Determination of Viscosity of Newtonian Liquids in a Capillary Flow between Horizontal, Parallel Flat Plates

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The viscosities of some Newtonian liquids were measured using a simple apparatus, whose construction is based on the principles of capillarity and flow between two parallel flat plates. The Navier–Stokes equations, with simplifying assumptions, were used to derive a relationship between fluid velocity and its viscosity and surface tension for the given geometry. Viscosity was the "unknown" variable in the experiments reported in this work. Motor oils and other liquids used in the experiments yielded viscosities ranging from 20 cp to 12,000 cp. Deviations from true viscosity, as determined by a Haake Rotovisco instrument, were within 5% for most cases.

INTRODUCTION

The dynamics of capillary flow, particularly rate of rise, has been of some interest for practical as well as academic reasons. Capillarity is important in the study of adhesives, soil mechanics, catalysis, absorption, rate of wetting and other topics in surface science. The first satisfactory formulation of the dyanmics of capillary flow is credited to Washburn (1), although others had been active in the field previously; see, for instance, Brittin (2). Washburn's equation (3) is a combination of Poiseuille's Law and the Young–Laplace equation. It describes how the volumetric flow rate varies with time in a cylindrical, capillary tube geometry. The development of this equation involves a number of assumptions. Many of these have been discussed in subsequent literature (4–8). Experiments with capillary tubes have generally confirmed the theoretical work (1, 5, 7, 8).

The subject of this paper is a theoretical and experimental treatment of capillary flow in horizontal slits (between parallel flat plates).

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The theoretical equation, similar to Washburn's formulation for cylindrical capillaries, can be found from Bikerman (4) and Newman (6). In this work the theoretical equation is developed from the Navier–Stokes equations (9, 10) and tested experimentally. The results are then used to determine the viscosity of a liquid, when its surface tension is known. The success of this leads to the expectation that a capillary flow between horizontal, parallel flat plates may prove useful as a simple, inexpensive viscometer for Newtonian liquids.

EXPERIMENTAL

The capillary flow viscometer consists of two glass microscope slides sandwiched together. The slides are separated by two thin, narrow strips of plastic sheet placed lengthwise along the outer edge of the long sides. Epoxy glue is applied along these edges to form a rigid, leakproof assembly. The "sandwich" is clamped until the epoxy cures. The slides are staggered lengthwise relative to each other, so that a small amount of liquid (2–4 cm³) can be accommodated on the open face. This area serves as a reservoir for the fluid to be tested.
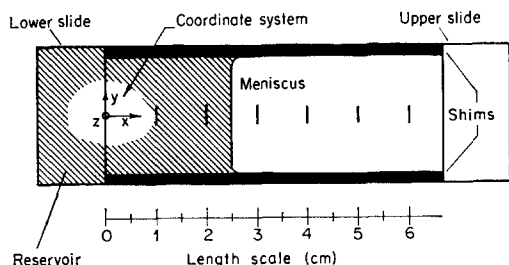
FIG. 1. Schematic of viscometer in operation.

The operation of this device is simple. The slide assembly is placed horizontally on top of a suitably drawn scale or grid. Horizontal placement (checked with a bubble level) eliminates the influence of gravity on flow. The grid markings serve to measure the length of the fluid from entrance to meniscus. An eye-dropper is used to move the fluid to be tested into the reservoir area on the open face. This must be done carefully to assure uniform contact of the fluid with the entrance to the capillary. Surface tension (capillary action) draws the liquid through the rectangular opening into the space between the slides. This motion continues until the meniscus reaches the other end of the slide "sandwich." Data consist of distance-time measurements on the movement of the meniscus. Clearly, the meniscus must be perpendicular to the long edges of the slides for uniform measurements. Figure 1 shows a schematic diagram of the apparatus in operation. A digital timer was used to obtain time readings as the meniscus reached the marked-off distances. Experiments were carried out at room temperature (25°C ± 0.5°C).

