The Physics GRE Solution Guide

# Sample, GR8677, GR9277, GR9677 and GR0177 Tests



http://groups.yahoo.com/group/physicsgre_v2

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# Preface

This solution guide initially started out on the Yahoo Groups web site and was pretty successful at the time. Unfortunately, the group was lost and with it, much of the the hard work that was put into it. This is my attempt to recreate the solution guide and make it more widely avaialble to everyone. If you see any errors, think certain things could be expressed more clearly, or would like to make suggestions, please feel free to do so.

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## Document Changes

**05-11-2009**
1. Added diagrams to GR0177 test questions 1-25
2. Revised solutions to GR0177 questions 1-25

**04-15-2009** First Version

# Contents

# List of Tables

# List of Figures

# Chapter 1

# Classical Mechanics

## 1.1 Kinematics

### 1.1.1 Linear Motion

**Average Velocity**

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{1.1.1}$$

**Instantaneous Velocity**

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = v(t) \tag{1.1.2}$$

**Kinematic Equations of Motion**

The basic kinematic equations of motion under constant acceleration, $a$, are

$$v = v_0 + at \tag{1.1.3}$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \tag{1.1.4}$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \tag{1.1.5}$$

$$x - x_0 = \frac{1}{2}\left(v + v_0\right)t \tag{1.1.6}$$

### 1.1.2 Circular Motion

In the case of **Uniform Circular Motion**, for a particle to move in a circular path, a radial acceleration must be applied. This acceleration is known as the **Centripetal Acceleration**

**Centripetal Acceleration**

$$a = \frac{v^2}{r} \tag{1.1.7}$$

**Angular Velocity**

$$\omega = \frac{v}{r} \tag{1.1.8}$$

We can write eq. (1.1.7) in terms of $\omega$

$$a = \omega^2 r \tag{1.1.9}$$

**Rotational Equations of Motion**

The equations of motion under a constant angular acceleration, $\alpha$, are

$$\omega = \omega_0 + \alpha t \tag{1.1.10}$$

$$\theta = \frac{\omega + \omega_0}{2} t \tag{1.1.11}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \tag{1.1.12}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \tag{1.1.13}$$

## 1.2 Newton's Laws

### 1.2.1 Newton's Laws of Motion

**First Law** A body continues in its state of rest or of uniform motion unless acted upon by an external unbalanced force.

**Second Law** The net force on a body is proportional to its rate of change of momentum.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a} \tag{1.2.1}$$

**Third Law** When a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA} \tag{1.2.2}$$

### 1.2.2 Momentum

$$\mathbf{p} = m\mathbf{v} \tag{1.2.3}$$

### 1.2.3 Impulse

$$\Delta\mathbf{p} = \mathbf{J} = \int \mathbf{F} dt = \mathbf{F}_{avg} dt \tag{1.2.4}$$

## 1.3 Work & Energy

### 1.3.1 Kinetic Energy

$$K \equiv \frac{1}{2}mv^2 \tag{1.3.1}$$

### 1.3.2 The Work-Energy Theorem

The net Work done is given by

$$W_{\text{net}} = K_f - K_i \tag{1.3.2}$$

### 1.3.3 Work done under a constant Force

The work done by a force can be expressed as

$$W = F\Delta x \tag{1.3.3}$$

In three dimensions, this becomes

$$W = \mathbf{F} \cdot \Delta\mathbf{r} = F\Delta r \cos\theta \tag{1.3.4}$$

For a non-constant force, we have

$$W = \int_{x_i}^{x_f} F(x)dx \tag{1.3.5}$$

### 1.3.4 Potential Energy

The Potential Energy is

$$F(x) = -\frac{dU(x)}{dx} \tag{1.3.6}$$

for conservative forces, the potential energy is

$$U(x) = U_0 - \int_{x_0}^{x} F(x')dx' \tag{1.3.7}$$

### 1.3.5 Hooke's Law

$$F = -kx \tag{1.3.8}$$

where $k$ is the spring constant.

### 1.3.6 Potential Energy of a Spring

$$U(x) = \frac{1}{2}kx^2 \tag{1.3.9}$$

## 1.4 Oscillatory Motion

### 1.4.1 Equation for Simple Harmonic Motion

$$x(t) = A\sin(\omega t + \delta) \tag{1.4.1}$$

where the Amplitude, $A$, measures the displacement from equilibrium, the phase, $\delta$, is the angle by which the motion is shifted from equilibrium at $t = 0$.

### 1.4.2 Period of Simple Harmonic Motion

$$T = \frac{2\pi}{\omega} \tag{1.4.2}$$

### 1.4.3 Total Energy of an Oscillating System

Given that

$$x = A\sin(\omega t + \delta) \tag{1.4.3}$$

and that the Total Energy of a System is

$$E = KE + PE \tag{1.4.4}$$

The Kinetic Energy is

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m\frac{dx}{dt} \\ &= \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \delta) \end{aligned} \tag{1.4.5}$$

The Potential Energy is

$$\begin{aligned} U &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}kA^2\sin^2(\omega t + \delta) \end{aligned} \tag{1.4.6}$$

Adding eq. (1.4.5) and eq. (1.4.6) gives

$$E = \frac{1}{2}kA^2 \tag{1.4.7}$$

### 1.4.4 Damped Harmonic Motion

$$\mathbf{F}_d = -b\mathbf{v} = -b\frac{d\mathbf{x}}{dt} \tag{1.4.8}$$

where $b$ is the damping coefficient. The equation of motion for a damped oscillating system becomes

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \tag{1.4.9}$$

Solving eq. (1.4.9) goves

$$x = Ae^{-\alpha t}\sin(\omega' t + \delta) \tag{1.4.10}$$

We find that

$$\alpha = \frac{b}{2m} \tag{1.4.11}$$

$$\begin{aligned}\omega' &= \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \\ &= \sqrt{\omega_0^2 - \frac{b^2}{4m^2}} \\ &= \sqrt{\omega_0^2 - \alpha^2}\end{aligned} \tag{1.4.12}$$

### 1.4.5 Small Oscillations

The Energy of a system is

$$E = K + V(x) = \frac{1}{2}mv(x)^2 + V(x) \tag{1.4.13}$$

We can solve for $v(x)$,

$$v(x) = \sqrt{\frac{2}{m}\left(E - V(x)\right)} \tag{1.4.14}$$

where $E \geq V(x)$ Let the particle move in the potential valley, $x_1 \leq x \leq x_2$, the potential can be approximated by the Taylor Expansion

$$V(x) = V(x_e) + (x - x_e)\left[\frac{dV(x)}{dx}\right]_{x=x_e} + \frac{1}{2}(x - x_e)^2\left[\frac{d^2V(x)}{dx^2}\right]_{x=x_e} + \cdots \tag{1.4.15}$$

At the points of inflection, the derivative $dV/dx$ is zero and $d^2V/dx^2$ is positive. This means that the potential energy for small oscillations becomes

$$V(x) \cong V(x_e) + \frac{1}{2}k(x - x_e)^2 \tag{1.4.16}$$

where

$$k \equiv \left[\frac{d^2V(x)}{dx^2}\right]_{x=x_e} \geq 0 \tag{1.4.17}$$

As $V(x_e)$ is constant, it has no consequences to physical motion and can be dropped. We see that eq. (1.4.16) is that of simple harmonic motion.

### 1.4.6 Coupled Harmonic Oscillators

Consider the case of a simple pendulum of length, $\ell$, and the mass of the bob is $m$[1]. For small displacements, the equation of motion is

$$\ddot{\theta} + \omega_0\theta = 0 \tag{1.4.18}$$

[1] Add figure with coupled pendulum-spring system

We can express this in cartesian coordinates, $x$ and $y$, where

$$
\begin{aligned}
x &= \ell \cos\theta \approx \ell && (1.4.19)\\
y &= \ell \sin\theta \approx \ell\theta && (1.4.20)
\end{aligned}
$$

eq. (1.4.18) becomes

$$\ddot{y} + \omega_0 y = 0 \tag{1.4.21}$$

This is the equivalent to the mass-spring system where the spring constant is

$$k = m\omega_0^2 = \frac{mg}{\ell} \tag{1.4.22}$$

This allows us to to create an equivalent three spring system to our coupled pendulum system. The equations of motion can be derived from the Lagrangian, where

$$
\begin{aligned}
L &= T - V \\
&= \frac{1}{2}m\dot{y}_1^2 + \frac{1}{2}m\dot{y}_2^2 - \left(\frac{1}{2}ky_1^2 + \frac{1}{2}\kappa\left(y_2 - y_1\right)^2 + \frac{1}{2}ky_2^2\right) \\
&= \frac{1}{2}m\left(\dot{y}_1{}^2 + \dot{y}_2{}^2\right) - \frac{1}{2}\left(k\left(y_1^2 + y_2^2\right) + \kappa\left(y_2 - y_1\right)^2\right) && (1.4.23)
\end{aligned}
$$

We can find the equations of motion of our system

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_n}\right) = \frac{\partial L}{\partial y_n} \tag{1.4.24}$$

The equations of motion are

$$
\begin{aligned}
m\ddot{y}_1 &= -ky_1 + \kappa\left(y_2 - y_1\right) && (1.4.25)\\
m\ddot{y}_2 &= -ky_2 + \kappa\left(y_2 - y_1\right) && (1.4.26)
\end{aligned}
$$

We assume solutions for the equations of motion to be of the form

$$
\begin{array}{ll}
y_1 = \cos(\omega t + \delta_1) & y_2 = B\cos(\omega t + \delta_2) \\
\ddot{y}_1 = -\omega y_1 & \ddot{y}_2 = -\omega y_2
\end{array} \tag{1.4.27}
$$

Substituting the values for $\ddot{y}_1$ and $\ddot{y}_2$ into the equations of motion yields

$$\left(k + \kappa - m\omega^2\right)y_1 - \kappa y_2 = 0 \tag{1.4.28}$$

$$-\kappa y_1 + \left(k + \kappa - m\omega^2\right)y_2 = 0 \tag{1.4.29}$$

We can get solutions from solving the determinant of the matrix

$$\begin{vmatrix} (k + \kappa - m\omega^2) & -\kappa \\ -\kappa & (k + \kappa - m\omega^2) \end{vmatrix} = 0 \tag{1.4.30}$$

Solving the determinant gives

$$\left(m\omega^2\right)^2 - 2m\omega^2\left(k + \kappa\right) + \left(k^2 + 2k\kappa\right) = 0 \tag{1.4.31}$$

This yields

$$\omega^2 = \begin{cases} \dfrac{k}{m} & = \dfrac{g}{\ell} \\ \dfrac{k+2\kappa}{m} & = \dfrac{g}{\ell} + \dfrac{2\kappa}{m} \end{cases} \tag{1.4.32}$$

We can now determine exactly how the masses move with each mode by substituting $\omega^2$ into the equations of motion. Where

$\omega^2 = \dfrac{k}{m}$ We see that

$$k + \kappa - m\omega^2 = \kappa \tag{1.4.33}$$

Substituting this into the equation of motion yields

$$y_1 = y_2 \tag{1.4.34}$$

We see that the masses move in phase with each other. You will also notice the absense of the spring constant term, $\kappa$, for the connecting spring. As the masses are moving in step, the spring isn't stretching or compressing and hence its absence in our result.

$\omega^2 = \dfrac{k+\kappa}{m}$ We see that

$$k + \kappa - m\omega^2 = -\kappa \tag{1.4.35}$$

Substituting this into the equation of motion yields

$$y_1 = -y_2 \tag{1.4.36}$$

Here the masses move out of phase with each other. In this case we see the presence of the spring constant, $\kappa$, which is expected as the spring playes a role. It is being stretched and compressed as our masses oscillate.

### 1.4.7 Doppler Effect

The Doppler Effect is the shift in frequency and wavelength of waves that results from a source moving with respect to the medium, a receiver moving with respect to the medium or a moving medium.

**Moving Source** If a source is moving towards an observer, then in one period, $\tau_0$, it moves a distance of $v_s\tau_0 = v_s/f_0$. The wavelength is decreased by

$$\lambda' = \lambda - \frac{v_s}{f_0} - \frac{v - v_s}{f_0} \tag{1.4.37}$$

The frequency change is

$$f' = \frac{v}{\lambda'} = f_0\left(\frac{v}{v - v_s}\right) \tag{1.4.38}$$

**Moving Observer** As the observer moves, he will measure the same wavelength, $\lambda$, as if at rest but will see the wave crests pass by more quickly. The observer measures a modified wave speed.

$$v' = v + |v_r| \tag{1.4.39}$$

The modified frequency becomes

$$f' = \frac{v'}{\lambda} = f_0\left(1 + \frac{v_r}{v}\right) \tag{1.4.40}$$

**Moving Source and Moving Observer** We can combine the above two equations

$$\lambda' = \frac{v - v_s}{f_0} \tag{1.4.41}$$

$$v' = v - v_r \tag{1.4.42}$$

To give a modified frequency of

$$f' = \frac{v'}{\lambda'} = \left(\frac{v - v_r}{v - v_s}\right) f_0 \tag{1.4.43}$$

## 1.5 Rotational Motion about a Fixed Axis

### 1.5.1 Moment of Inertia

$$I = \int R^2 dm \tag{1.5.1}$$

### 1.5.2 Rotational Kinetic Energy

$$K = \frac{1}{2} I\omega^2 \tag{1.5.2}$$

### 1.5.3 Parallel Axis Theorem

$$I = I_{\text{cm}} + Md^2 \tag{1.5.3}$$

### 1.5.4 Torque

$$\tau = \mathbf{r} \times \mathbf{F} \tag{1.5.4}$$

$$\tau = I\alpha \tag{1.5.5}$$

where $\alpha$ is the angular acceleration.

### 1.5.5 Angular Momentum

$$\mathbf{L} = I\omega \tag{1.5.6}$$

we can find the Torque

$$\tau = \frac{d\mathbf{L}}{dt} \tag{1.5.7}$$

### 1.5.6 Kinetic Energy in Rolling

With respect to the point of contact, the motion of the wheel is a rotation about the point of contact. Thus

$$K = K_{\text{rot}} = \frac{1}{2} I_{\text{contact}} \omega^2 \tag{1.5.8}$$

$I_{\text{contact}}$ can be found from the Parallel Axis Theorem.

$$I_{\text{contact}} = I_{\text{cm}} + MR^2 \tag{1.5.9}$$

Substitute eq. (1.5.8) and we have

$$\begin{aligned} K &= \frac{1}{2}\left(I_{\text{cm}} + MR^2\right)\omega^2 \\ &= \frac{1}{2} I_{\text{cm}}\omega^2 + \frac{1}{2}mv^2 \end{aligned} \tag{1.5.10}$$

The kinetic energy of an object rolling without slipping is the sum of hte kinetic energy of rotation about its center of mass and the kinetic energy of the linear motion of the object.

## 1.6 Dynamics of Systems of Particles

### 1.6.1 Center of Mass of a System of Particles

**Position Vector of a System of Particles**

$$\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + \cdots + m_N\mathbf{r}_N}{M} \tag{1.6.1}$$

**Velocity Vector of a System of Particles**

$$\begin{aligned} \mathbf{V} &= \frac{d\mathbf{R}}{dt} \\ &= \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 + \cdots + m_N\mathbf{v}_N}{M} \end{aligned} \tag{1.6.2}$$

**Acceleration Vector of a System of Particles**

$$\begin{aligned} \mathbf{A} &= \frac{d\mathbf{V}}{dt} \\ &= \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3 + \cdots + m_N\mathbf{a}_N}{M} \end{aligned} \tag{1.6.3}$$

## 1.7 Central Forces and Celestial Mechanics

### 1.7.1 Newton's Law of Universal Gravitation

$$\mathbf{F} = -\left(\frac{GMm}{r^2}\right)\hat{r} \tag{1.7.1}$$

### 1.7.2 Potential Energy of a Gravitational Force

$$U(r) = -\frac{GMm}{r} \tag{1.7.2}$$

### 1.7.3 Escape Speed and Orbits

The energy of an orbiting body is

$$\begin{aligned} E &= T + U \\ &= \frac{1}{2}mv^2 - \frac{GMm}{r} \end{aligned} \tag{1.7.3}$$

The escape speed becomes

$$E = \frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R_E} = 0 \tag{1.7.4}$$

Solving for $v_{\text{esc}}$ we find

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R_e}} \tag{1.7.5}$$

### 1.7.4 Kepler's Laws

**First Law** The orbit of every planet is an ellipse with the sun at a focus.

**Second Law** A line joining a planet and the sun sweeps out equal areas during equal intervals of time.

**Third Law** The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

$$\frac{T^2}{R^3} = C \tag{1.7.6}$$

where $C$ is a constant whose value is the same for all planets.

### 1.7.5 Types of Orbits

The Energy of an Orbiting Body is defined in eq. (1.7.3), we can classify orbits by their eccentricities.

**Circular Orbit** A circular orbit occurs when there is an eccentricity of 0 and the orbital energy is less than 0. Thus

$$\frac{1}{2}v^2 - \frac{GM}{r} = E < 0 \tag{1.7.7}$$

The Orbital Velocity is

$$v = \sqrt{\frac{GM}{r}} \tag{1.7.8}$$

**Elliptic Orbit** An elliptic orbit occurs when the eccentricity is between 0 and 1 but the specific energy is negative, so the object remains bound.

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)} \tag{1.7.9}$$

where $a$ is the semi-major axis

**Parabolic Orbit** A Parabolic Orbit occurs when the eccentricity is equal to 1 and the orbital velocity is the escape velocity. This orbit is not bounded. Thus

$$\frac{1}{2}v^2 - \frac{GM}{r} = E = 0 \tag{1.7.10}$$

The Orbital Velocity is

$$v = v_{\text{esc}} = \sqrt{\frac{2GM}{r}} \tag{1.7.11}$$

**Hyperbolic Orbit** In the Hyperbolic Orbit, the eccentricity is greater than 1 with an orbital velocity in excess of the escape velocity. This orbit is also not bounded.

$$v_\infty = \sqrt{\frac{GM}{a}} \tag{1.7.12}$$

### 1.7.6 Derivation of Vis-viva Equation

The total energy of a satellite is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \tag{1.7.13}$$

For an elliptical or circular orbit, the specific energy is

$$E = -\frac{GMm}{2a} \tag{1.7.14}$$

Equating we get

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{1.7.15}$$

## 1.8 Three Dimensional Particle Dynamics

## 1.9 Fluid Dynamics

When an object is fully or partially immersed, the buoyant force is equal to the weight of fluid displaced.

### 1.9.1 Equation of Continuity

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \tag{1.9.1}$$

### 1.9.2 Bernoulli's Equation

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{a constant} \tag{1.9.2}$$

## 1.10 Non-inertial Reference Frames

## 1.11 Hamiltonian and Lagrangian Formalism

### 1.11.1 Lagrange's Function ($L$)

$$L = T - V \tag{1.11.1}$$

where $T$ is the Kinetic Energy and $V$ is the Potential Energy in terms of Generalized Coordinates.

### 1.11.2 Equations of Motion(Euler-Lagrange Equation)

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \tag{1.11.2}$$

### 1.11.3 Hamiltonian

$$\begin{aligned} H &= T + V \\ &= p\dot{q} - L(q, \dot{q}) \end{aligned} \tag{1.11.3}$$

where

$$\frac{\partial H}{\partial p} = \dot{q} \tag{1.11.4}$$

$$\begin{aligned} \frac{\partial H}{\partial q} &= -\frac{\partial L}{\partial x} \\ &= -\dot{p} \end{aligned} \tag{1.11.5}$$

# Chapter 2

# Electromagnetism

## 2.1 Electrostatics

### 2.1.1 Coulomb's Law

The force between two charged particles, $q_1$ and $q_2$ is defined by Coulomb's Law.

$$\boxed{\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\right)\hat{\mathbf{r}}_{\mathbf{12}}} \tag{2.1.1}$$

where $\epsilon_0$ is the permitivitty of free space, where

$$\epsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2\mathrm{N.m}^2 \tag{2.1.2}$$

### 2.1.2 Electric Field of a point charge

The electric field is defined by mesuring the magnitide and direction of an electric force, $\mathbf{F}$, acting on a test charge, $q_0$.

$$\mathbf{E} \equiv \frac{\mathbf{F}}{q_0} \tag{2.1.3}$$

The Electric Field of a point charge, $q$ is

$$\boxed{\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}} \tag{2.1.4}$$

In the case of multiple point charges, $q_i$, the electric field becomes

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i} \tag{2.1.5}$$

**Electric Fields and Continuous Charge Distributions**

If a source is distributed continuously along a region of space, eq. (2.1.5) becomes

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r^2}\hat{\mathbf{r}}dq} \tag{2.1.6}$$

If the charge was distributed along a line with linear charge density, $\lambda$,

$$\lambda = \frac{dq}{dx} \tag{2.1.7}$$

The Electric Field of a line charge becomes

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\text{line}} \frac{\lambda}{r^2}\hat{\mathbf{r}}dx} \tag{2.1.8}$$

In the case where the charge is distributed along a surface, the surface charge density is, $\sigma$

$$\sigma = \frac{Q}{A} = \frac{dq}{dA} \tag{2.1.9}$$

The electric field along the surface becomes

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\text{Surface}} \frac{\sigma}{r^2}\hat{\mathbf{r}}dA} \tag{2.1.10}$$

In the case where the charge is distributed throughout a volume, $V$, the volume charge density is

$$\rho = \frac{Q}{V} = \frac{dq}{dV} \tag{2.1.11}$$

The Electric Field is

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\text{Volume}} \frac{\rho}{r^2}\hat{\mathbf{r}}dV} \tag{2.1.12}$$

### 2.1.3 Gauss' Law

The electric field through a surface is

$$\boxed{\Phi = \oint_{\text{surface } S} d\Phi = \oint_{\text{surface } S} \mathbf{E}\cdot d\mathbf{A}} \tag{2.1.13}$$

The electric flux through a closed surface encloses a net charge.

$$\boxed{\oint \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\epsilon_0}} \tag{2.1.14}$$

where $Q$ is the charge enclosed by our surface.

### 2.1.4 Equivalence of Coulomb's Law and Gauss' Law

The total flux through a sphere is

$$\oint \mathbf{E} \cdot d\mathbf{A} = E(4\pi r^2) = \frac{q}{\epsilon_0} \tag{2.1.15}$$

From the above, we see that the electric field is

$$E = \frac{q}{4\pi\epsilon_0 r^2} \tag{2.1.16}$$

### 2.1.5 Electric Field due to a line of charge

Consider an infinite rod of constant charge density, $\lambda$. The flux through a Gaussian cylinder enclosing the line of charge is

$$\Phi = \int_{\text{top surface}} \mathbf{E} \cdot d\mathbf{A} + \int_{\text{bottom surface}} \mathbf{E} \cdot d\mathbf{A} + \int_{\text{side surface}} \mathbf{E} \cdot d\mathbf{A} \tag{2.1.17}$$

At the top and bottom surfaces, the electric field is perpendicular to the area vector, so for the top and bottom surfaces,

$$\mathbf{E} \cdot d\mathbf{A} = 0 \tag{2.1.18}$$

At the side, the electric field is parallel to the area vector, thus

$$\mathbf{E} \cdot d\mathbf{A} = EdA \tag{2.1.19}$$

Thus the flux becomes,

$$\Phi = \int_{\text{side sirface}} \mathbf{E} \cdot d\mathbf{A} = E \int dA \tag{2.1.20}$$

The area in this case is the surface area of the side of the cylinder, $2\pi rh$.

$$\Phi = 2\pi rhE \tag{2.1.21}$$

Applying Gauss' Law, we see that $\Phi = q/\epsilon_0$. The electric field becomes

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}} \tag{2.1.22}$$

### 2.1.6 Electric Field in a Solid Non-Conducting Sphere

Within our non-conducting sphere or radius, $R$, we will assume that the total charge, $Q$ is evenly distributed throughout the sphere's volume. So the charge density of our sphere is

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} \tag{2.1.23}$$

The Electric Field due to a charge $Q$ is

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \tag{2.1.24}$$

As the charge is evenly distributed throughout the sphere's volume we can say that the charge density is

$$dq = \rho dV \tag{2.1.25}$$

where $dV = 4\pi r^2 dr$. We can use this to determine the field inside the sphere by summing the effect of infinitesimally thin spherical shells

$$\begin{aligned} E = \int_0^E dE &= \int_0^r \frac{dq}{4\pi\epsilon r^2} \\ &= \frac{\rho}{\epsilon_0}\int_0^r dr \\ &= \frac{Qr}{\frac{4}{3}\pi\epsilon_0 R^3} \end{aligned} \tag{2.1.26}$$

### 2.1.7 Electric Potential Energy

$$U(r) = \frac{1}{4\pi\epsilon_0} q q_0 r \tag{2.1.27}$$

### 2.1.8 Electric Potential of a Point Charge

The electrical potential is the potential energy per unit charge that is associated with a static electrical field. It can be expressed thus

$$U(r) = qV(r) \tag{2.1.28}$$

And we can see that

$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{2.1.29}$$

A more proper definition that includes the electric field, **E** would be

$$\boxed{V(\mathbf{r}) = -\int_C \mathbf{E}\cdot d\ell} \tag{2.1.30}$$

where $C$ is any path, starting at a chosen point of zero potential to our desired point.

The difference between two potentials can be expressed such

$$\begin{aligned} V(\mathbf{b}) - V(\mathbf{a}) &= -\int^{\mathbf{b}} \mathbf{E}\cdot d\ell + \int^{\mathbf{a}} \mathbf{E}\cdot d\ell \\ &= -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E}\cdot d\ell \end{aligned} \tag{2.1.31}$$

This can be further expressed

$$V(\mathbf{b}) - V(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} (\nabla V) \cdot d\ell \tag{2.1.32}$$

And we can show that

$$\boxed{\mathbf{E} = -\nabla V} \tag{2.1.33}$$

### 2.1.9 Electric Potential due to a line charge along axis

Let us consider a rod of length, $\ell$, with linear charge density, $\lambda$. The Electrical Potential due to a continuous distribution is

$$\boxed{V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}} \tag{2.1.34}$$

The charge density is

$$dq = \lambda dx \tag{2.1.35}$$

Substituting this into the above equation, we get the electrical potential at some distance $x$ along the rod's axis, with the origin at the start of the rod.

$$\begin{aligned} dV &= \frac{1}{4\pi\epsilon_0}\frac{dq}{x} \\ &= \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{x} \end{aligned} \tag{2.1.36}$$

This becomes

$$V = \frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{x_2}{x_1}\right] \tag{2.1.37}$$

where $x_1$ and $x_2$ are the distances from $O$, the end of the rod.

Now consider that we are some distance, $y$, from the axis of the rod of length, $\ell$. We again look at eq. (2.1.34), where $r$ is the distance of the point $P$ from the rod's axis.

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \\ &= \frac{1}{4\pi\epsilon_0}\int_0^{\ell} \frac{\lambda dx}{(x^2+y^2)^{\frac{1}{2}}} \\ &= \frac{\lambda}{4\pi\epsilon_0}\ln\left[x + \left(x^2+y^2\right)^{\frac{1}{2}}\right]_0^{\ell} \\ &= \frac{\lambda}{4\pi\epsilon_0}\ln\left[\ell + \left(\ell^2+y^2\right)^{\frac{1}{2}}\right] - \ln y \\ &= \frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{\ell + (\ell^2+y^2)^{\frac{1}{2}}}{d}\right] \end{aligned} \tag{2.1.38}$$

## 2.2 Currents and DC Circuits

2

## 2.3 Magnetic Fields in Free Space

3

## 2.4 Lorentz Force

4

## 2.5 Induction

5

## 2.6 Maxwell's Equations and their Applications

6

## 2.7 Electromagnetic Waves

7

## 2.8 AC Circuits

8

## 2.9 Magnetic and Electric Fields in Matter

9

## 2.10 Capacitance

$$Q = CV \tag{2.10.1}$$

## 2.11 Energy in a Capacitor

$$\begin{aligned} U &= \frac{Q^2}{2C} \\ &= \frac{CV^2}{2} \\ &= \frac{QV}{2} \end{aligned} \tag{2.11.1}$$

## 2.12 Energy in an Electric Field

$$u \equiv \frac{U}{\text{volume}} = \frac{\epsilon_0 E^2}{2} \tag{2.12.1}$$

## 2.13 Current

$$I \equiv \frac{dQ}{dt} \tag{2.13.1}$$

## 2.14 Current Destiny

$$I = \int_A \mathbf{J} \cdot d\mathbf{A} \tag{2.14.1}$$

## 2.15 Current Density of Moving Charges

$$\mathbf{J} = \frac{I}{A} = n_e q \mathbf{v}_d \tag{2.15.1}$$

## 2.16 Resistance and Ohm's Law

$$R \equiv \frac{V}{I} \tag{2.16.1}$$

## 2.17 Resistivity and Conductivity

$$R = \rho \frac{L}{A} \tag{2.17.1}$$

$$E = \rho \mathbf{J} \tag{2.17.2}$$

$$\mathbf{J} = \sigma \mathbf{E} \tag{2.17.3}$$

## 2.18 Power

$$P = VI \tag{2.18.1}$$

## 2.19 Kirchoff's Loop Rules

**Write Here**

## 2.20 Kirchoff's Junction Rule

**Write Here**

## 2.21 RC Circuits

$$\mathscr{E} - IR - \frac{Q}{C} = 0 \tag{2.21.1}$$

## 2.22 Maxwell's Equations

### 2.22.1 Integral Form

**Gauss' Law for Electric Fields**

$$\int_{\text{closed surface}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0} \tag{2.22.1}$$

**Gauss' Law for Magnetic Fields**

$$\int_{\text{closed surface}} \mathbf{B} \cdot d\mathbf{A} = 0 \tag{2.22.2}$$

**Ampère's Law**

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} \tag{2.22.3}$$

**Faraday's Law**

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int_{\text{surface}} \mathbf{B} \cdot d\mathbf{A} \tag{2.22.4}$$

### 2.22.2 Differential Form

**Gauss' Law for Electric Fields**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{2.22.5}$$

**Gauss' Law for Magnetism**

$$\nabla \cdot \mathbf{B} = 0 \tag{2.22.6}$$

**Ampère's Law**

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{2.22.7}$$

**Faraday's Law**

$$\nabla \cdot \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.22.8}$$

## 2.23 Speed of Propagation of a Light Wave

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \tag{2.23.1}$$

In a material with dielectric constant, $\kappa$,

$$c\sqrt{\kappa} = \frac{c}{n} \tag{2.23.2}$$

where $n$ is the refractive index.

## 2.24 Relationship between E and B Fields

$$E = cB \tag{2.24.1}$$

$$\mathbf{E} \cdot \mathbf{B} = 0 \tag{2.24.2}$$

## 2.25 Energy Density of an EM wave

$$u = \frac{1}{2}\left(\frac{B^2}{\mu_0} + \epsilon_0 E^2\right) \tag{2.25.1}$$

## 2.26 Poynting's Vector

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \tag{2.26.1}$$

# Chapter 3

# Optics & Wave Phonomena

## 3.1 Wave Properties

1

## 3.2 Superposition

2

## 3.3 Interference

3

## 3.4 Diffraction

4

## 3.5 Geometrical Optics

5

## 3.6 Polarization

6

## 3.7 Doppler Effect

7

## 3.8 Snell's Law

### 3.8.1 Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{3.8.1}$$

### 3.8.2 Critical Angle and Snell's Law

The critical angle, $\theta_c$, for the boundary seperating two optical media is the smallest angle of incidence, in the medium of greater index, for which light is totally refelected.

From eq. (3.8.1), $\theta_1 = 90$ and $\theta_2 = \theta_c$ and $n_2 > n_1$.

$$\begin{aligned} n_1 \sin 90 &= n_2 sin\theta_c \\ \sin\theta_c &= \frac{n_1}{n_2} \end{aligned} \tag{3.8.2}$$

# Chapter 4

# Thermodynamics & Statistical Mechanics

## 4.1 Laws of Thermodynamics

1

## 4.2 Thermodynamic Processes

2

## 4.3 Equations of State

3

## 4.4 Ideal Gases

4

## 4.5 Kinetic Theory

5

## 4.6 Ensembles

6

## 4.7 Statistical Concepts and Calculation of Thermodynamic Properties

7

## 4.8 Thermal Expansion & Heat Transfer

8

## 4.9 Heat Capacity

$$Q = C\left(T_f - T_i\right) \tag{4.9.1}$$

where $C$ is the Heat Capacity and $T_f$ and $T_i$ are the final and initial temperatures respectively.

## 4.10 Specific Heat Capacity

$$Q = cm\left(T_f - t_i\right) \tag{4.10.1}$$

where $c$ is the specific heat capacity and $m$ is the mass.

## 4.11 Heat and Work

$$W = \int_{V_i}^{V_f} PdV \tag{4.11.1}$$

## 4.12 First Law of Thermodynamics

$$dE_{int} = dQ - dW \tag{4.12.1}$$

where $dE_{int}$ is the internal energy of the system, $dQ$ is the Energy added to the system and $dW$ is the work done by the system.

### 4.12.1 Special Cases to the First Law of Thermodynamics

**Adiabatic Process** During an adiabatic process, the system is insulated such that there is no heat transfer between the system and its environment. Thus $dQ = 0$, so

$$\Delta E_{int} = -W \tag{4.12.2}$$

If work is done on the system, negative $W$, then there is an increase in its internal energy. Conversely, if work is done by the system, positive $W$, there is a decrease in the internal energy of the system.

**Constant Volume (Isochoric) Process** If the volume is held constant, then the system can do no work, $\delta W = 0$, thus

$$\Delta E_{int} = Q \tag{4.12.3}$$

If heat is added to the system, the temperature increases. Conversely, if heat is removed from the system the temperature decreases.

**Closed Cycle** In this situation, after certain interchanges of heat and work, the system comes back to its initial state. So $\Delta E_{int}$ remains the same, thus

$$\Delta Q = \Delta W \tag{4.12.4}$$

The work done by the system is equal to the heat or energy put into it.

**Free Expansion** In this process, no work is done on or by the system. Thus $\Delta Q = \Delta W = 0$,

$$\Delta E_{int} = 0 \tag{4.12.5}$$

## 4.13 Work done by Ideal Gas at Constant Temperature

Starting with eq. (4.11.1), we substitute the Ideal gas Law, eq. (4.15.1), to get

$$\begin{aligned} W &= nRT \int_{V_i}^{V_f} \frac{dV}{V} \\ &= nRT \ln \frac{V_f}{V_i} \end{aligned} \tag{4.13.1}$$

## 4.14 Heat Conduction Equation

The rate of heat transferred, $H$, is given by

$$H = \frac{Q}{t} = kA\frac{T_H - T_C}{L} \tag{4.14.1}$$

where $k$ is the thermal conductivity.

## 4.15 Ideal Gas Law

$$PV = nRT \tag{4.15.1}$$

where

$$\begin{aligned} n &= \text{Number of moles} \\ P &= \text{Pressure} \\ V &= \text{Volume} \\ T &= \text{Temperature} \end{aligned}$$

and $R$ is the **Universal Gas Constant**, such that

$$R \approx 8.314 \text{ J/mol. K}$$

We can rewrite the Ideal gas Law to say

$$PV = NkT \tag{4.15.2}$$

where $k$ is the **Boltzmann's Constant**, such that

$$k = \frac{R}{N_A} \approx 1.381 \times 10^{-23} \text{ J/K}$$

## 4.16 Stefan-Boltzmann's FormulaStefan-Boltzmann's Equation

$$P(T) = \sigma T^4 \tag{4.16.1}$$

## 4.17 RMS Speed of an Ideal Gas

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \tag{4.17.1}$$

## 4.18 Translational Kinetic Energy

$$\bar{K} = \frac{3}{2}kT \tag{4.18.1}$$

## 4.19 Internal Energy of a Monatomic gas

$$E_{\text{int}} = \frac{3}{2}nRT \tag{4.19.1}$$

## 4.20 Molar Specific Heat at Constant Volume

Let us define, $C_V$ such that

$$Q = nC_V\Delta T \tag{4.20.1}$$

Substituting into the First Law of Thermodynamics, we have

$$\Delta E_{\text{int}} + W = nC_V\Delta T \tag{4.20.2}$$

At constant volume, $W = 0$, and we get

$$C_V = \frac{1}{n}\frac{\Delta E_{\text{int}}}{\Delta T} \tag{4.20.3}$$

Substituting eq. (4.19.1), we get

$$C_V = \frac{3}{2}R = 12.5 \text{ J/mol.K} \tag{4.20.4}$$

## 4.21 Molar Specific Heat at Constant Pressure

Starting with

$$Q = nC_p\Delta T \tag{4.21.1}$$

and

$$\begin{aligned} \Delta E_{\text{int}} &= Q - W \\ \Rightarrow nC_V\Delta T &= nC_p\Delta T + nR\Delta T \\ \therefore C_V &= C_p - R \end{aligned} \tag{4.21.2}$$

## 4.22 Equipartition of Energy

$$C_V = \left(\frac{f}{2}\right)R = 4.16f \text{ J/mol.K} \tag{4.22.1}$$

where $f$ is the number of degrees of freedom.

| | Degrees of Freedom | | | Predicted Molar Specific Heats | |
|---|---|---|---|---|---|
| Molecule | Translational | Rotational | Total ($f$) | $C_V$ | $C_P = C_V + R$ |
| Monatomic | 3 | 0 | 3 | $\frac{3}{2}R$ | $\frac{5}{2}R$ |
| Diatomic | 3 | 2 | 5 | $\frac{5}{2}R$ | $\frac{7}{2}R$ |
| Polyatomic | 3 | 3 | 6 | $3R$ | $4R$ |

Table 4.22.1: Table of Molar Specific Heats

## 4.23 Adiabatic Expansion of an Ideal Gas

$$PV^{\gamma} = \text{a constant} \tag{4.23.1}$$

where $\gamma = \frac{C_P}{C_V}$.
We can also write

$$TV^{\gamma-1} = \text{a constant} \tag{4.23.2}$$

## 4.24 Second Law of Thermodynamics

Something.

# Chapter 5

# Quantum Mechanics

## 5.1 Fundamental Concepts

1

## 5.2 Schrödinger Equation

Let us define $\Psi$ to be

$$\Psi = Ae^{-i\omega\left(t-\frac{x}{v}\right)} \tag{5.2.1}$$

Simplifying in terms of Energy, $E$, and momentum, $p$, we get

$$\Psi = Ae^{-\frac{i\left(Et-px\right)}{\hbar}} \tag{5.2.2}$$

We obtain Schrödinger's Equation from the Hamiltonian

$$H = T + V \tag{5.2.3}$$

To determine $E$ and $p$,

$$\frac{\partial^2\Psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\Psi \tag{5.2.4}$$

$$\frac{\partial\Psi}{\partial t} = \frac{iE}{\hbar}\Psi \tag{5.2.5}$$

and

$$H = \frac{p^2}{2m} + V \tag{5.2.6}$$

This becomes

$$E\Psi = H\Psi \tag{5.2.7}$$

$$E\Psi = -\frac{\hbar}{i}\frac{\partial\Psi}{\partial t} \qquad p^2\Psi = -\hbar^2\frac{\partial^2\Psi}{\partial x^2}$$

The **Time Dependent Schrödinger's Equation** is

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi \tag{5.2.8}$$

The **Time Independent Schrödinger's Equation** is

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi \tag{5.2.9}$$

### 5.2.1 Infinite Square Wells

Let us consider a particle trapped in an infinite potential well of size $a$, such that

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a \\ \infty & \text{for } |x| > a, \end{cases}$$

so that a nonvanishing force acts only at $\pm a/2$. An energy, $E$, is assigned to the system such that the kinetic energy of the particle is $E$. Classically, any motion is forbidden outside of the well because the infinite value of $V$ exceeds any possible choice of $E$.

Recalling the Schrödinger Time Independent Equation, eq. (5.2.9), we substitute $V(x)$ and in the region $(-a/2, a/2)$, we get

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \tag{5.2.10}$$

This differential is of the form

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \tag{5.2.11}$$

where

$$k = \sqrt{\frac{2mE}{\hbar^2}} \tag{5.2.12}$$

We recognize that possible solutions will be of the form

$$\cos kx \quad \text{and} \ \sin kx$$

As the particle is confined in the region $0 < x < a$, we say

$$\psi(x) = \begin{cases} A\cos kx + B\sin kx & \text{for } 0 < x < a \\ 0 & \text{for } |x| > a \end{cases}$$

We have known boundary conditions for our square well.

$$\psi(0) = \psi(a) = 0 \tag{5.2.13}$$

It shows that

$$\begin{aligned} \Rightarrow A\cos 0 + B\sin 0 &= 0 \\ \therefore A &= 0 \end{aligned} \tag{5.2.14}$$

We are now left with

$$\begin{aligned} B \sin ka &= 0 \\ ka &= 0; \pi; 2\pi; 3\pi; \cdots \end{aligned} \tag{5.2.15}$$

While mathematically, $n$ can be zero, that would mean there would be no wave function, so we ignore this result and say

$$k_n = \frac{n\pi}{a} \quad \text{for } n = 1, 2, 3, \cdots$$

Substituting this result into eq. (5.2.12) gives

$$k_n = \frac{n\pi}{a} = \frac{\sqrt{2mE_n}}{\hbar} \tag{5.2.16}$$

Solving for $E_n$ gives

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \tag{5.2.17}$$

We cna now solve for $B$ by normalizing the function

$$\begin{aligned} \int_0^a |B|^2 \sin^2 kx dx &= |A|^2 \frac{a}{2} = 1 \\ \text{So} \quad |A|^2 &= \frac{2}{a} \end{aligned} \tag{5.2.18}$$

So we can write the wave function as

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \tag{5.2.19}$$

### 5.2.2 Harmonic Oscillators

Classically, the harmonic oscillator has a potential energy of

$$V(x) = \frac{1}{2}kx^2 \tag{5.2.20}$$

So the force experienced by this particle is

$$F = -\frac{dV}{dx} = -kx \tag{5.2.21}$$

where $k$ is the spring constant. The equation of motion can be summed us as

$$m\frac{d^2x}{dt^2} = -kx \tag{5.2.22}$$

And the solution of this equation is

$$x(t) = A\cos\left(\omega_0 t + \phi\right) \tag{5.2.23}$$

where the angular frequency, $\omega_0$ is

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{5.2.24}$$

The Quantum Mechanical description on the harmonic oscillator is based on the eigenfunction solutions of the time-independent Schrödinger's equation. By taking $V(x)$ from eq. (5.2.20) we substitute into eq. (5.2.9) to get

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}\left(\frac{k}{2}x^2 - E\right)\psi = \frac{mk}{\hbar^2}\left(x^2 - \frac{2E}{k}\right)\psi$$

With some manipulation, we get

$$\frac{\hbar}{\sqrt{mk}}\frac{d^2\psi}{dx^2} = \left(\frac{\sqrt{mk}}{\hbar}x^2 - \frac{2E}{\hbar}\sqrt{\frac{m}{k}}\right)\psi$$

This step allows us to to keep some of constants out of the way, thus giving us

$$\xi^2 = \frac{\sqrt{mk}}{\hbar}x^2 \tag{5.2.25}$$

$$\text{and} \quad \lambda = \frac{2E}{\hbar}\sqrt{\frac{m}{k}} = \frac{2E}{\hbar\omega_0} \tag{5.2.26}$$

This leads to the more compact

$$\frac{d^2\psi}{d\xi^2} = \left(\xi^2 - \lambda\right)\psi \tag{5.2.27}$$

where the eigenfunction $\psi$ will be a function of $\xi$. $\lambda$ assumes an eigenvalue anaglaous to $E$.

From eq. (5.2.25), we see that the maximum value can be determined to be

$$\xi^2_{\text{max}} = \frac{\sqrt{mk}}{\hbar}A^2 \tag{5.2.28}$$

Using the classical connection between $A$ and $E$, allows us to say

$$\xi^2_{\text{max}} = \frac{\sqrt{mk}}{\hbar}\frac{2E}{k} = \lambda \tag{5.2.29}$$

From eq. (5.2.27), we see that in a quantum mechanical oscillator, there are non-vanishing solutions in the forbidden regions, unlike in our classical case.

A solution to eq. (5.2.27) is

$$\psi(\xi) = e^{-\xi^2/2} \tag{5.2.30}$$

where

$$\frac{d\psi}{d\xi} = -\xi e^{-\xi^2/2}$$

$$\text{and} \quad \frac{d^\psi}{d\xi^2} = \xi^2 e^{-xi^2/2} - e^{-\xi^2/2} = \left(\xi^2 - 1\right)e^{-\xi^2/2}$$

This gives is a special solution for $\lambda$ where

$$\lambda_0 = 1 \tag{5.2.31}$$

Thus eq. (5.2.26) gives the energy eigenvalue to be

$$E_0 = \frac{\hbar\omega_0}{2}\lambda_0 = \frac{\hbar\omega_0}{2} \tag{5.2.32}$$

The eigenfunction $e^{-\xi^2/2}$ corresponds to a normalized stationary-state wave function

$$\Psi_0(x,t) = \left(\frac{mk}{\pi^2\hbar^2}\right)^{\frac{1}{8}} e^{-\sqrt{mk}\,x^2/2\hbar} e^{-iE_0t/\hbar} \tag{5.2.33}$$

This solution of eq. (5.2.27) produces the smallest possibel result of $\lambda$ and $E$. Hence, $\Psi_0$ and $E_0$ represents the ground state of the oscillator. and the quantity $\hbar\omega_0/2$ is the zero-point energy of the system.

### 5.2.3 Finite Square Well

For the Finite Square Well, we have a potential region where

$$V(x) = \begin{cases} -V_0 & \text{for } -a \le x \le a \\ 0 & \text{for } |x| > a \end{cases}$$

We have three regions

**Region I:** $x < -a$ In this region, The potential, $V = 0$, so Schrödinger's Equation becomes

$$\begin{aligned} -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} &= E\psi \\ \Rightarrow \frac{d^2\psi}{dx^2} &= \kappa^2\psi \\ \text{where} \quad \kappa &= \frac{\sqrt{-2mE}}{\hbar} \end{aligned}$$

This gives us solutions that are

$$\psi(x) = A\exp(-\kappa x) + B\exp(\kappa x)$$

As $x \to \infty$, the $\exp(-\kappa x)$ term goes to $\infty$; it blows up and is not a physically realizable function. So we can drop it to get

$$\psi(x) = Be^{\kappa x} \quad \text{for } x < -a \tag{5.2.34}$$

**Region II:** $-a < x < a$ In this region, our potential is $V(x) = V_0$. Substitutin this into the Schrödinger's Equation, eq. (5.2.9), gives

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi$$
$$\text{or} \quad \frac{d^2\psi}{dx^2} = -l^2\psi$$
$$\text{where} \quad l \equiv \frac{\sqrt{2m\,(E+V_0)}}{\hbar} \tag{5.2.35}$$

We notice that $E > -V_0$, making $l$ real and positive. Thus our general solution becomes

$$\psi(x) = C\sin(lx) + D\cos(lx) \quad \text{for } -a < x < a \tag{5.2.36}$$

**Region III:** $x > a$ Again this Region is similar to Region III, where the potential, $V = 0$. This leaves us with the general solution

$$\psi(x) = F\exp(-\kappa x) + G\exp(\kappa x)$$

As $x \to \infty$, the second term goes to infinity and we get

$$\psi(x) = Fe^{-\kappa x} \quad \text{for } x > a \tag{5.2.37}$$

This gives us

$$\psi(x) = \begin{cases} Be^{\kappa x} & \text{for } x < a \\ D\cos(lx) & \text{for } 0 < x < a \\ Fe^{-\kappa x} & \text{for } x > a \end{cases} \tag{5.2.38}$$

### 5.2.4 Hydrogenic Atoms

c

## 5.3 Spin

3

## 5.4 Angular Momentum

4

## 5.5 Wave Funtion Symmetry

5

## 5.6 Elementary Perturbation Theory

6

# Chapter 6

# Atomic Physics

## 6.1 Properties of Electrons

1

## 6.2 Bohr Model

To understand the Bohr Model of the Hydrogen atom, we will take advantage of our knowlegde of the wavelike properties of matter. As we are building on a classical model of the atom with a modern concept of matter, our derivation is considered to be 'semi-classical'. In this model we have an electron of mass, $m_e$, and charge, $-e$, orbiting a proton. The cetripetal force is equal to the Coulomb Force. Thus

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{m_e v^2}{r} \tag{6.2.1}$$

The Total Energy is the sum of the potential and kinetic energies, so

$$E = K + U = \frac{p^2}{2m_e} - |frace^2 4\pi\epsilon_0 r \tag{6.2.2}$$

We can further reduce this equation by subsituting the value of momentum, which we find to be

$$\frac{p^2}{2m_e} = \frac{1}{2}m_e v^2 = \frac{e^2}{8\pi\epsilon_0 r} \tag{6.2.3}$$

Substituting this into eq. (6.2.2), we get

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r} \tag{6.2.4}$$

At this point our classical description must end. An accelerated charged particle, like one moving in circular motion, radiates energy. So our atome here will radiate energy and our electron will spiral into the nucleus and disappear. To solve this conundrum, Bohr made two assumptions.

1. The classical circular orbits are replaced by stationary states. These stationary states take discreet values.

2. The energy of these stationary states are determined by their angular momentum which must take on quantized values of $\hbar$.

$$L = n\hbar \tag{6.2.5}$$

We can find the angular momentum of a circular orbit.

$$L = m_3 v r \tag{6.2.6}$$

From eq. (6.2.1) we find $v$ and by substitution, we find $L$.

$$L = e\sqrt{\frac{m_3 r}{4\pi\epsilon_0}} \tag{6.2.7}$$

Solving for $r$, gives

$$r = \frac{L^2}{m_e e^2/4\pi\epsilon_0} \tag{6.2.8}$$

We apply the condition from eq. (6.2.5)

$$r_n = \frac{n^2\hbar^2}{m_e e^2/4\pi\epsilon_0} = n^2 a_0 \tag{6.2.9}$$

where $a_0$ is the Bohr radius.

$$a_0 = 0.53 \times 10^{-10}\ \text{m} \tag{6.2.10}$$

Having discreet values for the allowed radii means that we will also have discreet values for energy. Replacing our value of $r_n$ into eq. (6.2.4), we get

$$E_n = -\frac{m_e}{2n^2}\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right) = -\frac{13.6}{n^2}\ \text{eV} \tag{6.2.11}$$

## 6.3 Energy Quantization

3

## 6.4 Atomic Structure

4

## 6.5 Atomic Spectra

### 6.5.1 Rydberg's Equation

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \tag{6.5.1}$$

where $R_H$ is the Rydberg constant.

For the Balmer Series, $n' = 2$, which determines the optical wavelengths. For $n' = 3$, we get the infrared or Paschen series. The fundamental $n' = 1$ series falls in the ultraviolet region and is known as the Lyman series.

## 6.6 Selection Rules

6

## 6.7 Black Body Radiation

### 6.7.1 Plank Formula

$$u(f,T) = \frac{8\pi\hbar}{c^3}\frac{f^3}{e^{hf/kT} - 1} \tag{6.7.1}$$

### 6.7.2 Stefan-Boltzmann Formula

$$P(T) = \sigma T^4 \tag{6.7.2}$$

### 6.7.3 Wein's Displacement Law

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m.K} \tag{6.7.3}$$

### 6.7.4 Classical and Quantum Aspects of the Plank Equation

**Rayleigh's Equation**

$$u(f,T) = \frac{8\pi f^2}{c^3}kT \tag{6.7.4}$$

We can get this equation from Plank's Equation, eq. (6.7.1). This equation is a classical one and does not contain Plank's constant in it. For this case we will look at the situation where $hf < kT$. In this case, we make the approximation

$$e^x \simeq 1 + x \tag{6.7.5}$$

Thus the demonimator in eq. (6.7.1) becomes

$$e^{hf/kT} - 1 \simeq 1 + \frac{hf}{kT} - 1 = \frac{hf}{kT} \tag{6.7.6}$$

Thus eq. (6.7.1) takes the approximate form

$$u(f,T) \simeq \frac{8\pi h}{c^3} f^3 \frac{kT}{hf} = \frac{8\pi f^2}{c^3} kT \tag{6.7.7}$$

As we can see this equation is devoid of Plank's constant and thus independent of quantum effects.

**Quantum**

At large frequencies, where $hf > kT$, quantum effects become apparent. We can estimate that

$$e^{hf/kT} - 1 \simeq e^{hf/kT} \tag{6.7.8}$$

Thus eq. (6.7.1) becomes

$$u(f,T) \simeq \frac{8\pi h}{c^3} f^3 e^{-hf/kT} \tag{6.7.9}$$

## 6.8 X-Rays

### 6.8.1 Bragg Condition

$$2d \sin\theta = m\lambda \tag{6.8.1}$$

for constructive interference off parallel planes of a crystal with lattics spacing, $d$.

### 6.8.2 The Compton Effect

The Compton Effect deals with the scattering of monochromatic X-Rays by atomic targets and the observation that the wavelength of the scattered X-ray is greater than the incident radiation. The photon energy is given by

$$\mathscr{E} = h\upsilon = \frac{hc}{\lambda} \tag{6.8.2}$$

The photon has an associated momentum

$$\mathscr{E} = pc \tag{6.8.3}$$

$$\Rightarrow p = \frac{\mathscr{E}}{c} = \frac{h\upsilon}{c} = \frac{h}{\lambda} \tag{6.8.4}$$

The Relativistic Energy for the electron is

$$E^2 = p^2c^2 + m_e^2c^4 \tag{6.8.5}$$

where

$$\mathbf{p} - \mathbf{p}' = \mathbf{P} \tag{6.8.6}$$

Squaring eq. (6.8.6) gives

$$p^2 - 2\mathbf{p}\cdot\mathbf{p}' + p'^2 = P^2 \tag{6.8.7}$$

Recall that $\mathscr{E} = pc$ and $\mathscr{E}' = cp'$, we have

$$\begin{aligned} c^2p^2 - 2c^2\mathbf{p}\cdot\mathbf{p}' + c^2p'^2 &= c^2P^2 \\ \mathscr{E}^2 - 2\mathscr{E}\mathscr{E}'\cos\theta + \mathscr{E}'^2 &= E^2 - m_e^2c^4 \end{aligned} \tag{6.8.8}$$

Conservation of Energy leads to

$$\mathscr{E} + m_ec^2 = \mathscr{E}' + E \tag{6.8.9}$$

Solving

$$\mathscr{E} - \mathscr{E}' = E - m_ec^2$$

$$\mathscr{E}^2 - 2\mathscr{E}\mathscr{E}' + \mathscr{E}' = E^2 - 2Em_ec^2 + m_e^2c^4 \tag{6.8.10}$$

$$2\mathscr{E}\mathscr{E}' - 2\mathscr{E}\mathscr{E}'\cos\theta = 2Em_ec^2 - 2m_e^2c^4 \tag{6.8.11}$$

Solving leads to

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_ec}(1 - \cos\theta) \tag{6.8.12}$$

where $\lambda_c = \frac{h}{m_ec}$ is the Compton Wavelength.

$$\lambda_c = \frac{h}{m_ec} = 2.427 \times 10^{-12}\text{m} \tag{6.8.13}$$

## 6.9 Atoms in Electric and Magnetic Fields

### 6.9.1 The Cyclotron Frequency

A test charge, $q$, with velocity $\mathbf{v}$ enters a uniform magnetic field, $\mathbf{B}$. The force acting on the charge will be perpendicular to $\mathbf{v}$ such that

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \tag{6.9.1}$$

or more simply $F_B = qvB$. As this traces a circular path, from Newton's Second Law, we see that

$$F_B = \frac{mv^2}{R} = qvB \tag{6.9.2}$$

Solving for $R$, we get

$$R = \frac{mv}{qB} \tag{6.9.3}$$

We also see that

$$f = \frac{qB}{2\pi m} \tag{6.9.4}$$

The frequency is depends on the charge, $q$, the magnetic field strength, $B$ and the mass of the charged particle, $m$.

### 6.9.2 Zeeman Effect

The Zeeman effect was the splitting of spectral lines in a static magnetic field. This is similar to the Stark Effect which was the splitting in the presence in a magnetic field.

