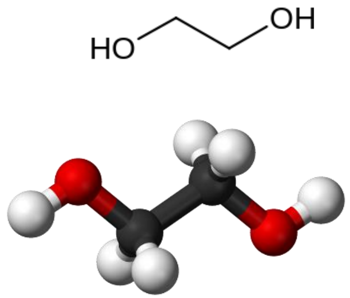
Fluids, fluid mechanics, fluid properties and forces in fluids

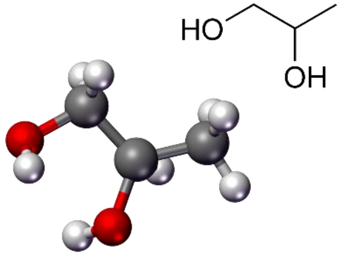
Applications of forces in fluids, viscosity, surface tension, pressure and contact angle

Liquids and gases comprise the fluids used for the delivery of heating or cooling or just to transport materials. The most common liquids are water (or brines, aqueous solutions of ethylene glycol or propylene glycol) and refrigerants (R134a, NH_3-H_2O, BrLi-$H_2$0, RXYZα) whereas the most common gas is atmospheric air (either dry or wet air). In this slides we review the key features of fluid mechanics that pertain to common applications. The goals of this topic are to

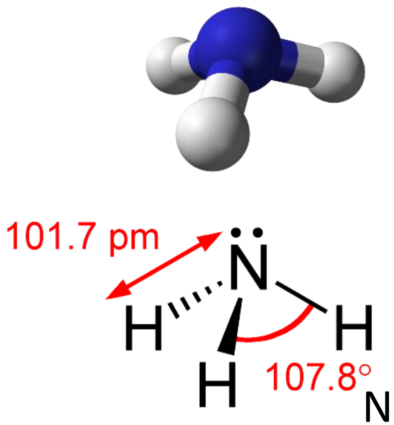
- Review the essentials of incompressible fluid mechanics as they apply to common engineering systems.
- Provide basic data needed to calculate flow rates and pressure drops, stresses, forces, torques, energy and power requirements, in ductwork, flue lines, piping systems, micro channels, porous media, and transport systems.



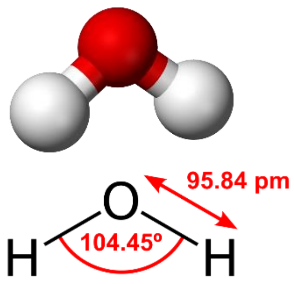
HO–CH_2CH_2–OH



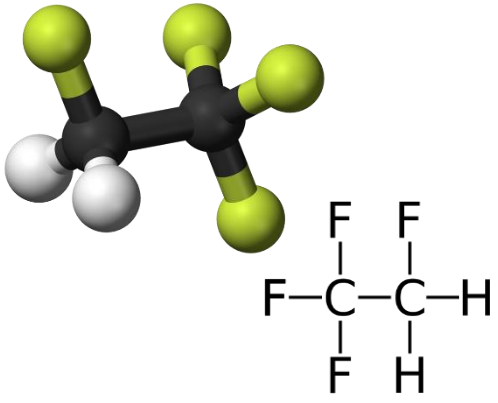
HO–$CH_2CH(OH)$-CH_3
$C_3H_8O_2$



NH_3



H_2O



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Introduction

Mechanics: The oldest physical science that deals with both stationary and moving bodies under the influence of forces or force fields.

Statics: The branch of mechanics that deals with bodies at rest (caution!, motion respect to an interacting boundary, or respect to its constituting molecules or fluid parcels).

Dynamics: The branch that deals with bodies in motion.

Fluid mechanics: The science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solids or other fluids at their respective boundaries.

Fluid dynamics: Fluid mechanics is also referred to as fluid dynamics by considering fluids at rest as a special case of motion with zero velocity.

Fluid mechanics deals with liquids and or gases in motion or at rest, or colloidal dispersions where the external phase is either gas or liquid, the fluid can also be made of plasma (of course in any context), usually if the constative equations describing the relationship among strain rate and strain with stress is complex fluid mechanics turns into rheology, specially when the material combines solid and liquid like properties .

“Fluide“ = That flow. Substance whose particles can move about with complete freedom (ideal fluids) or restricted freedom (real fluids)

What is a Fluid?

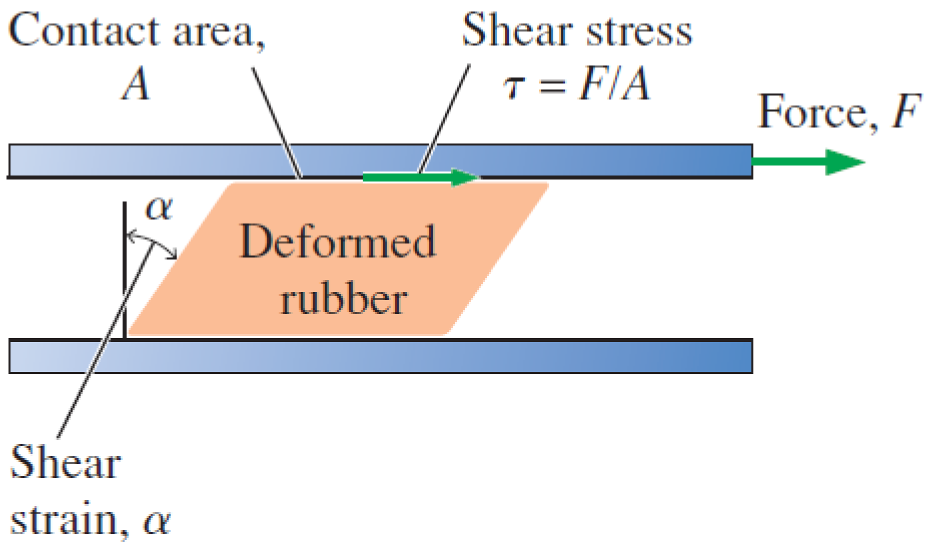
Fluid: A substance in the liquid and or gas phase or colloidal dispersion with either gas or liquid in the external phase.

A solid can resist an applied shear stress by deforming.

A fluid deforms continuously under the influence of a shear stress, no matter how small.

In solids, stress **is function** of *strain*, but in fluids, stress **is function of** *strain rate*.

When a constant shear force is applied, a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant *rate* of strain.



Deformation of a rubber block placed between two parallel plates under the influence of a shear force. The shear stress shown is that on the rubber—an equal but opposite shear stress acts on the upper plate.

Stress: Force per unit area.

Normal stress: The normal component of a force acting on a surface per unit area.

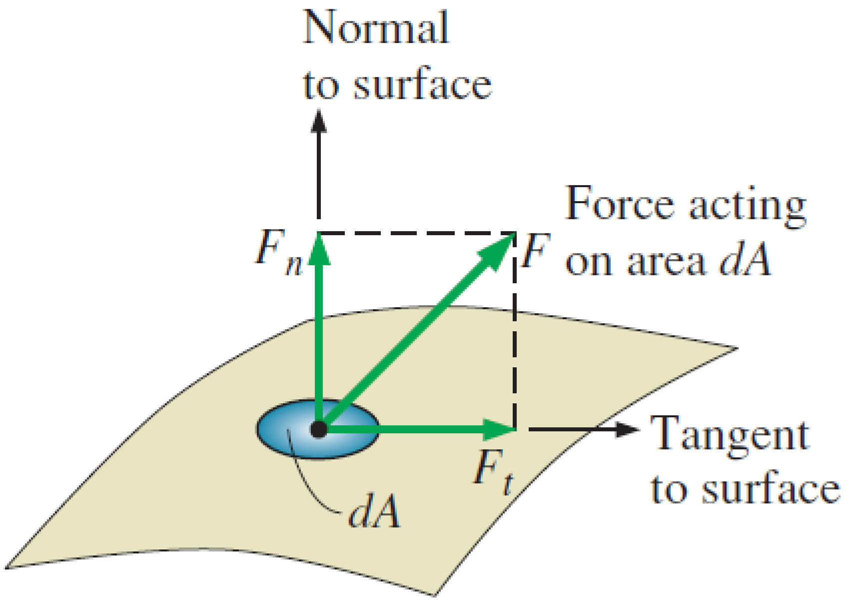
Shear stress: The tangential component of a force acting on a surface per unit area.

Pressure: The normal stress in a fluid at rest.

Zero shear stress: A fluid at rest is at a state of zero shear stress.

When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to re-establish a ~~horizontal~~ free surface. ???

"Yield stress "



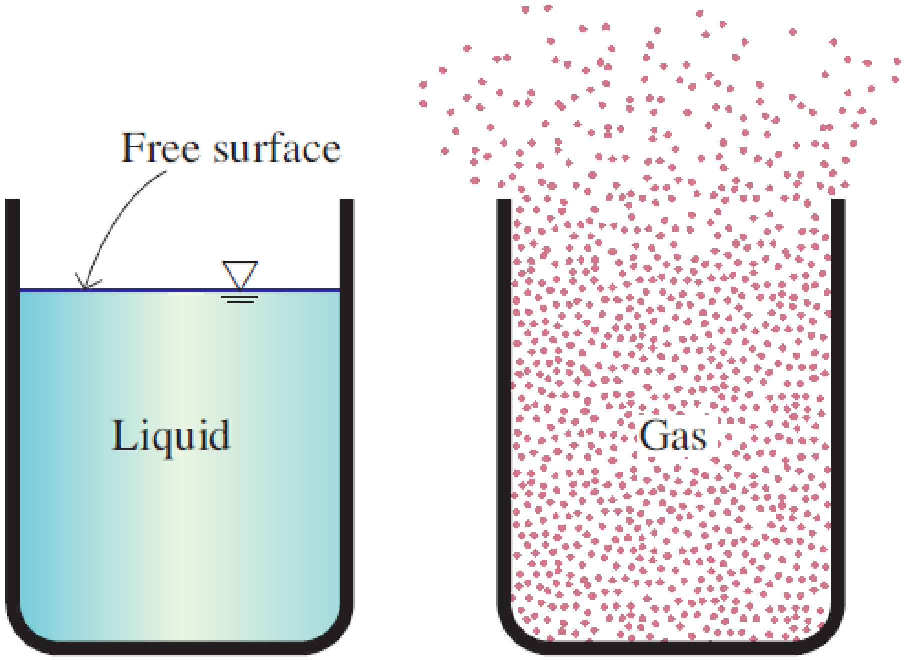
$$\text{Normal stress: } \sigma = \frac{F_n}{dA}$$

$$\text{Shear stress: } \tau = \frac{F_t}{dA}$$

The normal stress and shear stress at the surface of a fluid element. For fluids at rest, the shear stress is zero and pressure is the only normal stress.

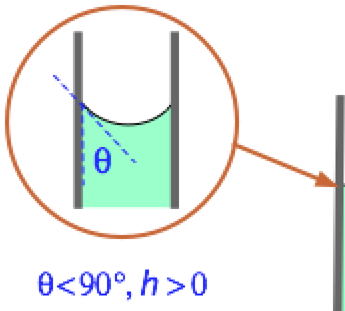
In a **liquid**, groups of molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules. As a result, a liquid takes the shape of the container it is in, and it forms a free surface, this free surface is horizontal in a larger container in a gravitational field as long as the Bond number (Eötvös number) is large (i.e. capillary effects are negligible).

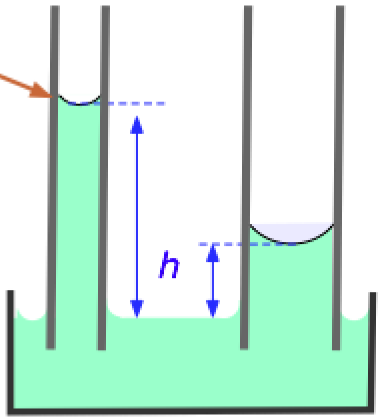
A **gas** expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. Unlike liquids, a gas in an open container cannot form a free surface.



Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.

$$Bo = \frac{\Delta\rho\, g\, r}{\sigma}$$



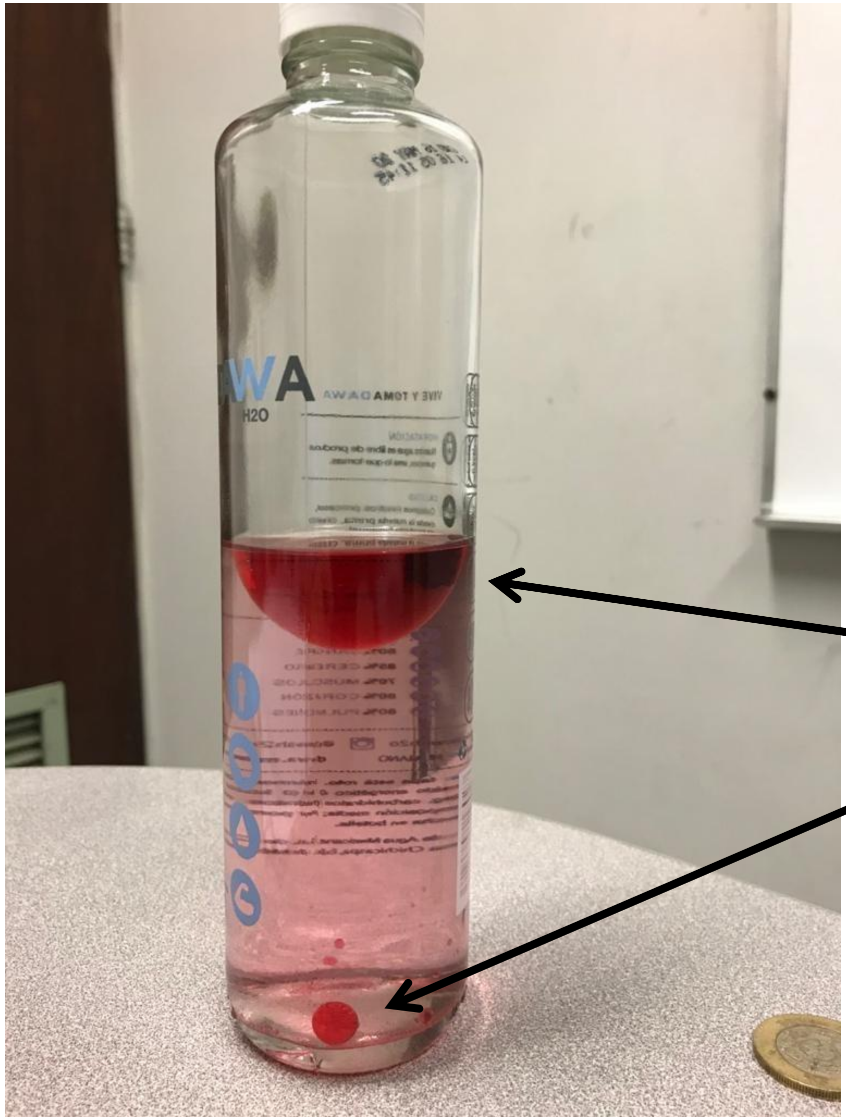


$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

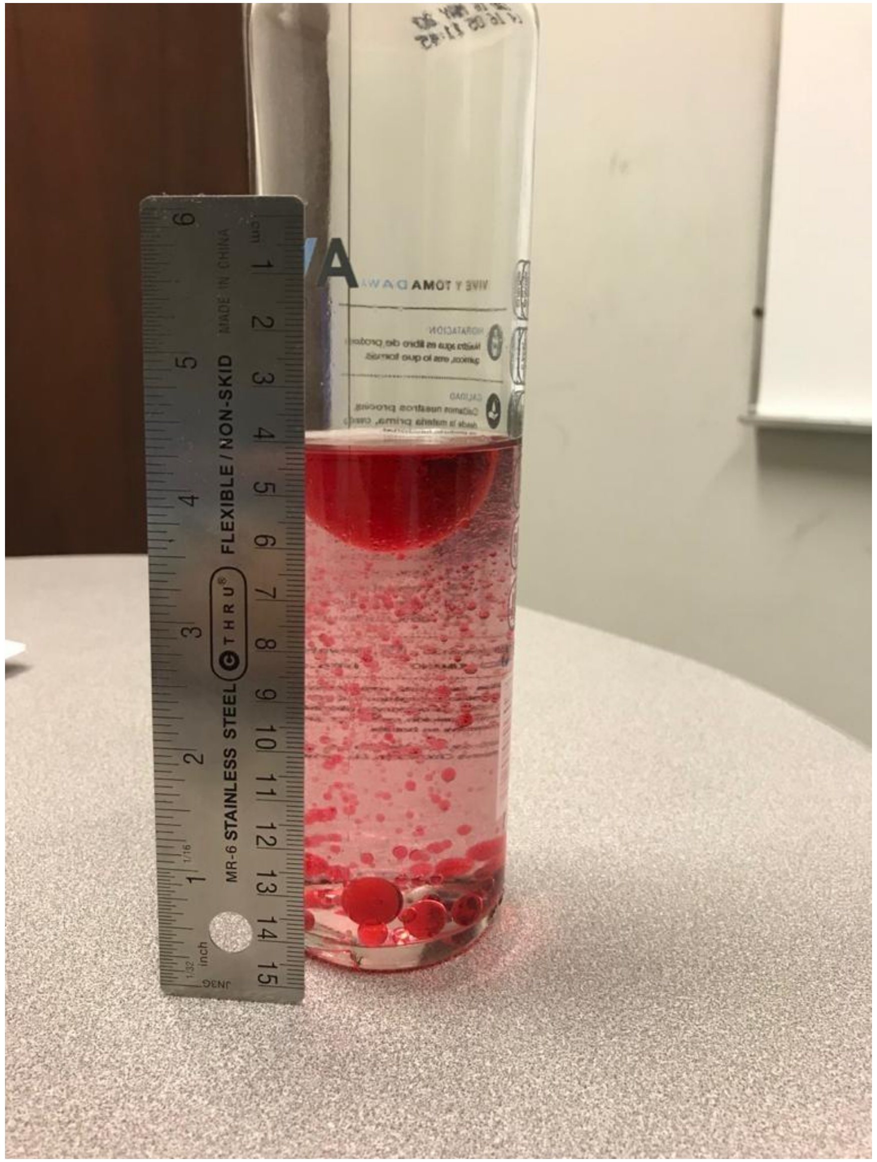
$\Delta\rho$ = Density difference between fluids
g = gravitational field
r= capillary radius
σ= Surface tension
h=capillary rise
Bo = Bond number

A **liquid** in a gravitational field will form a free surface if unconfined from above. Under mechanical equilibrium this free surface must be perpendicular to the forces acting on the liquid; if not there would be a force along the surface, and the liquid would flow in that direction. Thus, on the surface of the Earth, all free surfaces of liquids are horizontal unless disturbed (except near solids dipping into them, where surface tension distorts the surface in a region called the meniscus).

In a free liquid that is not affected by outside forces such as a gravitational field or centrifugal acceleration, internal attractive forces only play a role (e.g. Van der Waals forces, hydrogen bonds). Its free surface will assume the shape with the least surface area for its volume: a perfect sphere. Such behavior can be expressed in terms of surface tension. It can be demonstrated experimentally by observing a large globule of oil placed below the surface of a mixture of water and alcohol having the same density so the oil has neutral buoyancy.



