TEAM **5**

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Assessment 2- Part 2 (Team 5)

Due any time before 17:00 hrs on July 13th, 2020

Instructions

This assessment will require to work again in the first part (the one you sent to me early on) but as a team. Besides, in the next page there are two problems requiring calculations

Instructions for working again in the Assessment 2-Part 1

1) **You should get together with your teammates to solve as a group, the questions you answered in the Assessment 2-Part 1 and have to write down for each question the following:**
   A. *Rephrase the problem indicating very clearly what you have been asked to do.*
   B. *List all the data provided.*
   C. *Make a list of the assumptions. justifying each of them.*
   D. *Write down an algorithm for the solution you are proposing (no calculations are needed at this stage)*
   E. *Answer the question*
   F. *if needed check in the web for technical papers to support your answer.*
   G. *List the references used in the solution of the problem.*

Question 1

1. Your company has been using polypropylene for injection molding and "suddenly" the part you were injecting presented a flash problem. List at least three possible causes and how you could correct the problem. (5pts)
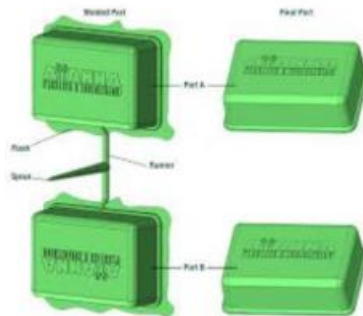
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Rephrase the problem

We are asked to mention three potential causes for a flash defect in the parts, and their possible solutions.
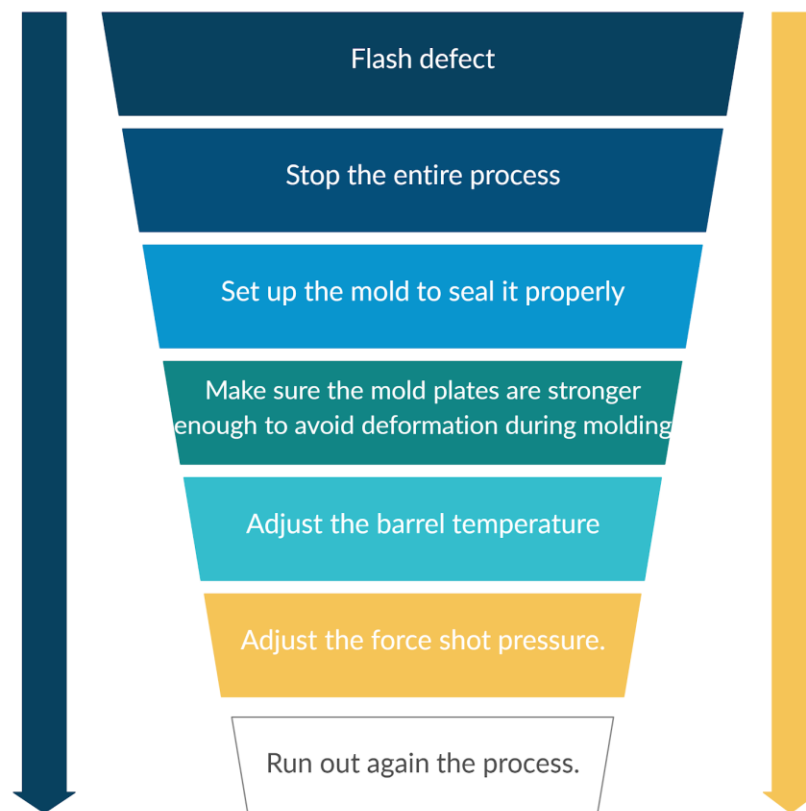
List all the data provided

- Problem: Flash defect
- Polymer: Polypropylene
- Process: Injection molding

Make a list of the assumptions. justifying each of them

3 possible causes could be errors in the injection molding operation.

1. The pressure or temperature of the injection is higher than needed: higher pressure and temperature in the injection shot allows an easier flow in the mold, causing the problem.
2. Gap within the mold: Improper molding conditions could also cause the flash defect.
   a. Not having enough clamping force in the mold allows an excess of material in the part.
3. Other causes: higher temperature than needed in the mold, higher processing time, improper mold design, the mold might be old and have cavities.

Write down an algorithm for the solution you are proposing

Flash defect

Stop the entire process

Set up the mold to seal it properly

Make sure the mold plates are stronger enough to avoid deformation during molding

Adjust the barrel temperature

Adjust the force shot pressure.

Run out again the process.

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Answer the question

In general, the problem needs an analysis for the entire process, while focusing on the following:

- Adjustment of the screw temperature and the injection pressure to reduce the injection force and the polymer flow speed into the mold.
- Increasing the clamping force 10-20% to ensure the mold is not going to leave any gaps while receiving the injection pressure, without affecting the mold.

References

Rosato, D. V., & Rosato, M. G. (2012). Injection molding handbook. Springer Science & Business Media.

Question 2

2. There has been an explosion in a residential area affecting a dozen of houses and some people got injured. The explosion occurred due to a gas leak from a high density polyethylene (HDPE) pipe used for residential natural gas distribution. You have been asked to act as a *polymer expert* and the forensic evidence you found was that the pipe has a 10 meters long rupture parallel to the length of the pipe. What could you say to the judge about the probable cause rupture? (5 pts)
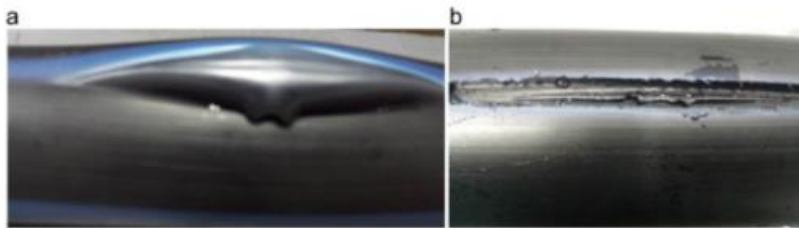
Figure from: HDPE pipes failure analysis and damage modeling. Engineering Failure Analysis Volume 71, January 2017, Pages 157-165

Rephrase the problem

Ten meters long rupture of a high density polyethylene pipe by natural gas distribution. What could we say about the possible cause?

List all the data provided

- Problem: The explosion occurred due to a gas leak from HDPE pipe used for residential natural gas distribution.
- Polymer: High density Polyethylene
- Uses: Pipes for residential natural gas distribution.
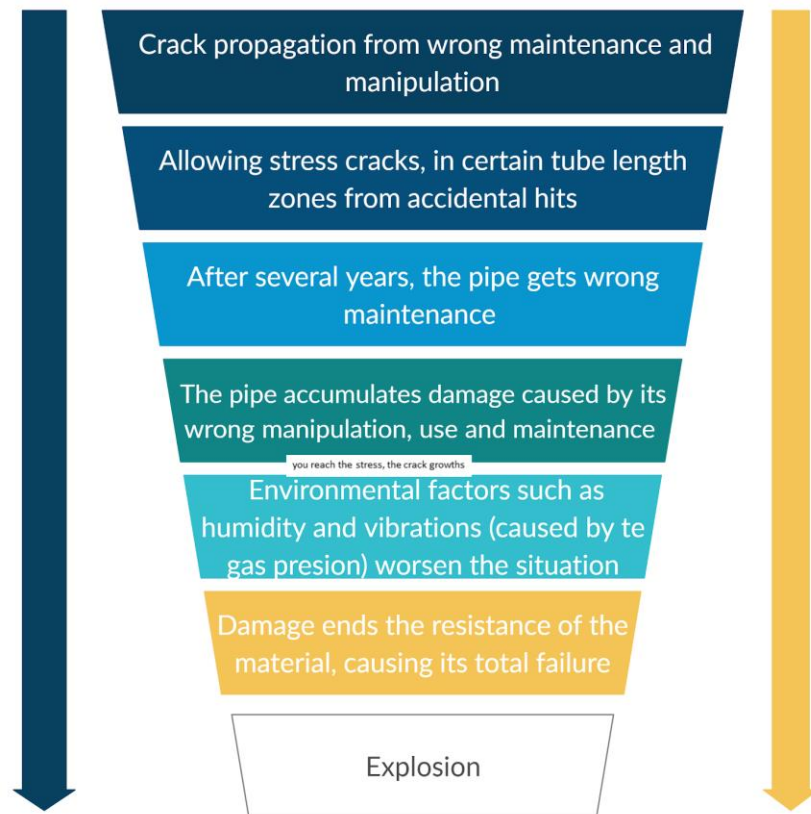- Other: The pipe has a 10 meters long rupture parallel to its length.

Make a list of the assumptions. justifying each of them

1. Damage during maintenance and manipulation:
   a. The material was correctly manufactured but incorrectly manipulated: allowing stress cracks, in certain tube length zones from accidental hits.
   b. Wrong maintenance: Assuming that the pipe was used normally after certain period of time, the leak occurred at a location where the maintenance was done.
2. Damage during pipe's manufacture:
   a. An inappropriate mixture of the polymer during the process.
      i. Pellets with different sizes: It produces a mix between melted and solid polymer, allowing some regions weaker than others.
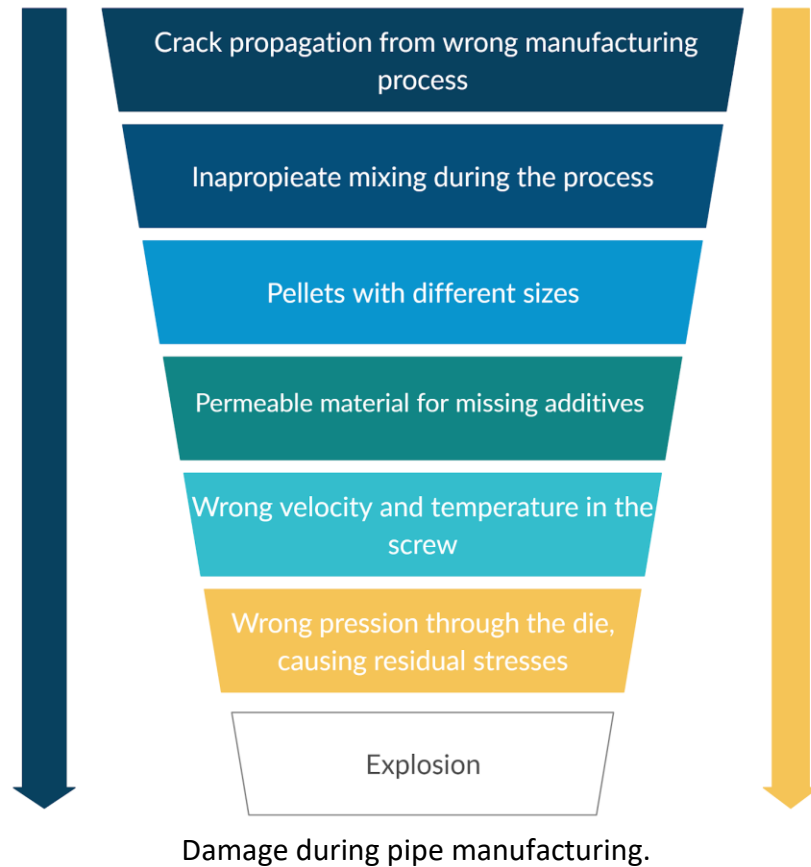   ii. Additives missing: Allows material to become permeable under severe environmental and operating conditions.

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   iii. Wrong velocity and temperature in the screw: Similar to pellets with different sizes; unbalance mixing and melted times produce defects in the final product.

  b. The way the material is pushed into the die.

   i. Temperature, pression and velocity affect the polymer through the die.

Write down an algorithm for the solution you are proposing

Crack propagation from wrong maintenance and manipulation

Allowing stress cracks, in certain tube length zones from accidental hits

After several years, the pipe gets wrong maintenance

The pipe accumulates damage caused by its wrong manipulation, use and maintenance

you reach the stress, the crack growths

Environmental factors such as humidity and vibrations (caused by te gas presion) worsen the situation

Damage ends the resistance of the material, causing its total failure

Explosion

Damage during maintenance and handling.

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Damage during pipe manufacturing.

Answer the question

There are two possible major reasons:

1. From the maintenance and handling perspective:
   a. Wrong handling and maintenance cause damage in areas where there was no damage, producing microstructural cracks that later get worse with the gas pressure. Also, wear due to pressure flow, and environmental factors such as humidity and vibrations make the situation worse. Eventually, damage ends the resistance of the material, causing the explosion.
2. From the production perspective:
   a. There was an improper mixture of the polymer, creating clusters in the semi-crystalline structure of the material, which when extruded, leave some regions weaker than others. This causes the polymeric chain alignment to go in the same direction as the pressure of the water because of the extrusion, thus breaking the pipe in a longitudinal manner. Also, uneven cooling during extrusion may have left residual stresses that with time and usage become cracks.