In the Zeeman experiment, a sodium flame was placed in a magnetic field and its spectrum observed. In the presence of the field, a spectral line of frequency, $v_0$ was split into three components, $v_0 - \delta v$, $v_0$ and $v_0 + \delta v$. A classical analysis of this effect allows for the identification of the basic parameters of the interacting system.

The application of a constant magnetic field, $\mathbf{B}$, allows for a direction in space in which the electron motion can be referred. The motion of an electron can be attributed to a simple harmonic motion under a binding force $-kr$, where the frequency is

$$v_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m_e}} \tag{6.9.5}$$

The magnetic field subjects the electron to an additional Lorentz Force, $-e\mathbf{v} \times \mathbf{B}$. This produces two different values for the angular velocity.

$$v = 2\pi r v$$

The cetripetal force becomes

$$\frac{m_e v^2}{r} = 4\pi^2 v^2 r m_e$$

Thus the certipetal force is

$$\begin{aligned} 4\pi^2 v^2 r m_e &= 2\pi v r e B + kr \quad \text{for clockwise motion} \\ 4\pi^2 v^2 r m_e &= -2\pi v r e B + kr \quad \text{for counterclockwise motion} \end{aligned}$$

We use eq. (6.9.5), to emiminate $k$, to get

$$\begin{aligned} v^2 - \frac{eB}{2\pi m_e} v - v_0 &= 0 \quad \text{(Clockwise)} \\ v^2 + \frac{eB}{2\pi m_e} v - v_0 &= 0 \quad \text{(Counterclockwise)} \end{aligned}$$

As we have assumed a small Lorentz force, we can say that the linear terms in $v$ are small comapred to $v_0$. Solving the above quadratic equations leads to

$$v = v_0 + \frac{eB}{4\pi m_e} \quad \text{for clockwise motion} \tag{6.9.6}$$

$$v = v_0 - \frac{eB}{4\pi m_e} \quad \text{for counterclockwise motion} \tag{6.9.7}$$

We note that the frequency shift is of the form

$$\delta v = \frac{eB}{4\pi m_e} \tag{6.9.8}$$

If we view the source along the direction of $\mathbf{B}$, we will observe the light to have two polarizations, a closckwise circular polarization of $v_0 + \delta v$ and a counterclosckwise circular polarization of $v_0 - \delta v$.

### 6.9.3 Franck-Hertz Experiment

The Franck-Hertz experiment, performed in 1914 by J. Franck and G. L. Hertz, measured the colisional excitation of atoms. Their experiement studied the current of electrons in a tub of mercury vapour which revealed an abrupt change in the current at certain critical values of the applied voltage.[1] They interpreted this observation as evidence of a threshold for inelastic scattering in the colissions of electrons in mercury atoms.The bahavior of the current was an indication that electrons could lose a discreet amount of energy and excite mercury atoms in their passage through the mercury vapour. These observations constituted a direct and decisive confirmation of the existence os quantized energy levels in atoms.



[1]Put drawing of Franck-Hertz Setup

# Chapter 7

# Special Relativity

## 7.1 Introductory Concepts

### 7.1.1 Postulates of Special Relativity

1. The laws of Physics are the same in all inertial frames.

2. The speed of light is the same in all inertial frames.

We can define

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{7.1.1}$$

## 7.2 Time Dilation

$$\Delta t = \gamma \Delta t' \tag{7.2.1}$$

where $\Delta t'$ is the time measured at rest relative to the observer, $\Delta t$ is the time measured in motion relative to the observer.

## 7.3 Length Contraction

$$L = \frac{L'}{\gamma} \tag{7.3.1}$$

where $L'$ is the length of an object observed at rest relative to the observer and $L$ is the length of the object moving at a speed $u$ relative to the observer.

## 7.4 Simultaneity

## 7.5 Energy and Momentum

### 7.5.1 Relativistic Momentum & Energy

In relativistic mechanics, to be conserved, momentum and energy are defined as

**Relativistic Momentum**

$$\bar{p} = \gamma m\bar{v} \tag{7.5.1}$$

**Relativistic Energy**

$$E = \gamma mc^2 \tag{7.5.2}$$

### 7.5.2 Lorentz Transformations (Momentum & Energy)

$$p'_x = \gamma\left(p_x - \beta\frac{E}{c}\right) \tag{7.5.3}$$
$$p'_y = p_y \tag{7.5.4}$$
$$p'_z = p_z \tag{7.5.5}$$
$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \beta p_x\right) \tag{7.5.6}$$

### 7.5.3 Relativistic Kinetic Energy

$$K = E - mc^2 \tag{7.5.7}$$
$$= mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \tag{7.5.8}$$
$$= mc^2\left(\gamma - 1\right) \tag{7.5.9}$$

### 7.5.4 Relativistic Dynamics (Collisions)

$$\Delta P'_x = \gamma\left(\Delta P_x - \beta\frac{\Delta E}{c}\right) \tag{7.5.10}$$
$$\Delta P'_y = \Delta P_y \tag{7.5.11}$$
$$\Delta P'_z = \Delta P_z \tag{7.5.12}$$
$$\frac{\Delta E'}{c} = \gamma\left(\frac{\Delta E}{c} - \beta\Delta P_x\right) \tag{7.5.13}$$

## 7.6 Four-Vectors and Lorentz Transformation

We can represent an event in $S$ with the column matrix, $s$,

$$s = \begin{bmatrix} x \\ y \\ z \\ ict \end{bmatrix} \tag{7.6.1}$$

A different Lorents frame, $S'$, corresponds to another set of space time axes so that

$$s' = \begin{bmatrix} x' \\ y' \\ z' \\ ict' \end{bmatrix} \tag{7.6.2}$$

The Lorentz Transformation is related by the matrix

$$\begin{bmatrix} x' \\ y' \\ z' \\ ict' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ict \end{bmatrix} \tag{7.6.3}$$

We can express the equation in the form

$$s' = \mathscr{L} s \tag{7.6.4}$$

The matrix $\mathscr{L}$ contains all the information needed to relate position four–vectors for any given event as observed in the two Lorentz frames $S$ and $S'$. If we evaluate

$$s^T s = \begin{bmatrix} x & y & z & ict \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ ict \end{bmatrix} = x^2 + y^2 + z^2 - c^2 t^2 \tag{7.6.5}$$

Similarly we can show that

$$s'^T s' = x'^2 + y'^2 + z'^2 - c^2 t'^2 \tag{7.6.6}$$

We can take any collection of four physical quantities to be four vector provided that they transform to another Lorentz frame. Thus we have

$$b = \begin{bmatrix} b_x \\ b_y \\ b_z \\ ib_t \end{bmatrix} \tag{7.6.7}$$

this can be transformed into a set of quantities of $b'$ in another frame $S'$ such that it satisfies the transformation

$$b' = \mathscr{L} b \tag{7.6.8}$$

Looking at the momentum-Energy four vector, we have

$$p = \begin{bmatrix} p_x \\ p_y \\ p_z \\ iE/c \end{bmatrix} \tag{7.6.9}$$

Applying the same transformation rule, we have

$$p' = \mathscr{L}p \tag{7.6.10}$$

We can also get a Lorentz-invariation relation between momentum and energy such that

$$p'^T p' = p^T p \tag{7.6.11}$$

The resulting equality gives

$$p_x'^2 + p_y'^2 + p_z'^2 - \frac{E'^2}{c^2} = p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2} \tag{7.6.12}$$

## 7.7 Velocity Addition

$$v' = \frac{v - u}{1 - \frac{uv}{c^2}} \tag{7.7.1}$$

## 7.8 Relativistic Doppler Formula

$$\bar{v} = v_0\sqrt{\frac{c+u}{c-u}} \qquad \text{let } r = \sqrt{\frac{c-u}{c+u}} \tag{7.8.1}$$

We have

$$\bar{v}_{\text{receding}} = rv_0 \qquad \text{red-shift (Source Receding)} \tag{7.8.2}$$

$$\bar{v}_{\text{approaching}} = \frac{v_0}{r} \qquad \text{blue-shift (Source Approaching)} \tag{7.8.3}$$

## 7.9 Lorentz Transformations

Given two reference frames $S(x, y, z, t)$ and $S'(x', y', z', t')$, where the $S'$-frame is moving in the $x$-direction, we have,

$$x' = \gamma\,(x - ut) \qquad x = (x' - ut') \tag{7.9.1}$$

$$y' = y \qquad y = y' \tag{7.9.2}$$

$$z' = y \qquad y' = y \tag{7.9.3}$$

$$t' = \gamma\left(t - \frac{u}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{u}{c^2}x'\right) \tag{7.9.4}$$

## 7.10 Space-Time Interval

$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2 (\Delta t)^2 \tag{7.10.1}$$

Space-Time Intervals may be categorized into three types depending on their separation. They are

**Time-like Interval**

$$c^2 \Delta t^2 > \Delta r^2 \tag{7.10.2}$$

$$\Delta S^2 > 0 \tag{7.10.3}$$

When two events are separated by a time-like interval, there is a cause-effect relationship between the two events.

**Light-like Interval**

$$c^2 \Delta t^2 = \Delta r^2 \tag{7.10.4}$$

$$S^2 = 0 \tag{7.10.5}$$

**Space-like Intervals**

$$c^2 \Delta t^2 < \Delta r^2 \tag{7.10.6}$$

$$\Delta S < 0 \tag{7.10.7}$$

# Chapter 8

# Laboratory Methods

## 8.1 Data and Error Analysis

### 8.1.1 Addition and Subtraction

$$x = a + b - c \tag{8.1.1}$$

The Error in $x$ is

$$(\delta x)^2 = (\delta a)^2 + (\delta b)^2 + (\delta c)^2 \tag{8.1.2}$$

### 8.1.2 Multiplication and Division

$$x = \frac{a \times b}{c} \tag{8.1.3}$$

The error in $x$ is

$$\left(\frac{\delta x}{x}\right)^2 = \left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta c}{c}\right)^2 \tag{8.1.4}$$

### 8.1.3 Exponent - (No Error in $b$)

$$x = a^b \tag{8.1.5}$$

The Error in $x$ is

$$\frac{\delta x}{x} = b\left(\frac{\delta a}{a}\right) \tag{8.1.6}$$

### 8.1.4 Logarithms

**Base e**

$$x = \ln a \tag{8.1.7}$$

We find the error in $x$ by taking the derivative on both sides, so

$$\begin{aligned}\delta x &= \frac{d\ln a}{da}\cdot\delta a\\ &= \frac{1}{a}\cdot\delta a\\ &= \frac{\delta a}{a}\end{aligned} \tag{8.1.8}$$

**Base 10**

$$x = \log_{10} a \tag{8.1.9}$$

The Error in $x$ can be derived as such

$$\begin{aligned}\delta x &= \frac{d(\log a)}{da}\delta a\\ &= \frac{\frac{\ln a}{\ln 10}}{da}\delta a\\ &= \frac{1}{\ln 10}\frac{\delta a}{a}\\ &= 0.434\frac{\delta a}{a}\end{aligned} \tag{8.1.10}$$

### 8.1.5 Antilogs

**Base e**

$$x = e^a \tag{8.1.11}$$

We take the natural log on both sides.

$$\ln x = a\ln e = a \tag{8.1.12}$$

Applaying the same general method, we see

$$\begin{aligned}\frac{d\ln x}{dx}\delta x &= \delta a\\ \Rightarrow \frac{\delta x}{x} &= \delta a\end{aligned} \tag{8.1.13}$$

**Base 10**

$$x = 10^a \tag{8.1.14}$$

We follow the same general procedure as above to get

$$\begin{aligned}\log x &= a\log 10\\ \frac{\log x}{dx}\delta x &= \delta a\\ \frac{1}{\ln 10}\frac{d\ln a}{dx}\delta x &= \delta a\\ \frac{\delta x}{x} &= \ln 10\delta a\end{aligned} \tag{8.1.15}$$

## 8.2 Instrumentation

2

## 8.3 Radiation Detection

3

## 8.4 Counting Statistics

Let's assume that for a particular experiment, we are making countung measurements for a radioactive source. In this experiment, we recored $N$ counts in time $T$. The counting rate for this trial is $R = N/T$. This rate should be close to the average rate, $\bar{R}$. The standard deviation or the uncertainty of our count is a simply called the $\sqrt{N}$ rule. So

$$\sigma = \sqrt{N} \tag{8.4.1}$$

Thus we can report our results as

$$\text{Number of counts} = N \pm \sqrt{N} \tag{8.4.2}$$

We can find the count rate by dividing by $T$, so

$$R = \frac{N}{T} \pm \frac{\sqrt{N}}{T} \tag{8.4.3}$$

The fractional uncertainty of our count is $\frac{\delta N}{N}$. We can relate this in terms of the count rate.

$$\begin{aligned}\frac{\delta R}{R} = \frac{\frac{\delta N}{T}}{\frac{N}{T}} &= \frac{\delta N}{N} \\ &= \frac{\sqrt{N}}{N} \\ &= \frac{1}{N}\end{aligned} \tag{8.4.4}$$

We see that our uncertainty decreases as we take more counts, as to be expected.

## 8.5 Interaction of Charged Particles with Matter

5

## 8.6 Lasers and Optical Interferometers

6

## 8.7 Dimensional Analysis

Dimensional Analysis is used to understand physical situations involving a mis of different types of physical quantities. The dimensions of a physical quantity are associated with combinations of mass, length, time, electric charge, and temperature, represented by symbols $M$, $L$, $T$, $Q$, and $\theta$, respectively, each raised to rational powers.

## 8.8 Fundamental Applications of Probability and Statistics

8

# Chapter 9

# Sample Test

## 9.1 Period of Pendulum on Moon

The period of the pendulum, $T$, is

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}} \tag{9.1.1}$$

where $\ell$ is the length of the pendulium string. The relationship between the weight of an object on the Earth, $W_e$, and the Moon, $W_m$, is

$$W_m = \frac{W_e}{6} \tag{9.1.2}$$

From eq. (9.1.2), we can determine the acceleration due to gravity on the Moon and on the Earth; we use the same subscript notation as above.

$$g_m = \frac{g_e}{6} \tag{9.1.3}$$

On Earth, the period of the pendulum, $T_e$, is one second. From eq. (9.1.1), the equation for the pendulum's period on Earth is

$$T_e = 2\pi\sqrt{\frac{\ell}{g_e}} = 1\,\mathrm{s} \tag{9.1.4}$$

and similarly for the moon, the period becomes

$$T_m = 2\pi\sqrt{\frac{\ell}{g_m}} \tag{9.1.5}$$

Substituting eq. (9.1.3) into eq. (9.1.5) gives

$$\begin{aligned} T_m &= 2\pi\sqrt{\frac{\ell}{g_m}} \\ &= \sqrt{6}\,T_e = \sqrt{6}\,\mathrm{s} \end{aligned}$$

**Answer: (D)**

## 9.2 Work done by springs in series

Hooke's Law tells us that the extension on a spring is proportional to the force applied.

$$\boxed{F = -kx} \tag{9.2.1}$$

Springs in series follow the same rule for capacitors, see section 13.90.2. The spring constants are related to each other by

$$k_1 = \frac{1}{3}k_2 \tag{9.2.2}$$

The springs are massless so we can assume that the weight is transmitted evenly along both springs, thus from Hooke's Law the extension is

$$F_1 = -k_1x_1 = F_2 = -k_2x_2 \tag{9.2.3}$$

where $k_1$ and $k_2$ are the spring constants for the springs $S_1$ and $S_2$ respectively. Thus we see

$$\frac{k_1}{k_2} = \frac{x_2}{x_1} = \frac{1}{3} \tag{9.2.4}$$

The work done in stretching a spring or its potential energy is

$$\boxed{W = \frac{1}{2}kx^2} \tag{9.2.5}$$

Thus

$$\begin{aligned}\frac{W_1}{W_2} &= \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2}\\ &= \frac{k_1}{k_2}\cdot\left(\frac{x_1}{x_2}\right)^2\\ &= 3\end{aligned} \tag{9.2.6}$$

**Answer: (D)**

## 9.3 Central Forces I

We are given a central force field where

$$\boxed{V(r) = -\frac{k}{r}} \tag{9.3.1}$$

The Angular Momentum of an object is

$$\boxed{\mathbf{L} = \mathbf{r}\times\mathbf{p}} \tag{9.3.2}$$

and the torque is defined

$$\boxed{\tau = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times \dot{\mathbf{p}}} \tag{9.3.3}$$

From eqs. (9.3.2) and (9.3.3), we see that

$$\boxed{\tau = \frac{d\mathbf{L}}{dt}} \tag{9.3.4}$$

We see that if $\tau = 0$, then $\mathbf{L}$ is constant and therefore conserved. This can occur if $\dot{\mathbf{r}} = 0$, $\dot{\mathbf{F}} = 0$ or $\mathbf{F} \propto \mathbf{r}$.

From 9.3.1, we can determine the force acting on the object since

$$F = -\frac{dV}{dr} = \frac{k}{r^2} \tag{9.3.5}$$

As our force is a central force, the force acts in the direction of our radius vector. Thus the torque becomes

$$\begin{aligned} \tau &= \mathbf{r} \times \mathbf{F} = rF\cos 0 \\ &= 0 \end{aligned}$$

We see that this means that our angular momentum is constant.

$$\mathbf{L} = \text{constant} \tag{9.3.6}$$

A constant angular momentum means that $\mathbf{r}$ and $\mathbf{v}$ remain unchanged. The total mechanical energy is the sum of the kinetic and potential energies.

$$\begin{aligned} E &= \text{KE} + \text{PE} \\ &= \frac{1}{2}mv^2 + \frac{k}{r^2} \end{aligned} \tag{9.3.7}$$

Both the kinetic and potential energies will remain constant and thus the total mechanical energy is also conserved.

**Answer: (C)**

## 9.4 Central Forces II

The motion of particle is governed by its potential energy and for a conservative, central force the potential energy is

$$V(r) = -\frac{k}{r} \tag{9.4.1}$$

we have shown in the above question that the angular momentum, $\mathbf{L}$, is conserved. We can define three types of orbits given $k$ and $E$.

| Orbit | $k$ | Total Energy |
|---|---|---|
| Ellipse | $k > 0$ | $E < 0$ |
| Parabola | $k > 0$ | $E = 0$ |
| Hyperbola | $k > 0$ or $k < 0$ | $E > 0$ |

Table 9.4.1: Table of Orbits

From, table 9.4.1, we expect the orbit to be elliptical; this eliminates answers **(C)**, **(D)** and **(E)**.

For an elliptical orbit, the total energy is

$$E = -\frac{k}{2a} \tag{9.4.2}$$

where $a$ is the length of the semimajor axis. In the case of a circular orbit of radius, $r$, eq. (9.4.2) becomes

$$E = -\frac{k}{2r} \tag{9.4.3}$$

Recalling eq. (9.3.1), we see

$$E = \frac{1}{2}V(r) = -K \tag{9.4.4}$$

This is the minimum energy the system can have resulting in a circular orbit.

**Answer: (A)**

## 9.5 Electric Potential I

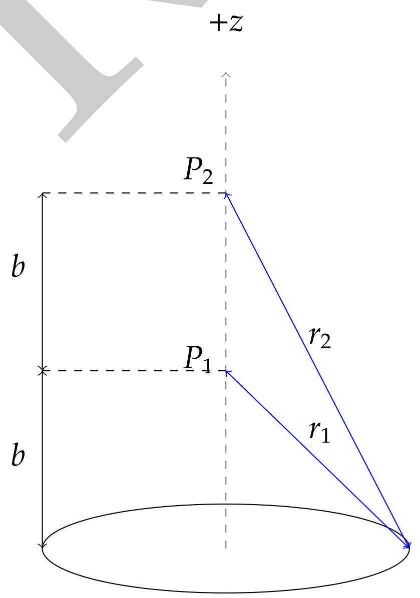



Figure 9.5.1: Diagram of Uniformly Charged Circular Loop

The Electric Potential of a charged ring is given by[1]

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2 + z^2}}} \tag{9.5.1}$$

where $R$ is the radius of our ring and $x$ is the distance from the central axis of the ring. In our case, the radius of our ring is $R = b$.

The potential at $P_1$, where $z = b$ is

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{b^2 + b^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{b\sqrt{2}} \tag{9.5.2}$$

The potential at $P_2$, where $z = 2b$ is

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{b^2 + (2b)^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{b\sqrt{5}} \tag{9.5.3}$$

Dividing eq. (9.5.3) by eq. (9.5.2) gives us

$$\frac{V_2}{V_1} = \sqrt{\frac{2}{5}} \tag{9.5.4}$$

**Answer: (D)**

## 9.6 Electric Potential II

The potential energy, $U(r)$, of a charge, $q$, placed in a potential, $V(r)$, is[1]

$$U(\mathbf{r}) = qV(\mathbf{r}) \tag{9.6.1}$$

The work done in moving our charge through this electrical field is

$$\begin{aligned} W &= U_2 - U_1 \\ &= qV_2 - qV_1 \\ &= q\left(V_2 - V_1\right) \end{aligned} \tag{9.6.2}$$

**Answer: (E)**

## 9.7 Faraday's Law and Electrostatics

We notice that our answers are in the form of differential equations and this leads us to think of the differential form of Maxwell's equations[2]. The electrostatics form of Maxwell's Equations are[3]

[1]Add Derivation

**Gauss's Law**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{9.7.1}$$

**Maxwell-Faraday Equation**

$$\nabla \times \mathbf{E} = 0 \tag{9.7.2}$$

**Gauss' Law for Magnetism**

$$\nabla \cdot \mathbf{B} = 0 \tag{9.7.3}$$

Ampère's Law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{9.7.4}$$

Comparing our answers, we notice that eq. (9.7.2) corresponds to **Answer: (C)** .
**Answer: (C)**

## 9.8 AC Circuits: RL Circuits

An inductor's characteristics is opposite to that of a capacitor. While a capacitor stores energy in the electric field, essentially a potential difference between its plates, an inductor stores energy in the magnetic field, which is produced by a current passing through the coil. Thus inductors oppose changes in currents while a capacitor opposes changes in voltages. A fully discharged inductor will initially act as an open circuit with the maximum voltage, $V$, across its terminals. Over time, the current increases and the potential difference across the inductor decreases exponentially to a minimum, essentiall behaving as a short circuit. As we do not expect this circuit to oscillate, this leaves us with choices **(A)** and **(B)**. At $t = 0$, we expect the voltage across the resistor to be $V_R = 0$ and increase exponentially. We choose **(A)**.

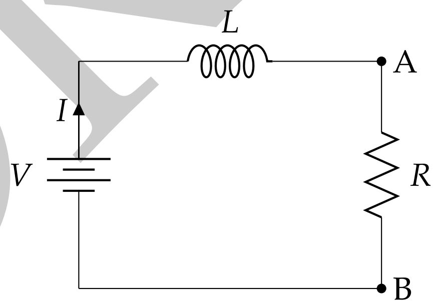



Figure 9.8.1: Schematic of Inductance-Resistance Circuit

We can see from the above schematic,

$$V = V_L + V_R \tag{9.8.1}$$

where $V_L$ and $V_R$ are the voltages across the inductor and resistor respectively. This can be written as a first order differential equation

$$V = L\frac{dI}{dt} + \frac{R}{L}I \tag{9.8.2}$$

Dividing by $L$ leaves

$$\frac{V}{L} = \frac{dI}{dt} + \frac{R}{L}I \tag{9.8.3}$$

The solution to eq. (9.8.3) leaves

$$\begin{aligned} I &= \frac{\int \frac{V}{L}\exp\left(\frac{Rt}{L}\right)dt + k}{\exp\left(\frac{Rt}{L}\right)} \\ &= \frac{V}{R} + k\exp\left(-\frac{Rt}{L}\right) \end{aligned} \tag{9.8.4}$$

Multiplying eq. (9.8.4) by $R$ gives us the voltage across the resistor

$$V_R = V + kR\exp\left(-\frac{Rt}{L}\right) \tag{9.8.5}$$

at $t = 0$, $V_R = 0$

$$\begin{aligned} 0 &= V + kR \\ \therefore k &= -\frac{V}{R} \end{aligned} \tag{9.8.6}$$

Substituting $k$ into eq. (9.8.5) gives us

$$V_R(t) = V\left[1 - \exp\left(-\frac{Rt}{L}\right)\right] \tag{9.8.7}$$

where $\tau = L/R$ is the time constant. Where $\tau = 2\,\mathrm{s}$

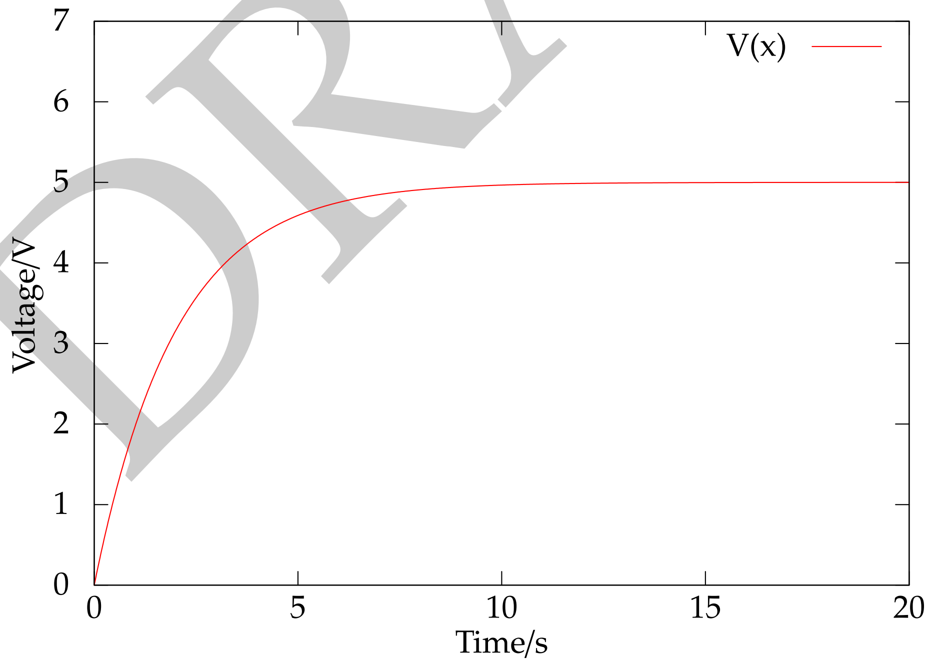



Figure 9.8.2: Potential Drop across Resistor in a Inductor-Resistance Circuit

**Answer: (A)**

## 9.9 AC Circuits: Underdamped RLC Circuits

When a harmonic oscillator is underdamed, it approaches zero much more quickly than a critically damped oscillator but it also oscillates about that zero. A quick examination of our choices means we can eliminate all but choices **(C)** and **(E)**. The choice we make takes some knowledge.

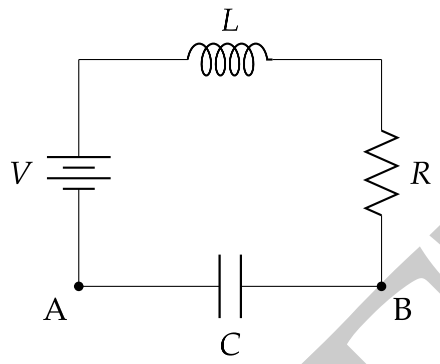



Figure 9.9.1: LRC Oscillator Circuit

The voltages in the above circuit can be written

$$\begin{aligned} V(t) &= V_L + V_R + V_C \\ &= L\frac{dI(t)}{dt} + RI(t) + \frac{1}{C}q(t) \end{aligned} \tag{9.9.1}$$

which can be written as a second order differential equation

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = V(t) \tag{9.9.2}$$

or as

$$\frac{d^2q(t)}{dt^2} + \gamma\frac{dq(t)}{dt} + \omega_0^2 q(t) = V(t) \tag{9.9.3}$$

This can be solved by finding the solutions for nonhomogenoeus second order linear differential equations. For any driving force, we solve for the undriven case,

$$\frac{d^2z}{dt^2} + \gamma\frac{dz}{dt} + \omega_0^2 = 0 \tag{9.9.4}$$

where for the underdamped case, the general solution is of the form

$$z(t) = A\exp(-\alpha t)\sin(\beta t + \delta) \tag{9.9.5}$$

where

$$\alpha = -\frac{\gamma}{2} \tag{9.9.6}$$

$$\beta = \frac{\sqrt{4\omega_0^2 - \gamma^2}}{2} \tag{9.9.7}$$

In the case of a step response,

$$V(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \tag{9.9.8}$$

The solution becomes

$$q(t) = 1 - \exp\left(-\frac{R}{2L}t\right) \frac{\sin\left(\sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}\, t + \delta\right)}{\sin\delta} \tag{9.9.9}$$

where the phase constant, $\delta$, is

$$\cos\delta = \frac{R}{2\omega_0^2 L} \tag{9.9.10}$$

where $\omega_0 \approx 3.162\,\mathrm{kHz}$ and $\gamma = 5\,\Omega\mathrm{H}^{-1}$

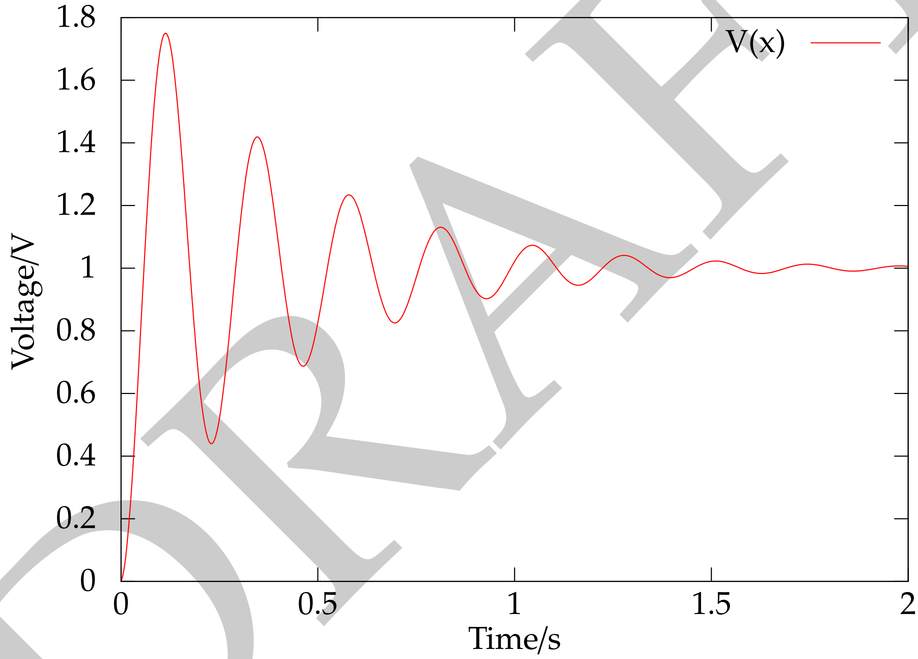



Figure 9.9.2: Forced Damped Harmonic Oscillations

This resembles choice **(C)**.
**Answer: (C)**

## 9.10 Bohr Model of Hydrogen Atom

Bohr's theory of the atom proposed the existence of stationary states by blending new quantum mechanics ideas with old classical mechanics concepts.

Bohr's model of the hydrogen atom starts as a system of two bodies bound together by the Coulomb attraction. The charges and mass of one particle is $-e$ and $m$ and $+Ze$

and $M$ for the other. In the case of our hydrogen atom system, $Z = 1$, $M = m_p$ and $m = m_e$. We will be taking into account the motion of both particles in our analysis. Normally, we expect the mass, $M$, to be stationary where $M/m \to \infty$ and as the proton-to-electron mass ration is very large, we approximate to this limiting condition.

$$\frac{m_p}{m_e} = 1836 \tag{9.10.1}$$

This effect is detectable and should be retained as a small correction. As the effects can be incorporated with little difficulty, we shall do so.[2]

We take the center of mass to be at th origin,[3]. We can reduce our two body system to an equivalent one body decription in terms of a single vector given by a relative coorindate.

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \tag{9.10.2}$$

As the center of mass is located at the origin

$$m\mathbf{r}_1 + M\mathbf{r}_2 = 0 \tag{9.10.3}$$

Solving eqs. (9.10.2) and (9.10.3) gives us

$$\mathbf{r}_1 = \frac{M}{M+m}\mathbf{r} \tag{9.10.4}$$

and

$$\mathbf{r}_2 = -\frac{m}{M+m}\mathbf{r} \tag{9.10.5}$$

Differentiating eqs. (9.10.4) and (9.10.5), gives us the corresponding velocities

$$\mathbf{v}_1 = \frac{M}{M+m}\mathbf{v} \tag{9.10.6}$$

and

$$\mathbf{v}_2 = -\frac{m}{M+m}\mathbf{v} \tag{9.10.7}$$

where the relative velocity is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \tag{9.10.8}$$

The total energy can be found from eqs. (9.10.6) and (9.10.7)

$$\begin{aligned} K &= \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \\ &= \frac{1}{2}\frac{mM}{M+m}v^2 \end{aligned} \tag{9.10.9}$$

We can reduce this to the equivalent of a one body system where the reduced mass factor is

$$\mu = \frac{mM}{M+m} \tag{9.10.10}$$

[2]Put figure here
[3]as seen in diagram

Equation (9.10.9) becomes

$$K = \frac{1}{2}\mu v_2 \tag{9.10.11}$$

As the Coulomb Force is a central force, the total angular momentum of the system will be constant.

$$\begin{aligned} L &= mv_1r_1 + Mv_2r_2 \\ &= m\left(\frac{M}{M+m}\right)^2 vr + M\left(\frac{m}{M+m}\right)^2 vr \\ &= \mu vr \end{aligned} \tag{9.10.12}$$

The centripetal force of the system is equal to the Coulomb force, thus

$$F = \frac{mv_1^2}{r_1} = \frac{Mv_2^2}{r_2} = \frac{\mu v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} \tag{9.10.13}$$

The potential energy of the system comes from the Coulomb potential energy

$$V = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r} \tag{9.10.14}$$

The total energy of the system can be found by adding eqs. (9.10.11) and (9.10.14)

$$\begin{aligned} E &= K + V \\ &= \frac{1}{2}\mu v^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r} \end{aligned} \tag{9.10.15}$$

Substituting eq. (9.10.13) into eq. (9.10.15) gives

$$E = -\frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r} \tag{9.10.16}$$

We expect the total energy of the system to be negative as it is a bound system.

We have, so far, adhered to the principles of classical mechanics up to this point. Beyond this point, we must introduce quantum mechanical concepts. To produce the stationary states he was seeking, Bohr introduced the hypothesis that the angular momentum is quantized.

$$L = n\hbar \tag{9.10.17}$$

Equating this with eq. (9.10.12) and substitution into eq. (9.10.13) gives discrete values for the orbital radius.

$$r_n = \frac{4\pi\epsilon_0}{Ze^2}\frac{n^2\hbar^2}{\mu} \tag{9.10.18}$$

We can rewrite the above equation

$$r_n = \frac{n^2 m_e}{\mu Z}a_0 \tag{9.10.19}$$

where

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2 m_e} \tag{9.10.20}$$

is the Bohr Radius.

As the orbital radii is discrete we expect the various orbital energies to also be discrete. Substitution of eq. (9.10.18) into eq. (9.10.16) gives

$$\begin{aligned} E_n &= -\frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r_n} \\ &= -\frac{Z^2}{n^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{\mu}{2\hbar^2} \end{aligned} \tag{9.10.21}$$

or

$$E_n = -\frac{Z^2}{n^2}\frac{\mu}{m_e}E_0 \tag{9.10.22}$$

where

$$E_0 = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{m_e}{2\hbar^2} = 13.6\,\mathrm{eV} \tag{9.10.23}$$

We see that this analysis eliminates all but one answer.

**Answer: (A)**

## 9.11 Nuclear Sizes

We know from electron scattering experiments, the nucleus is roughly spherical and uniform density[4]. The Fermi model gives us an expression

$$r = r_0 A^{\frac{1}{3}} \tag{9.11.1}$$

where $r_0 = 1.2 \times 10^{-15}$ m and $A$ is the mass number. In the case of hydrogen, we recall the Bohr radius to be $a_0 = 0.0592$ nm.

So in the case of hydrogen, $A = 1$,

$$r = r_0 = 1.2 \times 10^{-15}\,\mathrm{m} \tag{9.11.2}$$

Thus

$$\begin{aligned} \frac{r_0}{a_0} &= \frac{1.2 \times 10^{-15}}{0.0592 \times 10^{-9}} \\ &= 2.02 \times 10^{-5} \end{aligned} \tag{9.11.3}$$

**Answer: (B)**

[4]Add diagram of nuclear and atomic sizes here

## 9.12 Ionization of Lithium

The ionization energy of an electron is the energy to kick it off from its present state to infinity. It can be expressed as

$$\begin{aligned} E_{\text{ionization}} &= E_\infty - E_n \\ &= \frac{Z^2}{n^2}\frac{\mu}{m_e}E_0 \end{aligned} \tag{9.12.1}$$

where $E_0 = 13.6$ eV and $\mu$ is the reduced mass where

$$\frac{\mu}{m_e} = \frac{M}{M + m_e} \tag{9.12.2}$$

In the case of atoms, the above ratio is close to one and hence we can ignore it for this case.

Lithium has an atomic number, $Z = 3$ so its electron structure is[5]

$$1s^2, 2s^1 \tag{9.12.3}$$

So the total ionization energy will be the total energy needed to completly remove each electron. This turns out to be

$$\begin{aligned} E &= \left[\frac{3^2}{1^2} + \frac{3^2}{1^2} + \frac{3^2}{2^2}\right] 13.6 \text{ eV} \\ &\approx 20 \times 13.6 \text{ eV} \\ &= 272.0 \text{ eV} \end{aligned} \tag{9.12.4}$$

**Answer: (C)**

## 9.13 Electron Diffraction

We recall that in optics, one of the criteria for diffraction is a monochromatic wave. We expect the electron beam to also have wavelike effects. The de Broglie relations show that the wavelength is inversely proportional to the momentum of a particle and that the frequency is directly proportional to the particle's kinetic energy.

$$\lambda = \frac{h}{p} \quad \text{and} \quad E = hf$$

Thus, for electron we expect the beam to be monoenergetic.

**Answer: (B)**

[5]Draw Lithium atom and its electrons

## 9.14 Effects of Temperature on Speed of Sound

The speed of sound is determined by its Bulk Modulus and its density

$$v = \sqrt{\frac{B}{\rho}} \tag{9.14.1}$$

in the case of gases, the Bulk Modulus can be expressed

$$B = \gamma P \tag{9.14.2}$$

where $\gamma$ is the adiabatic ratio and $P$ is the pressure of the gas. For an ideal gas

$$PV = nRT \tag{9.14.3}$$

Thus our speed of sound equation becomes

$$v = \sqrt{\frac{n\gamma RT}{M}} \tag{9.14.4}$$

So we see that

$$v \propto T^{\frac{1}{2}} \tag{9.14.5}$$

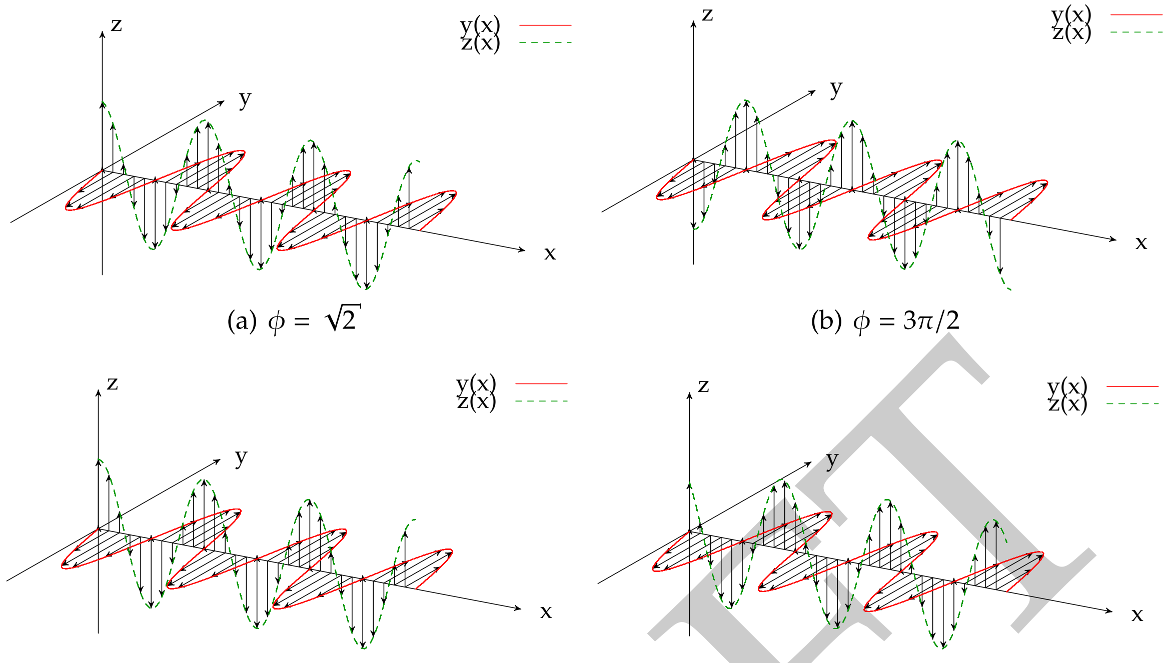
**Answer: (B)**

## 9.15 Polarized Waves

Given the equations

$$\begin{aligned} y &= y_0 \sin(\omega t - kx) \\ z &= z_0 \sin\left(\omega t - kx - \phi\right) \end{aligned} \tag{9.15.1}$$

A plane polarized wave will occur when $\phi = 0$.




(a) $\phi = \sqrt{2}$ (b) $\phi = 3\pi/2$

(c) $\phi = \pi/2$ (d) $\phi = \pi/4$

Figure 9.15.1: Waves that are not plane-polarized

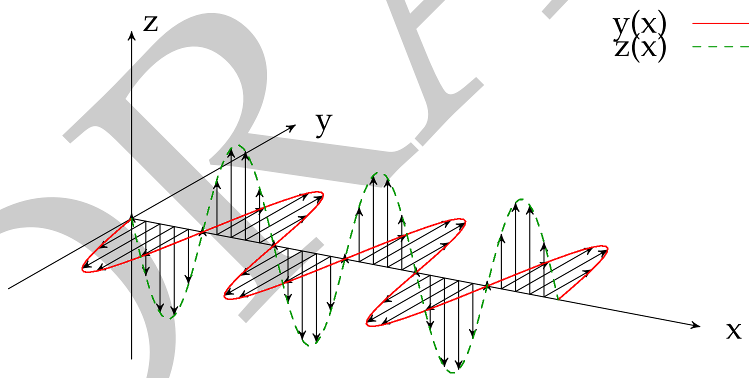



Figure 9.15.2: $\phi = 0$

**Answer: (E)**

## 9.16 Electron in symmetric Potential Wells I

As our potential is symmetric about the $V$-axis, then we will expect our wave function to also be symmetric about the $V$-axis.

**Answer: (E)**

## 9.17 Electron in symmetric Potential Wells II

If the electrons do not interact, we can ignore Pauli's Exclusion Principle. As a result they will not have spatially antisymmetric states but will have the same spatial wave functions.

**Answer: (B)**

## 9.18 Relativistic Collisions I

The Relativistic Momentum equation is

$$\boxed{p = \frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}}} \tag{9.18.1}$$

given that $p = mc/2$,

$$\begin{aligned} \frac{mc}{2} &= \frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \\ \Rightarrow \frac{c}{2} &= \frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \\ \therefore v &= \frac{c}{\sqrt{5}} \end{aligned} \tag{9.18.2}$$

**Answer: (D)**

## 9.19 Relativistic Collisions II

Momentum is conserved. So in the horizontal direction,

$$p = 2p_f \cos 30 \tag{9.19.1}$$

Solving this shows

$$p_f = \frac{mc}{2\sqrt{3}} \tag{9.19.2}$$

**Answer: (B)**

## 9.20 Thermodynamic Cycles I

We have a three stage cyclic process where

$$A(P_1, V_1, T_1) \quad , \quad B(P_2, V_2, T_2) \quad , \quad C(P_3, V_3, T_3) \tag{9.20.1}$$

**Adiabatic Expansion,** $A \longrightarrow B$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \tag{9.20.2}$$

Given that $V_2 = 2V_1$, we have

$$P_2 = 2^{-\gamma} P_1 \tag{9.20.3}$$

and

$$T_2 = 2^{1-\gamma} T_1 \tag{9.20.4}$$

**Isochoric Expansion,** $B \longrightarrow C$ We have

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \tag{9.20.5}$$

where $T_3 = T_1$, we have

$$P_3 = 2^{\gamma-1} P_2 \tag{9.20.6}$$

$$= \frac{1}{2} P_1 \tag{9.20.7}$$

This becomes

$$A(P_1, V_1, T_1) \quad , \quad B(2^{-\gamma} P_1, 2V_1, 2^{1-\gamma} T_1) \quad , \quad C(\frac{P_1}{2}, 2V_1, T_1) \tag{9.20.8}$$

On a $PV$-graph, we see that this makes a clockwise cycle, indicating that positive work is done by the gas on the environment.

**Answer: (A)**

## 9.21 Thermodynamic Cycles II

We recall **Calusius's Therorem**

$$\boxed{\oint \frac{dQ}{T} = 0} \tag{9.21.1}$$

for a reversible cycle, the change in entropy is zero.

**Answer: (C)**

## 9.22 Distribution of Molecular Speeds

The distribution of speeds of molecules follows the Maxwell-Boltzmann distribution, which has the form

$$f(v) = 4\pi \left[\frac{M}{2\pi RT}\right]^{\frac{3}{2}} v^2 \exp\left[-\frac{Mv^2}{2RT}\right] \tag{9.22.1}$$

where $R$ is the gas constant and $M$ is the molar mass of the gas. The speed distribution for noble gases at $T = 298.15\,\mathrm{K}$ looks like

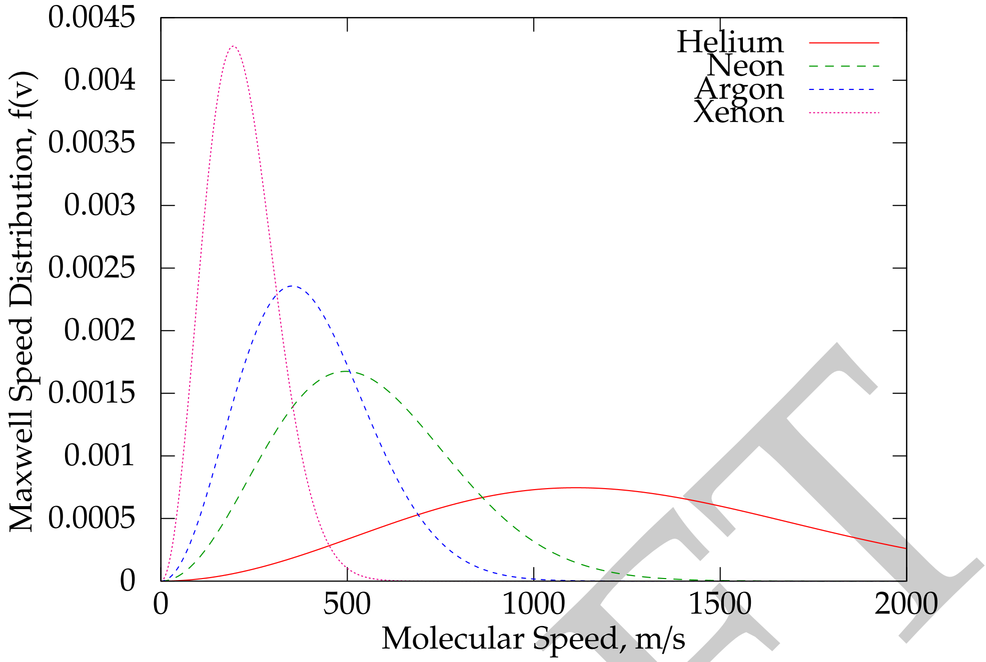



Figure 9.22.1: Maxwell-Boltzmann Speed Distribution of Nobel Gases

**Answer: (D)**

## 9.23 Temperature Measurements

All the thermometers won't be able to survive that high a temperature except for the optical pyrometer. Of course, a little knowledge always helps.

**Optical Pyrometer** Optical pyrometers work by using the human eye to match the brightness of a hot object to a calibrated lamp filament inside the instrument.

**Carbon Resistor** These thermometers are typically used for very low temperatures and not high ones. One of their main advantages is their sensitivity, their resistance increases exponentially to decreasing temperature and are not affected by magnetic fields.

**Gas-Bulb Thermometer** May also be known as the constant volume gas thermometer. Doubtful the glass bulb will survive such high temperatures.

**Mercury Thermometer** The boiling point of mercury is about 360 °C. This thermometer will be a gas before you even had a chance to think about getting a temperature reading.

**Thermocouple** Thermocouples are typically used in industry to measure high temperatures, usually in the order $\sim$ 1800 °C.

Even if we knew nothing about any of the above thermometers, we could have still taken a stab at it. We should probaby guess that at that high a temperature we won't want to make physical contact with what we are measuring. The only one that can do this is the **optical pyrometer**.

**Answer: (A)**

## 9.24 Counting Statistics

**NOT FINSIHED**

**Answer: (D)**

## 9.25 Thermal & Electrical Conductivity

A metal is a lattice of atoms, each with a shell of electrons. This forms a positive ionic lattice where the outer electrons are free to dissociate from the parent atoms and move freely through the lattic as a 'sea' of electrons. When a potential difference is applied across the metal, the electrons drift from one end of the conductor to the other under the influence of the electric field. It is this free moving electron 'sea' that makes a metal an electrical conductor.

These free moving electrons are also efficient at transferring thermal energy for the same reason. Thermal and electrical conductivity in metals are closely related to each other as outlined in the Wiedemann-Franz Law.

$$\frac{\kappa}{\sigma} = LT \tag{9.25.1}$$

where the Lorenz number, $L = 2.44 \times 10^{-8}\,\mathrm{W\Omega K^{-1}}$ and $\kappa$ and $\sigma$ are the thermal and electrical conductivities respectively. This corelation does not apply to non-metals due to the increased role of phonon carriers.

**Answer: (E)**

## 9.26 Nonconservation of Parity

**NOT FINSIHED**

**Answer: (B)**

## 9.27 Moment of Inertia

The moment of inertia is

$$\boxed{I = \int r^2 dm} \tag{9.27.1}$$

In the case of a hoop about its center axis,

$$I = MR^2 \tag{9.27.2}$$

From eq. (9.27.1), we see that the moment of inertia deals with how the mass is distributed along its axis.[6] So we see that

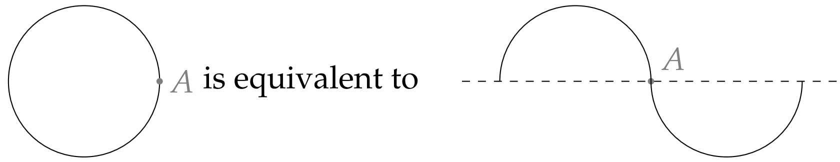



Figure 9.27.1: Hoop and S-shaped wire

Thus, see fig. 9.27.1, the moment of inertia of our S-shaped wire can be found from a hoop with its axis or rotation at its radius. This can be calculated by using the Parallel Axis Theorem

$$I = I_{\mathrm{CM}} + Md^2 \tag{9.27.3}$$

where $d^2$ is the distance from the center of mass. This becomes

$$I = MR^2 + MR^2 = 2MR^2 \tag{9.27.4}$$

**Answer: (E)**

## 9.28 Lorentz Force Law I

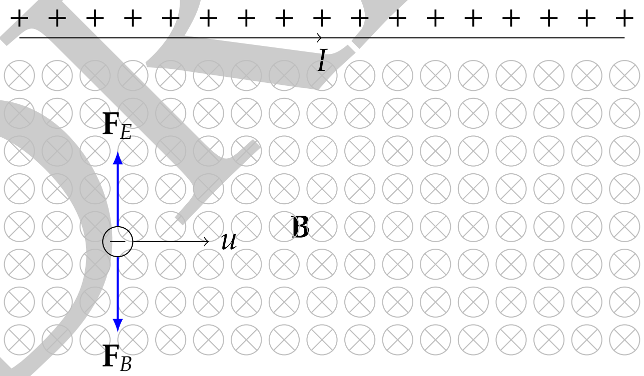



Figure 9.28.1: Charged particle moving parallel to a positively charged current carrying wire

The force on the charged particle is determined by the Lorentz Force Equation

$$\boxed{\mathbf{F} = e\left[\mathbf{E} + \mathbf{u} \times \mathbf{B}\right]} \tag{9.28.1}$$

[6]The moment of inertia of a 1 kg mass at a distance 1m from the axis of rotation is the same as a hoop with the same mass rotating about its central axis.

where $\mathbf{F}_E = e\mathbf{E}$ and $\mathbf{F}_B = e(\mathbf{u} \times \mathbf{B})$. For our charged particle to travel parallel to our wire, $F_E = F_B$.[7]

$$\boxed{E = \frac{\lambda \ell}{2\pi\epsilon_0 r}} \tag{9.28.2}$$

and the magnetic field can be determined from Ampère's Law

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}}} \tag{9.28.3}$$

In the case of our wire

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \tag{9.28.4}$$

Plugging eqs. (9.28.2) and (9.28.4) into eq. (9.28.1) gives

$$F = e\left[\frac{\lambda \ell}{2\pi\epsilon_0 r} + u\frac{\mu_0 I}{2\pi r}\right] = 0 \tag{9.28.5}$$

For the particle to be undeflected, $\mathbf{F}_E + \mathbf{F}_B = 0$

$$\begin{aligned} \mathbf{F}_E + \mathbf{F}_B &= 0 \\ \frac{\lambda \ell}{2\pi\epsilon_0 r} - u\frac{\mu_0 I}{2\pi r} &= 0 \end{aligned} \tag{9.28.6}$$

Now we can go about eliminating choices.

**Doubling charge per unit length** We see from eq. (9.28.6), halving the current, $I$, and doubling the linear charge density, $\lambda$, will not allow the particle to continue undeflected.

$$\frac{2\lambda \ell}{2\pi\epsilon_0 r} - u\frac{\mu_0 I/2}{2\pi r} = 2F_E - \frac{F_B}{2} \neq 0 \tag{9.28.7}$$

**Doubling the charge on the particle** We see from, eq. (9.28.6) that the charge on the particle, $e$, has no effect on the particle's trajectory. We would be left with

$$\frac{\lambda \ell}{2\pi\epsilon_0 r} - u\frac{\mu_0 I/2}{2\pi r} = F_E - \frac{F_B}{2} \neq 0 \tag{9.28.8}$$

**Doubling both the charge per unit length on the wire and the charge on the particle** As shown above, the particle's charge has no effect on the trajectory. This leaves us with the charge per unit length, $\lambda$ and as we have seen before, this will change the particle's trajectory, see eq. (9.28.7).

**Doubling the speed of the particle** If we double the particle's speed we will get

$$\begin{aligned} \frac{\lambda \ell}{2\pi\epsilon_0 r} - (2u)\frac{\mu_0 I/2}{2\pi r} &= \frac{\lambda \ell}{2\pi\epsilon_0 r} - u\frac{\mu_0 I}{2\pi r} \\ \therefore F_E &= F_B \end{aligned}$$

**This is our answer**

[7] Add derivation in a section

**Introducing an additional magnetic field parallel to the wire** Recalling eq. (9.28.1), the force due to the magnetic field is a cross product between the velocity and the field. A charged particle moving in the same direction as the field will experience no magnetic force.

$$\begin{aligned} \mathbf{F}_B &= e\left[\mathbf{u} \times \mathbf{B}\right] \\ &= uB \sin 0 \\ &= 0 \end{aligned} \tag{9.28.9}$$

**Answer: (D)**

## 9.29 Lorentz Force Law II

As we can see from eq. (9.28.5), the forces due to the electric and magnetic fields are equal.

$$F = e\left[\frac{\lambda\ell}{2\pi\epsilon_0 r} + u\frac{\mu_0 I}{2\pi r}\right] = 0 \tag{9.29.1}$$

If we move our charged particle a distance $2r$ from the wire with a speed $nu$, 9.28.5 becomes

$$\begin{aligned} e\left[\frac{\lambda\ell}{2\pi\epsilon_0(2r)} + u\frac{\mu_0 I}{2\pi(2r)r}\right] &= e\left(\frac{1}{2}\right)\left[\frac{\lambda\ell}{2\pi\epsilon_0 r} + nu\frac{\mu_0 I}{2\pi r}\right] \\ &= e\left(\frac{1}{2}\right)\left[F_E - nF_B\right] \\ &= 0 \end{aligned} \tag{9.29.2}$$

Thus

$$\begin{aligned} \left[F_E - nF_B\right] = F_E - F_B &= 0 \\ \Rightarrow n &= 1 \end{aligned} \tag{9.29.3}$$

The speed of the particle is $u$.