Mineral oil red dyed, in a mixture of IPA aqueous solution, both phases have the same density. Spherical shape is caused by the requirement of minimize area. The upper hemisphere resulted because surface tension of organic phase is smaller than the aqueous phase



Classification of materials

Purely elastic solids
Elastic deformations store energy

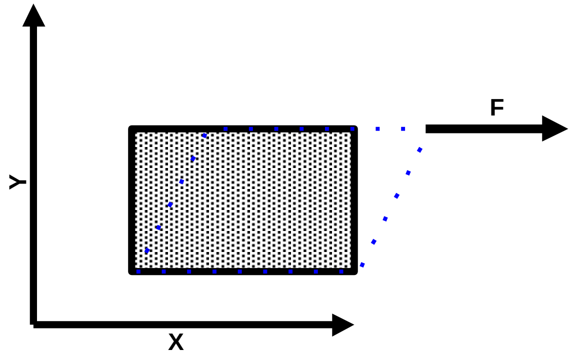
Rigid solid **(Euclidian)**	Liner elastic solid **(Hookean)**	Nonliner elastic solid **(non-Hookean)**
$\gamma = 0$	$\tau = G\gamma$	$\tau = f(\gamma)$
	$G = \tau / \gamma$	$G = \tau / \gamma$
	Shear modulus constant	Shear modulus function of τ or γ

Viscoelastic fluids and solids (Nonlinear)

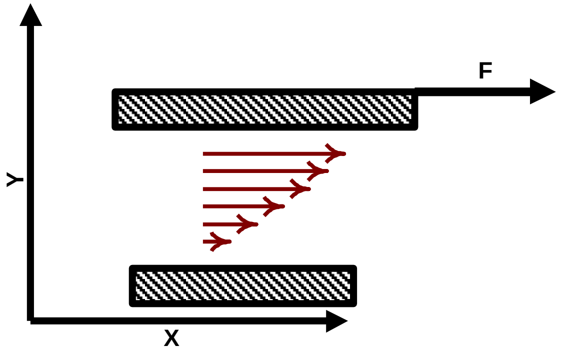
Purely viscous fluids
Viscous deformation dissipate energy

Nonlinear viscous fluid **(non-Newtonian)**	Linear viscous fluid **(Newtonian)**	Inviscid fluid **(Pascalian)**
$\tau = g(\dot{\gamma})$	$\tau = \mu \dot{\gamma}$	$\tau = 0$
$\eta = \tau/\dot{\gamma}$	$\mu = \tau/\dot{\gamma}$	
Viscosity function of stress or rate of strain	Viscosity- constant	

$$\tau = h(t, \gamma, \dot{\gamma}, \ddot{\gamma}, \cdots)$$



$$\gamma = \frac{dx}{dy}$$



$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{d\,v_x}{dy}$$

Ron Darby, 2001

Additional information

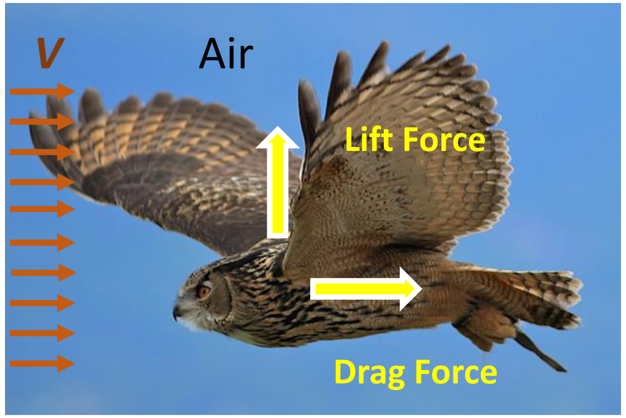
- ✓Gases, liquid water, and simple molecules in liquid phase (e.g. ammonia, alcohol, benzene, petroleum, chloroform, butane, R134a, ethylene glycol, propylene glycol, NH_3+H_2O, $HOCH_2CH_2OH+H_2O$, etc) are Newtonian.

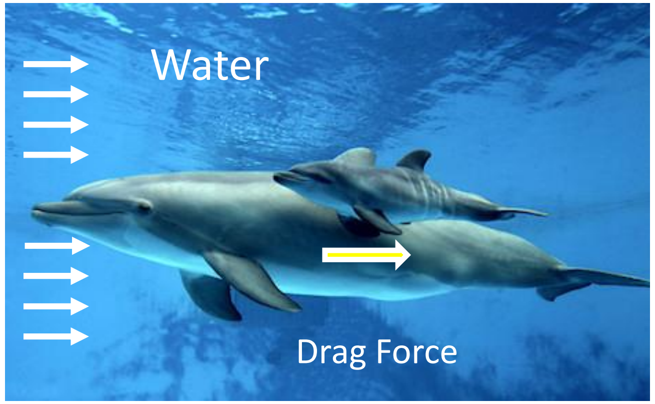
- ✓Muds, emulsions, biological fluids, polymers, solids in suspension, colloidal dispersions, and complex mixtures of immiscible materials may be non-Newtonians.

VISCOSITY

Viscosity: A property that represents the internal resistance of a fluid to motion or the “fluidity”. Is the ratio between the shear stress and the strain rate.

Drag force /Lift Force: The force a flowing fluid exerts on a body in the flow direction (if a solid is moving in a fluid, is the force in the direction of the resultant relative motion between solid and fluid). The magnitude of this force depends, in part, on viscosity, geometry and hydrodynamics/aerodynamics. As the drag force in nature, but perpendicular to the relative velocity, there is a force called lift force, and its direction depends on the geometry respect to the relative velocity, and a net effect observed whenever there is no symmetry of the object respect to the velocity field.



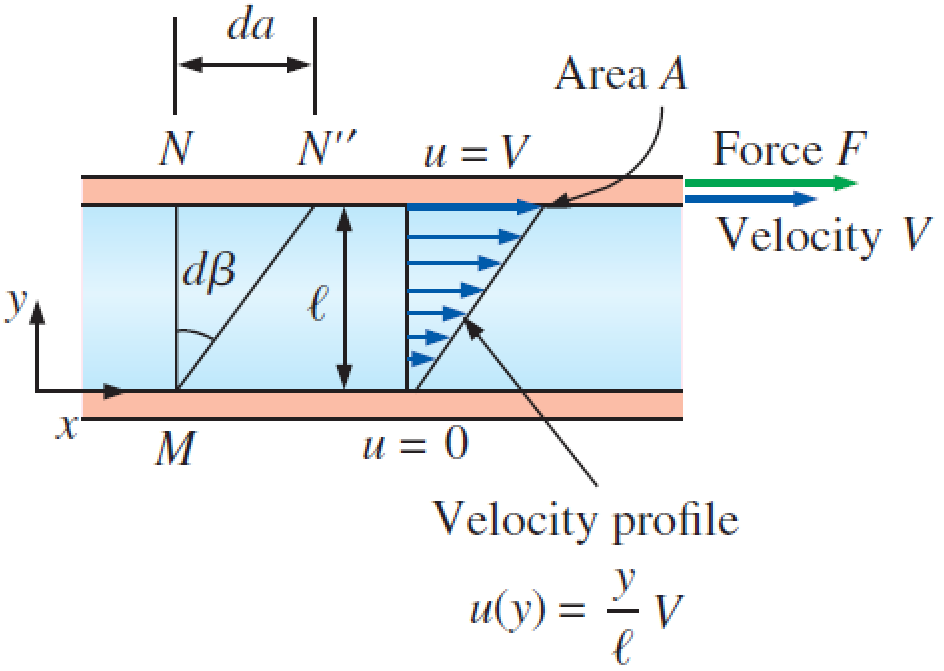


The viscosity of a fluid is a measure of its “*resistance to deformation*.”

Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.

Here hydrodynamics/aerodynamics is in the context of flow conditions, flow regime, e.g. Laminar-Turbulent, Sonic-subsonic, etc (i.e. relationship of the magnitude of the different forces involved in the process)



The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

$$\tau = \frac{F}{A} \qquad u(y) = \frac{y}{\ell} V \qquad \text{and} \qquad \frac{du}{dy} = \frac{V}{\ell}$$

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt \qquad \frac{d\beta}{dt} = \frac{du}{dy}$$

Newtonian fluids: Fluids for which the rate of deformation is proportional to the shear stress.

$$\boxed{\tau \propto \frac{d(d\beta)}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy}}$$

$$\boxed{\tau = \mu \frac{du}{dy} \qquad (\mathrm{N/m^2})}$$

Shear stress

Shear force

$$\boxed{F = \tau A = \mu A \frac{du}{dy} \qquad (\mathrm{N})}$$

μ coefficient of viscosity

Dynamic (absolute) viscosity

kg/m · s or N · s/m^2 or Pa · s

1 poise = 0.1 Pa · s

Classification of solids

Eucledian

$$\gamma = 0$$

No deformation, even under regular level of stress

Hooke

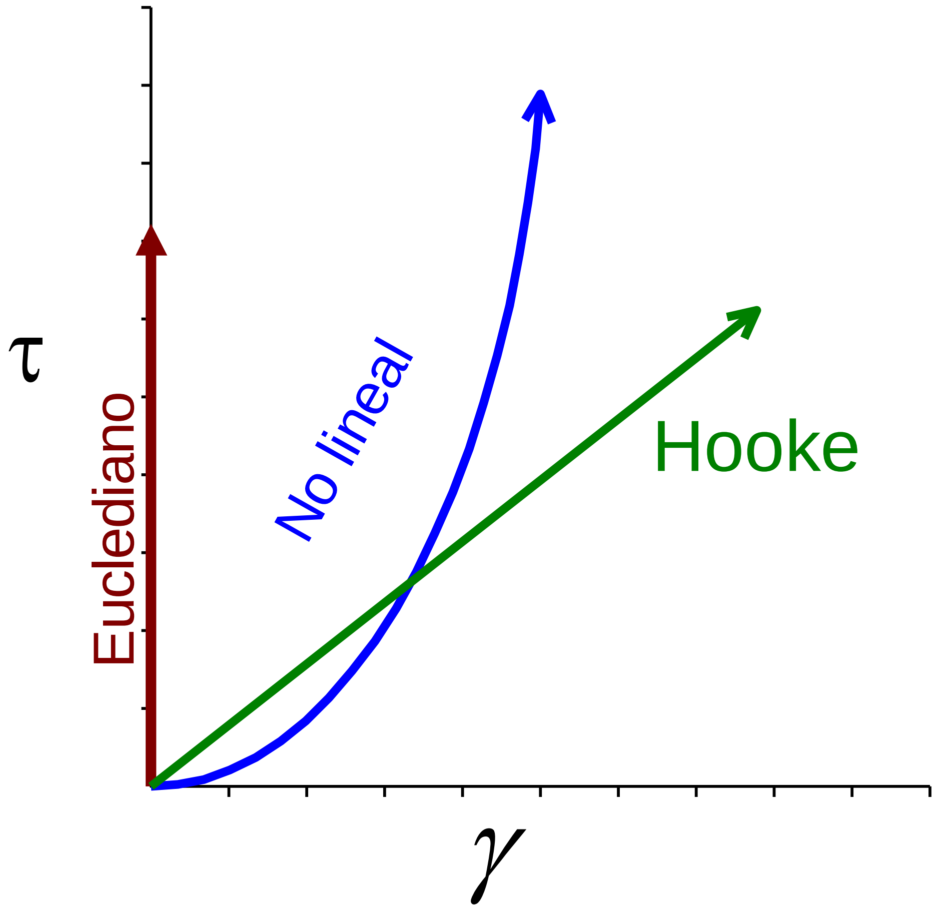
$$\tau = G\,\gamma$$

stress proportional to deformation

Nonlinear

$$\tau = \tau(\gamma)$$

stress is related to deformation, but they are not proportional



Classification of fluids

Pascal $\tau = 0$ Negligible stress, even under flowing fluid

Newtonian

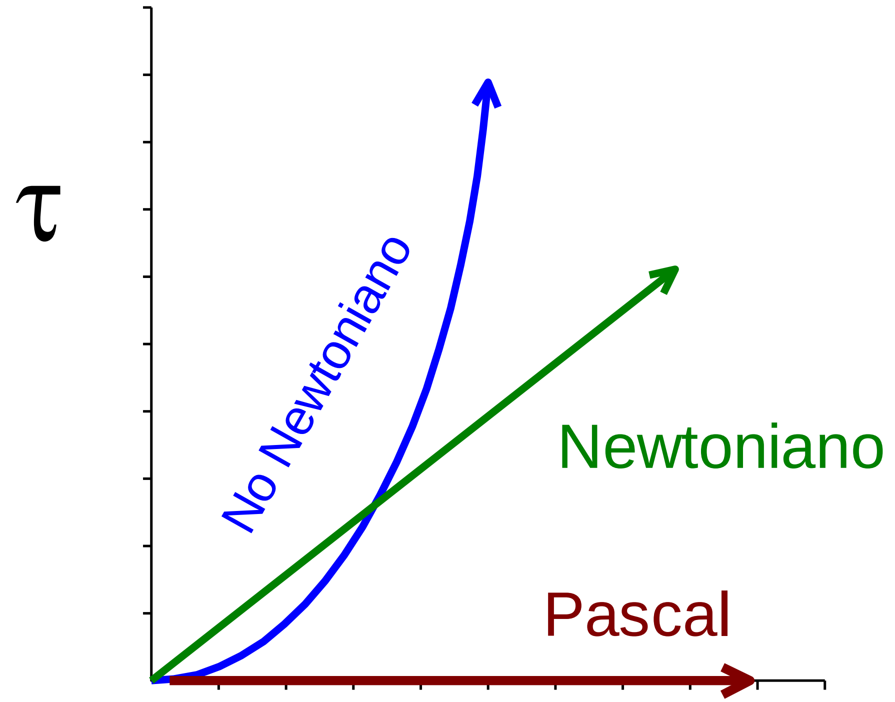
$$\tau = \mu \,\dot{\gamma} = \mu \,\frac{d\gamma}{dt} = \mu \,\frac{d\, v_x}{dy}$$

linear relation between strain rate and stress

no Newtonian

$$\tau = \tau(\dot{\gamma})$$

No linear relation between strain rate and stress

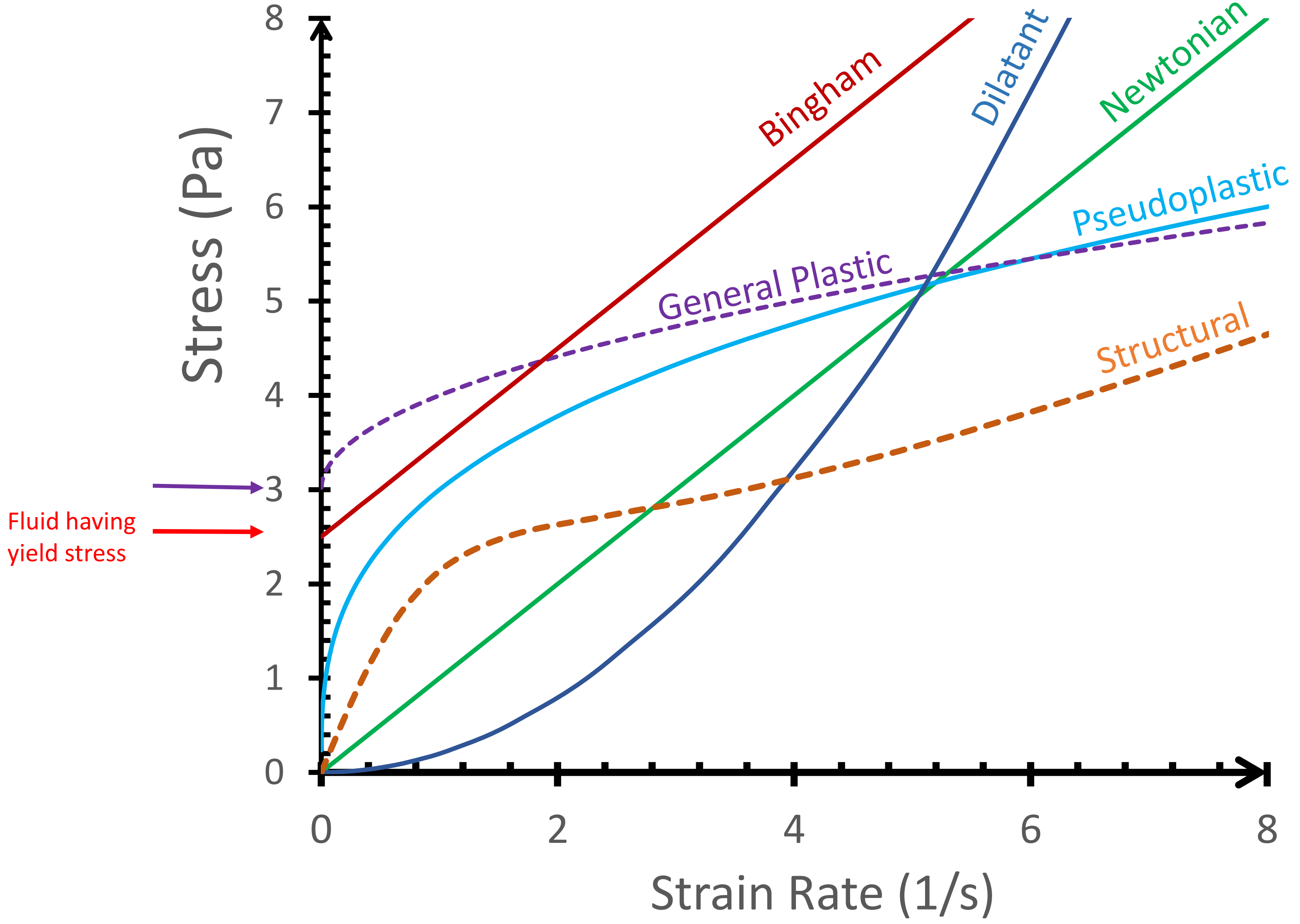


$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{d\, v_x}{dy}$$

Fluid classification

no time dependent

- ✓Newtonian
- ✓Bingham
- ✓Plastic
- ✓Pseudoplastic
- ✓Dilatant
- ✓Structural



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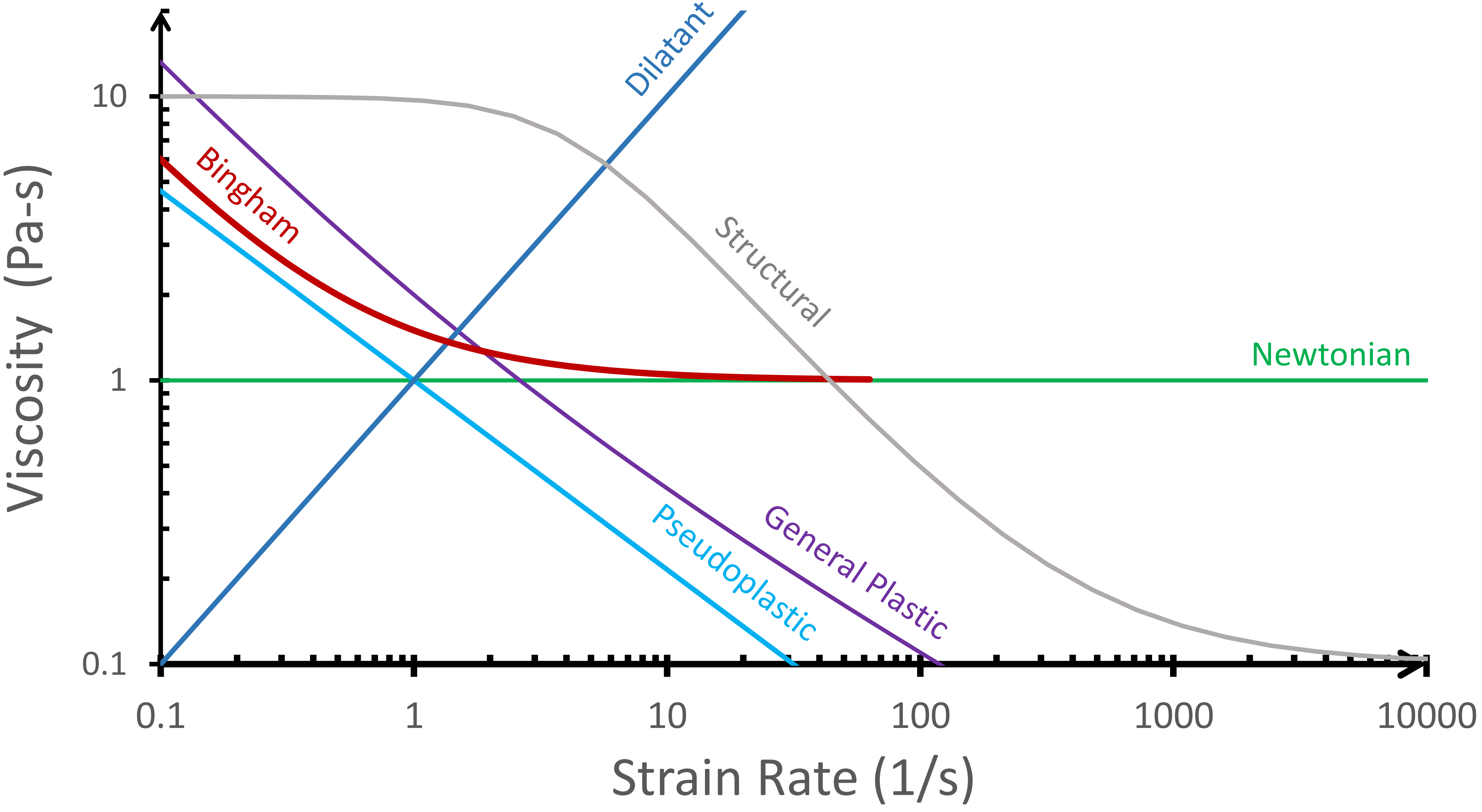

Bingham Plastics. The relationship between strain rate and shear ($d\gamma/dt$) stress (τ) is linear, but they have yield stress (τ_0).

$$\tau = \tau_o + \mu\,\dot{\gamma}$$

- Toothpaste
- Butter
- Peanut butter
- Mustard
- Mayonnaise
- Chocolate.
- Clay mixed with water
- Concentrated slurries
- Suspensions
- Muds
- Foams

- *Catsup*
- *Ketchup*
- *Paint*
- *Asphalt*
- *Wax*
- *Potter's clay*
- *Carbon ashes mixed with water*
- *Sediments of residual water*

Dilatant Fluids (Ostwald, n>1), also known as **shear thickening**. This behavior is relatively rare. Primarily for some concentrated suspensions of very small particles (e.g. starch suspensions) and some unusual polymeric fluids.

$$\tau = K\dot{\gamma}^n$$

- *Quicksand*
- *Starch in water*
- *Wet beach sand*
- *(fine solids) Solid in liquid emulsions*
- *Corn starch in ethylene glycol*
- *TiO_2 in solvent*
- *Thick starch solutions*
- *Fine powders in suspensions*
- *etc.*

Pseudo plastic fluid (Ostwald, $n<1$), also known as **shear thinning**. Most of the non-Newtonian fluids are shear thinning.