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References

Jones Jr, R. E., & Bradley, W. L. (1993). Failure of a Squeeze-Clamped Polyethylene Natural Gas Pipeline. ASM International, Handbook of Case Histories in Failure Analysis., 2, 482-486.
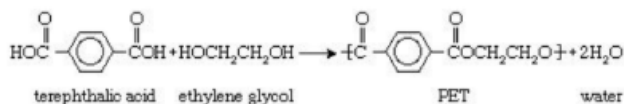
Naebe, M., Abolhasani, M. M., Khayyam, H., Amini, A., & Fox, B. (2016). Crack damage in polymers and composites: A review. Polymer reviews, 56(1), 31-69.

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Question 3

3. You started a business for recycling polyethylene terephthalate (PET) coming from drinking water bottles. The company cut the plastic into flakes, wash the plastic in a mild sodium hydroxide solution to get rid of the glue used in the labels. Afterwards they are washed with water, dried and pelletized. However, the MFI of the final resin has increased a lot. What do you think this is happening and how could you prevent it from happening? (5 pts)

   Note: The formula for the PET is the one presented in the right side of the chemical reaction that occurs when PET is produced:

$$HOC\text{-}C_6H_4\text{-}COH + HOCH_2CH_2OH \longrightarrow [C\text{-}C_6H_4\text{-}COCH_2CH_2O] + 2H_2O$$

terephthalic acid   ethylene glycol   PET   water

Rephrase the problem

PET water bottles are being recycled from different sources, then cut into flakes, washed and turned into pellets. New water bottles are being made from the resulting pellets but the melt-flow index is too high, meaning that the viscosity has decreased.
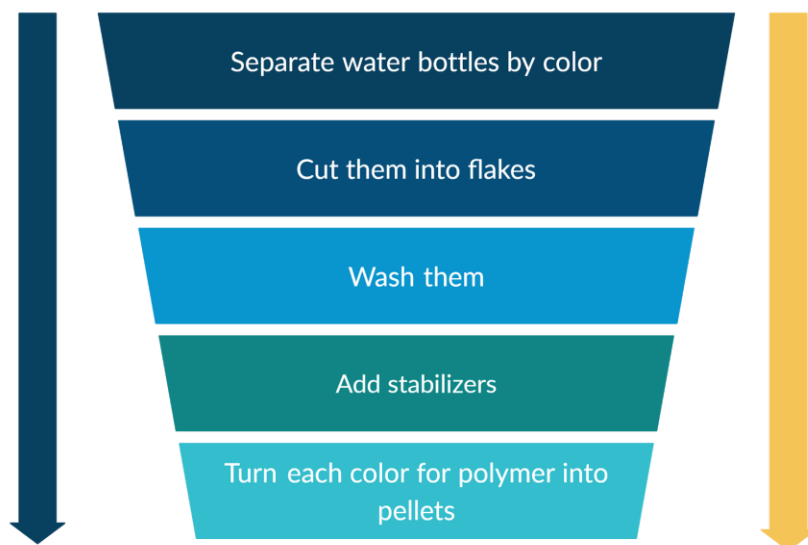
List all the data provided

- Problem: High melt flow index
- Material: Polyethylene terephthalate
- Source: Recycled water bottles

Make a list of the assumptions. justifying each of them

1. The resulting resin is being used to make new water bottles because they are recycling PET exclusively from drinking water bottles.

2. Manufacturing conditions are properly applied, because otherwise there would be other defects present in the resulting water bottles.

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Write down an algorithm for the solution you are proposing

Separate water bottles by color

Cut them into flakes

Wash them

Add stabilizers

Turn each color for polymer into pellets

Answer the question

Due to the recycled resin coming from different manufacturers, each type of water bottle has their own additives, concentrations, and treatments. Mixing them into one resin may cause it to have unexpected results in terms of properties and probably branching copolymerization.

Our solution proposal is to separate water bottles by colors and make resins of each kind while adding stabilizers like new PET (25% new PET with 75% recycle PET) to avoid having unexpected reactions.

References
Koltzenburg, S., Maskos, M., & Nuyken, O. (2017). Polymer Chemistry. Berlin, Germany: Springer. https://doi.org/10.1007/978-3-662-49279-6

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Question 4

4. Your company is injecting a plastic part like the one given below (black marks indicated by the arrows), and you are asked to solve the problem. What would you do? (5 pts)

Rephrase the problem
The plastic part injected is having black marks in the edges (corners) and needs to be solved.

List all the data provided
- Problem: Black marks.
- Process: Injection molding.
- Material: Plastic.

Make a list of the assumptions. justifying each of them
1. The part is made of a thermoset polymer.
2. The black marks are due to a Diesel effect.
3. All processing conditions are well established.
4. There is an air venting problem.

Write down an algorithm for the solution you are proposing
```
If vents are clogged {
    Clean vents
} else {
    Redesign the vent within the die
}
Install vacuum system {aid the air to escape the mold}
```

Answer the question
As it was mentioned in the assumptions, the problem is most likely to air and other gases not escaping properly due to vents being clogged. When injecting polymer into a mould, air and other gases need to escape the mould through vents. If the air/gas does not escape fast enough, pressure builds up inside the mould. Polymers under pressurized conditions heat up and the "diesel effect" takes place, carbonizing some of the polymer. The result is charcoal-like edges in the final product. The solution would be to redesign/clean the air vent and slow the injection

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process to allow proper air circulation, and if possible, add a vacuum system to extract the air from the mold, which with time would be less expensive than slowing down the process.

References

Plastics Technology. How to Fix Outgassing Problems in Injection Molding (2018)
https://www.ptonline.com/articles/how-to-fix-outgassing-problems-in-injection-molding#:~:text=Gas%20and%20air%20entrapment%20can,output%20in%20the%20molding%20process.

Injection molding presentation by Dr. Jaime Bonilla Ríos

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Question 5

5. You have been interviewed by a company producing polypropylene and they are asking you what could happen If they add peroxide while they extruded the polymer to produce pellets that will be sold for the production of fibers. What would be your answer? (5 pts)
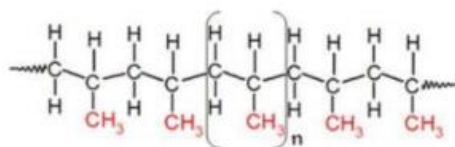
Rephrase the problem

A company wants to add peroxide to polypropylene while extruding it, to produce pellets for fibers. What will happen if they do this?
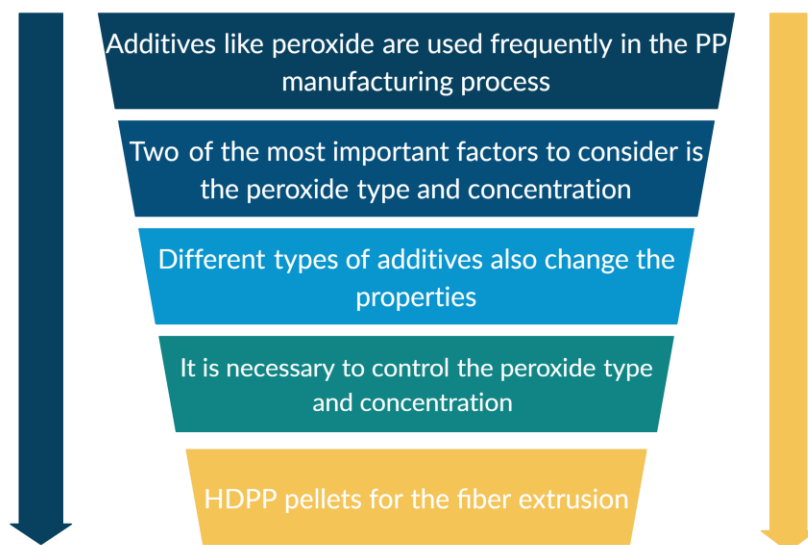
List all the data provided

- Process: Extrusion.
- Material: Polypropylene.
- Additive: Peroxide.
- Purpose: Pellet production for fibers.

Make a list of the assumptions. justifying each of them

- The modification of PP's molecular architecture with peroxides is focused on obtaining a more adequate rheological response for those processes that require high resistance to melt. In few words, they want to improve the stability of the polymer for future fiber's manufacturing, by adding peroxides.
- There are different types of peroxides, we will assume we are using the most compatible type for polypropylene.

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Write down an algorithm for the solution you are proposing

Additives like peroxide are used frequently in the PP manufacturing process

Two of the most important factors to consider is the peroxide type and concentration

Different types of additives also change the properties

It is necessary to control the peroxide type and concentration

HDPP pellets for the fiber extrusion

Answer the question

Free radical polymerization will take place. Peroxides are organic radical sources. During free radical polymerization, a radical (created from the decay of peroxides) adds to the double carbon to carbon bonds of a monomer, resulting in a new radical extending from the monomer unit. The result would be polymeric pellets with a higher molecular weight.

Nevertheless, one of the most important factors to consider is the peroxide concentration:

- An excessive amount in it could allow the breaking of PP chains instead of causing branching reactions, thus leading to a decrease in the weight average molecular weight of PP and also, changing the desired mechanical properties.
- At low peroxide concentrations, coupling reactions are observed, which can give non-desired properties.

References

Polyvel. Peroxide Rheology (Melt Flow) Modifiers.
https://www.polyvel.com/products/peroxides/

SALAZAR, M. L. G. (2014). Modificación estructural de polipropileno empleando iniciadores cíclicos multifuncionales triperóxido de dietilcetona (TPDEC) y diperóxido de pinacolona (DPP), en conjunto con extensores de cadena del tipo acrilato.

Kim, B. K., & Kim, K. J. (1993). Cross-Linking of polypropylene by peroxide and multifunctional monomer during reactive extrusion. Advances in Polymer Technology: Journal of the Polymer Processing Institute, 12(3), 263-269

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Question 6

6. You have a **blown film** business and you use HDPE resins. A vendor wants to become your supplier and ask you to consider his HDPE resin. He claims that the resin he is trying to sell to you has the same MFI and the same shear viscosity curve as the one you are using. The price of the resin is 15% cheaper than the one you are using. Would you buy the resin?

   a. In order to answer this question you need to justify it from the processing as well as from the rheological point of view

Rephrase the problem

We use HDPE resins in a blown film industry. A vendor is offering us a resin with, allegedly, the same MFI and same shear viscosity curve as the one we have, but 15% cheaper. Would we buy the resin? We should justify the answer from the processing and rheological point of view.
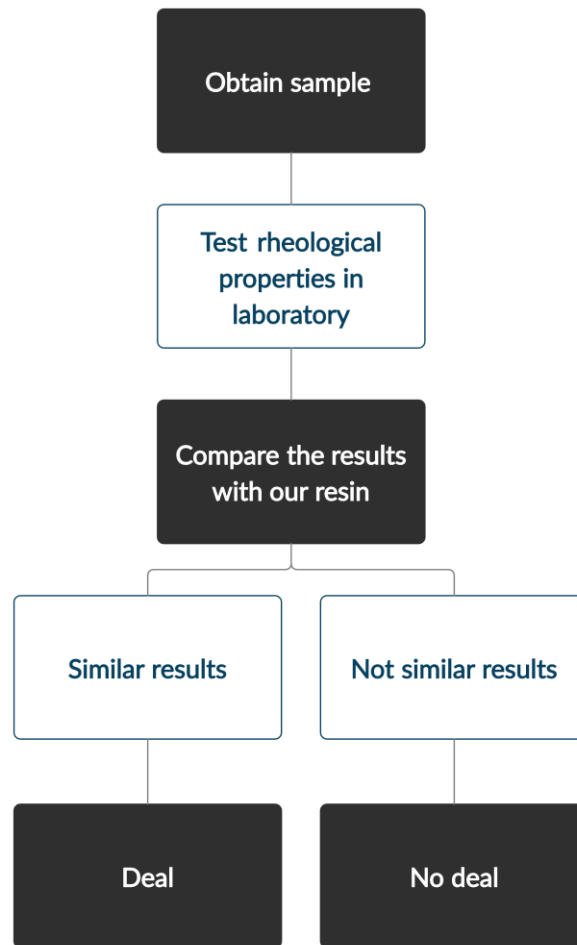
List all the data provided

- Process: Blown film.
- Material: HDPE resin.
- Material characteristics: "Same resin, MFI, shear viscosity curve", and cheaper.

Make a list of the assumptions. justifying each of them

1. Our enterprise has the necessary equipment to test a sample of the resin, just for proving its properties and compare them before doing the purchase.
2. It doesn't matter if the vendor is already known in the polymer's market for quality products, or not.
3. Resin has the same shear viscosity curve and same MFI of our resin.

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Write down an algorithm for the solution you are proposing

```
         ┌──────────────────┐
         │   Obtain sample  │
         └──────────────────┘
                  │
         ┌──────────────────┐
         │  Test rheological│
         │   properties in  │
         │    laboratory    │
         └──────────────────┘
                  │
         ┌──────────────────┐
         │ Compare the results│
         │   with our resin │
         └──────────────────┘
             │          │
   ┌──────────────┐  ┌──────────────────┐
   │Similar results│  │Not similar results│
   └──────────────┘  └──────────────────┘
          │                   │
   ┌──────────────┐  ┌──────────────────┐
   │    Deal      │  │     No deal      │
   └──────────────┘  └──────────────────┘
```

Answer the question

We would request a sample from him to run some rheology tests to ensure that his claims are true. If indeed they are true, we would buy his resin to reduce production costs while maintaining the same properties in our products.

