Q4001 Thermodynamics of Materials
Homework 3
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In [1]:
```
#*******************************************************************************#
# Futures
%matplotlib inline
# from __future__ import unicode_literals
# from __future__ import print_function

# Generic/Built-in
import datetime
import argparse

# Other Libs
from IPython.display import display, Image
import matplotlib.pyplot as plt
import numpy as np

# Owned
# from nostalgia_util import log_utils
# from nostalgia_util import settings_util
__authors__    = ["Osamu Katagiri - A01212611@itesm.mx"]
__copyright__ = "None"
__credits__    = ["Marcelo Videa - mvidea@itesm.mx"]
__license__    = "None"
__status__     = "Under Work"
#*******************************************************************************#
```

Exercise 1

In [6]:
```
display(Image(filename='./directions/1.jpg'))
```

1. Calculate the work that 1 mol of a perfect gas ($\overline{C}_V = 25\,\mathrm{J/mol\,K}$) produces when it expands from an initial volume of 10 L and 31.5°C to:

   (a) a final volume of 50 L, reversibly and isothermically.
   (b) irreversibly and isothermically against an external pressure of 0.5 atm.
   (c) a final volume of 50 L reversibly and adiabatically.
   (d) irreversibly and adiabatically against an external pressure of 0.5 atm.
   (e) For all cases above, determine the final state after the expansion.

For 1.a & 1.e (Isothermal Reversible Expansion)

$$W_{\text{REV}} = -RnT\ln\left(\frac{V_f}{V_i}\right)$$

where:
$n = 1mol$
$R = 8.314JK^{-1}mol^{-1}$
$T = 31.5^{\circ}C$
$V_i = 10L$
$V_f = 50L$

$$W_{\text{REV}} = -4076.481J$$

$$P_f = P_iV_iV_f^{-1} = 50657.202Pa$$
$$V_f = 50L$$
$$T_f = T_i = 304.65K$$

For 1.b & 1.e (Isothermal Irreversible Expansion)

$$W_{\text{IRREV}} = -P_{\text{ext}}\left(V_f - V_i\right)$$

where:
$P_{\text{ext}} = 0.5atm$
$V_f = 50L$
$V_i = 10L$

$$W_{\text{IRREV}} = -20atmL$$

as: $1atmL = 101.3J$

$$W_{\text{IRREV}} = -2027J$$

$$P_f = P_{ext} = 0.5atm = 50700Pa$$
$$V_f = 50L$$
$$T_f = T_i = 304.65K$$

For 1.c & 1.e (Adiabatic Reversible Expansion)

$$P_i = \frac{nRT_i}{V_i} = \frac{(1mol)(8.314JK^{-1}mol^{-1})(31.5^{\circ}C)}{(10L)}$$
$$P_i = 253286.01Pa$$
$$\gamma = \frac{\overline{C_p}}{\overline{C_v}} = \frac{\overline{C_v} + R}{\overline{C_v}} = \frac{(25JK^{-1}mol^{-1}) + (8.314JK^{-1}mol^{-1})}{25JK^{-1}mol^{-1}}$$
$$\gamma = 1.33256$$
$$P_f = \frac{P_iV_i^{\gamma}}{V_f^{\gamma}} = \frac{(253286Pa)\,(10L)^{1.33256}}{(50L)^{1.33256}}$$
$$P_f = 29661.406Pa$$
$$W_{\text{REV}} = \frac{P_fV_f - P_iV_i}{\gamma - 1} = \frac{(29661.406Pa)(50x10^{-3}m^{-3}) - (253286.01Pa)(10x10^{-3}m^{-3})}{1.33256 - 1}$$

$$W_{\text{REV}} = -3156.693J$$

$$P_f = 29661.406Pa$$

$$V_f = 50L$$
$$T_f = T_i V_i^{\gamma-1} \left(V_f^{\gamma-1}\right)^{-1} = 178.382K$$