Analysis of the flow using the Navier–Stokes equations (9, 10) and a force-balance form of the Young–Laplace equation (11) leads one to conclude that the velocity of flow (distance traveled per unit time) is governed by geometry, surface tension and the viscosity of the fluid. Thus, if surface tension and geometrical dimensions are known, viscosity can be calculated from experimental data. Conversely, if geometrical dimensions and viscosity are known, surface tension can be obtained, and so forth. In the experiments reported here,

viscosity was the "unknown" variable. Gap thickness was measured by a micrometer, and surface tension was measured by a Fisher Surface Tensiometer (ring method). In order to check the reliability of the experimental technique, fluid viscosities were measured using a Haake Rotovisco (MVI system-concentric cylinders).

THEORY

For a slow, viscous flow between horizontal, parallel flat plates, the gravity and inertia terms in the Navier–Stokes equations can be neglected. This assumption is justified, so long as the flow is horizontal and the Reynolds number is much less than unity. The velocity vector has only one non-zero component, v_x, the flow velocity. This component is a function of the z-coordinate and time. When the Navier–Stokes equations in cartesian coordinates are written out in component form, the inertia terms are identically zero, except for the acceleration term, $\rho(\partial v_x/\partial t)$. We choose to ignore this term on the basis of relative importance in the flow. The Navier–Stokes equations then reduce to the x-component equation:

$$\partial P/\partial x = \mu \partial^2 v_x/\partial z^2, \qquad [1]$$

where P is pressure and μ is viscosity.

At this point, some quasi steady-state reasoning can be used to obtain an expression for the velocity profile. The pressure gradient $\partial P/\partial x$ results from the difference between atmospheric pressure and the pressure created behind the meniscus due to surface curvature (surface tension). The net pressure force acting across the meniscus is balanced by the component of the surface tension force in the flow direction. The component of the surface tension force is given by $\gamma(\cos\theta)(2W)$, where W is the width of the slit, and γ is the surface tension. The pressure force is given by $\Delta P(A_x)$, where A_x is the cross-sectional area of the slit. This area is given by $W\delta$, where δ is the gap separation between the two plates. Thus

$$\Delta PW\delta = 2W\gamma \cos\theta. \qquad [2]$$

We make the assumption that $\cos\theta \simeq 1.0$, so that

$$\Delta P = 2\gamma/\delta. \qquad [3]$$

The term ΔP represents the difference between atmospheric pressure and the pressure in the fluid just behind the meniscus. Atmospheric pressure exists in the fluid at the reservoir end, and thus a pressure gradient driving force is established. This causes fluid motion in the capillary. Note that ΔP is a constant determined by surface tension and gap distance alone. This pressure difference acts along the length of the fluid from the capillary entrance to the meniscus. This distance $l(t)$ and therefore the pressure gradient, changes as the meniscus moves. Thus, the pressure gradient term can be rewritten as

$$\partial P/\partial x = \Delta P/l(t) = -2\gamma/\delta l(t). \qquad [4]$$

The minus sign indicates that $\partial P/\partial x$ is negative, $P_{\text{atm}} > P_{\text{meniscus}}$.

The LHS of Eq. (1) is not a function of z and therefore acts as a constant with respect to any integration in z. If we replace the partial derivatives with total derivatives, integration is straightforward. Substituting into Eq. (1), we have

$$-2\gamma/\delta l(t) = \mu d^2 v_x/dz^2. \qquad [5]$$

Using the boundary conditions of zero velocity at the wall and zero velocity gradient at the centerline of the flow, this integrates directly to the expression for the velocity profile:

$$v_x = [\gamma/\mu\delta l(t)][(\delta/2)^2 - z^2]. \qquad [6]$$

The expression for the average velocity $\bar{v}_x$ follows from the definition (9),

$$\bar{v}_x = \frac{\displaystyle\int v_x dA}{\displaystyle\int dA} = \frac{1}{W\delta} \int_{-W/2}^{+W/2} \int_{-\delta/2}^{+\delta/2} v_x dydz. \qquad [7]$$

Substituting Eq. (6) into Eq. (7) and integrating yields the expression for the average velocity.

$$\bar{v}_x = \gamma\delta/6\mu l(t). \qquad [8]$$

The average velocity is $\frac{2}{3}$ of the maximum (6, 11).