Blown film processes are used to manufacture very thin films. It is not possible to make the profile of the die as thin as needed without the back pressure becoming too high, these thin films cannot be extruded directly, in such a way that the rheological properties of the melt play an important role. As soon as the film leaves the die it is inflated by pressurized air. Depending on the speed of the extrusion and the pressure of the air flow, the film becomes thinner. The tubular film is then cooled while maintaining a blown form. The viscosity of the polymer melt dictates the size of the die and therefore the features of the final product, in this case it is imperative to ensure the MFI is the same, thus, having the same rheological properties as our resin.

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References

Plastics Technologies. A Film Processor's Guide to Understanding Materials & Equipment. (2016) https://www.ptonline.com/articles/a-film-processors-guide-to-understanding-materials-equipment

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Question 7

7. The company you are working on decides to switch resins and instead of using a Polypropylene resin they decide to go for a Polyester resin. What type of recommendations, if any, would you give to insure that the product will keep meeting the quality control specifications such as mechanical properties.

Rephrase the problem

The company wants to switch resins from polypropylene to polyester. What type of recommendations would we give them to ensure quality control of the product, such as mechanical properties?

List all the data provided

- Material: From polypropylene to polyester
- Problem: Ensure mechanical properties

Make a list of the assumptions. justifying each of them

Despite the difference in properties, the use of both resins can be justified for similar products with different mechanical properties, specifications and prices. Nevertheless, it is necessary to take the next considerations:

1. The company has the necessary resources and laboratory equipment.
2. Switching the resin to produce a different product:
   a. If the manufacture process is already adapted for polypropylene, it would be necessary to adapt it again for polyester, too (velocity, temperature and type of screw, pression, cooling system, additives, mold, die, etc.). The properties of the desire product also will change, but changes ensure that the quality is going to be maintained.
3. Switching the resin to produce the same product:
   a. If the above is not possible, it would be necessary to modify the rheological properties of the polyester resin (by using additives) to adapt it to the process and the desired properties of the final product.

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Write down an algorithm for the solution you are proposing
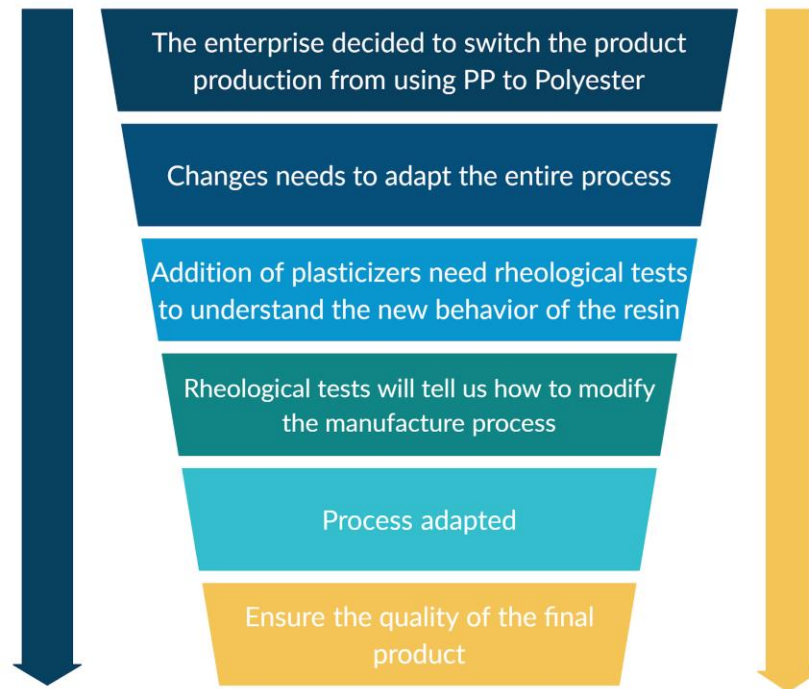
Answer the question

The use of additives to adapt the process will change the rheological (Tg, Mw, MFI and DPI) and mechanical properties of polyester making them similar to those of polypropylene. Understanding their new properties will be critical to ensure the quality of the final product while adjusting the manufacturing process to match the new rheological and mechanical properties.

References

No references needed for this module.

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Question 8

8. You have a company that produces a polymer pipe and one of your clients is complaining about the diameter of the product being out of specifications. This customer is very important since 40% of your sells go to that company.
   a. What would you do to get the product back into specification without compromising the pipe high pressure specifications? Otherwise you could be sued by the final user, if a gas leak explosion, God forbids, occurs.

Rephrase the problem

The company produces polymer pipes and a client is not satisfied with the diameter, claiming that is out of specifications. 40% of the sells go to that client, what can we do to ensure the product is back into specification without compromising high pressure specifications?
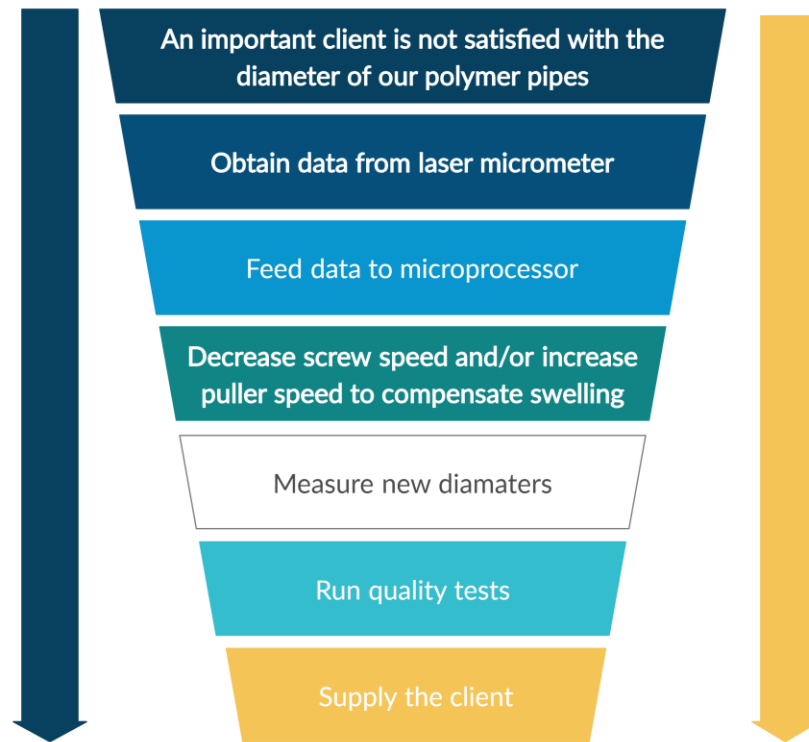
List all the data provided

- Application: Pipes
- Problem: Diameter out of specifications
- Ensure high pressure specifications

Make a list of the assumptions. justifying each of them

1. The pipes are made through extrusion.
2. The diameter is bigger than specified.
3. The polymer is swelling after extrusion being the most probable cause for the unmet specifications.
4. The pipes are made through extrusion.
5. We have a laser micrometre that feeds a signal to the microprocessor which adjusts the screw or puller speeds.

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Write down an algorithm for the solution you are proposing

An important client is not satisfied with the diameter of our polymer pipes

Obtain data from laser micrometer

Feed data to microprocessor

Decrease screw speed and/or increase puller speed to compensate swelling

Measure new diamaters

Run quality tests

Supply the client

Answer the question

We proposed a cost-effective solution, instead of changing the die (which is very expensive), we will adjust the screw and puller speeds to compensate for the diameter changes. If the diameter is too large the extruder screw speed has to decrease, or the puller speed has to increase. A balance between pulling speed and screw speed needs to be optimized to achieve the desired pipe diameters without overheating the screw or an over alignment of crystalline structures within the polymer.

References

Giles Jr, H. F., Mount III, E. M., & Wagner Jr, J. R. (2004). Extrusion: the definitive processing guide and handbook. William Andrew.

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Question 9

9. Order the resins,
   a. From the highest to the lowest molecular weight and
   b. From the highest to the less shear thinning

Resin	Zero Shear Viscosity (Pa.s)	Mw	PDI	shear thinning
A	1.8×10^5		2	
B	2.1×10^6		4	
C	3.1×10^4		14	
D	8.9×10^3		6	
E	2.5×10^4		8	

Rephrase the problem
Order the resins from highest to lowest molecular weight and from highest to less shear thinning.
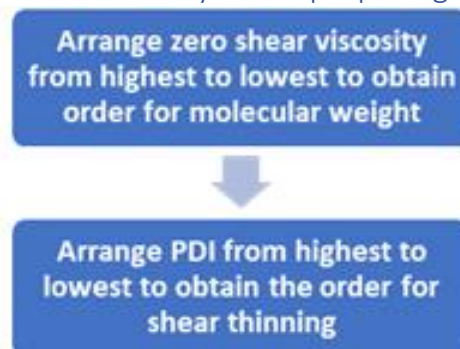
List all the data provided
- Zero shear viscosity
- Polydispersity Index

Make a list of the assumptions. justifying each of them
- If the PDI is higher then, the shear-thinning behavior is higher, because there is more variation of chain lengths.

Write down an algorithm for the solution you are proposing

Arrange zero shear viscosity from highest to lowest to obtain order for molecular weight

↓

Arrange PDI from highest to lowest to obtain the order for shear thinning

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Answer the question

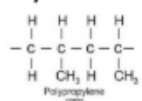| Resin	Zero Shear Viscosity (Pa.s)	Mw	PDI	shear thinning
A	1.8×10^5	2	2	5
B	2.1×10^6	1	4	4
C	3.1×10^4	3	14	1
D	8.9×10^3	5	6	3
E	2.5×10^4	4	8	2

References
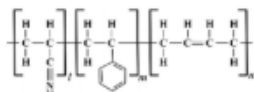Chemical nature of polymers presentation by Dr. Jaime Bonilla-Ríos

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Question 10

10. If you recycle Polypropylene (PP), Acryl-Butadiene-Styrene (ABS) and Polyethylene(PE)

PP

ABS

PE

a. what type of problems you might run into,
b. how would you solve them?

Rephrase the problem

We are recycling PP, ABS, and PE. What problems could we encounter and how would we solve them?

List all the data provided

- Source: Recycled materials
- Material: PP, ABS, PE
- Problem: MFI increase

Make a list of the assumptions. justifying each of them

1. We are recycling these polymers together, even though each one has different ways for being recycled.
2. It is not necessary to recycle the same quantity of them at the same time; we could vary the percentage.
3. We have the necessary equipment and additives to force the polymer blend.

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Write down an algorithm for the solution you are proposing

We are recycling PP, ABS, and PE

Due to the differences between them we'll have problems on the compatibility

It is necessary to force the union

Mechanical pretreatment for those polymers with a high number on the recycling scale, for ensuring de mixing

Varying the amount of each of them to improve mixing efficiency, like follows: PE > PP > ABS

Sufficient amount of plasticizers, as well as temperature and mixing force needed to keep the mixture stable

Mix achieved efficiently

Answer the question

The nature of each polymer in recyclability scale is as follows:

Polyethylene (No. 2): is the only polymer in our blend that can be recycled easily and does not represent a problematic process.

Polypropylene (No. 5): has a low rate of recyclability because there are not many municipal recycling centers that accept them. For recycle it, is necessary to have special equipment. The best option is to reuse them as many times as possible before disposing of them.

ABS (No. 7): is made from a variety of resins. Unfortunately, those mixtures make these products undesirable for recycling and forces number 7 and codeless items to go directly to the landfill. We will recycle small quantities of ABS because of the complexity it represents for recycling. Consumers can return these items to the manufacturers and reuse SAN and ABS plastics to reduce landfill waste. If it wasn't necessary to recycle the polymers together, we could recycle ABS on its own to produce other ABS products.

Due to the differences between them we will have problems on the compatibility; different surface tensions make the materials immiscible. So, it is necessary to force the union using both:

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types and sufficient plasticizers, as well as temperature and the amount of mixing force needed to keep the mixture stable during the process.

Also, because each polymer has a different nature, it would be best to vary the amount of each of them to improve mixing efficiency, like follows PE > PP > ABS. Finally, a mechanical treatment (crushing for example), prior to the recycling process, can improve the efficiency of the final mixture for those polymers with the highest number of recyclability scale.

References

Recycle Nation, The Most Difficult Plastics to Recycle. (2011)
https://recyclenation.com/2011/03/difficult-plastics-recycle/

Dr. Jaime Bonilla's class from July 13th

S. Koltzenburg, M. Maskos, O. Nuyken, Polymer Chemistry, Springer Berlin Heidelberg, Berlin, Heidelberg, 2017. https://doi.org/10.1007/978-3-662-49279-6.

