TEAM 5

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FINAL - Plastics and Composites Engineering

Due any time before 22:00 hrs on August 1st, 2020

# Instructions

Solve the problems following the next steps:

A. Rephrase the problem indicating very clearly what you have been asked to do.
B. List all the data provided.
C. Make a list of the assumptions, justifying each of them.
D. Write down an algorithm for the solution you are proposing (no calculations are needed at this stage)
E. Solve the problem.
F. Ask yourself if the result is reasonable and, if needed check in the web for technical papers to support your answer.
G. List the references used in the solution of the problem.

    This procedure is a must for this part of the exam to be considered for grading.

A. Work by yourself from A to D, afterwards you will work E to G with your team.
B. Upload your individual proposal in the Google Drive in the folder 10 Final Assessment in your personal folder, as: MY NAME. FINAL EXAMINATION PART ABCD.
C. Upload your team´s solution with the Subject: TEAM NUMBER FINAL EXAMINATION PART EFG

Problems (60 points)

Q1: The manufacturing plant where you work, The Mexico Supply Chain, has been requested, by the China Car Company, to inject a flat piece with the following dimensions (W=10 cm, L=20 cm y H= 0.5 cm) {20 pts}
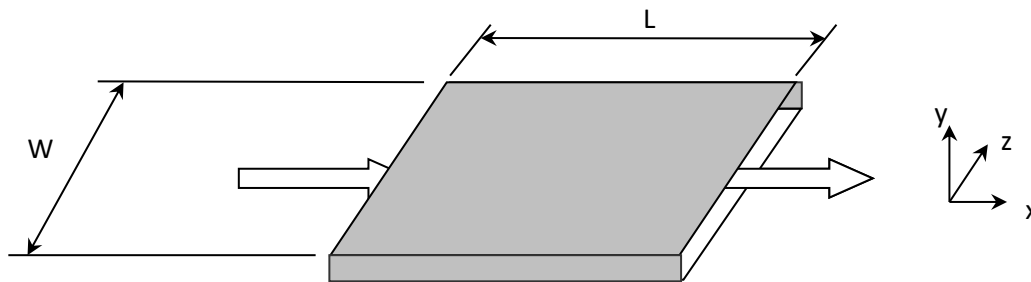
The injection can be done in a multicavity die and the plant manager wants to produce 10 specimens per minute, considering two different type of resins, but before he accepts the request of the China Car Company, he asks you:
a) To determine the pressure required to produce those pieces using the resin that requires the lowest pressure drop.
b) To propose he best multicavity configuration (justify your answer)

Consider that the injection happens at 200°C and that the density of the material at that temperature is 0.75; the viscosity can be represented by a power law and their respective parameters are given below.

Resin	MFI	η_0 (poises)	Critical shear rate (s^{-1})	Power law index
1	1.5	54000	1.5	0.48
2	2.5	29000	2.5	0.48

A. REPHRASE

We have been requested to produce 10 flat pieces per minute by injection molding. It can be done in multicavity die and we have 2 different resins. We need to a) determine the pressure required to produce ten flat pieces using the resin that requires the lowest pressure drop; and b) propose the best multicavity configuration.

B. AVAILABLE DATA

- Process: injection moulding.
- Mould: Multicavity.
- Dimensions of the flat piece: W=10 cm, L=20 cm and H= 0.5 cm.
- Pieces per minute: 10.
- Polymer: 2 resins.
- Injection temperature: 200 C.
- Densities, MFI, Viscosities, critical shear rates, power law indexes.
- Number of cavities: to determine.
- Arrangement of cavities: to determine.
- Pressure required: to be determined from resin with less pressure drop.

C. ASSUMPTIONS

- The Power Law model fits well the viscosity modulus of the resins.
- The plant manager is interested in spending the least amount of energy in the production of the pieces.
- The process is carried out at a constant pressure, constant temperature.
- The material density remains constant through the process.
- Isothermal, laminar and steady state flow.
- The filling time.
- The surface of the mold cavity reproduces its condition on a molded part. A significant advantage of the molding processes is the fact that different surface polishes and textures can be moulded into the part, and we will assume that no secondary surface-finishing operations are required.
- The surface finish of the runner system should be as good as that in the cavity. A good surface finish not only keeps the pressure drop low, but also prevents the tendency of the runner to stick to either half of the mold. Such sticking would aggravate the high stress in the area of the gate [2].

D. ALGORITHM

1. Obtain viscosity of each resin using the Power Law equation for different shear rates
2. Graph the viscosity curves of each resin
3. Select the resin with the lowest zero-shear viscosity, it will require the least pressure drop
4. Design the runner diameter to match the part and provide a proper filling
5. Scale the mould into a multicavity configuration and select the best runner lengths
6. Calculate the pressure needed to produce the pieces.

7. Plot the different pressures obtained at different filling times.

E. SOLUTION

Using the Power Law formula, we calculated the viscosity curves for each resin.

$$\eta = \eta_0 \left(\frac{\dot{\gamma}}{\dot{\gamma}_0} \right)^{n-1}$$

Where η_0 zero shear viscosity, $\dot{\gamma}_0$ critical shear rate, n power law index. We converted the poises to Pa s.

Power Law
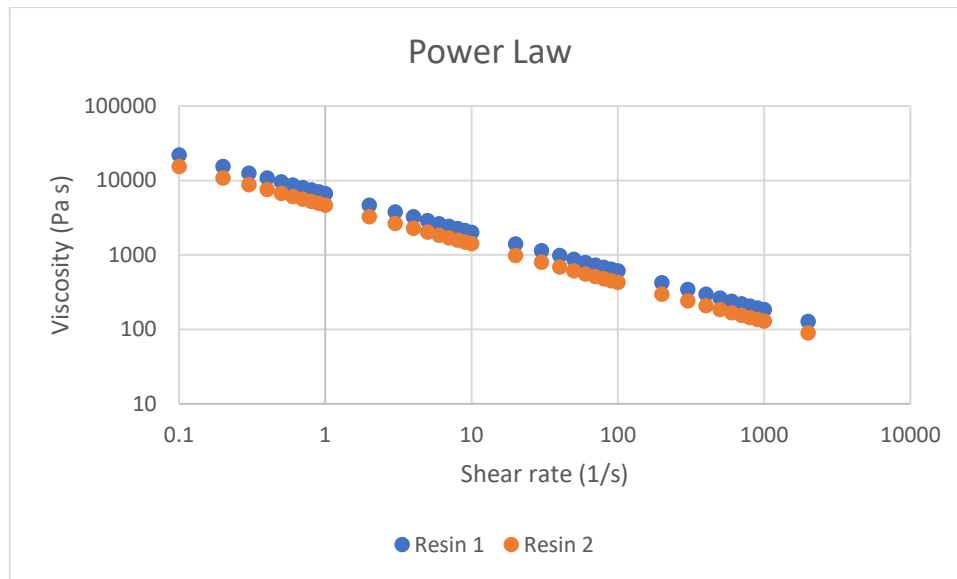
From this plot we can observe that the Resin 2 has the lowest viscosity, which means it will yield the least pressure drop. So we select it for our process.

Considering the typical injection cycle is from 20-30 seconds we propose a mold of 5 pieces so we can run 2 cycles and obtain the 10 pieces per minute [3].

Multi-cavity configuration based on the Sprue-Runner-Gate System

The sprue is the channel, cut in the stationary platen that transports the melt from the plasticizing unit through to the runner system or direct to the cavity. The runner system basically transports the melt from the sprue to the gate and the gate transports the melt into the cavity.

Cavity Evaluation

One of the most important aspects of multiple-cavity mold design is the layout of the feed to the cavities. The cavities should be arranged around the primary sprue so that each receives its full and equal share of the total pressure available, through its own runner system (so-called balanced runner system). This requires the shortest possible distance between cavities and primary sprue, and equal runner dimensions. When practical, a correct arrangement of cavities will avoid differences in product dimensions, mold release problems, defects, etc. Also, multicavity molds should contain cavities of identical shape [2].
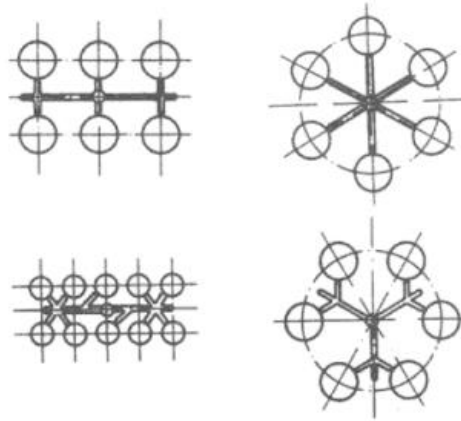
In the above image we can observe different cavity arrangements, the down-right design is not considered balanced. According to many sources, symmetric multi-cavity molds provide the best arrangements.

Runner

Systems Cavities should be placed so that:

1. The runner is short and, if possible, free of bends.
2. The supply of material to each cavity is balanced.

This means that the runners must be identical in both shape and. A balanced supply ensures that any change made in any one of the molding parameters will affect all cavities the same way.

The runners in multicavity molds must be large enough to convey the plastic melt rapidly to the gates without excessive chilling by the relatively cool mold for thermoplastics. Runner cross sections that are too small require higher injection pressure and more time to fill the cavities. Large runners produce a better finish on the molded parts and minimize weld lines, flow lines, sink marks, and internal stresses. However, excessively large runners should be avoided [2].

