# TTSfixed

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## 1 TTS Data of HDPE Resins at 5 temperatures

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[1]:
```
# PYTHON LIBRARIES
%matplotlib inline

import pandas as pd
import numpy as np
import datetime
import matplotlib as mpl
import matplotlib.dates as mdates
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings('ignore')
pd.options.display.max_columns = None

from adjustText import adjust_text
from sklearn.preprocessing import LabelEncoder
number = LabelEncoder()
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()
from matplotlib.axes._axes import _log as matplotlib_axes_logger
matplotlib_axes_logger.setLevel('ERROR')
from sklearn.linear_model import LinearRegression
```

## 2 Question/Problem to be solved:

### 2.1 Get the master curve at $210C$ and get the complex viscosity at $T = 180C$ and $T = 225C$

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## 3 List all the data provided:

### 3.1 I have the elastic ($G'(\omega)$) and the loss ($G''(\omega)$) moduli for a HDPE at five temperatures: $170$, $190$, $210$, $230$, $250C$.

[2]:
```python
# Let's read and look at the provided data

df = pd.read_csv("./data.csv", delimiter=",");

df = df.reindex(sorted(df.columns), axis=1);
print("./data.csv"); display(df);
```

./data.csv

| | G1_Sh170oC | G1_Sh190oC | G1_Sh210oC | G1_Sh230oC | G1_Sh250oC | G2_Sh170oC | \ |
|---|---|---|---|---|---|---|---|
| 0 | 343000.0 | 298000.0 | 288545.00 | 251301.00 | 218582.00 | 175000.0 | |
| 1 | 279000.0 | 241000.0 | 231412.00 | 199744.00 | 179203.00 | 155000.0 | |
| 2 | 226000.0 | 192000.0 | 182623.00 | 156375.00 | 146499.00 | 136000.0 | |
| 3 | 179000.0 | 151000.0 | 141545.00 | 120746.00 | 118516.00 | 118000.0 | |
| 4 | 139000.0 | 116000.0 | 108073.00 | 91516.80 | 94743.10 | 101000.0 | |
| 5 | 107000.0 | 87900.0 | 81183.40 | 68305.90 | 74972.20 | 83700.0 | |
| 6 | 80300.0 | 65500.0 | 60144.90 | 50295.80 | 58668.40 | 68100.0 | |
| 7 | 59600.0 | 48100.0 | 43975.30 | 36564.70 | 45439.30 | 54500.0 | |
| 8 | 43600.0 | 34800.0 | 31760.00 | 26301.90 | 34878.80 | 42700.0 | |
| 9 | 31500.0 | 24900.0 | 22724.80 | 18728.20 | 26547.30 | 33000.0 | |
| 10 | 22600.0 | 17600.0 | 16066.50 | 13228.90 | 20058.60 | 25000.0 | |
| 11 | 16000.0 | 12400.0 | 11355.50 | 9269.49 | 15067.60 | 18700.0 | |
| 12 | 11200.0 | 8590.0 | 7857.97 | 6441.07 | 11244.90 | 13900.0 | |
| 13 | 7840.0 | 5920.0 | 5467.97 | 4457.20 | 8336.15 | 10100.0 | |
| 14 | 5450.0 | 4060.0 | NaN | 3065.34 | 6159.25 | 7350.0 | |
| 15 | NaN | 2760.0 | NaN | 2134.04 | 4534.14 | NaN | |
| 16 | NaN | 1880.0 | NaN | 1377.14 | 3324.96 | NaN | |
| 17 | NaN | 1320.0 | NaN | NaN | 2418.17 | NaN | |
| 18 | NaN | 944.0 | NaN | NaN | 1766.42 | NaN | |