**Answer: (C)**

## 9.30 Nuclear Angular Moment

**NOT FINISHED**

**Answer: (B)**

## 9.31 Potential Step Barrier

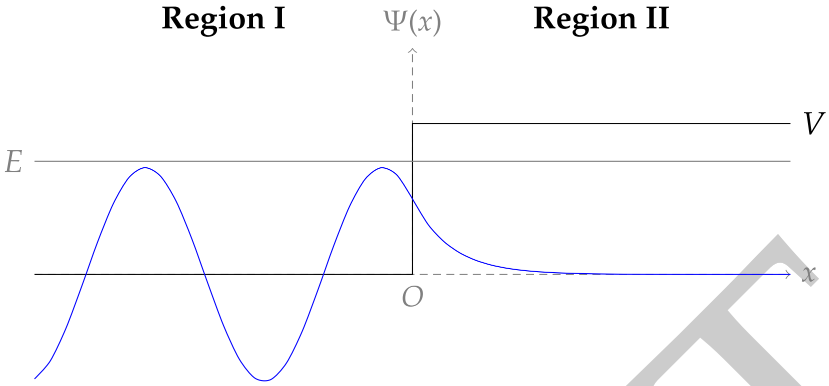



Figure 9.31.1: Wavefunction of particle through a potential step barrier

The point, $x = 0$, divides the region into two regions, **Region I**, where classical motion is allowed and, **Region II**, where classical motion is forbidden. The barrier potential is

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V & \text{for } x > 0 \end{cases} \tag{9.31.1}$$

For, $x < 0$, the eigenfunction satisfies

$$\frac{d^2\psi}{dx^2} = -k_1^2\psi \tag{9.31.2}$$

where

$$k_1^2 = \frac{2m}{\hbar^2}E \tag{9.31.3}$$

The general form of the eigenfunction is

$$\psi_I = Ae^{ik_1x} + Be^{-ik_1x} \tag{9.31.4}$$

For, $x > 0$, the eigenfunction satisfies

$$\frac{d^2\psi}{dx^2} = k_2^2\psi \tag{9.31.5}$$

where

$$k_2^2 = \frac{2m}{\hbar^2}(V - E) \tag{9.31.6}$$

The general form of the eigenfunction becomes

$$\psi_{II} = Ce^{-k_2x} + De^{k_2x} \tag{9.31.7}$$

As $x \to \infty$, the $e^{k_2x}$ term blows up. So to allow eq. (9.31.7) to make any physical sense we set $D = 0$, thus

$$\psi_{II} = Ce^{-k_2x} = Ce^{-\alpha x} \tag{9.31.8}$$

We can continue solving for $A$, $B$ and $C$ but for the purposes of the question we see from eq. (9.31.8) that $\alpha$ is both real and positive.

**Answer: (C)**

# Chapter 10

# GR8677 Exam Solutions

## 10.1 Motion of Rock under Drag Force

From the information provided we can come up with an equation of motion for the rock.

$$m\ddot{\mathbf{x}} = -mg - k\mathbf{v} \tag{10.1.1}$$

If you have seen this type of equation, and solved it, you'd know that the rock's speed will asymtotically increase to some max speed. At that point the drag force and the force due to gravity will be the same. We can best answer this question through analysis and elimination.

**A** Dividing eq. (10.1.1) by $m$ gives

$$\ddot{x} = -g - \frac{k}{m}\dot{x} \tag{10.1.2}$$

We see that this only occurs when $\dot{x} = 0$, which only happens at the top of the flight. So **FALSE**.

**B** From eq. (10.1.2), we see that this is **TRUE**.

**C** Again from eq. (10.1.2) we see that the acceleration is dependent on whether the rock is moving up or down. If $\dot{x} > 0$ then $\ddot{x} < -g$ and if $\dot{x} < 0$ then $\ddot{x} > -g$. So this is also **FALSE**.

**D** If there was no drag (fictional) force, then energy would be conserved and the rock will return at the speed it started with but there is a drag force so energy is lost. The speed the rock returns is $v < v_0$. Hence **FALSE**.

**E** Again **FALSE**. Once the drag force and the gravitational force acting on the rock is balanced the rock won't accelerate.

**Answer: (B)**

## 10.2 Satellite Orbits

The question states that the astronaut fires the rocket's jets towards Earth's center. We infer that no extra energy is given to the system by this process. section 1.7.5, shows that the only other orbit where the specific energy is also negative is an elliptical one.

**Answer: (A)**

## 10.3 Speed of Light in a Dielectric Medium

Solutions to the Electromagnetic wave equation gives us the speed of light in terms of the electromagnetic permeability, $\mu_0$ and permitivitty, $\epsilon_0$.

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \tag{10.3.1}$$

where $c$ is the speed of light. The speed through a dielectric medium becomes

$$\begin{aligned} v &= \frac{1}{\sqrt{\mu\epsilon_0}} \\ &= \frac{1}{\sqrt{2.1\mu_0\epsilon_0}} \\ &= \frac{c}{\sqrt{2.1}} \end{aligned} \tag{10.3.2}$$

**Answer: (D)**

## 10.4 Wave Equation

We are given the equation

$$y = A\sin\left(\frac{t}{T} - \frac{x}{\lambda}\right) \tag{10.4.1}$$

We can analyze and eliminate from what we know about this equation

**A** The Amplitude, $A$ in the equation is the displacement from equilibrium. So this choice is incorrect.

**B** As the wave moves, we seek to keep the $\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ term constant. So as $t$ increases, we expect $x$ to increase as well as there is a negative sign in front of it. This means that the wave moves in the positive $x$-direction. This choice is also incorrect.

**C** The phase of the wave is given by $\left(\frac{t}{T} - \frac{x}{\lambda}\right)$, we can do some manipulation to show

$$\begin{aligned} \left(\frac{t}{T} - \frac{x}{\lambda}\right) &= 2\pi ft - kx \\ &= \omega t - kx \end{aligned} \tag{10.4.2}$$

Or rather

$$kx = \omega t \tag{10.4.3}$$

Differentiating eq. (10.4.3) gives us the phase speed, which is

$$v = \frac{\lambda}{T} \tag{10.4.4}$$

This is also incorrect

**E** From eq. (10.4.4) the above we see that is the answer.

**Answer: (E)**

## 10.5 Inelastic Collision and Putty Spheres

We are told the two masses coalesce so we know that the collision is inelastic and hence, energy is not conserved. As mass $A$ falls, it looses Potential Energy and gains Kinetic Energy.

$$Mgh_0 = \frac{1}{2}Mv_0^2 \tag{10.5.1}$$

Thus

$$v_0^2 = 2gh_0 \tag{10.5.2}$$

Upon collision, momentum is conserved, thus

$$\begin{aligned} Mv_0 &= (3M + M)\, v_1 \\ &= 4Mv_1 \\ \Rightarrow v_1 &= \frac{v_0}{4} \end{aligned} \tag{10.5.3}$$

The fused putty mass rises, kinetic energy is converted to potential energy and we find our final height.

$$\begin{aligned} \frac{1}{2}(4M)\, v_1^2 &= 4Mgh \\ h &= \frac{v_1^2}{2g} \\ &= \frac{1}{2g}\left(\frac{v_0}{4}\right)^2 \\ &= \frac{h_0}{16} \end{aligned} \tag{10.5.4}$$

**Answer: (A)**

## 10.6 Motion of a Particle along a Track

As the particle moves from the top of the track and runs down the frictionless track, its Gravitational Potential Energy is converted to Kinetic Energy. Let's assume that the particle is at a height, $y_0$ when $x = 0$. Since energy is conserved, we get[1]

$$\begin{aligned} mgy_0 &= mg(y_0 - y) + \frac{1}{2}mv^2 \\ \Rightarrow \frac{1}{2}v^2 &= gy \end{aligned} \tag{10.6.1}$$

So we have a relationship between $v$ and the particle's position on the track.

The tangential acceleration in this case is

$$mg\cos\theta = \frac{mv^2}{r} \tag{10.6.2}$$

where $r$ is the radius of curvature and is equal to $\sqrt{x^2+y^2}$.

Substituting this into eq. (10.6.2) gives

$$\begin{aligned} g\cos\theta &= \frac{v^2}{r} \\ &= \frac{gx^2}{2\sqrt{x^2+y^2}} \\ &= \frac{gx}{\sqrt{x^2+4}} \end{aligned} \tag{10.6.3}$$

**Answer: (D)**

## 10.7 Resolving Force Components

This question is a simple matter of resolving the horizontal and vertical components of the tension along the rope. We have

$$T\sin\theta = F \tag{10.7.1}$$

$$T\cos\theta = mg \tag{10.7.2}$$

Thus we get

$$\begin{aligned} \tan\theta &= \frac{F}{mg} \\ &= \frac{10}{(2)(9.8)} \approx \frac{1}{2} \end{aligned} \tag{10.7.3}$$

**Answer: (A)**

[1]Insert Free Body Diagram of particle along track.

## 10.8 Nail being driven into a block of wood

We recall that

$$v^2 = v_0^2 + 2as \tag{10.8.1}$$

where $v$, $v_0$, $a$ and $s$ are the final speed, initial speed, acceleration and displacement that the nail travels. Now it's just a matter os plugging in what we know

$$\begin{aligned} 0 &= 100 + 2a(0.025) && (10.8.2)\\ \Rightarrow a &= -\frac{100}{2(0.025)} = -2000 \text{ m/s}^2 && (10.8.3) \end{aligned}$$

The Force on the nail comes from Newton's Second Law

$$\begin{aligned} F &= ma \\ &= 5 \cdot 2000 = 10000 \text{ N} && (10.8.4) \end{aligned}$$

**Answer: (D)**

## 10.9 Current Density

We can find the drift vleocity from the current density equation

$$\mathbf{J} = en\mathbf{v}_d \tag{10.9.1}$$

where $e$ is the charge of an electron, $n$ is the density of electrons per unit volume and $\vec{v}_d$ is the drift speed. Plugging in what we know

$$\begin{aligned} J &= \frac{I}{A} \\ I &= nAv_d e \\ v_d &= \frac{I}{nAe} \\ &= \frac{100}{(1 \times 10^{28})\frac{\pi \times 2 \times 10^{-4}}{4} 1.6 \times 10^{-19}} && (10.9.2) \end{aligned}$$

paying attention to the indices of the equation we get

$$2 - 28 + 4 + 19 = -4 \tag{10.9.3}$$

So we expect an answer where $v_d \approx 10^{-4}$.[2]

**Answer: (D)**

[2]It also helps if you knew that the electron drift velocity was slow, in the order of mm/s.

## 10.10 Charge inside an Isolated Sphere

You can answer this by thinking about Gauss' Law. The bigger the Gaussian surface the more charge it encloses and the bigger the electric field. Beyond the radius of the sphere, the field decreases exponentially.

We can calculate these relationships by using Gauss' Law.

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \tag{10.10.1}$$

where the current density, $\rho$ is

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{Q_{\text{enclosed}}}{\frac{4}{3}\pi r^3} \tag{10.10.2}$$

where $R$ is the radius of the sphere.

**for** $r < R$ The enclosed charge becomes

$$Q_{\text{enclosed}} = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{Qr^3}{R^3} \tag{10.10.3}$$

Gauss' Law becomes

$$E\left(4\pi r^2\right) = \frac{Qr^3}{\epsilon_0 R^3} \tag{10.10.4}$$

The Electric field is

$$\boxed{E_{(r<R)} = \frac{Qr}{4\pi\epsilon_0 R^3}} \tag{10.10.5}$$

This is a linear relationship with respect to r.

**for** $r \geq R$ The enclosed charge is

$$Q_{\text{enclosed}} = Q \tag{10.10.6}$$

Gauss' Law becomes

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \tag{10.10.7}$$

The Electric field is

$$\boxed{E_{(r\geq R)} = \frac{Q}{4\pi\epsilon_0 r^2}} \tag{10.10.8}$$

The linear increase is exhibited by choice **(C)**.

**Answer: (C)**

## 10.11 Vector Identities and Maxwell's Laws

We recall the vector identity

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0 \tag{10.11.1}$$

Thus

$$\begin{aligned} \nabla \cdot (\nabla \times \mathbf{H}) &= \nabla \cdot \left(\dot{\mathbf{D}} + \mathbf{J}\right) \\ &= 0 \end{aligned} \tag{10.11.2}$$

**Answer: (A)**

## 10.12 Doppler Equation (Non-Relativistic)

we recall the Doppler Equation[3]

$$f = f_0 \left(\frac{v - v_r}{v - v_s}\right) \tag{10.12.1}$$

where $v_r$ and $v_s$ are the observer and source speeds respectively. We are told that $v_r = 0$ and $v_s = 0.9v$. Thus

$$\begin{aligned} f &= f_0 \left(\frac{v}{v - 0.9v}\right) \\ &= 10 f_0 \\ &= 10 \text{ kHz} \end{aligned} \tag{10.12.2}$$

**Answer: (E)**

## 10.13 Vibrating Interference Pattern

Answering this question takes some analysis. The sources are coherent, so they will produce an interference pattern. We are also told that $\Delta f = 500$ Hz. This will produce a shifting interference pattern that changes too fast for the eye to see.[4]

**Answer: (E)**

## 10.14 Specific Heat at Constant Pressure and Volume

From section 4.20 and section 4.21, we see that

$$C_p = C_V + R \tag{10.14.1}$$

The difference is due to the work done in the environment by the gas when it expands under constant pressure.

[3] Add reference to Dopler Equations.
[4] Add Young's Double Slit Experiment equations.

We can prove this by starting with the First Law of Thermodynamics.

$$\boxed{dU = -dW + dQ} \tag{10.14.2}$$

Where $dU$ is the change in Internal Energy, $dW$ is the work done by the system and $dQ$ is the change in heat of the system.

We also recall the definition for Heat Capacity

$$\boxed{dQ = CdT} \tag{10.14.3}$$

At constant volume, there is no work done by the system, $dV = 0$. So it follows that $dW = 0$. The change in internal energy is the change of heat into the system, thus we can define, the heat capacity at constant volume to be

$$dU_V = C_V dT = dQ_V \tag{10.14.4}$$

At constant pressure, the change in internal energy is accompanied by a change in heat flow as well as a change in the volume of the gas, thus

$$\begin{aligned} dU_p &= -dW_p + dQ_p \\ &= -pdV + C_p dT \qquad \text{where} \qquad pdV = nRdT \\ &= -nRdT + C_p dT \end{aligned} \tag{10.14.5}$$

If the changes in internal energies are the same in both cases, then eq. (10.14.5) is equal to eq. (10.14.4). Thus

$$C_V dT = -nRdT + C_p dT$$

This becomes

$$\boxed{C_p = C_V + nR} \tag{10.14.6}$$

We see the reason why $C_p > C_V$ is due to the addition of work on the system; eq. (10.14.4) shows no work done by the gas while eq. (10.14.5) shows that there is work done.

**Answer: (A)**

## 10.15 Helium atoms in a box

Let's say the probability of the atoms being inside the box is 1. So the probability that an atom will be found outside of a $1.0 \times 10^{-6}$ cm$^3$ box is

$$P = 1 - 1.0 \times 10^{-6} \tag{10.15.1}$$

As there are $N$ atoms and the probability of finding one is mutually exclusive of the other, the probabolity becomes

$$P = \left(1 - 1.0 \times 10^{-6}\right)^N \tag{10.15.2}$$

**Answer: (C)**

## 10.16 The Muon

It helps knowing what these particles are

**Muon** The muon, is a lepton, like the electron. It has the ame charge, $-e$ and spin, $1/2$, as the electron execpt it's about 200 times heavier. It's known as a heavy electron.

**Electron** This is the answer.

**Graviton** This is a hypothetical particle that mediates the force of gravity. It has no charge, no mass and a spin of 2. Nothing like an electron.

**Photon** The photon is the quantum of the electromagnetic field. It has no charge or mass and a spin of 1. Again nothing like an electron.

**Pion** The Pion belongs to the meson family. Again, nothing like leptons.

**Proton** This ia a sub atomic particle and is found in the nucleus of all atoms. Nothing like an electron.

**Answer: (A)**

## 10.17 Radioactive Decay

From the changes in the Mass and Atomic numbers after the subsequent decays, we expect an $\alpha$ and $\beta$ decay.

**Alpha Decay**

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}\alpha \tag{10.17.1}$$

**Beta Decay**

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}X' + {}_{-1}e^{-} + \bar{\nu}_e \tag{10.17.2}$$

Combining both gives

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}\alpha \rightarrow {}^{A}_{Z-1}Y + {}_{-1}e^{-} + \bar{\nu}_e \tag{10.17.3}$$

**Answer: (B)**

## 10.18 Schrödinger's Equation

We recall that Schrödinger's Equation is

$$\boxed{E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi} \tag{10.18.1}$$

Given that

$$\psi(x) = A\exp\left\{-\frac{b^2x^2}{2}\right\} \tag{10.18.2}$$

We differentiate and get

$$\frac{\partial^2\psi}{\partial x^2} = \left(b^4x^2 - b^2\right)\psi \tag{10.18.3}$$

Plugging into Schrödinger's Equation, eq. (10.18.1), gives us

$$E\psi = -\frac{\hbar^2}{2m}\left(b^4x^2 - b^2\right)\psi + V(x)\psi \tag{10.18.4}$$

Applying the boundary condition at $x = 0$ gives

$$E\psi = -\frac{\hbar^2}{2m}b^2\psi \tag{10.18.5}$$

This gives

$$-\frac{\hbar^2 b^2}{2m}\psi = -\frac{\hbar^2}{2m}\left(b^4x^2 - b^2\right)\psi + v(x)\psi \tag{10.18.6}$$

Solving for $V(x)$ gives

$$V(x) = \frac{\hbar^2 b^4 x^2}{2m} \tag{10.18.7}$$

**Answer: (B)**

## 10.19 Energy Levels of Bohr's Hydrogen Atom

We recall that the Energy Levels for the Hydrogen atom is

$$\boxed{E_n = -\frac{Z^2}{n^2}13.6\text{ eV}} \tag{10.19.1}$$

where $Z$ is the atomic number and $n$ is the quantum number. This can easily be reduced to

$$E_n = -\frac{A}{n^2} \tag{10.19.2}$$

**Answer: (E)**

## 10.20 Relativistic Energy

The Rest Energy of a particle is given

$$E = mc^2 \tag{10.20.1}$$

The Relativistic Energy is for a relativistic particle moving at speed $v$

$$E = \gamma_v mc^2 \tag{10.20.2}$$

We are told that a kaon moving at relativistic speeds has the same energy as the rest mass as a proton. Thus

$$E_{K^+} = E_p \tag{10.20.3}$$

where

$$
\begin{aligned}
E_{K^+} &= \gamma_v m_{K^+} c^2 && (10.20.4)\\
E_p &= m_p c^2 && (10.20.5)
\end{aligned}
$$

Equating both together gives

$$
\begin{aligned}
\gamma_v &= \frac{m_p}{m_{K^+}} && (10.20.6)\\
&= \frac{939}{494} && (10.20.7)\\
&\approx \tfrac{940}{500} && (10.20.8)
\end{aligned}
$$

This becomes

$$\gamma_v \approx 1.9 \qquad (10.20.9)$$

Solving gives

$$v^2 = \frac{2.61}{3.61}c^2 \qquad (10.20.10)$$

This gives $v^2$ in the order of 0.7. Squaring will give an answer that's greater than 0.7, the only answer is $0.85c$.

**Answer: (E)**

## 10.21 Space-Time Interval

We recall the Space-Time Interval from section 7.10.

$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2 (\Delta t)^2 \qquad (10.21.1)$$

We get

$$
\begin{aligned}
\Delta S^2 &= (5-3)^2 + (3-3)^2 + (3-1)^2 - c^2(5-3)^2\\
&= 2^2 + 0^2 + 2^2 - 2^2\\
&= 2^2\\
\Delta S &= 2 && (10.21.2)
\end{aligned}
$$

**Answer: (C)**

## 10.22 Lorentz Transformation of the EM field

Lorentz transformations show that electric and magnetic fields are different aspects of the same force; the electromagnetic force. If there was one stationary charge in our rest frame, we would observe an electric field. If we were to move to a moving frame of reference, Lorentz transformations predicts the presence of an additional magnetic field.

**Answer: (B)**

## 10.23 Conductivity of a Metal and Semi-Conductor

More of a test of what you know.

**A** Copper is a conductor so we expect its conductivity to be much greater than that of a semiconductor. **TRUE**.

**B** As the temperature of the conductor is increased its atoms vibrate more and disrupt the flow of electrons. As a result, resistance increases. **TRUE**.

**C** Different process. As temperature increases, electrons gain more energy to jump the energy barrier into the conducting region. So conductivity increases. **TRUE**.

**D** You may have paused to think for this one but this is **TRUE**. The addition of an impurity causes an increase of electron scattering off the impurity atoms. As a result, resistance increases.[5]

**E** The effect of adding an impurity on a semiconductor's conductivity depends on how many extra valence electrons it adds or subtracts; you can either widen or narrow the energy bandgap. This is of crucial importance to electronics today. So this is **FALSE**.

**Answer: (E)**

## 10.24 Charging a Battery

The Potential Difference across the resistor, $R$ is

$$PD = 120 - 100 = 20 \text{ V} \tag{10.24.1}$$

The Total Resistance is

$$\begin{aligned} R + r &= \frac{V}{I} \\ &= \frac{20}{10} \\ R + 1 &= 2 \\ \Rightarrow R &= 1\Omega \end{aligned} \tag{10.24.2}$$

**Answer: (C)**

## 10.25 Lorentz Force on a Charged Particle

We are told that the charged particle is released from rest in the electric and magnetic fields. The charged particle will experience a force from the magnetic field only when

[5] There are one or two cases where this is not true. The addition of Silver increases the conductivity of Copper. But the conductivity will still be less than pure silver.

it moves perpendicular to the direction of the magnetic field lines. The particle will move along the direction of the electric field.

We can also anylize this by looking at the Lorentz Force equation

$$\boxed{\mathbf{F}_q = q\left[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right]} \tag{10.25.1}$$

$\mathbf{v}$ is in the same direction as $\mathbf{B}$ so the cross product between them is zero. We are left with

$$\mathbf{F}_q = q\mathbf{E} \tag{10.25.2}$$

The force is directed along the electrical field line and hence it moves in a straight line.

**Answer: (E)**

## 10.26 K-Series X-Rays

To calculate we look at the energy levels for the Bohr atom. As the Bohr atom considers the energy levels for the Hydrogen atom, we need to modify it somewhat

$$\boxed{E_n = Z^2_{\text{eff}}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) 13.6 \text{ eV}} \tag{10.26.1}$$

where $Z_{\text{eff}}$ is the effective atomic number and $n_f$ and $n_i$ are the energy levels. For the $n_f = 1$ transition

$$Z_{\text{eff}} = Z - 1 \tag{10.26.2}$$

where $Z = 28$ for nickle. As the electrons come in from $n_i = \infty$, eq. (10.26.1) becomes

$$E_1 = (28-1)^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] 13.6 \text{ eV} \tag{10.26.3}$$

This works out to

$$\begin{aligned} E_1 &= (27^2) 13.6 \text{ eV} \\ &\approx (30)^2 \times 13.6 \text{ eV} \end{aligned} \tag{10.26.4}$$

This takes us in the keV range.

**Answer: (D)**

## 10.27 Electrons and Spin

It helps if you knew some facts here.

**A** The periodic table's arrangement of elements tells us about the chemical properties of an element and these properties are dependent on the valent electrons. How these valent electrons are arranged is, of course, dependent on spin. So this choice is **TRUE**.

**B** The energy of an elecron is quantized and obey the Pauli's Exclusion Principle. All the electrons are accomodated from the lowest state up to the Fermi Level and the distribution among levels is described by the Fermi distribution function, $f(E)$, which defines the probability that the energy level, $E$, is occupied by an electron.

$$f(E) = \begin{cases} 1, & E < E_F \\ 0, & E > E_F \end{cases}$$

where $f(E)$ is the Fermi-Dirac Distribution

$$f(E) = \frac{1}{e^{E-E_F/kT} + 1} \tag{10.27.1}$$

As a system goes above 0K, thermal energy may excite to higher energy states but this energy is not shared equally by all the electrons. The way energy is distributed comes about from the exclusion principle, the energy an electron my absorb at room temperture is $kT$ which is much smaller than $E_F = 5$eV. We can define a Fermi Temperature,

$$E_F = kT_F \tag{10.27.2}$$

which works out to be, $T_F = 60000$K. Thus only electrons close to this temperature can be excited as the levels above $E_F$ are empty. This results in a small number of electrons being able to be thermally excited and the low electronic specific heat.

$$C = \frac{\pi^2}{2} Nk \left( \frac{T}{T_f} \right) \qquad \text{where} \qquad kT << E_F$$

So this choice is also **TRUE**.

**C** The Zeeman Effect describes what happens to Hydrogen spectral lines in a magnetic field; the spectral lines split. In some atoms, there were further splits in spctral lines that couln't be explained by magnetic dipole moments. The explanation for this additional splitting was discovered to be due to electron spin.[6]

**D** The deflection of an electron in a uniform magnetic field deflects only in one way and demonstrates none of the electron's spin properties. Electrons can be deflected depending on their spin if placed in a non-uniform magnetic field, as was demonstrated in the Stern-Gerlach Experiment.[7]

**E** When the Hydrogen spectrum is observed at a very high resolution, closely spaced doublets are observed. This was one of the first experimental evidence for electron spin.[8]

[6] Write up on Zeeman and anomalous Zeemen effects
[7] Write up on Stern-Gerlach Experiment
[8] Write up on Fine Structure

## 10.28 Normalizing a wavefunction

We are given

$$\psi(\phi) = Ae^{im\phi} \tag{10.28.1}$$

Normalizing a function means

$$\boxed{\int_{-\infty}^{\infty} |\Psi(x)|^2 \, dx = 1} \tag{10.28.2}$$

In this case, we want

$$\int_0^{2\pi} \left|\psi(\phi)\right|^2 d\phi = 1 \tag{10.28.3}$$

and that

$$\left|\psi(x)\right|^2 = \psi^*(x)\psi(x) \tag{10.28.4}$$

So

$$\begin{aligned} \Rightarrow \left|\psi(\phi)\right|^2 &= A^2 e^{im\phi}e^{-im\phi} \\ A^2 \int_0^{2\pi} d\phi &= 1 \\ A^2\left[2\pi - 0\right] &= 1 \\ \Rightarrow A &= \frac{1}{\sqrt{2\pi}} \end{aligned} \tag{10.28.5}$$

## 10.29 Right Hand Rule

First we use the 'Grip' rule to tell what direction the magnetic field lines are going. Assuming the wire and current are coming out of the page, the magnetic field is in a clockwise direction around the wire. Now we can turn to Fleming's Right Hand Rule, to solve the rest of the question.

As we want the force acting on our charge to be parallel to the current direction, we see that this will happen when the charge moves towards the wire[9].

**Answer: (A)**

## 10.30 Electron Configuration of a Potassium atom

We can alalyze and eliminate

**A** The $n = 3$ shell has unfilled d-subshells. So this is **NOT TRUE**.

**B** The 4$s$ subshell only has one electron. The $s$ subshell can 'hold' two electrons so this is also **NOT TRUE**.

**C** Unknown.

[9]Don't forget to bring your right hand to the exam

**D** The sum of all the electrons, we add all the superscripts, gives 19. As this is a ground state, a lone potassium atom, we can tell that the atomic number is 19. So this is **NOT TRUE**.

**E** Potassium has one outer electron, like Hydrogen. So it will also have a spherically symmetrical charge distribution.

## 10.31 Photoelectric Effect I

We are given

$$|eV| = h\nu - W \tag{10.31.1}$$

We recall that $V$ is the stopping potential, the voltage needed to bring the current to zero. As electrons are negatively charged, we expect this voltage to be negative.

**Answer: (A)**

## 10.32 Photoelectric Effect II

Some history needs to be known here. The photoelectric effect was one of the experiments that proved that light was absorbed in discreet packets of energy. This is the experimental evidence that won Einstein the Nobel Prize in 1921.

**Answer: (D)**

## 10.33 Photoelectric Effect III

The quantity $W$ is known as the work function of the metal. This is the energy that is needed to just liberate an electron from its surface.

**Answer: (D)**

## 10.34 Potential Energy of a Body

We recall that

$$F = -\frac{dU}{dx} \tag{10.34.1}$$

Given that

$$U = kx^4 \tag{10.34.2}$$

The force on the body becomes

$$\begin{aligned} F &= -\frac{d}{dx}kx^4 \\ &= -4kx^3 \end{aligned} \tag{10.34.3}$$

**Answer: (B)**

## 10.35 Hamiltonian of a Body

The Hamiltonian of a body is simply the sum of the potential and kinetic energies. That is

$$H = T + V \tag{10.35.1}$$

where $T$ is the kinetic energy and $V$ is the potential energy. Thus

$$H = \frac{1}{2}mv^2 + kx^4 \tag{10.35.2}$$

We can also express the kinetic energy in terms of momentum, $p$. So

$$H = \frac{p^2}{2m} + kx^4 \tag{10.35.3}$$

**Answer: (A)**

## 10.36 Principle of Least Action

Hamilton's Principle of Least Action[10] states

$$\Phi = \int_T \left(T\left(q(t), \dot{q}(t)\right) - V\left(q(t)\right)\right) dt \tag{10.36.1}$$

where $T$ is the kinetic energy and $V$ is the potential energy. This becomes

$$\Phi = \int_{t_1}^{t_2} \left(\frac{1}{2}mv^2 - kx^4\right) dt \tag{10.36.2}$$

**Answer: (A)**

## 10.37 Tension in a Conical Pendulum

This is a simple case of resolving the horizontal and vertical components of forces. So we have

$$T\cos\theta = mg \tag{10.37.1}$$
$$T\sin\theta = mr\omega^2 \tag{10.37.2}$$

Squaring the above two equations and adding gives

$$T^2 = m^2g^2 + m^2r^2\omega^4 \tag{10.37.3}$$

Leaving us with

$$T = m\left(g^2 + r^2\omega^4\right) \tag{10.37.4}$$

**Answer: (E)**

[10]Write something on this

## 10.38 Diode OR-gate

This is an OR gate and can be illustrated by the truth table below.

| Input 1 | Input 2 | Output |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Table 10.38.1: Truth Table for OR-gate

**Answer: (A)**

## 10.39 Gain of an Amplifier vs. Angular Frequency

We are given that the amplifier has some sort of relationship where

$$G = K\omega^a \tag{10.39.1}$$

falls outside of the amplifier bandwidth region. This is that 'linear' part of the graph on the log-log graph. From the graph, we see that, $G = 10^2$, for $\omega = 3 \times 10^5$ second$^{-1}$. Substituting, we get

$$\begin{aligned} 10^2 &= K\left(3 \times 10^5\right)^a \\ \therefore \log(10^2) &= a \log\left[K\left(3.5 \times 10^5\right)\right] \\ \Rightarrow a &\approx 2 - 5 \end{aligned} \tag{10.39.2}$$

We can roughly estimate by subtracting the indices. So our relationship is of the form

$$G = K\omega^{-2} \tag{10.39.3}$$

**Answer: (E)**

## 10.40 Counting Statistics

We recall from section 8.4 , that he standard deviation of a counting rate is $\sigma = \sqrt{N}$, where $N$ is the number of counts. We have a count of $N = 9934$, so the standard deviation is

$$\begin{aligned} \sigma = \sqrt{N} &= \sqrt{9934} \\ &\approx \sqrt{10000} \\ &= 100 \end{aligned} \tag{10.40.1}$$

**Answer: (A)**

## 10.41 Binding Energy per Nucleon

More of a knowledge based question. Iron is the most stable of all the others.[11]

**Answer: (C)**

## 10.42 Scattering Cross Section

We are told the particle density of our scatterer is $\rho = 10^{20}$ nuclei per cubic centimeter. Given the thickness of our scatterer is $\ell = 0.1$ cm, the cross sectional area is

$$\begin{aligned} \rho &= \frac{N}{V} \\ &= \frac{N}{A\ell} \\ \Rightarrow A &= \frac{N}{\rho\ell} \end{aligned} \tag{10.42.1}$$

Now the probability of striking a proton is 1 in a million. So

## 10.43 Coupled Oscillators

There are two ways this system can oscillate, one mass on the end moves a lot and the other two move out of in the opposite directions but not as much or the centermass can be stationary and the two masses on the end move out of phase with each other. In the latter case, as there isn't any energy transfer between the masses, the period would be that of a single mass-spring system. The frequency of this would simply be

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{10.43.1}$$

where $k$ is the spring constant and $m$ is the mass.

**Answer: (B)**

### 10.43.1 Calculating the modes of oscillation

In case you require a more rigorous approach, we can calculate the modes of oscillation. The Lagrangian of the system is

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}m\left[\dot{x}_1^2 + 2\dot{x}_2^2 + \dot{x}_3^2\right] - \frac{1}{2}k\left[(x_2 - x_2)^2 + (x_3 - x_2)^2\right] \end{aligned} \tag{10.43.2}$$

The equation of motion can be found from

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_n}\right) = \frac{\partial L}{\partial x_n} \tag{10.43.3}$$

[11]Write up on Binding Energy

The equations of motion are

$$m\ddot{x}_1 = k(x_2 - x_1) \tag{10.43.4}$$
$$2m\ddot{x}_2 = kx_1 - 2kx_2 + kx_3 \tag{10.43.5}$$
$$m\ddot{x}_3 = -k(x_3 - x_2) \tag{10.43.6}$$

The solutions of the equations are

$$\begin{array}{lll} x_1 = A\cos(\omega t) & x_2 = B\cos(\omega t) & x_3 = C\cos(\omega t) \\ \ddot{x}_1 = -\omega^2 x_1 & \ddot{x}_2 = -\omega^2 x_2 & \ddot{x}_3 = -\omega^2 x_3 \end{array} \tag{10.43.7}$$

Solving this, we get

$$\left(k - m\omega^2\right) x_1 - kx_2 = 0 \tag{10.43.8}$$
$$-kx_1 + \left(2k - 2m\omega^2\right) x_2 - kx_3 = 0 \tag{10.43.9}$$
$$-kx_2 + \left(k - m\omega^2\right) x_3 = 0 \tag{10.43.10}$$

We can solve the modes of oscillation by solving

$$\begin{vmatrix} k - m\omega^2 & -k & 0 \\ -k & 2k - 2m\omega^2 & -k \\ 0 & -k & k - m\omega^2 \end{vmatrix} = 0 \tag{10.43.11}$$

Finding the determinant results in

$$\left(k - m\omega^2\right)\left[2\left(k - m\omega^2\right)^2 - k^2\right] - k\left[k\left(k - m\omega^2\right)\right] \tag{10.43.12}$$

Solving, we get

$$\omega = \frac{k}{m}; \frac{k}{m} \pm \frac{\sqrt{2}k}{m} \tag{10.43.13}$$

Substituting $\omega = k/m$ into the equations of motion, we get

$$x_1 = -x_3 \tag{10.43.14}$$
$$x_2 = 0 \tag{10.43.15}$$

We see that the two masses on the ends move out of phase with each other and the middle one is stationary.

## 10.44 Collision with a Rod

Momentum will be conserved, so we can say

$$\begin{aligned} mv &= MV \\ V &= \frac{mv}{M} \end{aligned} \tag{10.44.1}$$

**Answer: (A)**

## 10.45 Compton Wavelength

We recall from section 6.8.2, the Compton Equation from eq. (6.8.12)

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)} \tag{10.45.1}$$

Let $\theta = 90°$, we get the Compton Wavelength

$$\lambda_c = \frac{h}{m_e c} = 2.427 \times 10^{-12}\text{m} \tag{10.45.2}$$

**Answer: (C)**

## 10.46 Stefan-Boltzmann's Equation

We recall the Stefan-Boltzmann's Equation, eq. (4.16.1)

$$\boxed{P(T) = \sigma T^4} \tag{10.46.1}$$

At temperature, $T_1$,

$$P_1 = \sigma T_1 = 10 \text{ mW} \tag{10.46.2}$$

We are given $T_2 = 2T_1$, so

$$\begin{aligned} P_2 &= \sigma T_2^4 \\ &= \sigma (2T_1)^4 \\ &= 16T_2^4 \\ &= 16P_1 = 160 \text{ mW} \end{aligned} \tag{10.46.3}$$

**Answer: (E)**

## 10.47 Franck-Hertz Experiment

The Franck-Hertz Experiment as seen in section 6.9.3 deals with the manner in which electrons of certain energies scatter or collide with Mercury atoms. At certain energy levels, the Mercury atoms can 'absorb' the electrons energy and be excited and this occurs in discreet steps.

**Answer: (C)**

## 10.48 Selection Rules for Electronic Transitions

We recall the selection rules for photon emission

| | |
|---|---|
| $\Delta\ell = \pm 1$ | Orbital angular momentum |
| $\Delta m_\ell = 0, \pm 1$ | Magnetic quantum number |
| $\Delta m_s = 0$ | Secondary spin quantum number, |
| $\Delta j = 0, \pm 1$ | Total angular momentum |

**NOT FINISHED**

**Answer: (D)**

## 10.49 The Hamilton Operator

The time-independent Schrödinger equation can be written

$$\hat{H}\psi = E\psi \tag{10.49.1}$$

We can determine the energy of a quantum particle by regarding the classical nonrelativistic relationship as an equality of expectation values.

$$\langle H \rangle = \left\langle \frac{p^2}{2m} \right\rangle + \langle V \rangle \tag{10.49.2}$$

We can solve this through the substition of a momentum operator

$$p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x} \tag{10.49.3}$$

Substituting this into eq. (10.49.2) gives us

$$\begin{aligned} \langle H \rangle &= \int_{-\infty}^{+\infty} \psi^* \left[ -\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi \right] dx \\ &= \int_{-\infty}^{+\infty} \psi^* i\hbar \frac{\partial}{\partial t}\psi dx \end{aligned} \tag{10.49.4}$$

So we can get a Hamiltonian operator

$$\boxed{H \rightarrow i\hbar\frac{\partial}{\partial t}} \tag{10.49.5}$$

**Answer: (B)**

## 10.50 Hall Effect

The Hall Effect describes the production of a potential difference across a current carrying conductor that has been placed in a magnetic field. The magnetic field is directed perpendicularly to the electrical current.

As a charge carrier, an electron, moves through the conductor, the Lorentz Force will cause a deviation in the carge carrier's motion so that more charges accumulate in one location than another. This asymmetric distribution of charges produces an electric field that prevents the build up of more electrons. This 'equilibrium' voltage across the conductor is known as the Hall Voltage and remains as long as a current flows through our conductor.

As the deflection and hence, the Hall Voltage, is determined by the sign of the carrier, this can be used to measure the sign of charge carriers.

An equilibrium condition is reached when the electric force, generated by the accumulated charge carriers, is equal the the magnetic force, that causes the accumulation of charge carriers. Thus

$$F_m = ev_d B \qquad F_e = eE \tag{10.50.1}$$

The current through the conductor is

$$I = nAv_d e \tag{10.50.2}$$

For a conductor of width, $w$ and thickness, $d$, there is a Hall voltage across the width of the conductor. Thus the electrical force becomes

$$\begin{aligned} F_e &= eE \\ &= \frac{EV_H}{w} \end{aligned} \tag{10.50.3}$$

The magnetic force is

$$F_m = \frac{BI}{neA} \tag{10.50.4}$$

eq. (10.50.3) is equal to eq. (10.50.4), thus

$$\begin{aligned} \frac{eV_H}{w} &= \frac{BI}{newd} \\ \therefore V_H &= \frac{BI}{ned} \end{aligned} \tag{10.50.5}$$

So for a measured magnetic field and current, the sign of the Hall voltage gives is the sign of the charge carrier.

**Answer: (C)**

## 10.51 Debye and Einstein Theories to Specific Heat

The determination of the specific heat capacity was first deermined by the Law of Dulong and Petite. This Law was based on Maxwell-Boltzmann statistics and was accurate in its predictions except in the region of low temperatures. At that point there is a departure from prediction and measurements and this is where the Einstein and Debye models come into play.

Both the Einstein and Debye models begin with the assumption that a crystal is made up of a lattice of connected quantum harmonic oscillators; **choice B**.

The Einstein model makes three assumptions

1. Each atom is a three-dimensional quantum harmonic oscillator.

2. Atoms do not interact with each other.

3. Atoms vibrate with the same frequency.

Einstein assumed a quantum oscillator model, similar to that of the black body radiation problem. But despite its success, his theory predicted an exponential decress in heat capacity towards absolute zero whereas experiments followed a $T^3$ relationship. This was solved in the Debye Model.

The Debye Model looks at phonon contribution to specific heat capacity. This theory correctly predicted the $T^3$ proportionality at low temperatures but suffered at intemediate temperatures.

**Answer: (B)**

## 10.52 Potential inside a Hollow Cube

By applying Gauss' Law and drawing a Gaussian surface inside the cube, we see that no charge is enclosed and hence no electric field[12]. We can realte the electric field to the potential

$$\boxed{\mathbf{E} = -\nabla V} \tag{10.52.1}$$

Where $V$ is the potential.

Gauss' Law shows that with no enclosed charge we have no electric field inside our cube. Thus

$$\mathbf{E} = -\nabla V = 0 \tag{10.52.2}$$

As eq. (10.52.1) is equal to zero, the potential is the same throughout the cube.[13]

**Answer: (E)**

## 10.53 EM Radiation from Oscillating Charges

As the charge particle oscillates, the electric field oscillates as well. As the field oscillates and changes, we would expect this changing field to affect a distant charge. If we consider a charge along the $xy$-plane, looking directly along the $x$-axis, we won't "see" the charge oscillating but we would see it clearly if we look down the $y$-axis. If we were to visualize the field, it would look like a doughnut around the $x$-axis. Based on that analysis, we choose **(C)**

**Answer: (C)**

## 10.54 Polarization Charge Density

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \tag{10.54.1}$$

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \epsilon_0 \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{P} \\ &= \frac{\epsilon_D \nabla \cdot \mathbf{E}}{\kappa} - \sigma_p\end{aligned}$$

[12] Draw Cube at potential $V$ with Gaussian Surface enclosing no charge

[13] As we expect there to be no Electric Field, we must expect the potential to be the same throughout the space of the cube. If there were differences, a charge place inside the cube would move.

**Answer: (E)**[14]

## 10.55 Kinetic Energy of Electrons in Metals

Electrons belong to a group known as fermions[15] and as a result obey the Pauli Exclusion Principle[16]. So in the case of a metal, there are many fermions present each with a different set of quantum numbers. The electron with the highest energy state is has an energy value known as the Fermi Energy.

**NOT FINSIHED**

**Answer: (B)**

## 10.56 Expectation or Mean Value

This is a definition question. The question states that for an operator $Q$,

$$\boxed{\langle Q \rangle = \int_{-\infty}^{+\infty} \psi^* Q \psi dx} \tag{10.56.1}$$

This is the very definition of the expectation or mean value of $Q$.

**Answer: (C)**

## 10.57 Eigenfuction of Wavefunction

We are given the momentum operator as

$$\boxed{p = -i\hbar \frac{\partial}{\partial x}} \tag{10.57.1}$$

With an eigenvalue of $\hbar k$. We can do this by trying each solution and seeing if they match[17]

$$-i\hbar \frac{\partial \psi}{\partial x} = \hbar k \psi \tag{10.57.2}$$

A: $\psi = \cos kx$ We expect $\psi$, to have the form of an exponential function. Substituting this into the eigenfuntion, eq. (10.57.2), we have

$$\begin{aligned} -i\hbar \frac{\partial}{\partial x} \cos kx &= -i\hbar \left(-k \sin kx\right) \\ &= i\hbar k \sin kx \neq \hbar k \psi \end{aligned}$$

$\psi$ does not surive our differentiation and so we can eliminate it.

[14] Check Polarization in Griffiths

[15] Examples of fermions include electrons, protons and neutrons

[16] The Pauli Exclusion Principle states that no two fermions may occupy the same quantum state

[17] We can eliminate choices **(A)** & **(B)** as we would expect the answer to be an exponential function in this case. These choices were just done for illustrative purposes and you should know to avoid them in the exam.

B: $\psi = \sin kx$ This is a similar case to the one above and we can eliminate for this reason.

$$\begin{aligned}
-i\hbar\frac{\partial}{\partial x}\sin kx &= -i\hbar\left(k\cos kx\right) \\
&= -i\hbar k\cos kx \neq \hbar k\psi
\end{aligned}$$

Again we see that $\psi$ does not survive when we apply our operator and so we can eliminate this choice as well.

C: $\psi = \exp -ikx$ Substituting this into eq. (10.57.2), gives

$$\begin{aligned}
-i\hbar\frac{\partial}{\partial x}e^{-ikx} &= -i\hbar\left(-ike^{-ikx}\right) \\
&= -\hbar ke^{-ikx} \neq \hbar k\psi
\end{aligned}$$

Close but we are off, so we can eliminate this choice as well.

D: $\psi = \exp ikx$ If the above choice didn't work, this might be more likely to.

$$\begin{aligned}
-i\hbar\frac{\partial}{\partial x}e^{ikx} &= -i\hbar\left(ike^{ikx}\right) \\
&= \hbar ke^{-ikx} = \hbar k\psi
\end{aligned}$$

**Success, this is our answer.**

E: $= \psi = \exp -kx$

$$\begin{aligned}
-i\hbar\frac{\partial}{\partial x}e^{-kx} &= -i\hbar\left(-ke^{-kx}\right) \\
&= -i\hbar ke^{-kx} \neq \hbar k\psi
\end{aligned}$$

Again this choice does not work, so we can eliminate this as well

**Answer: (D)**

## 10.58 Holograms

The hologram is an image that produces a 3-dimensional image using both the Amplitude and Phase of a wave. Coherent, monochromatic light, such as from a laser, is split into two beams. The object we wish to "photograph" is placed in the path of the illumination beam and the scattered light falls on the recording medium. The second beam, the reference beam is reflected unimpeded to the recording medium and these two beams produces an interference pattern.

The intensity of light recorded on our medium is the same as the scattered light from our object. The interference pattern is a result of phase changes as light is scattered off our object. Thus choices **(I)** and **(II)** are true.

**Answer: (B)**

## 10.59 Group Velocity of a Wave

We are given the dispersion relationship of a wave as

$$\omega^2 = \left(c^2k^2 + m^2\right)^{\frac{1}{2}} \tag{10.59.1}$$

The Group Velocity of a Wave is

$$\boxed{v_g = \frac{d\omega}{dk}} \tag{10.59.2}$$

By differentiating eq. (10.59.1) with respect to $k$, we can determine th group velocity

$$\begin{aligned} 2\omega d\omega &= 2c^2kdk \\ \Rightarrow \frac{d\omega}{dk} &= \frac{c^2k}{\omega} \\ &= \frac{c^2k}{\sqrt{c^2k^2 + m^2}} \end{aligned} \tag{10.59.3}$$

We want to examine the cases as $k \to 0$ and $k \to \infty$.

As $k \to 0$, we have

$$\begin{aligned} \frac{d\omega}{dk} &= \frac{c^2 0}{\sqrt{0 + m^2}} \\ &= 0 \end{aligned} \tag{10.59.4}$$

As $k \to \infty$, $c^2k^2 >> m^2$ the denominator becomes

$$\sqrt{c^2k^2 + m} \approx c^2k^2 \tag{10.59.5}$$

Replacing the denominator for our group velocity gives

$$\frac{d\omega}{dk} = \frac{c^2k}{ck} = c \tag{10.59.6}$$

**Answer: (E)**



## 10.60 Potential Energy and Simple Harmonic Motion

We are given a potential energy of

$$V(x) = a + bx^2 \tag{10.60.1}$$

We can determine the mass's spring constant, $k$, from $V''(x)$

$$\boxed{V''(x) = 2b = k} \tag{10.60.2}$$

The angular frequency, $\omega$, is

$$\omega^2 = \frac{k}{m} = \frac{2b}{m} \tag{10.60.3}$$

We see this is dependent on $b$ and $m$.

**Answer: (C)**

## 10.61 Rocket Equation I

We recall from the rocket equation that $u$ in this case is the speed of the exaust gas relative to the rocket.

**Answer: (E)**

## 10.62 Rocket Equation II

The rocket equation is

$$m\frac{dv}{dt} + u\frac{dm}{dt} = 0 \tag{10.62.1}$$

Solving this equation becomes

$$\begin{aligned} mdv &= udm \\ \int_0^v dv &= u\int_{m_0}^{m}\frac{dm}{m} \\ v &= u\ln\left(\frac{m}{m_0}\right) \end{aligned} \tag{10.62.2}$$

This fits none of the answers given.

**Answer: (E)**

## 10.63 Surface Charge Density

This question was solved as 'The Classic Image Problem'. Below is an alternative method but the principles are the same. Instead of determining the electrical potential, as was done by Griffiths, we will find the electrical field of a dipole and determine the surface charge density using

$$\boxed{E = \frac{\sigma}{\epsilon_0}} \tag{10.63.1}$$

Our point charge, $-q$ will induce a $+q$ on the grounded conducting plane. The resulting electrical field will be due to a combination of the real charge and the 'virtual' induced charge. Thus

$$\begin{aligned} \mathbf{E} = -E_y\hat{\mathbf{j}} &= (E_- + E_+)\,\hat{\mathbf{j}} \\ &= 2E_-\hat{\mathbf{j}} \end{aligned} \tag{10.63.2}$$

Remember the two charges are the same, so at any point along the $x$-axis, or rather our grounded conductor, the electrical field contributions from both charges will be the same. Thus

$$\begin{aligned} E_- &= \frac{q}{4\pi\epsilon r^2}\cos\theta \qquad \text{where } \cos\theta = \frac{d}{r} \\ &= \frac{qd}{4\pi\epsilon_0 r^3} \end{aligned} \tag{10.63.3}$$

Our total field becomes

$$E = \frac{2qd}{4\pi\epsilon_0 r^3} \tag{10.63.4}$$

You may recognize that $2qd$ is the electrical dipole moment. Now, putting eq. (10.63.4) equal to eq. (10.63.1) gives us

$$\frac{\sigma}{\epsilon_0} = \frac{qd}{2\pi\epsilon_0 r^3} \tag{10.63.5}$$

where $r = D$, we get

$$\sigma = \frac{qd}{2\pi D^2} \tag{10.63.6}$$

**Answer: (C)**

## 10.64 Maximum Power Theorem

We are given the impedance of our generator

$$Z_g = R_g + jX_g \tag{10.64.1}$$

For the maximum power to be transmitted, the maximum power theorem states that the load impedance must be equal to the complex conjugate of the generator's impedance.

$$\boxed{Z_g = Z_\ell^*} \tag{10.64.2}$$

Thus

$$\begin{aligned} Z_\ell &= R_g + jX_\ell \\ &= R_g - jX_g \end{aligned} \tag{10.64.3}$$

**Answer: (C)**

## 10.65 Magnetic Field far away from a Current carrying Loop

The Biot-Savart Law is

$$\boxed{d\mathbf{B} = \frac{\mu_0 i}{4\pi}\frac{d\ell \times \hat{\mathbf{r}}}{r^3}} \tag{10.65.1}$$

Let $\theta$ be the angle between the radius, $b$ and the radius vector, $r$, we get

$$\begin{aligned}
B &= \frac{\mu_0 i}{4\pi}\frac{r d\ell \cos\theta}{r^3} \qquad \text{where} \qquad \cos\theta = \frac{b}{r} \\
&= \frac{mu_0 i}{4\pi}\frac{d\ell\cos\theta}{r^2} \\
&= \frac{\mu_0 i}{4\pi}\frac{b d\ell}{r^3} \qquad \text{where} \qquad r = \sqrt{b^2+h^2} \\
&= \frac{\mu_0 i}{4\pi}\frac{b d\ell}{(b^2+h^2)^{\frac{3}{2}}} \qquad \text{where} \qquad d\ell = b\cdot d\theta \\
&= \frac{\mu 0 i}{4\pi}\cdot\frac{b^2}{(b^2+h^2)^{\frac{3}{2}}}\int_0^{2\pi} d\theta \\
&= \frac{\mu_0 i}{2}\frac{b^2}{(b^2+h^2)^{\frac{3}{2}}}
\end{aligned} \tag{10.65.2}$$

we see that

$$B \propto ib^2 \tag{10.65.3}$$

**Answer: (B)**

## 10.66 Maxwell's Relations

To derive the Maxwell's Relations we begin with the thermodynamic potentials

**First Law**

$$dU = TdS - PdV \tag{10.66.1}$$

**Entalpy**

$$\begin{aligned} H &= E + PV \\ \therefore dH &= TdS + VdP \end{aligned} \tag{10.66.2}$$

**Helmholtz Free Energy**

$$\begin{aligned} F &= E - TS \\ \therefore dF &= -SdT - PdV \end{aligned} \tag{10.66.3}$$

**Gibbs Free Energy**

$$\begin{aligned} G &= E - TS + PV \\ \therefore dG &= -SdT + VdP \end{aligned} \tag{10.66.4}$$

All of these differentials are of the form

$$\begin{aligned} dz &= \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \\ &= Mdx + Ndy \end{aligned}$$

For the variables listed, we choose eq. (10.66.1) and applying the above condition we get

$$T = \left(\frac{\partial U}{\partial S}\right)_V \qquad P = \left(\frac{\partial U}{\partial V}\right)_S \tag{10.66.5}$$

Thus taking the inverse of $T$, gives us

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V \tag{10.66.6}$$

**Answer: (E)**

## 10.67 Partition Functions

**NOT FINISHED**

## 10.68 Particle moving at Light Speed

**Answer: (A)**

## 10.69 Car and Garage I

We are given the car's length in its rest frame to be $L' = 5$ meters and its Lorentz Contracted length to be $L = 3$ meters. We can determine the speed from eq. (7.3.1)

$$\begin{aligned} L &= L'\sqrt{1 - \frac{v^2}{c^2}} \\ \left(\frac{3}{5}\right)^2 &= 1 - \frac{v^2}{c^2} \\ \Rightarrow v &= \frac{4}{5}c \end{aligned} \tag{10.69.1}$$

**Answer: (C)**

## 10.70 Car and Garage II

As the car approaches the garage, the driver will notice that things around him, including the garage, are length contracted. We have calculated that the speed that he is travelling at to be, $v = 0.8c$, in the previous section. We again use the Length Contraction formula, eq. (7.3.1), to solve this question.

$$\begin{aligned} L_g &= L'_g\left(1 - \frac{v^2}{c^2}\right) \\ &= 4\left(1 - 0.8^2\right) \\ &= 2.4 \text{ meters} \end{aligned} \tag{10.70.1}$$

**Answer: (A)**

## 10.71 Car and Garage III

This is more of a conceptual question. What happens depends on whose frame of reference you're in.

**Answer: (E)**

## 10.72 Refrective Index of Rock Salt and X-rays

No special knowledge is needed but a little knowledge always helps. You can start by eliminating choices when in doubt.

**Choice A NOT TRUE** Relativity says nothing about whether light is in a vacuum or not. If anything, this choice goes against the postulates of Special Relativity. The laws of Physics don't change in vacuum.

**Choice B NOT TRUE**. X-rays can "transmit" signals or energy; any waveform can once it is not distorted too much during propagation.

