• Eigenvalues problems

Find the corresponding eigen values and eigenvectors

a) $\begin{bmatrix} 1 & 2 \\ \emptyset & 3 \end{bmatrix}$ $\det(\lambda I - A)$ $\begin{bmatrix} \lambda & \emptyset \\ \emptyset & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ \emptyset & 3 \end{bmatrix} = \begin{bmatrix} \lambda-1 & -2 \\ \emptyset & \lambda-3 \end{bmatrix}$

$$\det(\lambda I - A) = (\lambda-1)(\lambda-3) - (-2 \cdot \emptyset) = \lambda^2 - 3\lambda - \lambda + 3 = \emptyset$$

$$\lambda^2 - 4\lambda + 3 = \emptyset$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-4) \pm \sqrt{-4^2 - 4(3)(1)}}{2(1)} \longrightarrow \lambda = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$\lambda = \frac{4 \pm \sqrt{4}}{2} \longrightarrow \lambda = \frac{4 \pm 2}{2} \longrightarrow \lambda_1 = \frac{4+2}{2} = \frac{6}{2} = 3$$

$$\lambda_2 = \frac{4-2}{2} = \frac{2}{2} = 1$$

$\lambda_1 = 3$

$$\begin{bmatrix} 3-1 & -2 \\ \emptyset & 3-3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{matrix} (3-1)x_1 - 2x_2 = \emptyset = 2x_1 - 2x_2 = \emptyset \\ \emptyset + (3-3)x_2 = \emptyset = \emptyset + \emptyset = 0 \end{matrix}$$

$X_1 = X_2$

$$t\begin{pmatrix} x_1 \\ X_2 \end{pmatrix} \rightarrow t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$\lambda_2 = 1$

$$\begin{bmatrix} 1-1 & -2 \\ \emptyset & 1-3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{matrix} (1-1)x_1 - 2x_2 = \emptyset = \emptyset - 2x_2 = \emptyset \\ \emptyset + (1-3)x_2 = \emptyset \quad \emptyset - 2x x_2 = \emptyset \end{matrix}$$

$X_1 = t \qquad X_2 = \emptyset$

$$t\begin{pmatrix} t \\ \emptyset \end{pmatrix} \longrightarrow t\begin{pmatrix} 1 \\ \emptyset \end{pmatrix}$$

b) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ $\det(\lambda I - A)$ $\begin{bmatrix} \lambda & \emptyset \\ \emptyset & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$\begin{bmatrix} \lambda-1 & -1 \\ 1 & \lambda-1 \end{bmatrix}$ $\det(\lambda I - A) = (\lambda-1)(\lambda-1) - (1 \cdot -1) = \lambda^2 - \lambda - \lambda + 1 + 1$

$\lambda^2 - 2\lambda + 2$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2(1)} \longrightarrow \lambda = \frac{2 \pm \sqrt{4-8}}{2}$$

$$\lambda = \frac{2 \pm \sqrt{-4}}{2} \longrightarrow \lambda = \frac{2 \pm \sqrt{4}\,\sqrt{-1}^{\,i}}{2} \longrightarrow \lambda = \frac{2 \pm 2i}{2}$$

$$\lambda_1 = \frac{2+2i}{2} = 1 + i \qquad \lambda_2 = \frac{2-2i}{2} = 1 - i$$

$\lambda_1 = 1 + i$

$\begin{bmatrix} 1+i-1 & -1 \\ -1 & 1+i-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} ix_1 - x_2 = \emptyset \\ x_1 - ix_2 = \emptyset \end{matrix}$ $\quad ix_1 = x_2 \quad x_2 = t$

$ix_1 = t$

$x_1 = t/i$

$\rightarrow t \begin{pmatrix} t/i \\ t \end{pmatrix} \rightarrow t \begin{pmatrix} 1/i \\ 1 \end{pmatrix}$

$\lambda_2 = 1 - i$

$\begin{bmatrix} 1-i-1 & -1 \\ 1 & 1-i-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} -ix_1 - x_2 = \emptyset \\ x_1 - ix_2 = \emptyset \end{matrix}$ $\quad x_1 = ix_2 \quad x_1 = t$

$t = ix_2$

$\frac{t}{i} = x_2$

$\rightarrow t \begin{pmatrix} t \\ t/i \end{pmatrix} \rightarrow t \begin{pmatrix} 1 \\ 1/i \end{pmatrix}$

c) $\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ When the matrix is upper, lower or diagonal, the eigenvalues are the entries of the main diagonal.

$$\lambda_1 = 3 \quad \lambda_2 = 4 \quad \lambda_3 = 1$$

$$(\lambda I - A)(x) \quad \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda-3 & -5 & -3 \\ 0 & \lambda-4 & -6 \\ 0 & 0 & \lambda-1 \end{bmatrix}$$

$\lambda_1 = 3$

$$\begin{bmatrix} \cancel{3-3}^{\,0} & -5 & -3 \\ 0 & 3-4 & -6 \\ 0 & 0 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \quad \begin{aligned} -5x_2 - 3x_3 &= 0 \\ -x_2 - 6x_3 &= 0 \\ 2x_3 &= 0 \end{aligned}$$

$$\begin{aligned} x_3 &= 0 \\ -x_2 - 6(0) &= 0 \\ -x_2 &= 0 \\ x_2 &= 0 \end{aligned}$$

$x_1 = t$, doesn't has any asigned value, so it can be any number

$$\to t\begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda_2 = 4$

$$\begin{bmatrix} 4-3 & -5 & -3 \\ 0 & \cancel{4-4}^{\,0} & -6 \\ 0 & 0 & 4-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \quad \begin{aligned} x_1 - 5x_2 - 3x_3 &= 0 \\ -6x_3 &= 0 \\ 3x_3 &= 0 \end{aligned}$$

$$\begin{aligned} x_3 &= 0 \\ x_2 &= t \end{aligned}$$

$x_2 = t$, So it doesn't has any asigned value, x_2 can be any number.

$$\begin{aligned} x_1 - 5t - 0 &= 0 \\ x_1 - 5t &= 0 \\ x_1 &= 5t \end{aligned}$$

$$\to t\begin{pmatrix} 5t \\ t \\ 0 \end{pmatrix} \to t\begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$$

$\lambda_3 = 1$

$$\begin{bmatrix} 1-3 & -5 & -3 \\ \emptyset & 1-4 & -6 \\ \emptyset & \emptyset & \cancel{1-1}\ \emptyset \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} =$$

$$-2x_1 - 5x_2 - 3x_3 = \emptyset$$
$$-3x_2 - 6x_3 = \emptyset$$
$$\emptyset x_3 = \emptyset$$

$X_3 = \emptyset$

$X_3 = t$, It doesn't has any asigned value, X_3 can be any number

$$-3x_2 - 6t = \emptyset$$
$$-3x_2 = 6t$$
$$x_2 = \frac{6t}{-3}$$
$$x_2 = -2t$$

$$\to t\begin{pmatrix} 7/2\, t \\ -2t \\ t \end{pmatrix} \to t\begin{pmatrix} 7/2 \\ -2 \\ 1 \end{pmatrix}$$

$$-2x_1 - 5(-2t) - 3t = \emptyset$$
$$-2x_1 + 10t - 3t = \emptyset$$
$$-2x_1 + 7t = \emptyset$$
$$-2x_1 = 7t$$
$$x_1 = \frac{7t}{2}$$