For 1.d & 1.e (Adiabatic Irreversible Expansion)

$$W_{\text{IRREV}} = \Delta\text{U} = -PdV = n\overline{C_v}dT$$

where:
$n = 1mol$
$\overline{C_v} = 25JK^{-1}mol^{-1}$
$dT = T_f - T_i$
$T_i = 304.65K$
$P_i = 253286.01Pa$
$P_f = P_ext = 0.5atm = 50700Pa$

$$T_f = \frac{T_i\left(\overline{C_v} + \frac{RP_f}{P_i}\right)}{\overline{C_p}}$$
$$T_f = 243.839K$$
$$W_{\text{IRREV}} = (1mol)(25JK^{-1}mol^{-1})(T_f - T_i)$$

$$W_{\text{IRREV}} = -1520.275J$$

$$P_f = P_{ext} = 0.5atm = 50700Pa$$
$$V_f = \frac{(1mol)(8.314JK^{-1}mol^{-1})(243.839K)}{50700Pa} = 40L$$
$$T_f = 243.839K$$

Exercise 2

In [8]:
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display(Image(filename='./directions/2.jpg'))
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2. A solar collector is used as a heat source for a Carnot engine with a heat sink at 300 K. The efficiency of the solar collector ε is defined as the fraction of the energy reaching the collector that is actually absorbed. It is related to the temperature of the collector as follows:

$$\varepsilon = 0.75 - 1.75\left(\frac{T}{300\,K} - 1\right)$$

Determine the best operating temperature of the collector.

Solar collector ε is given by:

$$\varepsilon = 0.75 - 1.75\left(\frac{T}{300K} - 1\right)$$

The Carnot engine efficiency η is given by:

$$\eta = \frac{T}{300K} - 1$$

The system's efficiency ϵ is given by:

$$\epsilon = \varepsilon\eta$$
$$\epsilon = -\frac{1.75T^2}{(300K)^2} + \frac{4.25T}{300K} - 1.5$$

Let's find $\frac{\partial \epsilon}{\partial T}$

$$\frac{\partial \epsilon}{\partial T} = -\frac{2(1.75)}{300^2}T + \frac{4.25}{300}$$
$$\frac{\partial \epsilon}{\partial T} = \frac{17}{1200} - \frac{7}{180000}T$$

Let's find where the function is the maximum by calculating T for $\frac{\partial \epsilon}{\partial T} = 0$

$$\frac{17}{1200} - \frac{7}{180000}T = 0$$
$$T = 354.286K$$
$$\varepsilon = 0.75 - 1.75\left(\frac{T}{300K} - 1\right)\text{; with } T = 354.286K$$
$$\varepsilon = 0.433$$

The operating temperature shall be $T = 354.286K$ to yeild the maximum power from the Cranot engine, with an efficiency of $\varepsilon = 0.433$

Exercise 3

In [9]:
```
display(Image(filename='./directions/3.jpg'))
```

3. Produce a single graph of P_f/P_i vs. V_f/V_i for the following expansion processes:

   (a) Reversible isothermic process.

   (b) Reversible adiabatic processes for monoatomic, diatomic and polyatomic perfect gases.

   (c) Irreversible adiabatic processes for monoatomic, diatomic and polyatomic perfect gases.

   Compare the plots in the graph and discuss the different behaviors observed. Choose the representation of your axes wisely.

Function to compute the "next" pressure value for REVERSIBLE processes

$$P = \frac{nRT^{\gamma}}{V^{\gamma}}$$

Function to compute the "next" pressure value for IRREVERSIBLE processes

$$\mathrm{dU} = -P\mathrm{dV} = n\overline{C_v}\mathrm{dT}$$

$$n\overline{C_v}\int_{T_i}^{T_f} dT = -P_f\int_{V_i}^{V_f} dV$$

$$n\overline{C_v}(T_f - T_i) = -P_f(V_f - V_i)$$

$$n\overline{C_v}\left(\frac{P_f V_f}{nR} - \frac{P_i V_i}{nR}\right) = -P_f(V_f - V_i)$$

$$\frac{n\overline{C_v}P_f V_f}{nR} - \frac{n\overline{C_v}P_i V_i}{nR} = -P_f(V_f - V_i)$$

$$P_f\left(\frac{n\overline{C_v}V_f}{nR} + V_f - V_i\right) = \frac{n\overline{C_v}P_i V_i}{nR}$$