The movement of the meniscus is directly related to the average velocity, because the meniscus movement reflects a volumetric flow, i.e., $Q = A\bar{v} = W\delta\bar{v}$. The speed of travel of the meniscus at any point in time is given by dl/dt; thus $dl/dt = \bar{v}_x$. Substituting this result into Eq. (8),

$$\bar{v}_x = dl/dt = \gamma\delta/6\mu l(t) \qquad [9]$$

and rearranging, leads to

$$ldl = (\gamma\delta/6\mu)dt \qquad [10]$$

which can be integrated directly, since γ, δ, and μ are all independent of time. Integration limits are from 0 to l and 0 to t. Thus, we have a form of the Washburn equation (4, 6):

$$l^2 = (\gamma\delta/3\mu)t \qquad [11]$$

or

$$l = [\gamma\delta/3\mu]^{\frac{1}{2}}t^{\frac{1}{2}}. \qquad [12]$$

For a liquid of constant surface tension and viscosity in a geometry of constant gap spacing, the distance traveled by the meniscus should vary directly as the square root of time. For the purpose of clarity, it is more convenient to plot the data as l vs $t^{\frac{1}{2}}$. This should yield a straight line of slope $(\gamma\delta/3\mu)^{\frac{1}{2}}$. Therefore, if any two of the quantities μ, δ, γ are known, the other can be calculated from experimental data. These results are subject to the following restrictions:

(1) low Re (to neglect inertia)
(2) laminar profile established very quickly
(3) side effects negligible, i.e., $W \gg \delta$.
(4) the meniscus movement reflects the average velocity.
(5) The contact angle is close to $0°$, i.e., $\cos\theta \simeq 1.0$.
(6) The flow is slow enough, so that instantaneous velocity profiles and contact angles are at their steady-state values. This means the dynamic contact angle (that exhibited when the fluid is moving) is always equal to the steady-state contact angle (that exhibited when the fluid
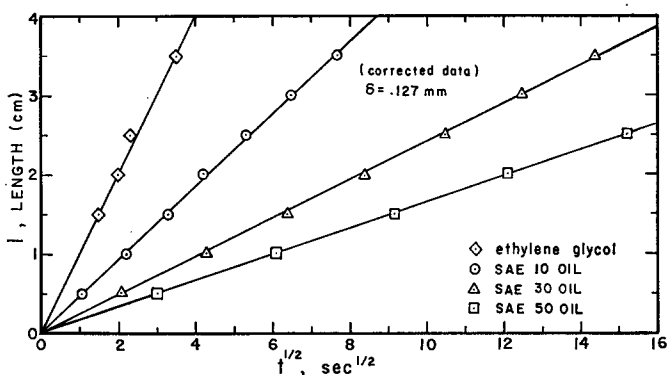
FIG. 2. Flow of different liquids in a capillary of 0.127 mm (0.005 in.) spacing.

is at rest). Also the shape of the velocity profile throughout the fluid (except for a small region directly behind the meniscus) is always parabolic.

(7) The energy loss at the entrance due to half a sudden contraction is negligible.

RESULTS AND DISCUSSION

Six different liquids and three gap spacings were used to test the theoretical expectation of Eq. (12). The liquids were a trio of motor oils (SAE 10, 30 and 50), STP oil additive, ethylene glycol and a mixture of sucrose,

FIG. 3. Flow of SAE 30 oil in three capillaries of different spacings.

glycerin and water. The three gap spacings used were 0.127 mm (0.005 in.), 0.203 mm (0.008 in.) and 0.254 mm (0.010 in.). Typical results are shown in Figs. 2 and 3. The data clearly show linear behavior in accordance with Eq. (12). Ideally, all lines should intersect at the origin, but it was not always possible to time the fluid's movement from the entrance to the capillary. The data shown in Figs. 2 and 3 have been shifted along the time scale, so that all lines pass through the origin. The slopes were unaffected by this procedure.