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Instructions for working again in the Assessment 2-Part 2

2) **You should get together with your teammates to solve as a group, the questions you answered in the Assessment 2-Part 1 and have to write down <u>for each question</u> the following:**

A. *Rephrase the problem indicating very clearly what you have been asked to do.*

B. *List all the data provided.*

C. *Make a list of the assumptions. justifying each of them.*

D. *Write down an algorithm for the solution you are proposing (no calculations are needed at this stage)*

E. *Solve the problem*

F. *Ask yourself if the result is reasonable and, if needed check in the web for technical papers to support your answer.*

G. *List the references used in the solution of the problem.*

I strongly suggest to you to work on your own in steps A to D so you can make an honest contribution to the team, *afterwards you can work E to G with the other members of the group.*

PROBLEMS

Problem 1 (Filling a mold)

1. Calculate the pressure required to fill the mold.
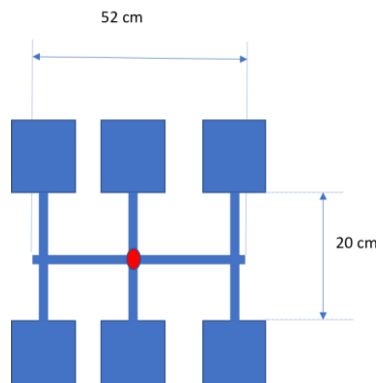
*   The dimensions (in cm) of each cavity are 10x10x1
*   The runners' diameter is 1 cm and their length are indicated in the drawing.
*   The polymer is a polypropylene (PP) with a density of 0.9 g/cm3
*   The viscosity curve for the resins is given in the document called PPVIS 2020:
    *   Groups 1 and 3 should work with resin PAR0
    *   Groups 2 and 4 should work with resin PAR3
    *   Groups 5 and 6 should work with resin PAR5
*   The injection is done at the red dot.

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Rephrase the problem

Calculate the necessary injection pressure to properly fill the given mold through the injection molding processing method.
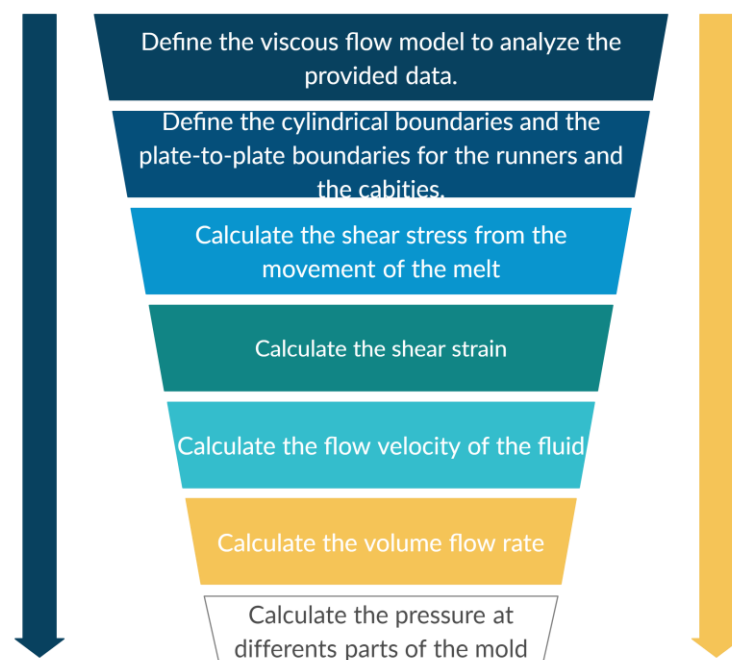
List all the data provided.

- Mould geometry (cavities and runners)
- Polymer: polypropylene (PP) density
- PP viscosity data
- Point/location of injection

Make a list of the assumptions, justifying each of them.

The following is assumed for simplicity and easy the analysis effort

- The polymer behaves as the Power Law model
- The material density remains constant through the whole process
- The melt flow is identical in the cross-sectional area (asymmetric flow)
- The melt flow is is fully developed (constant velocity at a given pressure)

Write down an algorithm for the solution you are proposing.

Define the viscous flow model to analyze the provided data.

Define the cylindrical boundaries and the plate-to-plate boundaries for the runners and the cabities.

Calculate the shear stress from the movement of the melt

Calculate the shear strain

Calculate the flow velocity of the fluid

Calculate the volume flow rate

Calculate the pressure at differents parts of the mold

Solve the problem

The following is adapted from the problem-solving reasoning of Dantzig et al. [1].

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The Viscous Flow Model

Several models have been proposed in literature with different number of fitting parameters. The purpose of these models is to obtain analytical solutions in polymer processing, and to measured data. The flow behavior of different fluids, different models better fit them as some fluids may be shear thinning and/or experience a yield stress.

Complex models such as the Carreau-Yasuda model better represent the rheological behavior of the polymer but can add significant difficulty to the analysis of the data. As depicted in the following figure, the Carreau-Yasuda model better fits the given data than the Power Law model.
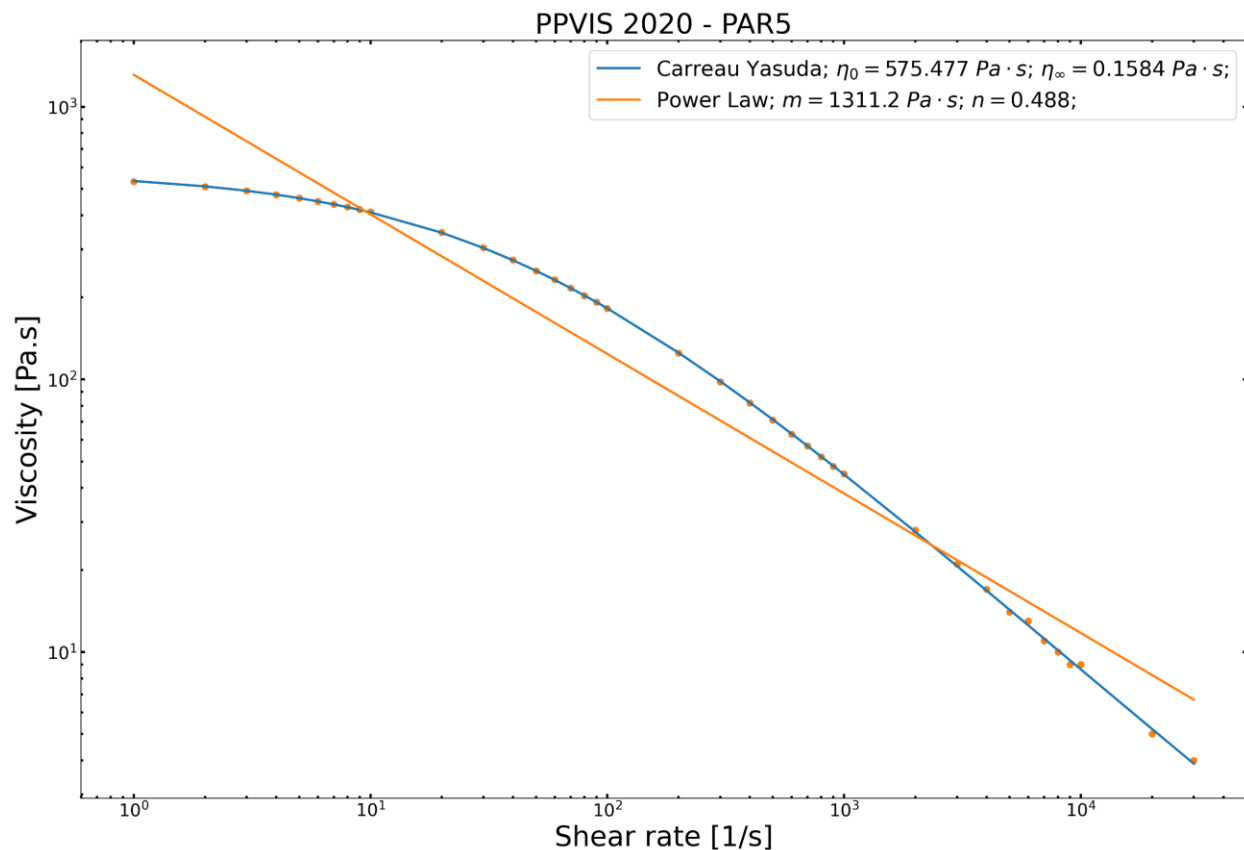
The Carreau-Yasuda Model

$$\eta = \frac{\eta_0 - \eta_\infty}{[1 + (\kappa\dot{\gamma})^a]^{\frac{1-n}{a}}} + \eta_\infty$$

Where:

η in $Pa \cdot s$ is the $viscosity$

$\dot{\gamma}$ in $1/s$ is the $shear$ $rate$

$\eta_\infty = 0.158429$ $Pa \cdot s$, is the $infinite$ $shear$ $rate$ $viscosity$

$\eta_0 = 575.477$ $Pa \cdot s$, is the $zero$-$shear$ $rate$ $viscosity$

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$\kappa = 0.0259115 \ N \ s^n/m^2$, *is the time constant*

$n = 0.244547$, *is the Power Law index*

$a = 0.716978$, *is the width of the transition region between the zero-shear viscosity and the Power Law region.*

The Power Law Model

$$\eta = m \ \dot{\gamma}^{n-1}$$

Where:

η is the viscosity

$\dot{\gamma}$ is the shear rate

$m = 1311.2 \ N \ s/m^2$, *is the consistency index*

$n = 0.488$, *is the Power Law index*

For this examination, the Power Law model is used for simplicity. Its imperative to mention that the Power Law model assumes that: *a) $\eta < \eta_0$, b) $\eta > \eta_\infty$, c) $\eta_\infty = 0$, d) $a = 1$, and e) $\kappa = m$* [2]. If n < 1 the fluid is shear thinning, and if n > 1 it has a shear-thickening behavior.

Pressure Flow Within the Runners
Define the Cylindrical Boundaries
From the power law model, let us consider the flow of a power law fluid in a circular tube to describe the mold runners and assume that the density of the material is constant. From Appendix B.2 in [1], let us define the tube boundaries by selecting the continuity equation for constant density, in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Where:

v_r is the flow velocity through the wall

v_θ is the flow velocity at the circular plane

v_z is the flow velocity in the z-direction (lengthwise)

Assuming the flow is identical in every θ plane (asymmetric flow), the second term $\frac{\partial v_\theta}{\partial \theta} = 0$, and in the assumption that the flow is fully developed (same/constant velocity along the cylinder at a given pressure), then $\frac{\partial v_z}{\partial z} = 0$. Giving: $\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = 0$. By integrating the previous, the following is given:

$$v_r = \frac{C_1}{r}$$

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Where the constant $C_1 = 0$ because v_r is zero since no fluid is leaking the boundary. Also, due to axisymmetric flow, $v_\theta = 0$. Therefore, the given assumptions and boundary conditions conclude that the only velocity is in the z-direction.

Calculate the shear stress through the movement of the fluid
Again, from Appendix B.2 in [1] let us define the momentum equation in the z-direction to describe the flow/motion of the fluid:

$$-\frac{\partial}{\partial z}p + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz} + \rho b_z = 0$$

Again, the asymmetry assumption makes $\frac{\partial}{\partial \theta}\tau_{\theta z} = 0$, and the fully developed flow assumption makes $\frac{\partial}{\partial z}\tau_{zz} = 0$. Also, $b_z = 0$ as the effect of gravity or electromagnetic fields are neglected. With a constant $\frac{\partial}{\partial z}p$ due to the fully developed flow assumption. Giving the following z-direction momentum equation:

$$-\frac{\partial}{\partial z}p + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = 0$$

Following the shape in the figure above, the change in pressure can be described as:

$$\frac{\partial}{\partial z}p = \frac{p_2 - p_1}{L} = -\frac{\Delta p}{L}$$

Where:
p_1 is the pressure leaving the cylinder
p_2 is the pressure entering the cylinder
L is the length of the cylinder
Δp is negative as the shear stress is not in the positive direction of z

By substituting the previous two equations:

$$-\frac{\Delta p}{L} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})$$

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Integrating the previous, gives: $\tau_{rz} = -\frac{\Delta p}{2L}r + \frac{c_2}{r}$. Where the constant $c_2 = 0$ to prevent the infinite values of shear stress when the radius $r = 0$. Such that the shear stress is given by the following for a steady, fully developed axisymmetric flow in cylindrical boundaries.

$$\tau_{rz} = -\frac{\Delta p}{2L}r$$

Calculate the strain rate

For cylindrical coordinates (Appendix B.3 in [1]) and the Power Law, the strain rate can be described as $\dot{\gamma} = \frac{\partial}{\partial r}v_z$, with $v_r = v_\theta = 0$ and v_z as a function of the radius r.