$$P_f = \frac{\overline{C_v}P_i V_i}{\overline{C_v}V_f + V_f R - V_i R}$$

In [5]:
```
# Function to compute the "next" pressure value for REVERSIBLE processes
def P_rev(n, R, T, V, gamma):
    return (n*R*(T**gamma))/(V**gamma);

# Function to compute the "next" pressure value for IRREVERSIBLE processes
def P_irrev(Cv, Pi, Vi, Vf, R):
    return (Cv*Pi*Vi)/(Cv*Vf+Vf*R-Vi*R);

# Draw the plot's workspace
n = 6;
plt.subplots(figsize=(3*n, 2*n))

# Initial State, Constants & Final Volume
Ti = 304.65; #K
Vi = 0.01; #10 L
Vf = 0.05; #50 L
V = np.linspace(Vi,Vf,1000);
n = 1; #mol
R = 8.314; #J/K mol)

# Reversible Isothemic Process
gamma = 1;
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_rev(n, R, Ti, V, gamma);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Reversible Isothemic Process');

# Reversible Adiabatic Process (monoatomic)
Cv = (3/2)*R;
Cp = Cv + R;
gamma = Cp/Cv;
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_rev(n, R, Ti, V, gamma);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Reversible Adiabatic Process (monoatomic)');

# Reversible Adiabatic Process (diatomic)
Cv = (5/2)*R;
Cp = Cv + R;
gamma = Cp/Cv;
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_rev(n, R, Ti, V, gamma);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Reversible Adiabatic Process (diatomic)');

# Reversible Adiabatic Process (polyatomic)
Cv = 3*R;
Cp = Cv + R;
gamma = Cp/Cv;
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_rev(n, R, Ti, V, gamma);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Reversible Adiabatic Process (polyatomic)');

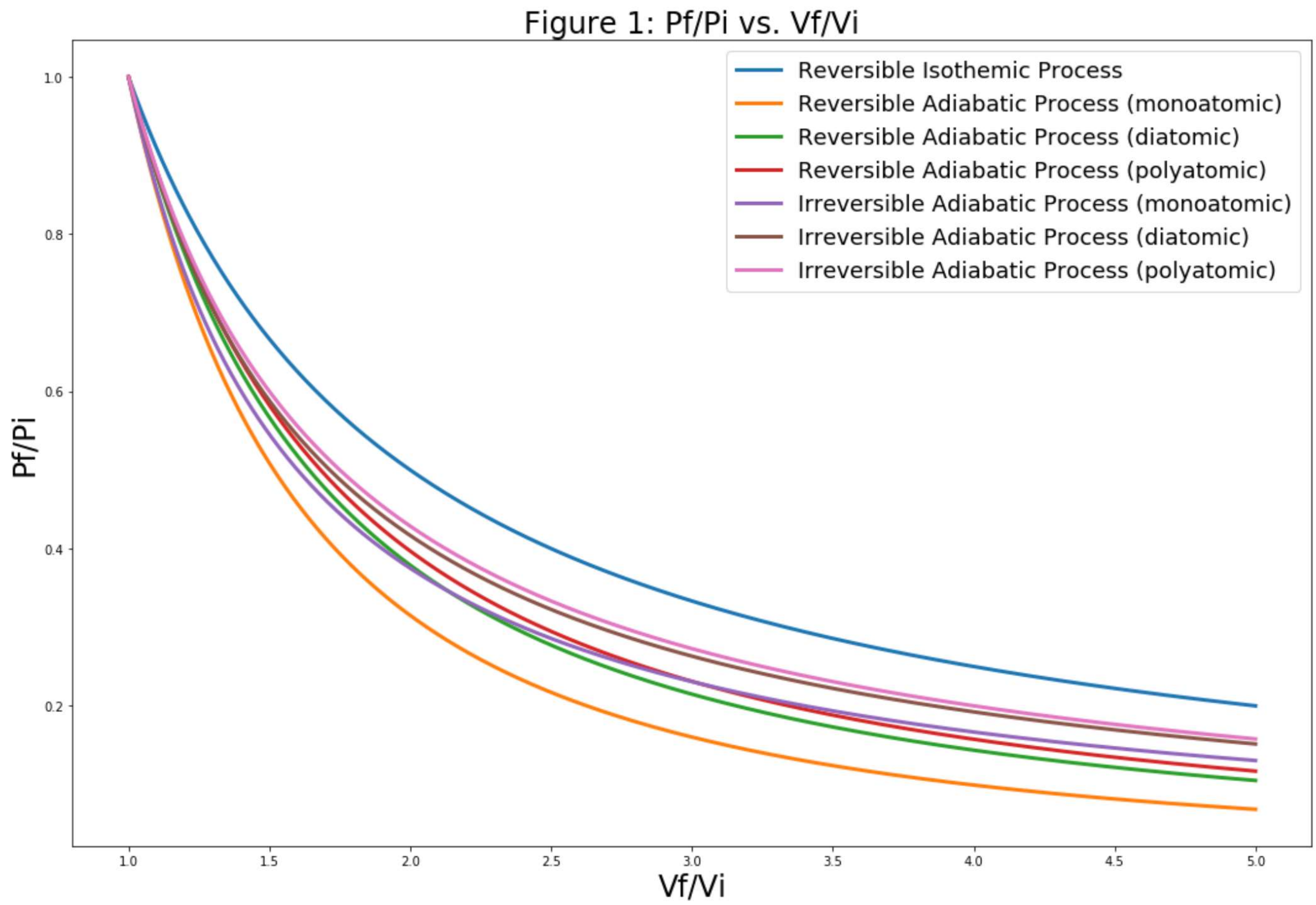
# Irreversible Adiabatic Process (monoatomic)
Cv = (3/2)*R;
Cp = Cv + R;
gamma = Cp/Cv;
#Tf = 243.839; #K (from exercice 1.d)
#Vf = (n*R*Tf)/(50700); #L
#V = np.linspace(Vi,Vf,1000);
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_irrev(Cv, Pi, Vi, Vf, R);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Irreversible Adiabatic Process (monoatomic)');

# Irreversible Adiabatic Process (diatomic)
Cv = (5/2)*R;
Cp = Cv + R;
gamma = Cp/Cv;
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_irrev(Cv, Pi, Vi, Vf, R);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Irreversible Adiabatic Process (diatomic)');
```

```
# Irreversible Adiabatic Process (polyatomic)
Cv = 3*R;
Cp = Cv + R;
gamma = Cp/Cv;
Vf = V;
Pi = P_rev(n, R, Ti, Vi, gamma);
Pf = P_irrev(Cv, Pi, Vi, Vf, R);
plt.plot(Vf/Vi, Pf/Pi, '-', linewidth=3, label='Irreversible Adiabatic Process (polyatomic)');

plt.xlabel('Vf/Vi', fontsize=24);
plt.ylabel('Pf/Pi', fontsize=24);
plt.title("Figure 1: Pf/Pi vs. Vf/Vi", size=24)
plt.legend(prop={'size': 18})
display(plt);
```

```
<module 'matplotlib.pyplot' from 'C:\\Users\\oskat\\Anaconda3\\lib\\site-packages\\matplotlib\\pyplo
t.py'>
```