**Choice C NOT TRUE**. Photons have zero rest mass. Though the tachyon, a hypothetical particle, has imaginary mass. This allows it to travel faster than the speed or light though they don't violate the principles of causality.

**Choice D NOT TRUE**. How or when we discover physical theories has no bearing on observed properties or behavior; though according to some it may seem so at times[18]

**Choice E** The phase and group speeds can be different. The phase velocity is the rate at which the crests of the wave propagate or the rate at which the phase of the wave is moving. The group speed is the rate at which the envelope of the waveform

[18]There is a quote by Douglas Adams,

> There is a theory which states that is ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable.
>
> There is another which states this has already happened.

So maybe the order in which discoveries are made matters. Who am I to question Douglas Adams?

is moving or rather it's the rate at which the amplitude varies in the waveform. We can use this principle of $n < 1$ materials to create X-ray mirrors using "total external reflection".

**Answer: (E)**

## 10.73 Thin Flim Non-Reflective Coatings

To analyze this system, we consider our lens with refractive index, $n_3$, being coated by our non-reflective coating of refractive index, $n_2$, and thickness, $t$, in air with refractive index, $n_1$, where

$$n_1 < n_2 < n_3 \tag{10.73.1}$$

As our ray of light in air strikes the first boundary, the coating, it moves from a less optically dense medium to a more optically dense one. At the point where it reflects, there will be a phase change in the reflected wave. The transmitted wave goes through without a phase change.

The refracted ray passes through our coating to strike our glass lens, which is optically more dense than our coating. As a result there will be a phase change in our reflected ray. Destructive interference occurs when the optical path difference, $2t$, occurs in half-wavelengths multiples. So

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \tag{10.73.2}$$

where $m = 0; 1; 2; 3$. The thinnest possible coating occurs at $m = 0$. Thus

$$t = \frac{1}{4}\frac{\lambda}{n_2} \tag{10.73.3}$$

We need a non-reflective coating that has an optical thicknes of a quarter wavelength.

**Answer: (A)**

## 10.74 Law of Malus

The **Law of Malus** states that when a perfect polarizer is placed in a polarized beam of light, the intensity $I$, is given by

$$I = I_0 \cos^2\theta \tag{10.74.1}$$

where $\theta$ is the angle between the light's plane of polarization and the axis of the polarizer. A beam of light can be considered to be a uniform mix of plane polarization angles and the average of this is

$$\begin{aligned} I &= I_0 \int_0^{2\pi} \cos^2\theta \\ &= \frac{1}{2} I_0 \end{aligned} \tag{10.74.2}$$

So the maximum fraction of transmitted power through all three polarizers becomes

$$I_3 = \left(\frac{1}{2}\right)^3 = \frac{I_0}{8} \tag{10.74.3}$$

**Answer: (B)**

## 10.75 Geosynchronous Satellite Orbit

We can relate the period or the angluar velocity of a satellite and Newton's Law of Gravitation

$$mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2 = \frac{GMm}{R^2} \tag{10.75.1}$$

where $M$ is the mass of the Earth, $m$ is the satellite mass and $R_E$ is the orbital radius. From this we can get a relationship between the radius of orbit and its period, which you may recognize as Kepler's Law.

$$R^3 \propto T^2 \tag{10.75.2}$$

We can say

$$R_E^3 \propto (80)^2 \tag{10.75.3}$$

$$R_S^3 \propto (24 \times 60)^2 \tag{10.75.4}$$

$$\tag{10.75.5}$$

Dividing eq. (10.75.4) and eq. (10.75.5), gives

$$\left(\frac{R_S}{R_E}\right)^3 = \left(\frac{24 \times 60}{80}\right)^2$$

$$R_S^3 = 18^2 R_E^3 \tag{10.75.6}$$

**Answer: (B)**

## 10.76 Hoop Rolling down and Inclined Plane

As the hoop rolls down the inclined plane, its gravitational potential energy is converted to translational kinetic energy and rotational kinetic energy

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \tag{10.76.1}$$

Recall that $v = \omega R$, eq. (10.76.1) becomes

$$MgH = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(MR^2\right)\omega^2 \tag{10.76.2}$$

Solving for $\omega$ leaves

$$\omega = \left(\frac{gh}{R^2}\right)^{\frac{1}{2}} \tag{10.76.3}$$

The angular momentum is

$$L = I\omega \tag{10.76.4}$$

Substituting eq. (10.76.3) gives us

$$\begin{aligned} L &= MR\left(\frac{gh}{R^2}\right)^{\frac{1}{2}} \\ &= MR\sqrt{gh} \end{aligned} \tag{10.76.5}$$

**Answer: (A)**

## 10.77 Simple Harmonic Motion

We are told that a particle obeys Hooke's Law, where

$$\mathbf{F} = -k\mathbf{x} \tag{10.77.1}$$

We can write the equation of motion as

$$m\ddot{\mathbf{x}} - k\mathbf{x} \quad \text{where} \quad \omega^2 = \frac{k}{m}$$

where

$$x = A\sin\left(\omega t + \phi\right) \tag{10.77.2}$$

$$\text{and} \quad \dot{x} = \omega A\cos\left(\omega t + \phi\right) \tag{10.77.3}$$

We are told that

$$\frac{1}{2} = \sin\left(\omega t + \phi\right) \tag{10.77.4}$$

We can show that

$$\cos\left(\omega t + \phi\right) = \frac{\sqrt{3}}{2} \tag{10.77.5}$$

Substituting this into eq. (10.77.3) gives

$$\begin{aligned} \dot{x} &= 2\pi f A \cdot \frac{\sqrt{3}}{2} \\ &= \sqrt{3}\pi f A \end{aligned} \tag{10.77.6}$$

**Amswer: (B)**

## 10.78 Total Energy between Two Charges

We are told three things

1. There is a zero potential energy, and

2. one particle has non-zero speed and hence kinetic energy.

3. No radiation is emitted, so no energy is lost.

The total energy of the system is

$$\begin{aligned} E &= \text{Potential Energy} + \text{ Kinetic Energy} \\ &= 0 + (\text{KE} > 0) \\ &> 0 \end{aligned} \tag{10.78.1}$$

Applying the three condition, we expect the total energy to be positive and constant.
**Answer: (C)**

## 10.79 Maxwell's Equations and Magnetic Monopoles

You may have heard several things about the $\nabla\cdot\mathbf{B} = 0$ equation in Maxwell's Laws. One of them is there being no magnetic monopoles or charges. There are some implications to this. No charge implies that the amount of field lines that enter a Gaussian surface must be equal to the amount of field lines that leave. So using this principle we know from the electric form of this law we can get an answer to this question.

**Choice A** The number of field lines that enter is the same as the number that leaves. So this does not violate the above law.

**Choice B** Again we see that the number of field lines entering is the same as the number leaving.

**Choice C** The same as above

**Choice D** In this case, we see that the field lines at the edge of the Gaussian Surface are all leaving; no field lines enter the surface. This is also what we'd expect the field to look like for a region bounded by a magnetic monopole.

**Choice E** The field loops in on itself, so the total number of field lines is zero. This fits with the above law.

**Answer: (D)**

## 10.80 Gauss' Law

To determine an electric field that could exist in a region of space with no charges we turn to Gauss' Law.

$$\boxed{\nabla \cdot \mathbf{E} = 0} \tag{10.80.1}$$

or rather

$$\boxed{\frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z = 0} \tag{10.80.2}$$

So we analyze each choice in turn to get our answer.

**Choice A**

$$\begin{aligned} \mathbf{E} &= 2xy\hat{i} - xy\hat{k} \\ \nabla \cdot \mathbf{E} &= \frac{\partial}{\partial x} 2xy + \frac{\partial}{\partial z}(-xz) \\ &= 2y + x \neq 0 \end{aligned} \tag{10.80.3}$$

**Choice B**

$$\begin{aligned} \mathbf{E} &= -xy\hat{j} + xz\hat{k} \\ \nabla \cdot \mathbf{E} &= \frac{\partial}{\partial y}(-xy) + \frac{\partial}{\partial z} xz \\ &= -x + x = 0 \end{aligned} \tag{10.80.4}$$

**Choice C**

$$\begin{aligned} \mathbf{E} &= xz\hat{i} + xz\hat{j} \\ \nabla \cdot \mathbf{E} &= \frac{\partial}{\partial x} xz + \frac{\partial}{\partial y} xz \\ &= z + 0 \neq 0 \end{aligned} \tag{10.80.5}$$

**Choice D**

$$\begin{aligned} \mathbf{E} &= xyz(\hat{i} + \hat{j}) \\ \nabla \cdot \mathbf{E} &= \frac{\partial}{\partial x} xyz + \frac{\partial}{\partial y} xyz \\ &= yz + xz \neq 0 \end{aligned} \tag{10.80.6}$$

**Choice E**

$$\begin{aligned} \mathbf{E} &= xyz\hat{i} \\ \nabla \cdot \mathbf{E} &= \frac{\partial}{\partial x} xyz \\ &= yz \neq 0 \end{aligned} \tag{10.80.7}$$

**Answer: (B)**

## 10.81 Biot-Savart Law

We can determine the magnetic field produced by our outer wire from the Biot-Savart Law

$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{d\ell \times \mathbf{r}}{r^3}} \tag{10.81.1}$$

As our radius and differential length vectors are orthogonal, the magnetic field works out to be

$$
\begin{aligned}
d\mathbf{B} &= \frac{\mu_0}{4\pi} I \frac{d\ell r}{r^3} \\
&= \frac{\mu_0 I}{4\pi} \cdot \frac{r d\theta}{r^2} \\
\mathbf{B} &= \frac{\mu_0 I}{4\pi r} \int_0^{2\pi} d\theta \\
&= \frac{\mu_0 I}{2b}
\end{aligned} \tag{10.81.2}
$$

We know from Faraday's Law, a changing magnetic flux induces a EMF,

$$
\mathscr{E} = \frac{d\Phi}{dt} \tag{10.81.3}
$$

where $\Phi = BA$. The magnetic flux becomes

$$
\Phi = \frac{\mu_0 I}{2b} \cdot \pi a^2 \tag{10.81.4}
$$

The induced EMF becomes

$$
\begin{aligned}
\mathscr{E} &= \frac{\mu_0 \pi}{2} \left( \frac{a^2}{b} \right) \frac{dI}{dt} \\
&= \frac{\mu_0 \pi}{2} \left( \frac{a^2}{b} \right) \omega I_0 \sin \omega t
\end{aligned} \tag{10.81.5}
$$

**Answer: (B)**

## 10.82 Zeeman Effect and the emission spectrum of atomic gases

Another knowledge based question best answered by the process of elimination.

**Stern-Gerlach Experiemnt** The Stern-Gerlach Experiment has nothing to do with spectral emissions. This experiment, performed by O. Stern and W. Gerlach in 1922 studies the behavior of a beam atoms being split in two as they pass through a non-uniform magnetic field.

**Stark Effect** The Stark Effect deals with the shift in spectral lines in the presence of electrical fields; not in magnetic fields.

**Nuclear Magnetic Moments of atoms** Close, the splitting seen in the Stern-Gerlach Experiment is due to this. Emission spectrum typically deals with electrons and so we would expect it to deal with electrons on some level.

**Emission lines are split in two** Closer but still not accurate. There is splitting but in some cases it may be more than two.

**Emission lines are greater or equal than in the absence of the magnetic field** This we know to be true.

The difference in the emission spectrum of a gas in a magnetic field is due to the Zeeman effect.

**Answer: (E)**

## 10.83 Spectral Lines in High Density and Low Density Gases

We expect the spectral lines to be broader in a high density gas and narrower in a low density gas ue to the increased colissions between the molecules. Atomic collisions add another mechanism to transfer energy.

**Answer: (C)**

## 10.84 Term Symbols & Spectroscopic Notation

To determine the term symbol for the sodium ground state, we start with the electronic configuration. This is easy as they have given us the number of electrons the element has thus allowing us to fill sub-shells using the Pauli Exclusion Principle. We get

$$1s^2, 2s^2, 2p^6, 3s^1 \tag{10.84.1}$$

We are most interested in the $3s^1$ sub-shell and can ignore the rest of the filled sub-shells. As we only have one valence electron then $m_s = +1/2$. Now we can calculate the total spin quantum number, **S**. As there is only one unpaired electron,

$$\mathbf{S} = \frac{1}{2} \tag{10.84.2}$$

Now we can calculate the total angular momentum quantum number, $\mathbf{J} = \mathbf{L} + \mathbf{S}$. As the $3s$ sub-shell is half filled then

$$\mathbf{L} = 0 \tag{10.84.3}$$

This gives us

$$\mathbf{J} = \frac{1}{2} \tag{10.84.4}$$

and as $L = 0$ then we use the symbol $S$. Thus our term equation becomes

$$^2S_{\frac{1}{2}} \tag{10.84.5}$$

**Answer: (B)**

## 10.85 Photon Interaction Cross Sections for Pb

Check Brehm p. 789

**Answer: (B)**

## 10.86 The Ice Pail Experiment

Gauss' law is equivalent to Coulomb's Law because Coulomb's Law is an inverse square law; testing one is a valid test of the other. Much of our knowledge of the consequences of the inverse square law came from the study of gravity. Jason Priestly knew that there is no gravitational field within a spherically symmetrical mass distribution. It was suspected that was the same reason why a charged cork ball inside a charged metallic container isn't attracted to the walls of a container.

**Answer: (E)**

## 10.87 Equipartition of Energy and Diatomic Molecules

To answer this question, we will turn to the equipartition of energy equation

$$\boxed{c_v = \left(\frac{f}{2}\right) R} \tag{10.87.1}$$

where $f$ is the number of degrees of freedom. In the case of Model I, we see that

| Degrees of Freedom | Model I | Model II |
|---|---|---|
| Translational | 3 | 3 |
| Rotational | 2 | 2 |
| Vibrational | 0 | 2 |
| Total | 5 | 7 |

Table 10.87.1: Specific Heat, $c_v$ for a diatomic molecule

So the specific heats for Models I & II are

$$c_{v_I} = \frac{5}{2}Nk \qquad c_{v_{II}} = \frac{7}{2}Nk$$

Now we can go about choosing our answer

**Choice A** From our above calculations, we see that $c_{v_I} = 5/2Nk$. So this choice is **WRONG**.

**Choice B** Again, our calculations show that the specific heat for Model II is larger than than of Model I. This is due to the added degrees of freedom (vibrational) that it possesses. So this choice is **WRONG**.

**C & D** They both contradict the other and they both contradict **Choice (E)**.

**E** This is **TRUE**. We know that at higher temperatures we have an additional degree of freedom between our diatomic molecule.

**Answer: (E)**

## 10.88 Fermion and Boson Pressure

To answer this question, we must understand the differences between fermions and bosons. Fermions follow Fermi-Dirac statistics and their behavior is obey the Pauli Exclusion Principle. Basically, this states that no two fermions may have the same quantum state. Bosons on the other hand follow Bose-Einstein statistics and several bosons can occupy the same quantum state.

As the temperature of a gas drops, the particles are going to fill up the available energy states. In the case of fermions, as no two fermions can occupy the same state, then these particles will try to occupy all the energy states it can until the highest is filled. Bosons on the other hand can occupy the same state, so they will all 'group' together for the lowest they can. Classically, we don't pay attention to this grouping, so based on our analyis, we expect,

$$P_F > P_C > P_B \tag{10.88.1}$$

where $P_B$ is the boson pressure, $P_C$ is the pressure with no quantum effects taking place and $P_F$ to be the fermion pressure.

**Answer: (B)**

## 10.89 Wavefunction of Two Identical Particles

We are given the wavefunction of two identical particles,

$$\boxed{\psi = \frac{1}{\sqrt{2}}\left[\psi_\alpha(x_1)\psi_\beta(x_2) + \psi_\beta(x_1)\psi_\alpha(x_2)\right]} \tag{10.89.1}$$

This is a symmetric function and satisfies the relation

$$\boxed{\psi_{\alpha\beta}(x_2, x_1) = \psi_{\alpha\beta}(x_1, x_2)} \tag{10.89.2}$$

Symmetric functions obey Bose-Einstein statistics and are known as bosons. Upon examination of our choices, we see that[19]

**electrons** fermion

**positrons** fermion

**protons** fermion

[19] You could have easily played the 'one of thes things is not like the other...' game

**neutron** fermion

**deutrons Boson**

Incidentally, a anti-symmetric function takes the form,

$$\boxed{\psi = \frac{1}{\sqrt{2}}\left[\psi_\alpha(x_1)\psi_\beta(x_2) - \psi_\beta(x_1)\psi_\alpha(x_2)\right]} \tag{10.89.3}$$

and satisfies the relation

$$\boxed{\psi_{\alpha\beta}(x_2, x_1) = -\psi_{\alpha\beta}(x_1, x_2)} \tag{10.89.4}$$

These obey Fermi-Dirac Statistics and are known as fermions.
**Answer: (E)**

## 10.90 Energy Eigenstates

We may recognize this wavefunction from studying the particle in an infinite well problem and see this is the $n = 2$ wavefunction. We know that

$$E_n = n^2 E_0 \tag{10.90.1}$$

We are given that $E_2 = 2$ eV. So

$$\begin{aligned} E_0 &= \frac{1}{n^2}E_2 \\ &= \frac{2}{4}\text{ eV} \\ &= \frac{1}{2}\text{ eV} \end{aligned} \tag{10.90.2}$$

**Answer: (C)**

## 10.91 Bragg's Law

We recall Bragg's Law

$$\boxed{2d \sin\theta = n\lambda} \tag{10.91.1}$$

Plugging in what we know, we determine $\lambda$ to be

$$\begin{aligned} \lambda &= 2(3\text{ Å})(\sin 30) \\ &= 2(3\text{ Å})(0.5) \\ &= 3\text{ Å} \end{aligned} \tag{10.91.2}$$

We employ the de Broglie relationship between wavelength and momentum

$$p = \frac{h}{\lambda} \tag{10.91.3}$$

We get

$$
\begin{aligned}
mv &= \frac{h}{\lambda} \\
\Rightarrow v &= \frac{h}{m\lambda} \\
&= \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(3 \times 10^{(} - 10))}
\end{aligned}
\tag{10.91.4}
$$

We can determine the order of our answer by looking at the relevant indices

$$
-34 - (-31) - (-10) = 7 \tag{10.91.5}
$$

We see that (D) is close to what we are looking for.

**Answer: (D)**

## 10.92 Selection Rules for Electronic Transitions

The selection rules for an electric dipole transition are

| | |
|---|---|
| $\Delta\ell = \pm 1$ | Orbital angular momentum |
| $\Delta m_\ell = 0, \pm 1$ | Magnetic quantum number |
| $\Delta m_s = 0$ | Secondary spin quantum number, |
| $\Delta j = 0, \pm 1$ | Total angular momentum |

We have no selection rules for spin, $\Delta s$, so we can eliminate this choice.

**Answer: (D)**

## 10.93 Moving Belt Sander on a Rough Plane

We know the work done on a body by a force is

$$
W = F \times x \tag{10.93.1}
$$

We can relate this to the power of the sander; power is the rate at which work is done. So

$$
\begin{aligned}
P &= \frac{dW}{dt} \\
&= F\frac{dx}{dt} = Fv
\end{aligned}
\tag{10.93.2}
$$

The power of the sander can be calculated

$$
P = VI \tag{10.93.3}
$$

where $V$ and $I$ are the voltage across and the current through the sander. By equating the Mechanical Power, eq. (10.93.2) and the Electrical Power, eq. (10.93.3), we can determine the force that the motor exerts on the belt.

$$\begin{aligned} F &= \frac{VI}{v} \\ &= \frac{120 \times 9}{10} \\ &= 108 \text{ N} \end{aligned} \tag{10.93.4}$$

The sander is motionless, so

$$F - \mu R = 0 \tag{10.93.5}$$

where $R$ is the normal force of the sander pushing against the wood. Thus the coefficient of friction is

$$\mu = \frac{F}{R} = \frac{108}{100} = 1.08 \tag{10.93.6}$$

**Answer: (D)**

## 10.94 RL Circuits

When the switch, $S$, is closed, a magnetic field builds up within the inductor and the inductor stores energy. The charging of the inductor can be derived from Kirchoff's Rules.

$$\mathscr{E} - IR - L\frac{dI}{dt} = 0 \tag{10.94.1}$$

and the solution to this is

$$\boxed{I(t) = I_0 \left[1 - \exp\left(\frac{R_1 t}{L}\right)\right]} \tag{10.94.2}$$

where the time constant, $\tau_1 = L/R_1$.

We can find the voltage across the resistor, $R_1$, by multiplying the above by $R_1$, giving us

$$\begin{aligned} V(t) &= R_1 \cdot I_0 \left[1 - \exp\left(\frac{R_1 t}{L}\right)\right] \\ &= \mathscr{E}\left[1 - \exp\left(\frac{R_1 t}{L}\right)\right] \end{aligned} \tag{10.94.3}$$

The potential at $A$ can be found by measuring the voltage across the inductor. Given that

$$\begin{aligned} \mathscr{E} - V_{R_1} - V_L &= 0 \\ \therefore V_L &= \mathscr{E} - V_{R_1} \\ &= \mathscr{E}\exp\left(\frac{R_1 t}{L}\right) \end{aligned} \tag{10.94.4}$$

This we know to be an exponential decay and (fortunately) limits our choices to either **(A)** or **(B)**[20]

The story doesn't end here. If the inductor was not present, the voltage would quickly drop and level off to zero but with the inductor present, a change in current means a change in magnetic flux; the inductor opposes this change. We would expect to see a reversal in the potential at $A$. Since both **(A)** and **(B)** show this flip, we need to think some more.

The energy stored by the inductor is

$$\boxed{U_L = \frac{1}{2}LI_0^2 = \frac{1}{2}L\left(\frac{\mathscr{E}}{R_1}\right)^2} \tag{10.94.5}$$

With $S$ opened, the inductor is going to dump its energy across $R_2$ and assuming that the diode has negligible resistance, all of this energy goes to $R_2$. Thus

$$U = \frac{1}{2}L\left(\frac{V_{R_2}}{R_2}\right)^2 \tag{10.94.6}$$

The above two equations are equal, thus

$$\begin{aligned} \frac{\mathscr{E}}{R_1} &= \frac{V_{R_2}}{R_2} \\ V_{R_2} &= 3\mathscr{E} \end{aligned} \tag{10.94.7}$$

We expect the potential at $A$ to be larger when $S$ is opened. Graph **(B)** fits this choice.

**Answer: B**

## 10.95 Carnot Cycles

The Carnot Cycle is made up of two isothermal transformations, $KL$ and $MN$, and two adiabatic transformations, $LM$ and $NK$. For isothermal transformations, we have

$$\boxed{PV = nRT = \text{a constant}} \tag{10.95.1}$$

For adiabatic transformations, we have

$$\boxed{PV^\gamma = \text{a constant}} \tag{10.95.2}$$

where $\gamma = C_P/C_V$.

For the $KL$ transformation, $dU = 0$.

$$\begin{aligned} Q_2 &= W_{K\to L} \\ \therefore W_{K\to L} &= \int_{V_K}^{V_L} PdV \\ &= nRT_2 \ln\left(\frac{V_K}{V_L}\right) \end{aligned} \tag{10.95.3}$$

[20] If you get stuck beyond this point, you can guess. The odds are now in your favor.

For the $LM$ transformation,

$$P_L V_L^\gamma = P_M V_M^\gamma \tag{10.95.4}$$

For the $MN$ transformation, $dU = 0$.

$$\begin{aligned} Q_1 &= W_{M\to N} \\ \therefore W_{M\to N} &= \int_{V_M}^{V_N} PdV \\ &= nRT_1 \ln\left(\frac{V_N}{V_M}\right) \end{aligned} \tag{10.95.5}$$

For the $NK$ transformation,

$$P_N V_N^\gamma = P_K V_K^\gamma \tag{10.95.6}$$

Dividing eq. (10.95.4) and eq. (10.95.6), gives

$$\begin{aligned} \frac{P_L V_L^\gamma}{P_K V_K^\gamma} &= \frac{P_M V_M^\gamma}{P_N V_N^\gamma} \\ \therefore \frac{V_L}{V_K} &= \frac{V_M}{V_N} \end{aligned} \tag{10.95.7}$$

The effeciency of an engine is defined

$$\boxed{\eta = 1 - \frac{Q_1}{Q_2}} \tag{10.95.8}$$

We get

$$\begin{aligned} \eta = 1 - \frac{Q_1}{Q_2} &= 1 - \frac{-W_{M\to N}}{W_{K\to L}} \\ &= 1 - \frac{nRT_1 \ln\left(\frac{VM}{V_N}\right)}{nRT_2 \ln\left(\frac{VK}{V_L}\right)} \\ &= 1 - \frac{T_1}{T_2} \end{aligned} \tag{10.95.9}$$

1. We see that

$$\begin{aligned} 1 - \frac{Q_1}{Q_2} &= 1 - \frac{T_1}{T_2} \\ \therefore \frac{Q_1}{Q_2} &= \frac{T_1}{T_2} \end{aligned} \tag{10.95.10}$$

   Thus choice **(A)** is true.

2. Heat moves from the hot reservoir and is converted to work and heat. Thus

$$Q_2 = Q_1 + W \tag{10.95.11}$$

   The entropy change from the hot reservoir

$$S = \frac{dQ_2}{T} \tag{10.95.12}$$

   As the hot reservoir looses heat, the entropy decreases. Thus choice **(B)** is true.

3. For a reversible cycle, there is no net heat flow over the cycle. The change in entropy is defined by Calusius's Theorem.

$$\boxed{\oint \frac{dQ}{T} = 0} \tag{10.95.13}$$

   We see that the entropy of the system remains the same. Thus choice **(C)** is false.

4. The efficieny is defined

$$\eta = \frac{W}{Q_2} \tag{10.95.14}$$

   This becomes

$$\begin{aligned}\eta &= 1 - \frac{Q_1}{Q_2} \\ &= \frac{Q_2 - Q_1}{Q_2}\end{aligned} \tag{10.95.15}$$

   Thus $W = Q_2 - Q_1$. So choice **(D)** is true,

5. The effeciency is based on an ideal gas and has no relation to the substance used. So choice **(E)** is also true.

**Answer: (C)**

## 10.96 First Order Perturbation Theory

Perturbation Theory is a procedure for obtaining approximate solutions for a perturbed state by studying the solutions of the unperturbed state. We can, and shouldn't, calculate this in the exam.

We can get the first order correction to be ebergy eigenvalue

$$\boxed{E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle} \tag{10.96.1}$$

From there we can get the first order correction to the wave function

$$\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{(E_n^0 - E_m^0)} \tag{10.96.2}$$

and can be expressed as

$$\psi_n^1 = \sum_{m \neq n} c_m^{(n)} \psi_m^0 \tag{10.96.3}$$

you may recognize this as a Fourier Series and this will help you knowing that the perturbing potential is one period of a saw tooth wave. And you may recall that the Fourier Series of a saw tooth wave form is made up of even harmonics.

**Answer: (B)**[21]

[21]Griffiths gives a similar problem in his text

## 10.97 Colliding Discs and the Conservation of Angular Momentum

As the disk moves, it possessed both angular and linear momentums. We can not exactly add these two as they, though similar, are quite different beasts. But we can define a linear angular motion with respect to some origin. As the two discs hit each other, they fuse. This slows the oncoming disc. We can calculate the linear angular momentum

$$\boxed{\mathbf{L} = \mathbf{r} \times \mathbf{p}} \tag{10.97.1}$$

where $p$ is the linear momentum and $r$ is the distance from the point $P$ to the center of disc I. This becomes

$$\begin{aligned} \mathbf{L}_{\mathbf{v_0}} &= M\mathbf{R} \times \mathbf{v_0} \\ &= -MRv_0 \end{aligned} \tag{10.97.2}$$

It's negative as the cross product of $\mathbf{R}$ and $\mathbf{v_0}$ is negative.

The Rotational Angular Momentum is

$$\boxed{\mathbf{L}_{\omega_0} = I\boldsymbol{\omega_0}} \tag{10.97.3}$$

Adding eq. (10.97.3) and eq. (10.97.2) gives the total angular momentum.

$$\begin{aligned} L &= L_{\omega_0} + L_{v_0} \\ &= I\omega_0 - MRv_0 \\ &= \frac{1}{2}MR^2\omega_0 - \frac{1}{2}MR^2\omega_0 \\ &= 0 \end{aligned}$$

Thus the total angular momentum at the point $P$ is zero.

**Answer: (A)**

## 10.98 Electrical Potential of a Long Thin Rod

We have charge uniformly distributed along the glass rod. It's linear charge density is

$$\lambda = \frac{Q}{\ell} = \frac{dQ}{dx} \tag{10.98.1}$$

The Electric Potential is defined

$$V(x) = \frac{q}{4\pi\epsilon_0 x} \tag{10.98.2}$$

We can 'slice' our rod into infinitesimal slices and sum them to get the potential of the rod.

$$dV = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{x} \tag{10.98.3}$$

We assume that the potential at the end of the rod, $x = \ell$ is $V = 0$ and at some point away from the rod, $x$, the potential is $V$. So

$$\begin{aligned}\int_0^V dV &= \frac{\lambda}{4\pi\epsilon_0}\int_\ell^x \frac{dx}{x}\\ &= \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{x}{\ell}\right)\end{aligned} \tag{10.98.4}$$

Where $x = 2\ell$, eq. (10.98.4) becomes

$$\begin{aligned}V &= \frac{Q}{\ell}\frac{1}{4\pi\epsilon_0}\ln\left(\frac{2\ell}{\ell}\right)\\ &= \frac{Q}{\ell}\frac{1}{4\pi\epsilon_0}\ln 2\end{aligned} \tag{10.98.5}$$

**Answer: (D)**

## 10.99 Ground State of a Positronium Atom

Positronium consists of an electron and a positron bound together to form an "exotic" atom. As the masses of the electron and positron are the same, we must use a reduced-mass correction factor to determine the enrgy levels of this system.[22]. The reduced mass of the system is

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p} \tag{10.99.1}$$

Thus $/mu$ is

$$\begin{aligned}\mu &= \frac{m_e \cdot m_p}{m_e + m_p}\\ &= \frac{m_e}{2}\end{aligned} \tag{10.99.2}$$

The ground state of the Hydrogen atom, in terms of the reduced mass is

$$\begin{aligned}E_1 &= -\frac{\mu}{m_e}E_0\\ &= -\frac{1}{2}E_0\end{aligned} \tag{10.99.3}$$

where $E_0 = 13.6$ eV.

**Answer: (B)**

## 10.100 The Pinhole Camera

A pinhole camera is simply a camera with no lens and a very small aperature. Light passes through this hole to produce an inverted image on a screen. For the photography

[22] Place cite here

buffs among you, you know that by varying the size of a camera's aperature can accomplish various things; making the aperature bigger allows more light to enter and produces a "brighter" picture while making the aperature smaller produces a sharper image.

In the case of the pinhole camera, making the pinhole, or aperature, smaller produces a sharper image because it reduces "image overlap". Think of a large hole as a set of tiny pinholes places close to each other. This results in an infinite amount of images overlapping each other and hence a blurry image. So to produce a sharp image, it is best to use the smallest pinhole possible, the tradeoff being an image that's not as "bright".

There are limits to the size of our pinhole. We can not say, for example, use an infinitely small pinhole the produce the sharpest possible image. Beyond some point diffraction effects take place and will ruin our image.

Consider a pinhole camera of length, $D$, with a pinhole of diameter, $d$. We know how much a beam of light will be diffracted through this pinhole by[23]

$$d \sin\theta = m\lambda \tag{10.100.1}$$

this is the equation for the diffraction of a single slit. As $\theta$ is small and we will consider first order diffraction effects, eq. (10.100.1) becomes

$$\begin{aligned} d\theta &= \lambda \\ \Rightarrow \theta &= \frac{\lambda}{d} \end{aligned} \tag{10.100.2}$$

The "size" of this spread out image is

$$\begin{aligned} y &= 2\theta D \\ &= \frac{2\lambda D}{d} \end{aligned} \tag{10.100.3}$$

So the 'blur' of our resulting image is

$$\begin{aligned} B &= y - d \\ &= \frac{2\lambda D}{d} - d \end{aligned} \tag{10.100.4}$$

We can see that we want to reduce $y$ as much as possible. i.e. make it $d$. So eq. (10.100.4) becomes

$$\begin{aligned} 0 &= \frac{2\lambda D}{d} - d \\ \therefore \frac{2\lambda D}{d} &= d \\ \text{Thus} \quad d &= \sqrt{2\lambda D} \end{aligned} \tag{10.100.5}$$

So we'd want a pinhole of that size to produce or sharpest image possible. This result is close to the result that Lord Rayleigh used, which worked out to be

$$d = 1.9\sqrt{D\lambda} \tag{10.100.6}$$

**Answer: (A)**

[23] Add image of pinhole camera

# Chapter 11

# GR9277 Exam Solutions

## 11.1 Momentum Operator

We recall the momentum operator is

$$p = \frac{\hbar}{i}\nabla \tag{11.1.1}$$

So the momentum of the particle is

$$\begin{aligned} p\psi &= \frac{\hbar}{i}\nabla\psi \\ &= \frac{\hbar}{i}\frac{\partial}{\partial x}\psi \\ &= \frac{\hbar}{i}\cdot ike^{i(kx-\omega t)} \\ &= \hbar k\psi \end{aligned} \tag{11.1.2}$$

**Amswer: (C)**

## 11.2 Bragg Diffraction

We know Bragg's Law to be

$$2d\sin\theta = m\lambda \tag{11.2.1}$$

where $d$ is the distance between the atomic layers and $\lambda$ is the wavelength of the incident X-ray beam. We know that $\sin\theta$ falls between the 0 and 1 and our longest wavelength will occur at our first order, $m = 1$. So

$$2d = \lambda \tag{11.2.2}$$

**Answer: (D)**

## 11.3 Characteristic X-Rays

Mosley[1] showed that when the square root of an element's characteristic X-rays are plotted agianst its atomic number we get a straight line. X-ray spectra is associated with atoms containg many electrons but in the X-ray regime, excitation removes tightly bound electrons from the inner orbit near the atom's nucleus. As these emitted X-rays are the result of transitions of a single electron, Bohr's Hydrogen model proves useful.

Thus Mosley's adaption of Bohr's model becomes

$$\boxed{\frac{1}{\lambda} = Z^2_{\text{eff}} R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)} \tag{11.3.1}$$

where $R_H$ is the Rydberg number and $Z_{\text{eff}}$ is the effective charge parameter that replaces the nuclear charge index, $Z$. The effective charge comes into play because the transition electron sees a nuclear charge that is smaller than $Ze$ as the other electrons shield the nucleus from view. Thus our effective "shielding" constant, $z_f$ is

$$Z_{\text{eff}} = Z - z_f \tag{11.3.2}$$

For the $K_\alpha$ series, $z_f = 1$, we get

$$E_{K_\alpha} = h\upsilon_{K_\alpha} = (Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right] 13.6 \text{ eV} = \frac{3}{4}(Z-1)^2\, 13.6\text{eV} \tag{11.3.3}$$

So

$$\begin{aligned} \frac{E_C}{E_{Mg}} &= \frac{(Z_C - 1)^2}{\left(Z_{Mg} - 1\right)^2} \\ &= \frac{(6-1)^2}{(12-1)^2} \\ &= \frac{25}{121} \approx \frac{1}{5} \end{aligned} \tag{11.3.4}$$

$\frac{1}{4}$ is our closest answer.

**Answer: (A)**

[1]Henry G. J. Moseley (1887-1915) was described by Rutherford as his most talented student. In his early 20's, he measured and plotted the X-ray frequencies for 40 elements of the periodic table and showed that the K-alpha X-rays followed a straight line when the atomic number Z versus the square root of frequency was plotted. This allowed for the sorting of the elements in the periodic table by atomic number and not mass as was popular at the time.

Moseley volunteered for combat duty with the Corps of Royal Engineers during World War I and was killed in action by a sniper at age 27 during the attack in the Battle of Gallipoli. It is widely speculated that because of his death, British and other world governments bagan a policy of no longer allowing scientists to enlist for combat.

## 11.4 Gravitation I

The Force due to gravity, Newton's Law of Gravitation, follows an inverse square law

$$F = \frac{GMm}{r^2} \tag{11.4.1}$$

or rather

$$F(R) \propto \frac{1}{R^2} \quad F(2R) \propto \frac{1}{2R} \tag{11.4.2}$$

Dividing, we get

$$\begin{aligned} \frac{F(R)}{F(2R)} &= \frac{(2R)^2}{R^2} \\ &= 4 \end{aligned} \tag{11.4.3}$$

**Answer: (C)**

## 11.5 Gravitation II

Newton's Law of Gravitation becomes a linear law inside a body. So

$$F \propto r \tag{11.5.1}$$

Thus

$$F(R) \propto R \quad F(2R) \propto 2R \tag{11.5.2}$$

Dividing the two equations gives us

$$\begin{aligned} \frac{F(R)}{F(2R)} &= \frac{R}{\frac{R}{2}} \\ &= 2 \end{aligned} \tag{11.5.3}$$

**Answer: (C)**

## 11.6 Block on top of Two Wedges

The principles of equilibrium of forces tells us that

1. All forces are balanced
2. Total torque is zero

We can determine the reaction on the two wedges to be

$$\begin{aligned} 2R &= 2mg + Mg \\ \Rightarrow R &= mg + \frac{Mg}{2} \end{aligned} \tag{11.6.1}$$

As the block rests on the wedges, its weight causes it to push out on the two blocks. There are horizontal and vertical components to this force; the horizontal component being of interest to us

$$F_B \cos\theta = Mg \tag{11.6.2}$$

where $\theta = 45°$. Since the wedges aren't moving, this is also equal to the frictional force. This force on the wedge has a horizontal component where

$$F_x = F\sin\theta = \frac{Mg}{2} \tag{11.6.3}$$

$$F_R = \mu R \tag{11.6.4}$$

We have, eq. (11.6.4) equal to eq. (11.6.3)

$$\begin{aligned} F_x &= F_R \\ \frac{Mg}{2} &= \mu\left[m + \frac{Mg}{2}\right] \\ \therefore M &= \frac{2\mu m}{1-\mu} \end{aligned} \tag{11.6.5}$$

**Answer: (D)**

## 11.7 Coupled Pendulum

From what we know about coupled pendulums, there are two modes in which this system can oscillate. The first is when the two pendulum masses oscillate out of phase with each other. As they oscillate, there is a torsional effect on the tube and we expect to see the effects of its mass, $M$ somewhere in the equation. The second occurs when the two masses swing in phase with each other. As they are in phase, there is no torsional effect on the connecting tube and the mode would be that of a single pendulum. So our modes of oscillation are

$$\omega = 0 \quad ; \quad \omega = \sqrt{\frac{g\,(M+2m)}{\ell M}} \quad \text{and} \quad \omega = \sqrt{\frac{g}{\ell}} \tag{11.7.1}$$

**Answer: (A)**

## 11.8 Torque on a Cone

The Torque is defined

$$\tau = \mathbf{r}\times\mathbf{F} \tag{11.8.1}$$

We are looking for a negative $\hat{\mathbf{k}}$ vector so we are looking for answers with $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ cross products. With this in mind we can easily eliminate answers **(A)**, **(B)** and **(E)**. From the above, the torque is defined

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \tag{11.8.2}$$

**Choice C**

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -b & 0 & c \\ 0 & a & 0 \end{vmatrix}$$
$$= -ac\hat{\mathbf{i}} - ab\hat{\mathbf{k}} \tag{11.8.3}$$

**This is the answer**

**Choice D**

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b & 0 & c \\ 0 & a & 0 \end{vmatrix}$$
$$= -ac\hat{\mathbf{i}} + ab\hat{\mathbf{k}} \tag{11.8.4}$$

**This is NOT the answer**

**Answer: (C)**

## 11.9 Magnetic Field outside a Coaxial Cable

If we were to draw an Amperian loop around the outside of the cable, the enclosed current is zero. We recall Ampere's Law,

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}}} \tag{11.9.1}$$

As $I_{\text{enclosed}} = 0$, then the magnetic induction at P($r > c$), is also zero[2].

**Answer: (A)**

## 11.10 Image Charges

The grounded conducting plate will act as a 'charge mirror' to our two positive charges, producing two negative charges, $-q$ at $-0.5a$ and $-2q$ at $-1.5a$. We can solve this using the vector form of Coulomb's Law but it is simpler to realize that the forces on the $q$ charge all act in the negative-x direction thusl allowing us to sum their magnitudes.

**Force between the $-2q$ and $q$ charges**

$$F_{(-2q)(q)} = \frac{-2q^2}{4\pi\epsilon_0(2.0a)^2} \tag{11.10.1}$$

**Force between the $-q$ and $q$ charges**

$$F_{(-q)(q)} = \frac{-q^2}{4\pi\epsilon_0(a)^2} \tag{11.10.2}$$

[2]Think of the two resulting fields cancelling each other out.

**Force between the $+2q$ and $q$ charges**

$$F_{(2q)(q)} = \frac{2q^2}{4\pi\epsilon_0 (a)^2} \tag{11.10.3}$$

Adding eq. (11.10.1), eq. (11.10.2) and eq. (11.10.3) gives

$$F = \frac{1}{4\pi\epsilon_0}\frac{7q^2}{2a^2} \tag{11.10.4}$$

**Answer: (E)**

## 11.11 Energy in a Capacitor

The Energy stored in a Capacitor is

$$\boxed{U = \frac{1}{2}CV^2} \tag{11.11.1}$$

The time for the potential difference across a capacitor to decrease is given by

$$\boxed{V = V_0 \exp\left(-\frac{t}{RC}\right)} \tag{11.11.2}$$

The energy stored in the capacitor is half of its initial energy, this becomes

$$U = \frac{1}{2}U_0 \tag{11.11.3}$$

where $U_0$ is the initial stored energy. We can find eq. (11.11.3) in terms of $C$ and $V$,

$$\begin{aligned} \frac{1}{2}CV^2 &= \frac{1}{2}CV_0^2 \\ \Rightarrow V^2 &= \frac{V_0}{2} \end{aligned} \tag{11.11.4}$$

Substituting eq. (11.11.2) into the above equation gives

$$V_0^2 \exp\left(-\frac{2t}{RC}\right) = \frac{V_0^2}{2}$$

Solving for $t$ gives

$$t = \frac{RC\ln 2}{2} \tag{11.11.5}$$

**Answer: (E)**

## 11.12 Potential Across a Wedge Capacitor

We are told that the plates of this capacitor is large which allows us to assume that the field is uniform between the plates except maybe at the edges. As the capacitor is sufficiently large enough we can ignore the edge effects.

At $\alpha$ the potential is $V_0$. As we have a linear relationship, the potential is proportional to the angle, $\varphi$. Thus

$$V = \frac{V_0 \varphi}{\alpha} \tag{11.12.1}$$

**Answer: (B)**

## 11.13 Magnetic Monopoles

The Maxwell Equations deal with electric and magnetic fields to the motion of electric charges and disallow for magnetic charges. If we were to allow for a 'magnetic charge' or magnetic monopole, we would also have to allow for a 'magnetic current'. As we do have electrical charges and currents and equations describing them, we can observe how they differ of our magnetic equations to come up with an answer.

**Equation I.: Ampère's Law** This relates the magnetic field to an electrical current and a changing electric field.

**Equation II: Faraday's Law of Induction** This equation is similar to Ampère's Law except there is no 'magnetic current' component. As we have stated above, the presence of a magnetic charge will lead us to assume a magnetic current, this is one of the equations that would be **INCORRECT**.

**Equation III.: Gauss' Law** Here we see that the distribution of an electric charge gives us an electric field.

**Equation IV.:Gauss' Law for Magnetism** This is similar to Gauss' Law above except that we have no 'magnetic charge'. If we did assume for monopoles, this equation would not be zero, so this equation would also be **INCORRECT**

**Answer: (D)**

## 11.14 Stefan-Boltzmann's Equation

The Stefan-Boltzmann's Equation says that the total energy emitted by a black body is proportional to the fourth power of its temperature, $T$.

$$\boxed{E = \sigma T^4} \tag{11.14.1}$$

If we were to double the temperature of this blackbody, the energy emitted would be

$$E_1 = \sigma(2T)^4 = 8\sigma T^4 = 8E \tag{11.14.2}$$

Let $C$ be the heat capacitance of a mass of water, the energy to change its temperature by a half degree is

$$E = C\Delta T \tag{11.14.3}$$

where $\delta T = 0.5$K. At $T_1 = 2T$, the energy used is $8E$, so

$$E_1 = 8E = 8C\Delta T \tag{11.14.4}$$

So the temperature change is eight degrees.

**Answer: (C)**

## 11.15 Specific Heat at Constant Volume

To determine the specific heat at constant volume we identify the degrees of freedom or the ways the molecule can move; translational and rotational. The question also adds that we are looking at high temperatures, so we have to add another degree of freedom; vibrational.

$$\begin{aligned} \text{Translational} &= 3 \\ \text{Rotational} &= 2 \\ \text{Vibrational} &= 2 \end{aligned}$$

We recall the formula, eq. (4.22.1) we used to determine the specific heat per mole at constant volume, $C_V$

$$\boxed{C_V = \left(\frac{f}{2}\right) R = 4.16f \text{ J/mol.K}} \tag{11.15.1}$$

where $f = 7$ Thus, $C_V$ is

$$C_V = \left(\frac{7}{2}\right) R \tag{11.15.2}$$

**Answer: (C)**

## 11.16 Carnot Engines and Effeciencies

The effeciency of an engine is the ratio of the work we get out of it to the energy we put in. So

$$\boxed{e = \frac{W_{\text{out}}}{E_{\text{in}}}} \tag{11.16.1}$$

A Carnot Engine is theoretically the most efficient engine. Its effeciency is

$$\boxed{e = 1 - \frac{T_c}{T_h}} \tag{11.16.2}$$

where $T_c$ and $T_h$ are the temperatures of our cold and hot reservoirs respectively. We must remember that this represents absolute temperature, so

$$T_c = 527 + 273 = 800 \text{ K}$$
$$T_h = 727 + 273 = 1000 \text{ K}$$

Plugging this into eq. (11.16.2), we get

$$e = 1 - \frac{800}{1000} = 0.2 \tag{11.16.3}$$

So the work our engine performs is

$$W_{\text{out}} = e \times E_{\text{in}} = 0.2 \times 2000 = 400 \text{ J} \tag{11.16.4}$$

**Answer: (A)**

## 11.17 Lissajous Figures

This question deals with Lissajous figures. These can be drawn graphically with the use of your favorite programming language or with the use of an oscilloscope. Before the days where digital frequency meters were prevalent, this was a common method to determine the frequency of a signal. To generate these figures, one signal was applied across the horizontal deflection plates of an oscilloscope and the other applied across the vertical deflection plates. The resulting pattern traces a design that is the ratio of the two frequencies.[3]

With this basic understanding we can determine signals in the X and Y inputs to our oscilloscope. Each figure represents a trace over a full period.

**Answer: (A)**

## 11.18 Terminating Resistor for a Coaxial Cable

Terminating a coaxial cable is important as it reduces refelctions and maximizes power transfer across a large bandwidth. The best way to think of this is to think of a wave propagating along a string, any imperfections will cause the wave's energy to either be attenuated or reflected across this boundary.[4]

**Choice A** The terminating resistor doesn't prevent leakage, the outer core of the cable was designed to confine the signal. Your coaxial cable is a waveguide.

[3] If you've never seen this in the lab, one of the best examples where this can be seen is during the opening sequence of the 1950's TV series, "The Outer Limits". *"We will control the horizontal. We will control the vertical. We can roll the image, make it flutter...."*

[4] You may be familiar or have seen the use of terminating resistors if you've dabbled in computers or electronics for the past couple of years. SCSI cables made use of terminating resistors as you daisy chained your drives across the cable. In the time when **10BASE2** ethernet networks were prevalent, the use of a 50 Ω BNC Terminator was of utmost importance or your computers would have lost connectivity.

**Choice B** The cable doesn't transmit enough power to cause over heating along it's length.

**Choice C** This is correct. The resistor essentially attenuates the remaining power across itself, making it seem that the wave gets propagated across an infinite length i.e. no reflections.

**Choice D** This won't prevent attenuation across the cable. The cable has a natural impedance which will attenuate the signal to a degree. The terminating resistor absorbs the remaining power so no signal gets refelcted back[5].

**Choice E** Improbable since the outer sheath's purpose is to cancel out these currents.

**Answer: (C)**

## 11.19 Mass of the Earth

There are many ways to tackle this question which depends on what you know. You may alrady know the mass of the earth for one, which may make thins convenient and a time saver. The denisty of the earth is approximately that of Iron, if you knew that and the volume of the earth, you would get an answer where

$$
\begin{aligned}
M &= \rho V \\
&= 7870 \times 109 \times 10^{21} && (11.19.1) \\
&= 8 \times 10^{24} \text{ kg} && (11.19.2)
\end{aligned}
$$

Of course, if you want to apply some real Phyics, we start with Newton's Gravitation Equation

$$g = \frac{GM}{R_E^2} \tag{11.19.3}$$

Solving for $M$, we get

$$
\begin{aligned}
M &= \frac{gR_E^2}{G} \\
&= \frac{(9.8)(6.4 \times 10^6)^2}{6.67 \times 10^{-11}} && (11.19.4)
\end{aligned}
$$

Adding the indices, gives

$$1 + 12 + 11 = 24 \tag{11.19.5}$$

So our answer is of the magnitude $10^{24}$ kg.

**Answer: (A)**

[5]Think Maximum Power Transfer

## 11.20 Slit Width and Diffraction Effects

We are familar with the Double slit experiment and interference and can recall the equation where

$$d \sin\theta = m\lambda \tag{11.20.1}$$

where $d$ is the distance between the slit centers and $m$ is the order maxima.

As the width of the slit gets smaller, the wave gets more and more spread out due to diffraction effects. If it gets wide enough, it will spread out the widths of the interference pattern and eventually 'erase' it. We recall the equation for the single slit interference

$$w \sin\theta = n\lambda \tag{11.20.2}$$

For this 'swamping out' effect to occur, the $m^{\text{th}}$ and $n^{\text{th}}$ orders must align. The ratios between them will be

$$\begin{aligned} \frac{d\sin\theta}{w\sin\theta} &= \frac{m\lambda}{n\lambda} \\ \therefore \frac{d}{w} &= \frac{m}{n} > 1 \end{aligned} \tag{11.20.3}$$

As $m$ and $n$ are integers and $m > n$, answer (D) fits this criteria.

**Answer: (D)**

## 11.21 Thin Film Interference of a Soap Film

Based on our knowledge of waves and interference we can eliminate choices

**Choice I** This is an impossibility. Light isn't 'absorbed'. Destructive interference can take place and energy is 'redistributed' to areas of constructive interferece. The soap film can't absorb energy without it going somewhere.

**Choice II** This is **CORRECT**. At the front of the soap film, there is a phase change of 180°as the soap film as a refractive index greater than air. The part that gets transmitted through this film gets reflected by the back part of the film with no phase change as air has a lower refractive index. This means that the two waves are out of phase with each other and interfere destructively.

**Choice III** Yes, this is true. Light comes from a less dense medium, air, and bounces off a more dense medium, soap, there is a phase change. There is no phase change for the transmitted wave through the soap film.

**Choice IV** Inside the soap film, the wave meets an interface from an optically more dense medium to a less dense one. There is no phase change.

From the above, we see that choices **II**, **III** and **IV** are all true.

**Answer: (E)**

## 11.22 The Telescope

The Magnification of the Telescope is

$$\boxed{M = \frac{f_o}{f_e}} \tag{11.22.1}$$

where $f_o$ and $f_e$ are the focal lengths of the objective and eyepiece lens respectively. From the information given to us, we know that

$$f_e = \frac{f_o}{10} = 0.1 \text{ m} \tag{11.22.2}$$

The optical path length is simply

$$\begin{aligned} d &= f_0 + f_e \\ &= 1.0 + 0.1 = 1.1 \text{ m} \end{aligned} \tag{11.22.3}$$

**Answer: (D)**

## 11.23 Fermi Temperature of Cu

The Fermi Temperature is related to the Fermi Energy by

$$E_F = kT_F \tag{11.23.1}$$

This is also the kinetic energy of the system,

$$E_F = \frac{1}{2} m_e v^2 \tag{11.23.2}$$

Solving for $v$

$$\begin{aligned} v &= \sqrt{\frac{2kT_F}{m_e}} \\ &= \sqrt{\frac{2(1.38 \times 10^{-23})(8 \times 10^{4})}{9.11 \times 10^{-31}}} \end{aligned} \tag{11.23.3}$$

Adding the indices, we get

$$\frac{-23 + 4 + 31}{2} = 6 \tag{11.23.4}$$

We are looking for speeds to be in the order of $10^6$ m/s.

**Answer: (E)**

## 11.24 Bonding in Argon

We recall that Argon is a noble gas. This means that its outermost electron shell is filled and thus unable to bond by conventional means. It won't bond Ionically as this method involves either giving up or receiving electrons. A full electron shell means that this will be difficult without some effort. In covalent bonds, electrons are shared between atoms. Again, because the shell is filled, there is no place to share any electrons. A Metallic bond involves a positive charge, the nucleus, 'swimming' in a sea of free electrons. Again, with Argon, all of its electrons are tightly bound and hence have no free electrons.

This leaves us with van der Waal bonds. van der Waal forces occur between atoms or molecules of the same type and occur due to variences in charge distribution in the atoms. This is our only obvious choice.

**Answer: (E)**

## 11.25 Cosmic rays

For our cosmic rays to reach deep underground, we are looking for particles that are essentiall massless and pass through matter easily.

**A** Alpha particles and neutrons have high kinetic energy but very short penetrating depth. This is primarily due to their masses. So we can eliminate this.

**B** protons and electrons don't penetrate much as they interact easily with matter.

**C** Iron and Carbon nuclei are very heavy and interact very easily with matter. We would not expect them to penetrate far, much less deep underground.

**D** muons and neutrinos. A muon is esseentially a 'heavy' electron they don't emit much brenstraalung radiation and hence are highly penetrating. Neutrinos have almost no mass and travel close to the speed of light. In the beginning it was dispited whether they had any mass at all. These two particles fit our choices.

**E** positrons and electrons. Thes two are highly interacting with matter.

**Answer: (D)**

## 11.26 Radioactive Half-Life

The radioactive decay is

$$\boxed{N = N_0 \exp[-\lambda t]} \tag{11.26.1}$$

Where $\lambda$ is the decay constant. At $t = 0$ on the graph, the count rate is $N = 6 \times 10^3$ counts per minute. We are looking for the time when $N = 3 \times 10^3$ counts per minute, which falls at around $t = 7$ minutes.[6]

**Answer: (B)**

[6]You may wonder what the presence of the $\log_{10} = 0.03$ and $\log_{10} e = 0.43$ is there for. It simply means that you will go down three divisions on the $y$-axis to get the half life count. We see that from

## 11.27 The Wave Function and the Uncertainty Principle

**NOT FINISHED**

## 11.28 Probability of a Wave function

**NOT FINISHED**

## 11.29 Particle in a Potential Well

This tests our knowledge of the properties of a wave function as well as what the wavefunction of a particle in a potential well looks like. We expect the wave function and its derivative to be continuous. This eliminates choices **(C)** & **(D)**. The first is not continuous and the second the derivative isn't continuous. We also expect the wave to be fairly localized in the potential well. If we were to plot the function $|\psi|^2$, we see that in the cases of choices **(A)** & **(E)**, the particle can exist far outside of the well. We expect the probability to decrease as we move away from the well. Choice **(B)** meets this. In fact, we recognize this as the $n = 2$ state of our wave function.

**Answer: (B)**

## 11.30 Ground state energy of the positronium atom

The positronium atom consists of a positron and an electron bound together. As their masses are the same, we can't look at this as a standard atom consisting of a proton and an electron. In our standard atom, it's center of mass is somewhere close to the center of mass of the proton. In the case of our positronium atom, it's center of mass is somewhere in between the electron and positron.