By substituting $\dot{\gamma}$ in the Power Law model, we have: $\tau_{rz} = \frac{\partial m v_z}{\partial r}\left(-\frac{\partial}{\partial r}v_z\right)^{n-1}$. On the other hand, the velocity at the tube wall v_r shall decrease from its maximum value (at $r = 0$) to zero, hence $\frac{\partial}{\partial r}v_z < 0$. Therefore, the shear stress can be written as

$$\tau_{rz} = -m\left(\frac{\partial v_z}{\partial r}\right)^n$$

Calculate flow velocity in the z-direction

By matching the values of τ_{rz} of the previous sections, we have: $-m\left(\frac{\partial v_z}{\partial r}\right)^n = -\frac{\Delta p}{2L}r$. If $s = n^{-1}$, then:

$$\frac{\partial v_z}{\partial r} = -\left(\frac{\Delta p}{2mL}\right)^s r^s$$

Integrating the radius r from 0 to R we have:

$$v_z = \frac{1}{s+1}\left(\frac{\Delta p}{2mL}\right)^s (R^{s+1} - r^{s+1})$$

According to the Power Law model fitting parameters we have $s = \frac{1}{n} = \frac{1}{0.488} = 2.049$ and $m = 1311.2\ Pa \cdot s$

For a Newtonian fluid, $s = 1$, in this case $s > 1$ as we are dealing with a shear thinning fluid.

Calculate volume flow rate Q passing through the runner

The calculation is to be done by integrating the velocity v_z over the cross section over θ, as follows:

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$$Q = \int_0^{2\pi} \int_0^R v_z(r) \, dr \, d\theta = \frac{\pi}{s+3} \left(\frac{\Delta p}{2mL}\right)^s R^{s+3}$$

The $n = 0.488$ represents a significant level of shear thinning. When $s = 2.049$, Q is proportional to $(\Delta p)^{2.049}$, meaning that doubling the pressure drop will quadruple the flow rate. The higher stress near the walls leads to a lower viscosity, and the lower viscosity requires an increased shear rate to sustain the higher shear stress. At the center of the tube the shear stress is small, so the viscosity is high in that point. Shear-thinning behavior is advantageous in the extrusion of polymers. Inside the runner the shear rate is large, so the polymer has a relatively low viscosity, and the pressure drop across the runner is not too high. However, once the polymer exits the die it is subjected only to stresses from its own weight, which are small. Under these low stresses the polymer has a high viscosity, which helps it holds its shape as it cools in the cavities.

Therefore, the pressure entering the runner p_2 can be solved from the previous equation as:

$$\sqrt[s]{Q \frac{s+3}{\pi R^{s+3}}} \, 2mL = \Delta p = p_2 - p_1$$

$$p_2 = \sqrt[s]{Q \frac{s+3}{\pi R^{s+3}}} \, 2mL + p_1 \tag{1}$$

Pressure Flow Within a Cavity (between parallel plates): velocity, shear stress & flow rate
Following a similar reasoning as in the running/cylindrical geometry, the pressure flow in the z-direction between parallel plates is described as follow: the plates are separated by a distance $2h$, the pressure difference Δp is applied over a distance L, and W is the width of the plates. The shear stress is described as:

$$\tau_{zx} = -\frac{\Delta p}{L} z$$

The velocity is in function of z and is given by:

$$v_x(z) = \frac{h}{s+1} \left(\frac{h\Delta p}{mL}\right)^s \left[1 - \left(\frac{z}{h}\right)^{s+1}\right]$$

The volume flow rate is given by:

$$Q = \frac{2W h^2}{s+2} \left(\frac{h\Delta p}{mL}\right)^s$$

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Therefore, the entering pressure to fill the cubic cavity can be solved from the previous equation as:

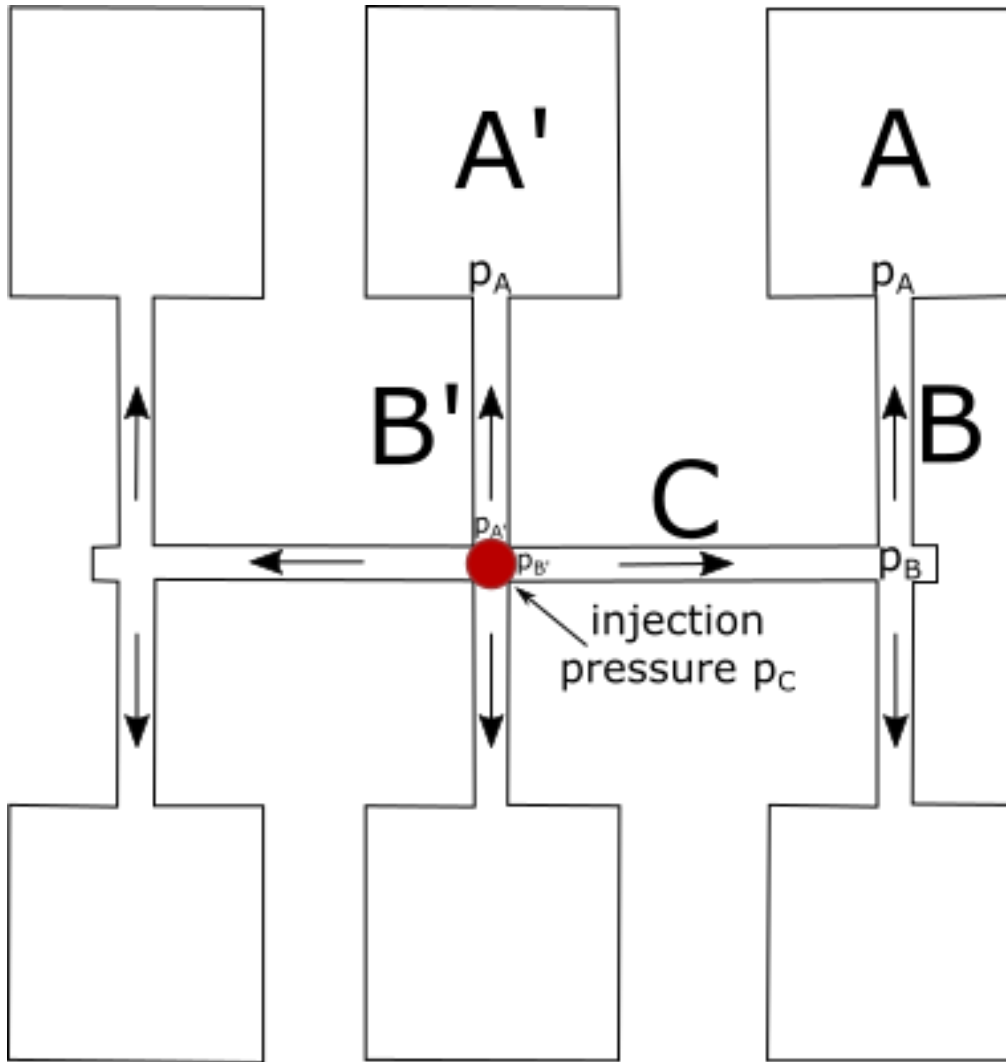$$\frac{\sqrt[s]{Q\frac{s+2}{2Wh^2}}}{h}mL = \Delta p = p_2 - p_1$$

$$p_2 = \frac{\sqrt[s]{Q\frac{s+2}{2Wh^2}}}{h}mL + p_1 \tag{2}$$

Analyze the pressures at different points of the mold

As stated by Farotti et al. [5], the typical volume flow rate Q is about $30\ \text{cm}^3/\text{s}$ with a packing pressure of $22\ MPa$.

Now let us calculate the pressures at each intersection within the mold, so let us dive the mold in sections as follows:

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1. Compute the pressure p_A using equation (2)

$$p_2 = \frac{\sqrt[s]{Q\,\dfrac{s+2}{2Wh^2}}}{h}\,mL + p_1$$

Where:

$p_2 = p_{A'}$
$Q = 30\ cm^3/s,$
$s = 2.049,$
$W = 10\ cm,$
$2h = 1\ cm,$
$m = 1311.2\ Pa \cdot s,$
$L = 10\ cm,$
$p_1 = p_C = 32\ MPa,$ *is the packing pressure.*

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$$p_2 = \frac{\sqrt[2.049]{30 \ cm^3/s \ \dfrac{2.049 + 2}{2(10cm)(0.5cm)^2}}}{0.5 \ cm}(1311.2Pa \cdot s)(10cm) + 22MPa$$

$$p_2 = \frac{\sqrt[2.049]{0.3 \times 10^{-6} \ m^3/s \ \dfrac{2.049 + 2}{2(0.01m)(0.005m)^2}}}{0.005 \ m}(1311.2Pa \cdot s)(0.1m) + 22MPa$$

$$p_2 = \frac{\sqrt[2.049]{0.3 \times 10^{-6} \ m^3/s \ \dfrac{2.049 + 2}{2(0.01m)(0.005m)^2}}}{0.005 \ m}(1311.2Pa \cdot s)(0.1m) + 22000kPa$$

$$p_2 = 22040.437 \ kPa$$

$$p_A = 22040.437 \ kPa$$

2. Compute the pressure p_B using equation (1)

$$p_2 = \sqrt[s]{Q \frac{s+3}{\pi \ R^{s+3}} 2mL} + p_1$$

Where:

$2p_2 = p_B$, we are multiplying by two since the pressure in p_B is divided into two equal cavities (up and down).

$Q = 30 \ cm^3/s,$

$s = 2.049,$

$R = 1 \ cm,$

$m = 1311.2 \ Pa \cdot s,$

$L = 10 \ cm,$

$p_1 = p_A = 32040.437 \ kPa.$

$$p_2 = \sqrt[2.049]{(30cm^3/s) \frac{2.049 + 3}{\pi \ (1cm)^{s+3}} 2(1311.2Pa \cdot s)(10cm)} + 22040.437kPa$$

$$p_2 = 22058.806 \ kPa$$

$$p_B = 2p_2 = 44117.612kPa$$

and

$$p_A{}' = p_2 = 22058.806 \ kPa$$

3. Compute the pressure p_B' using equation (1)

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$$p_2 = \sqrt[s]{Q\,\frac{s+3}{\pi\,R^{s+3}}\,2mL} + p_1$$

Where:

$p_2 = p_B'$,
$Q = 30 \; cm^3/s$,
$s = 2.049$,
$R = 1 \; cm$,
$m = 1311.2 \; Pa \cdot s$,
$L = 26 \; cm$,
$p_1 = p_B = 64117.612 \; kPa$.

$$p_2 = \sqrt[2.049]{(30cm^3/s)\frac{2.049+3}{\pi\,(1cm)^{s+3}}\,2(1311.2Pa \cdot s)(26cm)} + 44117.612kPa$$

$$p_2 = 44264.536 \; kPa$$
$$p_B' = p_2 = 44264.536kPa$$

4. Finally, the injection pressure is given by p_C:

$$p_C = 2p_A' + 2p_B'$$
$$p_C = 2(22058.806kPa) + 2(44264.536kPa)$$
$$\boldsymbol{p_C = 132.64 \; MPa}$$

Ask yourself if the result is reasonable

As stated by Farotti et al. [5] the typical packing pressure is about 32 MPa per cavity. The proper injection pressure depends largely on part size and configuration. Pressures usually range from 1,000 to 1,500 psi. Best results are obtained at higher pressures, up to about 75% of the press capacity. Pressures should be high enough to fill the part and to avoid problems with shrinkage, voids, sinks, and pigment dispersion. Too much pressure can cause parts to flash, burn, and stick in the mold. The calculated necessary injection pressure ensures a proper packing pressure of 22 MPa at each cavity.

References

1] J.A. Dantzig, C.L. Tucker, Modeling in Materials Processing, Cambridge University Press, 2001. https://doi.org/10.1017/CBO9781139175272.

[2] T. Osswald, N. Rudolph, T. Osswald, N. Rudolph, Generalized Newtonian Fluid (GNF) Models, Polym. Rheol. (2014) 59–99. https://doi.org/10.3139/9781569905234.003.

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[3] H. Zhou, Z. Hu, D. Li, Mathematical Models for the Filling and Packing Simulation, in: Comput. Model. Inject. Molding, John Wiley & Sons, Inc., Hoboken, NJ, USA, 2013: pp. 49–70. https://doi.org/10.1002/9781118444887.ch3.

[4] C. Fernandes, A.J. Pontes, J.C. Viana, A. Gaspar-Cunha, Modeling and Optimization of the Injection-Molding Process: A Review, Adv. Polym. Technol. 37 (2018) 429–449. https://doi.org/10.1002/adv.21683.

[5] E. Farotti, M. Natalini, Injection molding. Influence of process parameters on mechanical properties of polypropylene polymer. A first study., Procedia Struct. Integr. 8 (2018) 256–264. https://doi.org/10.1016/j.prostr.2017.12.027.