The liquid's viscosity was calculated from the slope of the plot l vs $t^{\frac{1}{2}}$ and Eq. (12). A summary of these results is given in Table I. These experimental values for viscosity are consistent with respect to different gap spacings and are in good agreement with the absolute viscosity values measured by the Rotovisco. In most cases, the experimental viscosity was within 5% of the true viscosity.

Restrictions

The restrictions mentioned previously must be met to insure that the results obtained are valid. Restrictions 1 and 2 can be examined together, because they involve calculation of the Reynolds number. The Reynolds number formulation must be modified, since the cross section is rectangular, not circular, and the average velocity is time-dependent. Thus, we substitute the hydraulic radius (9, 12) for diameter, and a "time averaged" average
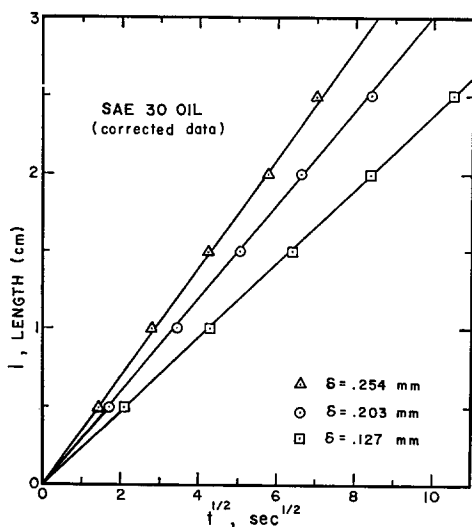
TABLE I

RESULTS OF VISCOSITY MEASUREMENTS

Liquid	γ, dyne/cm	η, cp(Rotovisco)	Experimental viscosity, cp		
			0.127 mm gap	0.203 mm gap	0.254 mm gap
SAE 10 oil	29.5	57.3	56.8	55.6	—
SAE 30 oil	31.0	221.0	232	227	217
SAE 50 oil	31.0	540.0	494	566	510
STP oil	29.6	11,335.0	—	—	11,100
Ethylene glycol	45.5	18.0	19.2	—	—
Sucrose–glycerin–water mixture	65.0	82.0	74.4	—	—

velocity for the average velocity.

$$\text{Re} = D\bar{v}\rho/\mu = 4R_h\langle\bar{v}\rangle_t\rho/\mu. \quad [13]$$

Here, the hydraulic radius is R_h, and the "time averaged" average velocity is $\langle\bar{v}\rangle_t$. The hydraulic radius is defined as the ratio of cross-sectional area to wetted perimeter, thus

$$R_h = W\delta/(2W + 2\delta) \simeq \delta/2. \quad [14]$$

The "time averaged" average velocity is, by definition,

$$\langle\bar{v}\rangle_t = \left(\int_0^t \bar{v}\,dt\right)\Big/\left(\int_0^t dt\right). \quad [15]$$

Since $\bar{v} = dl/dt$, and $l = Kt^{\frac{1}{2}}$ [by Eq. (12)], where K is the experimental slope or the analytic expression, $(\gamma\delta/3\mu)^{\frac{1}{2}}$, we can substitute

$$\bar{v} = \frac{dl}{dt} = \frac{d}{dt}(Kt^{\frac{1}{2}}) = \frac{K}{2}t^{-\frac{1}{2}} \quad [16]$$

into Eq. (15) and integrate. Thus,

$$\langle\bar{v}\rangle_t = \frac{K}{2t}\int_0^t t^{-\frac{1}{2}}dt = \frac{K}{t^{\frac{1}{2}}}, \quad [17]$$

and the Reynolds number becomes,

$$\text{Re} = (2\delta\rho/\mu)(K/t^{\frac{1}{2}}) = [4\rho^2\delta^3\gamma/3\mu^3 t]^{\frac{1}{2}}. \quad [18]$$