To calculate the energy levels of the positronium atom, we need to "reduce" this two mass system to an effective one mass system. We can do this by calculating its effective or reduced mass.

$$\boxed{\frac{1}{\mu} = \frac{1}{m_p} + \frac{1}{m_e}} \tag{11.30.1}$$

---

eq. (11.26.1),

$$dN = -\lambda N dt \tag{11.26.2}$$

Integrating gives us

$$\int_N^{\frac{N}{2}} \frac{dN}{N} = -\lambda \int_{t_0}^{t_1} dt$$

$$\ln 2 = t_2 - t_1$$

$$\therefore \frac{\log_{10} 2}{\log_{10} e} = t_2 - t_1 = \Delta t \tag{11.26.3}$$

So at any point on the graph, a change in three divisions on the $y$-axis will give us the half life.

where $\mu$ is the reduced mass and $m_p$ and $m_e$ are the masses of the positron and the electron respectively. As they are the same, $\mu$ is

$$\mu = \frac{m_e}{2} \tag{11.30.2}$$

We can now turn to a modified form of Bohr's Theory of the Hydrogen Atom to calculate our energy levels

$$\boxed{E_n = \frac{Z^2}{n_2} \cdot \frac{\mu}{m_e} 13.6 \text{ eV}} \tag{11.30.3}$$

where $Z$ is the atomic mass and $n$ is our energy level. We have calculated that the reduced mass of our system is half that of Hydrogen. So $Z = 1$. For the $n = 2$ state we have

$$E_2 = \frac{1}{2} \cdot \frac{13.6}{2^2} \text{ eV} = \frac{E_0}{8} \text{ eV} \tag{11.30.4}$$

**Answer: (E)**

## 11.31 Quantum Angular Momentum

**Not FINISHED**

## 11.32 Electrical Circuits I

The power dissipated by a resistor, $R$, is

$$P = I^2 R \tag{11.32.1}$$

where $I$ is the current through the resisitor. In this case the current through the $R_1$ resistor is the current from the battery, $I$. After the current passes through $R_1$, the current divides as it goes through to the other resistors, so the current passing through $R_1$ is the maximum current. As the other resistors are close to $R_1$ but have smaller currents passing through them, the power dissipated by the $R_1$ resistor is the largest.

**Answer: (A)**

## 11.33 Electrical Circuits II

We can find the voltage across the $R_4$ resisitor by determining how the voltage divids across each resisitors. $R_3$ and $R_4$ are in parallel and so the potential difference across both resistors are the same. The net resistance is

$$\begin{aligned} R_{3\|4} &= \frac{R_3 R_4}{R_3 + R_4} \\ &= \frac{60 \cdot 30}{90} = 20\Omega \end{aligned} \tag{11.33.1}$$

This is in parallel with the $R_5$ resistor, so

$$R_{5+(3\|4)} = 20 + 30 = 50\Omega \tag{11.33.2}$$

This net resistance is the same as the $R_2$ resistor. This reduces our circuit to one with two resistors in series where $R_T = 25\Omega$. The voltage across $R_T$ is found by using the voltage divider equation

$$V_T = \frac{25}{75} 3.0 \text{ V} = 1.0 \text{ V} \tag{11.33.3}$$

This means that the potential across the $R_3$, $R_4$ and $R_5$ combination is 1.0 V. The voltage across the $R_4$ resistor is the voltage across the $R_3 \parallel R_4$ resistor

$$V_4 = \frac{20}{50} 1.0 \text{ V} = 0.4 \text{ V} \tag{11.33.4}$$

**Answer: (A)**

## 11.34 Waveguides

**NOT FINISHED Answer: (D)**

## 11.35 Interference and the Diffraction Grating

Interference maxima for the diffraction grating is determined by the equation

$$d \sin \theta_m = m\lambda \tag{11.35.1}$$

where $d$ is the width of the diffraction grating. We are told that our grating has 2000 lines per cm. This works out to

$$d = \frac{1 \times 10^{-2}}{2000} = 0.5 \times 10^{-5} \text{ m} \tag{11.35.2}$$

As $\theta$ is very small, we can approximate $\sin \theta \approx \theta$. The above equation can be reduced to

$$d\theta = \lambda \tag{11.35.3}$$

Plugging in what we know

$$\begin{aligned} \theta &= \frac{\lambda}{d} \\ &= \frac{5200 \times 10^{-10}}{0.5 \times 10^{-5}} \\ &= 0.1 \text{ radians} \end{aligned} \tag{11.35.4}$$

As the answer choices is in degrees,

$$x = \frac{180}{\pi} \cdot 0.1 = \frac{18}{\pi} \approx 6^\circ \tag{11.35.5}$$

**Answer: (B)**

## 11.36 EM Boundary Conditions

**NOT FINSHED**

## 11.37 Decay of the $\pi^0$ particle

Relativity demands that th photon will travel at the speed of light.
**Answer: (A)**

## 11.38 Relativistic Time Dilation and Multiple Frames

For this question we recall the time dilation equation where

$$\Delta T' = \frac{\Delta T}{\sqrt{1-\frac{u^2}{c^2}}} \tag{11.38.1}$$

where $T$ is the time measure in the fram at rest and $T'$ is the time measured in the frame moving at speed $u$ relative to the rest frame. With the information given in the question we can see that

$$\Delta t_2 = \frac{\Delta t_1}{\sqrt{1-\frac{v_{12}^2}{c^2}}} \tag{11.38.2}$$

$$\Delta t_3 = \frac{\Delta t_1}{\sqrt{1-\frac{v_{13}^2}{c^2}}} \tag{11.38.3}$$

You may think that **Answer: (C)** is a possible answer but it would be incorrect the leptons are not in the $S_2$ frame, they are in the $S_1$ frame, so this possibility has no physical consequence.
**Answer: (B)**

## 11.39 The Fourier Series

In most cases, we rarely see pure sine waves in nature, it is often the case our waves are made up of several sine functions added together. As daunting as this question may seem, we just have to remember some things about square waves,

1. We see that our square wave is an odd function so we would expect it to be made up of sine functions. This allows us to eliminate all but two of our choices; answers **(A)** and **(B)**[7].

2. Square waves are made up of odd harmonics. In choice **(A)**, we see that both even and odd harmonics are included. In the case of choice **(B)**, only odd harmonics will make up the function.

[7]If you got stuck at this point, now would be a good time to guess. The odds are in your favor.

As choice (B) meets the criteria we recall, we choose this one.

**Answer: (B)**

### 11.39.1 Calculation

As we have the time, we see that the function of out square wave is

$$V(t) = \begin{cases} 1 & 0 < t < \frac{\pi}{\omega} \\ -1 & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases} \tag{11.39.1}$$

Our square wave will take the form,

$$\begin{aligned} V(t) = \frac{1}{2}a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \cdots + a_n \cos n\omega t + \\ + b_1 \sin \omega t + b_2 \sin 2\omega t + \cdots + b_n \sin n\omega t \end{aligned} \tag{11.39.2}$$

We can solve for $a_n$ and $b_n$, where

$$a_n = \frac{1}{\pi}\int_0^{2\pi/\omega} V(t) \cos n\omega t dt \tag{11.39.3}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi/\omega} V(t) \sin n\omega t dt \tag{11.39.4}$$

Solving for $a_n$, gives us

$$\begin{aligned} a_n &= \frac{1}{\pi}\left[\int_0^{\pi/\omega} (1)\cos(n\omega t)dt + \int_{\pi/\omega}^{2\pi/\omega} (-1)\cos(n\omega t)dt\right] \\ &= \begin{cases} \frac{1}{\pi}\left[\int_0^{\pi/\omega} dt - \int_{\pi/\omega}^{2\pi/\omega} dt\right] & \text{for } n = 0 \\ \frac{1}{\pi}\left[\left(\frac{1}{n\omega}\sin(n\omega t)\right)\Big|_0^{\frac{\pi}{\omega}} - \left(\frac{1}{n\omega}\sin(n\omega t)\right)\Big|_0^{\frac{\pi}{\omega}}\right] & \text{for } n \neq 0 \end{cases} \\ &= \begin{cases} \frac{1}{\pi}\left[\frac{\pi}{\omega} - \left(\frac{2\pi}{\omega} - \frac{\pi}{\omega}\right)\right] & \text{for } n = 0 \\ \left(\frac{1}{n\pi\omega}\right)\left[\sin(n\omega t)|_0^{\frac{\pi}{\omega}} - \sin(n\omega t)|_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}}\right] & \text{for } n \neq 0 \end{cases} \\ &= \begin{cases} 0 & \text{for } n = 0 \\ \frac{1}{n\pi\omega}\left[\sin(n\pi) - (\sin(2n\pi) - \sin(n\pi))\right] = 0 & \text{for } n \neq 0 \end{cases} \end{aligned}$$

We can see that[8]

$$a_0 = 0 \quad \text{and} \quad a_n = 0$$

[8]We expected this as we can see that the functions is an **odd** function. Odd functions are made up of sine functions.

Now we solve for $b_n$,

$$\begin{aligned} b_n &= \frac{1}{\pi}\left[\int_0^{\pi/\omega} 1\cdot\sin(n\omega t)dt - \int_0^{2\pi/\omega}\sin(n\omega t)dt\right] \\ &= \frac{1}{\pi}\left[\left(\frac{-1}{n\omega}\cos(n\omega t)\right)\Big|_0^{\frac{\pi}{\omega}} - \left(\frac{-1}{n\omega}\cos(n\omega t)\right)\Big|_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}}\right] \\ &= \frac{1}{n\pi\omega}\left[\cos(2n\pi) - 2\cos(n\pi) + 1\right] \\ &= \frac{2-2(-1)^n}{n\pi\omega} \\ &= \begin{cases} 0 & \text{for even } n \\ \frac{4}{n\pi\omega} & \text{for odd } n \end{cases} \end{aligned} \tag{11.39.5}$$

We can write this as

$$V(t) = \frac{4}{n\pi\omega}\sin(n\omega t) \quad \text{for all odd values of } n \tag{11.39.6}$$

or we can say

$$n = 2m+1 \quad \text{for all values of } m \tag{11.39.7}$$

This leads to[9]

$$V(t) = \frac{4}{\pi\omega}\sum_{m=0}^{\infty}\frac{1}{2m+1}\sin((2m+1)\omega t) \tag{11.39.8}$$

## 11.40 Rolling Cylinders

At the pont of contact, the cylinder is not moving. We do see the center of the cylinder moving at speed, $v$ and the top of the cylinder moving at speed, $2v$. Thus the acceleration acting at the point of contact is the cetripetal acceleration, which acts in an upwards direction.

**Answer: (C)**

## 11.41 Rotating Cylinder I

We recall that the change in kinetic energy of a rotating body is

$$\boxed{\Delta K = \frac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)} \tag{11.41.1}$$

This becomes

$$\begin{aligned} \Delta K &= \frac{1}{2}4\left(80^2 - 40^2\right) \\ &= 9600 \text{ J} \end{aligned} \tag{11.41.2}$$

**Answer: (D)**

[9]Definitely not something you have lots of time to do in the exam.

## 11.42 Rotating Cylinder II

Again we recall our equations of motion of a rotating body under constant angular acceleration. Fortunatley, they are similar to the equations for linear motion

$$\omega = \omega_0 + \alpha t \tag{11.42.1}$$

Plugging in the values we were given, we can find the angular acceleration, $\alpha$,

$$\alpha = -4 \text{ rad.s}^{-2} \tag{11.42.2}$$

The torque is

$$\tau = I\alpha \tag{11.42.3}$$

Which works out to be

$$\tau = 16 \text{ Nm} \tag{11.42.4}$$

**Answer: (D)**

## 11.43 Lagrangian and Generalized Momentum

We recall that the Lagrangian of a system is

$$\boxed{L = T - V} \tag{11.43.1}$$

and in generalized coordinates, this looks like

$$L = \frac{1}{2}m\dot{q}_n^2 - U(q_n) \tag{11.43.2}$$

We can find the equations of motion from this

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_n}\right) = \frac{\partial L}{\partial q_n}} \tag{11.43.3}$$

We are told that

$$\frac{\partial L}{\partial q_n} = 0 \tag{11.43.4}$$

or rather

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_n}\right) = 0 \tag{11.43.5}$$

We can see that the generalized momentum is

$$\frac{\partial L}{\partial \dot{q}_n} = m\dot{q}_n = p_n \tag{11.43.6}$$

and that

$$\frac{d}{dt}m\dot{q}_n = m\ddot{q}_n = 0 \tag{11.43.7}$$

So we expect the generalized momentum, $p_n$ is constant.

**Answer: (B)**

## 11.44 Lagrangian of a particle moving on a parabolic curve

As the particle is free to move in the x and y planes, Lagrangian of the system is

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{x}^2 - mgy \end{aligned} \tag{11.44.1}$$

We are given a realtinship between $y$ and $x$, where

$$y = ax^2 \tag{11.44.2}$$

Differentiating this with respect to time gives

$$\dot{y} = 2ax\dot{x} \tag{11.44.3}$$

Subsitutung this into eq. (11.44.1), gives us

$$L = \frac{1}{2}m\left[\dot{y}^2 + \frac{\dot{y}^2}{4ay}\right] - mgy \tag{11.44.4}$$

**Answer: (A)**

## 11.45 A Bouncing Ball

As the ball falls from a height of $h$, its potential energy s converted into kineic energy.

$$E = mgh = \frac{1}{2}mv_i^2 \tag{11.45.1}$$

Upon hitting the floor, the ball bounces but some of the energy is lost and its speed is 80% of what it was before

$$v_f = 0.8v_i \tag{11.45.2}$$

As it rises, its kinetic energy is converted into potential energy

$$\begin{aligned} \frac{1}{2}mv_f^2 &= mgh_2 \\ &= \frac{1}{2}m(0.8v_i)^2 \\ &= mgh_2 \end{aligned} \tag{11.45.3}$$

$$\Rightarrow h_2 = 0.64h \tag{11.45.4}$$

**Answer: (D)**

## 11.46 Phase Diagrams I

**NOT FINISHED**

**Answer: (B)**

## 11.47 Phase Diagrams II

**NOT FINISHED**
**Answer: (B)**

## 11.48 Error Analysis

The error for Newton's equation, $F = ma$, would be

$$\boxed{\left(\frac{\sigma_f}{F}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2} \tag{11.48.1}$$

**Answer: (C)**

## 11.49 Detection of Muons

The muons travel a distance of 3.0 meters. As muons move at relativistic speeds, near the speed of light, the time taken for a photon to traverse this distance is the time needed to distinguish between up travelling muons and down travelling muons.

$$t = \frac{x}{c} = \frac{3.0}{3.0 \times 10^8} \text{ seconds} \tag{11.49.1}$$

**Answer: (B)**

## 11.50 Quantum Mechanical States

**NOT FINISHED**

## 11.51 Particle in an Infintie Well

We recall that the momentum opertor is

$$\boxed{p = \frac{\hbar}{i}\frac{d}{dx}} \tag{11.51.1}$$

and the expectation value of the momentum is

$$\begin{aligned} \langle p \rangle &= \langle \psi_n^* | p | \psi_n \rangle \\ &= \int \psi_n^* \left( \frac{\hbar}{i} \frac{d}{dx} \psi_n \right) dx \end{aligned} \tag{11.51.2}$$

Given that

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \tag{11.51.3}$$

Equation [equation][2][1151]11.51.2 becomes

$$\begin{aligned}\langle p\rangle &= \frac{\hbar}{i}\frac{n\pi}{a}\sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{n\pi x}{a}\right)dx\\ &= 0\end{aligned} \tag{11.51.4}$$

**Answer: (A)**

## 11.52 Particle in an Infinite Well II

This is the very defintion of orthonormality.
**Answer: (B)**

## 11.53 Particle in an Infinite Well III

We recall that Schrödinger's Equation is

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x) = E\psi} \tag{11.53.1}$$

within the region $0 < x < a$, $V(x) = 0$, thus

$$\begin{aligned}-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} &= E\psi\\ \therefore \frac{d^2\psi}{dx^2} &= -k_n^2\psi = -\frac{2mE}{\hbar^2}\psi\end{aligned} \tag{11.53.2}$$

We see that

$$\boxed{E = \frac{k_n^2\hbar^2}{2m} = \frac{n^2\pi^2}{a^2}\frac{\hbar}{2m}} \tag{11.53.3}$$

where $n = 1, 2, 3\cdots$. So

$$E \geq \frac{\pi^2\hbar^2}{2ma^2} \tag{11.53.4}$$

**Answer: (B)**

## 11.54 Current Induced in a Loop II

Recalling Faraday's Law

$$\mathscr{E} = -\frac{d\Phi}{dt} \tag{11.54.1}$$

This gives the relationship between the induced EMF and the change in magnetic flux. The minus sign in the equation comes from Lenz's Law; the change in flux and the EMF have opposite signs.

If we use the Right Hand Grip Rule we see that the magnetic flux goes into the page around the loop. Pulling the loop to the right, moving away from the wire, reduced the magnetic flux inside the loop. As a result, the loop induces a current to counteract this reduction in flux and induces a current that will produce a magnetic field acting in the same direction. Again, using the Right Hand Grip Rule, this induced magnetic field acting into the page induces a **clockwise** current in the loop.

| Induced Current | Force on Left Side | Force on Right Side |
|---|---|---|
| Clockwise | To the left | To the right |

Table 11.54.1: Table showing something

Now we know the current directions, we can determine the forces on the wires. As we have a current moving through the wire, a motor, we use Fleming's Left Hand Rule. From this rule, we see the force on the wire is actin to the left. Using the same principle, on the right side, the force is acting on the right.

**Answer: (E)**

## 11.55 Current induced in a loop II

The magnetic force on a length of wire, $\ell$ is

$$\boxed{\mathbf{F}_B = I\ell \times \mathbf{B}} \tag{11.55.1}$$

We need to calculate the magnetic induction on the left and right sides of the loop. For this we turn to Ampere's Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} \tag{11.55.2}$$

On the left side of the loop,

$$\begin{aligned} B_L(2\pi r) &= \mu_0 I \\ B_L &= \frac{\mu_0 I}{2\pi r} \end{aligned} \tag{11.55.3}$$

On the right side of the loop, we have

$$\begin{aligned} B_R[2\pi(r+a)] &= \mu_0 I \\ B_R &= \frac{\mu_0 I}{2\pi(r+a)} \end{aligned} \tag{11.55.4}$$

The Magnetic force on the left side becomes

$$F_L = ib\left(\frac{\mu_0 I}{2\pi r}\right) \tag{11.55.5}$$

and the magnetic force on the right side is

$$F_R = ib\left(\frac{\mu_0 I}{2\pi\left(r+a\right)}\right) \tag{11.55.6}$$

We know that from the above question, these forces act in oppsoite directions, so

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_L - \mathbf{F}_R \\ &= \frac{\mu_0 iIb}{2\pi}\left[\frac{1}{r} - \frac{1}{r+1}\right] \\ &= \frac{\mu_0 iIb}{2\pi}\left[\frac{a}{r\left(r+a\right)}\right]\end{aligned} \tag{11.55.7}$$

**Answer: (D)**

## 11.56 Ground State of the Quantum Harmonic Oscillator

We recall that the ground state of the quantum harmonic oscillator to be $\frac{1}{2}h\nu$.
**Answer: (C)**

## 11.57 Induced EMF

The induced EMF follows Lenz Law

$$\mathscr{E} = -\frac{d\Phi}{dt} \tag{11.57.1}$$

where $\Phi = BA$, where $B$ is the magnetic field and $A$ is the area of the coil.As the coil cuts into the field, an EMF is induced. As it starts leaving, an EMF the EMFwill change polarity.
**Answer: (A)**

## 11.58 Electronic Configuration of the Neutral Na Atom

The atomic mass, $Z$, of the neutral Na atom is 11. We want our superscripts to ad to 11. Thus

$$1s^2, 2s^2, 2p^6, 3s^1 \tag{11.58.1}$$

**Answer: (C)**

## 11.59 Spin of Helium Atom

The electronic configuration of the He atom is

$$1s^2 \tag{11.59.1}$$

The spin of this is

$$S = \frac{1}{2} - \frac{1}{2} = 0 \tag{11.59.2}$$

representing one electron in the spin up and the other in the spin down directions. Thus the total spin is zero making it a spin singlet.

**Answer: (A)**

## 11.60 Cyclotron Frequency of an electron in metal

As a charged particle enters a transverse magnetic field, it experiences a centripetal force. Recalling the Lorents Force equation

$$\begin{aligned} \frac{mv^2}{r} &= Bev \qquad\qquad \text{where } \omega = \frac{v}{r} \\ \Rightarrow \omega_c &= \frac{Be}{m} \end{aligned} \tag{11.60.1}$$

Plugging in what we know, we get

$$\omega_c = \frac{1 \times 1.6 \times 10^{-19}}{0.1 \times 9.11 \times 10^{-31}}$$

Adding the indices of the equation gives an order of magnitude approximation

$$-19 + 31 = 12$$

Our closest match is $1.8 \times 10^{12}$ rad/s.

**Answer: (D)**

## 11.61 Small Oscillations of Swinging Rods

The period of an oscillator given its moment of inertia is[10]

$$\omega = \sqrt{\frac{mgd}{I}} \tag{11.61.1}$$

where $m$ is the mass, $r$ is the distance from the center of rotation and $I$ is the moment of inertia. The moment of inertia is essentially dependent on how the mass is distributed and is defined by

$$I = mr^2 \tag{11.61.2}$$

In the first case, we have two masses at a distance, $r$. So its moment of inertia becomes

$$I_1 = 2mr^2 \tag{11.61.3}$$

[10]Show derivation of this equation.

In the second case, we have one mass at the distance $\frac{r}{2}$ and another mass at $r$. We can get the total moment of inertia by adding the moment of inertias of both these masses

$$I_2 = mr^2 + m\left(\frac{r}{2}\right)^2 = \frac{5mr^2}{4} \tag{11.61.4}$$

Now the tricky part. The distance $d$ in eq. (11.61.1) is the distance from the pivot to the center of mass. In the first case it's the distance from the pivot to the two masses. In the second case the center of mass is between the two masses. The center of mass can be found by

$$r_{cm} = \frac{\sum m_i r_i}{\sum m_i} \tag{11.61.5}$$

Thus the center of mass for the second pendulum is

$$r_2 = \frac{mr + m\frac{r}{2}}{2m} = \frac{3}{4}r \tag{11.61.6}$$

Now we can determine the angular frequencies of our pendulums. For the first pendulum

$$\begin{aligned}\omega_1 &= \sqrt{\frac{2mgr}{2mr^2}}\\ &= \sqrt{\frac{g}{r}}\end{aligned} \tag{11.61.7}$$

For the second pendulum

$$\begin{aligned}\omega_2 &= \sqrt{\frac{(2m)g\left(\frac{3r}{4}\right)}{\frac{5}{4}mr^2}}\\ &= \sqrt{\frac{6g}{5r}}\end{aligned} \tag{11.61.8}$$

Dividing eq. (11.61.8) by eq. (11.61.7) gives

$$\frac{\omega_2}{\omega_1} = \left[\frac{\frac{6g}{5r}}{\frac{g}{r}}\right]^{\frac{1}{2}} = \left(\frac{6}{5}\right)^{\frac{1}{2}} \tag{11.61.9}$$

**Answer: (A)**

## 11.62 Work done by the isothermal expansion of a gas

The work done by the expansion of an ideal gas is

$$\boxed{W = \int_{V_0}^{V_1} P dV} \tag{11.62.1}$$

As the temperature is constant, we substitute the ideal gas equation

$$P = \frac{nRT}{V} \tag{11.62.2}$$

to give

$$\begin{aligned} W &= \int_{V_0}^{V_1} \frac{dV}{V} \\ &= nRT \ln\left[\frac{V_1}{V_0}\right] \end{aligned} \tag{11.62.3}$$

For one mole of gas, $n = 1$, gives us

$$W = RT_0 \ln\left[\frac{V_1}{V_0}\right] \tag{11.62.4}$$

This gives[11]
**Answer: (E)**

## 11.63 Maximal Probability

**NOT FINISHED**[12]
**Answer: (D)**

## 11.64 Gauss' Law

We recall the principle of Gauss' Law for electric fields.

$$\boxed{\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}} \tag{11.64.1}$$

The presence of a field means the presence of a charge somewhere.
**Answer: (B)**

## 11.65 Oscillations of a small electric charge

In this scenario, we have a charge, $-q$, placed between two charges, $+Q$. The net force on the small charge but if we were to slightly displace this charge it would be pulled back to its central axis. The force by which it is pulled back is

$$F = \frac{Qq}{4\pi\epsilon_0 R^2} + \frac{Qq}{4\pi\epsilon_0 R^2} = \frac{2Qq}{4\pi\epsilon_0 R^2} \tag{11.65.1}$$

[11] It seems the inclusion of the specific heat ratio was not needed and was there to throw you off. As Prof. Moody would say, "CONSTANT VIGILANCE!!!"
[12] Add something

For small oscillations

$$\boxed{k = \frac{d^2V}{dx^2}} \tag{11.65.2}$$

where $k$ is the spring constant.

We also recall that

$$\boxed{F = \frac{dV}{dx}} \tag{11.65.3}$$

Thus we can find $k$ by differentiating 11.65.1.

$$k = \frac{d}{dR}\left(\frac{2Qq}{4\pi\epsilon_0 R^2}\right) = \frac{Qq}{2\pi\epsilon_0 R^3} \tag{11.65.4}$$

The angular frequency, $\omega$, of our point charge system is

$$\omega^2 = \frac{k}{m} \tag{11.65.5}$$

where $m$ is the mass of the small particle.

Thus the angular frequency, $\omega$, is

$$\omega = \left[\frac{Qq}{2\pi\epsilon_0 m R^3}\right]^{\frac{1}{2}} \tag{11.65.6}$$

**Answer: (E)**

## 11.66 Work done in raising a chain against gravity

We recall our familar Work equation

$$W = \int_0^L F \cdot dx = \int_0^L mg \cdot dx \tag{11.66.1}$$

As the chain is pulled up, the mass changes with the length that is hanging. We are given the linear density so we know the relationship of the chain's mass to its length[13]

$$\rho = \frac{M}{L} = \frac{m}{x} \tag{11.66.2}$$

[13]There is another way to think of this problem. We are told that the steel chain is uniform, so its center of mass is in the middle of its length. The work done is the work done in raising the mass of the chain by this distance. Thus

$$W = Mg\left(\frac{L}{2}\right) = \rho L g \frac{L}{2} = 2 \cdot 10 \cdot 10 \cdot \frac{10}{2} = 1000\ \mathrm{J}$$

As you can see, we get the same result. You may find this solution quicker.

Substituting this into the above equation, we have

$$\begin{aligned} W &= \int_0^L \rho x \cdot dx \\ &= \rho g \left.\frac{x^2}{2}\right|_0^L \\ &= \frac{\rho g L^2}{2} = \frac{2(10)(100)}{2} = 1000 \text{ J} \end{aligned} \tag{11.66.3}$$

**Answer: (C)**

## 11.67 Law of Malus and Unpolarized Light

We suspect that the equation might have polarized and unpolarized components. Having no real idea and all the time in the world, we can derive the equations on what this might look like. We recall the Law of Malus

$$I_p = I_{po}\cos^2\theta = \frac{I_{po}}{2}\left[\cos 2\theta - 1\right] \tag{11.67.1}$$

Unpolarized light would have the same intensity through all $\theta$ so

$$I_u = I_{uo} \tag{11.67.2}$$

The total intensity is the sum of the two intensities

$$\begin{aligned} I &= I_u + I_p \\ &= I_{uo} + I_{po}\cos^2\theta = \frac{I_{po}}{2}\left[\cos 2\theta - 1\right] \\ &= \left[I_{uo} - \frac{I_{po}}{2}\right] + \frac{I_{po}}{2}\cos 2\theta \end{aligned} \tag{11.67.3}$$

where

$$A = I_{uo} - B \quad B = \frac{I_{po}}{2}$$

Given that $A > B > 0$, the above hypothesis holds.

**Answer: (C)**

## 11.68 Telescopes and the Rayleigh Criterion

The resolution limit of a telescope is determined by the Rayleign Criterion

$$\boxed{\sin\theta = 1.22\frac{\lambda}{d}} \tag{11.68.1}$$

where $\theta$ is the angular resolution, $\lambda$ is the wavelength of light and $d$ is the len's aperature diameter. As $\theta$ is small we can assume that $\sin\theta \approx \theta$. Thus

$$\begin{aligned} d &= 1.22\frac{\lambda}{\theta} \\ &= \frac{1.22 \times 5500 \times 10^{-10}}{8 \times 10^{-6}} \end{aligned} \tag{11.68.2}$$

After some fudging with indices we get something in the order of $10^{-2}$ m.

**Answer: (C)**

## 11.69 The Refractive Index

The refractive index is the ratio of the speed of light in vacuum to the speed of light in the medium.

$$n = \frac{c}{v} \tag{11.69.1}$$

Given $n = 1.5$,

$$v = \frac{c}{n} = \frac{2}{3}c \tag{11.69.2}$$

**Answer: (D)**

## 11.70 High Relativistic Energies

Recalling the equation for relativistic energy

$$E^2 = pc^2 + m^2c^4 \tag{11.70.1}$$

We are told that the energy of the particle is 100 times its rest energy, $E = 100mc^2$. Substituting this into the above equation gives

$$\left(100mc^2\right)^2 = pc^2 + \left(mc^2\right)^2 \tag{11.70.2}$$

As we are looking at ultra-high energies, we can ignore the rest energy term on the right hand side, so[14]

$$\begin{aligned} \left(100mc^2\right)^2 &= pc^2 \\ \therefore p &\approx 100mc \end{aligned} \tag{11.70.4}$$

**Answer: (D)**

[14]We can find a solution by relating the relativistic energy and momentum equations. Given

$$E = \gamma mc^2 \quad p = \gamma mc$$

This gives us

$$\begin{aligned} E &= pc \\ \Rightarrow p &= 100mc \end{aligned} \tag{11.70.3}$$

## 11.71 Thermal Systems I

**NOT FINISHED**

**Answer: (B)**

## 11.72 Thermal Systems II

**NOT FINISHED**

**Answer: (A)**

## 11.73 Thermal Systems III

**NOT FINSIHED**

**Answer: (C)**

## 11.74 Oscillating Hoops

The angular frequency of our hoop can be found by the equation

$$\omega = \sqrt{\frac{mgd}{I}} \tag{11.74.1}$$

where $m$ is the mass of the hoop, $d$ is the distance from the center of mass and $I$ is the moment of inertia. The Moment of Inertia of our hoop is

$$I_{\text{cm}} = Mr^2 \tag{11.74.2}$$

where $M$ is the mass of the hoop and $r$ is its radius. As the hoop is hanging from a nail on the wall, we use the Parallel Axis Theorem to determine its new Moment of Inertia

$$I = I_{\text{cm}} + Mr^2 = 2Mr^2 \tag{11.74.3}$$

we are given that

$$M_X = 4M_Y \quad d_X = 4d_Y$$

We can now find the moment of inertias of our two hoops

$$I_X = 2M_X R_X^2 \quad I_Y = 2M_Y R_Y^2$$

$$\begin{aligned} \omega_X &= \sqrt{\frac{M_X g R_X}{2M_X R_X^2}} \\ &= \sqrt{\frac{g}{2R_X}} \\ &= \frac{2\pi}{T} \end{aligned} \qquad \begin{aligned} \omega_Y &= \sqrt{\frac{M_Y g R_Y}{2M_Y R_Y^2}} \\ &= \sqrt{\frac{g}{2R_Y}} \\ &= 2\omega_X \end{aligned}$$

**Answer: (B)**

## 11.75 Decay of the Uranium Nucleus

When the Uranium nucleus decays from rest into two fissile nuclei, we expect both nuclei to fly off in opposite directions. We also expect momentum to be conserved thus

$$\begin{aligned} M_{Th}V_{Th} &= m_{He}v_{He} \\ \therefore v_{He} &= \frac{M_{Th}}{m_{He}}V_{Th} \end{aligned}$$

we see that $v_{He} \approx 60V_{Th}$. We can calculate the kinetic energies of the Thorium and Helium nuclei

$$K_{Th} = \frac{1}{2}M_{Th}V_{Th}^2 \qquad \begin{aligned} k_{He} &= \frac{1}{2}m_{He}v_{He}^2 \\ &= \frac{1}{2}\left(\frac{M_{Th}}{60}\right)(60V_{Th})^2 \\ &\approx 60K_{Th} \end{aligned}$$

Now we can go through our choices and find the correct one.

**A** This is clearly not the case. From the above, we see that $k_{He} = 60K_{Th}$.

**B** Again, this is clearly not the case. We see that $v_{He} \approx 60V_{Th}$.

**C** No this is not the case as momentum is conserved. For this to take place the two nuclei must fly off in opposite directions.

**D** Again, this is incorrect. Momentum is conserved so the momentum of both nuclei are equal.

**E** This is **correct**. We see from the above calculations that $k_{He} \approx 60K_{Th}$.

**Answer: (E)**

## 11.76 Quantum Angular Momentum and Electronic Configuration

The total angular momentum is

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \tag{11.76.1}$$

As none of the electron sub-shells are filled, we will have to add the individual angular momentum quantum numbers.

For the 1$s$ case, the spin, $s$, is

$$s_1 = \frac{1}{2} \tag{11.76.2}$$

As this is in the $s$ sub-shell, then the orbital quantum number is

$$\ell_1 = 0 \tag{11.76.3}$$

The total angular momentum for this electron is

$$\mathbf{J}_1 = \frac{1}{2} \tag{11.76.4}$$

For the other two electron shells we get

$$\begin{aligned} s_2 &= \frac{1}{2} \\ \ell_2 &= 1 \\ j_2 &= \ell_2 + s_2 = \frac{3}{2} \end{aligned} \tag{11.76.5}$$

and similarly for the third electron shell

$$\begin{aligned} s_3 &= \frac{1}{2} \\ \ell_3 &= 1 j_3 &= \ell_3 + s_3 = \frac{3}{2} \end{aligned}$$

Thus the total angulat momentum is

$$\begin{aligned} j &= j_1 + j_2 + j_3 \\ &= \frac{1}{2} + \frac{3}{2} + \frac{3}{2} = \frac{7}{2} \end{aligned} \tag{11.76.6}$$

**Answer: (A)**

## 11.77 Intrinsic Magnetic Moment

**NOT FINISHED**

**Answer: (E)**

## 11.78 Skaters and a Massless Rod

As the two skaters move towards the rod, the rod will begin to turn about its center of mass. The skaters linear momentum is converted to a combination of linear and rotational momentum of the rod.

The rotational momentum can be calculated

$$\boxed{\mathbf{L} = m\left(\mathbf{r} \times \mathbf{V}\right)} \tag{11.78.1}$$

The total rorational momentum can be calculated,

$$\begin{aligned} L &= L_{\text{top}} + L_{\text{bottom}} \\ &= m\left(\frac{b}{2}\right)(2v) + m\left(\frac{b}{2}\right)v \\ &= mbv \end{aligned} \tag{11.78.2}$$

The rod will rotate with angular velocity, $\omega$,

$$\boxed{\mathbf{L} = I\omega} \tag{11.78.3}$$

where $I$ is the Moment of Inertia, where

$$I = m\left(\frac{b}{2}\right)^2 \tag{11.78.4}$$

Thus the angular frequency of the rod becomes

$$\omega = \frac{3v}{b} \tag{11.78.5}$$

As the two masses move towards the rod, the speed of the center of mass remains the same even after collision. This allows us to define the linear motion of the rod which we know to be at the center of the rod.

$$\boxed{v_{\text{center}} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}} \tag{11.78.6}$$

Thus the center of mass velocity is

$$v_{\text{center}} = \frac{m(2v) + m(-v)}{2m} = \frac{v}{2} \tag{11.78.7}$$

The poition of the mass at $b/2$ is a combination of the translational and rotational motions. The translatioal motion is

$$v_{\text{translational}} = 0.5vt \tag{11.78.8}$$

And the rotational motion is

$$v_{\text{rotational}} = 0.5b\sin(\omega t) \tag{11.78.9}$$

where $\omega = 3v/b$. This becomes

$$x = 0.5vt + 0.5b\sin\left(\frac{3vt}{b}\right) \tag{11.78.10}$$

**Answer: (C)**

## 11.79 Phase and Group Velocities

We recall that the group velocity is

$$v_g = \frac{d\omega}{dk} \tag{11.79.1}$$

and the phase velocity is

$$v_p = \frac{\omega}{k} \tag{11.79.2}$$

From the graph, we see that in the region $k_1 < k < k_2$, the region of the graph is a straight line with a negative gradient. So we can assume that $\frac{d\omega}{dk} < 0$ and that $\frac{\omega}{k} > 0$. Thus the two velocities are in opposite directions.

**A** This is correct as shown above.

**B** They can't be in the same directions. The phase velocity is moving in the opposite direction to the group velocity.

**C** Again incorrect, they are not travelling in the same direction.

**D** Also incorrect, the phase velocity is finite as $k \neq 0$.

**E** Again, they are not in the same direction.

**Answer: (A)**

## 11.80 Bremsstrahlung Radiation

As an electron is accelerated towards a target, it gets rapidly decelerated and emits emectromagnetic radiation in the X-ray spectrum. The frequency of this emitted radiation is determined by the magnitude of its decelertion. If it has been completly decelerated, then all of its kinetic energy is converted to EM radiation. We are told the kinetic energy of our accelerated electrons are $K = 25\,\text{keV}$. The energy of a photon is

$$E = hf = \frac{hc}{\lambda} = K \tag{11.80.1}$$

Solving for $\lambda$, we have

$$\begin{aligned}
\lambda &= \frac{hc}{K} \\
&= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{25 \times 10^{3} \times 1.60 \times 10^{-19}} \\
&= \frac{6.63 \times 3}{25 \times 1.6} \times 10^{-10} \\
&\approx \frac{12}{25} \times 10^{-10} = 0.5\,\text{Å}
\end{aligned} \tag{11.80.2}$$

**Answer: (B)**

## 11.81 Resonant Circuit of a RLC Circuit

The maximum steady state amplitude will occur at its resonant frequency. This can be see if one were to draw a graph of $\mathcal{E}$ vs. $\omega$. The resonant frequency occurs when the capacitive impedance, $X_C$ and the inductive impedance,$X_L$ are equal. Thus

$$X_L = \omega L \quad X_C = \frac{1}{\omega C}$$

Equating $X_L$ and $X_C$ together gives

$$\begin{aligned} \omega L &= \frac{1}{\omega C} \\ \omega &= \frac{1}{\sqrt{LC}} \end{aligned} \tag{11.81.1}$$

**Answer: (C)**

## 11.82 Angular Speed of a Tapped Thin Plate

Fortunately we have been given the angular impulse, **H**,

$$H = \int \tau dt \tag{11.82.1}$$

where *tau* is the torque on the plate. We recall that $\tau = I\alpha$ where $I$ is the moment of inertia and $\alpha$ is the angular acceleration. Thus the above equation becomes

$$\omega = \int \alpha dt = \frac{H}{I} \tag{11.82.2}$$

The moment of inertia of the plate is $I = \frac{1}{3}Md^2$[15]. Thus

$$\omega = \frac{3H}{md^2} \tag{11.82.3}$$

**Answer: (D)**

## 11.83 Suspended Charged Pith Balls

We can resolve the horizontal and vertical force components acting on the pith balls. We see that

$$\begin{aligned} \text{Horizontal Force:} &\qquad T\sin\theta = \frac{kq^2}{2d^2} \\ \text{Vertical Force:} &\qquad T\cos\theta = mg \end{aligned} \tag{11.83.1}$$

[15] Add derivation of moment of inertia for thin plate.

As we are looking at small values of $\theta$ we can make the following approximations

$$\begin{aligned} \sin\theta \approx \tan\theta &\approx \theta \\ \cos\theta &\approx 1 \end{aligned}$$

We see that $\sin\theta = \frac{d}{2L}$. Subsituting the above, we get

$$\begin{aligned} T &= mg \\ \Rightarrow T\sin\theta &= mg\left(\frac{d}{2L}\right) = \frac{kq^2}{2d^2} \\ \therefore d^3 &= \left(\frac{2kq^2L}{mg}\right) \end{aligned} \tag{11.83.2}$$

**Answer: (A)**

## 11.84 Larmor Formula

We recall the Larmor Formula which describes the total power of radiated EM radiation by a non-relativistic accelerating point charge.

$$\boxed{P = \frac{q^2a^2}{6\pi\epsilon_0 c^3}} \tag{11.84.1}$$

where $q$ is the charge and $a$ is the acceleration. We can use this to eliminate choices.

**A** This says

$$P \propto a^2 \tag{11.84.2}$$

We see that this is **TRUE** from the above equation.

**B** This says

$$P \propto e^2 \tag{11.84.3}$$

This is also **TRUE**.

**C** Also true.

**D** **False**

**E** True

**Answer: (D)**

## 11.85 Relativistic Momentum

We recall the Relativistic Energy Formula

$$E^2 = p^2c^2 + m^2c^4 \tag{11.85.1}$$

We are given $E = 1.5$ MeV. Plugging into the above equation yields

$$1.5^2 = p^2c^2 + 0.5^2 \tag{11.85.2}$$

We find $p^2 = 2$ MeV/c.

**Answer: (C)**

## 11.86 Voltage Decay and the Oscilloscope

We recall that the voltage decay across a capacitor follows an exponential decay, such that

$$V = V_0 \exp - \left[\frac{t}{RC}\right] \tag{11.86.1}$$

Solving for $C$, we see that

$$C = -\frac{t}{R} \ln \left[\frac{V_0}{V}\right] \tag{11.86.2}$$

We need to find $t$, which we can determine by how fast the trace sweeps, $s$. We need to find $R$, which we will be given. The ration $\frac{V_0}{V}$ can be read off the vertical parts of the scope.

**Answer: (B)**

## 11.87 Total Energy and Central Forces

The total energy of a system is

$$E = T + V \tag{11.87.1}$$

where $T$ is the kinetic energy and $V$ is the potential energy. We are told the particle moves under a circular orbit where

$$F = \frac{K}{r^3} = \frac{mv^2}{r} \tag{11.87.2}$$

The potential energy can be found

$$V = \int F dr = K \int \frac{dr}{r^3} = -\frac{1}{2}\frac{K}{r^2} \tag{11.87.3}$$

The kinetic Energy is

$$T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{K}{r^2} \tag{11.87.4}$$

Thus

$$E = \frac{1}{2}\frac{K}{r^2} - \frac{1}{2}\frac{K}{r^2} = 0 \tag{11.87.5}$$

**Answer: (C)**

## 11.88 Capacitors and Dielectrics

This question can be answered through the process of elimination and witout knowing exactly what the displacement vector is or what it does.

As the capacitor is connected to a battery with potential difference, $V_0$, an electric field, $E_0$ forms between the plates and a charge, $Q_0$ accumulates on the plates. We can relate this to the capacitance, $C_0$, of the capacitor

$$Q_0 = C_0 V_0 \tag{11.88.1}$$

While still connected to the battery, a dielectric is inserted between the plates. This serves to change the electric field between the plates and as a result the capacitance.

$$Q_f = C_f V_f \tag{11.88.2}$$

As we have not disconnected the battery,

$$\boxed{V_f = V_0} \tag{11.88.3}$$

The dieiectric has a dielectric constant, $\kappa\epsilon_0$, such that

$$\boxed{C_f = \kappa C_0} \tag{11.88.4}$$

It follows that

$$\boxed{Q_f = \kappa Q_0} \tag{11.88.5}$$

When a dielectric is placed inside an electric field, there is an induced electric field, that points in the opposite direction to the field from the battery.

$$\mathbf{E}_f = \mathbf{E}_0 + \mathbf{E}_{\text{induced}} \tag{11.88.6}$$

This results in

$$\boxed{\mathbf{E}_f = \frac{\mathbf{E}_0}{\kappa}} \tag{11.88.7}$$

From the above we can infer

1. $V_f = V_0$
2. $Q_f > Q_0$
3. $C_f > C_0$
4. $E_f < E_0$

Based on this we can eliminate all but choice **(E)**. In the case of the last choice, the effect of the electric field places charges on the plates of the capacitor. Gauss' Law tells us

$$\epsilon_0 \nabla \cdot \mathbf{E} = \rho \tag{11.88.8}$$

If we were to place the dielectric between the plates, the atoms in the dielectric would become polarized in the presence of the electric field. This would result in the accumulations of bound charges, $\rho_b$ within the dielectric. The total charge becomes

$$\rho = \rho_b + \rho f \tag{11.88.9}$$

This becomes

$$\epsilon_0 \nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{P} + \rho_f \tag{11.88.10}$$

This becomes

$$\nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) = \rho_f \tag{11.88.11}$$

Where the term $\epsilon_0 \mathbf{E} + \mathbf{P}$ is the displacemet vector. In the beginning, there is no polarization vector, $\mathbf{P}$ so

$$\mathbf{D}_0 = \epsilon_0 \mathbf{E} \tag{11.88.12}$$

But with the presence of the dielectric it becomes,

$$\mathbf{D}_f = \epsilon_0 \mathbf{E} + \mathbf{P} \tag{11.88.13}$$

Thus

$$D_f > D_0 \tag{11.88.14}$$

**Answer: (E)**

## 11.89 harmonic Oscillator

**NOT FINISHED**
**ANSWER: (E)**

## 11.90 Rotational Energy Levels of the Hydrogen Atom

**NOT FINSIHED**
**Answer: (B)**

## 11.91 The Weak Ineteraction

**NOT FINISHED**
**Answer: (D)**

## 11.92 The Electric Motor

For each rotation the wire makes, it crosses the the three pairs of magnets which results in three periods. Thus we expect

$$f = 3f_0 = 30\text{Hz} \tag{11.92.1}$$

**Answer: (D)**

## 11.93 Falling Mass connected by a string

When the mass is at the top of its swing, $\theta = 0°$, the only force acting on it will be the one due to gravity; the tangential force acting downward. As it is let go, we expect the total acceleration to increase and reach a maximum at $\theta = 90°$. We can use this to eliminate the choices given.

$g\sin\theta$ At $\theta = 0$, the total acceleration, $a$, is zero. At $\theta = 90$, the total acceleration is a maximum, $a = g$. We would expect this to be greater than $g$[16]**We can eliminate this choice.**

$2g\cos\theta$ At $\theta = 0$, the total acceleration is $2g$.A bit of an impossibility considering the mass is stationary. At $\theta = 90$ the total acceleration becomes zero. Another impossibility, we expect the total acceleration to increase. **We can eliminate this choice.**

$2g\sin\theta$ At $\theta = 0$, the total acceleration is zero. Another impossibility. At $\theta = 90$, the acceleration is a maximum at $2g$. **We can eliminate this choice.**

$g\sqrt{3\cos^2\theta + 1}$ At $\theta = 0$, our total acceleration is a maximum, $a = 2g$ and a minumum at the bottom, $a = g$. We don't expect this physically. **We can eliminate this.**

$g\sqrt{3\sin^2\theta + 1}$ At $\theta = 0$ the total accleration is $a = g$. This we expect. At $\theta = 90$, it is a maximum, $a = 2g$. Again this is what we expect. **We choose this one.**

**ANSWER: (E)**

### 11.93.1 Calculation

As the mass falls, its gravitaional potential energy is converted to kinetic energy. We can express this as a function of $\theta$.

$$mg\ell\sin\theta = \frac{1}{2}mv^2 \tag{11.93.1}$$

where $\ell$ is the length of the rod.

The radial or centripetal force, $a_r$ is

$$ma_r = \frac{mv^2}{\ell} \tag{11.93.2}$$

Solving for $a_r$ gives us

$$a_r = 2g\sin\theta \tag{11.93.3}$$

The tangential accelertion is a component of the gravitational acceleration, thus

$$a_t = g\cos\theta \tag{11.93.4}$$

[16]Think an amusement park ride, something along 'the Enterprise' ride manufactured by the *HUSS Maschinenfabrik* company. The g-forces are at the greatest at the bottom about $2g$s and lowest at the top. There are no restraints while you're inside; you're kept in place through centripetal forces. *Your faith in the force should dispel any fears.*

We can find the total acceleration, $a$, by adding our tangential and radial accelerations. As these accelerations are vectors and they are orthogonal

$$a = \sqrt{a_t^2 + a_r^2} \tag{11.93.5}$$

Plugging in and solving gives us

$$\begin{aligned} a &= \sqrt{4g^2 \sin^2\theta + g^2\cos^2\theta} \\ &= g\sqrt{3\sin^2\theta + 1} \end{aligned} \tag{11.93.6}$$

## 11.94 Lorentz Transformation

**NOT FINISHED**

**Answer: (C)**

## 11.95 Nuclear Scatering

**NOT FINISHED**

**ANSWER: (C)**

## 11.96 Michelson Interferometer and the Optical Path Length

The optical path length through a medium of of refractive index, $n$, and distance, $d$, is the length of the path it would take through a vacuum, $D$. Thus

$$D = nd \tag{11.96.1}$$

The Optical Path Difference is the difference in these two lengths. As the gas is evacuated, we observe 40 fringes move past our field of view. So our optical path difference is

$$\Delta = n\lambda \tag{11.96.2}$$

**NOT FINSIHED**

**ANswer: (C)**

## 11.97 Effective Mass of an electron

**NOT FINSIHED**

**Answer: (D)**

## 11.98 Eigenvalues of a Matrix

We are given a matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{11.98.1}$$

By finding the determinant of the characteristic matrix, we can find its eigenvalues

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0 \tag{11.98.2}$$

Solving for $\lambda$ yields

$$-\lambda[(-\lambda)(-\lambda) - (1)(0)] - 1[(0)(-\lambda) - (1)(1)] + 0[(0)(0) - (-\lambda)(1)] = 0 \tag{11.98.3}$$

Thus

$$\lambda^3 = 1 \tag{11.98.4}$$

This leaves us solutions where

$$\begin{aligned} \lambda^3 &= \exp[2\pi n i] \\ \Rightarrow \lambda &= \exp\left[\frac{i2\pi n}{3}\right] = 1 \end{aligned} \tag{11.98.5}$$

eq. (11.98.5) has solutions of the form

$$\lambda_n = \exp\left[\frac{i2\pi n}{3}\right] = \cos\left(\frac{2\pi n}{3}\right) + i\sin\left(\frac{2\pi n}{3}\right) \tag{11.98.6}$$

where $n = 1, 2, 3$.

This gives us solutions where

$$\begin{aligned} \lambda_1 &= \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) \\ &= -\frac{1}{2} + i\frac{\sqrt{3}}{2} \end{aligned} \tag{11.98.7}$$

$$\begin{aligned} \lambda_2 &= \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) \\ &= -\frac{1}{2} - i\frac{\sqrt{3}}{2} \end{aligned} \tag{11.98.8}$$

$$\begin{aligned} \lambda_3 &= \cos(2\pi) + i\sin(2\pi) \\ &= +1 \end{aligned} \tag{11.98.9}$$

Now we have our solutions we can eliminate choices,

**A** We see that

$$\begin{aligned} \lambda_1 + \lambda_2 + \lambda_3 &= 0 \\ \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) + 1 &= 0 \end{aligned}$$

This is **TRUE**

**B** We see that $\lambda_1$ and $\lambda_2$ are not real. This is **NOT TRUE**.

**C** We see that, in our case,

$$\begin{aligned}\lambda_1\lambda_2 &= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \\ &= \frac{1}{4} + \frac{3}{4} \\ &= 1\end{aligned} \tag{11.98.10}$$

So this is also true.

**D** This is also true

**E** Also TRUE

**Answer: (E)**

## 11.99 First Order Correction Perturbation Theory

**NOT FINSHED**
**Answer: (A)**

## 11.100 Levers

As our system is in equlibrium, we know that the sum of the moments is equal to zero. We are also told that the rod is uniform, so the center of mass is in the middle of the rod. Thus, taking the clockwise and anticlockwise moments about the pivot

$$20x + 20(x + 5) = 40y \tag{11.100.1}$$

where $x$ is the distance of the center of mass from the pivot and $y$ is the distance of the 40kg mass from the pivot. We know

$$10 = 5 + x + y \tag{11.100.2}$$

Solving for $x$ results in

$$x = 1.25 \text{ m} \tag{11.100.3}$$

**Answer: (C)**

# Chapter 12

# GR9677 Exam Solutions

## 12.1 Discharge of a Capacitor

The voltage of a capacitor follows an exponential decay

$$V(t) = V_0 \exp\left[-\frac{t}{RC}\right] \tag{12.1.1}$$

When the switch is toggled in the $a$ position, the capacitor is quickly charged and the potential across its plates is $V$. $r$ is small and we assume that the potential difference across it is negligible. When the switch is toggled on the $b$ position, the voltage across the capacitor begins to decay. We can find the current through the resistor, $R$, from Ohm's Law

$$I(t) = \frac{V(t)}{R} = V_0 \exp\left[-\frac{t}{RC}\right] \tag{12.1.2}$$

At $t = 0$ $V_0 = V$. **Graph B**, shows an exponential decay.

**Answer:(B)**

## 12.2 Magnetic Fields & Induced EMFs

We have a circuit loop that is placed in a decaying magnetic field where the field direction acts into the page. We have two currents in the circuit. The first is due to the battery and the other is an induced current from the changing magnetic field. We can easily determine the current of the cell from Ohm's Law.

$$I_c = \frac{V}{R} = \frac{5.0}{10}\text{ A} \tag{12.2.1}$$

The induced EMF from the magnetic field is found from Faraday's Law of Induction

$$\mathscr{E} = -\frac{d\Phi}{dt} \tag{12.2.2}$$

where $\Phi$ is the magnetic flux.

$$\frac{d\Phi}{dt} = A\frac{dB}{dt} \tag{12.2.3}$$

The area of our loop, $A = 10 \times 10\text{cm}^2$. So the induced EMF is

$$\mathscr{E} = -100 \times 10^{-4} \times 150 = 1.5 \text{ V} \tag{12.2.4}$$

The field acts into the page, we consider this a negative direction, it's decaying, also negative. So

$$\text{negative} \times \text{negative} = \text{positive} \tag{12.2.5}$$

Faraday's Law of Induction has a negative sign. So we expect our EMF to be negative. Using the Right Hand Grip Rule, and pointing our thumb into the page, our fingers curl in the clockwise direction. So we see that the current from our cell goes in the counter-clockwise direction and the induced current in the clockwise direction; they oppose each other. The total EMF is

$$V = 5.0 - 1.5 = 3.5 \text{ V} \tag{12.2.6}$$

The current through the resistor is

$$I = \frac{3.5}{10} = 0.35 \text{ A} \tag{12.2.7}$$

**Answer: (B)**

## 12.3 A Charged Ring I

The Electric Potential is

$$V = \frac{Q}{4\pi\epsilon_0 r} \tag{12.3.1}$$

The distance, $r$, of $P$ from the charged ring is found from the pythagorean theorem

$$r^2 = R^2 + x^2 \tag{12.3.2}$$

Plugging this into the above equation yields

$$V = \frac{Q}{4\pi\epsilon_0 \sqrt{R^2 + x^2}} \tag{12.3.3}$$

**Answer: (B)**

## 12.4 A Charged Ring II

The force a small charge, $q$ experiences if placed in the center of the ring can be found from Coulomb's Law

$$F = \frac{qQ}{4\pi\epsilon_0 R^2} \tag{12.4.1}$$

If it undergoes small oscillations, $R >> x$, then

$$F = mR\omega^2 \tag{12.4.2}$$

Equating the two equations, and solving for $\omega$, we get

$$\omega = \sqrt{\frac{qQ}{4\pi\epsilon_0 m R^3}} \tag{12.4.3}$$

**Answer: (A)**

## 12.5 Forces on a Car’s Tires

The horizontal force on the car’s tires is the sum of two forces, the cetripetal force and the frictional force of the road. The cetripetal force acts towards the center, $\mathbf{F}_A$, while the frictional force acts in the forwards direction, $\mathbf{F}_C$. If it’s not immediately clear why it acts in the forward direction, the tires, as they rotate, exert a backward force on the road. The road exerts an equal and opposite force on the tires, which is in the forward direction[1] So the force on the tires is the sum of these forces, $\mathbf{F}_A$ and $\mathbf{F}_C$, which is $\mathbf{F}_B$

**Answer: (B)**

## 12.6 Block sliding down a rough inclined plane

We are told several things in this question. The first is that the block attains a constant speed, so it gains no kinetic energy; all its potential energy is lost due to friction.[2]

**Answer: (B)**

## 12.7 Collision of Suspended Blocks

We are told that the ball collides elastically with the block, so both momentum and energy are conserved. As the ball falls from a height, $h$, its potential energy is converted to kinetic energy

$$\begin{aligned} mgh &= \frac{1}{2}mv_1^2 \\ v_1^2 &= 2gh \end{aligned} \tag{12.7.1}$$

[1]Sometimes the frictional force can act in the direction of motion. This is one such case.