Various shapes of runners are used and, as shown in the above figure, some are better than others. A full round (circular cross section) runner is preferred over any of the other cross-sectional shapes, as it provides the minimum contact surface of the hot plastic with the cool mold. So, in our design we will consider cylindrical runners.

Injection Times

Since the molding process cycles typically are carried out in between 15-30 seconds, we can consider 2 cycles in order to complete the 10 pieces per minute, giving a possible mold configuration design of 5 cavities [3].

Clamping Forces

The clamping force required to keep the mold closed during injection must exceed the force given by the product of the live cavity pressure and the total projected area of all impressions and runners. Consider including a safety factor of about 10 to 20% to ensure sufficient clamping pressure, particularly when one is not familiar with the operation.

Plasticizing Capacity

Another consideration is the maximum amount of plastic required to fill the mold, including any runner system with a sprue that solidifies. The amount is usually 50% of machine shot capacity, or at most 60 to 70%, to ensure proper plasticizing action.

Pressure Capacity

As stated by Farotti et al. [1] the typical packing pressure is about 32 MPa per cavity. The proper injection pressure depends largely on part size and configuration. Pressures usually range from 1,000 to 1,500 psi. The best results are obtained at higher pressures, up to about 75% of the press capacity. Pressures should be high enough to fill the part and to avoid problems with shrinkage, voids, sinks, and pigment dispersion. Too much pressure can cause parts to flash, burn, and stick in the mold.

Final multicavity mould design:
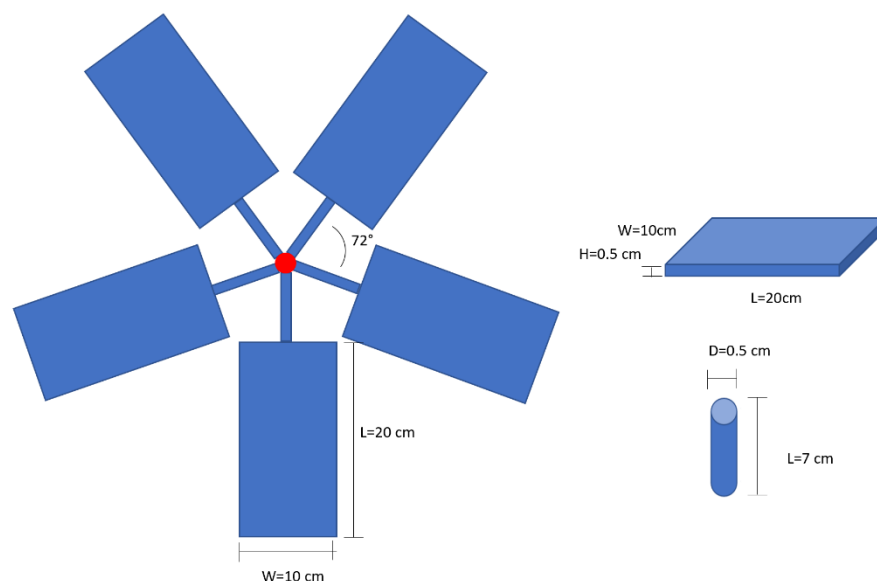
We calculated the volume of the rectangular piece

$$V = WLH$$

Where W=0.1 m, L=0.2 m, and H=0.005 m

$$V = 0.0001 \ [m^3]$$

And assuming the time to fill the part is t=1 s, we obtained the flow rate

$$Q = \frac{V}{t}$$

$$Q = 0.0001 \ \left[\frac{m^3}{s}\right]$$

Then obtain the shear rate with the following equation

$$\dot{\gamma} = \frac{6Q}{WH^2}$$

$$\dot{\gamma} = 240 \ \left[\frac{1}{s}\right]$$

After obtaining the shear rate, we can calculate the viscosity for the Resin 2 with the power law model

$$\eta = \ 270.158 \ Pa \cdot s$$

We then calculated the pressure drop ΔP in the one cavity for Resin 2 with

$$\Delta P = \frac{12QL\eta}{WH^3}$$

$$\Delta P_{cavity} = 5.18 [MPa]$$

For the runner calculations we used

$$V = \pi R^2 L$$

Where R=0.005 m and L=0.07 m

$$V = 5.497 \times 10^{-6} \ [m^3]$$

Assuming the filling time is also t=1 s, we then calculate the flow rate

$$Q = \frac{V}{t}$$

$$Q = 5.497 \times 10^{-6} \ \left[\frac{m^3}{s}\right]$$

For shear rate in the cylindrical runners

$$\dot{\gamma} = \frac{4Q}{\pi R^3}$$

$$\dot{\gamma} = 56 \left[\frac{1}{s}\right]$$

For the pressure drop for resin 2 is given by:

$$\eta = 575.79 \ Pa \cdot s$$

$$\Delta P_{runner} = \frac{8Q\eta}{\pi R^4}$$

$$\Delta P_{runner} = 12.898 \ [MPa]$$

So, the total pressure for the whole mould is:

$$\Delta P_{Tot} = \Delta P_{runner} + \Delta P_{cavity} = 18.084 \ MPa$$

$$F_{tot} = 5\left(P_{runner}A_{runner} + P_{cavity}A_{cavity}\right)$$
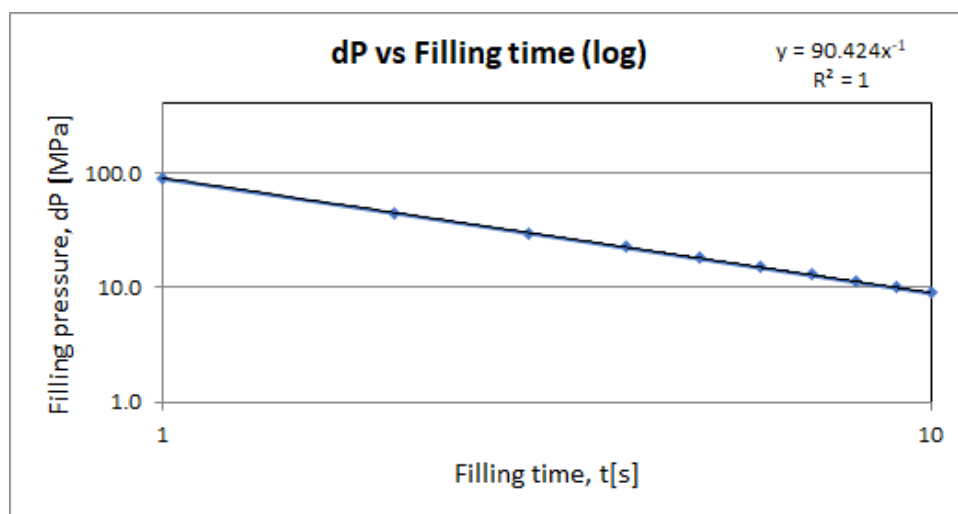
$$F_{tot} = 18032.512 \ N$$

T(s)	Q-CP(m3/s)	Q-RP(m3/s)	Y-CP(1/s)	Y-RP(1/s)	η-CP(pa/m)	η-RP(pa/m)	dP-CP[MPa]	dP-RP[MPa]	Total dP [mPa]
0.01	0.00055	0.01	5600.0	24000.0	52.5	24.6	1289.8	518.7	9042.4
0.02	0.000275	0.005	2800.0	12000.0	75.3	35.3	644.9	259.4	4521.2
0.03	0.000183	0.003333	1866.7	8000.0	93.0	43.6	429.9	172.9	3014.1
0.04	0.000137	0.0025	1400.0	6000.0	108.0	50.7	322.4	129.7	2260.6
0.05	0.00011	0.002	1120.0	4800.0	121.3	56.9	258.0	103.7	1808.5
0.06	9.16E-05	0.001667	933.3	4000.0	133.3	62.6	215.0	86.5	1507.1
0.07	7.85E-05	0.001429	800.0	3428.6	144.5	67.8	184.3	74.1	1291.8
0.08	6.87E-05	0.00125	700.0	3000.0	154.8	72.6	161.2	64.8	1130.3
0.09	6.11E-05	0.001111	622.2	2666.7	164.6	77.2	143.3	57.6	1004.7
0.1	5.5E-05	0.001	560.0	2400.0	173.9	81.6	129.0	51.9	904.2
0.2	2.75E-05	0.0005	280.0	1200.0	249.3	117.0	64.5	25.9	452.1
0.3	1.83E-05	0.000333	186.7	800.0	307.9	144.5	43.0	17.3	301.4
0.4	1.37E-05	0.00025	140.0	600.0	357.6	167.8	32.2	13.0	226.1
0.5	1.1E-05	0.0002	112.0	480.0	401.5	188.4	25.8	10.4	180.8
0.6	9.16E-06	0.000167	93.3	400.0	441.5	207.1	21.5	8.6	150.7
0.7	7.85E-06	0.000143	80.0	342.9	478.3	224.4	18.4	7.4	129.2
0.8	6.87E-06	0.000125	70.0	300.0	512.7	240.6	16.1	6.5	113.0
0.9	6.11E-06	0.000111	62.2	266.7	545.1	255.8	14.3	5.8	100.5
1	5.5E-06	0.0001	56.0	240.0	575.8	270.2	12.9	5.2	90.4
2	2.75E-06	0.00005	28.0	120.0	825.7	387.4	6.4	2.6	45.21
3	1.83E-06	3.33E-05	18.7	80.0	1019.5	478.3	4.3	1.7	30.14
4	1.37E-06	0.000025	14.0	60.0	1184.0	555.5	3.2	1.3	22.61
5	1.1E-06	0.00002	11.2	48.0	1329.6	623.9	2.6	1.0	18.08
6	9.16E-07	1.67E-05	9.3	40.0	1461.9	685.9	2.1	0.9	15.07