Use of the hydraulic radius in the Reynolds number expression is not recommended for laminar flow cases (9). An accurate equivalent diameter can be found by calculating the friction factor for slit flow. This expression is rearranged to match the form of the one for tube flow, i.e. $f = 16/Re$. Then, the equivalent diameter is easily calculated. For the slit flow case, the equivalent diameter was found to be $4\delta/3$. The equivalent diameter ($4R_h$) using the hydraulic radius is 2δ. Thus, the Reynolds numbers calculated using the hydraulic radius are slightly higher and therefore conservative for our purposes here.

In the fastest experiment, ethylene glycol took 11 seconds to travel about 3.5 cm in a capillary of 0.127 mm spacing. The Reynolds number for this experiment was about 0.045. All other Reynolds numbers were lower. For example, the velocity of flow for the SAE 30 oil in a capillary of 0.254 mm spacing was 3.5 cm in about 100 seconds. This results in a Reynolds number of about 0.0025. The "time averaged" average velocity for each experiment is given in Table II. At Reynolds numbers on the order of 10^{-2}, inertia forces can be neglected. The Reynolds numbers calculated here are averages, and a possibility for some inertia effect exists at the very early stages of flow, e.g. the first few millimeters. Thus, a relatively large flow length is important to minimize the contribution of inertia forces. Flow length to gap spacing should be about 100/1.

A large flow length to gap spacing ratio is also important for minimizing errors due to the entrance length, i.e. the length required to establish a laminar profile (9, 12). This entrance length (restriction 2) is given by $L_e = 0.05$ ReD, or for a rectangular channel $L_e = 0.2R_h$ Re. Since the maximum Reynolds

TABLE II

DATA FOR REYNOLDS NUMBER CALCULATION

Liquid	gap (mm)	ρ (g/cm³)	slope/$t^{\frac{1}{2}a}$	$\langle \bar{v} \rangle_t$ (cm/sec)
SAE 10	0.127	0.87	0.469/7.8	0.060
SAE 10	0.203	0.87	0.602/5.4	0.110
SAE 30	0.127	0.87	0.238/15.0	0.016
SAE 30	0.203	0.87	0.303/8.7	0.035
SAE 30	0.254	0.87	0.347/7.6	0.046
SAE 50	0.127	0.87	0.163/15.9	0.010
SAE 50	0.203	0.87	0.192/16.3	0.012
SAE 50	0.254	0.87	0.227/10.2	0.022
STP	0.254	0.90	0.047/45.4	0.001
ethylene glycol	0.127	1.12	1.02/3.6	0.283
sucrose-glycerin-water mixture	0.127	1.28	0.61/7.2	0.085

[a] Slopes are calculated from experimental data plotted as l vs $t^{\frac{1}{2}}$. The $t^{\frac{1}{2}}$ values are taken to be the longest time recorded for an experiment.

numbers (near the entrance) are on the order of 0.1, and since the hydraulic radius is half the thickness, it is obvious that the entrance length is negligibly small when compared to the flow length of approximately 3 cm.

Restrictions 3 and 4 are clearly met on the following bases: Side effects in the flow are negligible, since W/δ was on the order of 100/1 for all experiments. Also, it is obvious that the meniscus motion reflects the average velocity, since only one liquid–gas interface exists, and since fluid is continually entering the capillary from the reservoir.

For restriction 5, the contact angle for motor oils on glass is virtually 0°, so that cosθ would be unity for all cases. Ethylene glycol has a slightly higher contact angle than 0° based on visual observation, and cosθ would be slightly less than 1.0. The contact angle for the sucrose–glycerin–water solution was significantly greater than 0°. The flow of this solution was very irregular. The liquid tended to separate in the capillary; the meniscus was rarely perpendicular to the edges; and data were extremely difficult to obtain. The effect of small differences in contact angle among the oils and ethylene glycol was pretty much

buried in the measurement error of the experiment. A large difference in contact angle (from 0°) clearly affects the experiment and the results as evidenced by our problems with the sucrose–glycerin–water mixture. More detailed investigation into the effects of contact angle were not undertaken.