[2]If you’d like a more rigorous proof, not something you might do in the exam. The work done by the frictional force, $F_r$ is

$$W = \int F_r dx \tag{12.6.1}$$

$F_r$ acts along the direction of the incline and is equal to

$$F_r = mg\sin\theta \tag{12.6.2}$$

The distance the force acts is

$$x = \frac{h}{\sin\theta} \tag{12.6.3}$$

So the work done is

$$\begin{aligned} W &= F_r \cdot x \\ &= mg\sin\theta \times \frac{h}{\sin\theta} \\ &= mgh \end{aligned}$$

Momentum is conserved, so

$$\begin{aligned} mv_1 &= mv_2 + 2mv_3 \\ v_1 &= v_2 + 2v_3 \end{aligned} \tag{12.7.2}$$

Energy is also conserved, so

$$\begin{aligned} \frac{1}{2}mv_1^2 &= \frac{1}{2}mv_2^2 + \frac{1}{2}2mv_3^2 \\ v_1^2 &= v_2^2 + 2v_3^2 \end{aligned} \tag{12.7.3}$$

Squaring eq. (12.7.2) and equating with eq. (12.7.3) gives

$$\begin{aligned} v_1^2 &= (v_2 + 2v_3)^2 \\ \therefore 2v_2 &= -v_3 \end{aligned} \tag{12.7.4}$$

Substituting this into eq. (12.7.2) gives

$$\begin{aligned} v_1 &= v_2 + 2v_3 \\ &= 3v_2 \\ \Rightarrow v_1^2 &= 9v_2^2 \end{aligned} \tag{12.7.5}$$

The $2m$ block's kinetic energy is converted to potential energy as it rises to a height of $h_2$. Thus

$$\begin{aligned} mgh_2 &= \frac{1}{2}mv_2^2 \\ \therefore v_2^2 &= 2gh_2 \end{aligned} \tag{12.7.6}$$

We see that

$$\begin{aligned} \frac{2gh}{9} &= 2gh_2 \\ \Rightarrow \frac{h}{9} &= h_2 \end{aligned} \tag{12.7.7}$$

**Answer: (A)**

## 12.8 Damped Harmonic Motion

From section 1.4.4, we see that the frequency of a damped oscillator is

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \tag{12.8.1}$$

This shows that the damped frequency will be higher than the natural frequency, $\omega_0$, or its period, $T_0$, will be longer.

**Answer: (A)**

## 12.9 Spectrum of the Hydrogen Atom

The hydrogen spectrum can be found by the emperical Rydberg equation

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \tag{12.9.1}$$

where $n_i$ and $n_f$ are the intial and final states respectively. The longest wavelength, or the smallest energy transition, would represent the transition $n_1 = n_f + 1$.

For the Lyman series, $n_f = 1$, which lies in the ultra-violet spectrum, we have

$$\frac{1}{\lambda_L} = R_H \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} R_H \tag{12.9.2}$$

For the Balmer series, $n_f = 2$, which lies in the optical spectrum, we have

$$\frac{1}{\lambda_B} = R_H \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5}{36} R_H \tag{12.9.3}$$

Dividing eq. (12.9.3) by eq. (12.9.2), we get

$$\frac{\lambda_L}{\lambda_B} = \frac{\frac{5}{36} R_H}{\frac{3}{4} R_H} = \frac{5}{27} \tag{12.9.4}$$

**Answer: (A)**[3]

## 12.10 Internal Conversion

We are lucky that they actually tell us what the intenal conversion process is. From this we gather that the inner most electron has left its orbit and the most likely outcome will be for the remaining electrons to 'fall' in and take its place. These transitions will result in the emission of $X$-ray photons[4].

**Answer: (B)**

## 12.11 The Stern-Gerlach Experiment

The description of the experiment in the question is the Stern-Gerlach Experiment. In this experiment, we expect the electrons to be deflected vertically into two beams representing spin-up and spin-down electrons.

**Answer: (D)**

[3] The other transition, the Paschen series, $n_f = 3$, lies in the infra-red region of the spectrum.
[4] This can also result in the emission of an Auger Electron

## 12.12 Positronium Ground State Energy

The positronium atom consists of an electron and a positron in a bound state. Classically, this atom looks like two planets orbiting a central point or center of mass. We need to reduce this system to an equivalent one where an electron cirlces a central mass. We call this equivalent system the *reduced mass* of the two body system. This is

$$\mu = \frac{m_e M}{M + m_e} \tag{12.12.1}$$

The energy levels in terms of the reduced mass is defined as

$$\boxed{E_n = -\frac{Z^2}{n^2}\frac{\mu}{m_e}E_0} \tag{12.12.2}$$

The reduced mass of positronium is

$$\frac{\mu}{m_e} = \frac{m_e}{2m_e} = \frac{1}{2} \tag{12.12.3}$$

and the ground state of this atom, $Z = 1$ and $n = 1$. The ground state energy of Hydrogen is 13.6eV.

eq. (12.12.2) becomes

$$E_1 = -\frac{1}{2}13.6 \text{ eV} = -6.8 \text{ eV} \tag{12.12.4}$$

**Answer: (C)**

## 12.13 Specific Heat Capacity and Heat Lost

In this question, you are being asked to put several things together. Here, we are told, a heater is placed into the water but the water does not boil or change temperture. We can assume that all of the supplied heat by the heater is lost and we infer from the power of the heater that 100 Joules is lost per second.

The energy to change water by one degree is derived from its specific heat capacity.

$$E = \text{SHC} \times \text{Mass} \times \text{Temp. Diff.} = 4200 \times 1 \times 1 = 4200 \text{ J} \tag{12.13.1}$$

So the time to loose 4200 Joules of heat is

$$t = \frac{E}{P} = \frac{4200}{100} = 42 \text{ seconds} \tag{12.13.2}$$

**Answer: (B)**

## 12.14 Conservation of Heat

Assuming that little to no heat is lost to the environment, the two blocks will exchange heat until they are both in thermal equilibrium with each other. As they have the same masses we expect the final temperature to be 50°C. We can, of course, show this more rigorously where both blocks reach a final temperature, $T_f$. The intial temperatures of blocks I and II are $T_1 = 100°$Cand $T_2 = 0°$C, respectively.

$$\begin{aligned}\text{Heat Lost by Block I} &= \text{Heat gained by Block II}\\ 0.1 \times 10^3 \times 1 \times \left(100 - T_f\right) &= 0.1 \times 10^3 \times 1 \times \left(T_f - 0\right)\\ 2\left(0.1 \times 10^3\right) T_f &= 10 \times 10^3\\ \therefore T_f &= 50°\text{C}\end{aligned}$$

Thus the heat exchanged is

$$0.1 \times 10^3 \times 1 \times (100 - 50) = 5 \text{ kcal} \tag{12.14.1}$$

**Answer: (D)**

## 12.15 Thermal Cycles

We are told the cycle is reversible and moves from $ABCA$. We can examine each path and add them to get the total work done.

Path $A \to B$ is an isothermal process

$$\begin{aligned}W_{A\to B} &= \int_{V_1}^{V_2} P \cdot dV \quad \text{where } P = \frac{nRT}{V}\\ &= nRT_h \int_{V_1}^{V_2} \frac{dV}{V}\\ &= nRT_h \ln\left[\frac{V_2}{V_1}\right]\end{aligned} \tag{12.15.1}$$

Path $B \to C$ is an isobaric process

$$\begin{aligned}W_{B\to C} &= \int_{V_2}^{V_1} P_2 dV \quad \text{where } P_2V_2 = nRT_h\\ &= P_2\left(V_1 - V_2\right) \quad \text{and } P_2V_1 = nRT_c\\ &= nR\left(T_c - T_h\right)\end{aligned} \tag{12.15.2}$$

and Path $C \to A$

$$\begin{aligned}W_{C\to A} &= \int P_1 dV \quad \text{where} \quad dV = 0\\ &= 0\end{aligned} \tag{12.15.3}$$

Adding the above, we get

$$\begin{aligned} W &= W_{A\to B} + W_{B\to C} + W_{C\to A} \\ &= nRT_h \ln\left[\frac{V_2}{V_1}\right] + nR\left(T_c - T_h\right) \end{aligned} \quad (12.15.4)$$

where $n = 1$ mole.

$$W = RT_h \ln\left[\frac{V_2}{V_1}\right] - R\left(T_h - T_c\right) \quad (12.15.5)$$

**Answer: (E)**

## 12.16 Mean Free Path

The mean free path of a particle, be it an atom, molecule or photon, is the average distance travelled between collisions. We are given the equation as

$$\boxed{\ell = \frac{1}{\eta\sigma}} \quad (12.16.1)$$

where $\eta$ is the number desnity and $\sigma$ is the collision cross section. The number density works out to be

$$\eta = \frac{N}{V} \quad (12.16.2)$$

where $N$ is the number of molecules and $V$ is the volume. We can determine this from the ideal gas law,

$$\begin{aligned} PV &= NkT \\ \therefore \eta &= \frac{N}{V} = \frac{P}{kT} \end{aligned} \quad (12.16.3)$$

The collision cross section is the area through which a particle can not pass without colliding. This works out to be

$$\sigma = \pi d^2 \quad (12.16.4)$$

Now we can write eq. (12.16.1) in terms of variables we know

$$\ell = \frac{kT}{\pi P d^2} \quad (12.16.5)$$

As air is composed mostly of Nitrogen, we would have used the diameter of Nitrogen in our calculations. This is approximately $d = 3.1$Å. Plugging in the constants given we have

$$\begin{aligned} \ell &= \frac{(1.38 \times 10^{23})(300)}{\pi \times 1.0 \times 10^5 (3.1 \times 10^{10})^2} \\ &= 1.37 \times 10^{-7} \text{ m} \end{aligned}$$

As we don't have a calculator in the exam, we can estimate by adding the indices in our equation,

$$-23 + 2 - 5 + 20 = -6 \quad (12.16.6)$$

So we expect our result to be in the order of $10^{-6}$ m. We choose **(B)**.

**Answer: (B)**

## 12.17 Probability

The probability of finding a particle in a finite interval between two points, $x_1$ and $x_2$, is

$$P(2 \leq x \leq 4) = \int_2^4 |\Psi(x)|^2 \, dx \tag{12.17.1}$$

with the normalization condition,

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 \, dx = 1 \tag{12.17.2}$$

We can tally the values given to us on the graph The probability of finding the particle

| $x$ | $\Psi$ | $\Psi^2$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 2 | 4 |
| 4 | 3 | 9 |
| 5 | 1 | 1 |
| 6 | 0 | 0 |
| Total | | 16 |

Table 12.17.1: Table of wavefunction amplitudes

between $(2 \leq x \leq 4)$ is

$$\begin{aligned} P(2 \leq x \leq 4) &= \frac{1^2 + 2^2 + 3^2}{1^2 + 1^2 + 2^2 + 3^2 + 1^2 + 0^2} \\ &= \frac{4 + 9}{1 + 1 + 4 + 9 + 1 + 0} \\ &= \frac{13}{16} \end{aligned} \tag{12.17.3}$$

**Answer: (E)**

## 12.18 Barrier Tunneling

Classically, if a particle didn't have enough kinetic energy, it would just bounce off the wall but in the realm of Quantum Mechanics, there is a finite probability that the particle will tunnel through the barrier and emerge on the other side. We expect to see a few things. The wave function's amplitide will be decreased from $x > b$ and to decay exponentially from $a < x < b$. We see that choice **(C)** has these characteristics.

**Answer: (C)**

## 12.19 Distance of Closest Appraoch

This question throws a lot of words at you. The $\alpha$-particle with kietic energy 5 MeV is shot towards an atom. If it goes towards the atom it will slow down, loosing kinetic energy and gaining electrical potential energy. The $\alpha$-particle will then be repelled by the Ag atom. Thus

$$U = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{D} = KE \tag{12.19.1}$$

Where $D$ is the distance of closest approach, $q_1 = ze$ and $q_2 = Ze$. We are given $z = 2$ for the alpha particle and $Z = 50$ for the metal atom. Plugging in all of this gives us

$$\begin{aligned} D &= \frac{1}{4\pi\epsilon_0}\frac{(2e)(50e)}{5\times 10^6 e} \\ &= \frac{1}{4\pi\epsilon_0}\frac{100e}{5\times 10^6} \\ &= \frac{1}{4\pi\times 8.85\times 10^{-12}}\frac{100\times 1.6\times 10^{-19}}{5\times 10^6} \\ &\approx 0.3\times 10^{-13}\text{ m} \end{aligned} \tag{12.19.2}$$

**Answer: (B)**

## 12.20 Collisions and the He atom

As the collision is elastic, we know that both momentum and kinetic energy is conserved. So conservation of momentum shows

$$\begin{aligned} 4uv &= (-0.6)(4u)v + MV \\ \Rightarrow 6.4uv &= MV \end{aligned} \tag{12.20.1}$$

Conservation of Energy shows that

$$\begin{aligned} \frac{1}{2}(4u)v^2 &= \frac{1}{2}(4u)(0.6v)^2 + \frac{1}{2}MV^2 \\ 4u\left[0.64v^2\right] &= MV^2 \end{aligned} \tag{12.20.2}$$

Solving for $M$

$$M = \frac{6.4^2 u}{4(0.64)} = 16u \tag{12.20.3}$$

We see that this corresponds to an Oxygen atom, mass 16u.

**Answer: (D)**

## 12.21 Oscillating Hoops

We are given the period of our physical pendulum, where

$$T = 2\pi\sqrt{\frac{I}{mgd}} \tag{12.21.1}$$

where $I$ is the moment of inertia and $d$ is the distance of the pivot from the center of mass. The moment of inertia of our hoop is

$$I_{cm} = Mr^2 \tag{12.21.2}$$

The moment of inertia of the hoop hanging by a nail is found from the Parallex Axis Theorem

$$I = I_{cm} + Md^2 = Mr^2 + Mr^2 = 2Mr^2 \tag{12.21.3}$$

Plugging this into the first equation gives

$$\begin{aligned} T &= 2\pi\sqrt{\frac{2Mr^2}{Mgr}} \\ &= 2\pi\sqrt{\frac{2r}{g}} \\ &\approx 2\pi\sqrt{\frac{2 \times 20 \times 10^{-2}}{10}} \\ &= 4\pi \times 10^{-1} \approx 1.2 \text{ s} \end{aligned}$$

**Answer: (C)**

## 12.22 Mars Surface Orbit

If a body travels forward quickly enough that it follows the planet's curvature it is in orbit. We are told that in the case of Mars, there is a 2.0 meter drop for every 3600 meter horizontal distance. We are also told that the acceleration due to gravity on Mars is $g_M = 0.4g$. So the time to drop a distance of 2.0 meters is

$$\begin{aligned} s &= \frac{1}{2}g_M t^2 \\ \Rightarrow t &= 1 \text{ second} \end{aligned} \tag{12.22.1}$$

So the horizontal speed is

$$v_x = \frac{3600}{1} \text{ meters/second} \tag{12.22.2}$$

**Answer: (C)**

## 12.23 The Inverse Square Law

**Choice A** Energy will be conserved. This isn't dependent on an inverse square law.

**Choice B** Momentum is conserved. This also isn't dependent on the inverse square law.

**Choice C** This follows from Kepler's Law

$$mr\left(\frac{2\pi}{T}\right)^2 = \frac{Gm_1m_2}{r^{2+\epsilon}}$$
$$\Rightarrow T \propto r^{(3+\epsilon)/2} \tag{12.23.1}$$

**Choice D** This is FALSE. This follows from Bertrand's Theorem[5], which states that only two types of potentials produce stable closed orbits

1. An inverse square central force such as the gravitational or electrostatic potential.
$$V(r) = \frac{-k}{r} \tag{12.23.2}$$
2. The radial Harmonic Oscillator Potential
$$V(r) = \frac{1}{2}kr^2 \tag{12.23.3}$$

**Choice E** A stationary circular orbit occurs under special conditions when the central force is equal to the centripetal force. This is not dependent on an inverse square law but its speed.

**Answer: (D)**

## 12.24 Charge Distribution

An inportant thing to keep in mind is that charge will distribute itself evenly throughout the conducting spheres and that charge is conserved.

**Step I: Uncharged Sphere C touches Sphere A**

| $A$ | $B$ | $C$ |
|---|---|---|
| $\frac{Q}{2}$ | $Q$ | $\frac{Q}{2}$ |

**Step II: Sphere C is touched to Sphere B**

| $A$ | $B$ | $C$ |
|---|---|---|
| $\frac{Q}{2}$ | $\frac{3Q}{4}$ | $\frac{3Q}{4}$ |

The initial force between $A$ and $B$ is
$$F = \frac{kQ^2}{r^2} \tag{12.24.1}$$

The final force between $A$ and $B$ is
$$F_f = \frac{k\frac{Q}{2}\frac{3Q}{4}}{r^2} = \frac{3}{8}F \tag{12.24.2}$$

**Answer: (D)**

[5] Add reference here

## 12.25 Capacitors in Parallel

We have one capacitor, $C_1$ connected to a battery. This capacitor gets charged and stores a charge, $Q_0$ and energy, $U_0$.

$$Q_0 = C_1 V \tag{12.25.1}$$

$$U_0 = \frac{1}{2} C_1 V^2 \tag{12.25.2}$$

When the switch is toggled in the on position, the battery charges the second capacitor, $C_2$. As the capacitors are in parallel, the potential across them is the same. As $C_1 = C_2$, we see that the charges and the energy stored across each capacitor is the same. Thus

$$Q_1 = C_1 V_1 \qquad Q_2 = C_2 V_2$$

$$Q_2 = Q_1$$

$$U_1 = \frac{1}{2} C_1 V_1^2 \qquad U_2 = \frac{1}{2} C_2 V_2^2$$

$$U_2 = U_1$$

We see that

$$U_1 + U_2 = C_1 V_1^2 = 2U_0 \tag{12.25.3}$$

We can also analyze this another way. The two capacitors are in parallel, so their net capacitance is

$$C_T = C_1 + C_2 = 2C_1 \tag{12.25.4}$$

So the total charge and energy stored by this parallel arrangemt is

$$Q = C_T V_1 = 2C_1 V_1$$

$$U_T = \frac{1}{2} 2C_1 V_1^2 = 2U_0$$

Of all the choices, only **(E)** is incorrect.

**Answer: (E)**

## 12.26 Resonant frequency of a RLC Circuit

The circuit will be best 'tuned' when it is at its resonant frequence. This occurs when the impedances for the capacitor and inductor are equal. Thus

$$X_C = \frac{1}{\omega C} \quad \text{and} \quad X_L = \omega L \tag{12.26.1}$$

When they are equal

$$X_C = X_L$$

$$\frac{1}{\omega C} = \quad X_L = \omega L$$

$$\therefore \omega = \frac{1}{\sqrt{LC}} \tag{12.26.2}$$

Solving for $C$,

$$
\begin{aligned}
C &= \frac{\omega^2 L}{} \\
&= \frac{1}{4}\pi^2 \times (103.7 \times 10^6)^2 \times 2.0 \times 10^{-6} \\
&\approx 0.125 \times 10^{-11}\ \mathrm{F}
\end{aligned} \tag{12.26.3}
$$

**Answer: (C)**

## 12.27 Graphs and Data Analysis

It is best to analyse data is they are plotted on straight line graphs of the form, $y = mx+c$. This way we can best tell how well our data fits, etc.[6]

**A** We want a plot of activity, $\frac{dN}{dt}$ vs. time, $t$. If we were to plot this as is, we would get an exponential curve. To get the straight line graph best suited for further analysis, we take the logs on both sides.

$$
\begin{aligned}
\frac{dN}{dt} &\propto e^{-2t} \\
\log\left[\frac{dN}{dt}\right] &= \log e^{-2t} \\
\log\left[\frac{dN}{dt}\right] &= -2t
\end{aligned} \tag{12.27.1}
$$

We have a Semilog graph with a plot of $\log\left[\frac{dN}{dt}\right]$ on the y-axis, $t$ on the x-axis with a gradient of 2.

**B** This is already a linear equation we can plot with the data we already have. No need to manipulate it in any way.

**C** We take logs on bot sides of the equation to get

$$
\begin{aligned}
s &\propto t^2 \\
\log s &= 2\log t
\end{aligned} \tag{12.27.2}
$$

We can plot $\log s$ vs. $\log t$. This gives a linear equation with $\log s$ on the y-axis and $\log t$ on the x-axis and a gradient of 2.

**D** Again, we take logs on both sides of the equation

$$
\begin{aligned}
\frac{V_{\text{out}}}{V_{\text{in}}} &\propto \frac{1}{\omega} \\
\log\left[\frac{V_{\text{out}}}{V_{\text{in}}}\right] &= -\log\omega
\end{aligned}
$$

[6]This is of course with nothing but a sheet of graph paper and calculator and without the help of computers and data analysis software.

We have a log-log plot of $\log\left[\frac{V_{\text{out}}}{V_{\text{in}}}\right]$ on the y-axis and $\log\omega$ on the x-axis with a gradient of -1. We see that this choice is **INCORRECT**.

**E** As with the other choices, we take logs on both sides and get

$$P \propto T^4$$
$$\log P = 4\log T$$

This can be plotted on a log-log graph with $\log P$ on the y-axis and $\log T$ on the x-axis and a gradient of 4.

**Answer: (D)**

## 12.28 Superposition of Waves

As the question states, we can see the superposition of the two waves. For the higher frequency wave, we see that the period on the oscilloscope is about 1cm. This works out to be a period of

$$T = \frac{1\text{ cm}}{0.5\text{ cm/ms}} = 2.0\text{ ms} \tag{12.28.1}$$

The frequency is

$$\begin{aligned} f &= \frac{1}{2.0\times 10^{-3}} \\ &= 500\text{ Hz} \end{aligned} \tag{12.28.2}$$

We can measure the amplitude of this oscillation by measuring the distance from crest to trough. This is approximately $(2-1)/2$, thus[7]

$$\begin{aligned} A &= 1.\text{ cm}\times 2.0\text{ V/cm} \\ &\approx 2.0\text{ V} \end{aligned} \tag{12.28.3}$$

For the longer period wave, we notice that approximately a half-wavelngth is displayed, is $2(4.5-1.5) = 6$ cm. The period becomes

$$\begin{aligned} T &= \frac{6.0\text{ cm}}{0.5\text{ cm/ms}} \\ &= 12.0\text{ ms} \end{aligned} \tag{12.28.4}$$

Thus the frequency is

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{12.0\times 10^{-3}} = 83\text{ Hz} \end{aligned} \tag{12.28.5}$$

We see that **(D)** matches our calculations.

**Answer: (D)**

[7] If you happened to have worked this one first you'll notice that only choice **(D)** is valid. You can stop and go on to the next question.

## 12.29 The Plank Length

This question is best analysed through dimensional analysis; unless of course you're fortunate to know the formula for the Plank Length. We are told that

$$G = 6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$
$$\hbar = \frac{6.63 \times 10^{-34}}{2\pi}\text{Js}$$
$$c = 3.0 \times 10^8 \text{ m.s}^{-1}$$

We can substitute the symbols for Length, $L$, Mass, $M$ and Time, $T$. So the dimensions of our constants become

$$G = L^3M^{-1}T^{-2}$$
$$\hbar = ML^2T^{-1}$$
$$c = LT^{-1}$$
$$\ell_p = L$$

Our Plank Length is in the form

$$\ell_p = G^x\hbar^y c^z$$

Dimensional Analysis Gives

$$L = \left(L^3M^{-1}T^{-2}\right)^x \left(ML^2T^{-1}\right)^y \left(LT^{-1}\right)^z$$

We get

**L**

$$3x + 2y + z = 1$$

**M**

$$-x + y = 0$$

**T**

$$z = -3x$$

Solving, we get

$$x = \frac{1}{2} \quad y = \frac{1}{2} \quad z = -\frac{3}{2}$$

Thus

$$\ell_p = \sqrt{\frac{G\hbar}{c^3}}$$

**Answer: (E)**

## 12.30 The Open Ended U-tube

We recall that the pressure throughout a fluid is equal throughout the fluid. As the system is in equlibrium, the pressure on the left arm is equal to the pressure on the right arm. We can set up an equation such that

$$\rho_2 g 5 + \rho_1 g \left(h_1 - 5\right) = \rho_1 g h_2 \tag{12.30.1}$$

where whater, $\rho_1 = 1.0\ \text{g/cm}^3$, some immiscible liquid, $\rho_2 = 4.0\ \text{g/cm}^3$.

Solving, gives us

$$h_2 - h_1 = 15\ \text{cm} \tag{12.30.2}$$

Let' call the height of the water column on the left side of the tube, $x_1$. We get

$$\begin{aligned} h_2 - (x_1 + 5) &= 15 \\ \therefore h_2 - x_1 &= 20 \end{aligned}$$

We expect the water column to go down on the left side of the tube as it goes up on the right side of the tube; conservation of mass. So we infer the change in height on both sides is 10 cm. We conclude that since the intial height is 20cm, then $h_2 = 30$ cm and $x_1 = 10$ cm. So

$$\frac{h_2}{h_1} = \frac{30}{15} = 2 \tag{12.30.3}$$

**Answer: (C)**

## 12.31 Sphere falling through a viscous liquid

Our sphere falls through a viscous liquid under gravity and experiences a drag force, $bv$. The equation of motion can be expressed

$$ma = mg - bv \tag{12.31.1}$$

We are also told that the buoyant force is negligible. Armed with this information, we can analyze out choices and emiminate.

**A** This statement will be incorrect. We have been told to ignore the buoyant force, which if was present, would act as a constant retarding force and slow our sphere down and reduce its kinetic energy. **INCORRECT**

**B** This is also incorrect. In fact if you were to solve the above equation of motion, the speed, and hence kinetic energy, would monotonically increase and approach some terminal speed. It won't go to zero. **INCORRECT**

**C** It may do this if it was shot out of a gun, but we were told that it is released from rest. So it will not go past its terminal speed.

**D** The terminal speed is the point when the force due to gravity is balanced by the retarding force of the fluid. Setting $ma = 0$ in the above equation, we get

$$0 = mg - bv \tag{12.31.2}$$

Solving for $v$ yields,

$$v = \frac{mg}{b} \tag{12.31.3}$$

We see that our terminal velocity is dependent on both $b$ and $m$. This choice is **INCORRECT**

**E** From the above analysis, we choose this answer. **CORRECT**

**Answer: (E)**

## 12.32 Moment of Inertia and Angular Velocity

The moment of inertia of an object is

$$I = \sum_{i=1}^{N} m_i r_i^2$$

where $r_i$ is the distance from the point mass to the axis of rotation.

The moment of inertia about point $A$ is found by finding the distances of each of the three masses from that point. The distance between the mass, $m$ and $A$ is

$$r = \frac{\ell}{\sqrt{3}}$$

Thus the moment of inertia is

$$I_A = 3m\left(\frac{\ell}{\sqrt{3}}\right)^2 = m\ell^2$$

The Moment of Inertia about $B$ can be found by the Parallel Axis Theorem but it may be simpler to use the formula above. As the axis of rotation is about $B$, we can ignore this mass and find the distances of the other two masses from this point, which happens to be $\ell$. Thus

$$I_B = 2m\ell^2$$

The rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2$$

So the ratio of the kinetic energies at fixed, $\omega$ becomes

$$\frac{K_B}{K_A} = \frac{I_B}{I_A} = \frac{2m\ell^2}{m\ell^2} = 2$$

**Answer: (B)**

## 12.33 Quantum Angular Momentum

The probability is

$$P = \frac{3^2 + 2^2}{38} = \frac{13}{38} \tag{12.33.1}$$

**NOT FINISHED**

**Answer: (C)**

## 12.34 Invariance Violations

**Not FINISHED**

**Answer: (D)**

## 12.35 Wave function of Identical Fermions

The behavior of fermions are described by the Pauli Exclusion Principle, which states that no two fermions may have the same quantum state. This is a results in the anti-symmetry in the wave funtion.

**Answer: (A)**

## 12.36 Relativistic Collisions

We are told that no energy is radiated away, so it is conserved; all of it goes into the composite mass. The relativistic energy is

$$E = \gamma mc^2 \tag{12.36.1}$$

Given that $v = 3/5c$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{5}{4} \tag{12.36.2}$$

So the energy of the lump of clay is

$$E = \gamma mc^2 = \frac{5}{4}mc^2 \tag{12.36.3}$$

The composite mass can be found by adding the energies of the two lumps of clay

$$\begin{aligned} E_T &= 2E \\ Mc^2 &= \frac{10}{4}mc^2 \\ \therefore M &= 2.5m = 2.5 \times 4 = 10 \text{ kg} \end{aligned} \tag{12.36.4}$$

**Answer: (D)**

## 12.37 Relativistic Addition of Velocities

We recall that the relativistic addition formula

$$v' = \frac{u+v}{1+\frac{uv}{c^2}} \tag{12.37.1}$$

where $u = 0.3c$ and $v = 0.6c$. This becomes

$$v' = \frac{0.9c}{1+0.18} = \frac{0.9}{1.18}c \approx \frac{9}{12}c = 0.75c \tag{12.37.2}$$

**Answer: (D)**

## 12.38 Relativistic Energy and Momentum

The Relativistic Momentum and Energy equations are

$$p = \gamma m v \quad E = \gamma m c^2 \tag{12.38.1}$$

We can determine the speed by dividing the relativistic momentum by the relativistic energy equation to get

$$\begin{aligned}
\frac{p}{E} &= \frac{\gamma m v}{\gamma m c^2} \\
&= \frac{v}{c^2} \\
\therefore \frac{5\text{MeV/c}}{10\text{MeV}} &= \frac{v}{c^2} \\
\frac{5}{10c} &= \frac{v}{c^2} \\
\Rightarrow v &= \frac{1}{2}c
\end{aligned} \tag{12.38.2}$$

**Answer: (D)**

## 12.39 Ionization Potential

The Ionization Potential, or Ionization Energy, $E_I$, is the energy required to remove one mole of electrons from one mole of gaaseous atoms or ions. It is an indicator of the reactivity of an element.

$^2_4He$ The Helium atom is a noble gas and has filled outermost electron shells as well as its electrons being close to the nucleus. It would be very difficult to ionize.

$$He = 1s^2$$

$^7_{14}N$ Nitrogen has two outermost electrons.

$$N = 1s^2, 2s^2, 2p^6, 2s^2, 2p^2$$

$^{8}_{16}O$ Oxygen has four outermost electrons.

$$O = 1s^2, 2s^2, 2p^6, 2s^2, 2p^4$$

$^{18}_{40}Ar$ Another noble gas, this has filled outermost electrons and is not reactive.

$^{55}_{133}Cs$ We can see that Cs has a high atomic number and hence a lot of electrons. We expect the outmost electrons to be far from the nucleus and hence the attraction to be low. This will have a low ionization potential.

**Answer: (E)**

## 12.40 Photon Emission and a Singly Ionized He atom

The energy levels can be predicted by Bohr's model of the Hydrogen atom. As a Helium atom is more massive than Hydrogen, some corrections must be made to our model and equation. The changes can be written

$$\boxed{E_n = -\frac{Z^2}{n^2}\frac{\mu}{m_e}E_0} \tag{12.40.1}$$

where $Z$ is the atomic number, $n$ is the energy level, $E_0$ is the ground state energy level of the Hydrogen atom and $\mu/m_e$ is the reduced mas correction factor.

The emitted photon can also be found through a similar correction

$$\boxed{\Delta E = \frac{hc}{\lambda e} = Z^2\frac{\mu}{m_e}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]13.6} \tag{12.40.2}$$

As Helium's mass is concentrated in the center, it's reduced mass is close to unity[8].

$$\frac{\mu}{m_e} = \frac{Z}{Z + m_e} \approx 1 \tag{12.40.3}$$

Plugging in the values we know into eq. (12.40.2), we get

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{470 \times 10^{-9} \times 1.60 \times 10^{-19}} = 2^2 13.6\left[\frac{1}{n_f^2} - \frac{1}{4^2}\right] \tag{12.40.4}$$

After some fudging and estimation we get

$$\begin{aligned}\frac{6.63 \times 3}{470 \times 1.6} \times 10^2 &\approx \frac{19}{470 \times 1.6} \times 10^2 \\ &\approx \frac{20}{750} \times 10^2 \\ &= 0.026 \times 10^2 \text{ eV}\end{aligned} \tag{12.40.5}$$

[8]It is helpful to know that in the case of atoms, the reduced mass will be close to unity and can be ignored from calculation. In the case of smaller bodies, e.g. positronium, this correction factor can not be ignored.

and

$$\frac{2.6}{2^2 \cdot 13.6} \approx \frac{1}{20}$$

Solving for $n_f$, gives[9]

$$\begin{aligned}
\frac{1}{20} &= \frac{1}{n_f^2} - \frac{1}{4^2} \\
\frac{1}{n_f^2} &= \frac{1}{20} + \frac{1}{16} = \frac{9}{80} \\
&\approx \frac{1}{9} \\
\therefore n_f &= 3 \qquad (12.40.6)
\end{aligned}$$

Now we can calculate the energy level at $n = 3$ from eq. (12.40.1), which gives,

$$\begin{aligned}
E_3 &= -\frac{2^2}{3^2} \cdot 13.6 \\
&= -6.0 \text{ eV} \qquad (12.40.7)
\end{aligned}$$

We get $E_f = -6.0$ eV and $n_f = 3$. This corresponds to **(A)**.

**Answer: (A)**

## 12.41 Selection Rules

**NOT FINISHED**

**Answer: A**

## 12.42 Photoelectric Effect

This question deals with the photoelectric effect which is essentially an energy conservation equation. Energy of a photon strikes a metal plate and raises the electrons to where they can leave the surface. Any extra energy is then put into the kinetic energy of the electron. The photoelectric equation is

$$hf = eV_s + K \qquad (12.42.1)$$

As our choices are in electron-volts, our equation becomes

$$\frac{hc}{e\lambda} = eV_s + K \qquad (12.42.2)$$

where $K$ is the kinetic energy of our photoelectrons. Plugging in the values we were given and solving for $K$, we get

$$K = 0.2 \text{ eV} \qquad (12.42.3)$$

**Answer: (B)**

[9] As $n_f = 3$ is only in choice **(A)**, we can forego any further calculation and choose this one.

## 12.43 Stoke's Theorem

**NOT FINSIHED**

**Answer: (C)**

## 12.44 1-D Motion

A particle moves with the velocity

$$v(x) = \beta x^{-n} \tag{12.44.1}$$

To find the acceleration, $a(x)$, we use the chain rule

$$\begin{aligned} a(x) &= \frac{dv}{dx} = \frac{dx}{dt} \cdot \frac{dv}{dx} \\ &= v \cdot \frac{dv}{dx} \end{aligned} \tag{12.44.2}$$

Differentiating $v(x)$ with respect to $x$ gives

$$\frac{dv}{dx} = -n\beta x^{-n-1}$$

Thus, our acceleration, $a(x)$, becomes

$$\begin{aligned} a(x) &= \beta x^{-n} \cdot -n\beta x^{-n-1} \\ &= -n\beta^2 x^{-2n-1} \end{aligned} \tag{12.44.3}$$

**Answer: (A)**

## 12.45 High Pass Filter

Capacitors and Inductors are active components; their impedances vary with the frequency of voltage unlike an ohmic resistor whose resitance is pretty much the same no matter what. The impedances for capacitors and inductors are

$$X_C = \frac{1}{\omega C} \quad X_L = \omega L$$

We see that in the case of capacitors, there is an inverse relationship with frequency and a linear one for inductors. Simply put, at high frequencies capacitors have low impedances and inductors have high inductances.

**NOT FINSIHED**

**Answer: (E)**

## 12.46 Generators and Faraday's Law

The induced EMF in the loop follows Faraday's Law

$$\mathscr{E} = -\frac{d\Phi}{dt}$$

In this case, the magnetic field, $B$, is constant and the Cross Sectional Area, $A$, through which the magnetic field acts changes. Thus the above equation becomes

$$\mathscr{E}_0 \sin \omega t = -B\frac{dA}{dt} \tag{12.46.1}$$

Let's say that at $t = 0$ the loop is face on with the magnetic field,

$$A = \pi R^2 \cos \omega t \tag{12.46.2}$$

Substituting this into eq. (12.46.1) gives

$$\begin{aligned}\mathscr{E}_0 \sin \omega t &= -B \cdot \frac{dA}{dt} \\ &= -B \cdot \left(-\omega \pi R^2 \sin \omega t\right) \\ &= \omega B \pi R^2 \sin \omega t \end{aligned} \tag{12.46.3}$$

Solving for $\omega$ gives

$$\omega = \frac{\mathscr{E}_0}{B\pi R^2} \tag{12.46.4}$$

**Answer: (C)**

## 12.47 Faraday's Law and a Wire wound about a Rotating Cylinder

The induced EMF of our system can be found from Faraday's Law, where

$$\mathscr{E} = -\frac{d\Phi}{dt} \tag{12.47.1}$$

Here the flux changes because the number of loops enclosing the field increases, so

$$\Phi = NBA \tag{12.47.2}$$

Substituting this into Faraday's Equation we get

$$\begin{aligned}\mathscr{E} &= BA\frac{dN}{dt} \\ &= B\pi R^2 N\end{aligned} \tag{12.47.3}$$

**Answer: (C)**

## 12.48 Speed of $\pi^+$ mesons in a laboratory

As the $\pi^+$ meson travels through our laboratory and past the detectors, its half life is time dilated in our laboratory's rest frame. We can also look at things in the $\pi^+$ meson's rest frame. In this case, the distance it travels will be length contracted in it rest frame. The speed of our $\pi^+$ mesons is the length divided by the time dilation in the laboratory's rest frame or the length contraction in the $\pi^+$ meson's rest frame divided by its half life. In either case, we get

$$v = \frac{L}{T_{1/2}}\sqrt{1 - \frac{v^2}{c^2}}$$

Factorizing we get

$$v^2\left[1 + \frac{L^2}{c^2 T_{1/2}^2}\right] = \frac{L^2}{T_{1/2}^2} \tag{12.48.1}$$

We see that

$$\frac{L}{T_{1/2}} = 6 \times 10^8$$

Plugging this into eq. (12.48.1), the speed in terms of $c$

$$\frac{v^2}{c^2}\left[1 + \frac{36}{9}\right] = \frac{36}{9}$$

$$\frac{v^2}{c^2}(5) = 4$$

$$\Rightarrow v = \frac{2}{\sqrt{5}}c \tag{12.48.2}$$

**Answer: (C)**

## 12.49 Transformation of Electric Field

**NOT FINSHED**

**Answer: (C)**

## 12.50 The Space-Time Interval

We have two events, in the $S$-frame,

$$S_1(x_1, t) \quad \text{and} \quad S_2(x_2, t)$$

In the $S'$-frame, the co-ordintes are

$$S'_1(x'_1, t'_1) \quad \text{and} \quad S'_2(x'_2, t'_2)$$

The Space-Time Interval in the $S$-frame

$$\Delta S = \Delta x^2 = 3c \text{ minutes} \tag{12.50.1}$$

In the $S'$-frame, the Space-Time Interval is

$$\Delta S' = \Delta x'^2 - c^2 \Delta t'^2 = 5c \text{ minutes} \tag{12.50.2}$$

The Space-Time Interval is invariant across frames, so eq. (12.50.1) is equal to eq. (12.50.2)

$$\begin{aligned} (3c)^2 &= (5c)^2 - c^2 \Delta t^2 \\ \Rightarrow \Delta t &= 4 \text{ minutes} \end{aligned} \tag{12.50.3}$$

**Answer: (C)**

## 12.51 Wavefunction of the Particle in an Infinte Well

The wave function has zero probability density in the middle for even wave functions, $n = 2, 4, 6, \cdots$.

**Answer: (B)**

## 12.52 Spherical Harmonics of the Wave Function

**NOT FINSIHED**

**Answer: (C)**

## 12.53 Decay of the Positronium Atom

**NOT FINSHED**

**Answer: (C)**

## 12.54 Polarized Electromagnetic Waves I

We are given an electromagnetic wave that is the superposition of two independent orthogonal plane waves where

$$\mathbf{E} = \hat{\mathbf{x}} E_1 \exp\left[i\left(kz - \omega t\right)\right] + \hat{\mathbf{y}} E_2 \exp\left[i\left(kz - \omega t + \pi\right)\right] \tag{12.54.1}$$

As we are looking at the real components and $E_1 = E_2$, we have

$$\begin{aligned} \mathbf{E} &= \Re(E_1 e^{ikz} \cdot e^{-i\omega t})\hat{\mathbf{x}} + \Re(E_1 e^{ikz} \cdot e^{-i\omega t} \cdot e^{-i\pi})\hat{\mathbf{y}} \\ &= \Re(E_1 e^{ikz} \cdot e^{-i\omega t})\hat{\mathbf{x}} - \Re(E_1 e^{ikz} \cdot e^{-i\omega t})\hat{\mathbf{y}} \end{aligned}$$

We see that the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ vectors have the same magnitude but opposite sign; they are both out of phase with each other. This would describe a trajectory that is 135°to the $x$-axis.

**Answer: (B)**

## 12.55 Polarized Electromagnetic Waves II

**NOT FINISHED**

**Answer: (A)**

## 12.56 Total Internal Reflection

Total internal reflectance will occur when the incident beam, reaches a critical angle, $\theta_i$, such that the refracted angle just skims along the water's surface, $\theta_r = 90°$. We can find this critical angle using Snell's Law

$$n_2 \sin\theta_i = n_1 \sin 90$$

where $n_2 = 1.33$ and $n_1 = 1$, we have

$$\sin\theta_2 = \frac{1}{1.33} = \frac{3}{4} \tag{12.56.1}$$

We know that $\sin 30° = 1/2$ and $\sin 60° = \sqrt{3}/2$, so $30° < \theta < 60°$.

**Answer: (C)**

## 12.57 Single Slit Diffraction

For a single slit, diffraction maxima can be found from the formula

$$a\sin\theta = m\lambda \tag{12.57.1}$$

where $a$ is the slit width, $\theta$ is the angle between the minimum and the central maximum, and $m$ is the diffraction order. As $\theta$ is small and solving doe $d$, we can approximate the above equation to

$$\begin{aligned} d &= \frac{\lambda}{\theta} \\ &= \frac{400 \times 10^{-9}}{4 \times 10^{-3}} \\ &= 0.1 \times 10^{-3} \text{ m} \end{aligned} \tag{12.57.2}$$

**Answer: (C)**

## 12.58 The Optical Telescope

The magnification of the optical telescope can be found from the focal length of the eyepiece, $f_e$ and the objective, $f_o$. Thus

$$M = \frac{f_o}{f_e} = 10 \tag{12.58.1}$$

The focal length of the objective is

$$f_e = 10 \times 1.5 \text{ cm} = 15 \text{ cm} \tag{12.58.2}$$

To achieve this magnification, the lens must be placed in a position where the focal length of the eyepiece meets the focal length of the objective. Thus

$$D = f_e + f_o = 15.0 + 1.5 = 16.5 \text{ cm} \tag{12.58.3}$$

**Answer: (E)**

## 12.59 Pulsed Lasers

Lasers may operate in a pulsed mode to deliver more energy in a short space of time rather than deliver the same energy over a longer period of time in a continuous mode. While there are several methods to achieve a pulsed mode, beyond what is needed to answer this question, we can determine the number of photons delivered by such a device. The energy of a photon is

$$E = hf = \frac{hc}{\lambda} \tag{12.59.1}$$

The power is the energy delivered in one second. So for a 10kW laser, the total energy in $10^{-15}$ seconds is

$$\begin{aligned} E_L &= Pt = 10 \times 10^3 \times 10^{-15} \\ &= 10 \times 10^{-12} \text{ J} \end{aligned} \tag{12.59.2}$$

So the total number of photons is

$$\begin{aligned} n &= \frac{E_L}{E} \\ &= \frac{10 \times 10^{-12} \times \lambda}{hc} \\ &= \frac{10 \times 10^{-12} \times 600 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8} \\ &= \frac{10 \times 600}{6.63 \times 3} \times 10^6 \approx 3 \times 10^8 \end{aligned} \tag{12.59.3}$$

**Answer: (B)**

## 12.60 Relativistic Doppler Shift

**NOT FINSIHED**

**Answer: (B)**

## 12.61 Gauss' Law, the Electric Field and Uneven Charge Distribution

We can find the electric field in a non-conducting sphere by using Gauss' Law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \tag{12.61.1}$$

The enclosed charge can be found from the charge density, which is

$$\begin{aligned}\rho &= \frac{\text{Enclosed Charge}}{\text{Enclosed Volume}} \\ &= \frac{q}{\frac{4}{3}\pi r^3}\end{aligned} \tag{12.61.2}$$

We can find the enclosed charge by integrating within 0 to $R/2$. The charge density is

$$\begin{aligned}\rho &= \frac{dq}{dV} \\ &= \frac{dq}{4\pi r^2 dr}\end{aligned} \tag{12.61.3}$$

$$\begin{aligned}\therefore dq &= \rho 4\pi r^2 dr \\ &= 4\pi A r^4 dr\end{aligned} \tag{12.61.4}$$

Gauss' Law becomes

$$\begin{aligned}\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{q_{\text{enclosed}}}{\epsilon_0} \\ E\left(4\pi r^2\right) &= \int_0^{\frac{R}{2}} \frac{dq}{\epsilon_0} \\ &= \frac{4\pi A}{\epsilon_0} \int_0^{\frac{R}{2}} r^4 dr \\ &= \frac{4\pi A}{\epsilon_0} \left.\frac{r^5}{5}\right|_0^{\frac{R}{2}}\end{aligned} \tag{12.61.5}$$

$$\therefore E = \frac{A}{\epsilon_0} \left.\frac{r^3}{5}\right|_0^{\frac{R}{2}} \tag{12.61.6}$$

Solving gives

$$E = \frac{AR^3}{40\epsilon_0} \tag{12.61.7}$$

**Answer: (B)**

## 12.62 Capacitors in Parallel

We initially charge both of our capacitors in parallel across a 5.0V battery. The charge stored on each capacitor is

$$Q_1 = C_1 V \quad Q_2 = C_2 V$$

where $V = 5.0$ V, $C_1 = 1\mu$F and $C_2 = 2\mu$F.

The battery is then disconnected and the plates of opposite charges are connected to each other. This results in the excess charges cancelling each other out and then redistributing themselves until the potential across the new configuration is the same. The charge left after this configuration is

$$Q_A = Q_2 - Q_1 \tag{12.62.1}$$

The charges on each capacitor becomes

$$Q_{1_A} + Q_{2_A} = Q_A \tag{12.62.2}$$

where

$$Q_{1_A} = C_1 V_f \quad \text{and} \quad Q_{2_A} = C_2 V_f \tag{12.62.3}$$

Solving for $V_f$ gives us

$$V_f = \frac{(C_2 - C_1)V}{C_1 + C_2} = \frac{5}{3} = 1.67 \text{ V} \tag{12.62.4}$$

**Answer: (C)**

## 12.63 Standard Model

**NOT FINSIHED**

**Answer: (A)**

## 12.64 Nuclear Binding Energy

Typically a heavy nucleus contains $\sim 200$ nucleons. The energy liberated would be the difference in the binding energies 1 MeV $\times 200$.

**Answer: (C)**

## 12.65 Work done by a man jumping off a boat

The work the man does is the sum of the kinetic energies of both the boat and himself. We can find the speeds of the man and the boat because momentum is conserved.

$$mu = Mv$$
$$u = \frac{Mv}{m} \tag{12.65.1}$$

The total energy of the system, and hence the work the man does in jumping off the boat is

$$\begin{aligned} W &= \frac{1}{2}mu^2 + \frac{1}{2}Mv^2 \\ &= \frac{1}{2}Mv^2\left[\frac{M}{m} + 1\right] \end{aligned} \tag{12.65.2}$$

**Answer: (D)**

## 12.66 Orbits and Effective Potential

**NOT FINSHED**
**Answer: (E)**

## 12.67 Schwartzchild Radius

Any mass can become a black hole if it is compressed beyond its Schwarzschild Radius. Beyond this size, light will be unable to escape from it's surface or if a light beam were to be trapped within this radius, it will be unable to escape. The Schwarzschild Radius can be derive by putting the Gravitational Potential Energy equal to a mass of kinetic energy travelling at light speed.

$$\frac{GMm}{R} = \frac{1}{2}mc^2 \tag{12.67.1}$$

Solving for $R$ yields

$$R = \frac{2GM}{c^2} \tag{12.67.2}$$

Plugging in the values given, we get

$$R = \frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^24}{(3 \times 10^8)^2}$$

Our indices indicate we will get an answer in the order $\approx 10^{-3}$ meters.
**Answer: (C)**

## 12.68 Lagrangian of a Bead on a Rod

The Lagrangian of a system is defined

$$L = T - V \tag{12.68.1}$$

The rod can move about the length of the rod, $s$ and in circular motion along a radius of $s\sin\theta$. The Lagrangian of this system becomes

$$L = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(s\sin\theta)^2\omega^2 - mgs\cos\theta \tag{12.68.2}$$

**Answer: (E)**

## 12.69 Ampere's Law

We can use Ampere's Law to tell us the magnetic field at point $A$.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} \tag{12.69.1}$$

The point $A$ is midway between the center of the two cylinders and as the currents are in opposite directions, thier magnetic fields at $A$ point in the $+y$-direction. We can use the right hand grip rule to determine this. This leaves us with choices **(A)** or **(B)**.

The current density, $\mathbf{J}$ is

$$\begin{aligned} J &= \frac{I}{\text{Area}} \\ &= \frac{I}{\pi r^2} \end{aligned} \tag{12.69.2}$$

We draw an Amperian loop of radius, $r = d/2$, thus the magnetic field becomes,

$$\begin{aligned} \oint \mathbf{B} \cdot d\mathbf{s} &= \mu_0 I_{\text{enclosed}} \\ B_{\odot} \cdot (2\pi r) &= \mu_0 J \left(\pi r^2\right) \\ B &= \frac{\mu_0 \pi J r}{2\pi} \\ &= \frac{\mu_0 \pi J}{2\pi} \frac{d}{2} \end{aligned} \tag{12.69.3}$$

We expect $B_{\otimes}$ to be the same, thus

$$\begin{aligned} B &= B_{\odot} + B_{\otimes} \\ &= \left(\frac{\mu_0}{2\pi}\right) \pi d J \end{aligned} \tag{12.69.4}$$

**Answer: (A)**

## 12.70 Larmor Formula

The Larmor Formula is used to calculate total power radiated by an accelerating non-relativistic point charge.

$$\boxed{P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3}} \tag{12.70.1}$$

where $a$ is the acceleration. For particles $A$ & $B$, we are given

| | $A$ | $B$ |
|---|---|---|
| Charge | $q_a = q$ | $q_b = 2q$ |
| Mass | $m_a = m$ | $m_b = \frac{m}{2}$ |
| Velocity | $v_a = v$ | $v_b = 3v$ |
| Acceleration | $a_a = a$ | $a_b = 4a$ |

From the above, we see that

$$P_A \propto q^2a^2 \quad P_B = (2q)^2(4a^2)$$

Thus

$$\begin{aligned}\frac{P_B}{P_A} &= \frac{(2q)^2(4a^2)}{q^2a^2} \\ &= 64 \end{aligned} \tag{12.70.2}$$

**Answer: (D)**

## 12.71 The Oscilloscope and Electron Deflection

As the electron passes through the deflection plates, the electric field, $E$ exerts a force on the charge, pulling it up. Ignoring gravitational effects, the electric force is

$$F_e = qE = \frac{qV}{d} = ma_y \tag{12.71.1}$$

The time it takes to traverse this distance is

$$t = \frac{L}{v} \tag{12.71.2}$$

The deflection angle, $\theta$ is determined by

$$\tan\theta = \frac{v_y}{v_x} \tag{12.71.3}$$

Now

$$\begin{aligned} v_y &= a_y t \\ &= \frac{qV}{m_e d}t \\ &= \frac{qV}{m_e d}\frac{L}{v} \end{aligned} \tag{12.71.4}$$

The the horizontal speed is $v$ The angle of deflection becomes

$$\begin{aligned} \tan\theta &= \frac{\frac{qV}{md}\frac{L}{v}}{v} \\ &= \frac{qVL}{mdv^2} \end{aligned} \tag{12.71.5}$$

**Answer: (A)**

## 12.72 Negative Feedback

All amplifiers exhibit non-linear behavior of some sort. Negative feedback seeks to correct some of these effects by sending some of the output back and subtracting it from the input. This results in a decrease in gain. This tradeoff og gain improves linearity and hence the stability of the amplifier. This also allows for increased bandwidth response and decreased distortion.

**Answer: (A)**

## 12.73 Adiabatic Work of an Ideal Gas

The adiabatic condition states that

$$PV^{\gamma} = C \tag{12.73.1}$$

The work done by an ideal gas is

$$W = \int PdV \tag{12.73.2}$$

Substituting the adiabatic condition into the work equation yields

$$\begin{aligned} W &= C\int_{V_i}^{V_f} \frac{dV}{V^{\gamma}} \\ &= C\left.\frac{V^{-\gamma+1}}{1+\gamma}\right|_{V_i}^{V_f} \\ &= \frac{C}{1-\gamma}\left[V_f^{-\gamma+1} - V_i^{-\gamma+1}\right] \end{aligned} \tag{12.73.3}$$

The adiabatic condition is

$$C = PV^{\gamma} = P_iV_i^{\gamma} = P_fV_f^{\gamma} \tag{12.73.4}$$

Substituting this into eq. (12.73.3), we get

$$\begin{aligned} W &= \frac{CV_f^{-\gamma+1} - CV_i^{-\gamma+1}}{1-\gamma} \\ &= \frac{P_fV_f^{\gamma}V_f^{-\gamma+1} - P_fV_f^{\gamma}V_f^{-\gamma+1}}{1-\gamma} \\ &= \frac{P_fV_f - P_iV_i}{1-\gamma} \end{aligned} \tag{12.73.5}$$

**Answer: (C)**

## 12.74 Change in Entropy of Two Bodies

The change in entropy of a system is

$$dS = \frac{dQ}{T} \tag{12.74.1}$$

The change in heat of a system is

$$dQ = nCdT \tag{12.74.2}$$

Substituting this into the above equation, we get

$$\begin{aligned} dS &= mC \int_{T_i}^{T_f} \frac{dT}{T} \\ &= mC \ln\left[\frac{T_f}{T_i}\right] \end{aligned} \tag{12.74.3}$$

We are told that the two bodies are brought together and they are in thermal isolation. This means that heat is not absorbed or lost to the environment, only transferred between the two bodies. Thus

$$\begin{aligned} \text{Heat Lost by Body A} &= \text{Heat Gained by Body B} \\ mC\left(500 - T_f\right) &= mc\left(T_f - 100\right) \\ \therefore T_f &= 300 \text{ K} \end{aligned}$$

The Total Change in Entropy is the sum of the entropy changes of bodies $A$ and $B$. Thus

$$\begin{aligned} dS &= dS_A + dS_B \\ &= mC \ln\left[\frac{300}{500}\right] + mC \ln\left[\frac{300}{100}\right] \\ &= mC \ln\left[\frac{9}{5}\right] \end{aligned} \tag{12.74.4}$$

**Answer: (B)**

## 12.75 Double Pane Windows

The rate of heat transer is proportional to the temperature difference across the body, is stated in Newton's Law of Cooling.

$$\frac{dQ}{dt} = kxdT \tag{12.75.1}$$

where $k$ is the thermal conductivity, $x$ is the thickness of the material and $dT$ is the temperature difference across the material.