- *Pulp (paper)*
- *Paints*
- *Blood*
- *Melted Polyethylene*
- *Aqueous solutions of clay*
- *etc.*

Newtonian Fluids usually liquids made up of simple molecules and gases as well

$$\tau = \mu\,\dot{\gamma} = \mu\,\frac{d\gamma}{dt} = \mu\,\frac{d\,v_x}{dy}$$

Fluid	Density	Viscosity	Vapor Pressure
	(kg/m^3)	mPa-s	kPa
Water	998	1.006	2.339
Ammonia	612	0.219	857.5
CO_2	772	0.0703	5724
Glycerin	1264	1490	2.40E-05
R-12	1330	0.263	567
R-134a	1226	0.2618	572.1
R-22	1213	0.2096	909.6
Ethylene Glycol	1116	21.398	0.0266
Mercury	13550	1.545	1.71E-04
Galinstan	6440	2.4	1.33E-09

Viscosity of water and mercury at 2.2°C have the viscosity of 1.67 cP, but the mercury is much **denser** than water (ρ_{H2O} = 1000 kg/m^3, ρ_{Hg}=13589 kg/m^3).

Ethylene glycol has a density 11.6% higher than water, but its viscosity is 21-fold more viscous, this is ethylene glycol is **thicker** (more viscous) than water.

$$\tau = \mu\,\dot{\gamma} = \mu\,\frac{d\gamma}{dt} = \mu\,\frac{d\,v_x}{dy}$$

Shear stress

Viscosity

Strain rate

rate of deformation

Velocity gradient

Bingham Plastic

Fluid	τ_o (Pa)	μ (Pa-s)
Ketchup (30°C)	14	0.08
Mustard (30°C)	38	0.25
Oleomargarine (30°C)	51	0.72
Mayonnaise (30°C)	85	0.63
Toothpaste (30°C)	200	10
Soy butter	80	1
Lead (441°C)	0	2.12E-03
Butter (nearly melted)	10	
Butter (from the frig)	100	
Lead (20°C)	1.30E+07	
Stainless S (20°C)	5.00E+08	
Solder (20°C)	2.70E+07	
Copper (20°C)	7.00E+07	
Titanium alloy (20° C)	1.20E+09	

$$\tau = \tau_o + \mu \frac{d\, v_x}{dy}$$

Shear stress

yield stress

Plastic viscosity

Velocity gradient

Ostwald de Waele fluids

(both pseudoplastic and dilatant)

Flow behavior index

$$\tau = K \underbrace{\frac{dv_x}{dy}}\left|\underbrace{\frac{dv_x}{dy}}\right|^{n-1}$$

Shear stress

Fluid consistency index

Velocity gradient

Fluid	*n*	*K*	T(°C)
		Pa-s^n	
Banana puree	0.33	10.7	24
Banana puree	0.46	6.5	24
Applesauce	0.41	0.66	24
Applesauce	0.65	0.5	24
Human blood	0.89	0.00384	36
Soups and sauces	0.51	3.6-5.6	
Tomato juice			
Tomato juice (5.8% solids)	0.59	0.22	32
Tomato Juice (30% solids)	0.4	18.7	32
Honey	2	0.05102	30
Paper pulp in water (4%)	0.575	20.7	
Lime in water (33%)	0.171	7.16	
Cement (54.3%)	0.153	2.51	
15% CMC in water	0.554	3.13	

Cellulose Gum (carboxy-methyl-cellulose)

Some other models for fluids

τ = shear stress

$d\gamma/dt = (dv_x/dy)$ = strain rate

Pascal $\tau = 0$

Newtonian $\tau = \mu\dot{\gamma}$

Ostwald $\tau = K\dot{\gamma}|\dot{\gamma}|^{n-1}$

Sisko $\tau = \mu\,\dot{\gamma} + K\dot{\gamma}|\dot{\gamma}|^{n-1}$

Reiner-Philippoff $\tau = \left[\mu + \frac{\mu_0 - \mu}{1 + \tau^2/A}\right]\dot{\gamma}$

Ellis $\tau = \left[\frac{\dot{\gamma}}{1/\mu + K_e\tau^{\alpha-1}}\right]$

Carreau $\tau = \left[\mu_\infty + \frac{\mu_0 - \mu_\infty}{[1 + (\lambda\dot{\gamma})^2]^p}\right]\dot{\gamma}$

Eyring $\tau = arcsinh(\dot{\gamma}/B)$

Bingham $\tau = \tau_0 + \mu_\infty\,\dot{\gamma}$

Herschel-Bulkey $\tau = \tau_0 + K\dot{\gamma}|\dot{\gamma}|^{n-1}$

Casson $\tau^{1/2} = \tau_0^{1/2} + K\dot{\gamma}^{1/2}$

Casson G $\tau^{1/2} = \tau_0^{1/2} + K_G\dot{\gamma}^n$

Week No.2, Problem 1
Identify different kind of forces in systems under motion

Problem:

Review of some basic concepts of physics needed for this class.

(Objective: To be aware of which forces are present when you have fluids, and distinguish forces over systems consisting of just solid objects)

A wood block of 2-m width, 1-m breadth and 0.3-m height is to be used to smooth a 0.5-cm thick layer of fresh cement over a sidewalk at constant velocity of 0.5 m/s in horizontal direction.

a) Calculate the force required for this purpose , if the force is applied with an angle of 30 degrees below the horizontal direction.
b) What is the force needed if the cement is already dry. The friction coefficient between wood and dry cement is 0.68

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$$\tau = K\,\dot{\gamma}^n$$

K= 2.5 Pa-s$^{3/20}$ n= 3/20

Scenario I : Fluid between two solids boundaries

$\tau =$ Shear stress at the base of the block, or tangential viscous shear stress

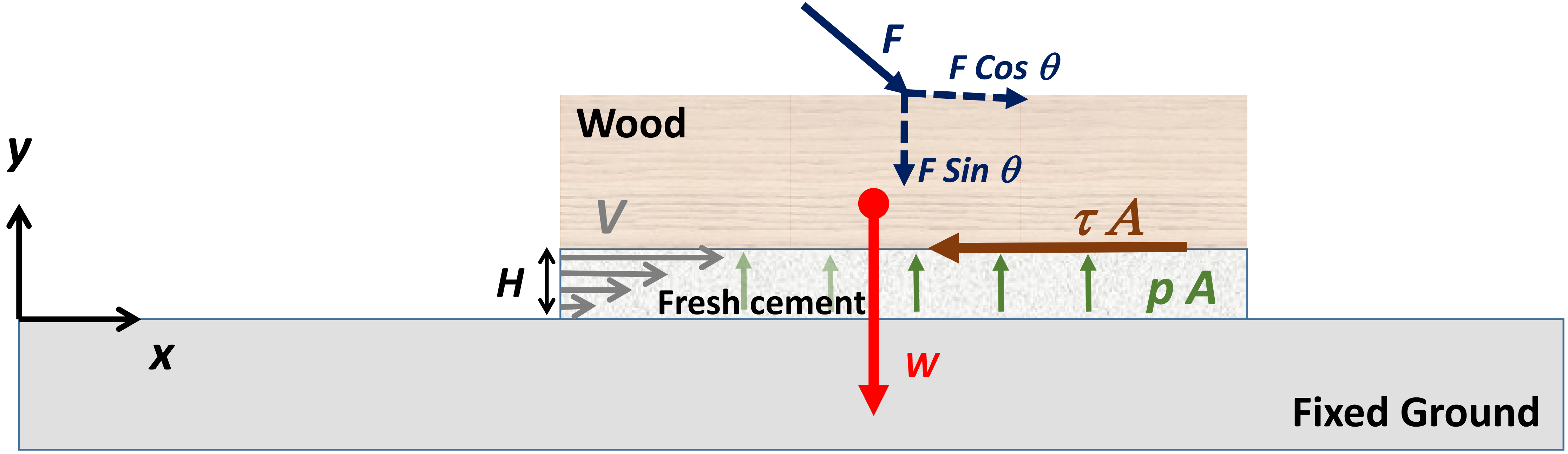
$p =$ pressure at the base of the block, or normal stress

W = Weight of the block

Friction force, or viscous force = τA

Pressure force, or compression force = $p\,A$

Gravitational force, or weight= $\rho\, g\, V$



Free Body Diagram

If you are certain about the direction of each force, you can write directly the force balances On each direction

Force balance in "x-direction"

$$F\ Cos\theta\ -\ \tau\ A = 0$$

Force balance in "y-direction"

$$-F\ Sin\theta\ -W + p\ A = 0$$

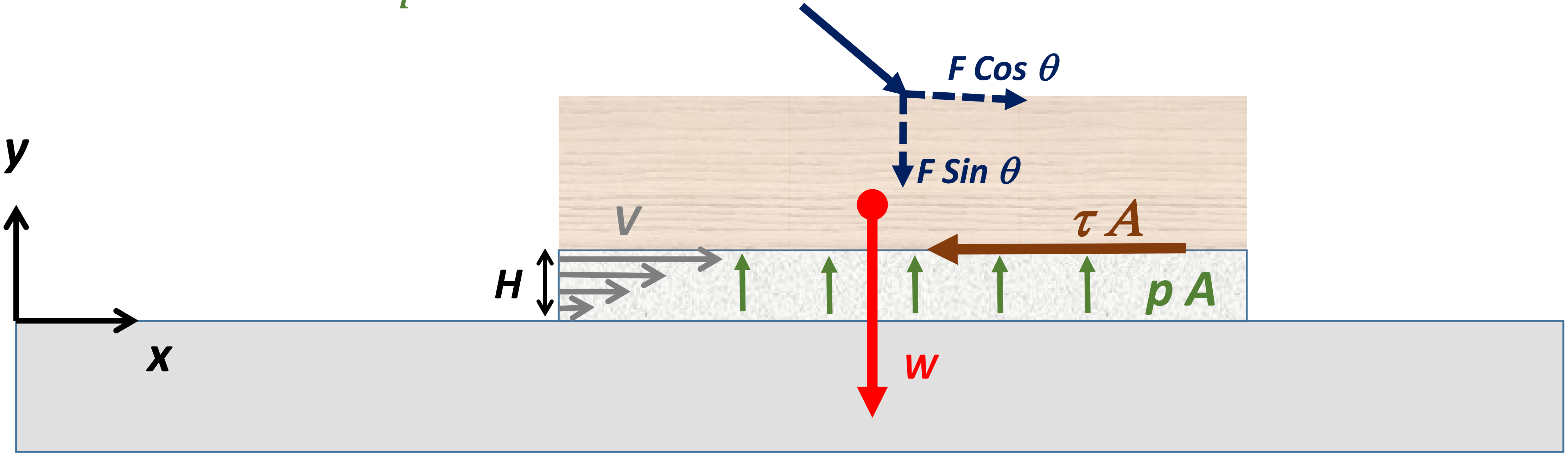
Viscous law or constitutive equation

$$\tau\ = K(\dot{\gamma})^n$$

K = Flow Consistency index

n = Flow index

$\dot{\gamma}$ = strain rate



Free Body Diagram

Assumptions or approximation

Boundary conditions: non- slip boundary condition

V_x $(y=0) = 0$

V_x $(y=H) = V$

$$\dot{\gamma} = \frac{dV_x}{dy} \sim \frac{\Delta V_x}{\Delta y} = \frac{V - 0}{H - 0}$$

Equations

Parameters for the viscosity Law Model

Viscosity Model: Power Law, or Ostwal-de Waele

Fluid Wet cement

$K = (5/2)\ \left[\mathrm{Pa \cdot s^{3/20}}\right]$

Flow consistency index

$n = 3/20$

flow index

Dimensions of the block

$L_1 = 1\ [\mathrm{m}]$

breadth of the block

$L_2 = 2\ [\mathrm{m}]$

with of the block

$L_3 = 0.3\ [\mathrm{m}]$

Height of the block

Layer thicness of cement

$H = 0.5 \times 10^{-2}\ [\mathrm{m}]$

Velocity of the block in the horizontal direction

$V = 0.5\ [\mathrm{m/s}]$

$\theta = 30\ [\mathrm{deg}]$

inclination of the force below the horizontal axis direction

Force balance in x-axis

$F \cdot \mathrm{Cos}(\theta) - \tau \cdot A = 0$

Force balance in y-axis

$-F \cdot \sin(\theta) - m \cdot g + p \cdot A = 0$

$m = \rho \cdot Vol$

mas of the block

$$Vol = L_1 \cdot L_2 \cdot L_3$$

Geometry: Parallelepiped, then to calculate the Volume of the block we use:

$$A = L_1 \cdot L_2$$

Area of the block's base

$$g = 9.80665 \ [\mathrm{m/s^2}]$$

Gravitational field

shear rate is the velocity gradient within the fluid

$$\dot{\gamma} = \frac{V_2 - V_1}{y_2 - y_1}$$

$$y_2 = H$$

Upper boundary of the fluid

$$y_1 = 0 \ [\mathrm{m}]$$

Lower boundary of the fluid

$V_2 = V$

no-slip boundary condition at the top of the cement

$V_1 = 0 \ [\text{m/s}]$

non-slip boundary condition, or ground at rest at the bottom of the cement layer

Relationship between shear stress and strain rate

$\tau = K \cdot (GAMMAdot)^n$

$T = 25 \ [\text{C}]$

$\rho = \rho \left(wood_{pine}, \ \mathrm{T} = T \right)$

Equation of state to determine the desnity of the solid, database values or calling built in functions

Solution

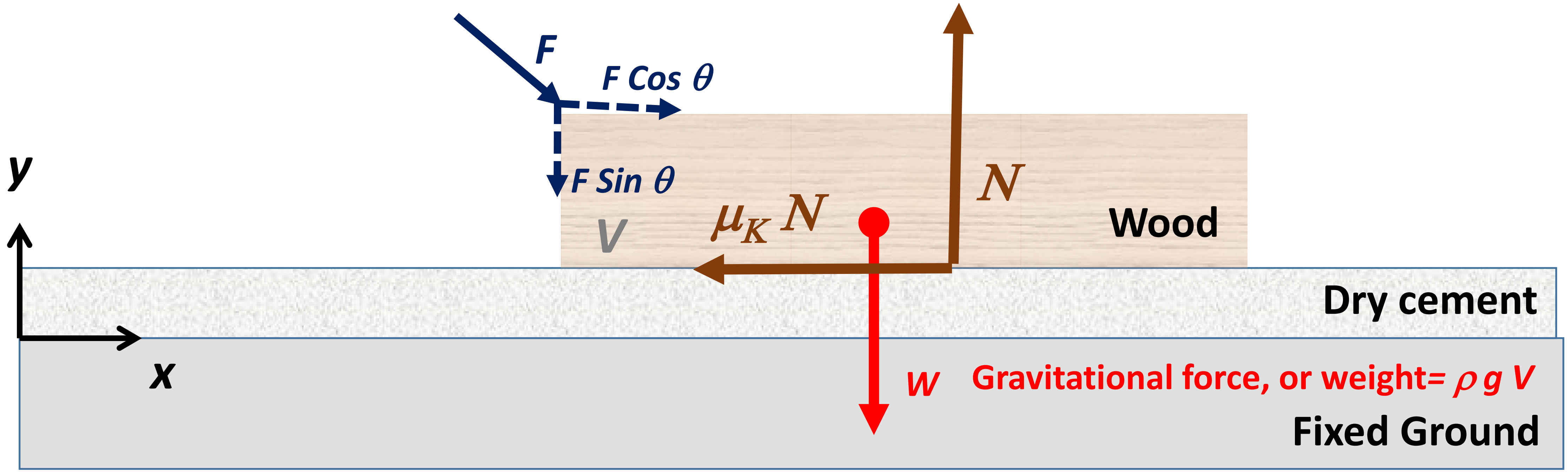
$A = 2\ [\mathrm{m}^2]$	$F = 11.52\ [\mathrm{N}]$
$g = 9.807\ [\mathrm{m/s}^2]$	$\dot{\gamma} = 100\ [1/\mathrm{s}]$
$H = 0.005\ [\mathrm{m}]$	$K = 2.5\ [\mathrm{Pa\text{-}s}^{(3}/20)]$
$L_1 = 1\ [\mathrm{m}]$	$L_2 = 2\ [\mathrm{m}]$
$L_3 = 0.3\ [\mathrm{m}]$	$m = 268.8\ [\mathrm{kg}]$
$n = 0.15$	$p = 1321\ [\mathrm{Pa}]$
$\rho = 447.9\ [\mathrm{kg/m}^3]$	$T = 25\ [\mathrm{C}]$
$\tau = 4.988\ [\mathrm{Pa}]$	$\theta = 30\ [\mathrm{deg}]$
$V = 0.5\ [\mathrm{m/s}]$	$Vol = 0.6\ [\mathrm{m}^3]$
$V_1 = 0\ [\mathrm{m/s}]$	$V_2 = 0.5\ [\mathrm{m/s}]$
$y_1 = 0\ [\mathrm{m}]$	$y_2 = 0.005\ [\mathrm{m}]$

Scenario II : Only solids involved

μ_K ***N*** = Friction Force

N = Normal force

W = Weight of the block



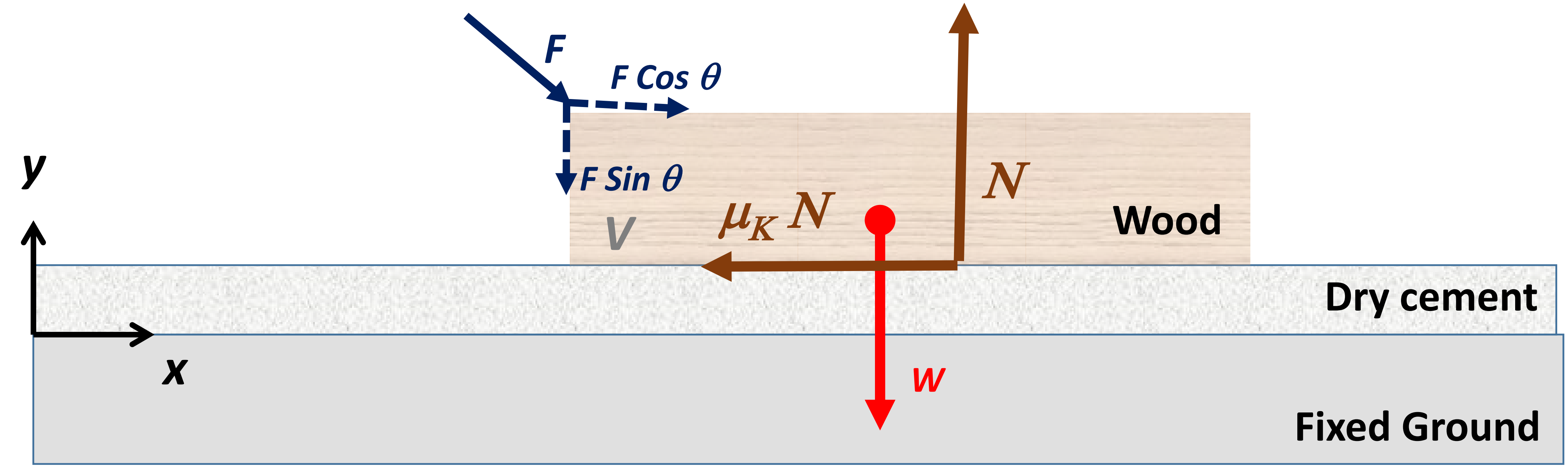
Free Body Diagram

Force balance in “x-direction”

$$F\ Cos\theta\ -\mu_K\ N = 0$$

Force balance in “y-direction”

$$-F\ Sin\theta\ - W + N = 0$$



Free Body Diagram

Second Scenario: Dry cement

Friction Coefficient

$$\mu_K = 0.68$$

Force balance in x-axis

$$F \cdot \text{Cos}(\theta) - \mu_K \cdot N_f = 0$$

Force balance in y-axis

$$-F \cdot \sin(\theta) - m \cdot g + N_f = 0$$

Solution

$A = 2$ [m^2]	$F = 3407$ [N]
$g = 9.807$ [m/s^2]	$H = 0.005$ [m]
$L_1 = 1$ [m]	$L_2 = 2$ [m]
$L_3 = 0.3$ [m]	$m = 268.7$ [kg]
$\mu_K = 0.68$	$N_f = 4339$ [N]
$\rho = 447.9$ [kg/m^3]	$\theta = 30$ [deg]
$V = 0.5$ [m/s]	$Vol = 0.6$ [m^3]

Problem-solving Technique

Step 1: Understand the problem statement

Step 2: Sketch (Diagram, process flow chart or free body diagram, always include the frame of reference)

Step 3: Assumptions, Approximations, Constraints and set time and space dependence (Boundary conditions, steady state, equilibrium, quasi-steady state, quasi-equilibrium, transient, start-up, etc.)

