* Poisson's and Laplace's Equations

$$\vec{\nabla}\cdot\vec{D}=\rho_v$$

$$\vec{D}=\varepsilon\vec{E}$$

$$\vec{E}=-\vec{\nabla}V$$

$$\vec{\nabla}\cdot\vec{D}=\vec{\nabla}\cdot(\varepsilon\vec{E})=-\vec{\nabla}\cdot(\varepsilon\vec{\nabla}V)=\rho_v$$

$\vec{\nabla}\cdot\vec{\nabla}V=-\frac{\rho_v}{\varepsilon}$ // for a homogeneous region in which ε is constant

↳ Poisson's equation

$$\vec{\nabla}\cdot\vec{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}V=\frac{\partial V}{\partial x}\vec{a_x}+\frac{\partial V}{\partial y}\vec{a_y}+\frac{\partial V}{\partial z}\vec{a_z}$$

$$\therefore\ \vec{\nabla}\cdot\vec{\nabla}V=\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\vec{\nabla}^2V=\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=-\frac{\rho_v}{\varepsilon}\ //$$

if $\rho_v=0$

$$\vec{\nabla}^2V=0\ //$$

↳ Laplace's equation

$$\vec{\nabla}^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \text{(cartesian)}$$

$$\vec{\nabla}^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \qquad \text{(cylindrical)}$$

$$\vec{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} \qquad \text{(spherical)}$$