For Window A

$$P_A = \frac{dQ_A}{dt} = 0.8 \times 4 \times 10^{-3} dT \tag{12.75.2}$$

For Window B

$$P_B = \frac{dQ_B}{dt} = 0.025 \times 2 \times 10^{-3} dT \tag{12.75.3}$$

The ratio of heat flow is

$$\frac{P_a}{P_B} = \frac{0.8 \times 4 \times 10^{-3} dT}{0.025 \times 2 \times 10^{-3} dT} = 16 \tag{12.75.4}$$

**Answer: (D)**

## 12.76 Gaussian Wave Packets

We have a Gaussian wave packet travelling through free space. We can best think of this as an infinite sum of a bunch of waves initially travelling together. Based on what we know, we can eliminate the choices given.

**I** The average momentum of the wave packet can not be zero as $\mathbf{p} = \hbar\mathbf{k}$. As we have a whole bunch of wave numbers present, the average can not be zero. **INCORRECT**

**II** Our wave packet contains a bunch of waves travelling together each with a differnt wave vector, $\mathbf{k}$. The speed of propagation of these individual wave vectors is defined by the group velocity, $v_g = d\omega/dk$. So some waves will travel, some slower than others. As a result of these different travelling rates our wave packet becomes spread out or 'dispersed'. This is the basis of our dispersion relation, $\omega(k)$, relative to the center of the wave packet. **CORRECT**

**III** As we expect the wave packet to spread out, as shown above, the amplitude will decrease over time. The energy that was concentrated in this packet gets spread out or dispersed. **INCORRECT**

**IV** This is true. This statement is the Uncertainty Principle and comes from Fourier Analysis. **CORRECT**

We see that choices **II** and **IV** are **CORRECT**.
**Answer: (B)**

## 12.77 Angular Momentum Spin Operators

**NOT FINSIHED**
**Answer: (D)**

## 12.78 Semiconductors and Impurity Atoms

**NOT FINISHED**
**Answer: (B)**

## 12.79 Specific Heat of an Ideal Diatomic Gas

The formula for finding the molar heat capacity at constant volume can be found by

$$c_v = \left(\frac{f}{2}\right) R \tag{12.79.1}$$

where $f$ is the number of degrees of freedom. At very low temperatures, there are only three translational degrees of freedom; there are no rotational degrees of freedom in this case. At very high temperatures, we have three translational degrees of freedom, two rotational and two vibrational, giving a total of seven in all.

| For Low Temperatures | | For High Temperatures | |
|---|---|---|---|
| Translational | 3 | Translational | 3 |
| Rotational | 0 | Rotational | 2 |
| Vibrational | 0 | Vibrational | 2 |
| Total($f$) | 3 | Total($f$) | 7 |

Table 12.79.1: Table of degrees of freedom of a Diatomic atom

We see that in the case of very low temperatures,

$$c_{v_l} = \frac{3}{2} R \tag{12.79.2}$$

and at very high temperatures,

$$c_{v_h} = \frac{7}{2} R \tag{12.79.3}$$

Thus, the ratio of molar heat capacity at constant volume at very high temperatures to that at very low temperatures is

$$\begin{aligned} \frac{c_{v_h}}{c_{v_l}} &= \frac{\frac{7}{2}R}{\frac{3}{2}R} \\ &= \frac{7}{3} \end{aligned} \tag{12.79.4}$$

**Answer: (D)**

## 12.80 Transmission of a Wave

**NOT FINSIHED**

**Answer: (C)**

## 12.81 Piano Tuning & Beats

The $D_2$ note has a frequency of 73.416 Hz and the $A_4$ note has a frequency of 440.000 Hz. Beats are produced when the two frequencies are close to each other. If they were the same there would be no beat frequency. So we can determine the $D_2$ where this happens by setting the beat frequency to zero.

$$440.000 - n(73.416) = 0 \tag{12.81.1}$$

This works out to be

$$\begin{aligned} n &= \frac{440.000}{73.416} \\ &\approx \frac{440}{72} = 6 \end{aligned} \tag{12.81.2}$$

Thus the closest harmonic will be the $6^{\text{th}}$ one. As we expect this to be very close to the $A_4$ frequency, we expect the number of beats to be small; close to zero. Answer (B) fits this.[10]

**Answer: (B)**

## 12.82 Thin Films

As light moves from the glass to air interface, it is partially reflected and partially transmitted. There is no phase change when the light is refected. In the case of the transmitted wave, when it reaches the air-glass interface, there is a change in phase of the reflected beam. Thus the condition for destructive interference is

$$\boxed{2L = \left(n + \frac{1}{2}\right)\lambda} \tag{12.82.1}$$

where $L$ is the thickness of the air film and $n = 0, 1, 2$ is the interference mode. Thus

$$L = \left(\frac{2n+1}{4}\right)\lambda \tag{12.82.2}$$

We get

$$L_0 = \frac{\lambda}{4} = 122 \text{ nm} \tag{12.82.3}$$

$$L_1 = \frac{3\lambda}{4} = 366 \text{ nm} \tag{12.82.4}$$

$$L_2 = \frac{5\lambda}{4} = 610 \text{ nm} \tag{12.82.5}$$

**Answer: (E)**

[10] Incidentally, you can multiply the $D_2$ frequency by six to determine the harmonic. This turns out to be

$$73.416 \times 6 = 440.496 \text{ Hz} \tag{12.81.3}$$

Subtracting this from the $A_2$ frequency gives

$$440.496 - 440.000 = 0.496 \text{ Hz} \tag{12.81.4}$$

## 12.83 Mass movng on rippled surface

**NOT FINSHED**

**Answer: (D)**

## 12.84 Normal Modes and Couples Oscillators

**NOT FINSIHED**

**Answer: (D)**

## 12.85 Waves

**NOT FINSIHED**

**Answer: (B)**

## 12.86 Charged Particles in E&M Fields

**NOT FINSIHED**

**Answer: (B)**

## 12.87 Charged Particles Something

**NOT FINSIHED**

**Answer: (A)**

## 12.88 Coaxial Cable

We expect there to be no magnetic field outside, $r > c$, the coaxial cable. Choices **(D)** and **(E)** make no sense. So choice **(B)** is our best one left.

We can also use Ampere's Law to show what the magnetic induction will look like as we ove away from the center. We recall Ampere's Law

$$\oint \mathbf{B} \cdot d\ell = \mu_0 I_{\text{enclosed}} \tag{12.88.1}$$

$0 < r < a$ The current enclosed by our Amperian Loop is

$$I_{\text{enclosed}} = I \frac{\pi r^2}{\pi a^2} \tag{12.88.2}$$

Ampere's Law shows us

$$B(2\pi r) = \mu_0 I \frac{\pi r^2}{\pi a^2}$$
$$B_{(0<r<a)} = \frac{\mu_0 I}{2\pi} \frac{r}{a^2} \tag{12.88.3}$$

We see the magnetic induction will increase linearly with respect to $r$.

$a < r < b$ Within the shaded region, the enclosed current is, $I_{\text{enclosed}} = I$. Ampere's Law becomes

$$\begin{aligned} B(2\pi r) &= \mu_0 I \\ B_{(a<r<b)} &= \frac{\mu_0}{2\pi}\frac{I}{r} \end{aligned} \tag{12.88.4}$$

The magnetic induction decreases inversely with respect to $r$.

$b < r < c$ The area of the outer sheath is

$$A = \pi\left(c^2 - b^2\right) \tag{12.88.5}$$

The enclosed current will be

$$\begin{aligned} I_{\text{enclosed}} &= I - I\left[\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right] \\ &= I\left[\frac{c^2 - r^2}{c^2 - b^2}\right] \end{aligned} \tag{12.88.6}$$

Ampere's Law becomes

$$\begin{aligned} B(2\pi r) &= \mu_0 I\left[\frac{c^2 - r^2}{c^2 - b^2}\right] \\ B_{(b<r<c)} &= \frac{\mu_0 I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right] \end{aligned} \tag{12.88.7}$$

This will fall to an inversely with respect to $r$.

$r > c$ We see that $I_{\text{enclosed}} = 0$, so from Ampere's Law

$$B_{(r>c)} = 0 \tag{12.88.8}$$

The graph shown in **(B)** fits this[11].

**Answer: (B)**

## 12.89 Charged Particles in E&M Fields

**NOT FINSIHED**

**Answer: (D)**

[11] This is one of the reasons we use coaxial cables to transmit signals. No external magnetic field from our signals means that we can, theoretically, eliminate electromagnetic interference.

## 12.90 THIS ITEM WAS NOT SCORED

## 12.91 The Second Law of Thermodynamics

This question deals with the Second Lar of Thermodynamics, which states that the entropy of an isloated system, which is not in equilibrium, will increase over time, reaching a maximum at equilibrium. An alternative but equivalent statement is the Clausius statement, "Heat can not flow from cold to hot without work input". Thusit would be impossible to attain the 900 K temperature the experimenter needs with a simple lens.

**Answer: (E)**

## 12.92 Small Oscillations

We are given a one dimensional potential function

$$V(x) = -ax^2 + bx^4 \tag{12.92.1}$$

We can find the points of stability by differentiating the above equation to get and setting it to zero.

$$\frac{dV}{dx} = -2ax + 4bx^3 = 0 \tag{12.92.2}$$

we see that $x = 0; a/2b$. Taking the second differential gives us the mass's spring constant, $k$

$$k = \frac{d^2V}{dx^2} = -2a + 12bx^2 \tag{12.92.3}$$

We can also use this to find the minimum and maximum points of inflection in our potential graph. We see that

$$k = \frac{d^2V(0)}{dx^2} = -2a \qquad k = \frac{d^2V(a/2b)}{dx^2} = 4a$$

We see that when $x = a/2b$, we are at a minima and hence at a point of stable equilibrium. We can now find the angular frequency

$$\begin{aligned}\omega &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\frac{4a}{m}} \\ &= 2\sqrt{\frac{a}{m}}\end{aligned} \tag{12.92.4}$$

**Answer: (D)**

## 12.93 Period of Mass in Potential

The total period of our mass will be the time it takes to return to the same point, say the origin, as it moves through the two potentials.

**for** $x < 0$ The potential is

$$V = \frac{1}{2}k^2 \tag{12.93.1}$$

The spring constant, $k$, is

$$k = \frac{d^2V}{dx^2} = k \tag{12.93.2}$$

Thus the period of oscillation would be

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{12.93.3}$$

But this represents the period of the mass could also swing from $x > 0$. As a result, our period would just be half this. So

$$T_{(x<0)} = \pi\sqrt{\frac{m}{k}} \tag{12.93.4}$$

**for** $x > 0$ The potential is

$$V = mgx \tag{12.93.5}$$

We may recognize this as the gravitational potential energy. In this case, the "period" would be the time for the mass to return the origin. This would simply be

$$s = v_0 t - \frac{1}{2}gt^2 \tag{12.93.6}$$

where $s = 0$. Solving for $t$, we get $t = 0$ and $t = 2v_0/g$ We can solve this in terms of the total energy, $E$ of the mass. The energy is $E = 1/2mv_0^2$. Our period works out to be

$$T_{(x>0)} = 2\sqrt{\frac{2E}{mg^2}} \tag{12.93.7}$$

Thus our total period is

$$\begin{aligned} T &= T_{(x<0)} + T_{(x>0)} \\ &= \pi\sqrt{\frac{m}{k}} + 2\sqrt{\frac{2E}{mg^2}} \end{aligned} \tag{12.93.8}$$

**Answer: (D)**

## 12.94 Internal Energy

**NOT FINISHED**

**Answer: (D)**

## 12.95 Specific Heat of a Super Conductor

**NOT FNISHED**

**Answer: (E)**

## 12.96 Pair Production

We are given the equation of pair production

$$\gamma \to e^- + e^+ \tag{12.96.1}$$

Pair production occurs when a $\gamma$ ray of high energy is absorbed in the vincinity of an atomic nucleus and particles are created from the absorbed photon's energy. This takes place in the Coulomb field of the nucleus; the nucleus acts as a massive body to ensure the conservation of momentum and energy. The nucleus is an essential part of this process; if the photon could spontaneously decay into an electron-positron pair in empty space, a Lorentz frame could be found where the electron and positron have equal and opposite momenta and the photon will be at rest. This is a clear violation of the principles of Special Relativity. We choose **(A)**.

Based on what we know, we can determine the maximum wavelength as a matter or interest. As the nucleus is massive, we will ignore its recoil and consider that all of the photon's energy goes into electron-positron creation and the particles' kinetic energy.

$$\begin{aligned} h\nu &= E^- + E^+ \\ &= \left(K^- + m_e c^2\right) + \left(K^+ + m_e c^2\right) \\ &= K^- + K^+ + 2m_e c^2 \end{aligned} \tag{12.96.2}$$

where $K^-$ and $K^+$ are the kinetic energies of the electron and positron respectively. Thus the minimum energy needed to initiate this process is

$$h\nu_{\text{min}} = 2m_e c^2 \tag{12.96.3}$$

The wavelength of this photon is

$$\boxed{\lambda_{\text{max}} = \frac{h}{2m_e c^2}} \tag{12.96.4}$$

which happens to be half the Compton wavelength.

**Answer: (A)**

## 12.97 I don't know

**NOT FINISHED**

**Answer: (E)**

## 12.98 Harmonic Oscillator

**Not FINISHED**
**Answer: (D)**

## 12.99 Metastable States of the LASER

**NOT FINISHED**
**Answer: (B)**

## 12.100 Quantum Oscillator

**NOT FINISHED**
**Answer: (C)**

# Chapter 13

# GR0177 Exam Solutions

## 13.1 Acceleration of a Pendulum Bob

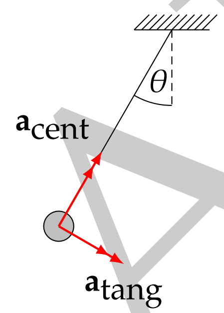



Figure 13.1.1: Acceleration components on pendulum bob

The acceleration of an object that rotates with variable speed has two components, a centripetal acceleration and a tangential acceleration. We can see this in the above diagram, fig. 13.1.1, where

**Centripetal Acceleration**

$$\mathbf{a}_{\text{cent}} = \frac{v^2}{r} = \omega^2 r \tag{13.1.1}$$

**Tangential Acceleration**

$$\mathbf{a}_{\text{tang}} = \alpha r \tag{13.1.2}$$

The net acceleration on the bob can be found by adding the cetripetal and tangential accelerations

$$\mathbf{a}_{\text{tang}} + \mathbf{a}_{\text{cent}} = \mathbf{a} \tag{13.1.3}$$

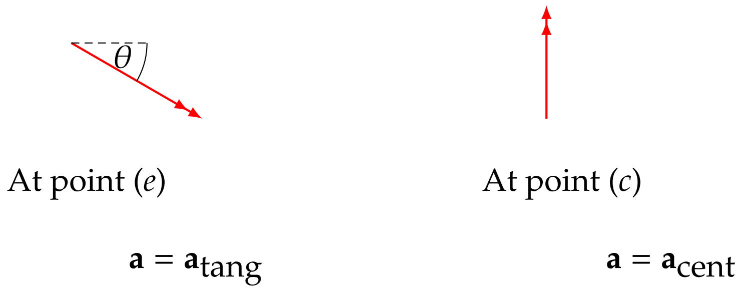



Figure 13.1.2: Acceleration vectors of bob at equilibrium and max. aplitude positions

At point *(e)*, $v = 0$, so

$$\mathbf{a}_{\text{cent}} = 0 \text{ and}$$
$$\mathbf{a}_{\text{tang}} = \mathbf{a}$$

There is only the tangential component to the bob's acceleration.

At point *(c)*, $\alpha = 0$, so

$$\mathbf{a}_{\text{cent}} = a \text{ and}$$
$$\mathbf{a}_{\text{tang}} = 0$$

There is only the cetripetal component to the bob's acceleration.

We see from **(C)**, the acceleration vectors point in the directions we expect.

**Answer: (C)**

## 13.2 Coin on a Turntable

The coin will stay in place as long as the certipetal force and the static friction force are equal. We can see that this is dependent on its position on the turntable, see fig. 13.2.1.

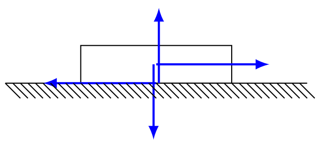


Figure 13.2.1: Free Body Diagram of Coin on Turn-Table

The coin is in equilbrium, we see that

$$mr\omega^2 - \mu R = 0$$
$$\therefore r = \frac{\mu g}{\omega^2} \tag{13.2.1}$$

From here it's just a matter of plugging what we know, $\omega = 33.3$ revolutions/min. For the sake of simplicity, we will assume that this is equal to 100/3 revolutions per minute. Thus

$$\begin{aligned}\omega &= 33.3\text{revolutions/min}\\ &= \frac{100}{3}\cdot\frac{2\pi}{60}\\ &= \frac{10\pi}{9}\,\mathrm{rad\,s^{-1}}\end{aligned}\tag{13.2.2}$$

Pluggin this into eq. (13.2.1), we get

$$\begin{aligned}r &= \frac{(0.3)\,(9.8)}{100\pi^2}81\\ &\approx \frac{0.3\cdot 10}{100\cdot\pi^2}\cdot 81\\ &\approx \frac{243}{100\cdot 10}\\ &\simeq 0.243\,\mathrm{m}\end{aligned}\tag{13.2.3}$$

**Answer: (D)**

## 13.3 Kepler's Law and Satellite Orbits

Kepler's Third Law, states, "The square of the orbital period of a planet is directly proportional to the third power of the semi-major axis of its orbit." This means

$$T^2 \propto r^3 \tag{13.3.1}$$

**Answer: (D)**

See section 1.7.4. If you're unable to remember Kepler's Law and its relationship between the period and orbital distance, some quick calculation will yield some results.

$$\begin{aligned}mr\omega &= \frac{GMm}{r^2}\\ mr\left(\frac{2\pi}{T}\right)^2 &= \frac{GM}{r^2}\\ \Rightarrow r^3\frac{(2\pi)^2}{GM} &= T^2\\ \therefore r^3 &= kT^2\end{aligned}$$

This is Kepler's Third Law.

## 13.4 Non-Elastic Collisions

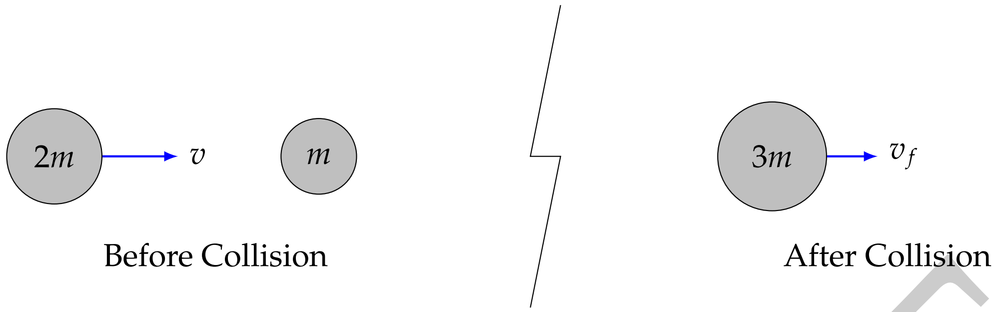



Figure 13.4.1: Inelastic collision between masses $2m$ and $m$

As the bodies fuse, see fig. 13.4.1, the resulting collision will be an inelastic one; kinectic energy is not conserved but momentum will be conserved. This allows us to immediately eliminate choice **(A)**[1].

Thus

$$\begin{aligned} \text{Momentum Before Collision} &= \text{Momentum After Collision} \\ 2mv &= 3mv_f \\ \Rightarrow v &= \frac{3}{2}v_f \end{aligned} \tag{13.4.1}$$

The kinectic energy before and after collision are

**Initial K.E.**

$$E = \frac{1}{2}(2m)v^2 = mv^2 \tag{13.4.2}$$

**Final K.E.**

$$\begin{aligned} E_f = \frac{1}{2}(3m)v_f^2 &= \frac{1}{2}(3m)\left(\frac{2}{3}v\right)^2 \\ &= \frac{2}{3}mv^2 \end{aligned} \tag{13.4.3}$$

Subtracting eq. (13.4.3) from eq. (13.4.2) gives us the energy lost in the collision

$$\begin{aligned} \Delta E &= mv^2 - \frac{2}{3}mv^2 \\ &= \frac{1}{3}mv^2 \end{aligned} \tag{13.4.4}$$

The fraction of initial kinetic energy lost in the collision is

$$\begin{aligned} \frac{\Delta E}{E} &= \frac{\frac{1}{3}mv^2}{mv^2} \\ &= \frac{1}{3} \end{aligned} \tag{13.4.5}$$

[1]Not much help but the elimination of just one choice may work to our advantage.

**Answer: (C)**

## 13.5 The Equipartition Theorem and the Harmonic Oscillator

The average total energy of our oscillator is determined by the Equipartition Theorem, see section 4.22, where

$$C_V = \left(\frac{f}{2}\right) R = 4.16\,\mathrm{J\,mol^{-1}\,K^{-1}} \tag{13.5.1}$$

where $f$ is the number of degrees of freedom.

The average total energy is

$$\begin{aligned} Q &= nC_V T \\ &= n\left(\frac{f}{2}\right) RT \\ &= N\left(\frac{f}{2}\right) kT \end{aligned} \tag{13.5.2}$$

We are told this is a three dimensional oscillator so we have $f = 6$ degrees of freedom and $N = 1$. So eq. (13.5.2) becomes

$$Q = \left(\frac{6}{2}\right) kT = 3kT\mathrm{J} \tag{13.5.3}$$

**Answer: (D)**

## 13.6 Work Done in Isothermal and Adiabatic Expansions

This is a simple question if you remember what isothermal and adiabatic processes look like on a $P - V$ graph. The areas under the curves tell us the work done by these processes and since the adiabatic curve is less steep than the isothermal one, then the work done by this process is less[2].
So we get $0 < W_a < W_i$.
**Answer: (E)**

## 13.7 Electromagnetic Field Lines

The two poles are of the same polarity so we expect the filed lines to repel.[3] **Answer: (B)**

[2]Get P-V diagram with isothermal and adiabatic expansions
[3]Get diagram with magneti field lines

## 13.8 Image Charges

We can use the "Method of Image Charges" to solve this question. We have a positive charge near the plate so this will induce an equal and opposite charge in the plate.[4]
But, let's for the sake of argument say that you didn't know of this 'method' and needed to figure it out, we know a few things. We know that the plate is grounded. So if we were to bring a charge near to the plate, an equal but opposite charge will be induced. In this case, negative charges in the plate are attracted to the nearby charge and positive ones are repelled. As the positive ones want to "get away", they succeed in doing so through the ground, leaving only the negative charges behind. Thus the plate is left with a net negative charge.
**Answer: (D)**

## 13.9 Electric Field Symmetry

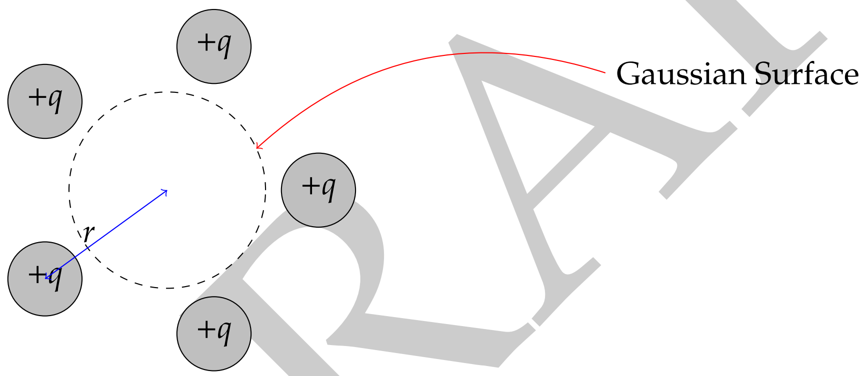



Figure 13.9.1: Five charges arranged symmetrically around circle of radius, $r$

Gauss's Law states that "The electric flux through any closed surface is proportional to the enclosed electric charge".

$$\boxed{\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}} \qquad (13.9.1)$$

If we draw a Gaussian Surface at the center of our arrangement, see fig. 13.9.1, we notice there are no charges enclosed and thus no electric field.[5]
**Answer: (A)**

[4] Put wikipedia reference here
[5] This makes sense, there is no electric field inside a conductor because all the charges reside on the surface.

## 13.10 Networked Capacitors

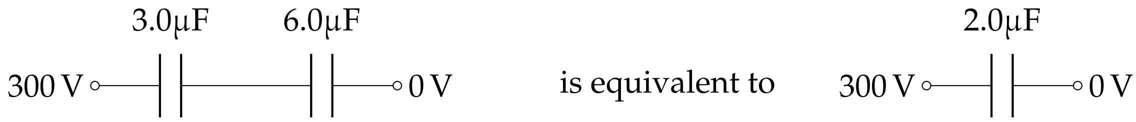



Figure 13.10.1: Capacitors in series and its equivalent circuit

The Energy stored in a Capacitor is

$$E = \frac{1}{2}CV^2 \tag{13.10.1}$$

Our networked capacitors, as shown in fig. 13.10.1, can be reduced to a circuit with only one capacitor. The equivalence capacitance of two Capacitors in series is

$$\begin{aligned} C_T &= \frac{C_1 C_2}{C_1 + C_2} \\ &= \frac{(3)(6)}{9} \\ &= 2\,\mu\text{F} \end{aligned}$$

Substitute into eq. (13.10.1) we can find the energy stored

$$\begin{aligned} E &= \frac{1}{2}\left(2 \times 10^{-6}\right)(300)^2 \\ &= 9 \times 10^{-2}\,\text{J} \end{aligned}$$

**Answer: (A)**

## 13.11 Thin Lens Equation

Using the Len's Maker Equation (Thin Lens). We have

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \qquad \text{where} \qquad \begin{aligned} S &= \text{Object Distance} \\ S' &= \text{Image Distance} \\ f &= \text{Focal Length} \end{aligned}$$

Solving for the first lens, we have

$$\begin{aligned} \frac{1}{40} + \frac{1}{I_1} &= \frac{1}{20} \\ \Rightarrow \frac{1}{I_1} &= \frac{1}{20} - \frac{1}{40} \\ &= \frac{1}{40} \end{aligned}$$

The resulting image is 40cm from the first lens which forms a virtual image that is 10cm to the right of the second lens. We get,[6]

$$\frac{1}{-10} + \frac{1}{I_2} = \frac{1}{10}$$
$$\implies \frac{1}{I_2} = \frac{1}{-5}$$

The image is located 5cm to the right of the second lens. **Answer: (A)**

## 13.12 Mirror Equation

For a concave mirror, we know that if an object is before the focal length, then image is virtual If object is after focal length, the image is real.

The Mirror Equation is

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i} \tag{13.12.1}$$

We can show that

$$\frac{1}{F} = \frac{1}{O} + \frac{1}{d_i} \tag{13.12.2}$$
$$\Rightarrow d_i = \frac{FO}{O - F} \tag{13.12.3}$$

Here we see that $O < F$, so $d_i$ is negative. The image is virtual and at point $V$.

We can also keep in mind that for a concave lens, if the object is between the focal point and the lens, the image is virtual and enlarged. Think of what happens when you look at a makeup mirror.[7]

**Answer: (E)**

## 13.13 Resolving Power of a Telescope

The Resolving Power of a Tesescope is

$$R = 1.22\frac{\lambda}{D} \tag{13.13.1}$$

Solving for $D$ and substuting $\lambda$ and $R$, we get

$$\Rightarrow D = 1.22\frac{\lambda}{R}$$
$$= 1.22 \times \frac{600 \times 10^{-19}}{3 \times 10^{-5}}$$
$$= 1.22 \times 200 \times 10^{-4}$$
$$= 2.44 \times 10^{-2}$$

**Answer: (B)**

[6] Get book/internet references for this equation.
[7] Get references for this as well

## 13.14 Radiation detected by a NaI(Tl) crystal

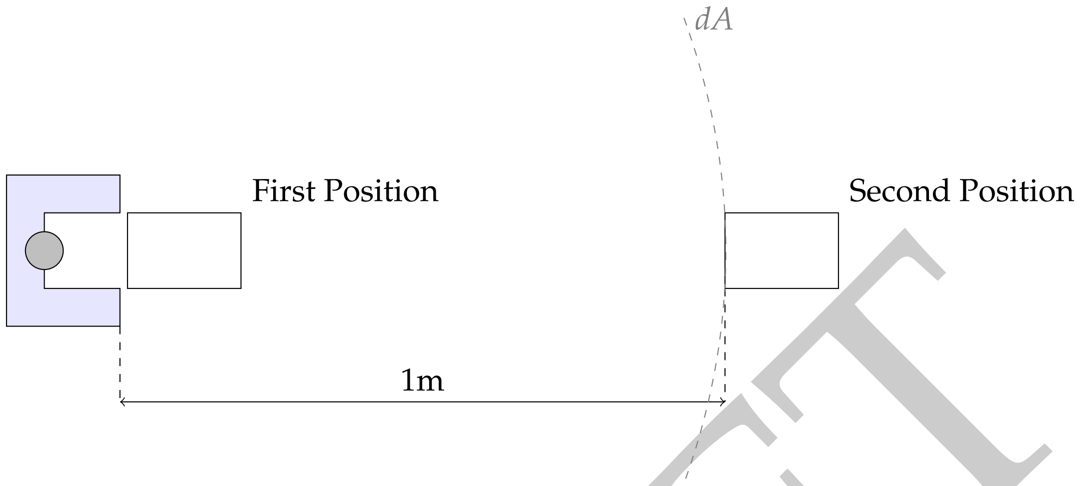



Figure 13.14.1: Diagram of NaI(Tl) detector postions

Thallium doped Sodium Iodine crystals are used in scintillation detectors, usually found in hospitals. These crystals have a high light output and are usually coupled to photomultiplier tubes. No emitted power is lost to the surrounding medium. Thus, the net power radiated by our source is

$$P = \int I dA \tag{13.14.1}$$

where $P$ is the radiated power, $I$ is the intensity and $dA$ is a differential element of a closed surface that contains the source.

The emitted power at the first position is

$$P_1 = I_1 A_1 \tag{13.14.2}$$

and the emitted power at the second position is

$$P_2 = I_2 A_2 \tag{13.14.3}$$

As no power is lost to the environment, then eqs. (13.14.2) and (13.14.3) are equal, thus

$$\begin{aligned} P_1 &= P_2 \\ I_1 A_1 &= I_2 A_2 \\ \Rightarrow \frac{I_2}{I_1} &= \frac{A_1}{A_2} \end{aligned} \tag{13.14.4}$$

In the first case, as the detector is right up to the radioactive source, the area, $A_1$ is the cross sectional area of the NaI(Tl) crystal.

$$A_1 = \frac{\pi d^2}{4} \tag{13.14.5}$$

In the second position, the area is the surface area of the sphere of emitted radiation.

$$A_2 = 4\pi D^2 \tag{13.14.6}$$

where $D$ is the distance of the crystal detector from the radioactive source.

The ratio between $I_2$ and $I_1$ can be found from eq. (13.14.4),

$$\begin{aligned}
\frac{I_2}{I_1} &= \frac{A_1}{A_2} \\
&= \frac{\pi d^2}{4}\frac{1}{4\pi D^2} \\
&= \frac{d^2}{16D^2} \\
&= \frac{\left(8\times 10^{-2}\right)^2}{16} \\
&= 4\times 10^{-2}
\end{aligned}$$

**Answer: (C)**

## 13.15 Accuracy and Precision

From the graphs, the accuracy is how close the peak is to the reference value while precision deals with how narrow the peak is. So we look for the graph with the narrowest peak.

**Answer: (A)**

## 13.16 Counting Statistics

From the data given, we have $N = 20$ counts in $T = 10$ seconds. Our uncertainty or counting error is $\sqrt{N} = \sqrt{20}$. So we can express our uncertainty in the number of counts as

$$N = 20 \pm \sqrt{20} \tag{13.16.1}$$

The rate is the number of counts per unit time. So the uncertainty in the rate is

$$R = \frac{20}{10} \pm \frac{\sqrt{20}}{10} \tag{13.16.2}$$

We can see that the error in the rate is $\delta R = \frac{\delta N}{T}$. Our uncertainty can be expressed

$$\begin{aligned}
\frac{\delta R}{R} &= \frac{\frac{\delta N}{T}}{\frac{N}{T}} \\
&= \frac{\delta N}{N} \\
&= \frac{1}{\sqrt{N}}
\end{aligned} \tag{13.16.3}$$

The question wants an uncertainty of 1%. From the above, we see that

$$\frac{1}{\sqrt{N}} = 0.01 \tag{13.16.4}$$

But $N$ is the number of counts. We want to know how long making these counts will take us. As $R = \frac{N}{T}$, we see that

$$\frac{1}{\sqrt{2T}} = 0.01 \tag{13.16.5}$$

Solving for $T$ gives

$$T = \frac{1}{2(0.01)^2} = 5000 \tag{13.16.6}$$

**Answer: (D)**

## 13.17 Electron configuration

We know the Energy Sub-Shells are

$$1s^2, 2s^2, sp^6, 3s^2, 3p^3 \tag{13.17.1}$$

**Answer: (B)**

## 13.18 Ionization Potential (He atom)

From Bohr's Theory, the energy needed to completly ionize an atom is

$$E = -13.6\frac{Z^2}{n^2}\text{eV} \tag{13.18.1}$$

The total Energy to ionize the atom is

$$E = E_1 + E_2 \tag{13.18.2}$$

We expect the energy to remove the first electron to be less than the second; there are more positive charges in the nucleus holding the second electron in place. We can use the above equation to find out the energy to remove this second electron.

$$\begin{aligned} E_2 &= -13.6\frac{2^2}{1^2}\text{eV} \\ &= 54.4\text{eV} \end{aligned} \tag{13.18.3}$$

Now we can determine the energy needed to remove the first electron

$$\begin{aligned} E_1 &= E - E_2 \\ &= 79.0 - 54.4 \\ &= 24.6\,\text{eV} \end{aligned} \tag{13.18.4}$$

**Answer: (A)**

## 13.19 Nuclear Fusion

The are several fusion reactions by which stars convert Hydrogen into Helium and release energy. One of the main reactions is the proton-proton chain reaction and looks something like this

$$4{}_1^1H \rightarrow_2^4 He + 2\left({}_1^0e +_0^0 \gamma +_v^0\right) \tag{13.19.1}$$

Four Hydrogen atoms combine to give one Helium atom, the difference in masses being released as energy.

**Answer: (B)**

## 13.20 Bremsstrahlung

A definition question. It may help to know that Bremsstrahlung is German for 'braking radiation'. This is the radiation that is released as the electron slows down or is "braked" and results in a continous spectra.

**Answer: (E)**

## 13.21 Atomic Spectra

The Rydberg Formula describes the spectral wavelengths of chemical elements. For the Hydrogen atom, the equation is

$$\boxed{\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \tag{13.21.1}$$

where $\lambda$ is the wavlength of the emitted light, $R_H$ is the Rydberg constant for Hydrogen, $n_1$ and $n_2$ are the electron orbital numbers.

For the Lyman-$\alpha$ emission, electrons jump from $n_2 = 2$to the $n_1 = 1$ orbital. This gives

$$\begin{aligned}\frac{1}{\lambda_1} &= R_H\left(\frac{1}{1} - \frac{1}{2}\right)\\ &= \frac{1}{2}R_H\end{aligned} \tag{13.21.2}$$

For the Balmer-$\alpha$ emission, electrons jump from $n_2 = 3$ to $n_1 = 2$ orbital. This gives

$$\begin{aligned}\frac{1}{\lambda_2} &= R_H\left(\frac{1}{2} - \frac{1}{3}\right)\\ &= \frac{1}{6}\end{aligned} \tag{13.21.3}$$

Dividing eq. (13.21.3) by eq. (13.21.2), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{3} \tag{13.21.4}$$

**Answer: (C)**

## 13.22 Planetary Orbits

Newton's Law of Universal Gravitation can be expressed

$$\frac{mv^2}{r} = G\frac{Mm}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2 \tag{13.22.1}$$

We can use the information above to eliminate choices.

**Mass of he Moon** We see that in all cases, the mass of the moon, $m$, cancels out. We can not find the mass of the moon from the astronomer's observations.

**Mass of the Planet** We can determine the mass of the planet, $M$, from the data.

$$\frac{v^2}{r} = \frac{GM}{r^2} \tag{13.22.2}$$

We will need the distances and the moon's orbital speed.

**Minimum Speed of the Moon** The speed of the Moon, $v$, also does not cancel out in any of our equations. So we can find this out.

**Period of Orbit** As the period, $T$, does not cancel out, we can also determine this.

$$r\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^2} \tag{13.22.3}$$

Semimajor axis of orbit The semimajor axis is the longest distance from the center of an ellipse. The distance, $r$, in our equations are a measure of the semimajor axis.

We see that the mass, $m$, the mass of the moon cancels out. Everything else mentioned remains. Thus

**Answer: (A)**

## 13.23 Acceleration of particle in circular motion

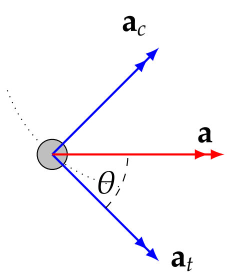



Figure 13.23.1: Acceleration components of a particle moving in circular motion

Since the particle's speed increases as it moves in a circle, it is going to have two accelerations acting on it; a centripetal acceleration and a tangential acceleration, see fig. 13.23.1. The net acceleration of our particle can be found by adding the centripetal and tangential components, so

$$\mathbf{a} = \mathbf{a}_c + \mathbf{a}_t \tag{13.23.1}$$

where

$$a_c = \frac{v^2}{r} \tag{13.23.2}$$
$$a_t = \alpha r \tag{13.23.3}$$

The angle between the particle's acceleration, **a** and its velocity, $v$, is

$$\tan\theta = \frac{a_c}{a_t} \tag{13.23.4}$$

The velocity vector points in the same direction as $\mathbf{a}_t$, hence the reason why we use the above calculation.

Given that $r = 10$ meters, $v = 10$ meters per second, we calculate $a_c$ to be

$$a_c = 10\text{meters per second per second} \tag{13.23.5}$$

Plugging this into eq. (13.23.4), we get

$$\begin{aligned}\tan\theta &= \frac{a_c}{a_t}\\ &= \frac{10}{10} = 1\end{aligned} \tag{13.23.6}$$

Thus

$$\theta = 45° \tag{13.23.7}$$

**Answer: (C)**

## 13.24 Two-Dimensional Trajectories

In the stone's horizontal velocity, there is no resistive force, so it remains unchanged during the stone's trajectory. Graph II represents this.

Gravity acts on the stone's vertical motion, slowing it down as it moves upwards until it reaches zero and then causing its speed to increase its speed in the opposite direction. Graph III, illustrates this.

**Answer: (C)**

# 13.25 Moment of inertia of pennies in a circle

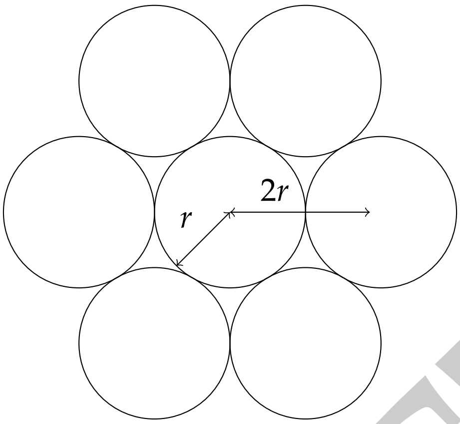



Figure 13.25.1: Seven pennies in a hexagonal, planar pattern

The Moment of inertia of a penny is the same for a disc or cylinder

$$I_{\mathbf{penny}} = I_{\mathrm{cm}} = \frac{1}{2}Mr^2 \tag{13.25.1}$$

For the other pennies, we find their Moments of Inertia by using the Parallel Axis Theorem.

$$I_T = I_{\mathrm{cm}} + Md^2 \tag{13.25.2}$$

where $d = 2r$. This becomes

$$\begin{aligned} I_T &= \frac{1}{2}Mr^2 + M(2r)^2 \\ &= \frac{9}{2}Mr^2 \end{aligned} \tag{13.25.3}$$

The Moment of Inertia depends on the mass distribution. Whether the six pennies were stacked on top of each other next or arranged in a hexagonal pattern, the Moment of Inertia will be the same. The total moment of inertia is found by adding eqs. (13.25.1) and (13.25.3).

$$\begin{aligned} I &= \frac{1}{2}Mr^2 + 6 \times \frac{9}{2}Mr^2 \\ &= \frac{55}{2}Mr^2 \end{aligned} \tag{13.25.4}$$

**Answer: (E)**

## 13.26 Falling Rod

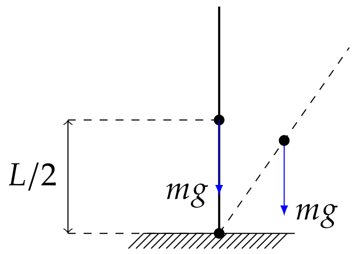



Figure 13.26.1: Falling rod attached to a pivot point

As the rod falls, its Gravitational Potential Energy is converted to Rotational Kinetic Energy. In this case, we will need to calculate the Moment of Inertia of the rod about its point of rotation. For this, we turn to the Parallel Axis Theorem. The Moment of Inertia of the rod is

$$I_{cm} = \frac{1}{12}ML^2 \tag{13.26.1}$$

The Parallel Axis Theorem gives us the Moment of Inertia about the pivot point

$$\begin{aligned} I &= I_{cm} + Md^2 \\ &= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 \\ &= \frac{1}{3}ML^2 \end{aligned} \tag{13.26.2}$$

The rod is uniform, so its Center of Mass is in the middle of the rod. Its Gravitational Potential Energy while standing upright is

$$PE = Mg\left(\frac{L}{2}\right) \tag{13.26.3}$$

The Rotational Kinetic Energy is

$$KE = \frac{1}{2}I\omega^2 \tag{13.26.4}$$

As energy is conserved, eqs. (13.26.3) and (13.26.4) are equal.

$$\begin{aligned} Mg\frac{L}{2} &= \frac{1}{2}\left(\frac{1}{3}ML^2\right)\left(\frac{v}{L}\right)^2 \\ v &= \sqrt{3gL} \end{aligned} \tag{13.26.5}$$

**Answer: (C)**

## 13.27 Hermitian Operator

Another definition question. The eigenvalues are real and hence observable.

**Answer: (A)**

## 13.28 Orthogonality

Two functions are orthogonal if their inner or dot product is zero . On the other hand they are orthonormal if their inner product is one. Thus

$$\langle \psi_1 | \psi_2 \rangle = 0 \tag{13.28.1}$$
$$\langle 1|1\rangle = \langle 2|2\rangle = \langle 3|3\rangle = 1 \tag{13.28.2}$$

We get,

$$\langle \psi_1 | \psi_2 \rangle = (5)\,(1)\,\langle 1|1\rangle + (-3)\,(-5)\,\langle 2|2\rangle + \\ + (2)\,(x)\,\langle 3|3\rangle \tag{13.28.3}$$

This gives

$$\begin{aligned} 5 + 15 + 2x &= 0 \\ \Rightarrow x &= -10 \end{aligned} \tag{13.28.4}$$

**Answer: (E)**

## 13.29 Expectation Values

The Expectation Value is defined

$$\langle O \rangle = \langle \psi | O | \psi \rangle \tag{13.29.1}$$

Where

$$\psi = \frac{1}{\sqrt{6}}\psi_{-1} + \frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{3}}\psi_2 \tag{13.29.2}$$

Thus

$$\begin{aligned} \langle O \rangle &= \frac{1}{6} + \frac{1}{2} + \frac{1}{3} \\ &= 1 \end{aligned} \tag{13.29.3}$$

**Answer: (C)**

## 13.30 Radial Wave Functions

We need to be thinking of something that decays exponentially. So

**I.** $e^{-br}$ This decays exponentially. As $r \to \infty$ then $\psi \to 0$.

**II.** $A \sin br$ This doesn't go to zero as $r \to \infty$.

**III.** $\frac{A}{r}$ This does become zero as $r \to \infty$ but there is no realistic value at $r = 0$. It blows up.

**Answer: (A)**

## 13.31 Decay of Positronium Atom

Positronium(Ps) is a quasiatomic structure where an electron, $e^-$, and a positron, $e^+$, are bound together by Coulomb attraction. Positronium has a short lifetime due to pair-annihilation; the system usually lasts for $10^{-10}$ s before decaying into two photons.

As this is a two-body system, we need to find the reduced-mass of the system. This will allow us to solve this problem as if it was a one body problem. The Reduced Mass of the System is

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \tag{13.31.1}$$

Substituting for the masses of the electron and the positron, we get

$$\begin{aligned}\mu &= \frac{m_e m_p}{m_e + m_p} \\ &= \frac{m_e}{2} \end{aligned} \tag{13.31.2}$$

We then apply the Bethe-Salpeter equation,

$$E_n = -\frac{\mu}{8h^2\varepsilon_0^2}\frac{q_e^4}{n^2} \tag{13.31.3}$$

In most two body atom systems, the reduced-mass factor is close to unity but as the masses of the electron and the positron are equal, the reduced-mass has an appreciable effect on the energy levels. Substituting, we get,

$$\begin{aligned}E_n &= -\frac{1}{2}\frac{m_e q_e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2} \\ &= \frac{-6.8}{2} \qquad (13.31.4) \\ &= -3.4\,\text{eV} \qquad (13.31.5)\end{aligned}$$

**Answer: (A)**

## 13.32 Relativistic Energy and Momentum

The rest energy of our particle is

$$E_{\text{rest}} = mc^2 \tag{13.32.1}$$

The particle's total energy is given by

$$E^2 = c^2p^2 + m^2c^4 \tag{13.32.2}$$

We are told that

$$E = 2E_{\text{rest}} \tag{13.32.3}$$

Substuting eq. (13.32.3) in eq. (13.32.2) gives

$$\begin{aligned} E &= 2E_{\text{rest}} \\ \therefore 4m^2c^4 &= p^2c^2 + m^2c^4 \\ \Rightarrow p &= \sqrt{3}\,mc \end{aligned} \tag{13.32.4}$$

**Answer: (D)**

## 13.33 Speed of a Charged pion

The speed that is measured is the same in either frame, so we can say

$$v = \frac{L}{\Delta t} = \frac{L'}{\Delta t'} \tag{13.33.1}$$

We know that from the pi-meson's point of view the distance is length contracted.

$$L' = L\sqrt{1 - \frac{v^2}{c^2}} \tag{13.33.2}$$

We can alternatively look at things from the laboratory's point of view, in this case we will be using the Relativistic Time Dilation Formula. Substuting eq. (13.33.2) into eq. (13.33.1), gives us

$$v = \frac{L}{\Delta t}\sqrt{1 - \frac{v^2}{c^2}} \tag{13.33.3}$$

With some manipulation, we simplify eq. (13.33.3) to give

$$\begin{aligned} v^2 &= \frac{L^2}{L^2 + c^2\left(\Delta t'\right)^2} \\ &= \frac{900}{900 + 9} \\ &= \frac{100}{101} \end{aligned} \tag{13.33.4}$$

We can surmise that the result of eq. (13.33.4) will be closer to **(D)** than **(C)**.
**Answer: (D)**

## 13.34 Simultaneity

The Space-Time Interval of an event is

$$\Delta S^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 \tag{13.34.1}$$

The Space-Time interval is a Lorentz-invariant quantity which means that it has the same value in all Lorentz frames. Depending on the two events, the interval can be positive, negative or zero. So

**Time-Like** If $\Delta S < 0$, the two events occur in the same space but at different times.

**Space-Like** If $\Delta S > 0$, the two events occur at the same time (simultaneously) but are seperated spatially.

**Light-like** If $\Delta = 0$, the two events are connected by a signal moving at light speed.

So we are looking for a situation where $\Delta S$ is positive. Thus

$$\begin{aligned} \Delta S &= \Delta x^2 - c^2 \Delta t^2 > 0 \\ \Rightarrow \frac{\Delta x}{\Delta t} &> c \end{aligned} \tag{13.34.2}$$

**Answer: (C)**

## 13.35 Black-Body Radiation

The energy radiated by a black body is given by the Stefan–Boltzmann's Law

$$\boxed{u = \sigma T^4} \tag{13.35.1}$$

Let

$$u_1 = \sigma T_1^4 \tag{13.35.2}$$

The temperature of the object increases by a factor of 3, thus

$$\begin{aligned} T_2 &= 3T_1 \\ u_2 &= \sigma \left(3T_1\right)^4 \end{aligned} \tag{13.35.3}$$

and we get

$$\begin{aligned} u_2 &= 81\sigma T_1^4 \\ &= 81 u_1 \end{aligned} \tag{13.35.4}$$

It increases by a factor of 81.
**Answer: (E)**

## 13.36 Quasi-static Adiabatic Expansion of an Ideal Gas

This is more of a defintion question that we can best answer through the process of elimination.

**(A)** This is **TRUE**. The expansion is quasi-static, which means that it happens very slowly and hence at equilibrium. Hence no heat is exchanged.

**(B)** Again this is **TRUE**. The entropy is defined

$$\boxed{dS = \frac{dQ}{T}} \tag{13.36.1}$$

As $dQ = 0$, then there is no change in entropy.

**(C)** The First Law of Thermodynamics says

$$\boxed{dU = -dW + dQ} \tag{13.36.2}$$

where $dQ = 0$, we see that

$$dU = -dW \tag{13.36.3}$$

The work done by the gas is

$$\boxed{dW = \int pdV} \tag{13.36.4}$$

Thus

$$dU = -\int PdV \tag{13.36.5}$$

**(D)** We see from eq. (13.36.4) that this is **TRUE**.

**(E)** The temperature of the gas is not constant. For an adiabatic process

$$\boxed{PV^{\gamma} = \text{ constant}} \tag{13.36.6}$$

Given $PV = nRT$, substituting this into the above equation gives

$$\boxed{TV^{\gamma-1} = \text{ constant}} \tag{13.36.7}$$

So

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \tag{13.36.8}$$

The temperature of the gas is not constant. So this is **NOT TRUE**.

**Answer: (E)**

## 13.37 Thermodynamic Cycles

Thermodynamic Work is defined

$$W = -\int_{V_i}^{V_f} PdV \tag{13.37.1}$$

The Total Work is calculated along each path. Thus,

$$W = W_{C\to A} + W_{A\to B} + W_{B\to C} \tag{13.37.2}$$

Work along path $C \to A$ is an isochoric process, $dV = 0$.

$$W_{C\to A} = 0 \tag{13.37.3}$$

Work along path $A \to B$ is an isobaric process

$$\begin{aligned} W_{A\to B} &= P \cdot dV \\ &= P(V_B - 2) \end{aligned} \tag{13.37.4}$$

Work along path $B \to C$ is an isothermal process

$$\begin{aligned} W_{B\to C} &= \int_{V_B}^{V_C} PdV \quad \text{where} \quad P = \frac{nRT}{V} \\ &= nRT\int_{V_B}^{V_A} \frac{dV}{V} \\ &= nRT \ln\left[\frac{V_A}{V_B}\right] \end{aligned} \tag{13.37.5}$$

The path along $BC$ is an isotherm and this allows us to find the volume at point $B$.

$$\begin{aligned} P_B V_B &= P_C V_C = nRT \\ 200 V_B &= 500 \cdot 2 \\ \therefore V_B &= 5 \end{aligned} \tag{13.37.6}$$

Plugging in what we know, we add eq. (13.37.4), eq. (13.37.5) and eq. (13.37.3) to get the total work.

$$\begin{aligned} W &= W_{C\to A} + W_{A\to B} + W_{B\to C} \\ &= 0 + P(V_B - V_A) + P_C V_C ln\left[\frac{V_A}{V_B}\right] \\ &= 200(5-2) + (500)(2)\ln\left[\frac{2}{5}\right] \\ &= 600 + 1000\ln\left[\frac{2}{5}\right] \end{aligned} \tag{13.37.7}$$

Now $\ln(2/5) > -1$, so we expect

$$W = 600 + 1000\ln\left[\frac{2}{5}\right] > -400 \text{ kJ} \tag{13.37.8}$$

**Answer: (D)**

## 13.38 RLC Resonant Circuits

The current will be maximized when the inductive and capacitive reactances are equal in magnitude but cancel each other out due to being 180°out of phase. The Inductive Impedance is

$$X_L = \omega L \tag{13.38.1}$$

And the Capacitive Impedance is

$$X_C = \frac{1}{\omega C} \tag{13.38.2}$$

We let, eq. (13.38.2) = eq. (13.38.1)

$$\begin{aligned}\omega L &= \frac{1}{\omega C}\\ \Rightarrow C &= \frac{1}{\omega^2 L}\\ &= \frac{1}{25\times 10^{-3}}\\ &= 40\mu\text{F}\end{aligned} \tag{13.38.3}$$

**Answer: (D)**

## 13.39 High Pass Filters

We recall that the Inductive Impedance is

$$X_L = \omega L \tag{13.39.1}$$

and the Capacitive Impedance to be

$$X_C = \frac{1}{\omega C} \tag{13.39.2}$$

If we look at eq. (13.39.1), we see there is a linear relationship between $L$ and $X_L$; an increase in $L$ results in an increase in $X_L$.

We also see from eq. (13.39.2) that there is an inverse relationship between $C$ and $X_C$; an increase in $C$ decreases $X_C$.

Recall the Voltage Divider Equation

$$V_{\text{Out}} = \frac{X_2}{X_1 + X_2} V_{In} \tag{13.39.3}$$

We will use this to help us solve the question.

**Circuit 1** As $\omega$ increases, the impedance of the inductor increases; $X_1$ becomes large. Think of this as a very large resistor where the inductor is. We see from eq. (13.39.3)

$$\begin{aligned}V_{\text{Out}} &= \frac{X_2}{\infty + X_2} V_{\text{In}}\\ &= 0 V_{\text{In}}\end{aligned} \tag{13.39.4}$$

. This is a **Low-Pass Filter**.

**Circuit II** As $\omega$ increases, $X_L$ becomes large. We will say that $X_2 >> X_1$. Again eq. (13.39.3) shows that

$$\begin{aligned}V_{\text{Out}} &= \frac{X_2}{X_1 + X_2} V_{\text{In}}\\ &\approx \frac{1}{1} V_{\text{In}}\end{aligned} \tag{13.39.5}$$

This is one of the **High-Pass Filters**.

**Circuit III** As $\omega$ increases, $X_C$ decreases. At high $\omega$, $X_C = X_1 \approx 0$. Again eq. (13.39.3) shows that

$$\begin{aligned} V_{\text{Out}} &= \frac{X_2}{0 + X_2} V_{\text{In}} \\ &= V_{\text{In}} \end{aligned} \tag{13.39.6}$$

This is the other **High-Pass Filter**.

**Circuit IV** Using what we know, eq. (13.39.3) tells us that

$$\begin{aligned} V_{\text{Out}} &= \frac{0}{X_1 + 0} V_{\text{In}} \\ &= 0 \end{aligned} \tag{13.39.7}$$

This is a **Low-Pass Filter**.

Thus **Circuits II & III** are our High-Pass Filters.
**Answer: (D)**

## 13.40 RL Circuits

As an EMF is introduced in the circuit, there is going to be a slowly rising (or falling) current. If the inductor was not present, the current would rapidly rise to a steady state current of $\frac{\mathscr{E}}{R}$. The inductor produces a self-induced EMF, $\mathscr{E}_L$, in the circuit; from Lens's Law. This EMF is

$$\mathscr{E}_L = -L\frac{di}{dt} \tag{13.40.1}$$

Applying Kirchoff's Voltage law gives

$$\mathscr{E} = iR + L\frac{di}{dt} \tag{13.40.2}$$

This differential equation can be solved such that

$$i = \frac{\mathscr{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \tag{13.40.3}$$

Let $\tau_L = \frac{L}{R}$, we can rewrite eq. (13.40.3), to say

$$i = \frac{\mathscr{E}}{R}\left(1 - e^{-\frac{t}{\tau_L}}\right) \tag{13.40.4}$$

The time constant is the time to fall to $\frac{1}{e}$ of its original value.

$$\begin{aligned} \tau_L &= \frac{10\text{mH}}{2\Omega} \\ &= 2 \text{ milli-seconds} \end{aligned} \tag{13.40.5}$$

As we expect the EMF to decay across the inductor, we choose
**Answer: (D)**[8]

[8] We note that choices **D)** and **E)** both decay exponentially and that milli-second decay times are standard with the usual components you find in a lab. A 200 sec decay time is unusual given the "normal" electronic components in the question.

