ITESM Campus Monterrey
Mathematical Physical Modelling F4005
HW2: Row echelon form and linear systems
Due Date: Tuesday 29-2019, 23:59 hrs.
Professor: Ph.D Daniel López Aguayo

Full names of team members: ______________________________

Instructions: Please write neatly on each page of your homework and send it in pdf format to dlopez.aguayo@tec.mx. Typed solutions in LaTeX (only) will be given extra credit; no late homework will be accepted. Each team should consist (of at most) 5 students.

I. In Exercises $1-6$, determine which equations are linear equations in the variables x, y and z. If any equation is not linear, please explain why not.

1. $x - \pi y + \sqrt[3]{5} \cdot z = 0$
2. $x^2 + y^2 + z^2 = 1$
3. $x^{-1} + 7y + z = \sin^2\left(\frac{\pi}{9}\right)$
4. $x + 7y + z = sin\left(\frac{\pi}{9}\right)$
5. $3 \cdot cos(x) - 4y + z = \sqrt{3}$
6. $cos(3) \cdot x - 4y + z = \sqrt{3}$

II. In Exercises $7-9$, draw graphs (you are allowed to use Mathematica) corresponding to the given linear systems. Determine geometrically whether each system has a unique solution, infinitely many solutions or no solutions. Then solve each system algebraically to confirm your answer.

7. $$\begin{cases} x + y = 0 \\ 2x + y = 3 \end{cases}$$

8. $$\begin{cases} x - 2y = 7 \\ 3x + y = 7 \end{cases}$$

9. $$\begin{cases} 3x - 6y = 3 \\ -x + 2y = 1 \end{cases}$$

III. In Exercises $10-12$, solve the given system by back substitution.

10. $$\begin{cases} x - y + z = 0 \\ 2y - z = 1 \\ 3z = -1 \end{cases}$$

11. $$\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ -5x_2 + 2x_3 = 0 \\ 4x_3 = 0 \end{cases}$$

12. $$\begin{cases} x_1 + x_2 - x_3 - x_4 = 1 \\ x_2 + x_3 + x_4 = 0 \\ x_3 - x_4 = 0 \\ x_4 = 1 \end{cases}$$

IV. In Exercises $13-14$, the systems of equations are nonlinear. Find substitutions (changes of variables) that convert each system into a linear system and use this linear system to solve the original system. Also, verify your answer with Mathematica.

13. $$\begin{cases} \frac{2}{x} + \frac{3}{y} = 0 \\ \frac{3}{x} + \frac{4}{y} = 1 \end{cases}$$

14. $$\begin{cases} -2^a + 2 \cdot 3^b = 1 \\ 3 \cdot 2^a - 4 \cdot 3^b = 1 \end{cases}$$

V. In Exercises 15 – 18, determine whether the given matrix is in row echelon form (and justify your answer). If it is, state whether it is also in reduced row echelon form.

15. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}$

16. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

17. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

18. $\begin{bmatrix} 1 & 2 & 0 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

VI. Use Gaussian elimination to find the rank of the following matrices and verify your answer with Mathematica.

19. $\begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$

VII. In Exercises 21 – 23, solve the given system of equations using Gaussian elimination; then verify your answer with Mathematica.

21.

$$\begin{cases} x_1 + 2x_2 - 3x_3 = 9 \\ 2x_1 - x_2 + x_3 = 0 \\ 4x_1 - x_2 + x_3 = 4 \end{cases}$$

22.

$$\begin{cases} 2r + s = 3 \\ 4r + s = 7 \\ 2r + 5s = -1 \end{cases}$$

23.

$$\begin{cases} w + x + 2y + z = 1 \\ w - x - y + z = 0 \\ x + y = -1 \\ w + x + z = 2 \end{cases}$$

VIII. For what value(s) of k, if any, will the following system have (a) no solution and (b) infinitely many solutions? No guessing is allowed! (i.e use an algebraic method)

24.

$$\begin{cases} kx + y = -2 \\ 2x - 2y = 4 \end{cases}$$

IX. Give an example of three planes that intersect in a single point. *Hint*: Keep it simple! think about familiar planes you learned in Mathematics III.

X. Let n be an arbitrary positive integer. Find the rank of the identity matrix I_n and justify your answer.

XI. For each of the following problems, write the corresponding linear system and solve it using Mathematica (**not by hand!**).

25. A coffee merchant sells three blends of coffee. A bag of the house blend contains 300 grams of Colombian beans and 200 grams of French roast beans. A bag of the special blend contains 200 grams of Colombian beans, 200 grams of Kenyan beans, and 100 grams of French roast beans. A bag of the gourmet blend contains 100 grams of Colombian beans, 200 grams of Kenyan beans, and 200 grams of French roast beans. The merchant has on hand 30 kilograms of Colombian beans, 15 kilograms of Kenyan beans, and 25 kilograms of French roast beans. If he wishes to use up all of the beans, how many bags of each type of blend can be made?

26. There are two fields whose total area is 1800 square yards. One field produces grain at the rate of $\frac{2}{3}$ bushel per square yard; the other field produces grain at the rate of $\frac{1}{2}$ bushel per square yard. If the total yield is 1100 bushels, what is the area of each field?

27. Find a parabola with an equation of the form $y = ax^2 + bx + c$ that passes through $(0, 1)$, $(-1, 4)$ and $(2, 1)$.