Step 4: Physical Laws (Fundamental Laws, and constitutive equations or rate laws), write or list all of them, even if is the same equation used in one section, specially if is under different context or subsystem.

Step 5: Properties and correlations (Thermodynamic relations, equations of state, empirical correlations resulting from dimensional analysis), must be included if used.

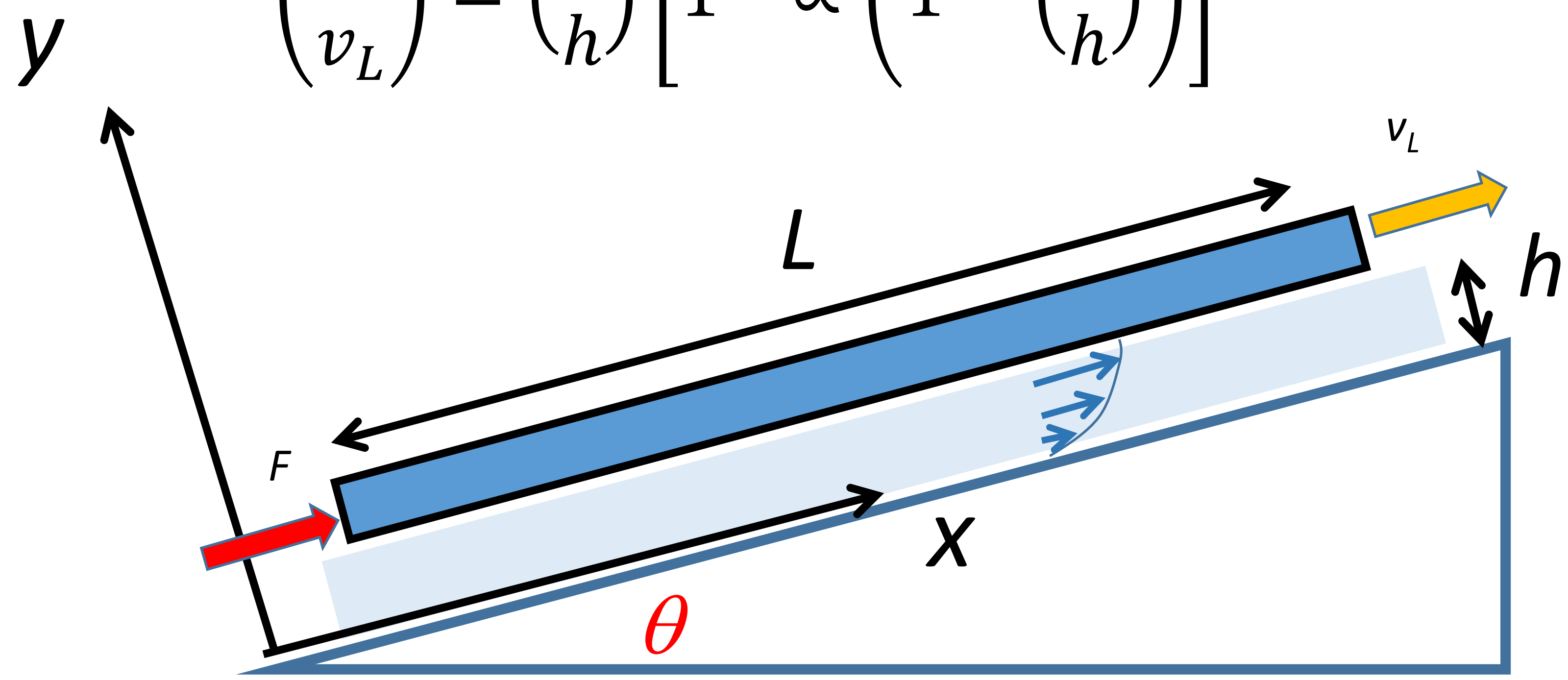
Step 6: Calculations (neat work, always show the units during operations)

Step 7: Reasoning (Sensitivity analysis, what if), Verification (context), and Discussion

Label each variable correctly (Use either IUPAP or SPE nomenclature if possible or the one used in class, if you create your own nomenclature you must include a nomenclature table, with dimensions and units)

Problem. (What happens when the velocity of the liquid is not uniform ?) When you move a plate up hill in a slope, the velocity profile takes the form:

$$\left(\frac{v}{v_L}\right) = \left(\frac{y}{h}\right)\left[1 - \propto \left(1 - \left(\frac{y}{h}\right)\right)\right]$$



a) Calculate the stress over the the ramp, stress over the plate, liquid flow rate, and average liquid velocity. b) Write all your results in dimensionless form:

If the fluid (lubricant is a Newtonian Fluid) obeys Newton's viscosity law:

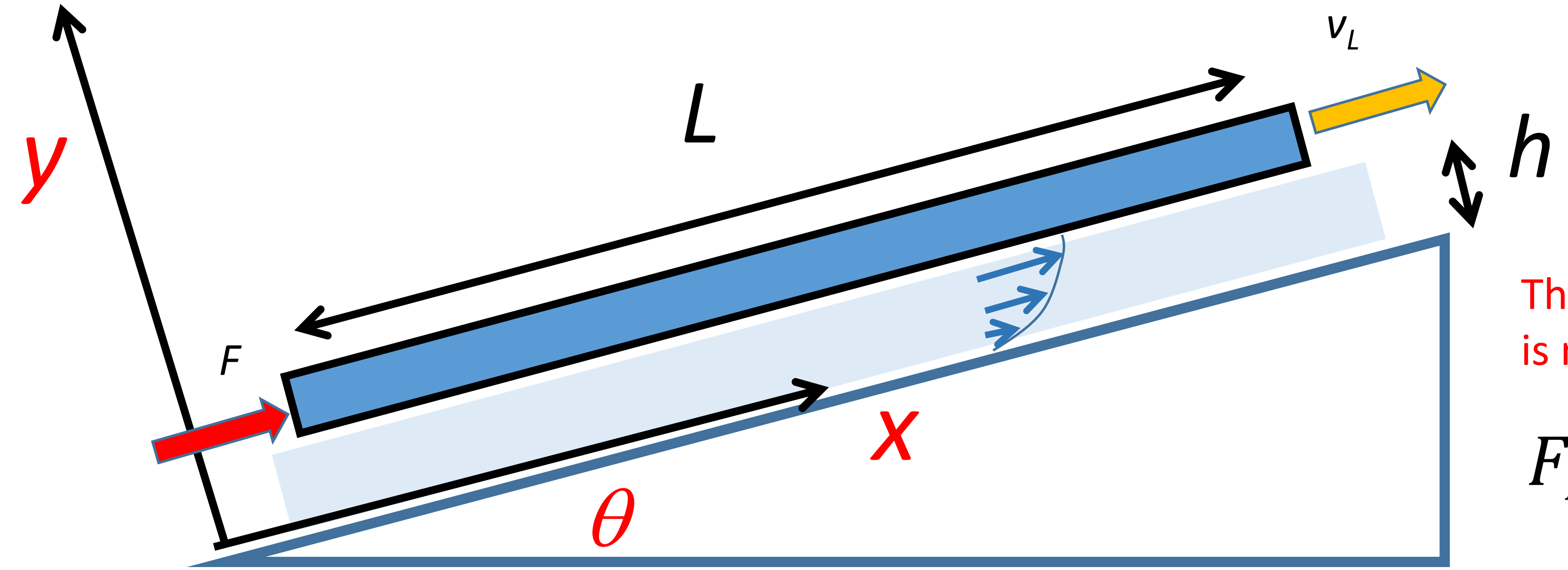
$$v = v_L\left(\frac{y}{h}\right)\left[1 - \propto\left(1 - \left(\frac{y}{h}\right)\right)\right]$$

The shear stress is calculated as:

$$\tau = \mu\frac{dv}{dy} \qquad \tau = \mu\left(\frac{v_L}{h}\right)\left[1 - \propto\left(1 - 2\left(\frac{y}{h}\right)\right)\right]$$

The shear stress over the plate is (this is at *y*=*h*):

$$\tau_H = \mu\left(\frac{v_L}{h}\right)[1 + \propto]$$



The force applied over the plate if its mass is negligible, should be:

$$F_H = \mu\left(\frac{v_L}{h}\right)[1 + \propto]WL$$

To calculate the flow rate , we integrate the velocity profile across the thickness

$$\dot{V} = \int_0^h v\, dA = \int_o^h v\, d[Wy] = W \int_0^h v\, dy \qquad v = v_L \left(\frac{y}{h}\right)\left[1 - \propto \left(1 - \left(\frac{y}{h}\right)\right)\right]$$

Velocity is function of height, then integration is needed to calculate the volume flow rate

$$\dot{V} = W v_L \int_0^h \left(\frac{y}{h}\right)\left[1 - \propto \left(1 - \left(\frac{y}{h}\right)\right)\right] dy$$

$$\dot{V} = W h v_L \left[\frac{3 - \alpha}{6}\right]$$

The average velocity is the volumetric flow divided by area

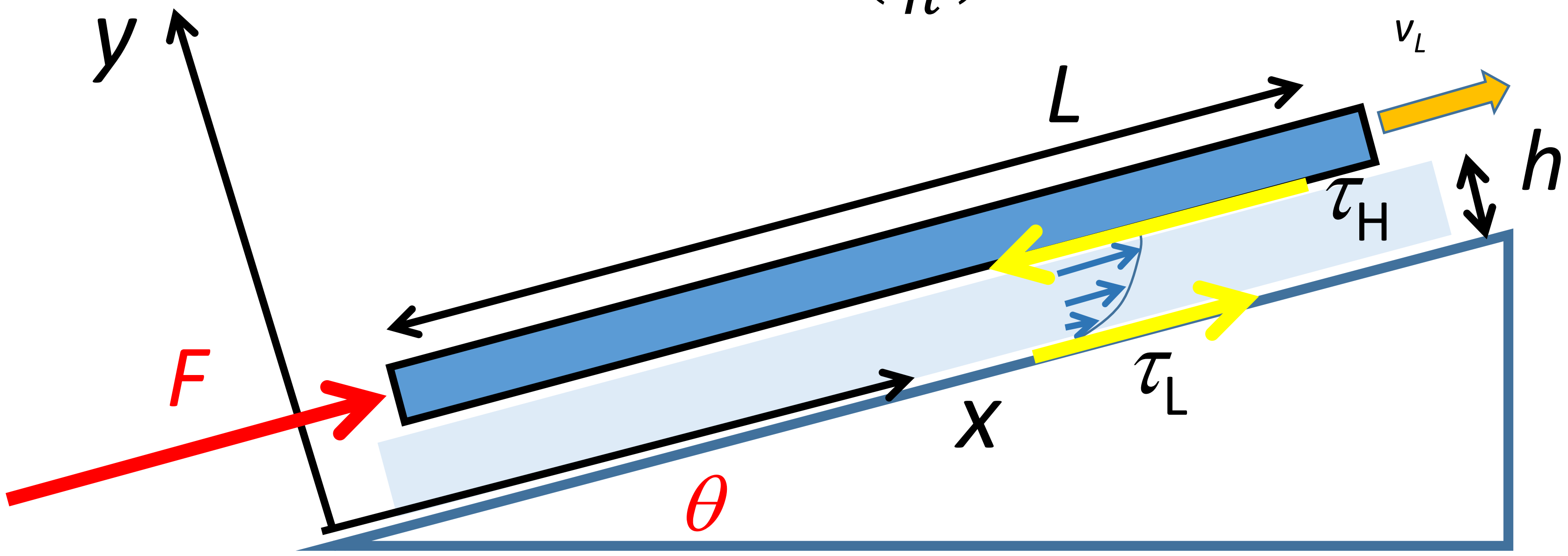
$$\langle v \rangle = v_L \left[\frac{3 - \alpha}{6}\right]$$

If he shear stress is calculated as:

$$\tau = \mu \frac{dv}{dy} \qquad \tau = \mu \left(\frac{v_L}{h}\right)\left[1 - \propto \left(1 - 2\left(\frac{y}{h}\right)\right)\right]$$

The shear stress over the ramp (*y*=0):

$$\tau_L = \mu \left(\frac{v_L}{h}\right)[1 - \propto]$$



There is a dimensionless number called friction factor, which is calculated as the ratio between the stress, and the inertial force per unit area. Evaluate the friction factor for both boundaries, the upper one (f_H) and the lower one (f_L)

$$f = \frac{\tau}{\frac{1}{2}\rho\langle v\rangle^2}$$

$$f_H = \frac{\tau_H}{\frac{1}{2}\rho\langle v\rangle^2} = \frac{\mu}{\rho v_L h}\left[\frac{72(1+\alpha)}{(3-\alpha)^2}\right] = \frac{1}{Re_L}\left[\frac{72(1+\alpha)}{(3-\alpha)^2}\right]$$

$$f_L = \frac{\tau_L}{\frac{1}{2}\rho v_L^2} = 2(1+\alpha)\frac{\mu}{\rho v_L h} = \frac{2(1+\alpha)}{Re_L}$$

For sake of science: The value of α is a measure of the ratio between gravitational forces and viscous forces. All the properties in the expression of α are liquid properties.

$$\alpha = \frac{\rho v_L h}{\mu}\left[\frac{sin\theta}{2}\right]\frac{gh}{{v_L}^2} = \frac{Re_L}{Fr_L}\frac{sin\theta}{2}$$

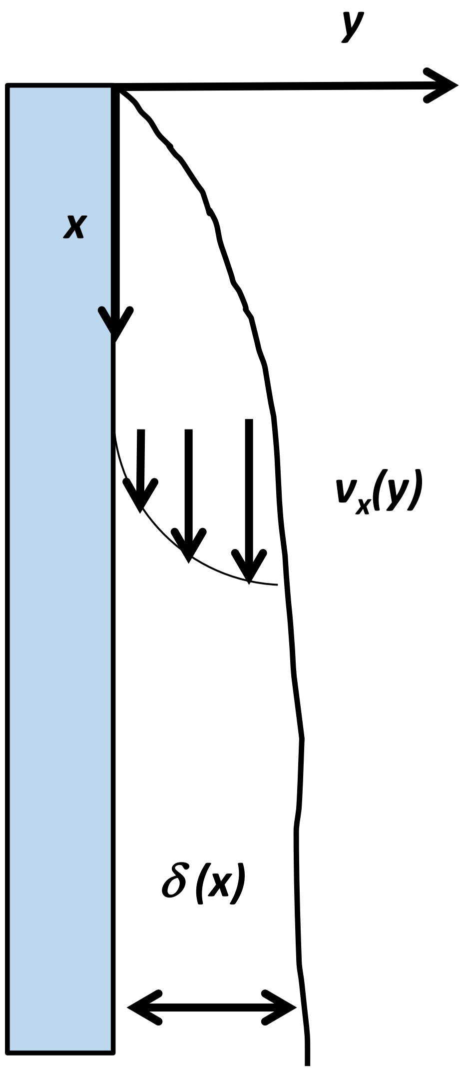
It is more practical to express everything in terms of measurable variables rather than average values, so the previous equations are practical, re write the previous equations in terms of the average velocity instead of the velocity of the upper plate.

Please reflect and discuss about the direction of the stress tensor and the direction of the force per unit area and it direction.

In film condensation along a vertical plate in a vapor atmosphere, Nusselt found out that in laminar flow the velocity profile at a station *x* is.

$$v_x(y) = \frac{(\rho_l - \rho_v) g \delta^2}{\mu} \left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 \right]$$

Where ρ_{l} and ρ_{v} are the density of the fluid in the liquid and vapor phase, respectively, and μ the liquid viscosity. Find the flow rate per unit width at any value of *x*, and the shear stress at the wall (interface wall-liquid) and at the interface water-air.



To calculate volumetric flow, the velocity profile is integrated through the cross sectional area, perpendicular to the velocity (You can assume that the width of the wall is *W*)

$$\dot{V} = \int_{z=0}^{z=W} \int_{y=0}^{y=\delta} v_x \, dz \, dy = \int_{y=0}^{y=\delta} W v_x(y) dy$$

$$\int_0^{\delta} W v_x(y) dy = \int_0^{\delta} \frac{W(\rho_l - \rho_v) g \delta^2}{\mu} \left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 \right] dy = \frac{W \Delta\rho g \delta^3}{\mu} \int_0^1 \left[\eta - \frac{\eta^2}{2} \right] d\eta$$

$$\dot{V} = \frac{W \Delta\rho \, g \, \delta^3}{\mu} \left[\frac{\eta^2}{2} - \frac{\eta^3}{6} \right] \Bigg|_0^1 = \frac{W \, \Delta\rho \, g \, \delta^3}{3\mu}$$

To calculate shear rate over the wall, we use Newton's viscosity law

$$\tau = \mu \frac{dv_x(y)}{dy} \qquad \tau = \mu \frac{dv_x(y)}{dy} = (\rho_l - \rho_v) g \delta^2 \left[\frac{1}{\delta} - \frac{y}{\delta^2} \right]$$

Then we evaluate the stress in the wall, i.e y=0

$$\tau_w = \mu \frac{dv_x(y)}{dy}\Bigg|_{y=0} = (\rho_l - \rho_v) g \delta$$

Finally the stress at the water-air interface, i.e $y=\delta$

$$\tau_w = \mu \frac{dv_x(y)}{dy}\Bigg|_{y=\delta} = 0$$

In a nutshell whenever you have a non uniform velocity profile, integration will be needed. Engineers will subside this issue by using average properties.

$$\langle \phi \rangle = \frac{\int \phi \, dA}{\int dA}$$

Average is different form mean. Mean can be arithmetic, weighted, geometric, harmonic, power, *f* or function, truncated, interquartile, and so on.