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Problem 2 (Capillary rheometer correction)

2. You have the following information from a capillary rheometer for a HDPE resin (this is real data so several runs were made at the same velocity due to possible variabilities in the instrument)

Velocidad (in/s)	Average load (lbf) for the L/D=40 die	Average load (lbf) for the L/D=20 die	Average load (lbf) for the L/D=10 die
0.061	170.3	92.6	55.9
0.061	171.7	92.6	54.1
0.307	420.5	216.9	121.3
0.307	417.5	214.4	119.4
0.614	579.1	296.5	164.6
0.614	575.8	294.1	162.9
3.07	848.4	390.7	261.6
6.14	626.8	360.6	233.8
6.14	623.6	377.8	264.3
13.8	939.4	552.4	368.5
18.4	1098.4	640.3	418.0

- Some of the dimensions are:
- Barrel diameter 0.68 cm
- Capillary diameter: 0.05 inches
- Die Lengths: 2 in; 1 in; 0.5 in
- The piston moves at constant velocity (inches/second)
- The load is given pound force (lb$_f$) and the force sensor is at the top of the piston.

You are asked to:

a) Get the real shear viscosity
b) The pressure at the entrance
c) The elongational viscosity

Rephrase the problem

Obtain the real shear viscosity η_s, the pressure at the entrance Pe, and the elongational viscosity η_s with the information given from a capillary rheometer for an HDEP resin.

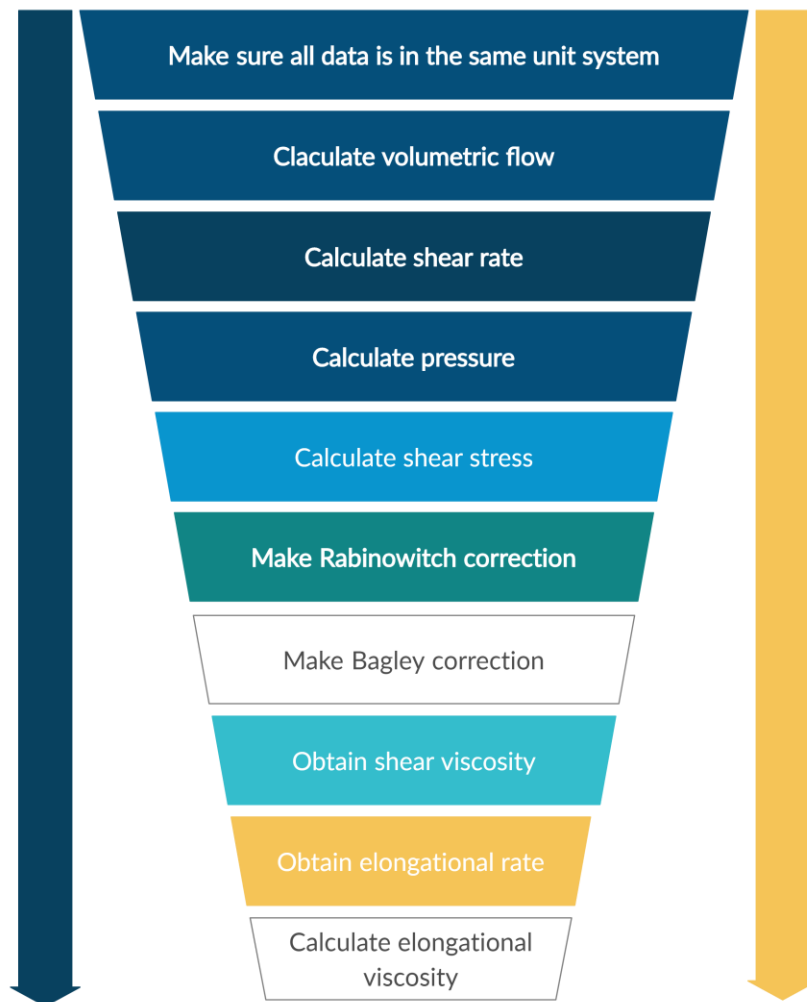## List all the data provided.
- Velocities (constant)

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- Average load
- Barrel diameter
- Die diameter
- Die lengths

Make a list of the assumptions, justifying each of them.

No assumptions needed for this module.

Write down an algorithm for the solution you are proposing.

Make sure all data is in the same unit system

Claculate volumetric flow

Calculate shear rate

Calculate pressure

Calculate shear stress

Make Rabinowitch correction

Make Bagley correction

Obtain shear viscosity

Obtain elongational rate

Calculate elongational viscosity

Solve the problem

Volumetric flow

The first thing we need to do is calculate the volumetric flow (Q) which is given by the piston velocity (V) times the piston cross-section area (A).

$$Q = V * A$$

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To obtain A we have the barrel diameter (0.68 cm) we convert it to inches dividing by 2.54 which gives us a diameter (d) of 0.2575 in.

$$A = \pi * r^2 = \pi \left(\frac{d}{2}\right)^2$$

$$A = \pi \left(\frac{0.2575}{2}\right)^2$$

$$A = 0.0521 \ in^2$$

Since we have different velocities, we will obtain different volumetric flows, we will enumerate the different sample trials (by velocity) to classify them easily.

Table 1. Volumetric flows from velocities given

No.	V (in/s)	Q (in³/s)
1	0.061	0.003179
2	0.307	0.015997
3	0.614	0.031994
4	3.07	0.15997
5	6.14	0.31994
6	13.8	0.719083
7	18.4	0.958777

Apparent shear rate and shear stress at the wall

Once we have the volumetric flows for each velocity used, we can obtain the apparent shear rate (Γ) given by:

$$\Gamma = \frac{4 * Q}{\pi R^3}$$

Where the die radius R = 0.025 in.

Table 2. Apparent shear rates from volumetric flows.

No.	Q (in³/s)	Γ (1/s)
1	0.0034	279.81
2	0.0173	1408.21
3	0.0346	2816.43
4	0.1728	14082.14
5	0.3456	28164.29
6	0.7768	63300.84
7	1.0358	84401.12

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To avoid discrepancies, we obtained the following table by averaging the different load forces (F) for same velocities.

Table 3. Load forces applied for every die configuration

No.	F (lb$_f$) L/D=40	F (lb$_f$) L/D=20	F (lb$_f$) L/D=10
1	171.0	92.6	55.0
2	419.0	215.7	120.4
3	577.45	295.3	163.75
4	848.4	390.7	261.6
5	625.2	369.2	249.05
6	939.4	552.4	368.5
7	1098.4	640.3	418

To obtain the load pressure (P) we need the piston area (A) and the load force (F) applied.

$$P = F/A$$

Table 4. Pressures obtained from the forces and area of the piston

No.	P (lb$_f$/in^2) L/D=40	P (lb$_f$/in^2) L/D=20	P (lb$_f$/in^2) L/D=10
1	3281.7	1777.1	1055.5
2	8041.1	4138.6	2309.7
3	11081.9	5667.1	3142.5
4	16281.7	7498.0	5020.4
5	11998.3	7085.4	4779.5
6	18028.1	10601.2	7071.9
7	21079.5	12288.1	8021.9

Once we have all the apparent shear rates, and pressures we need to obtain the apparent shear stress at the wall (τw)

$$\tau_w = \frac{RP}{2L}$$

So we will obtain a τw for every die configuration and for every P.

Table 5. Die configurations

L/D	40.0	20.0	10.0
R (in)	0.025	0.025	0.025
L (in)	2	1	0.5

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Table 6. Shear stress at the wall for the different die configurations and pressures

No.	τ_w (lb$_f$/in^2) L/D=40	τ_w (lb$_f$/in^2) L/D=20	τ_w (lb$_f$/in^2) L/D=10
1	18.99	20.56	24.43
2	46.52	47.89	53.45
3	64.11	65.57	72.72
4	94.20	86.76	116.18
5	69.42	81.98	110.61
6	104.30	122.67	163.66
7	121.96	142.18	185.64

Rabinowitch correction

The treatment above is for a Newtonian fluid, but the shear rates at which these measures were made in the capillary rheometer follow a non-Newtonian behavior, so we have to perform a correction. For this correction we have to plot the log(Γ) vs log(τw) to obtain the slope (b) and follow the Rabinowitch correction equation to obtain the real shear rate at the wall ($\dot{\gamma}_w$).

$$\dot{\gamma}_w = \left(\frac{3+b}{4}\right)\Gamma$$

Table 7. Rabinowitch correction log values for slope calculation

No.	log(Γ)	log(τ_w) L/D=40	log(τ_w) L/D=20	log(τ_w) L/D=10
1	2.447	1.278	1.313	1.388
2	3.149	1.668	1.680	1.728
3	3.450	1.807	1.817	1.862
4	4.149	1.974	1.938	2.065
5	4.450	1.841	1.914	2.044
6	4.801	2.018	2.089	2.214
7	4.926	2.086	2.153	2.269

In the next figure, a plot of apparent shear rate vs shear stress at the wall is shown, where we obtain the slopes to apply the Rabinowitch correction.

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Rabinowitch correction log(Γ) vs log(τw)
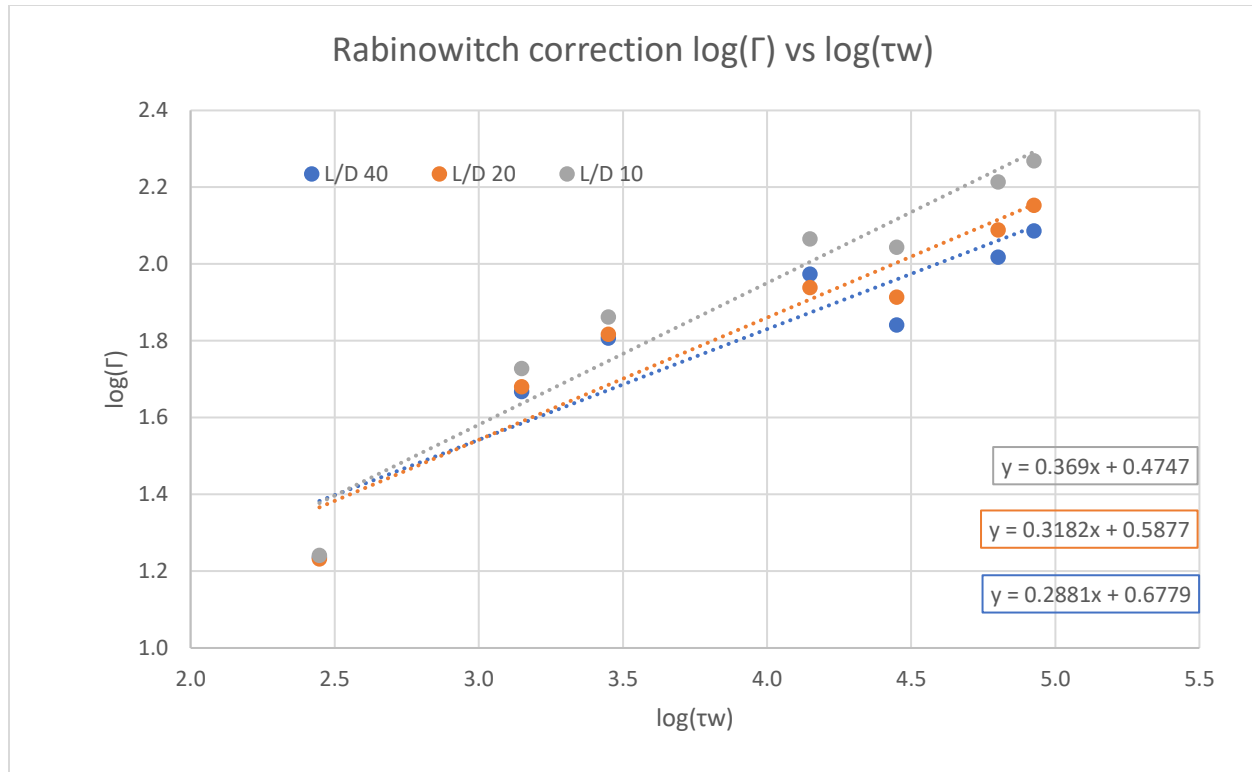
Once we obtained the slope b for each die configuration, we can proceed to substitute values in the Rabinowitch equation stated above and obtain the real shear rate ($\dot{\gamma}_w$) for each die configuration.

Table 8. Slopes obtained for each die configuration

L/D	b
40	0.288
20	0.318
10	0.369

Table 9. Real shear rates obtained through the Rabinowitch equation

No.	Γ (1/s)	$\dot{\gamma}_w$ (1/s) L/D=40	$\dot{\gamma}_w$ (1/s) L/D=20	$\dot{\gamma}_w$ (1/s) L/D=10
1	279.81	230.00	232.10	235.67
2	1408.21	1157.55	1168.11	1186.07
3	2816.43	2315.10	2336.23	2372.14
4	14082.14	11575.52	11681.14	11860.68
5	28164.29	23151.04	23362.27	23721.37
6	63300.84	52033.29	52508.04	53315.13
7	84401.12	69377.72	70010.73	71086.84

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Bagley correction

Now that we have the shear rate correction, we must do the correction for the pressure. The instrument gives us the total pressure P which is given by the viscous pressure and the entrance pressure which is the one we need to calculate. To obtain this entrance pressure we need to extrapolate L/D in to find a theoretical pressure when L/D is 0.