7	7.85E-07	1.43E-05	8.0	34.3	1583.9	743.1	1.8	0.7	12.92
8	6.87E-07	1.25E-05	7.0	30.0	1697.8	796.6	1.6	0.6	11.30
9	6.11E-07	1.11E-05	6.2	26.7	1805.0	846.9	1.4	0.6	10.05
10	5.5E-07	0.00001	5.6	24.0	1906.6	894.6	1.3	0.5	9.04
20	2.75E-07	0.000005	2.8	12.0	2734.0	1282.8	0.6	0.3	4.52
30	1.83E-07	3.33E-06	1.9	8.0	3375.8	1583.9	0.4	0.2	3.01
40	1.37E-07	2.5E-06	1.4	6.0	3920.5	1839.5	0.3	0.1	2.26
50	1.1E-07	0.000002	1.1	4.8	4402.8	2065.8	0.3	0.1	1.81
60	9.16E-08	1.67E-06	0.9	4.0	4840.7	2271.2	0.2	0.1	1.51
70	7.85E-08	1.43E-06	0.8	3.4	5244.7	2460.8	0.2	0.1	1.29
80	6.87E-08	1.25E-06	0.7	3.0	5621.8	2637.7	0.2	0.1	1.13
90	6.11E-08	1.11E-06	0.6	2.7	5976.9	2804.3	0.1	0.1	1.00
100	5.5E-08	0.000001	0.6	2.4	6313.5	2962.2	0.1	0.1	0.90
200	2.75E-08	5E-07	0.3	1.2	9053.3	4247.7	0.1	0.0	0.45

And we obtain the following pressure curve at the different filling times
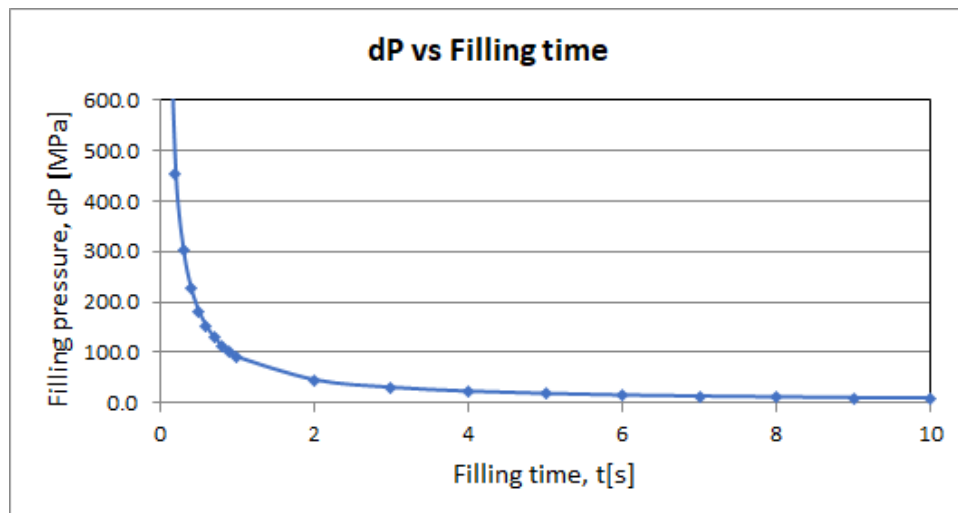

And in the logarithmic scale

We can see that we need a pressure of 18.084 MPa and a force of 18032.512 N to fill up the mould just in 1 second. This is dependent on the filling time, which is shown in the above figures, where we can observe the relation between the filling pressure vs filling time. The figure shows that at short times, it needs very high pressures. When we compare the pressure with the filling time, we can see that it's not needed to waste a lot of energy while applying those quantities of pressure. In this case, while the pressure slows down, the amount of time required for injection increases. In order to complete 10 pieces per minute, we consider that the injection cycle can last 1 seconds because the amount of pressure needed is 18.084 MPa/sec giving us plenty of time to fulfil the full cycle under 30 seconds.

Analysis of pressures at different points of the mould in 1 second of flow
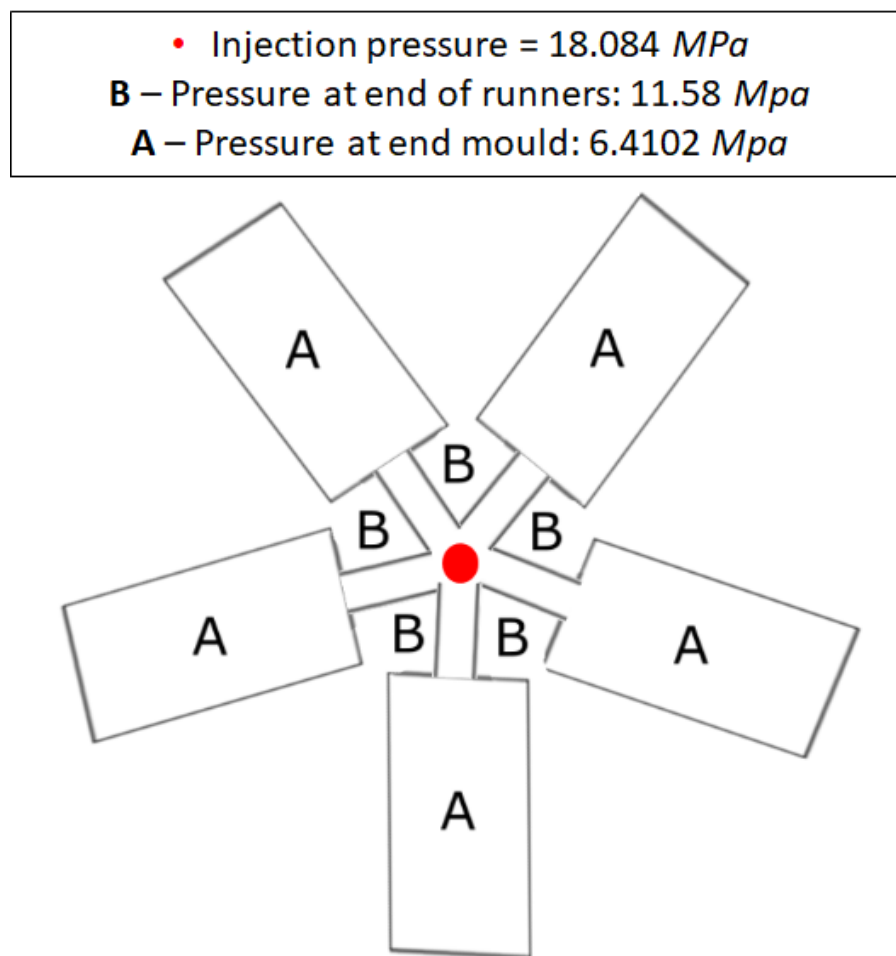
F. VALIDATION

We believe our results are reasonable because we obtained a pressure drop equivalent to the ones mentioned in the literature, for example the data obtained by Jones, D. R., & Ashby, M. F. [4] means that our results are within the average needed to give good detail for the injection processes and also, the curve of pressures versus filling times depict the obtained pressures at times which seem reasonable.

G. REFERENCES

[1] E. Farotti, M. Natalini, Injection molding. Influence of process parameters on mechanical properties of polypropylene polymer. A first study., Procedia Struct. Integr. 8 (2018) 256–264. https://doi.org/10.1016/j.prostr.2017.12.027

[2] Rosato, D. V., & Rosato, M. G. (2012). *Injection molding handbook*. Springer Science & Business Media.

[3]www.quora.com/How-fast-does-an-average-injection-molding-machine-produce-parts#:~:text=Injection%20molding%20is%20indeed%20very,processes%20for%20long%20manufacturing%20runs!

[4] Jones, D. R., & Ashby, M. F. (2012). Engineering materials 2: An introduction to microstructures and processing. Butterworth-Heinemann.

Q2: You have decided to start a plastic recycling company and Mr. Good Die is selling you an extruder with a 500 holes die, each of them has a length of 1.25 inches and an radius of 0.05 inches **{20 pts}**
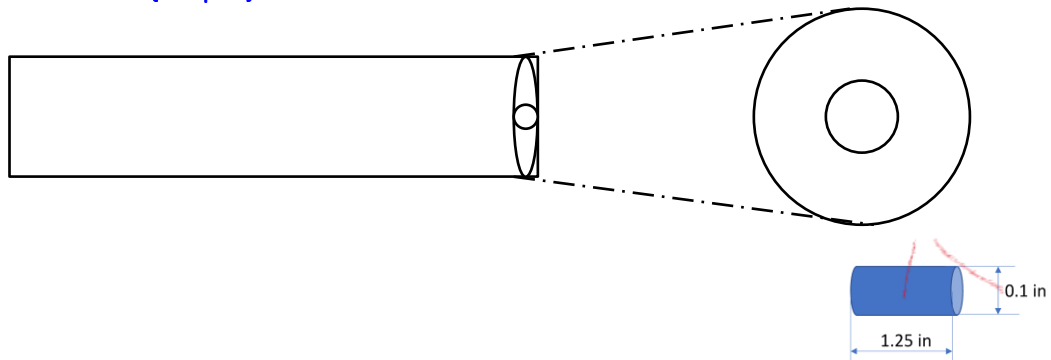
You need to *estimate the pressure drop* across the die since that information is needed to determine how much energy/ton of recycled plastic you have to pay. You will use that information together with other data (energy for plasticizing the plastic, extruder maintenance, etc.) to estimate the investment and the ROI. You want to make sure that can get recycled pellets at 1,000 kgs/hour. The viscosity curve is that given in the first problem for resin 2.