Physical meaning of different fluxes and expression of flow

Flux	property	Flow symbol	Units of flow	Expression for flow
$\underline{v}$	Volume	$\dot{V}$	m³/s	$\int \underline{v} \cdot \underline{n}\, dA$
$\rho\, \underline{v}$	mass	$\dot{m}$	kg/s	$\int \rho\, \underline{v} \cdot \underline{n}\, dA$
$\phi\, \rho\, \underline{v}$	Φ	$\dot{\Phi}$	prop/s	$\int \rho\, \phi\, \underline{v} \cdot \underline{n}\, dA$
$\underline{q} = \underline{\dot{Q}_A}$	heat	$\dot{Q}$	J/s	$\int \underline{q} \cdot \underline{n}\, dA = \int \underline{\dot{Q}_A} \cdot \underline{n}\, dA$
$\rho \underline{v}\, \underline{v}$	momentum	$\dot{m}\langle \underline{v} \rangle \beta$	N	$\int \rho \underline{v}\, \underline{v} \cdot \underline{n}\, dA$
$\rho \hat{E}\, \underline{v}$	Energy, *E*	$\dot{m}\hat{E}$	J/s	$\int \rho \hat{E} \underline{v} \cdot \underline{n}\, dA$

$\phi = \hat{\Phi}$ $\qquad \beta = \dfrac{\langle v^2 \rangle}{\langle v \rangle^2}$ $\qquad \hat{E} = \hat{U} + g\langle z \rangle + \dfrac{\langle v^3 \rangle}{2\langle v \rangle}$

This is expression for flow not flux, in math jargon the term flux and flow are interchangeable

$$\dot{\Phi} = \frac{d\Phi}{dt}$$

$$\hat{\Phi} = \frac{d\Phi}{dm}$$

$$\langle \phi \rangle = \frac{\int \phi\, dA}{\int dA}$$

Basic Flow assumptions and Their Mathematical Statements

Flow assumption:

- Time-dependence
- Dimensionality
- Directionality
- Unidirectional flow
- Development phase
- Symmetry

Consequence:

$$\begin{cases} \dfrac{\partial B}{\partial t} = 0 & \text{i.e. steady state} \\ \underline{v} = \underline{v}(t) & \text{i.e. transient flow} \end{cases}$$

Required number of space variables $\underline{x} = (x, y, z)$

Required number of velocity components $\underline{v} = (u, v, w)$

Special case when all but one velocity component are zero

$$\frac{\partial v}{\partial s} = 0$$

i.e. fully developed flow, where s is the axial coordinate

$$\begin{cases} \left.\dfrac{\partial B}{\partial n}\right|_{n=0} = 0 & \text{Mid plane (n is the normal coordinate)} \\ \dfrac{\partial B}{\partial \theta} = 0 & \text{axisymmetric} \end{cases}$$

Week No.2, Problem 3

Identify different kind of forces in systems under motion

A 50-cm x 40-cm x 30 cm block (density 740 kg/m^3) is moved at constant velocity on an inclined surface, by applying a horizontal force of 160 N. To accomplish this task a 1-mm thick layer of 5 mPa-s wax is used. Determine:

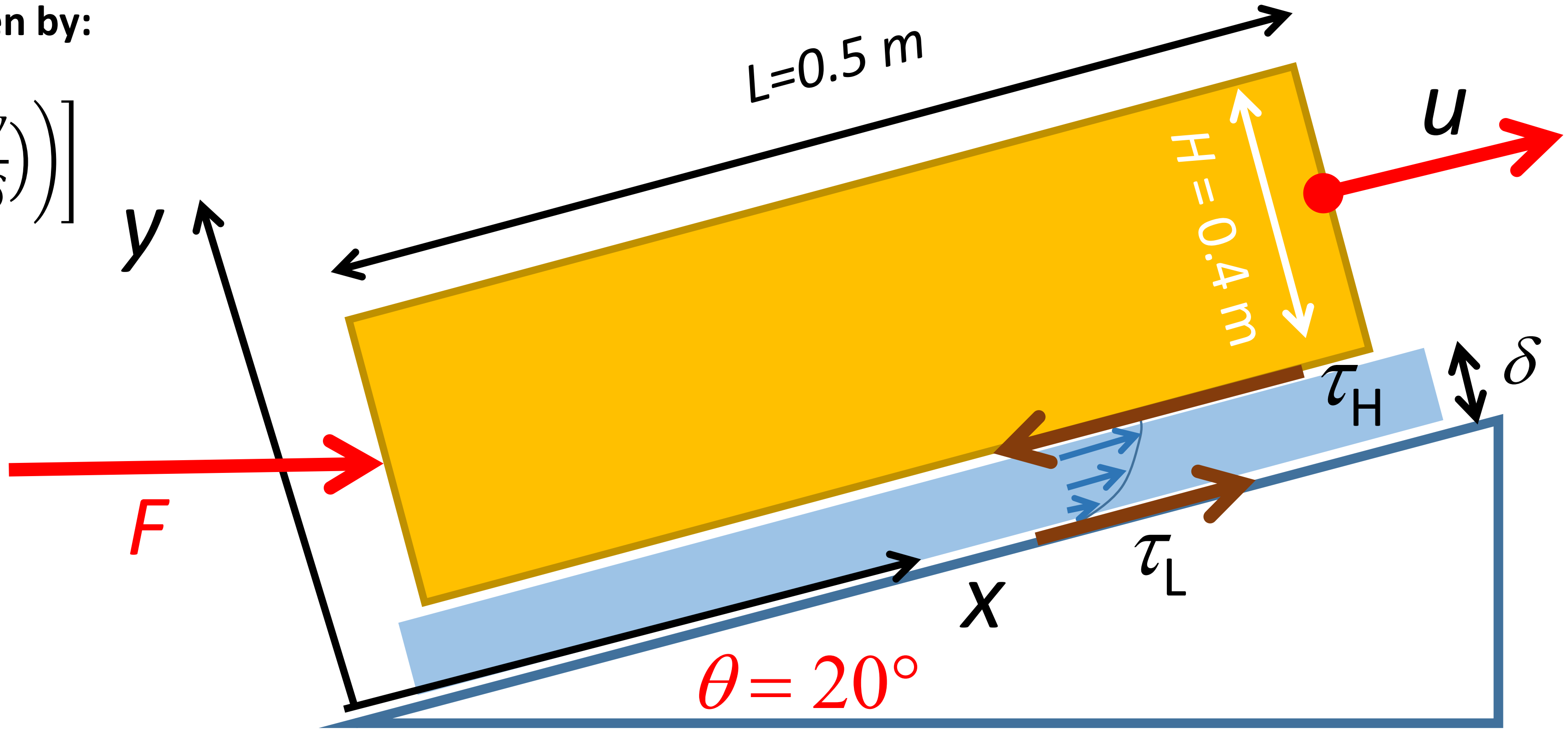
a) The velocity "*u*" of the block moving uphill.
b) The pressure "*p*" at the base of the block.
c) Calculate the heat per unit volume (W/m^3), generated by viscous dissipation assuming that can be calculated by the equation $\mu\,(u/\delta)^2\,(1+\alpha^2\,/\,3)$
d) Recalculate all if the density of the wax is 800 kg/m^3, and reflect about this fact.

Note: μ= Viscosity, δ= wax layer thickness, F = Horizontal force, θ= Inclination angle. Assume density of the wax is 740 kg/m^3.

The velocity in the wax is given by:

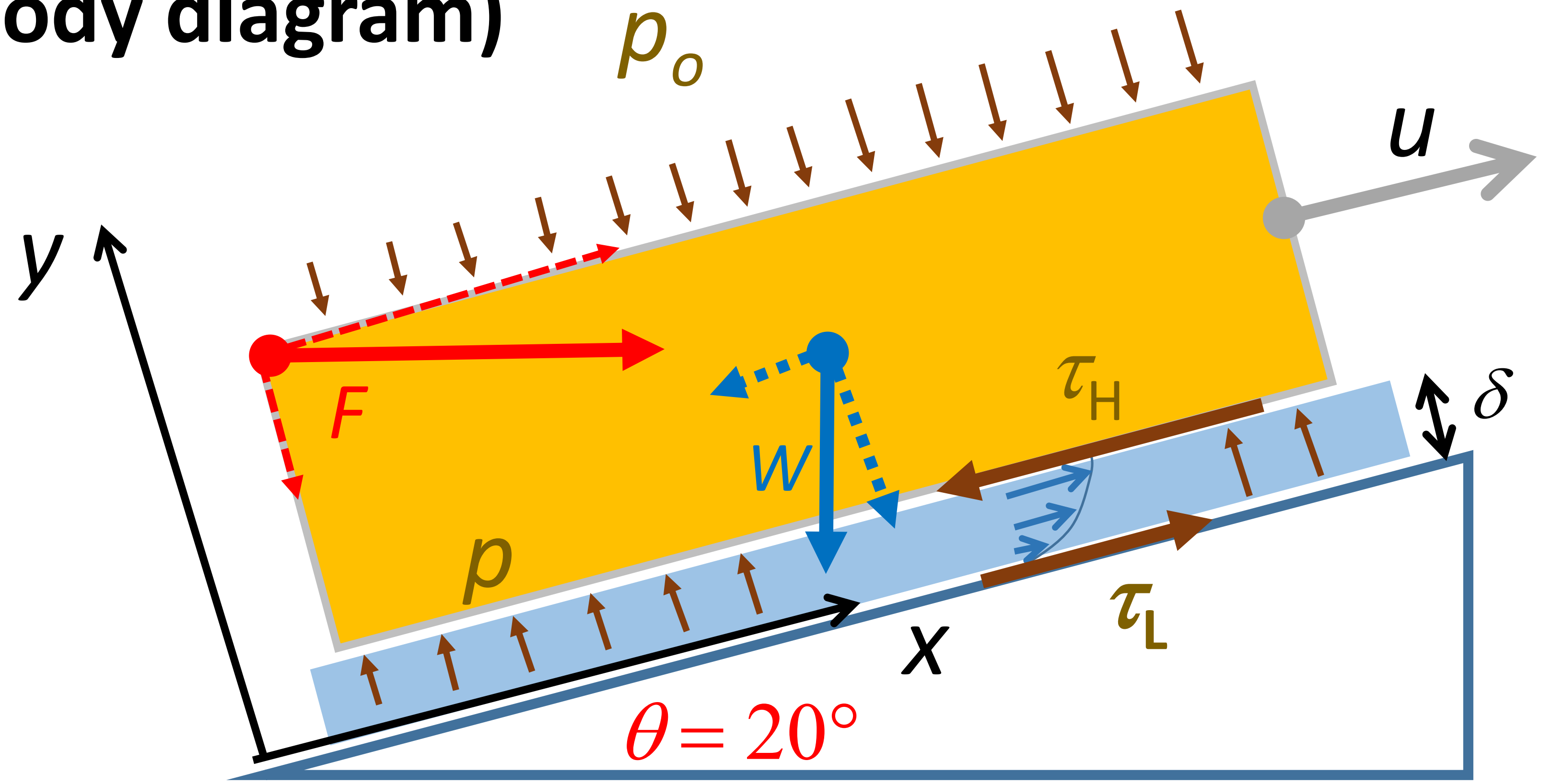
$$v = u\left(\frac{y}{\delta}\right)\left[1 - \alpha\left(1 - \left(\frac{y}{\delta}\right)\right)\right]$$

$$\alpha = \frac{\rho\, u\, \delta}{\mu}\left[\frac{\sin\theta}{2}\right]\frac{g\,\delta}{u^2}$$



Problem 3

FBD (Free body diagram)



Linear momentum balances (or force balances)

$$F\ \cos\theta\ - W\ \sin\theta - \ \tau_H\ A = 0 \qquad x\text{-axis}$$

$$-F\ \sin\theta\ - W\ \cos\theta + \ (p - p_O)A = 0 \qquad y\text{-axis}$$

To estimate the shear strain rate, we need to identify the type of fluid (in this case Newtonian), then if we know the velocity field, with the mathematical model obtain the strain rate.

Velocity field

$$v = u\left(\frac{y}{\delta}\right)\left[1 - \propto \left(1 - \left(\frac{y}{\delta}\right)\right)\right]$$

Relationship between **shear stress** and strain rate

$$\tau = \mu\dot{\gamma} = \mu\frac{dv}{dy}$$

$$\tau = \mu\dot{\gamma} = \mu\frac{dv}{dy} = \mu\frac{d}{dy}\left[u\left(\frac{y}{\delta}\right)\left[1 - \propto \left(1 - \left(\frac{y}{\delta}\right)\right)\right]\right]$$

$$\tau = \frac{\mu\, u}{\delta}\frac{d}{dy}\left[y - \propto \left(y - \left(\frac{y^2}{\delta}\right)\right)\right] = \frac{\mu\, u}{\delta}\left[1 - \propto \left(1 - 2\left(\frac{y}{\delta}\right)\right)\right]$$

$$\tau_H = \frac{\mu\, u}{\delta}\left[1 - \propto \left(1 - 2\left(\frac{y}{\delta}\right)\right)\right]\Bigg|_{y=\delta} = \frac{\mu\, u}{\delta}(1 + \alpha)$$

shear stress in the upper boundary (with wood block)

$$\tau_L = \frac{\mu\, u}{\delta}\left[1 - \propto \left(1 - 2\left(\frac{y}{\delta}\right)\right)\right]\Bigg|_{y=0} = \frac{\mu\, u}{\delta}(1 - \alpha)$$

shear stress in lower boundary (with the ramp)

Recall that stress is a tensor, in this case we are calculating the force per unit area, force in *x*-axis on the face perpendicular to y-axis, this is expressed as τ_{yx}, then that particular component of the stress tensor has the form.

Relationship between stress and strain rate

$$\tau = \mu\dot{\gamma} = \mu\frac{dv}{dy}$$

Components of the stress tensor

$$\tau_{yx} = \mu\frac{dv_x}{dy}\,\hat{j}\hat{\imath}$$

Relationship between stress and Force

$$\underline{F} = \underline{n}\cdot\underline{\underline{\tau}}\,A$$

$$\tau_H = \frac{\mu\,u}{\delta}(1+\alpha)$$

$$\tau_L = \frac{\mu\,u}{\delta}(1-\alpha)$$

Analyzing the fluid within the ramp and the wood block, the normal vectors at the top and at the base are:

$$\underline{n}_H = \hat{j} \qquad \underline{n}_L = -\hat{j}$$

$$\underline{\underline{\tau}}_H = \frac{\mu\,u}{\delta}(1+\alpha)\,\hat{j}\hat{\imath} \qquad \underline{\underline{\tau}}_L = \frac{\mu\,u}{\delta}(1-\alpha)\,\hat{j}\hat{\imath}$$

The typical form to write the model or the relationship between strain rate and shear stress, is to consider the fluid as a system, some book may write the models for the boundaries interacting with the fluid:

$\tau = \mu\dot{\gamma}$ Fluid oriented model

$\tau = -\mu\dot{\gamma}$ Boundary oriented model

Calculating the force of each boundary over the fluid, we have: $\underline{F} = \underline{n} \cdot \underline{\underline{\tau}}\, A$

$\underline{\underline{\tau}}_H = \frac{\mu\, u}{\delta}(1+\alpha)\, \hat{j}\hat{i}$

$\underline{n}_H = \hat{j}$

$\underline{\underline{\tau}}_L = \frac{\mu\, u}{\delta}(1-\alpha)\, \hat{j}\hat{i}$

$\underline{n}_L = -\,\hat{j}$

$$\underline{F}_H = \underline{n}_H \cdot \underline{\underline{\tau}}_H = A\frac{\mu\, u}{\delta}(1+\alpha)\,\hat{\mathrm{j}} \cdot \hat{\mathrm{j}}\hat{\mathrm{i}} = A\frac{\mu\, u}{\delta}(1+\alpha)\,\hat{\mathrm{i}}$$

$$\underline{F}_L = \underline{n}_L \cdot \underline{\underline{\tau}}_L = \mathrm{A}\frac{\mu\, u}{\delta}(1-\alpha)\left(-\hat{\mathrm{j}}\right) \cdot \hat{\mathrm{j}}\hat{\mathrm{i}} = -A\frac{\mu\, u}{\delta}(1-\alpha)\,\hat{\mathrm{i}}$$

Writing the force balance in x-axis for the fluid :

$$-\rho\, A\, \delta\, g\, \sin\theta + F_H + F_L = \frac{d[\rho\, A\, \delta\, v_x]}{dt}$$

Writing the force balance in x-axis for the fluid :

$$\rho\, A\, \delta\, g_x + F_{Hx} + F_{Lx} = \frac{d[\rho\, A\, \delta\, v_x]}{dt}$$

$$\underline{g} = g_x\, \hat{\imath} + g_y\, \hat{\jmath} = -g \sin\theta\ \hat{\imath} + g\ \cos\theta\, \hat{\jmath}$$

$$\underline{F_H} = \underline{n_H} \cdot \underline{\underline{\tau_H}} = A\frac{\mu\, u}{\delta}(1+\alpha)\,\hat{\jmath}\cdot\hat{\jmath}\,\hat{\imath} = A\frac{\mu\, u}{\delta}(1+\alpha)\,\hat{\imath}$$

$$\underline{F_L} = \underline{n_L} \cdot \underline{\underline{\tau_L}} = \mathrm{A}\frac{\mu\, u}{\delta}(1-\alpha)(-\hat{\jmath})\cdot\hat{\jmath}\,\hat{\imath} = -A\frac{\mu\, u}{\delta}(1-\alpha)\,\hat{\imath}$$

$$-\rho\, A\, \delta\, g \sin\theta\, \hat{\imath} + A\frac{\mu\, u}{\delta}(1+\alpha)\,\hat{\imath} - A\frac{\mu\, u}{\delta}(1-\alpha)\,\hat{\imath} = \frac{d[\rho\, A\, \delta\, v_x]}{dt}$$

$$-\rho\, A\, \delta\, g \sin\theta\, \hat{\imath} + 2\, A\frac{\mu\, u}{\delta}\alpha\ \hat{\imath} = \frac{d[\rho\, A\, \delta\, v_x]}{dt}$$

Under the quasi-steady state assumption, i.e. no accumulation of linear momentum in x-axis, it can be proven that the value of "α" coincides with the one given in the problem statement.

$$\alpha = \frac{\rho\, u\, \delta}{\mu}\left[\frac{sin\theta}{2}\right]\frac{g\, \delta}{u^2}$$

Data

Height or Breadth

$H = 0.4$ [m]

Length

$L = 0.5$ [m]

Width

$W = 0.3$ [m]

Volume of the block

$V = H \cdot L \cdot W$

Area of the boundary solid-wax

$A = L \cdot W$

Thickness of the wax layer

$\delta = 1 \ [mm] \cdot \left| 0.001 \cdot \frac{m}{mm} \right|$

Viscosity of the wax

$\mu = 5 \ [milliPa\text{-}s] \cdot \left| 0.001 \cdot \frac{Pa\text{–}s}{milliPa\text{–}s} \right|$

Density of the wood

$\rho_w = 740$ [kg/m^3]

Horizontal Force

$F = 160$ [N]

Inclination angle, also the angle of the frame of reference respect to the horizontal

$\theta = 20$ [deg]

Wax density

$$\rho = 800 \quad [kg/m^3]$$

Atmospheric pressure at sea level

$$p_o = 101325 \quad [Pa]$$

gravitational field, or gravity

$$g = 9.81 \quad [m/s^2]$$

Linear momentum balances

x-axis

$$F \cdot \cos(\theta) - \rho_w \cdot g \cdot V \cdot \sin(\theta) - \tau_H \cdot A = 0$$

y-axis

$$-F \cdot \sin(\theta) - \rho_w \cdot g \cdot V \cdot \cos(\theta) + (p - p_o) \cdot A = 0$$

Relationship between stress, and velocity

$$\tau_H = \mu \cdot u \cdot \left[\frac{1 + \alpha}{\delta}\right]$$

$$\alpha = Re \cdot \frac{\sin(\theta)}{2 \cdot Fr}$$

Froude Number

$$Fr = \frac{u^2}{g \cdot \delta}$$

Reynolds Number

$$Re = \rho \cdot u \cdot \frac{\delta}{\mu}$$

Sterss at the ramp

$$\tau_L = \mu \cdot u \cdot \left[\frac{1 - \alpha}{\delta}\right]$$

SOLUTION

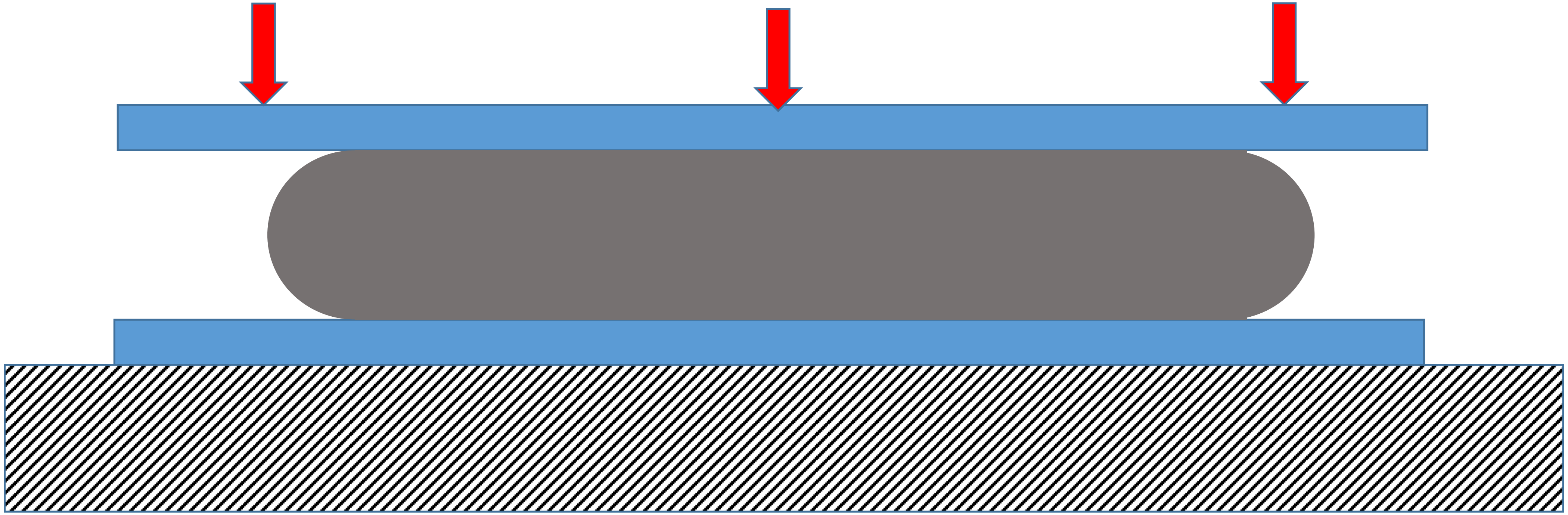
Unit Settings: SI C kPa kJ mass deg

$A = 0.15$ [m^2]	$\boxed{\alpha = 0.1709}$	$\delta = 0.001$ [m]	$F = 160$ [N]	$Fr = 251.4$	$g = 9.81$ [m/s^2]
$H = 0.4$ [m]	$L = 0.5$ [m]	$\mu = 0.005$ [Pa-s]	$\boxed{p = 104418 \text{ [Pa]}}$	$p_o = 101325$ [Pa]	$Re = 251.3$
$\rho = 800$ [kg/m^3]	$\rho_w = 740$ [kg/m^3]	$\tau_H = 9.194$ [Pa]	$\tau_L = 6.51$ [Pa]	$\theta = 20$ [deg]	$\boxed{u = 1.57 \text{ [m/s]}}$
$V = 0.06$ [m^3]	$W = 0.3$ [m]				

Reflecting about the value of α ....