For this correction we ignore the Rabinowitch correction in order to maintain the same shear rate, using Γ, for every die configuration L/D.

Table 10. Pressures obtained for different shear rates and configurations

No.	Γ (1/s)	P (lb_f/in^2) L/D=40	P (lb_f/in^2) L/D=40	P (lb_f/in^2) L/D=40
1	279.81	3037.8	1645.0	977.1
2	1408.21	7443.4	3831.0	2138.0
3	2816.43	10258.3	5245.9	2909.0
4	14082.14	15071.6	6940.7	4647.3
5	28164.29	11106.5	6558.8	4424.3
6	63300.84	16688.2	9813.3	6546.3
7	84401.12	19512.8	11374.8	7425.7

Then we plot the data so that we can do a linear regression and find the pressure at L/D=0, which will be given at the y intercept. In the figure we can observe the linear fit for different velocities to determine the entrance pressure Pe.

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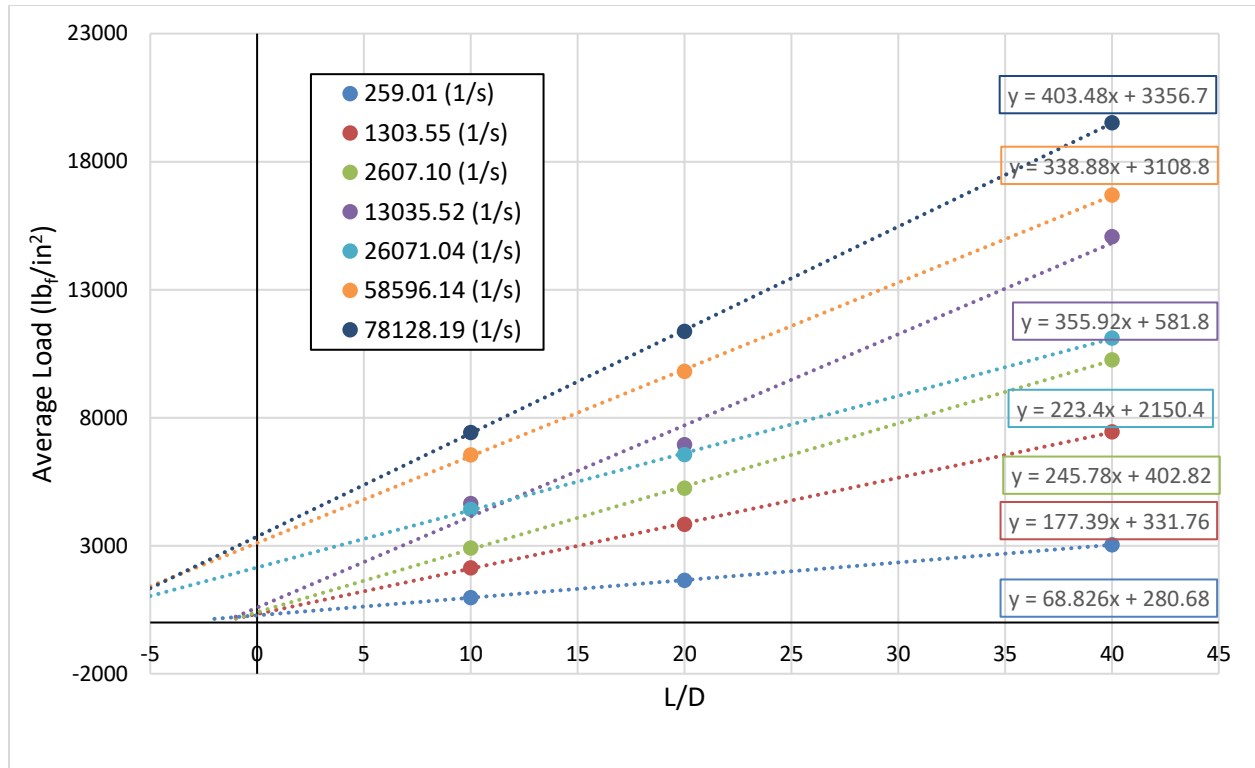
From the figure we can obtain the y intercepts which are considered the entrance pressure Pe at every shear rate evaluated.

Table 11. Entrance pressures by shear rates

No.	Γ (1/s)	P_e (lb$_f$/in^2)
1	279.808	280.68
2	1408.21	331.76
3	2816.429	402.82
4	14082.14	581.8
5	28164.29	2150.4
6	63300.84	3108.8
7	84401.12	3356.7

And by subtracting the entrance pressure to the total pressure we obtain the viscous pressure

Table 12. Viscous pressures obtained by subtracting P-P$_e$

No.	P_v (lbf/in2) L/D=40	P_v (lbf/in2) L/D=20	P_v (lbf/in2) L/D=10
1	2757.1	1364.3	696.4
2	7111.7	3499.2	1806.2
3	9855.4	4843.1	2506.2
4	14489.8	6358.9	4065.5
5	8956.1	4408.4	2273.9
6	13579.4	6704.5	3437.5

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| 7 | 16156.1 | 8018.1 | 4069.0 |

And with the viscous pressure we can obtain the real sheer stress τ.

Table 13. Real sheer stress from P_v

No.	τ (lb_f/in^2) L/D=40	τ (lb_f/in^2) L/D=20	τ (lb_f/in^2) L/D=10
1	17.23	17.05	17.41
2	44.45	43.74	45.16
3	61.60	60.54	62.65
4	90.56	79.49	101.64
5	55.98	55.10	56.85
6	84.87	83.81	85.94
7	100.98	100.23	101.72

In the following figure we plotted the apparent shear stress (left) and the corrected shear stress vs apparent shear rate to show the changes.
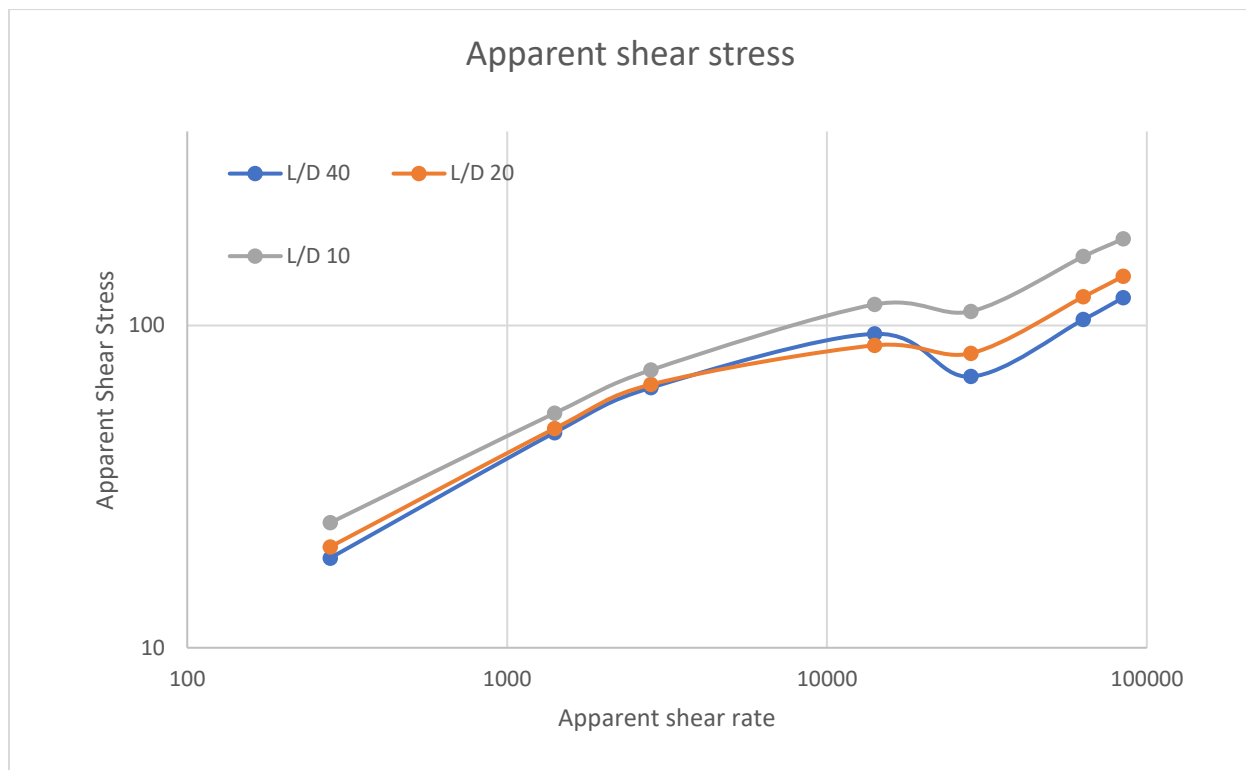
TEAM **5**
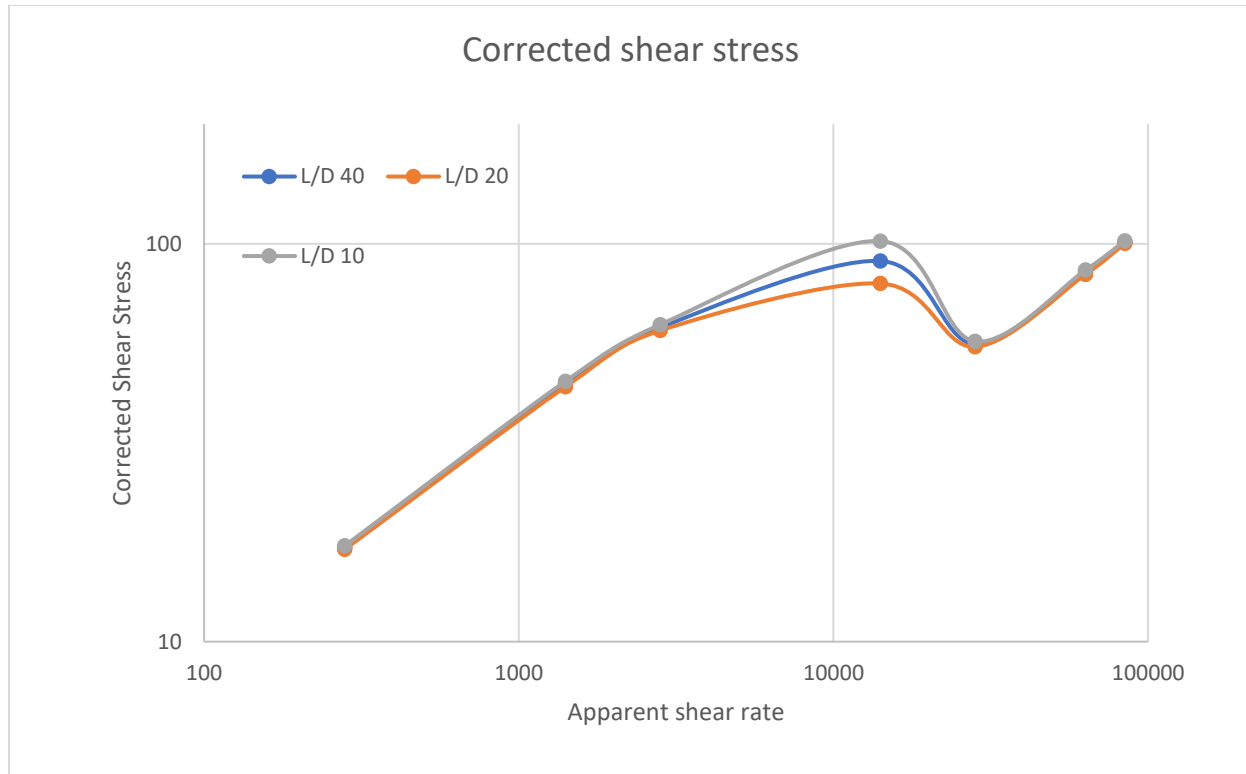
Shear viscosity

For obtaining the real shear viscosity ηs we need to divide the real shear stress over the real shear rate.

$$\eta_s = \frac{\tau}{\dot{\gamma}}$$

The real shear viscosity can be found in the following table.