Restriction 6 is met on the basis of low Reynolds numbers in our experiments. Also, the region just behind the capillary where a nonparabolic profile might be expected is estimated to be no more than 0.1 cm. This is quite small in relation to the length of the fluid in the capillary (0.5 to 4.0 cm) during the recording of the data.

Restriction 7 is not a factor, because fluid friction in a highly viscous, Newtonian flow predominates to such an extent (9, 12) as to make the entrance loss numerically negligible.

Sources of Error

The major source of experimental error was capillary misalignment. Misalignments can occur in three coordinate planes. The most serious misalignment was found to be a change in spacing along the flow direction (see Fig. 4). This resulted from cleaning procedures. The misalignment affected the data by inducing curvature in the l vs $t^{\frac{1}{2}}$ plot. Difference in spacing was 10% over a distance of about
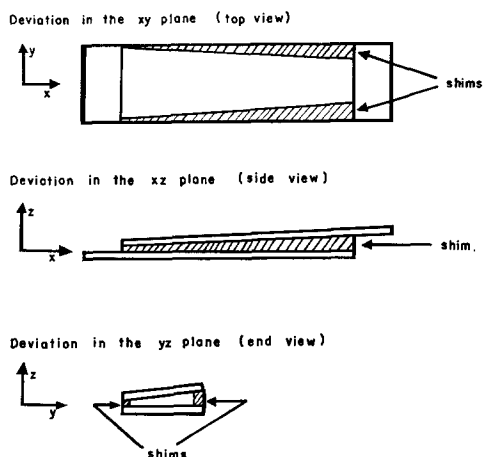

FIG. 4. Possible misalignments in construction of the capillary.

6 cm for one slide "sandwich." Measurements were made with a micrometer.

Hydrostatic head at the entrance to the capillary might also cause a significant error. The height difference was about 0.1 cm. This increases the pressure at the entrance to the capillary by ρgh, or about 80 dyne/cm² for the liquids used. The overall ΔP, from atmospheric to meniscus, was about 3000 dyne/cm² for most experiments. Thus head effects are negligible.

CONCLUSIONS

The instrument works according to the principles developed in the theory section. Reasonably accurate results have been obtained for Newtonian fluids with viscosities in the range 20 to 12,000 cp. Above 12,000 cp uniform deposition of the fluid at the capillary entrance becomes difficult. Below 20 cp the meniscus moves too fast for accurate recording by stop timer and human eye. A smaller gap spacing could be used in this case, however.

Application of the capillary flow concept to non-Newtonian viscosity measurement appears to be a formidable challenge. Several problems arise: first, the shear rate is constantly changing; second, the range of shear rates may be too narrow to be of interest; and third, a Rabinowitsch-type formulation (10) requires multiple differentiation of the data, which reduces both accuracy and simplicity.

On the practical side, this viscometer could find useful application as a quality control instrument, where precision is a secondary consideration. This instrument has several advantages besides its simplicity and low cost. Even though the flow is unsteady, shear stresses and shear rates, or their average, can be calculated with reasonable accuracy using standard procedures from capillary viscometry (10). Also, errors in the geometry are easily

detectable and readily corrected, an impossibility with capillary tubes of untrue bore.

The operation of such an instrument can be simplified by making two marks on the glass surface and timing the fluids motion past each. Say the marks are at 2 cm and 1 cm from the reservoir edge, and the elapsed time is Δt. Since time varies directly as l^2, the slope $\Delta l^2/\Delta t$ is always constant for Newtonian fluids, and is equal to the quantity $\gamma\delta/3\mu$. Thus, viscosity can be calculated from a simple timed interval. Charts could also be constructed for a standard instrument(s) to eliminate any calculations.

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