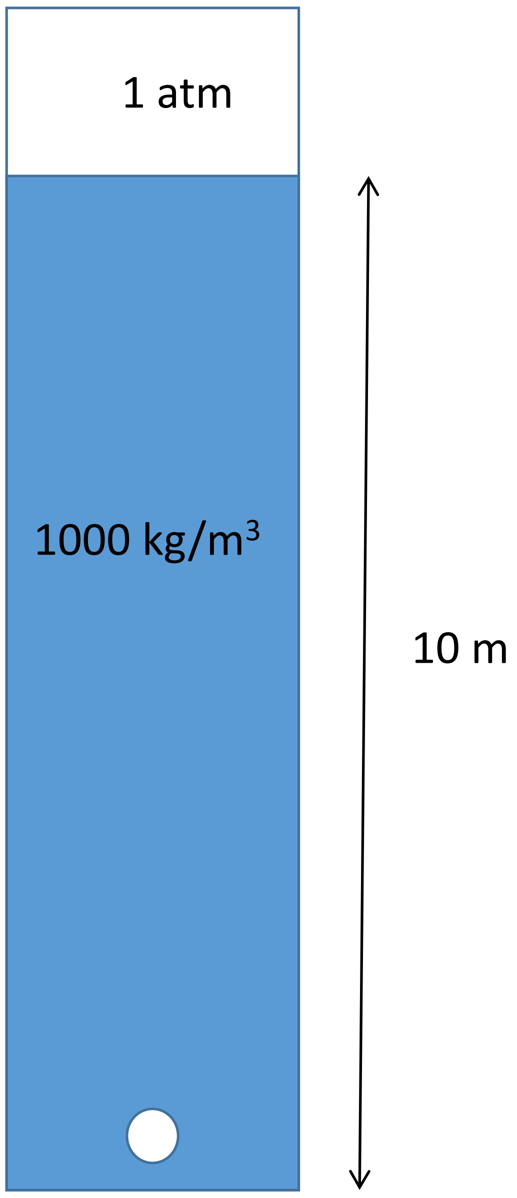
Is the ratio between gravitational and viscous forces

Problem 4



Two 10-cm × 10-cm 6mm-thick glass plates (2400 kg/m^3) are used to compress galinstan (Density = 6440 kg/m^3, Surface tension 0.535 N/m). If the original volume of the liquid metal is 10 cm^3, calculate the separation between plates, at equilibrium conditions.

Note: as a first approach you can assume the contact angle to be 180°, and neglect gravity effects over the fluid.



Problem No.5 An air bubble at the bottom of a tank has a diameter of 0.5 mm, calculate:

a) The pressure inside the bubble at the bottom of the tank

b) The diameter of the spherical bubble at the center of the tank (depth 5m)

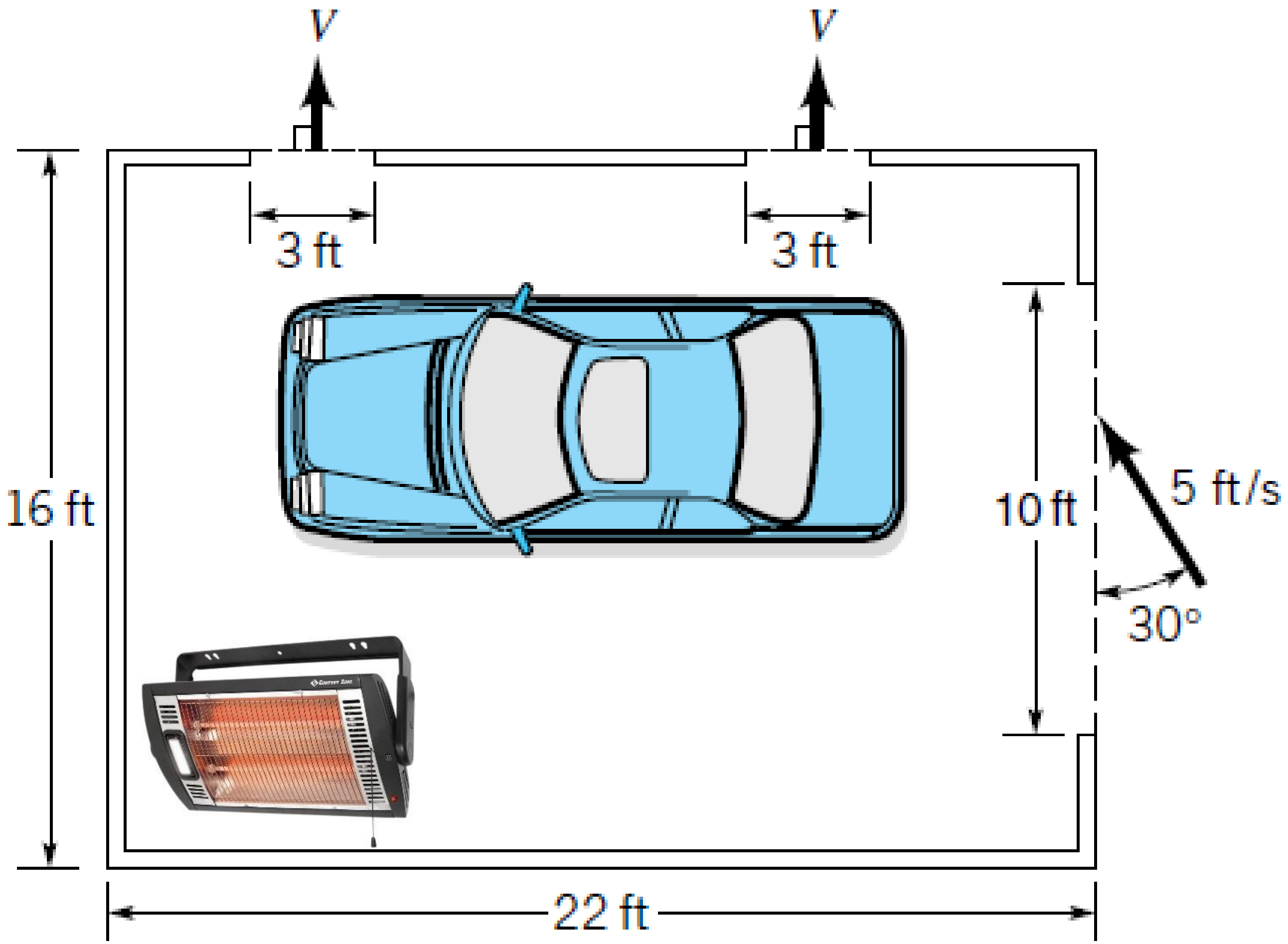
c) The pressure inside the bubble just before reaching the Surface.

For water-air σ=72.8×10^{-3} N/m

Problem No.6 The wind blows through a 7 ft × 10 Ft garage door opening with a speed of 5 ft/s and 32°F, as shown in the figure.

(a) Determine the average speed, *V*, of the air through the two 3 ft × 4 ft opening in the windows, if a heater located inside the garage keeps an average temperature of 70°F. All the walls, ceiling and floor are insulated.

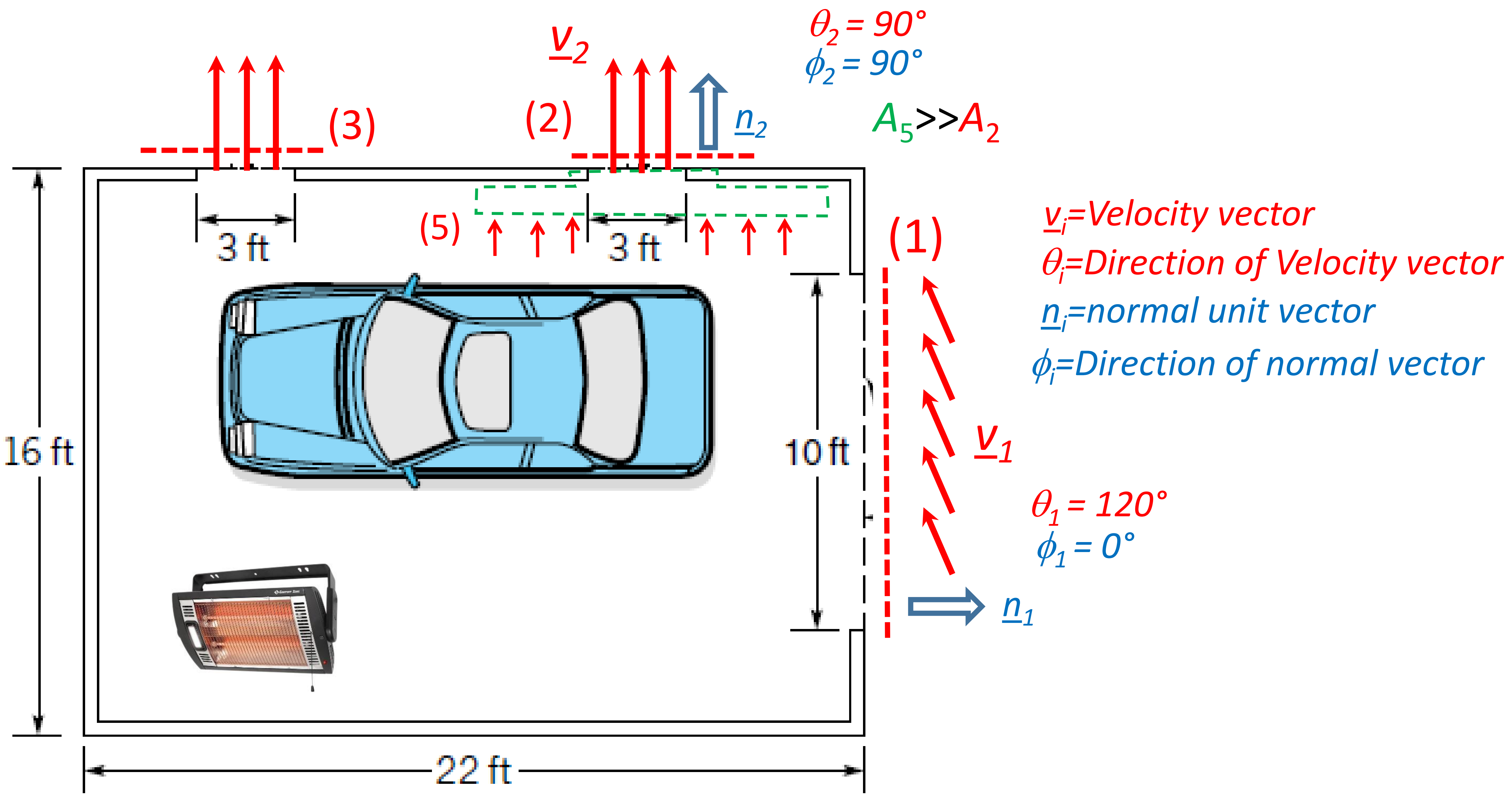
b) Calculate the pressure inside the garage.



Hint: Air flow can be analyzed as incompressible fluid as long as the velocity satisfies *Ma* < 0.3. Mach Number is the ratio of actual velocity respect to the speed of sound in the media, calculated with the equation:

$$Ma = \frac{v}{c} \qquad c \approx \sqrt{\frac{k\,p}{\rho}}$$

k = 1.4 For air, which is the specific heat capacity ratio ($k = C_p/C_v$)



$\theta_2 = 90°$
$\phi_2 = 90°$
$\underline{v}_2$
(3)
(2)
$\underline{n}_2$
$A_5 >> A_2$
3 ft
(5)
3 ft
(1)
$\underline{v}_i$=Velocity vector
θ_i=Direction of Velocity vector
$\underline{n}_i$=normal unit vector
ϕ_i=Direction of normal vector
16 ft
10 ft
$\underline{v}_1$
$\theta_1 = 120°$
$\phi_1 = 0°$
$\underline{n}_1$
22 ft

Problem No.7 Balloons are often filled with helium gas because it weights only about one-seventh of what air weights under identical conditions. The buoyancy force, which can be expressed as $\boldsymbol{F}_b = \rho_{air}\, g\, V_{balloon}$, will push the balloon upward. If the balloon has a diameter of 12 m and carries two people, 70 kg each, determine the acceleration of the balloon when it is first released. Assume the density of air is ρ = 1.16 kg/m^3, and neglect the weight of the ropes and cage.

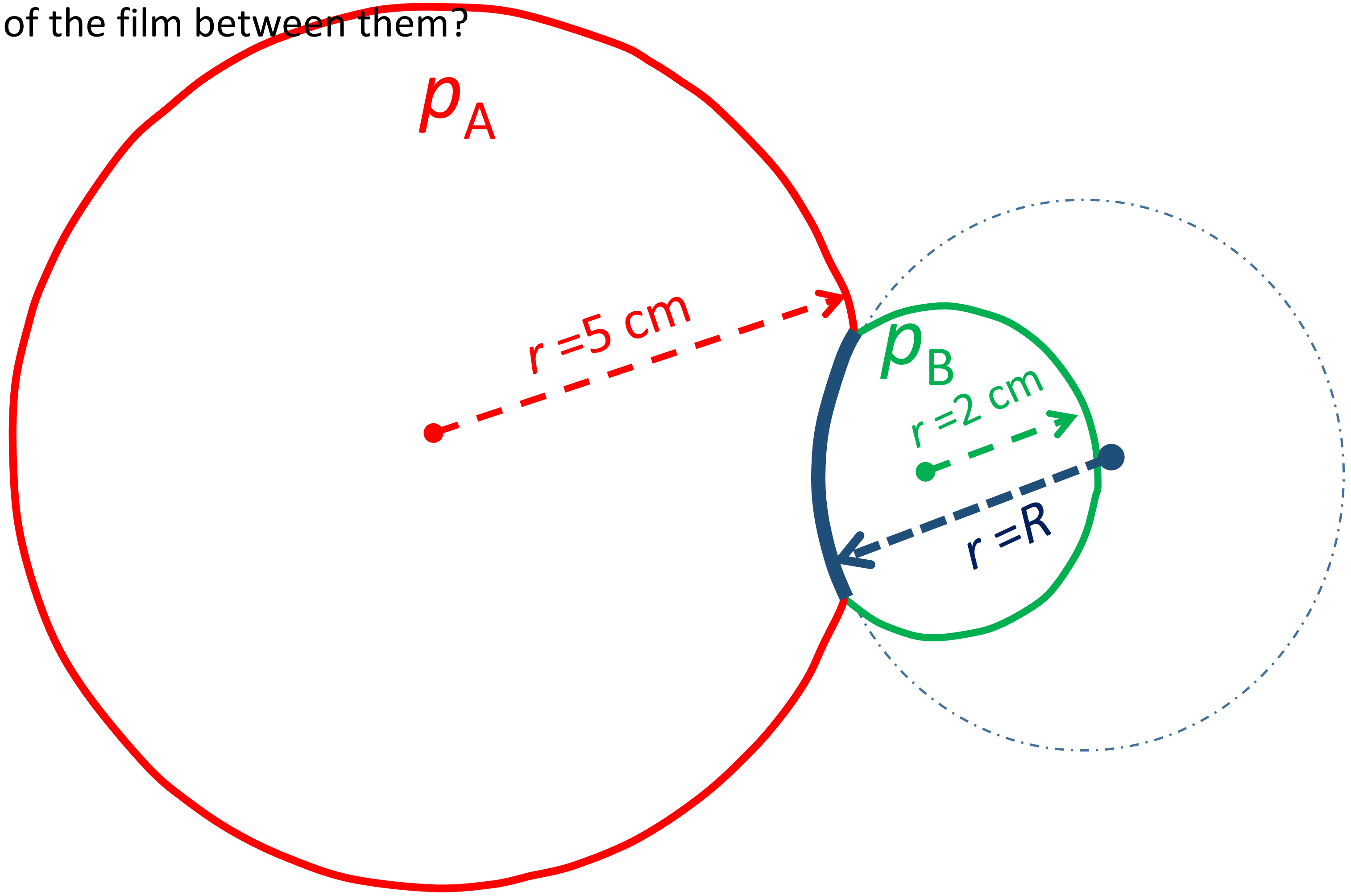
b) Calculate the height if the balloon, if after 1 h the sensors are measuring a pressure of 680 Torr. (the balloon was released at sea level where the pressure is ca 760 Torr)