A. PARAPHRASE

We will recycle plastics (resin 2 from problem 1). We have to calculate the pressure drop across an extruder die with 500 holes, to determine the energy/ton of the recycled plastic we need to pay. With that and other information we will obtain the investment and the ROI. We want to recycle 1,000 kgs of pellets per hour with a good ROI balance.

B. AVAILABLE DATA

- Process: extrusion.
- Polymer: Recycled material (resin 2).
- Die: 500 holes.
- Single hole dimensions: length of 1.25 in and radius of 0.05 in.
- Recycle 1,000 kgs of pellets per hour
- Viscosity curve: resin 2.
- Pressure drop across die: obtain.
- Energy/ton of recycled plastic to pay: obtain.
- Investment: obtain.
- ROI: obtain.

C. ASSUMPTIONS

- The applied pressure is constant.
- The resin density remains constant.
- The flow in the extruder is at constant velocity, laminar, steady state.
- Viscous heating at the wall and entrance effects as the flow changes are neglected.
- Use of plasticizers and other elements.
- The energy needed increases with the shear stress into the polymer.

- Making less than 1,000 kgs of recycled pellets per hour is not worth the investment.
- We have all the money needed to do the investment.
- We will take the energy costs of Mexico.
- We will sell the pellets at a selected price, and they will be bought.
- The extruder will be operating only for 8 hours during workdays. (20 days a month)
- The typical entrance pressure is 14 MPa as in reference [8]

D. ALGORITHM

1. Plot the viscosity vs. shear rate curve at 200°C from previous problem.
2. Obtain the flow rate and shear rate for each hole.
3. Calculate the pressure drop of each hole.
4. Calculate the energy consumption with the flow and the pressure drop.
5. Estimate cost of additives.
6. Estimate total investment.
7. Sell pellets.
8. Calculate ROI.

E. SOLUTION

Area of 1 hole: 5.06707×10^{-06} m^2

Volume to fill 1 hole: 1.6088×10^{-07} m^3

Flow rate: $Q = \dfrac{V}{t} = 1.6088 x 10^{-7}\ m^3$

Apparent shear rate: $\Gamma = \dfrac{4Q}{\pi R^3} = 100\quad 1/s$, where $R\ =\ 0.00127\ m$

Viscosity: $\eta = \eta_0 \left(\dfrac{\dot{\gamma}}{\dot{\gamma}_0}\right)^{n-1}\ =\ 425.9187924$ Pa $\cdot$ s

$$\Delta P_{oneHole} = \frac{8\eta}{\pi R^4} Q\ = 67.0737 \text{ MPa for 1 hole}$$

$$F = P A_{500holes}$$

$$A_{500holes} = 2.533535 x 10^{-3} m^2$$

$$F = \Delta P\ A\ =\ 169746.845\ N$$

To obtain the energy consumption of the extruder can be calculated with the following equation:

$$Energy\ consumption = \frac{Q\ \Delta P\ t}{1000}$$

Where Q is the flow rate of the 500 holes, ΔP is the pressure drop in the 500 holes, and t is measured in hours.

Considering the extruder will be functioning 8 hours a day, 5 days per week, and 4 weeks per month, we will calculate the energy consumption per month.

We obtained an energy consumption per month of 172.653 kWh

From the Federal Commission of Electricity (CFE) we obtained that the cost of each kWh is 2.941 MXN [3].

Multiplying the cost of each kWh times the total energy consumed per month we get a cost of operating the extruder of 507 MXN.

With this viscosity and this flow rate the production is not enough to obtain the 1000 kg/h, we need very high pressures to obtain them. So, we need to add plasticizers to make it less viscous and peroxide to reduce the molecular weight distribution broadness and the molecular weight.

For each 1000 kg batch we need 200 Lt of peroxide, for a month we need 32,000 lt, considering that the price of peroxide is 73.48 per lt, the cost of a month worth of peroxide is: 2,372,640 MXN per month [7].

Since we are planning on selling these pellets for plastics that will be in contact with food (mainly), we will use Di-n-butyl phthalate as the plasticizer for this application. 18 lt of this plasticizer cost 7,688 MXN, if we will use 50 lt per 1000 kg pellet, then the cost for a month is: 3,416,888 MXN.

We want to sell our pellets at a competitive price, given the market offer and demand we can sell 500 g at 3,400 MXN [4], since we are producing 160,000 kg of pellets per month, the total return is: 1,088,000,000 MXN.

ROI = INCOME – INVESTMENT

INVESTMENT = 2,372,640 + 507 + 3,416,888 = 5,790,035 MXN

INCOME = 1,088,000,000 MXN

ROI = 1,088,000,000 – 5,790,035 = 1,082,209,965 MXN

F. VALIDATION

According to the source found, the pressure drop obtained seems reasonable for the inlet pressure that we proposed.

Our results were not convincing to us in regards of the consumed energy because we could not find the dimensions of the extruder, its motor, etc. Also, we found that the flow rate values were too small but we were not able to find the exact error due to a lack of time.

G. REFERENCES

[1] Bonilla-Rios Jaime, Capillary rheometry. (2020).

[2] https://app.cfe.mx/aplicaciones/ccfe/tarifas/tarifas/tarifas_casa.aspTarifa=DACTAR1&Anio=2014&mes=6&imprime=#

[3] https://www.infobae.com/america/mexico/2020/01/03/en-2019-aumento-48-la-tarifa-electrica-en-2020-subira-a-5/#:~:text=Datos%20de%20la%20Comisi%C3%B3n%20Federal,el%20rango%20de%20consumo%20b%C3%A1sico.

[4] https://www.sigmaaldrich.com/catalog/search?term=9002-88-4&interface=CAS%20No.&N=0&mode=partialmax&lang=es&region=MX&focus=product

[5] http://www.plasticextrusion.info/?gclid=Cj0KCQjwpZT5BRCdARIsAGEX0znm2omNttB2pI70pOeqJ-ZMN_cJ_lVwN6PcLqd2-lmPURXlg3iXqagaAhSEALw_wcB

[6] https://www.sigmaaldrich.com/catalog/product/aldrich/524980?lang=es&region=MX

[7] https://www.quiminet.com/productos/peroxido-de-dicumilo-227234525/precios.htm

[8] Rosato, D. V., & Rosato, D. V. (1990). Extrusion. In *Plastics Processing Data Handbook* (pp. 91-169). Springer, Dordrecht.

Q3: You are requested to get the master curve of the HDPE resins at the reference temperature of T=210°C and get the complex viscosity curve at 200°C. Also, using the master curve of the G"(ω) at T=210°C , you are requested to get the First normal stress difference at steady state, for shear rates 0.1, 1, 10 and 100 1/s, using the Wagner Model. Make any comments/observations you consider important. **(20 pts)**

A. PARAPHRASE

Describe the TTS and modelling procedure to get the first normal stress difference at steady state from complex viscosity curves, with a reference temperature of 210°C from a PP-HDPE Copolymer with an MFI of 2 g/10 min.

B. AVAILABLE DATA

- Process: TTS
- Polymers: PP-HDPE copolymer
- MFI: 2g/10min
- Objective temperature: 200°
- Reference temperature: 210°C
- Time: infinite
- Shear rates: 0.1, 1, 10 and 100 1/s
- Master curves: obtain G* and G''
- N1: obtain

C. ASSUMPTIONS

- The given data was obtained under the same conditions at different temperatures
- There will be no environmental activity on the polymer that affects the performance in time
- The data from shear viscosity comes from a capillary test and the relaxation modulus comes from an oscillatory test

D. ALGORITHM

1. Calculate tan delta
2. Plot tan delta vs. frequency to get the horizontal shift factors (a_T) by superposing manually the curves to the chosen reference temperature
3. Plot tan delta vs. G* to get the vertical shift factors (b_T) manually
4. Apply the shift to the data at each temperature to obtain the master curve at reference temperature
5. Obtain activation energies to get the horizontal and vertical shifts for the objective temperature
6. Apply the shifts obtained to the master curve to get the master curve at the objective temperature
7. Fit the new master curve into the Wagner model
8. From Wagner fit obtain the fitting parameters: as, lambdas, n1, n2 and f1

9. With the fitting parameters, calculate N1 at the steady state for the given shear rates.

E. SOLUTION

The complete solution (source code) can be found at https://tecmx-my.sharepoint.com/:u:/g/personal/a01212611_itesm_mx/EdsBCPRq6sVAtk95i6zAsPIBO364 BytEiHC1XD8xH8xgfQ?e=1x6iNp, the following explains the problem-solving process.