Table 14. Real shear viscosities

No.	η_s (lbf s/in²) L/D=40	η_s (lbf s/in²) L/D=20	η_s (lbf s/in²) L/D=10
1	0.07492	0.073478	0.073873
2	0.038398	0.037445	0.038072
3	0.026606	0.025913	0.026412
4	0.007824	0.006805	0.008569
5	0.002418	0.002359	0.002396
6	0.001631	0.001596	0.001612
7	0.001455	0.001432	0.001431

Converting to Pa·s we obtain the following viscosities:

Table 15. Real shear viscosities in Pa s

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No.	η_s (Pa·s) L/D=40	η_s (Pa·s) L/D=20	η_s (Pa·s) L/D=10
1	52.67415	51.65983	51.93788
2	26.99661	26.32649	26.76708
3	18.70611	18.2187	18.56979
4	5.500478	4.784144	6.024736
5	1.699917	1.658322	1.684893
6	1.146774	1.122136	1.133267
7	1.023281	1.0065	1.006082
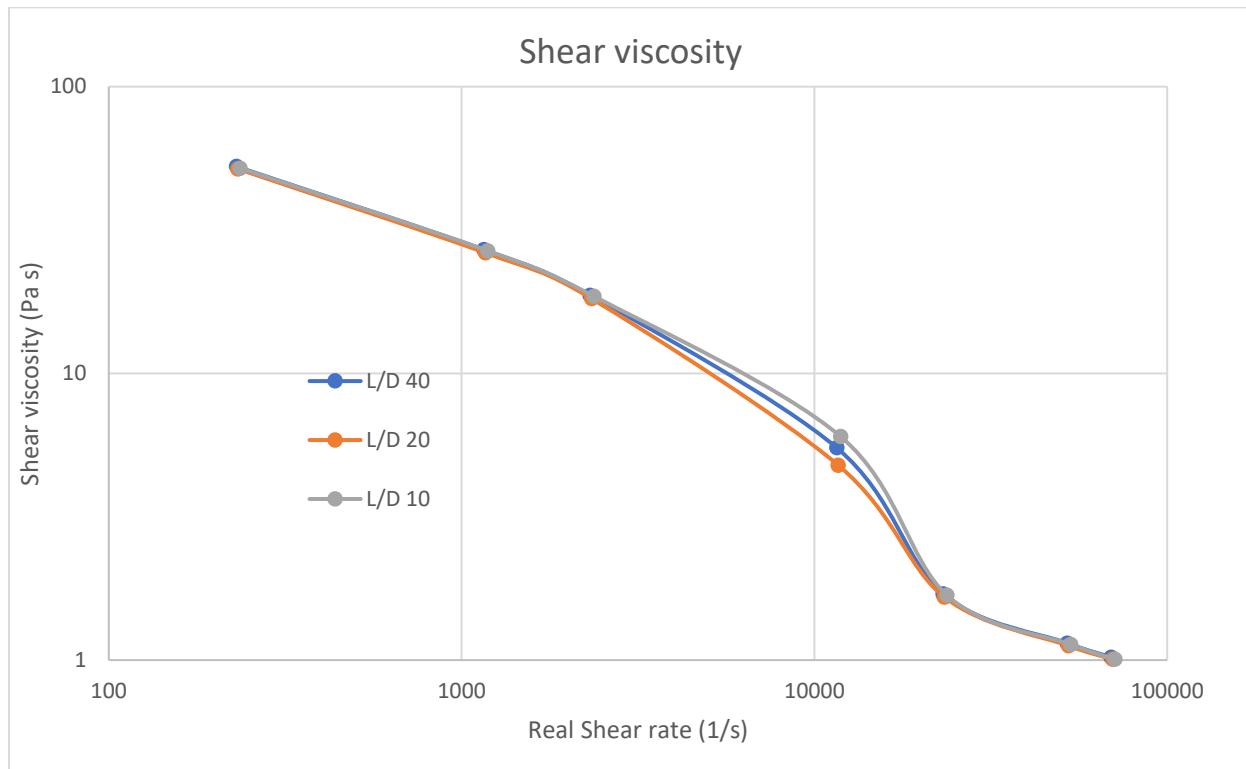
We compared our results with the Carreau-Yasuda model to be certain of our calculations, in the following figure we plotted the fitted model alongside our findings, and as can be seen they are very similar thus our calculations are a close representation of the model.

TEAM **5**
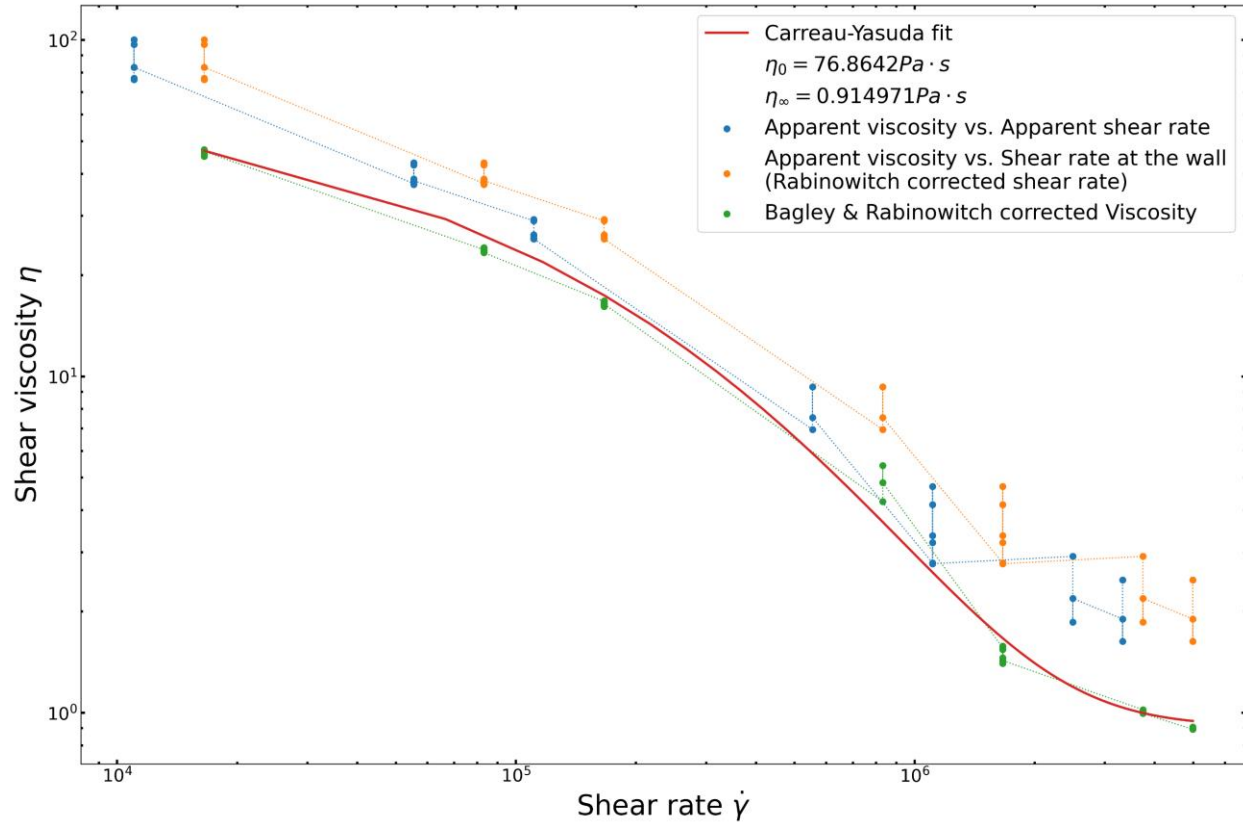
Elongational viscosity

The elongational viscosity is defined as the ratio of elongational stress to elongational strain rate. It measures the resistance of a fluid against elongational deformation [1].

To measure the elongational viscosity η_e, first we need to calculate the Elongational rate $\dot\varepsilon$ which is given by,

$$\dot\varepsilon = \frac{4\eta_s \Gamma^2}{3(n+1)P_e}$$

Where η_A is the apparent shear viscosity, Γ the apparent shear rate, Pe the entrance pressure, and n the power law index [1].

We know that using Carreau-Yasuda model for this resin we obtain n=0.48 so we obtain the following values

Table 16. Elongational rates

No.	$\dot\varepsilon$ (in/s) L/D=40	$\dot\varepsilon$ (in/s) L/D=20	$\dot\varepsilon$ (in/s) L/D=10
1	18.82721	18.46466	18.56405
2	206.777	201.6443	205.0189

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3	472.008	459.7094	468.5683
4	2402.391	2089.525	2631.366
5	803.4992	783.8387	796.398
6	1894.015	1853.324	1871.707
7	2782.648	2737.014	2735.879

And then we proceed to calculate η_e with the following equation,

$$\eta_e = \frac{3(n+1)P_e}{8\dot{\varepsilon}}$$

Which results can be found in the following Table 14.

Table 17. Elongational viscosities

| No.	η_e (lbf s/in²) L/D=40	η_e (lbf s/in²) L/D=40	η_e (lbf s/in²) L/D=40
1	7.882716	8.03749	7.99446
2	0.848344	0.869939	0.855619
3	0.451245	0.463317	0.454557
4	0.12805	0.147223	0.116908
5	1.41509	1.450584	1.427708
6	0.86788	0.886935	0.878224
7	0.63783	0.648464	0.648733

Converting to Pa·s we obtain the following viscosities:

Table 18. Elongational viscosities in Pa s

No.	η_e (Pa·s) L/D=40	η_e (Pa·s) L/D=20	η_e (Pa·s) L/D=10
1	5542.088	5650.904	5620.652
2	596.4441	611.6262	601.5586
3	317.2558	325.7433	319.5848
4	90.02806	103.508	82.19404
5	994.9052	1019.86	1003.776
6	610.179	623.5761	617.4514
7	448.4378	455.9145	456.1037

In the following plot, we can find the behavior of the elongational viscosity.

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Elongational viscosity
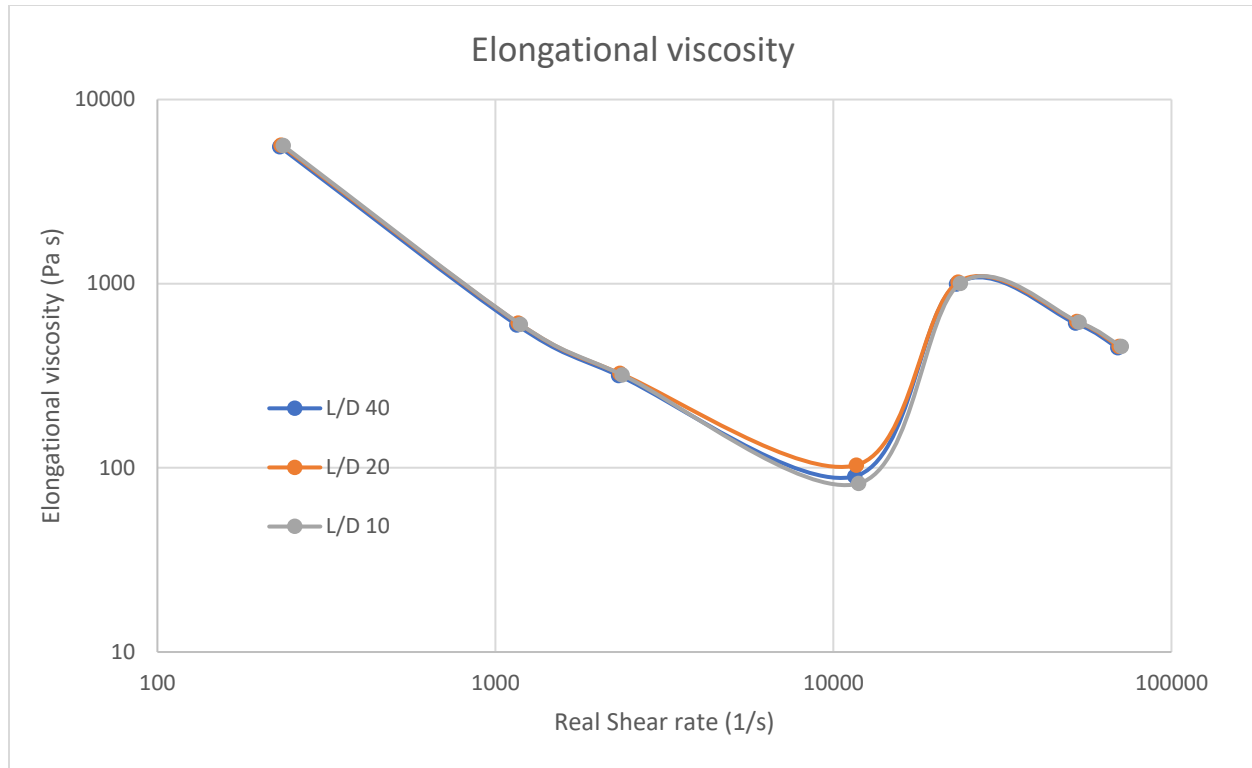
Ask yourself if the result is reasonable

We believe our results of the shear viscosity are reasonable because they were comparable to those of the reviewed article by Lin et al. For the elongational viscosity we followed the equations from Sarkar and Gupta but could not find a comparable graph to validate our results.

References

[1] D. Sarkar and M. Gupta, "Estimation of Elongational Viscosity," Asme, vol. 90, pp. 309–318, 2000.

[2] M. Gupta, Estimation of elongational viscosity of polymers from entrance loss data using individual parameter optimization, Adv. Polym. Technol. 21 (2002) 98–107. https://doi.org/10.1002/adv.10017.

[3] X. Lin, A. Kelly, M. Woodhead, D. Ren, K. Wang, P. Coates, Capillary study on geometrical dependence of shear viscosity of polymer melts, J. Appl. Polym. Sci. 131 (2014). https://doi.org/10.1002/app.39982.