Homework No.1

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What will I achieve?

With this homework you will practice the use of the concepts and knowledge acquired throughout the corresponding topics.

Instructions

Part A

Find five differential equations found in fluid mechanics, heat transfer, mass transfer, bioengineering or reaction engineering. Three of them must be PDE (Partial differential equations), and you must explain each term, the physical meaning of each term, and what they represent.

Conservation of Mass : Continuity Equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \tag{1}$$

The conservation of mass equation is obtained by replacing B in the Reynolds transport theorem by mass m, and b by 1 (m per unit mass = 1).

Conservation of Momentum : Bernoulli equation

$$\frac{P}{\rho} + \frac{v^2}{2} + gz = constant \tag{2}$$

The Bernoulli equation is valid in inviscid regions of flow where net viscous forces are negligible compared to inertial, gravitational, or pressure forces.

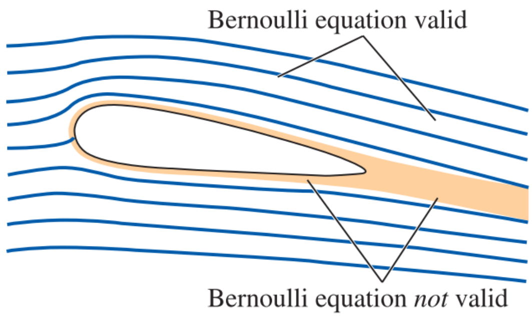

Figure 1: Adapted from [2]

This equation describes a balance between pressure (flow energy) $\frac{P}{\rho}$, velocity (kinetic energy) $\frac{V^2}{2}$, and position (potential energy) gz of fluid particles relative to the gravity vector (potential energy) in regions of a fluid flow where frictional force on fluid particles is negligible compared to pressure force in that region of the flow (see inviscid flow). There are multiple forms of the Bernoulli equation for incompressible vs. compressible, steady vs. nonsteady, and derivations through Newton's law vs. the first law of thermodynamics. The most commonly used forms are for steady incompressible fluid flow derived through conservation of momentum.

Conservation of Energy : 1st Law of Thermodynamics

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial \rho e}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v} e) = \frac{\mathrm{d}}{\mathrm{d}t} \iiint e\rho \, \mathrm{d}V + \oiint e\rho(\vec{v} \cdot \vec{n}) \, \mathrm{d}A \tag{3}$$

The total energy within a system is affected by work transfer W and heat transfer Q. Since energy is conserved, the total energy of the system is $E = Q + W$. [1, 2]

Heat Transfer Q : The internal energy, heat or thermal energy tends to move from a high temperature body to a low temperature body. The temperature difference dictates the rate of heat transfer. At higher temperature differences, the higher is the rate of heat transfer. Once the temperature is balanced between the two bodies, heat transfer stops.

Work Transfer W : Work is an energy interaction related to a force. A system may involve several forms of work, in such a way that: $W = W_{motor} + W_{pressure} + W_{viscous} +$work done by other forces.... W_{motor} is the work done by a rotating shaft, $W_{pressure}$ is the work transmitted by pressure forces, and $W_{viscous}$ are the normal and shear stress components of viscous forces. W can also be comprised by work done by other forces such as surface tension, magnetic and electric. $W_{viscous}$ is only considered when the moving walls are part of the control surface.

The relationship between conservation of energy for a control volume is obtained by the Reynolds transport theorem, replacing B with the total energy E and $\hat{B}$ with the total energy per unit mass, which is $e = u + \frac{v^2}{2} + gz$. u, $\frac{v^2}{2}$, and gz are the internal, kinetic, and potential energies per unit mass respectively. In other words, $\frac{\mathrm{d}E}{\mathrm{d}t}$ can be described as follows:

$$\left(\begin{array}{c}\text{The rate of}\\ \text{energy transfer by}\\ \text{heat and work}\end{array}\right) = \left(\begin{array}{c}\text{The time rate of}\\ \text{change in energy}\end{array}\right) + \left(\begin{array}{c}\text{The flow rate of}\\ \text{energy out of}\\ \text{the surface}\end{array}\right)$$

Linear Momentum : Cauchy's equation

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \vec{\nabla} \cdot (\rho \vec{v} \vec{v}) = \rho \vec{g} + \vec{\nabla} \cdot \sigma_{ij} \tag{4}$$

Linear Momentun : Navier-Stokes Equation

$$\rho \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\vec{\nabla} P + \rho \vec{g} + \mu \nabla^2 \vec{v} \tag{5}$$

Part B

Select one problem of any of the fields listed above, and solve it, following the steps given below.

1. *Read problem statement, and collect the information that may be needed.*
2. *Make a Sketch (Diagram, process flow chart), indicating mass, linear or angular momentum (i.e. forces and torques) and energy interaction, and label each stream and boundaries as well.*
3. *List Assumptions and Approximations (sometimes they may be inferred by the sketch, but make them explicit) supported by equations if possible (geometric relationships, or fundamental equations).*
4. *Physical Laws (Fundamental Laws) must be written in full form, and terms can be dropped by the right selection of frame of reference, operating conditions, assumptions, simplifications or constraints.*
5. *Physical constants should be obtained from a reliable source (knowing this information by heart is always helpful), geometric relations and formulae must be included as part of your analysis.*
6. *Physical transport or thermodynamic properties (Thermodynamic relations) should be evaluated, approximated, calculated or obtained from a reliable source.*
7. *Calculations are done including units. Any algebraic manipulation is recommended in few cases, because limits the step 8, but if needed should be done before using numerical values of constants, properties or variables.*
8. *Reasoning (Sensitivity analysis, what if), Verification (context), and Discussion should always be part of your answer to any problem, regardless the task requested.*

References

[1] F. Moukalled, L. Mangani, and M. Darwish, *The Finite Volume Method in Computational Fluid Dynamics: An Advanced Introduction with OpenFOAM® and Matlab®*. Springer International Publishing, 2016, p. 816, ISBN: 978-3-319-16873-9.

[2] M. F. White, *Fluid Mechanics*, 7th. McGraw-Hill Series in Mechanical Engineering, Oct. 2011, p. 885, ISBN: 978-0-07-352934-9.