1. Calculate $tan\delta = \frac{G''(\omega)}{G'(\omega)}$ and $G^*(\omega) = [G''(\omega)^2 + G'(\omega)^2]^{0.5}$, and graph $tan\delta$ vs. frequency and $tan\delta$ vs. $G^*(\omega)$.
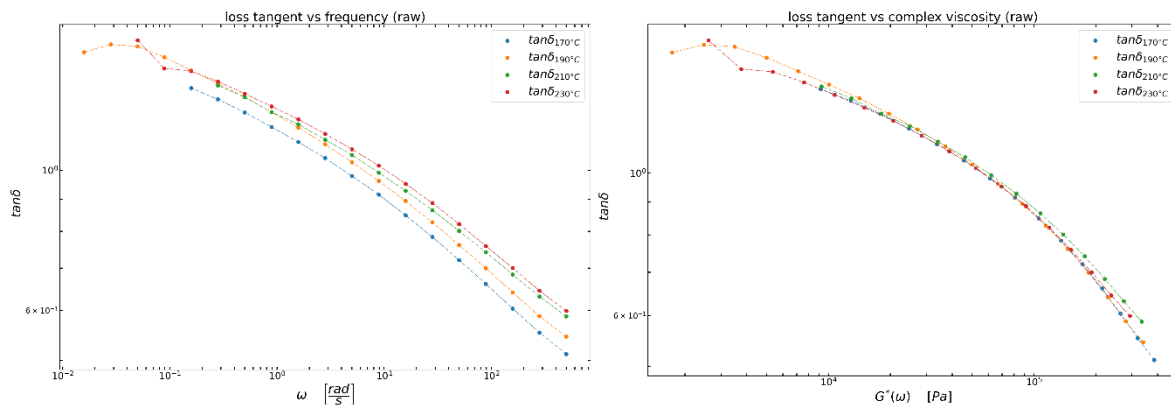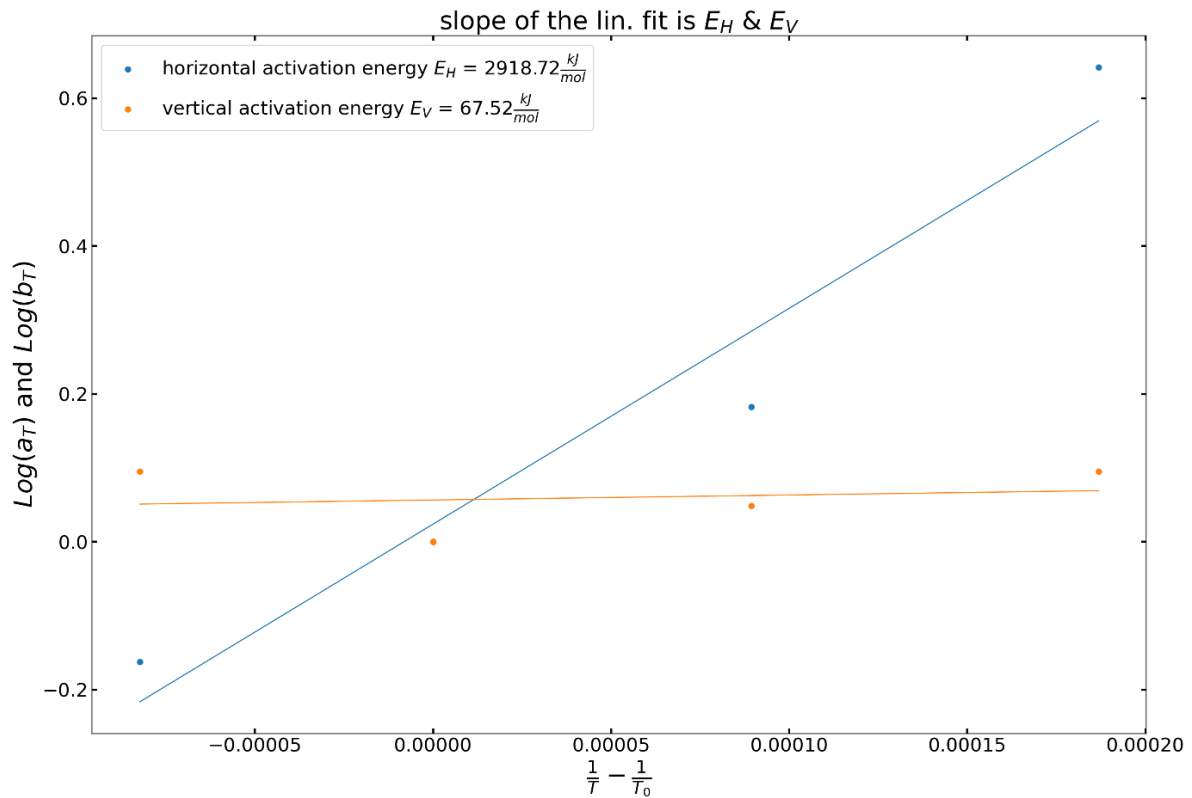

2. Calculate the vertical and horizontal shift factors by hand until all the curves overlap the 210°C curve.
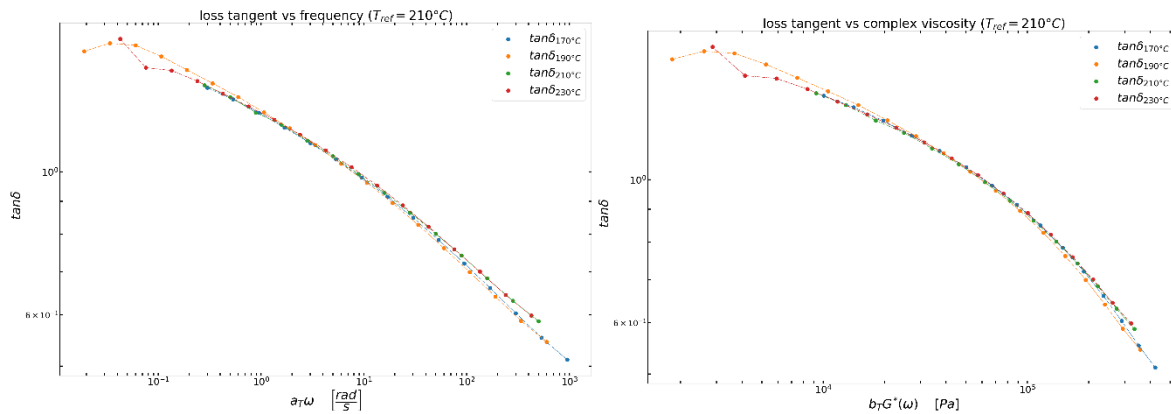
| Temperature [°C]	Horizontal shift factor aT	Vertical shift factor bT
170	1.900000	1.100000
190	1.200000	1.050000
210	1.000000	1.000000
230	0.850000	1.100000

3. Get the activation energies E_H and E_V by plotting $\log a_T$ & $\log b_T$ vs. $\left(\frac{1}{T} - \frac{1}{T_0}\right)$. Then do a linear regression as the slopes of the linear fit are the activation energies.

slope of the lin. fit is E_H & E_V

4. Compute the shift factor a_T and b_T for an arbitrary temperature of 200°C with the Williams-Landell-Ferry (WLF) equation $a_T = \exp\left[E_H\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$, $and\ b_T = \exp\left[E_V\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$

| Temperature [°C]	Horizontal shift factor aT	Vertical shift factor bT
200	1.136186	1.002958

5. Calculate the complex viscosity master curve at 210°C using the $G^*(\omega)$ master curve with: $\eta^*(\omega) = \frac{G^*(\omega)}{\omega}$

Master curve for complex viscosity vs frequency (T_{ref} = 210°C)
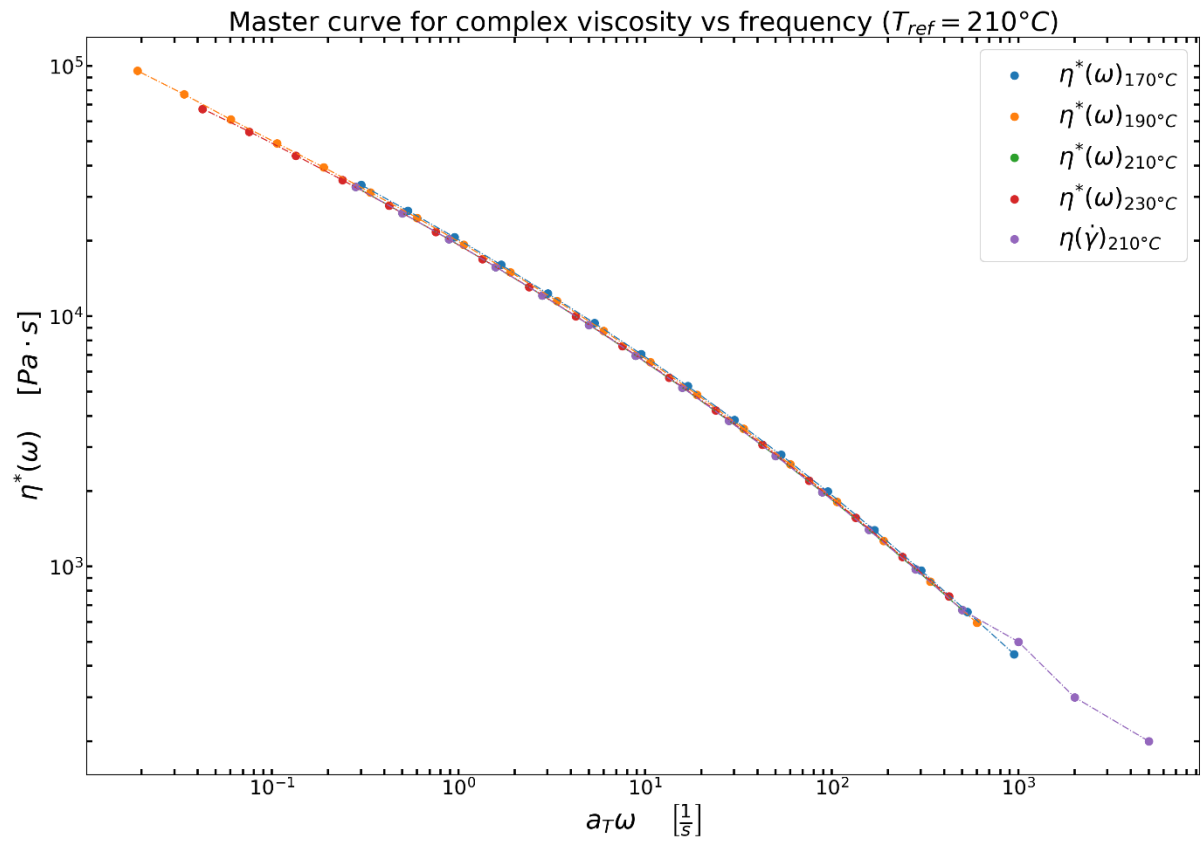
Note that the $\eta(\dot{\gamma})_{210°C}$ data is plotted without shifting as it is already measured at the reference temperature.