Exercise 4

In [10]:
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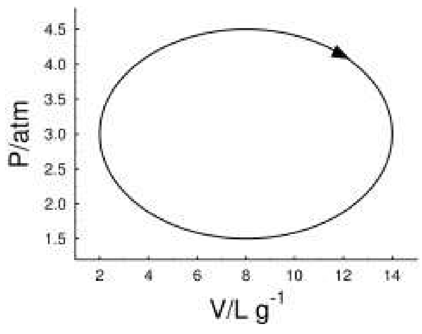
display(Image(filename='./directions/4.jpg'))
```

4. A PV diagram of an imaginary heat engine is represented in the following figure.



(a) Find the work performed per cycle by 1 g of working fluid.

(b) Find the engine efficiency if it rejects 5.7 kJ/g during each cycle

The work performed per cycle is given by the area within the PV curve.

$$W_{performed} = \pi(14-8)(4.5-3.0)atmLg^{-1}$$
$$W_{performed} = 9\pi atmLg^{-1}$$

as: $1atmL = 101.3J$

$$W_{performed} = \frac{9117}{10}\pi Jg^{-1} = 2864.190Jg^{-1}$$

$$\eta_{engine} = \frac{W_{performed}}{W_{performed} + W_{rejected}}$$

$$\eta_{engine} = \frac{2864.190Jg^{-1}}{2864.190Jg^{-1} + 5700Jg^{-1}} = 0.334$$

Exercise 5

In [3]:
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display(Image(filename='./directions/5.jpg'))
```

