ITESM Campus Monterrey
Mathematical Physical Modelling F4005
Cheat sheet for the first partial exam

Instructions: you are allowed to consult this sheet during the whole time duration of the first partial exam; you **cannot** add any annotations on it.

1. The rank of a matrix is the number of nonzero rows in its row echelon form (or row reduced echelon form).

2. **Theorem**. Let A be a square matrix of size n. A is invertible if and only if the reduced row echelon form of A is I_n, where I_n denotes the identity matrix of size n.

3. **Theorem**. Let $AX = B$ be a linear system, where A is a matrix of size $m \times n$ (not necessarily square).

   (a) If $\text{rank}(A) < \text{rank}([A|B])$, then the system has **no solution**.

   (b) If $\text{rank}(A) = \text{rank}([A|B])$ and **both** are less than n, then the system has **infinitely many solutions**.

   (c) If $\text{rank}(A) = \text{rank}([A|B]) = n$, then the system has a **unique solution**.

   (d) If the system is homogeneous and $n > m$ (more variables than equations), then the system has **infinitely many solutions**.

4. A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is called a **linear transformation** if

   (a) $T(u + v) = T(u) + T(v)$ for all $u, v \in \mathbb{R}^n$ and

   (b) $T(cv) = cT(v)$ for all $v \in \mathbb{R}^n$ and all scalars c.

5. If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then $T(0_n) = 0_m$ where 0_n is the zero vector corresponding to $\mathbb{R}^n$ and 0_m is the zero vector corresponding to $\mathbb{R}^m$.

6. **Theorem**. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y) = (ax + by, cx + dy)$ where a, b, c, d are arbitrary real numbers.

   (a) If $a = b = c = d = 0$, then the range of T is simply the origin in $\mathbb{R}^2$.

   (b) If $ad - bc \neq 0$, then the range of T is the whole plane $\mathbb{R}^2$.

   (c) If $ad - bc = 0$, and if at least one of the constants a, b, c, d is non-zero, then the range of T is a line through the origin (either a diagonal line, the y-axis or the x-axis).