6. Plot the complex viscosity curve against shear rate at 200°C

Complex viscosity vs Shear rate ($T_{ref} = 210°C$)
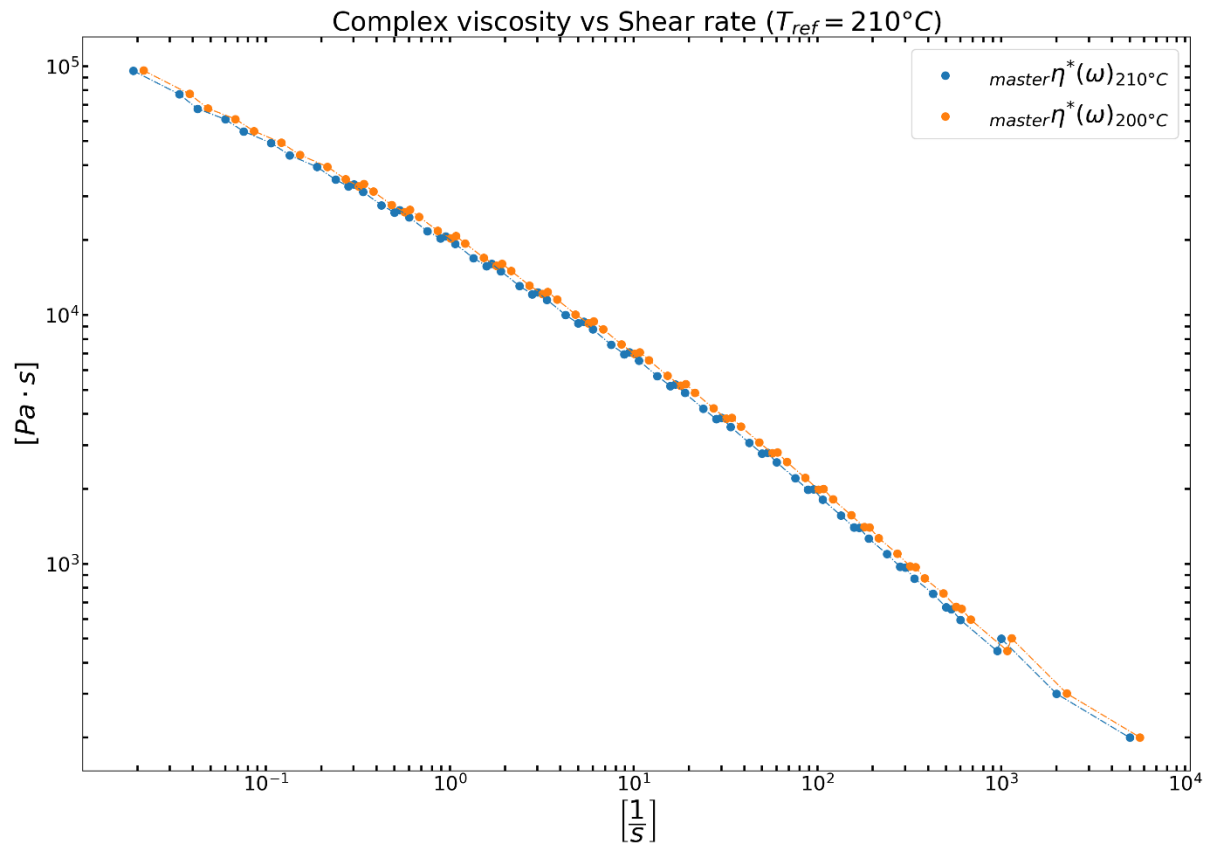
Notice that the lower temperature curve (200°C) as higher viscosity values at a given shear rate.

7.   Loss modulus master curve at 210°C

Loss Modulus vs frequency ($T_{ref} = 210°C$)
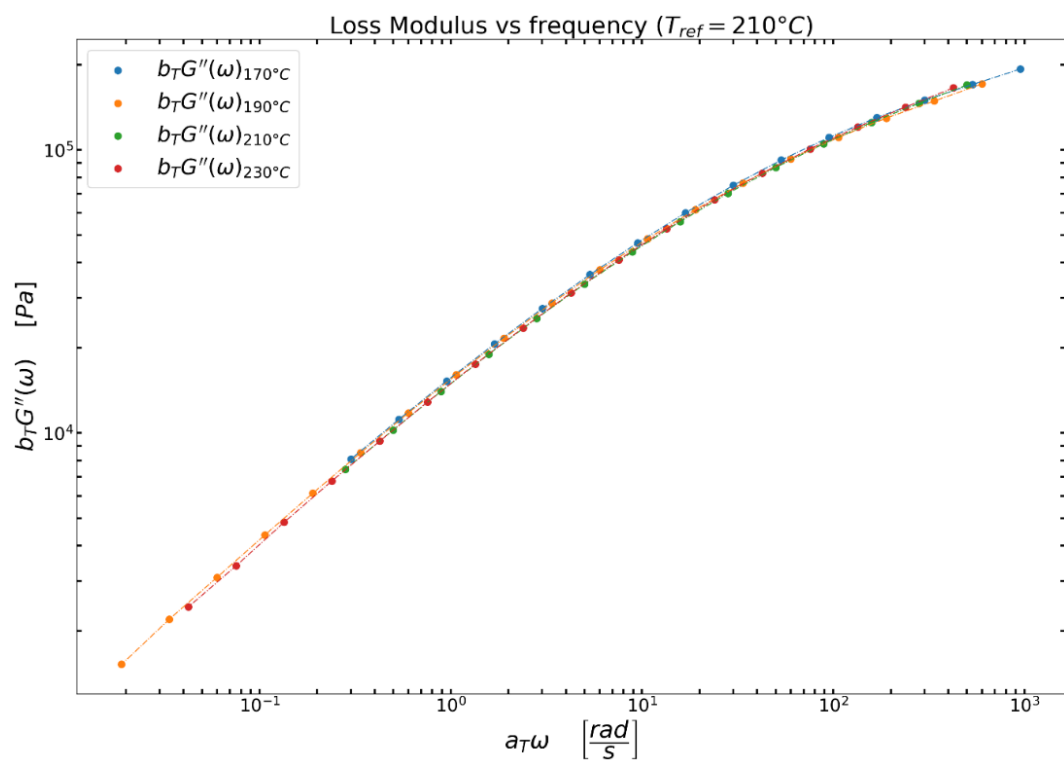
Maxwell fit of loss modulus with 5 elements. It was attempted to run the Relaxation Spectra.exe tool with more than 5 Maxwell elements, however the tool is unable to compute due to short frequency ranges.

$$G^{''} = \sum_{i=1}^{n} \frac{\eta_i \omega}{1 + \omega^2 \lambda_i^2}$$

Maxwell fit - loss modulus
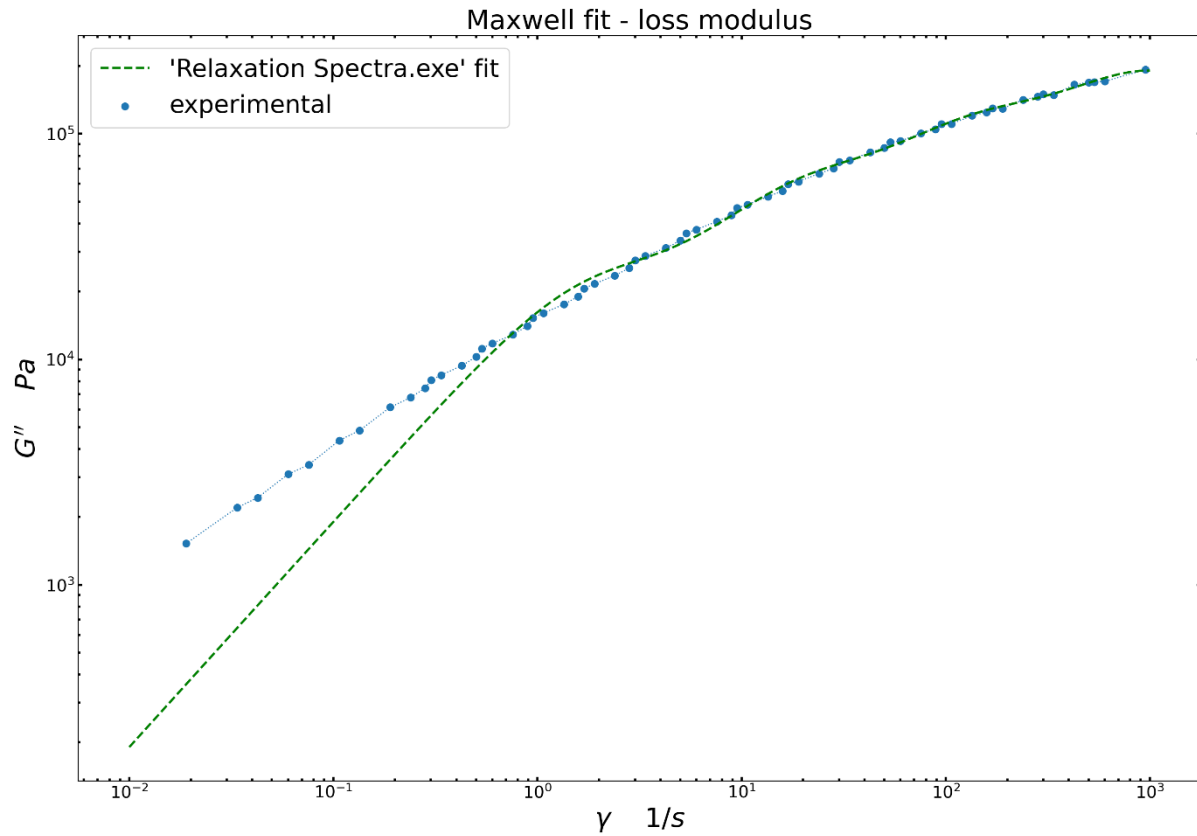
The fitting parameters are verified by plotting the storage modulus model along with the experimental data (storage master curve)