## 13.41 Maxwell's Equations

Maxwell's Equations relate electric and magnetic fields to the motion of electric charges. These equations allow for electric charges and not for magnetic charges. One can write symmetric equations that allow for the possibility of "magnetic charges" that are similar to electric charges. With the inclusion of these so called "magnetic charges", $\rho_m$, we must also include a magnetic current, $\mathbf{j_m}$. These new Maxwell equations become

**Gauss' Law** This equation relates the distribution of electric charge to the resulting electric field.

$$\nabla \cdot \mathbf{E} = 4\pi\rho_e \tag{13.41.1}$$

**Gauss' Law for Magnetism** Here we assume that there are no magnetic charges, so the equation that we know and have studies is

$$\nabla \cdot \mathbf{B} = 0 \tag{13.41.2}$$

But if we assume for magnetic charges, the equation will become

$$\nabla \cdot \mathbf{B} = 4\pi\rho_m \tag{13.41.3}$$

Here we have used a symmetric argument from Gauss' Law to get this equation.

**Ampère's Law** This equation relates the magnetic field to a current. With Maxwell's displacement current, $\mathbf{j_e}$, we have

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + 4\pi\mathbf{j_e} \tag{13.41.4}$$

**Faraday's Law of Induction** This equation relates a changing Magnetic field to an Electric Field. The equation is

$$\nabla \cdot \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \tag{13.41.5}$$

Again we will use a symmetric argument to "derive" the magnetic monopole case. As in Ampère's Law where there exists an electric displacement current, we postulate a "magnetic displacement current". This becomes

$$\nabla \cdot \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + 4\pi\mathbf{j_m} \tag{13.41.6}$$

We see the changes are in equations **II** and **III**.

**Answer: (E)**[9]

[9] It's easy to answer this question if you think along symmetry arguments. How do the presence of electric charges and currents affect the Maxwell's equations and by noticing the differences with magnetism you can come up with an answer.

## 13.42 Faraday's Law of Induction

Faraday's Law states that the induced EMF is equal to the rate of change of magnetic flux.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \tag{13.42.1}$$

The minus sign is from Lenz's Law which indicates that the induced EMF and the changing magnetic flux have opposite signs. Thinking of it as a system that induces an opposing force to resist the change of this changing flux is one of the best analogies.

**Loop A** In this case, the flux increases as the current carrying loop approaches. So to 'compensate' for this increase, Loop A, induces a current in the opposite direction to prevent this increase. Thus the induced current wil be in the **clock-wise** direction.

**Loop B** As the current carying loop moves away from Loop B, the magnetic flux will decrease. Loop B wants to prevent this decrease by inducing an increasing current. The induced current will be in the **clock-wise** direction.

**Answer: (C)**

## 13.43 Quantum Mechanics: Commutators

We recall our commutator relations

$$[B, AC] = A\,[B, C] + [B, A]\,C \tag{13.43.1}$$
$$[A, B] = -\,[B, A] \tag{13.43.2}$$

Thus

$$\begin{aligned}\left[L_x L_y, L_z\right] &= -\left[L_z, L_x L_y\right] \\ &= -\left(-L_x\left[L_z, L_y\right] + [L_z, L_x]\,L_y\right) \\ &= -\left(-L_x\,(i\hbar L_x) + \left(i\hbar L_y\right)L_y\right) \\ &= i\hbar\left(L_x^2 + L_y^2\right)\end{aligned} \tag{13.43.3}$$

**Answer: (D)**

## 13.44 Energies

We are given that

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \tag{13.44.1}$$

where $n = 1, 2, 3, \cdots$ So the possible energy values are $E_2 = 4E_1, E_3 = 9E_1, E_4 = 16E_1, \cdots$. Possible answers are of the form

$$E_n = n^2 E_1 \tag{13.44.2}$$

textbfD) follows where $n = 3$. All the rest don't.[10]

**Answer: (D)**

## 13.45 1-D Harmonic Oscillator

We are given that

$$H|n\rangle = \hbar\omega\left(n + \frac{1}{2}\right) \tag{13.45.1}$$

and that

$$|\psi\rangle = \frac{1}{\sqrt{14}}|1\rangle - \frac{2}{\sqrt{14}}|2\rangle + \frac{3}{\sqrt{14}}|3\rangle \tag{13.45.2}$$

For the Energy eigenstates, we calculate

$$H|1\rangle = \frac{3}{2}\hbar\omega|1\rangle \tag{13.45.3}$$

$$H|2\rangle = \frac{5}{2}\hbar\omega|2\rangle \tag{13.45.4}$$

$$H|3\rangle = \frac{7}{2}\hbar\omega|3\rangle \tag{13.45.5}$$

The Expectation Value is

$$\begin{aligned}\langle\psi|H|\psi\rangle &= \frac{1}{14}\frac{3}{2}\hbar\omega + \frac{4}{14}\frac{5}{2}\hbar\omega + \frac{9}{14}\frac{7}{2}\hbar\omega \\ &= \frac{3}{28}\hbar\omega + \frac{20}{28}\hbar\omega + \frac{63}{28}\hbar\omega \\ &= \frac{43}{14}\hbar\omega\end{aligned} \tag{13.45.6}$$

**Answer: (B)**

## 13.46 de Broglie Wavelength

The Energy of a particle can be related to its momentum by

$$E = \frac{p^2}{2m} \tag{13.46.1}$$

The de Broglie Relationship is

$$\lambda = \frac{h}{p} \tag{13.46.2}$$

Substituting eq. (13.46.1) into eq. (13.46.2), yields

$$\lambda = \frac{h}{\sqrt{2mE}} \tag{13.46.3}$$

[10]This question seems to be designed to trip you up and make you focus on irrelevant details.

The particle enters a region of potential, $V$. So

$$E' = E - V \tag{13.46.4}$$

This changes the de Broglie Wavelength of the particle such that

$$\lambda' = \frac{h}{\sqrt{2mE'}} \tag{13.46.5}$$

Dividing eq. (13.46.5) by eq. (13.46.3), yields

$$\begin{aligned}
\frac{\lambda'}{\lambda} &= \frac{h}{\sqrt{2m(E-V)}}\frac{\sqrt{2mE}}{h} \\
\Rightarrow \lambda' &= \frac{\lambda\sqrt{2mE}}{\sqrt{2m(E-V)}} \\
&= \lambda\left(1 - \frac{V}{E}\right)^{-\frac{1}{2}}
\end{aligned} \tag{13.46.6}$$

**Answer: (E)**

## 13.47 Entropy

Our container is sealed and thermall insulated. This means that the temperature throughout the process remains the same. You may recall that the work done by an isothermal process is

$$\boxed{W = nRT\ln\left[\frac{V_f}{V_i}\right]} \tag{13.47.1}$$

Where $V_f = 2V_i$. As the temperature remains the same, there is no change in internal energy. The First Law of Thermodynamics says

$$\boxed{dU = -dW + dQ} \tag{13.47.2}$$

where $dU = 0$, so

$$dQ = dW \tag{13.47.3}$$

The Entropy of a system is defined as

$$\boxed{dS = \frac{dQ}{T}} \tag{13.47.4}$$

eq. (13.47.4) becomes

$$\begin{aligned}
dS &= \frac{nRT\ln(2)}{T} \\
&= nR\ln 2
\end{aligned} \tag{13.47.5}$$

**Answer: (B)**

## 13.48 RMS Speed

The $v_{\text{rms}}$ of a gas is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \tag{13.48.1}$$

There is an inverse relationship between the rms speed,$v_{\text{rms}}$, and the molar mass, $M$. The Molar Masses of Oxygen and Nitrogen are $64u$ and $56u$ respectively.

$$\begin{aligned} v_{\text{rms}} &\propto \sqrt{\frac{1}{M}} \\ \Rightarrow \frac{v_{\text{rms}}(N_2)}{v_{\text{rms}}(O_2)} &= \sqrt{\frac{M_{O_2}}{M_{N_2}}} \\ &= \sqrt{\frac{64}{56}} \\ &= \sqrt{\frac{8}{7}} \end{aligned} \tag{13.48.2}$$

Answer: C)

## 13.49 Partition Function

The Partition Function is defined

$$Z = \sum_j g_j \cdot e^{-\beta E_j} \tag{13.49.1}$$

where

$$\beta = \frac{1}{k_B T}$$

$$g_j = \text{degeneracy for each state}$$

So

$$\begin{aligned} Z &= 2e^{-\frac{\epsilon}{k_B T}} + 2e^{-\frac{2\epsilon}{k_B T}} \\ &= 2\left[e^{-\frac{\epsilon}{k_B T}} + e^{-\frac{2\epsilon}{k_B T}}\right] \end{aligned} \tag{13.49.2}$$

**Answer: (E)**

## 13.50 Resonance of an Open Cylinder

We don't need to recall the resonance formula for an Open Cylinder to solve this problem. We do need to realize that the wavelength of the soundwave will remain the same as we are assuming that the dimensions of the cylinder will not change. We know

$$v = f\lambda \tag{13.50.1}$$

At 20°C, we have

$$v_1 = f_1\lambda \tag{13.50.2}$$

The speed of sound is 3% lower, so

$$v_2 = 0.97v_1 \tag{13.50.3}$$

The resonant frequency of the pipe on a cold day becomes

$$\begin{aligned} v_2 &= f_2\lambda \\ 0.97v_1 &= f_2\lambda \\ f_2 &= 0.97f_1 \\ &= 427\text{Hz} \end{aligned} \tag{13.50.4}$$

**Answer: (B)**

## 13.51 Polarizers

The Law of Malus gives us the intensity of a beam of light after it passes through a polarizer. This is given by

$$I = I_0\cos^2\theta_i \tag{13.51.1}$$

A beam of light is a mixture of polarizations at all possible angles. As it passes through a polarizer, half of these vectors will be blocked. So the intensity after passing through the first polarizer is

$$\frac{I_1}{I_0} = \frac{1}{2} \tag{13.51.2}$$

Each polarizer reduces the intensity of the light beam by a factor of $\frac{1}{2}$. With $n$ polarizers, we can say

$$\frac{I_n}{I_0} = \left(\frac{1}{2}\right)^n \tag{13.51.3}$$

Where $n = 3$, we substitute into eq. (13.51.3) and get

$$\begin{aligned} \frac{I}{I_0} &= \left(\frac{1}{2}\right)^3 \\ &= \frac{1}{8} \end{aligned} \tag{13.51.4}$$

**Answer: (B)**

## 13.52 Crystallography

We are told that the volume of the cube is

$$V = a^3 \tag{13.52.1}$$

For a cube, each corner has 1/8 of an atom. In the BCC case, we also have an atom in the center. So there are a total of two atoms in our BCC crystal's primitive unit cell. The volume of this primitive unit cell is $V/2 = a^3/2$.

**Answer: (C)**

## 13.53 Resistance of a Semiconductor

To best answer this it helps to know some things about semicondictors. Semiconductors are more closer to insulators than conductors, the only difference being their energy levels. Typically, an insulator requires a lot of energy to break an electron free from an atom (typically about 10eV), while a semiconductor requires about 1eV.

When a semiconductor is 'cold', all its electrons are tightly held by their atoms. When the substance is heated, the energy liberates some electrons and he substance has some free electrons; it conducts. The more energy the more electrons freed. So we are looking for a relationship where **the conductivity increases with temperature**.

**Answer: (A)**

## 13.54 Impulse

The Impulse is defines as

$$\mathbf{J} = \int \mathbf{F} \cdot dt \tag{13.54.1}$$

On a $F$ vs. $t$ graph, the Impulse will be the area under the curve.

The area under the graph is

$$\mathbf{J} = \frac{2 \times 2}{2} = 2\text{kg.m/s} \tag{13.54.2}$$

**Answer: (C)**

## 13.55 Fission Collision

Once masses split up or fuse energy is not conserved but we know that momentum is always conserved. Horizontal Momentum

$$\begin{aligned} mv &= 2mv' \cos\theta \\ \Rightarrow v' &= \frac{v}{2\cos\theta} \end{aligned} \tag{13.55.1}$$

Vertical Momentum

$$0 = mv' \sin\theta - mv' \sin\theta \tag{13.55.2}$$

The value of $\theta$ can be

$$0^\circ \leqslant \theta \tag{13.55.3}$$

Plugging eq. (13.55.3) into eq. (13.55.1), and we see that

$$v' \geqslant \frac{v}{2} \tag{13.55.4}$$

**Answer: (E)**

## 13.56 Archimedes' Principal and Buoyancy

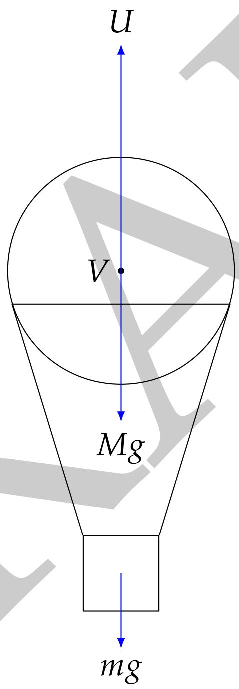



Figure 13.56.1: Diagram of Helium filled balloon attached to a mass

Archimedes' Principle states that when an object is fully or partiall immersed in a fluid, the upthrust acting on it is equal to the weight of fluid displaced. If we neglected the weight of the balloon, we see from fig. 13.56.1, for the helium balloon to just float our mass

$$U - Mg - mg = 0 \tag{13.56.1}$$

where $U$ is the upthrust, $M$ is the mass of helium and $m$ is the mass to be suspended. Given the density of helium, $\rho_{\mathrm{He}} = 0.18\,\mathrm{kg\,m^{-3}}$ and the density of air, $\rho_{\mathrm{air}} = 1.29\,\mathrm{kg\,m^{-3}}$, we have

$$U = \rho_{\mathrm{air}} V g \tag{13.56.2}$$

and

$$M = \rho_{\mathrm{He}} g \tag{13.56.3}$$

where $V$ is the volume of Helium used and air displaced. Substituting eqs. (13.56.2) and (13.56.3) into eq. (13.56.1) and simplifying, we get

$$V = \frac{m}{\rho_{\mathrm{air}} - \rho_{\mathrm{He}}} \tag{13.56.4}$$

which works out to be

$$\begin{aligned} V &= \frac{300}{1.29 - 0.18} \\ &= 270\,\mathrm{m}^3 \end{aligned} \tag{13.56.5}$$

**Answer: (D)**

## 13.57 Fluid Dynamics

The force on the wall is found from Newtin's Second Law

$$F = \frac{dp}{dt} \tag{13.57.1}$$

The momentum is defined as

$$p = mv \tag{13.57.2}$$

So

$$\begin{aligned} F &= \frac{dp}{dt} \\ &= \overbrace{m \cdot dv}^{=0} + v \cdot dm \\ &= v \cdot dm \end{aligned} \tag{13.57.3}$$

We need to calculate $dm$, the density of fluid is

$$\rho = \frac{M}{V} = \frac{dm}{dV} \tag{13.57.4}$$

Substituting this into eq. (13.57.3), we get

$$\begin{aligned} F &= v\rho\frac{dV}{dt} \\ &= v\rho A\frac{dx}{dt} \\ &= v^2\rho A \end{aligned} \tag{13.57.5}$$

**Answer: (A)**

## 13.58 Charged Particle in an EM-field

The Forces on a negatively charged particle in Electric and Magnetoic Fields are described by the Lorentz law.

$$\mathbf{F} = q\left(\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right) \tag{13.58.1}$$

In the first case, the electron is undeflected, so we can write

$$\mathbf{F_1} = e\left[\mathbf{E} + \mathbf{v_1} \times \mathbf{B}\right] = 0 \tag{13.58.2}$$

Vectorially our directions are

$$\mathbf{E} = E\hat{\mathbf{i}} \tag{13.58.3}$$

$$\mathbf{v} = v\hat{\mathbf{k}} \tag{13.58.4}$$

$$\mathbf{B} = B\hat{\mathbf{j}} \tag{13.58.5}$$

eq. (13.58.2) becomes

$$\begin{aligned}\mathbf{F_1} &= e\left[E\hat{\mathbf{i}} + \left(v_1\hat{\mathbf{k}} \times B\hat{\mathbf{j}}\right)\right]\\ &= e\left[E\hat{\mathbf{i}} + v_1 B\left(\hat{\mathbf{k}} \times \hat{\mathbf{j}}\right)\right]\\ &= e\left[E\hat{\mathbf{i}} - v_1 B\hat{\mathbf{i}}\right]\\ &= 0\end{aligned} \tag{13.58.6}$$

eq. (13.58.6) is balanced so

$$E - v_1 B = 0 \tag{13.58.7}$$

We are told that the accelerating potential is doubled, so the speed at which the electron enters is

$$\begin{aligned}\frac{1}{2}mv_1^2 &= eV\\ \Rightarrow v_1 &= \sqrt{\frac{2eV}{m}}\\ \therefore v_2 &= v_1\sqrt{2}\end{aligned} \tag{13.58.8}$$

Since $v_2 > v_1$, we can see that

$$\mathbf{F_2} = e\left[E\hat{\mathbf{i}} - v_1 B\sqrt{2}\,\hat{\mathbf{i}}\right] < 0\hat{i} \tag{13.58.9}$$

The electron will move in the negative x-direction.

**Answer: (B)**

## 13.59 LC Circuits and Mechanical Oscillators

We are given

$$L\ddot{Q} + \frac{1}{C}Q = 0 \tag{13.59.1}$$

For a mechanical oscillator,

$$m\ddot{x} + kx = 0 \tag{13.59.2}$$

Comparing both equations we see that

$$L = m \tag{13.59.3}$$

$$\frac{1}{C} = k \tag{13.59.4}$$

$$\text{and } Q = x \tag{13.59.5}$$

**Answer: (B)**

## 13.60 Gauss' Law

Gauss' Law states that the electric flux through any Gaussian surface is proportional to the charge it encloses.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{Enclosed}}}{\epsilon_0} \tag{13.60.1}$$

The charge density, $\sigma$, is

$$\sigma = \frac{Q}{A} \tag{13.60.2}$$

We need to find the area that the Gaussian Surface encloses on the carged sheet. The Gaussian Surface encompasses a circle of radius $(R^2 - x^2)$. So the charge enclosed is

$$\begin{aligned} Q_{\text{Enclosed}} &= \sigma A \\ &= \sigma\pi\left(R^2 - x^2\right) \end{aligned} \tag{13.60.3}$$

The Electric Flux is

$$\begin{aligned} \Phi &= \frac{Q_{\text{Enclosed}}}{\epsilon_0} \\ &= \frac{\sigma\pi\left(R^2 - x^2\right)}{\epsilon_0} \end{aligned} \tag{13.60.4}$$

**Answer: (C)**

## 13.61 Electromagnetic Boundary Conditions

The boundary conditions for Electrodynamics can be expressed

$$\begin{aligned} \epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} &= \sigma \quad \mathbf{E_1^{\parallel}} - \mathbf{E_2^{\parallel}} &&= 0 \\ B_1^{\perp} - B_2^{\perp} &= 0 \quad \frac{1}{\mu_1}\mathbf{B_1^{\parallel}} - \frac{1}{\mu_2}\mathbf{B_2^{\parallel}} &&= 0 \end{aligned}$$

We are given

$$E = E_0 \cos(kx - \omega t) \tag{13.61.1}$$

We are told that we have a perfect conductor

## 13.62 Cyclotron Frequency

The cetripetal force is equal to the transverse magnetic field. So

$$BQv = mr\omega^2 \tag{13.62.1}$$

Solving for $m$, gives

$$m = \frac{BQ}{2\pi f} \tag{13.62.2}$$

Plugging what we know and we get
**Answer: (A)**

## 13.63 Wein's Law

Wein's Law tells us there is an inverse relationship between the peak wavelength of a blackbody and its temperature. It says

$$\boxed{\lambda_{\text{Max}} T = \text{const.}} \tag{13.63.1}$$

From the graph, we see that the peak wavelength is approximately $2\mu$m. Plugging this into eq. (13.63.1), we get

$$\begin{aligned} T &= \frac{2.9 \times 10^{-3}}{2.0 \times 10^{-6}} \\ &= 1.45 \times 10^{3}\text{K} \end{aligned} \tag{13.63.2}$$

**Answer: (D)**

## 13.64 Electromagnetic Spectra

This question tests your knowledge of Electromagnetic Radiation and its properties.

**A** Infra-red, Ultraviolet and Visible Light emissions occur at the electron level. You would typically expect higher EM radiation levels to occur at the nuclear level. **NOT CORRECT**

**B** The wavelengths in the absorbtion spectra are the same for emission. They are in a sense, the negative image of each other. **Correct**

**C** This is true. We do analyse the spectral output of stars to determine its composition. **Correct**

**D** Again this is also true. Once it interacts with photons we can detect it. **Correct**

**E** Molecular Spectral lines are so close to each other they often appear to be bands of 'color'; their interaction is much richer than that of elements which often appear as spectral lines. **Correct**

So by the process of knowledgeable elimination, we have
**Answer: (A)**

## 13.65 Molar Heat Capacity

We are given that Einstein's Formula for Molar Heat Capacity is

$$C = 3kN_A\left(\frac{h\upsilon}{kT}\right)^2\frac{e^{\frac{h\upsilon}{kT}}}{\left(e^{\frac{h\upsilon}{kT}}-1\right)^2} \tag{13.65.1}$$

We recall that

$$e^x \approx 1 + x \tag{13.65.2}$$

So we can simplify

$$\frac{e^{\frac{h\upsilon}{kT}}}{\left(e^{\frac{h\upsilon}{kT}}-1\right)^2} = \frac{1+\frac{h\upsilon}{kT}}{\left(\frac{h\upsilon}{kT}\right)^2} \tag{13.65.3}$$

Plugging in eq. (13.65.3) into eq. (13.65.1), we have

$$C = 3kN_A\left[1+\frac{h\upsilon}{kT}\right] \tag{13.65.4}$$

As $T \to \infty$, $\frac{h\upsilon}{kT} \to 0$,

$$C = 3kN_A \tag{13.65.5}$$

## 13.66 Radioactive Decay

The total decay rate is equal to the sum of all the probable decay rates. If you didn't know this, some quick calculation would show this. So for an exponential decay

$$\frac{dN}{dt} = -\lambda N \tag{13.66.1}$$

Solving this, we have

$$N = N_0 e^{-\lambda T} \tag{13.66.2}$$

Let's say that there are two decay modes or channels along which our particle can decay, we have

$$\begin{aligned}\frac{dN}{dt} &= -\lambda_1 N - \lambda_2 N \\ &= -N\left(\lambda_1+\lambda_2\right)\end{aligned} \tag{13.66.3}$$

Solving gives us,

$$N = N_0 e^{-(\lambda_1+\lambda_2)T} \tag{13.66.4}$$

From the above two equations, we can see that we add the decay constants,

$$\lambda_C = \lambda_1 + \lambda_2 + \cdots \tag{13.66.5}$$

We also know that the mean time, $\tau$, is

$$\tau = \frac{1}{\lambda} \tag{13.66.6}$$

So

$$\frac{1}{\tau_C} = \frac{1}{\tau_1} + \frac{1}{\tau_2} \tag{13.66.7}$$

This gives us

$$\begin{aligned}\frac{1}{\tau} &= \frac{1}{24} + \frac{1}{36} \\ \tau &= \frac{\tau_1\tau_2}{\tau_1 + \tau_2} \\ &= \frac{24 \cdot 36}{24 + 36} \\ &= 14.4\end{aligned} \tag{13.66.8}$$

**Answer: (D)**

## 13.67 Nuclear Binding Energy

Nuclei are made up of protons and neutrons but the sum of the individual masses is less than the actual mass of the nucleus. The Energy of this 'missing' mass is what holds the nucleus together and is known as the Binding Energy. As a heavy nucleus splits or undergoes fission, some of this energy is released.

$$U_i - U_f = K = 200 \text{ MeV} \tag{13.67.1}$$

So

$$\begin{aligned}U_f &= U_i - K \\ &= 238(7.8) - 200\end{aligned} \tag{13.67.2}$$

eq. (13.67.2) refers to the total energy holding the nucleus together. To find the Binding Energy per nucleon we divide $U_f$ by 238. To make this simpler and to save precious time, let's say there were 240 nucleons and their binding energy was 8 Mev/nucleon[11] Thus

$$\frac{(240)(8) - 200}{240} \tag{13.67.3}$$

We can see the binding energy for a nucleus, $A = 120$, is less than 8MeV/nucleon.

**Answer: (D)**

## 13.68 Radioactive Decay

We are told that Beryllium decays to Lithium. By looking at this process we expect it to be some sort of $\beta$-decay process. Electron Capture is a type of a $\beta$-decay. The decay looks like this

$${}^7_4Be + {}^0_{-1}e \longrightarrow {}^7_3Li + \nu \tag{13.68.1}$$

[11] This actually works out to be about 6.96MeV/nucleon. Binding Energy peaks around Iron which has a binding energy of 8.8 MeV/nucleon and an atomic mass, $A = 55$. If you knew this you won't have had to work anything out.

where $v$ is a neutrino.

**Answer: (E)**

## 13.69 Thin Film Interference

Since the refractive index of glass is higher than that of oil, the maxima can be found by

$$2nL_{min} = m\lambda \tag{13.69.1}$$

where $n$ is the refractive index of the oil film and $m$ to its order. Thus

$$\begin{aligned} 2(1.2)L_{min} &= 1(480 \times 10^{-9}) \\ \therefore L_{min} &= 200 \times 10^{-9} \text{ m} \end{aligned} \tag{13.69.2}$$

**Answer: (B)**

## 13.70 Double Slit Experiment

Young's Double Slit Equation states for constructive interference

$$d \sin\theta = m\lambda \tag{13.70.1}$$

If we take into account the distance, $D$, of the screen and finge seperation, $\Delta y$, we get

$$\Delta y = \frac{m\lambda D}{d} \tag{13.70.2}$$

In terms of frequency, $v$, there is an inverse relationship with the fringe seperation.

$$\Delta y = \frac{mcD}{dv} \tag{13.70.3}$$

So doubling the frequency, $v$, will half the finge seperation.

**Answer: (B)**

## 13.71 Atomic Spectra and Doppler Red Shift

The Relativistic Dopler Shift (Red) is

$$\bar{v} = v_0 \sqrt{\frac{1-\beta}{1+\beta}} \tag{13.71.1}$$

where $\beta = \frac{u}{c}$ and let $r = \sqrt{\frac{1-\beta}{1+\beta}}$

$$\begin{aligned} 121.5r &= 607.5 \\ r &\approx 5 \\ \therefore v &\approx \frac{12}{13}c \end{aligned} \tag{13.71.2}$$

As the wavelength is longer, it is red-shifted and thus moving away from the observer. We also expect the speed to be close to $c$.

**Answer: (D)**

## 13.72 Springs, Forces and Falling Masses

Before the string is cut, the system is in equilibrium. Let us call the tension on the top string, $T_1$ and the tension on the spring, $T_2$. Let's also call the top and bottom masses, $M_1$ and $M_2$ respectively. Thus

$$T_1 - M_1 g - M_2 g = 0 \tag{13.72.1}$$

$$T_2 = M_2 g \tag{13.72.2}$$

After the string is cut, $T_1$ is now zero. The spring, pulls on the top mass with the froce due to its extension. Thus

$$\begin{aligned} M_1 a &= -M_1 g - T_2 \\ &= -M_1 g - M_2 g \\ \Rightarrow a &= -2g \end{aligned} \tag{13.72.3}$$

**Answer: (E)**

## 13.73 Blocks and Friction

The Force used to push the blocks is

$$F = (M_A + M_B)\, a \tag{13.73.1}$$

The Reaction on the block, $B$, is

$$R = M_B a \tag{13.73.2}$$

Since block $B$ doesn't move, we can say

$$M_B g - \mu R = 0 \tag{13.73.3}$$

Substituting eq. (13.73.1) and eq. (13.73.2) into the equation gives us

$$M_B g - \mu M_B \left[\frac{F}{M_A + M_B}\right] = 0$$

$$\Rightarrow F = \frac{g\,(M_A + M_B)}{\mu} \tag{13.73.4}$$

We get $F = 40g$

**Answer: (D)**

## 13.74 Lagrangians

The Langrangian for the system is

$$L = a\dot{q}^2 + bq^4 \tag{13.74.1}$$

The equation of motion is

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \tag{13.74.2}$$

Solving, we get

$$\frac{\partial L}{\partial q} = 4bq^3 \tag{13.74.3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 2a\ddot{q} \tag{13.74.4}$$

eq. (13.74.3) is equal to eq. (13.74.4)

$$\begin{aligned} 2a\ddot{q} &= 4bq^3 \\ \ddot{q} &= \frac{2b}{a}q^3 \end{aligned} \tag{13.74.5}$$

**Answer: (D)**

## 13.75 Matrix Transformations & Rotations

We notice that in the transformation matrix, that $a_{33} = 1$. The coordinate in the z-axis remains unchanged and thus we expect a rotation about this point. The Rotation Matrix about the z-axis is of the form

$$A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13.75.1}$$

From the above, we see that

$$\cos\theta = \frac{1}{2} \tag{13.75.2}$$

$$\sin\theta = \frac{\sqrt{3}}{2} \tag{13.75.3}$$

Solving

$$\theta = 60^\circ \tag{13.75.4}$$

As positive rotations are in the counter-clockwise direction
**Answer: (E)**

## 13.76 Fermi Gases

Electrons are fermions and follow Fermi-Dirac Statistics. This means that they follow the Pauli Exclusion Principle; every fermion must have a unique quantum state. This means that the total energy of the Fermi gas at zero temperature will be larger than the product of the number of particles and the single-particle ground state energy; the fermions will occupy all states from ground state up until all the quantum states are occupied.

**Answer: ((C)**

## 13.77 Maxwell-Boltzmann Distributions

Recall the Partition Function

$$Z = \sum_j g_j \cdot e^{-\frac{\epsilon}{kT}} \tag{13.77.1}$$

The degeneracies, $g_j$, are the same for both states. So the ratio between bot states becomes

$$\frac{Z_a = e^{-\frac{\epsilon_b+0.1}{kT}}}{Z_b = e^{-\frac{0.1}{kT}}} \tag{13.77.2}$$

Simplifying, this becomes

$$\begin{aligned} \frac{Z_a}{Z_b} &= e^{-\frac{0.1}{kT}} \\ &= e^{-4} \end{aligned} \tag{13.77.3}$$

**Answer: (E)**

## 13.78 Conservation of Lepton Number and $\mu^-$ Decay

Muons are elementary particles, similar to electrons that decay via the weak interaction

$$\mu^- = e^- + v_\mu + v_e \tag{13.78.1}$$

We can analyze each choice in turn and eliminate

**Charge** The muon is best described as a heavy electron. The neutrino on the other hand has no charge. So the charges in the above reaction are

$$-1 = -1 + 0 \tag{13.78.2}$$

They balance, so charge is conserved.

**Mass** This one is a bit trickier. We know the muon mass is about 200 times the electron. Neutrinos on the other hand have a small but nonzero mass. Is it enough to complete this reaction? Maybe

**Energy and Momentum** It may be possible to kinematically make this work.

**Baryon Number** Baryons are a list of composite particles; they are made up of quarks. Muons, neutrinos, and the other particles mentioned in this question are elementary particles. Thus, the Baryon number for all are, $B = 0$. So

$$0 = 0 + 0 \tag{13.78.3}$$

So there is conservation of Baryon number.

Incidentally, the Baryon number can be found by knowing the component quarks and antiquarks.

$$B = \frac{n_q - n_{\bar{q}}}{3} \tag{13.78.4}$$

where $n_q$ is the number of constituent quarks and $n_{\bar{q}}$ the number of constituent antiquarks.

**Lepton Number** There are several ways that lepton number must be conserved in a reaction. We can add/subtract the number of leptons and antileptons at the beginning and end of the reaction and see if it is conserved.

**Answer: ((E)**

## 13.79 Rest Mass of a Particle

The Relativistic Energy is the sum of its Rest Mass, $m_0$, and its momentum, $p$. Thus

$$E = \sqrt{p^2c^2 + m_0^2c^4} \tag{13.79.1}$$

We are given that

$$\begin{aligned} E &= 10 \text{ GeV} \\ p &= 8 \text{ GeV/c} \end{aligned}$$

Substituting $E$ and $p$ into eq. (13.79.1), we have

$$\begin{aligned} 10^2 &= 8^2 + m_0^2c^2 \\ 100 - 64 &= m_0^2c^2 \\ \Rightarrow m_0 &= 6 \text{ GeV/c}^2 \end{aligned} \tag{13.79.2}$$

**Answer: (D)**

## 13.80 Relativistic Addition of Velocities

The Relativistic Addition of Velocities is

$$u_x = \frac{u_s' + v}{1 + \frac{u_s' v}{c^2}} \tag{13.80.1}$$

Substituting

$$v = \frac{c}{2} \tag{13.80.2}$$
$$u'_x = \frac{c}{n} = \frac{3c}{4} \tag{13.80.3}$$

We get

$$u_x = \frac{10c}{11} \tag{13.80.4}$$

**Answer: (D)**

## 13.81 Angular Momentum

The orbital angular momentum is

$$L^2 = \ell(\ell+1)\hbar^2 \tag{13.81.1}$$

and the z-component of the angular momentum in terms of the magnetic quantum number is

$$L_z = m_\ell \hbar \tag{13.81.2}$$

We are told that

$$L^2 = 6\hbar^2 \tag{13.81.3}$$
$$L_z = -\hbar \tag{13.81.4}$$

Solving for the above equations gives us

$$\ell(\ell+1) = 6 \tag{13.81.5}$$
$$m = -1 \tag{13.81.6}$$

Solving for $\ell$, gives

$$\ell = -3; 2 \tag{13.81.7}$$

It's not possible to have negative numbers for $\ell$, so we are left with $\ell = 2$ and $m = -1$. This gives us $Y_2^{-1}(\theta, \phi)$.

**Answer: (B)**

## 13.82 Addition of Angular Momentum

Addition of Angular Momentum

**NOT FINISHED pp.185**

## 13.83 Spin Basises

Spin Basises **NOT FINISHED**

## 13.84 Selection Rules

The selection rules for an electric-dipole transition are

| | |
|---|---|
| $\Delta\ell = \pm 1$ | Orbital angular momentum |
| $\Delta m_\ell = 0, \pm 1$ | Magnetic quantum number |
| $\Delta m_s = 0$ | Secondary spin quantum number |
| $\Delta j = 0, \pm 1$ | Total angular momentum |

We can examine the transitions to see which are valid

**Transition A** This transition goes from

$$n = 2 \qquad \text{to} \quad n = 1$$

and

$$\ell = 0 \quad \text{to} \quad \ell = 0$$

We see that this transition is forbidden[12].

**Transition B** This transition goes from

$$\ell = 1 \quad \text{to} \quad \ell = 0$$

and

$$j = \frac{3}{2} \quad \text{to} \quad j = \frac{1}{2}$$

This leaves us with the transitions

$$\Delta\ell = -1 \quad \text{and} \quad \Delta j = -1$$

These are allowed transitions.

**Transition C** The transition goes from

$$j = \frac{1}{2} \quad \text{to} \quad j = \frac{1}{2}$$

This leaves us with the transition

$$\Delta j = 0$$

Which is a valid transition[13].

From the above, we see that transitions **B** & **C** are the only valid ones.

**Answer: (D)**

[12] Not forbidden really, just highly unlikely.

[13] $\Delta j = 0$ is a valid transition as long as you don't have the $j = 0 \rightarrow j = 0$ transition. The vector angular momentum must change by one unit in a electronic transition and this can't happen when $j = 0 \rightarrow j = 0$ because there is no total angular momentum to re-orient to get a change of 1

## 13.85 Resistivity

Ohm's Law gives the Resistance to be

$$R = \frac{\rho L}{A} \tag{13.85.1}$$

We can use this to calculate the individual resistances of the two wires in series.

$$R_1 = \rho \frac{2L}{A} \tag{13.85.2}$$
$$R_2 = \rho \frac{L}{2A} \tag{13.85.3}$$

Circuit [14] The Potential Difference across $R_1$ is

$$\left(\frac{R_1}{R_1 + R_2}\right)\Delta V = V_1 \tag{13.85.4}$$

$V_1$ is the potential difference across $R_1$. To get the Potential at $A$, we let

$$V_1 = 8.0 - V \tag{13.85.5}$$

Substituting eq. (13.85.5) into eq. (13.85.4), we get

$$\begin{aligned} 8.0 - V &= \left(\frac{R_1}{R_1 + R_1}\right)\Delta V \\ 8.0 - V &= \frac{2}{3}(7) \\ V &= 3\frac{1}{3} \end{aligned} \tag{13.85.6}$$

**Answer: (B)**

## 13.86 Faraday's Law

Faraday's Law tells us that the induced EMF is equal to the rate of change of Magnetic Flux through a circuit. Thus

$$E = -\frac{d\Phi}{dt} \tag{13.86.1}$$

The Magnetic Flux is defined as

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A} \tag{13.86.2}$$

At $t = 0$, the plane of the coil is in the $Y$ direction. So

$$B = B_0 \sin \omega t \tag{13.86.3}$$

[14] Put Schematic of Resistors in Series here

So

$$\Phi = \int B_0 \sin \omega t \cdot \pi r^2 \tag{13.86.4}$$

Plugging this into Faraday's Law, we have

$$E = \omega \pi B r^2 \cos \omega t \tag{13.86.5}$$

Each loop has an induced EMF as described in eq. (13.86.5). So $n$ loops will have a total EMF of $nE$. We can use Ohm's Law to find the current,

$$\begin{aligned} I &= \frac{nE}{R} \\ &= \frac{n\omega B r^2 \pi}{R} \cos \omega t \\ &= 250\pi \times 10^{-2} \cos \omega t \end{aligned} \tag{13.86.6}$$

**Answer: (E)**

## 13.87 Electric Potential

The Electric Potential inside a charged sphere is zero as there is no Electric Field present. So the Electric Force exerted on the positive charge, $Q$, by the sphere is also zero. So we just have to consider the force exerted by the opposite sphere. The distance of the charge, $Q$, from the center of the opposing sphere is

$$x = 10d - \frac{d}{2} \tag{13.87.1}$$

The Electric Field is defined by Coulomb's Law

$$F = \frac{1}{4\pi\epsilon_0} \frac{qQ}{x^2} \tag{13.87.2}$$

So the force exerted on the charge is

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{qQ}{\left(10d - \frac{d}{2}\right)} \\ &= \frac{qQ}{361\pi\epsilon_0 d^2} \end{aligned} \tag{13.87.3}$$

Both charges $q$ and $Q$ are the same so the force is repulsive, i.e. acts to the left.
**Answer: (A)**

## 13.88 Biot-Savart Law

The Biot-Savart Law is

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\ell \times \mathbf{r}}{r^2} \tag{13.88.1}$$

Along the straight parts, the wire is parallel to the radius vector and hence the cross product is zero; there is no magnetic contribution. But along the arc, the wire's path is perpendicular to the radius vector, so there is contribution. Hence we will only look at the arc. The length of arc, $d\ell$, is

$$d\ell = R\theta \tag{13.88.2}$$

Substituting into the Biot-Savart Equation gives

$$\begin{aligned} B = \int_0^\theta dB &= \frac{\mu_0 I}{4\pi} \cdot \frac{R\theta}{R^2} \\ &= \frac{\mu_0 I}{4\pi} \frac{\theta}{R} \end{aligned} \tag{13.88.3}$$

**Answer: (C)**

## 13.89 Conservation of Angular Momentum

We can find the Total Moment of Inertia of the child–merry-go-round system by using the Parallel Axis Theorem.

$$\begin{aligned} I_i &= I_d + I_c \\ &= \frac{1}{2} M_d R^2 + M_c R^2 \\ &= R^2 \left[ \frac{1}{2} M_d + M_c \right] \\ &= 400 \cdot 2.5^2 \end{aligned} \tag{13.89.1}$$

The final Moment of Inertia is just that of a disc. Moment of Inertia deals with how a mass is distributed and taking the child as a point mass means that his rotation can be ignored. Thus

$$\begin{aligned} I_f &= \frac{1}{2} M_d R^2 \\ &= 100 \cdot 2.5^2 \end{aligned} \tag{13.89.2}$$

Angular Momentum is conserved, thus

$$\begin{aligned} I_i \omega_i &= I_f \omega_f \\ \Rightarrow \omega_f &= \frac{I_i}{I_f} \omega_i \\ &= \frac{140}{100} 2 = 2.8 \text{ rad/s} \end{aligned} \tag{13.89.3}$$

**Answer: (E)**

## 13.90 Springs in Series and Parallel

The Period of a Mass-Spring System is

$$\omega = \sqrt{\frac{k_e}{m}} \tag{13.90.1}$$

Now it comes down to solving the effective spring constants for springs in series and parallel. The addition of springs in series and parallel are the same as capacitors but if you didn't recall this you can solve the relationships.

### 13.90.1 Springs in Parallel

In the parallel case, both springs extend by the same amount, $x$. The Forces on both springs also add up sych that

$$\begin{aligned} F &= F_1 + F_2 \\ &= -k_1 x - k_2 x \end{aligned} \tag{13.90.2}$$

This Parallel arrangement is the same as a Mass-Spring system with only one spring of spring constant, $k_e$. We have

$$F = -k_e x \tag{13.90.3}$$

(eq. (13.90.2)) = (eq. (13.90.3)), gives

$$\boxed{k_e = k_1 + k_2} \tag{13.90.4}$$

### 13.90.2 Springs in Series

Springs in Series is a bit more challenging. For this case, we will assume light springs such that the tension throughout the springs is constant. So we have

$$\begin{aligned} F = -k_1 x_1 &= -k_2 x_2 \\ \Rightarrow \frac{k_1}{k_2} &= \frac{x_2}{x_1} \end{aligned} \tag{13.90.5}$$

This is equivalent to a single Spring system where we again have a single Spring System but

$$x = x_1 + x_2 \tag{13.90.6}$$

Since the Forces are equal, we can say

$$\begin{aligned} kx &= k_2 x_2 \\ k\left(x_1 + x_2\right) &= k_2 x_2 \\ \Rightarrow k &= \frac{k_1 k_2}{k_1 + k_2} \end{aligned} \tag{13.90.7}$$

We have thus shown that

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}} \tag{13.90.8}$$

The period of the Mass-Spring Sytem for the Series Arrangement becomes

$$\begin{aligned} k_e &= k_1 + k_2 = 2k \\ T_s &= 2\pi\sqrt{\frac{m}{k_e}} \\ &= 2\pi\sqrt{\frac{m}{2k}} \end{aligned} \tag{13.90.9}$$

The period for the Mass-Spring System in the Parallel arrangement becomes

$$\begin{aligned} \frac{1}{k} &= \frac{1}{k} + \frac{1}{k} \\ &= \frac{2}{k} \\ T_p &= 2\pi\sqrt{\frac{2m}{k}} \end{aligned} \tag{13.90.10}$$

The ratio between $T_p$ and $T_s$ is

$$\begin{aligned} \frac{T_p}{T_s} &= \frac{2\pi\sqrt{\frac{2m}{k}}}{2\pi\sqrt{\frac{m}{2k}}} \\ &= 2 \end{aligned} \tag{13.90.11}$$

**Answer: (E)**[15]

## 13.91 Cylinder rolling down an incline

As the cylinder rolls down the hill, Gravitational Potential Energy is converted to Translational Kinetic Energy and Rotational Kinectic Energy. This can be expressed as

$$\begin{aligned} PE_{\text{gravity}} &= KE_{\text{translational}} + KE_{\text{rotational}} \\ mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \end{aligned} \tag{13.91.1}$$

Solving for $I$, we have

$$I = \frac{MR^2}{v^2}\left(2gH - v^2\right) \tag{13.91.2}$$

Plugging in for $v$, we get

$$\begin{aligned} I &= MR^2\frac{7}{8gH}\left(2gH - \frac{8gH}{7}\right) \\ &= \frac{3}{4}MR^2 \end{aligned} \tag{13.91.3}$$

**Answer: (B)**

[15] Springs are like Capacitors; when in parallel, their spring constants add and when in series, the inverse of the total spring constant is the sum of the inverse of the individual ones.

## 13.92 Hamiltonian of Mass-Spring System

The Hamiltonian of a System is

$$\boxed{H = T + V} \tag{13.92.1}$$

where $T_i = \frac{p_i^2}{2m}$ and $V = V(q)$. So the Hamiltonian is the sum of the kinetic energies of the partiles and the energy stroed in the spring. Thus

$$H = \frac{1}{2}\left\{\frac{p_1^2}{m} + \frac{p_2^2}{m} + k\left(\ell - \ell_0\right)\right\} \tag{13.92.2}$$

**Answer: (E)**

## 13.93 Radius of the Hydrogen Atom

The radial probability density for the ground state of the Hydrogen atom is found by multiplying the square of the wavefunction by the spherical shell volume element.

$$\boxed{P_r = \int \left|\psi_0\right|^2 dV} \tag{13.93.1}$$

The Volume of a sphere is $V = \frac{4}{3}\pi r^3$, so $dV = 4\pi r^2$. From the above equation we see that

$$\frac{dP_r}{dr} = \frac{e^{-\frac{2r}{a_0}}}{\pi a_0^3} \cdot 4\pi r^2 \tag{13.93.2}$$

We find the maxima and thus the most probable position by determining $\frac{d^2P_r}{dr^2} = 0$. Differentiating eq. (13.93.2) gives

$$\frac{d^2P}{dr^2} = \frac{4}{a_0^3}\left[2r \cdot e^{-\frac{2r}{a_0}} - r^2 \cdot \frac{2}{a_0} \cdot e^{-\frac{2r}{a_0}}\right] = 0 \tag{13.93.3}$$

Solving for $r$ gives

$$r = a_0 \tag{13.93.4}$$

This is Bohr's Radius which was found using semi-classical methods. In this case, Schroedinger's Equation confirms the first Bohr radius as the most probable radius and more; the semi-classical Bohr's Theory does not.

**Answer: (C)**

## 13.94 Perturbation Theory

Perturbation Theory **NOT FINISHED**

## 13.95 Electric Field in a Dielectric

The Electric Field has magnitude

$$\boxed{E = \frac{1}{4\pi\kappa\epsilon_0}\frac{q}{r^2} = \frac{\sigma}{\kappa\epsilon_0}} \tag{13.95.1}$$

In a vacuum, $\kappa = 1$ and the strength of the Electric Field is $E_0$. So

$$E = \frac{E_0}{\kappa} \tag{13.95.2}$$

**Answer: (A)**

## 13.96 EM Radiation

Though the size of the sphere oscillates between $R_1$ and $R_2$, the charge remains the same. So the power radiated is zero.

**Answer: (E)**

## 13.97 Dispersion of a Light Beam

The Angular Spread of the light beam can be calculated by using Snell's Law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{13.97.1}$$

For an Air-Glass system, this becomes

$$\sin\theta_1 = n \sin\theta' \tag{13.97.2}$$

We know that for dispersion to take place then

$$\theta' = \theta'(\lambda) \tag{13.97.3}$$

$$n = n(\lambda) \tag{13.97.4}$$

Differentiating eq. (13.97.2) with respect to $\lambda$, we have

$$\begin{aligned}
\frac{d}{d\lambda}(\sin\theta) &= \frac{d}{d\lambda}(n\sin\theta') \\
0 &= \frac{d}{d\lambda}(n\sin\theta') \\
&= \frac{dn}{d\lambda}\sin\theta' + n\frac{d\theta'}{d\lambda}\cos\theta' \\
\Rightarrow \frac{d\theta'}{d\lambda} &= \frac{1}{n}\frac{dn}{d\lambda}\tan\theta' \\
\therefore \delta\theta' &= \left|\frac{1}{n}\frac{dn}{d\lambda}\tan\theta'\delta\lambda'\right| \qquad (13.97.5)
\end{aligned}$$

**Answer: (E)**

## 13.98 Average Energy of a Thermal System

The thermodynamic total energy is simply the expected value of the energy; this is the sum of the microstate energies weighed by their probabilities. This look like

$$\boxed{\langle E\rangle = \frac{\sum_i E_i e^{-E_i/kT}}{\sum_i e^{-E_i/kT}}} \tag{13.98.1}$$

**Answer: (A)**

## 13.99 Pair Production in vincinity of an electron

The familiar pair production reaction takes place in the Coulomb field of a massive atom. As this nucleus is massive, we can ignore any recoil action of this spectator to calculate the minimum energy needed for our photon. This time, our pair production process takes place in the neighbourhood of an electron thus forcing us to take the momenta and energies of all participants preset[16]

Our pair production process is

$$\gamma + e^- \longrightarrow e^- + e^- + e^+$$

### 13.99.1 Solution 1

Momentum and Energy is conserved during the process. The energy of our photon is, $\mathscr{E}$. Conservation of Momentum shows us

$$\frac{\mathscr{E}}{c} = \frac{3m_e v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \tag{13.99.1}$$

The left hand side of the equation is the momentum of our photon and the right hand side is the momentum of all our electrons[17]. We assume that their momenta is the same for all. Energy conservation gives us

$$\mathscr{E} + m_e c^2 = \frac{3m_e c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \tag{13.99.2}$$

Dividing eq. (13.99.1) by eq. (13.99.2) gives us

$$\frac{\mathscr{E}}{\mathscr{E} + m_e c^2} = \left(\frac{v}{c}\right) \tag{13.99.3}$$

[16]This question was covered as an example question here.

[17]I am using electrons to indicate both electrons & positrons. As they have the same rest mass we can treat them the same. We don't have to pay attention to their charges.

Substituting eq. (13.99.3) into eq. (13.99.1) yields

$$\frac{\mathscr{E}}{c} = 3m_e c \frac{\mathscr{E}}{\mathscr{E} + m_e c^2} \frac{\mathscr{E} + m_e c^2}{\sqrt{2\mathscr{E} m_e c^2 + (m_e c^2)^2}} \tag{13.99.4}$$

After some very quick simplification, we get

$$\mathscr{E} = 4m_e c^2 \tag{13.99.5}$$

### 13.99.2 Solution 2

You may find the above a bit calculation intensive; below is a somewhat quicker solution but the principle is exactly the same. We use the same equations in a different form. The total relativistic energy before our collision is

$$\mathscr{E}_i = \mathscr{E} + m_e c^2 \tag{13.99.6}$$

After collision, the relativistic energy of one electron is

$$E_e^2 = (p_e c)^2 + \left(m_e c^2\right)^2 \tag{13.99.7}$$

We have three electrons so the final energy is

$$\begin{aligned} E_f &= 3E_e \\ &= 3\sqrt{(p_e c)^2 + (m_e c^2)^2} \end{aligned} \tag{13.99.8}$$

Thus we have

$$\mathscr{E} + m_e c^2 = 3\sqrt{(p_e c)^2 + (m_e c^2)^2} \tag{13.99.9}$$

As momentum is conserved, we can say

$$p_e = \frac{p}{3} \tag{13.99.10}$$

where $p$ is the momentum of the photon, which happens to be

$$\mathscr{E} = pc \tag{13.99.11}$$

Substituting eq. (13.99.11) and eq. (13.99.10) into eq. (13.99.9) gives us

$$\mathscr{E} = 4m_e c^2 \tag{13.99.12}$$

Which is exactly what we got the first time we worked it out[18].

**Answer: (D)**

[18]The maximum wavelength of this works out to be

$$\lambda = \frac{h}{4m_e c} = \frac{\lambda_c}{4} \tag{13.99.13}$$

where $\lambda_c$ is the Compton Wavelength

## 13.100 Michelson Interferometer

A fringe shift is registered when the movable mirror moves a full wavelength. So we can say

$$d = m\lambda \tag{13.100.1}$$

where $m$ is the number of fringes. If $m_g$ and $m_r$ are the number of fringes for green and red light respectively, the wavelength of green light will be

$$\lambda_g = \frac{m_r \lambda_r}{m_g} \tag{13.100.2}$$

This becomes

$$\begin{aligned} \lambda_g &= \frac{(85865)(632.82)}{100000} \\ &\approx \frac{86000 \cdot 630}{100000} \\ &= 541.\cdots \end{aligned} \tag{13.100.3}$$

**Answer: (B)**

# Appendix A

# Constants & Important Equations

## A.1 Constants

| Constant | Symbol | Value |
|---|---|---|
| Speed of light in a vacuum | $c$ | $2.99 \times 10^{8}$ m/s |
| Gravitational Constant | $G$ | $6.67 \times 10^{-11}$ m$^3$/kg.s$^2$ |
| Rest Mass of the electron | $m_e$ | $9.11 \times 10^{-31}$ kg |
| Avogadro's Number | $N_A$ | $6.02 \times 10^{23}$ mol$^{-1}$ |
| Universal Gas Constant | $R$ | 8.31 J/mol.K |
| Boltzmann's Constant | $k$ | $1.38 \times 10^{-23}$ J/K |
| Electron charge | $e$ | $1.60 \times 10^{-9} C$ |
| Permitivitty of Free Space | $\epsilon_0$ | $8.85 \times 10^{-12}$ C$^2$/N.m$^2$ |
| Permeability of Free Space | $\mu_0$ | $4\pi \times 10^{-7}$ T.m/A |
| Athmospheric Pressure | 1 atm | $1.0 \times 10^{5}$ M/m$^2$ |
| Bohr Radius | $a_0$ | $0.529 \times 10^{-10}$ m |

Table A.1.1: Something

## A.2 Vector Identities

### A.2.1 Triple Products

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \tag{A.2.1}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}\,(\mathbf{A} \cdot \mathbf{B}) \tag{A.2.2}$$

### A.2.2 Product Rules

$$\nabla(fg) = f(\nabla g) + g(\nabla f) \tag{A.2.3}$$
$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} \tag{A.2.4}$$
$$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f) \tag{A.2.5}$$
$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \tag{A.2.6}$$
$$\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f) \tag{A.2.7}$$
$$\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) \tag{A.2.8}$$

### A.2.3 Second Derivatives

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0 \tag{A.2.9}$$
$$\nabla\times(\nabla f) = 0 \tag{A.2.10}$$
$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \tag{A.2.11}$$

## A.3 Commutators

### A.3.1 Lie-algebra Relations

$$[A,A] = 0 \tag{A.3.1}$$
$$[A,B] = -[B,A] \tag{A.3.2}$$
$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0 \tag{A.3.3}$$

### A.3.2 Canonical Commutator

$$[x,p] = i\hbar \tag{A.3.4}$$

### A.3.3 Kronecker Delta Function

$$\delta_{mn} = \begin{cases} 0 & \text{if } m \neq n; \\ 1 & \text{if } m = n; \end{cases}$$

For a wave function

$$\int \psi_m(x)^*\psi_n(x)dx = \delta_{mn} \tag{A.3.5}$$

## A.4 Linear Algebra

### A.4.1 Vectors

#### Vector Addition

The sum of two vectors is another vector

$$|\alpha\rangle + |\beta\rangle = |\gamma\rangle \tag{A.4.1}$$

**Commutative**

$$|\alpha\rangle + |\beta\rangle = |\beta\rangle + |\alpha\rangle \tag{A.4.2}$$

**Associative**

$$|\alpha\rangle + (|\beta\rangle + |\gamma\rangle) = (|\alpha\rangle + |\beta\rangle) + |\gamma\rangle \tag{A.4.3}$$

**Zero Vector**

$$|\alpha\rangle + |0\rangle = |\alpha\rangle \tag{A.4.4}$$

**Inverse Vector**

$$|\alpha\rangle + |-\alpha\rangle = |0\rangle \tag{A.4.5}$$

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# Index