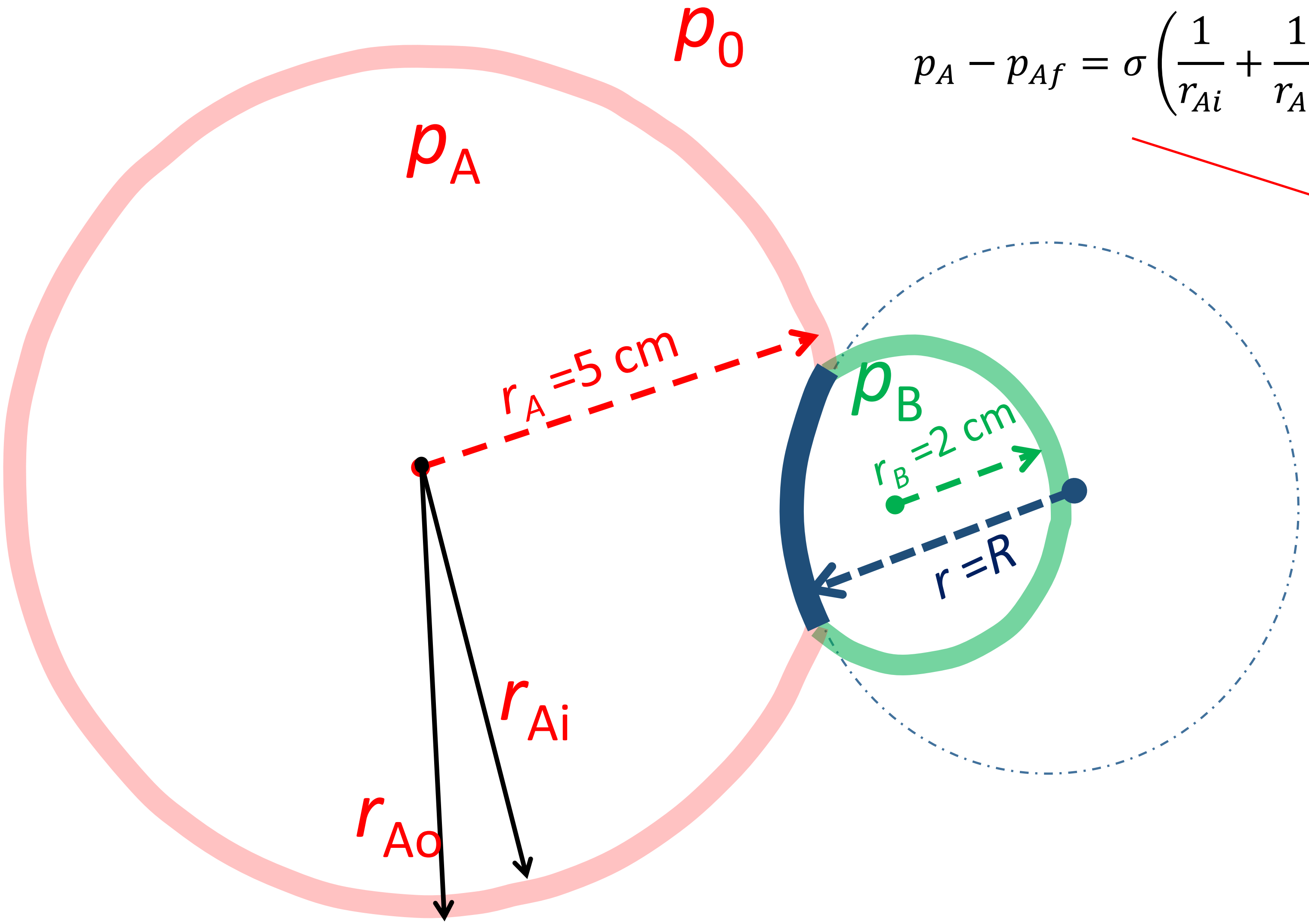
There is a World-Wide Shortage of Helium

Problem No.8 A Scientist is studying the lifetime of soap bubbles, and for this purpose, he needs to calculate the pressure inside them, and the radius of curvature of the film between them when two of them stick together. If the surface tension of an aqueous surfactant solution is 30×10^{-3} N/m and the radii are 5-cm and 2-cm for each bubble.

a) What is pressure inside each bubble?

b) What is radius of curvature of the film between them?





$$p_{Af} - p_0 = \sigma\left(\frac{1}{r_{Ao}} + \frac{1}{r_{Ao}}\right)$$

Pressure difference between film of bubble A and atmosphere

$$p_A - p_{Af} = \sigma\left(\frac{1}{r_{Ai}} + \frac{1}{r_{Ai}}\right)$$

Pressure difference between gas trapped within bubble A and the film of bubble A

Adding both to cancel the pressure within the film

$$p_A - p_0 = \sigma\left(\frac{2}{r_{Ai}} + \frac{2}{r_{Ao}}\right) \sim \frac{4\sigma}{r_A}$$

$$p_B - p_0 = \sigma\left(\frac{2}{r_{Bi}} + \frac{2}{r_{Bo}}\right) \sim \frac{4\sigma}{r_B}$$

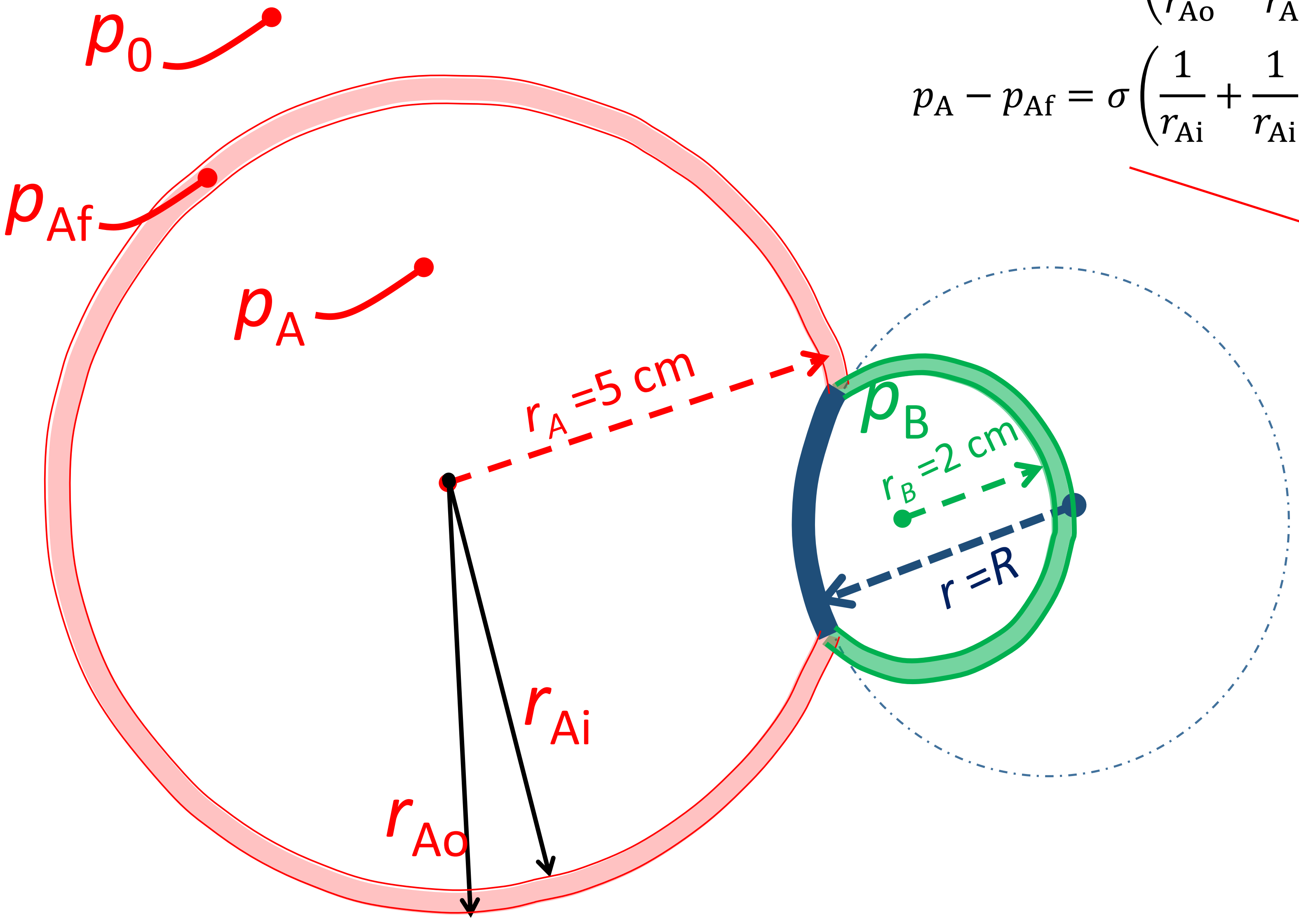
$$p_B - p_A = \sigma\left(\frac{2}{R_i} + \frac{2}{R_o}\right) \sim \frac{4\sigma}{R}$$

$p_A - p_0 = 2.4$ Pa

$p_B - p_0 = 6$ Pa

$p_B - p_A = 3.6$ Pa

R=3.33 cm



$$p_{\mathrm{Af}} - p_0 = \sigma\left(\frac{1}{r_{\mathrm{Ao}}} + \frac{1}{r_{\mathrm{Ao}}}\right)$$

Pressure difference between film of bubble A and atmosphere

$$p_{\mathrm{A}} - p_{\mathrm{Af}} = \sigma\left(\frac{1}{r_{\mathrm{Ai}}} + \frac{1}{r_{\mathrm{Ai}}}\right)$$

Pressure difference between gas trapped within bubble A and the film of bubble A

Adding both to cancel the pressure within the film

$$p_{\mathrm{A}} - p_0 = \sigma\left(\frac{2}{r_{\mathrm{Ai}}} + \frac{2}{r_{\mathrm{Ao}}}\right) \sim \frac{4\sigma}{r_{\mathrm{A}}}$$

$$p_{\mathrm{B}} - p_0 = \sigma\left(\frac{2}{r_{\mathrm{Bi}}} + \frac{2}{r_{\mathrm{Bo}}}\right) \sim \frac{4\sigma}{r_{\mathrm{B}}}$$

$$p_{\mathrm{B}} - p_{\mathrm{A}} = \sigma\left(\frac{2}{R_i} + \frac{2}{R_o}\right) \sim \frac{4\sigma}{R}$$

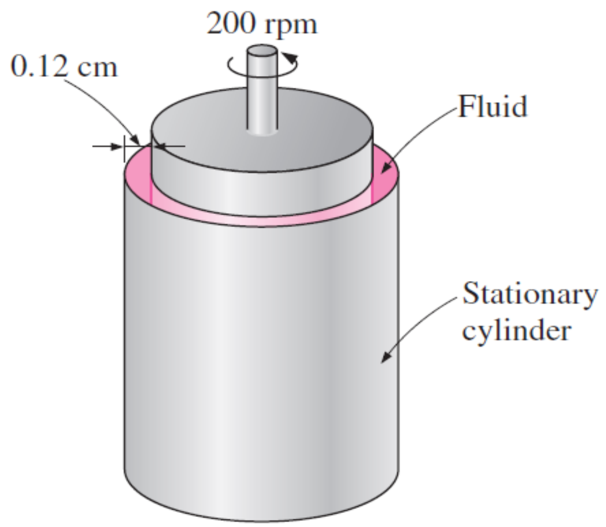
$p_{\mathrm{A}} - p_0 = 2.4$ Pa

$p_{\mathrm{B}} - p_0 = 6$ Pa

$p_{\mathrm{B}} - p_{\mathrm{A}} = 3.6$ Pa

R=3.33 cm

Problem 9. The viscosity of a fluid is to be measured by a viscometer constructed of two 75-cm-long concentric cylinders (***L***=0.75 m). The outer diameter of the inner cylinder is 30 cm (***R***=0.15 m), and the gap between the two cylinders is 1.2 mm. (Δr = 1.2×10^{-3} m) The inner cylinder is rotated at 200 rpm (Ω=20.944 rad/s), and the torque is measured to be 0.8 N-m (***T*** = 0.8 N-m)



a) Prove that the stress can be calculated by the equation:

$$\tau = \frac{T}{2\pi R^2 L}$$

$$\tau = \frac{F}{A} = \frac{F}{2\ \pi\ R\ L} = \frac{F\ R}{2\ \pi\ R\ L\ R} = \frac{T}{2\pi R^2 L}$$

b) Prove that the strain rate can be calculated approximated with the equation (list the assumption needed):

$$\dot{\gamma} = \frac{\Omega\ R}{\Delta r}$$

$$\dot{\gamma} = \frac{dv}{dr} \sim \frac{\Delta v}{\Delta r} = \frac{\Omega\ R}{\Delta r}$$

c) Calculate the viscosity of the fluid in Pa-s

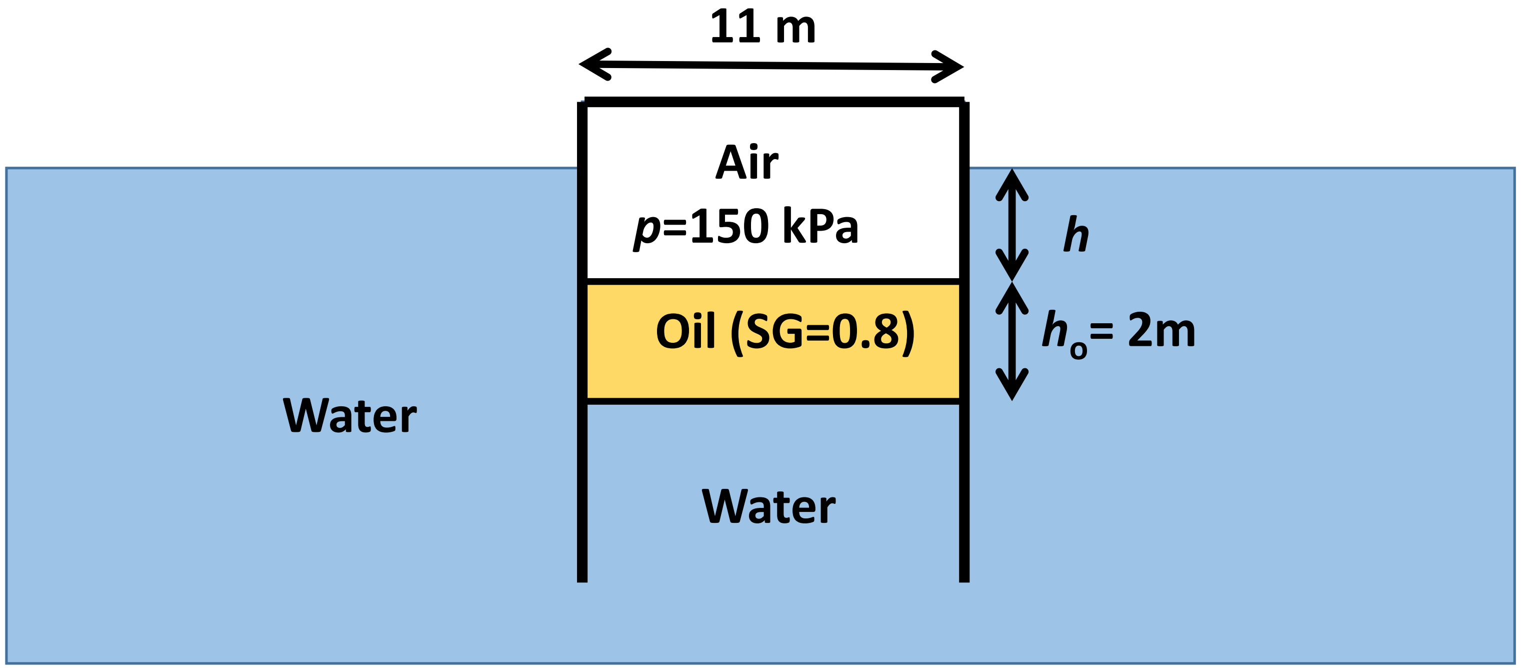
$$\mu = \tau / \dot{\gamma}$$

$$\tau = 7.5451\ \mathrm{Pa} \qquad \dot{\gamma} \sim 2618\ \mathrm{s}^{-1}$$

$$\mu = 0.0029\ \mathrm{Pa-s} \qquad \mu = 2.9\ \mathrm{cP}$$

Problem No.10 A container with a square cross section measuring 11 m by 11m is floating in water as shown in the figure. Assume that tank wall has negligible thickness compared with dimensions shown. Calculate:

a) the distance *h* (m)
b) The weight of the tank (N)



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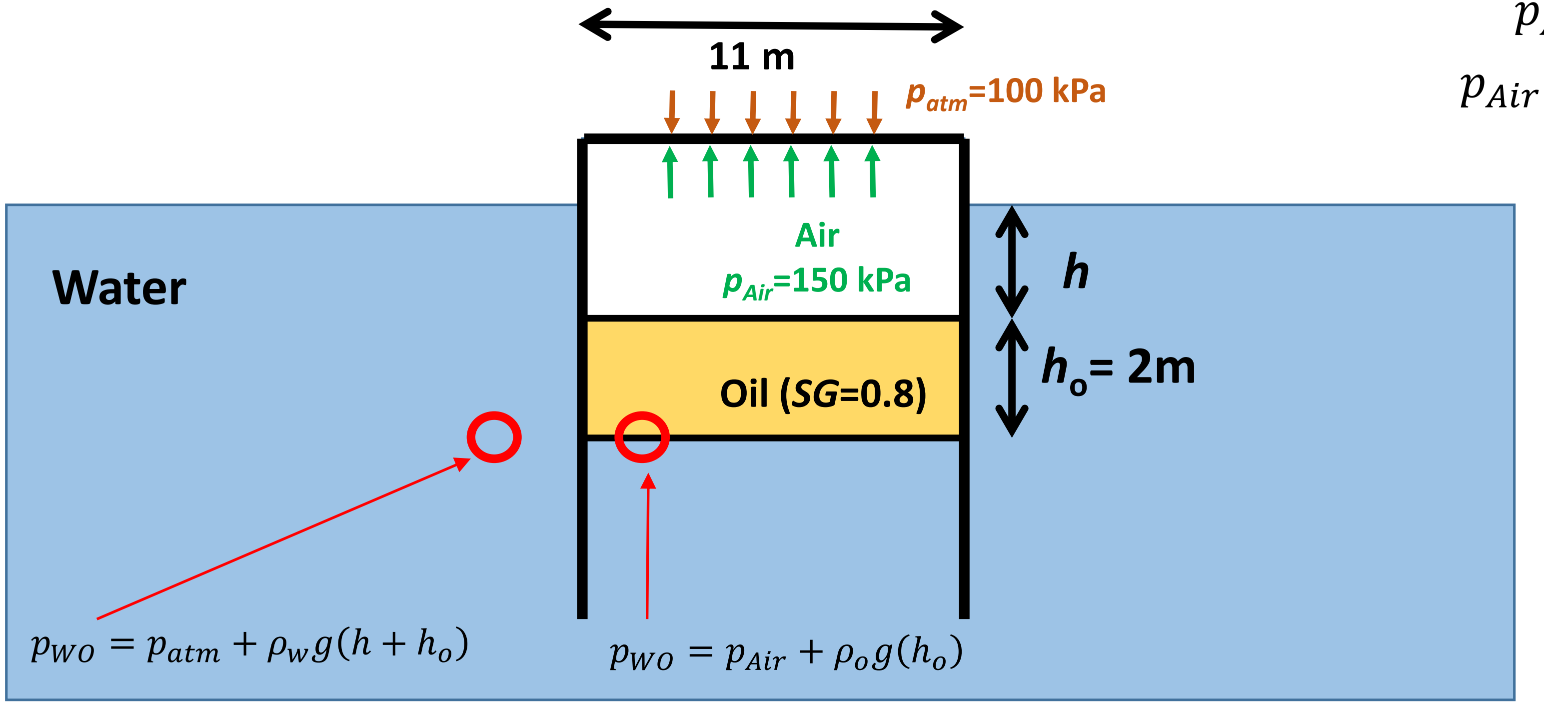


$$p_{Air} + \rho_o g(h_o) = p_{atm} + \rho_W g(h + h_o)$$

$$p_{Air} - p_{atm} = \rho_W g(h + h_o) - \rho_o g(h_o)$$

$$p_{Air} - p_{atm} - (\rho_W - \rho_o)\, g\, h_o = \rho_W\, g\, h$$

$$\frac{p_{Air} - p_{atm} - (\rho_W - \rho_o)\, g\, h_o}{\rho_W\, g} = h$$

$$\boxed{\frac{p_{Air} - p_{atm} - \rho_W (1 - SG)\, g\, h_o}{\rho_W\, g} = h}$$

h=4.697 m

Force balance in y-axis $\quad (p_{Air} - p_{atm})A - W = 0$

$$W = (p_{Air} - p_{atm})A = (150000 - 100000)\text{Pa}\ (121\ \text{m}^2) = 6.05\ \text{MN}$$

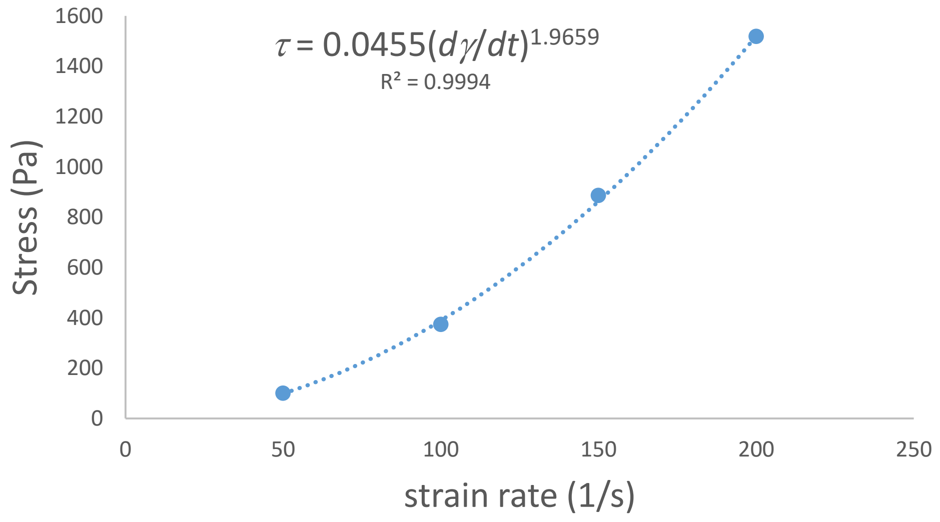
Problem No.11 Fluid for which the shearing stress is not linearly related to the rate of shearing strain are called non-Newtonian fluids. Experimental data for a fluid obtained in the spindle-bob rheometer are given in the following table.

a) Tell if the fluid can be classified as Newtonian

b) Fit the experimental data using at least two models seen in class, and give the parameters.