$$G^{'} = \sum_{i=1}^{n} \frac{\eta_i \lambda_i \omega^2}{1 + \omega^2 \lambda_i^2}$$

Maxwell fit - storage modulus
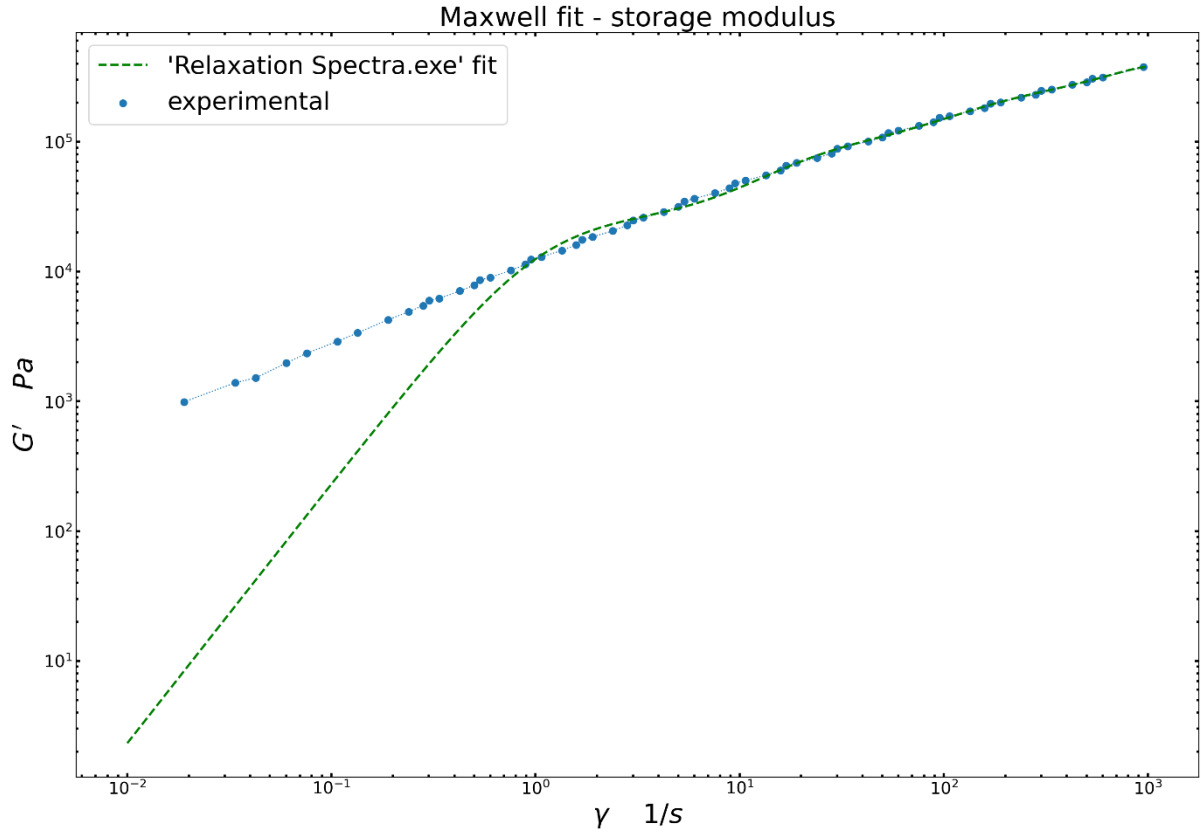
8. Compute fitting curve with the Wagner model with the master curve at $210°C$. The shear viscosity as a function of shear rate $\dot{\gamma}_o$, is:

$$\eta(t, \dot{\gamma}_o) = f_1 \sum_{i=1}^{n} \frac{a_i}{\alpha_i{}^2}\left(1 - e^{1-\alpha_i t}(1 - \dot{\gamma}_o n_1 \lambda_i \alpha_i t)\right) + f_2 \sum_{i=1}^{n} \frac{a_i}{\beta_i{}^2}\left(1 - e^{1-\beta_i t}(1 - \dot{\gamma}_o n_2 \lambda_i \beta_i t)\right)$$

Where:

$$f_2 = 1 - f_1$$

$$\alpha_i = \frac{1 + n_1 \lambda_i \dot{\gamma}_o}{\lambda_i}$$

$$\beta_i = \frac{1 + n_2 \lambda_i \dot{\gamma}_o}{\lambda_i}$$

$\eta(t, \dot{\gamma}_o) = shear\ viscosity$

$t\ = time$

$\dot{\gamma}_o = shear\ rate$

$a_i = the\ i^{th}\ elastic\ value\ of\ the\ Maxwell\ element$

$\lambda_i = the\ i^{th}\ characteristic\ time\ of\ the\ Maxwell\ element$

f_1, f_2, n_1, n_2 are fitting parameters

The Wagner model gives the shear viscosity vs. time of a single shear rate. The steady state shear viscosity at several shear rates at $t = \infty$ is given by:

$$\eta(t = \infty, \dot{\gamma}_o) = f_1 \sum_{1=1}^{n} \frac{a_i}{{\alpha_i}^2} + f_2 \sum_{1=1}^{n} \frac{a_i}{{\beta_i}^2}$$

The following if the Wagner fit using the Relaxation Spectra.exe tool, plotting the loss modulus against frequency. As depicted in the figure, the bottom-left values are not well fitted, therefore the fitting was also performed with the scipy library to look for a better fitting. The scipy fitting (see the figure below), was fitted with the viscosity Wagner model in function of frequency (omega).
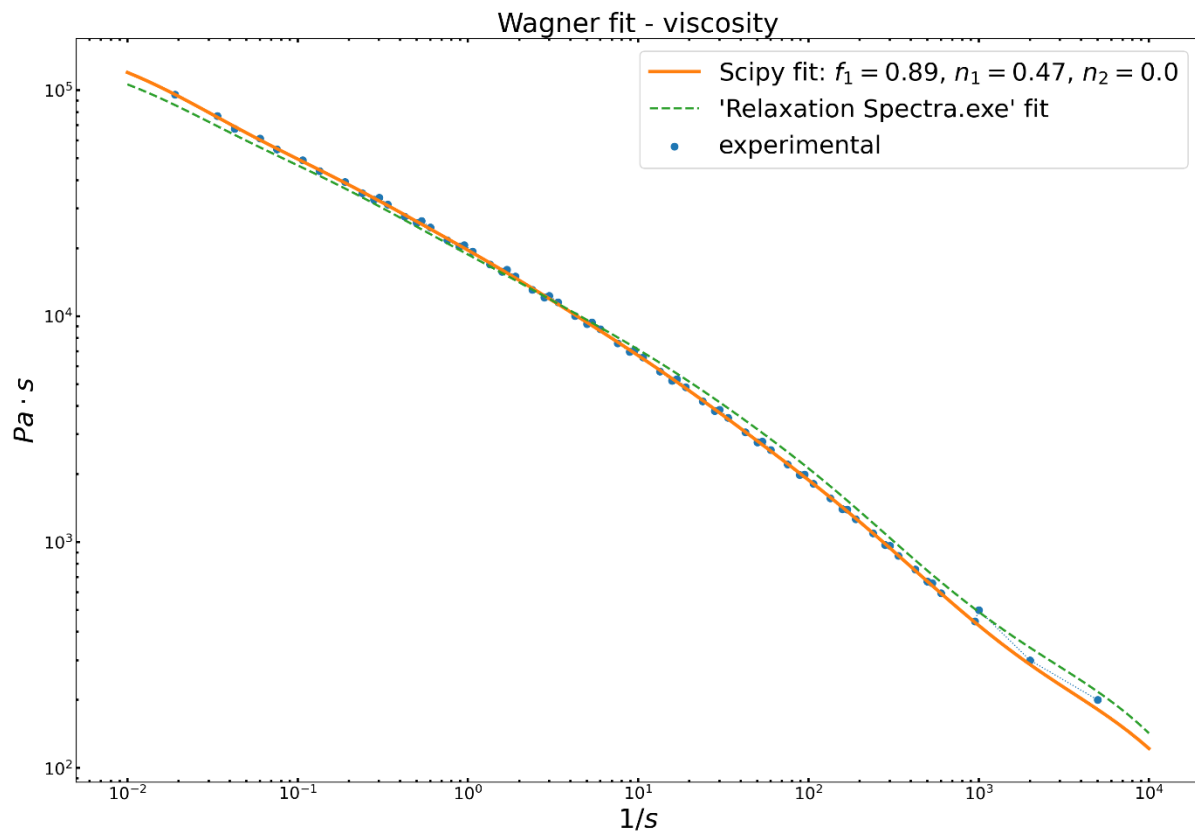
The following are the fitting values for the Wagner model (with five Maxwell elements) to match the experimental data within the master curve at 210°C.

