the methodology

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