5. A heat engine, of which all steps in a cycle are reversible, absorbs thermal energy from a high-temperature reservoir, performs an amount of net work w_{net} , and rejects thermal energy into a lowtemperature reservoir. Initially, the reservoirs are at temperatures T_1 and T_2. Their heat capacities are constant with values C_1 and C_2, respectively. Calculate what will be the final temperatures for the heat reservoirs and the maximum amount of work produced by the engine.

The following is extracted from: Mungan, C. E. (2009). Heat engine with finite thermal reservoirs and nonideal efficiency. Latin-American Journal of Physics Education, 3(2), 239–242.

Let's denote some variables:
dQ_H = heat transfer out of the hot reservoir
dQ_C = heat transfer into the cold reservoir
dW = work output by the engine
dT_H = change in temperature of the hot reservoir ($dT_H < 0$ as it's cooling down)
T_{Ho} = T_1 = initial temperature of the hot reservoir
dT_C = change in temperature of the cold reservoir ($dT_C > 0$ as it warms up)
T_{Co} = T_2 = initial temperature of the cold reservoir
T = final temperature of both reservoirs
η = 2nd law efficiency given

As both reservoirs have temperature-independent heat capacities C_1 and C_2, then it follows that:

$$dQ_H = -C_1 T_H$$
$$dQ_C = C_2 T_C$$

The engine efficiency can be defined as:

$$\varepsilon = \frac{dW}{dQ_H} = 1 - \frac{dQ_C}{dQ_H} = 1 + \frac{dT_C}{dT_H}$$

With the following Carnot limiting efficiency:

$$\varepsilon_c = 1 - \frac{T_C}{T_H}$$

The 2nd law efficiency η is assumed to be constant.

$$\eta = \frac{W_{net,out}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

Rearranging terms, T_C can be defined as a function of T_H:

$$T_H \frac{dT_C}{dT_H} + \eta T_C = (\eta - 1) T_H$$

The complementary solution of the previous is $AT_H^{-\eta}$ where A is a constant to taylor the initial conditions. On the other hand, by trying a particular solution of the equation that is proportional to T_H, the following is obtained $(\eta - 1)T_H/(\eta + 1)$. Adding together these complementary and particular solutions and fitting to the initial temperatures of the two reservoirs gives:

$$\frac{T_C}{T_{\mathrm{Ho}}} = \left(\frac{T_{\mathrm{Ho}}}{T_H}\right)^{\eta} \left\{ \frac{T_{\mathrm{Co}}}{T_{\mathrm{Ho}}} + \frac{1-\eta}{1+\eta}\left(1 - \left(\frac{T_H}{T_{\mathrm{Ho}}}\right)^{\eta+1}\right) \right\}$$

Let's set $T_C = T_H = T$ to find the common final temperature of the two reservoirs:

$$\frac{T}{T_{Ho}} = \left(\frac{(T_{Co} + T_{Ho}) + \eta(T_{Co} - T_{Ho})}{2T_{Ho}}\right)^{\frac{1}{1+\eta}}$$

$$T = T_{Ho}\left(\frac{(T_{Co} + T_{Ho}) + \eta(T_{Co} - T_{Ho})}{2T_{Ho}}\right)^{\frac{1}{1+\eta}}$$

where:
$T_{\min} = \sqrt{T_{\mathrm{Co}} T_{\mathrm{Ho}}}$ is the geometric average of the initial temperatures
$T_{\max} = \frac{T_{Co} + T_{Ho}}{2}$ is their aithmetic average

The total work output by the engine is the difference in heat transfers for the two reservoirs, as follows:

$$W = Q_H - Q_C = C_1(T_{Ho} - T) - C_2(T - T_{Co})$$

And the maximum amount of work is given by:

$$W_{max} = C_1 C_2 (T_{max} - T_{min})$$

The maximum work is achieved for reversible operation of the engine when $\eta = 1$. On the other hand, zero work is output for maximally irreversible operation of the engine when $\eta = 0$.

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In [ ]:
```