$\dot{\gamma}(1/s)$	0	50	100	150	200
τ (Pa)	0	101	374	886	1518

Excel approach



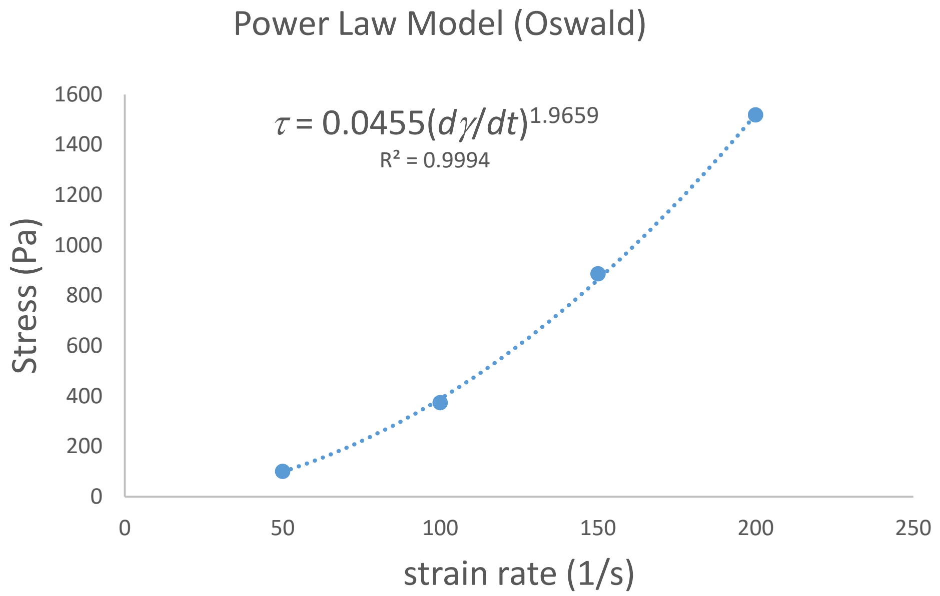
Power law model:

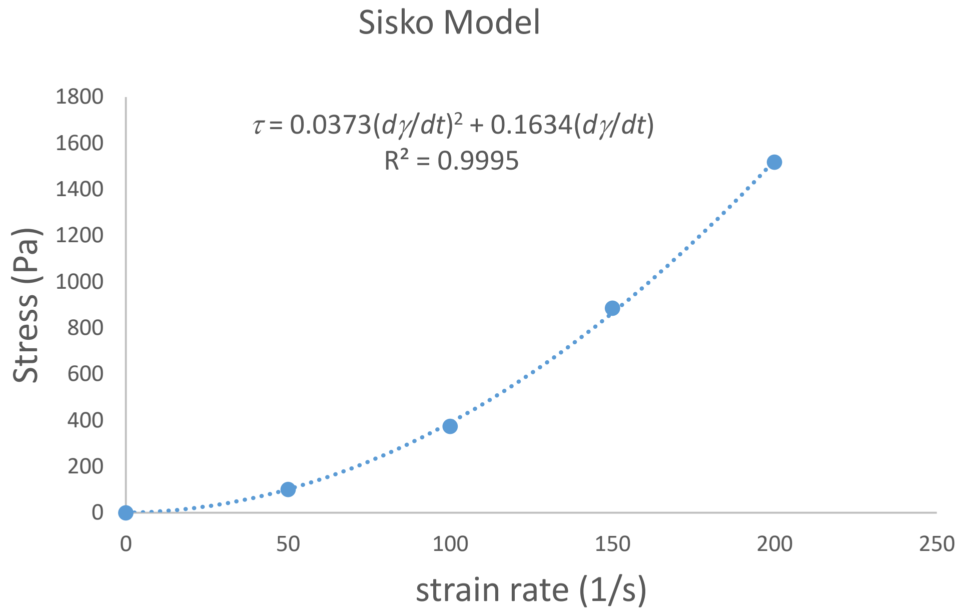
$$\tau = K\,\dot{\gamma}^n$$

$K = 0.045516$ Pa s^n

n=1.9659

It is a non-Newtonian Fluid, fits Power Law and Sisko Models





$$\tau = K\,\dot{\gamma}^{n}$$

K = 0.045516 Pa s^n

n=1.9659

$$\tau = \mu\dot{\gamma} + K\,\dot{\gamma}^{n}$$

K = 0.0373 Pa s^2

n=2

μ = 0.1634 Pa s

Problem:

Review of some basic concepts of physics needed for this class.

(Objective: To be aware of which forces are present when you have fluids, and distinguish forces over systems consisting of just solid objects)

A wood block of 2-m width, 1-m breadth and 0.3-m height is to be used to smooth a 0.5-cm thick layer of fresh cement over a sidewalk at constant velocity of 0.5 m/s in horizontal direction.

a) Calculate the force required for this purpose , if the force is applied with an angle of 30 degrees below the horizontal direction.
b) What is the force needed if the cement is already dry. The friction coefficient between wood and dry cement is 0.68

$$\tau = K\,\dot{\gamma}^{n}$$

K= 2.5 Pa-s$^{3/20}$ n= 3/20

A 50-cm x 40-cm x 30 cm block (density 740 kg/m^3) is moved at constant velocity on an inclined surface, by applying a horizontal force of 160 N. To accomplish this task a 1-mm thick layer of 5 mPa-s wax is used. Determine:

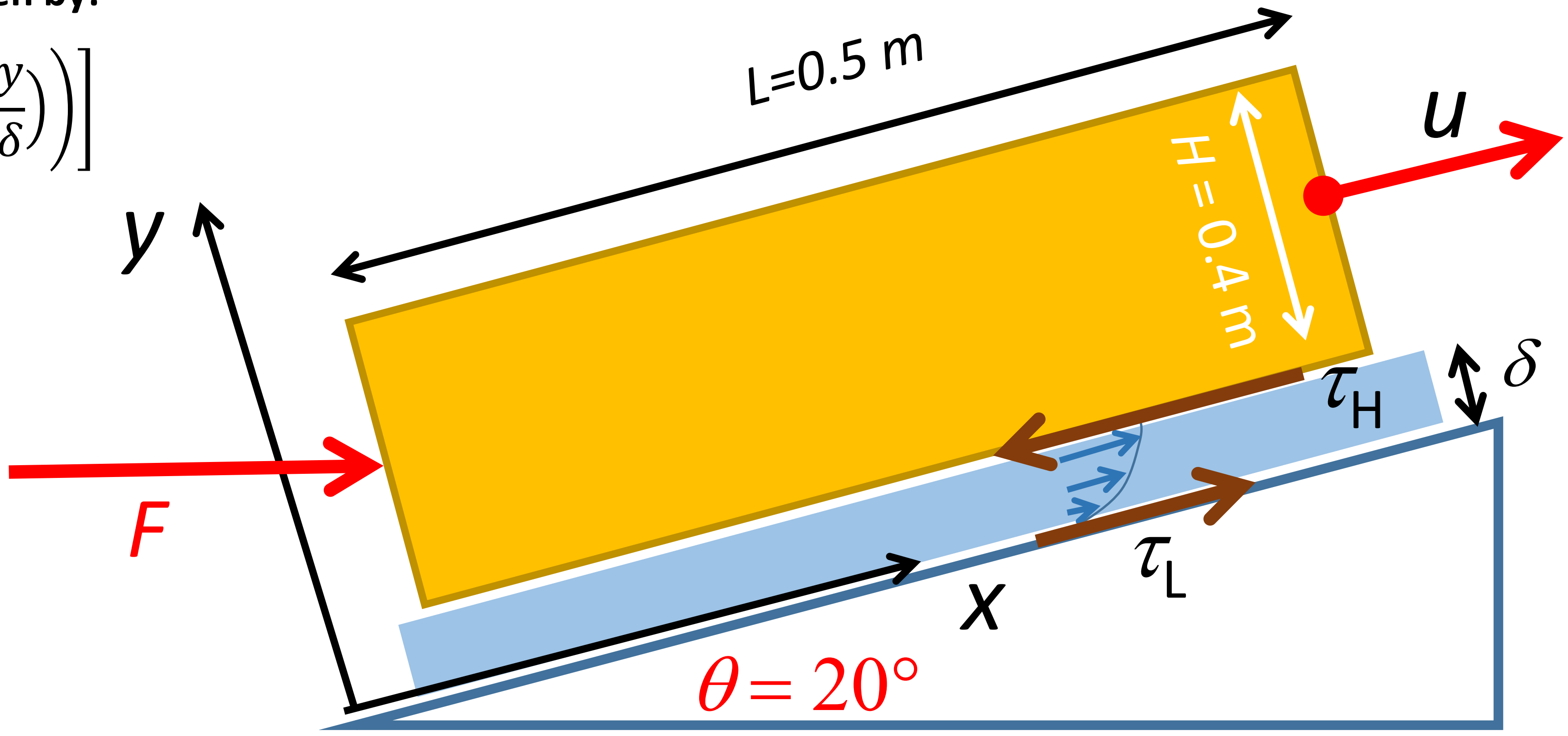
a) The velocity "*u*" of the block moving uphill.
b) The pressure "*p*" at the base of the block.
c) Calculate the heat per unit volume (W/m^3), generated by viscous dissipation assuming that can be calculated by the equation $\mu\,(u/\delta)^2\,(1+\alpha^2\,/\,3)$

Note: μ= Viscosity, δ= wax layer thickness, F = Horizontal force, θ= Inclination angle

The velocity in the wax is given by:

$$v = u\left(\frac{y}{\delta}\right)\left[1 - \propto\left(1 - \left(\frac{y}{\delta}\right)\right)\right]$$

$$\alpha = \frac{\rho\, u\, \delta}{\mu}\left[\frac{\sin\theta}{2}\right]\frac{g\,\delta}{u^2}$$



Problem 3

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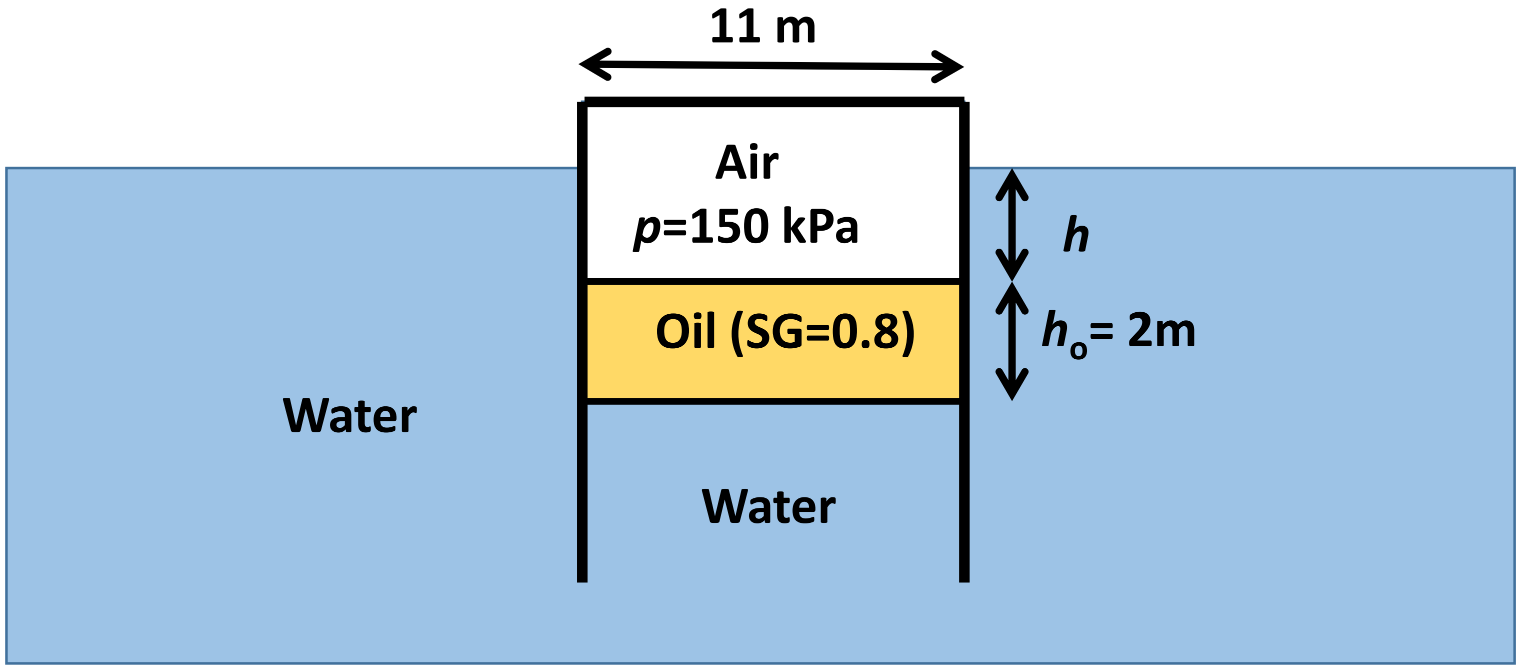

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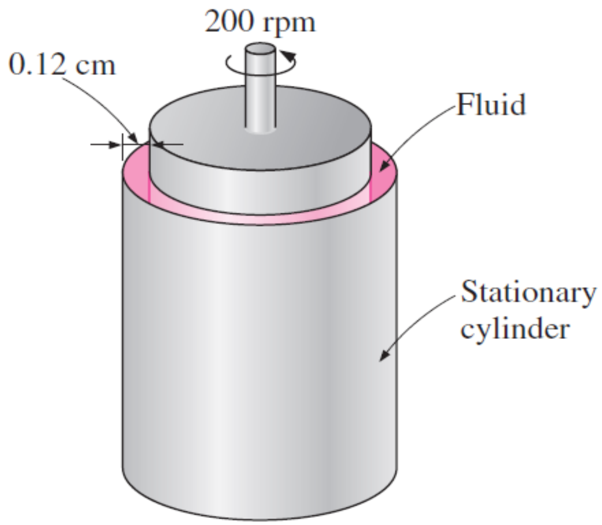

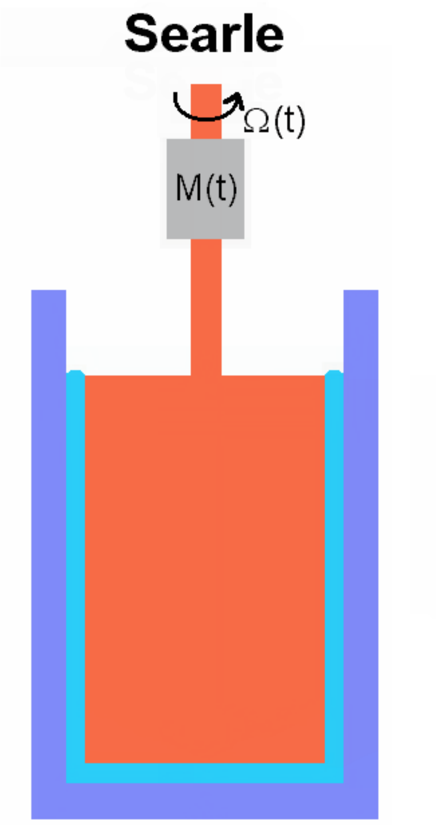

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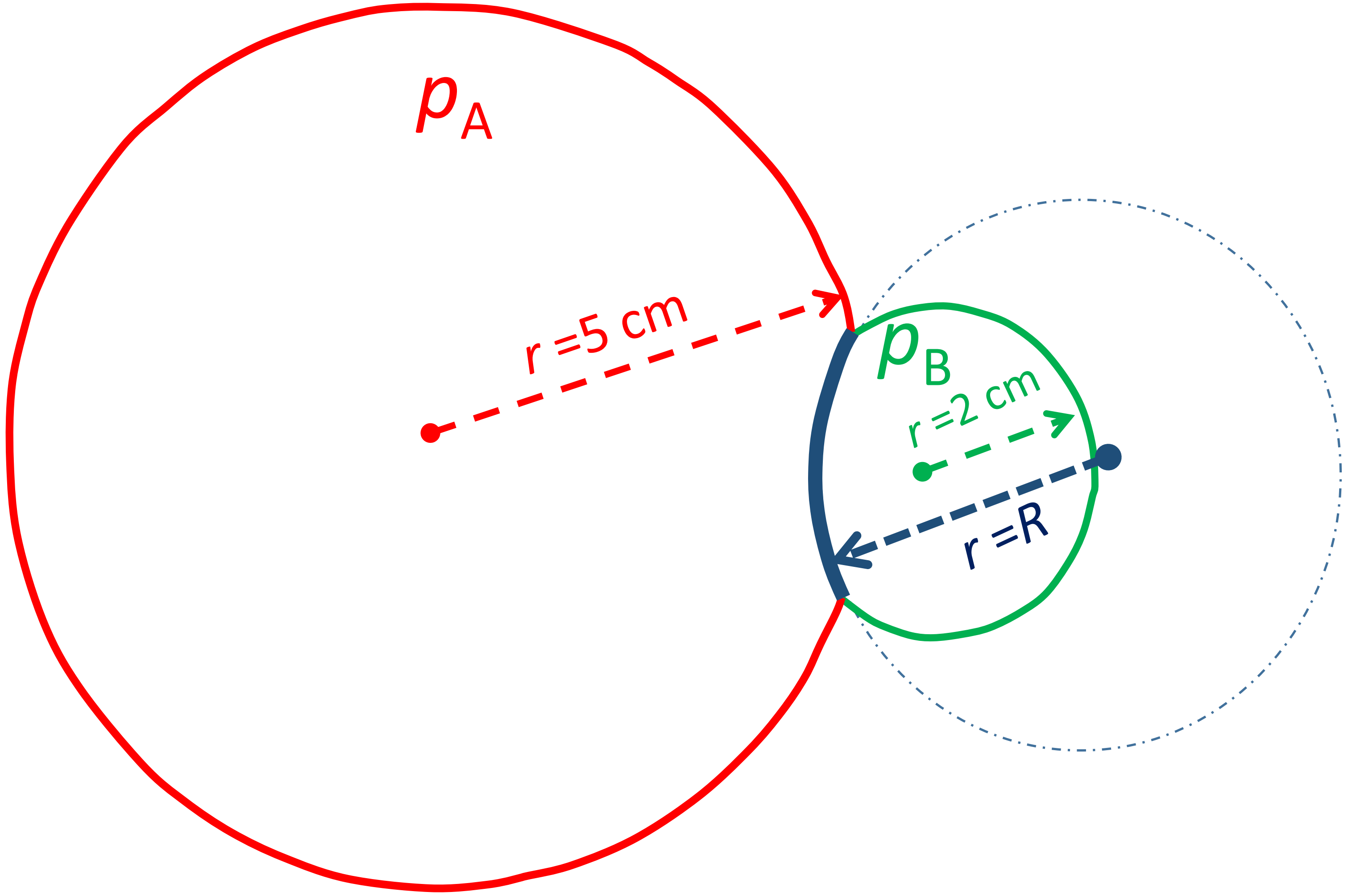

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[There is a World-Wide Shortage of Helium]

By: Dr. José Luis López Salinas
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