$a_1 =$ 2.68425342e+08 $\lambda_1 =$ 8.96633937e-03
$a_2 =$ 1.09191259e+06 $\lambda_2 =$ 1.74375415e-01
$a_3 =$ 2.57623389e+05 $\lambda_3 =$ 1.74521550e-01
$a_4 =$ 9.67069909e+03 $\lambda_4 =$ 6.30348573e+00
$a_5 =$ 2.04877643e+02 $\lambda_5 =$ 8.07822062e+01
$f_1 =$ 8.89712169e-01
$n_1 =$ 4.69519418e-01
$n_2 =$ 4.83767742e-03

The fitting parameters above (λ_i & a_i) are computed with the python scipy package (orange line), see the Problem3.html and jbr.py for more details (specifically to the `plot_Wagner_fit` function). The scipy fitting values are comparable with the ones obtained with the provided "Relaxation Spectra.exe" tool as they are alike. The small discrepancy can be contributed to the fact that only twelve data points were provided to the "Relaxation Spectra.exe" tool. Since the "Relaxation Spectra.exe" fitting curve (green line) ignores some data points, the scipy results are used to calculate the normal stress difference.

```
C:\Users\oskat\OneDrive - Instituto Tecnologico y de Estudios Superiores de Monterrey\Documents\MNT_ITESM_courses\3a.1.PlasticsAndCompositesE...   —   □   ×
3-26

The Discrete Relaxation Spectrum

 lamda #0 = 0.010357    eta #0 = 165.786734
 lamda #1 = 0.165265    eta #1 = 9761.608180
 lamda #2 = 0.183612    eta #2 = 248532.480687
 lamda #3 = 6.212577    eta #3 = 1182801.688356
 lamda #4 = 81.691297   eta #4 = 259334433.275553
Press any key to see statistical results

Residual 0 = 32.469355
Residual 1 = 9.489482
Residual 2 = 2.660423
Residual 3 = 0.952538
Residual 4 = 2.854023
Residual 5 = 5.416978
Residual 6 = 6.935212
Residual 7 = 3.794354
Residual 8 = 1.743035
Residual 9 = 5.034154
Residual 10 = 4.879913
Residual 11 = 1.604921
SS = 147071899.164380
SSmean = 47908632912.130287
Spread = 4583.696250
R^2 = 0.996930

Press any key to continue
```
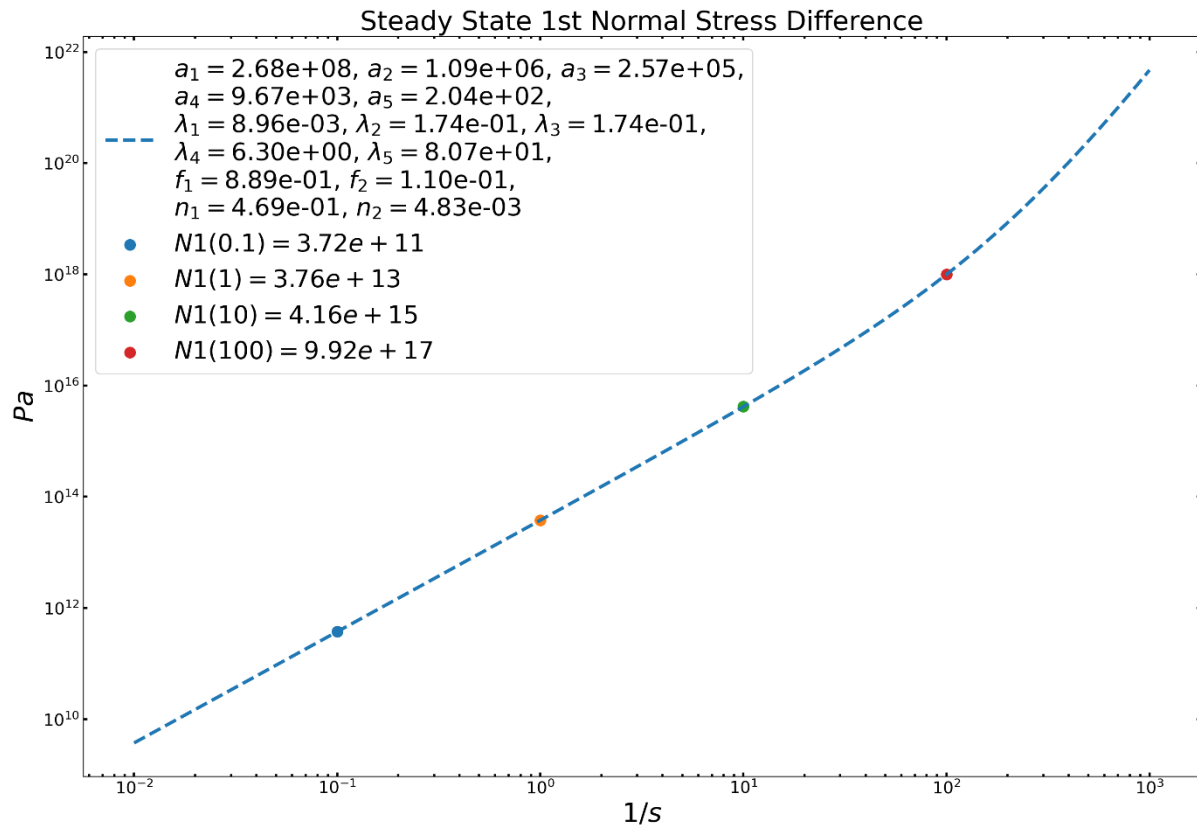
9. Calculate the steady state first normal stress difference $N1$ at $t = \infty$

$$N1(t, \dot{\gamma}_o) = \dot{\gamma}_o{}^2 \left\{ f_1 \sum_{i=1}^{n} a_i \alpha_i^3 \left(1 - e^{-\alpha_i t}\left[1 + \alpha_i t - \alpha_i{}^2 \left(n_1 \lambda_i \frac{\dot{\gamma}_o}{2}\right) t^2\right]\right) \right.$$

$$\left. + f_2 \sum_{i=1}^{n} a_i \beta_i^3 \left(1 - e^{-\beta_i t}\left[1 + \beta_i t - \beta_i{}^2 \left(n_2 \lambda_i \frac{\dot{\gamma}_o}{2}\right) t^2\right]\right) \right\}$$

$$N1(t = \infty, \dot{\gamma}_o) = \dot{\gamma}_o{}^2 \left\{ f_1 \sum_{i=1}^{n} a_i \alpha_i^3 + f_2 \sum_{i=1}^{n} a_i \beta_i^3 \right\}$$

The figure above plots $N1(t = \infty, \dot{\gamma}_o)$ and displays the first normal stress difference at steady state, for shear rates 0.1, 1, 10 and 100 1/s.

F. VALIDATION

Based on the fact that the provided data is similar to the one in the TTS class activity, we conclude the following: The given solution is feasible as the resulted normal stress difference curve and master curves are similar to the ones obtained during the course class activities

G. REFERENCES

Bonilla-Rios Jaime, ESTIMATION OF THE FIRST NORMAL STRESS DIFFERENCE (N1) AND CREEP COMPLIANCE(J(t)) OF POLYPROPYLENE (PP) RESINS USING A CONSTITUTIVE EQUATION. (2020)

J. Ahmed, Time–Temperature Superposition Principle and its Application to Biopolymer and Food Rheology, in: Adv. Food Rheol. Its Appl., Elsevier, 2017: pp. 209–241. https://doi.org/10.1016/B978-0-08-100431-9.00009-7.

A. Oseli, A. Aulova, M. Gergesova, I. Emri, Time-Temperature Superposition in Linear and Non-linear Domain, Mater. Today Proc. 3 (2016) 1118–1123. https://doi.org/10.1016/j.matpr.2016.03.059.

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Q4: What type of melt distortions can be observed in polymers, give some examples on different polymer extrusión processeses and how can be avoided. {10 pts} BONUS

Defect	Solution
Bubbles	Bubbles are cause by humidity; therefore, it would be necessary to dry the material
Shrinkage	The cause is due to excessive stress in the material. Slower cooling, lower speeds and a die with lower drag may solve this issue
pimples	The decrease of extrusion speed sorts out this problem, which is caused by the use of gels
Shark skin	Due to melt overheating, solution would be to increase the die temperature and decrease the extrusion speed
Surging	The puller may be at irregular speeds; the screw rotation would be at inconsistent velocities; or due to uneven polymer feed
Lumpiness	Due to low temperature. Rising the temperature would solve the problem.