| | G2_Sh190oC | G2_Sh210oC | G2_Sh230oC | G2_Sh250oC | frecuencia_Sh170oC | \ |
|---|---|---|---|---|---|---|
| 0 | 163000.0 | 169359.00 | 150515.00 | 137616.00 | 500.000 | |
| 1 | 142000.0 | 146049.00 | 128738.00 | 120310.00 | 281.000 | |
| 2 | 123000.0 | 124870.00 | 109503.00 | 105154.00 | 158.000 | |
| 3 | 105000.0 | 105000.00 | 91613.60 | 91057.70 | 88.900 | |
| 4 | 88500.0 | 86653.80 | 75213.80 | 77870.90 | 50.000 | |
| 5 | 72700.0 | 70188.50 | 60615.80 | 65873.40 | 28.100 | |
| 6 | 58500.0 | 55798.10 | 47883.30 | 54882.00 | 15.800 | |
| 7 | 46200.0 | 43587.80 | 37176.90 | 45151.80 | 8.890 | |
| 8 | 35800.0 | 33575.10 | 28394.40 | 36729.00 | 5.000 | |
| 9 | 27400.0 | 25403.80 | 21383.50 | 29542.20 | 2.810 | |
| 10 | 20600.0 | 18996.10 | 15923.30 | 23515.30 | 1.580 | |
| 11 | 15300.0 | 14030.40 | 11700.70 | 18530.00 | 0.889 | |

```
12     11200.0    10253.40     8512.25    14497.10               0.500
13      8110.0     7445.87     6158.84    11208.90               0.281
14      5840.0         NaN     4399.15     8641.05               0.158
15      4160.0         NaN     3093.10     6595.03                 NaN
16      2950.0         NaN     2211.87     5019.15                 NaN
17      2100.0         NaN         NaN     3805.66                 NaN
18      1450.0         NaN         NaN     2858.24                 NaN

    frecuencia_Sh190oC  frecuencia_Sh210oC  frecuencia_Sh230oC  \
0             500.0000          500.000000          500.000000
1             281.0000          281.172000          281.172000
2             158.0000          158.117000          158.117000
3              88.9000           88.916000           88.916000
4              50.0000           50.002000           50.002000
5              28.1000           28.118700           28.118700
6              15.8000           15.812500           15.812500
7               8.8900            8.892090            8.892090
8               5.0000            5.000490            5.000490
9               2.8100            2.812010            2.812010
10              1.5800            1.581330            1.581330
11              0.8890            0.889252            0.889252
12              0.5000            0.500061            0.500061
13              0.2810            0.281212            0.281212
14              0.1580                 NaN            0.158138
15              0.0889                 NaN            0.088928
16              0.0500                 NaN            0.050009
17              0.0281                 NaN                 NaN
18              0.0158                 NaN                 NaN

    frecuencia_Sh250oC
0           500.000000
1           315.477000
2           199.051000
3           125.592000
4            79.242200
5            49.998000
6            31.546400
7            19.904300
8            12.558600
9             7.923950
10            4.999630
11            3.154540
12            1.990390
13            1.255830
14            0.792374
15            0.499954
16            0.315445
17            0.199032
```

```
18              0.125580
```

---

## 4 List of assumptions, justifying each of them:

All the specimens were run at the same % strain and using the same geometry and gap between the plates.

The polymer was protected with antioxidants, so decomposition is not present.

Assume that the lowest temparture is greater than Tg+100 so I can use the Arrhenius equation for the calculation of the shift factors

Linear polymers (HDPE, LLDPE, PP) and EVOH do not require a vertical shift, but long chain branched polymers do (LDPE, EVA)

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## 5 Algorithm for the solution

Decide the reference temperature

Do the calculations of tan del and $G^*$ where: $\tan\delta = \frac{G''(\omega)}{G'(\omega)}$ $G^*(\omega) = \left[G''(\omega)^2 + G'(\omega)^2\right]^{0.5}$

According to the abstract of the Mavridis paper

Get the $\tan\delta$ vs. frequency using the data at each $T$ to get $a_T$

Get the loss tangent vs. $G^*$ to get $b_T$

Plot $\tan\delta$ vs. $\omega$ for all temperatures in the same graph.

Multiply all the frequecies of a given temperature by a factor $a_T$ until it coincides with the reference plotted data.

Do that for each temperature

Plot $tan\delta$ vs. $G^*(\omega)$

If needed multiply $G^*(\omega)$ by a $b_T$ like in point 4

## 6 Solution

### 6.1

$$tan\delta = \frac{G''(\omega)}{G'(\omega)}$$

[3]:
```
def _tand(G1, G2):
    tand = G2 / G1;
    return tand;
```

```
# Iterate the data per sample
sample_id = '';
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Compute tan d
        df['tand_' + sample_id] = _tand(df['G1_' + sample_id], df['G2_' +␣
↪sample_id]);

df = df.reindex(sorted(df.columns), axis=1);
print("./data.csv"); display(df);
```

```
./data.csv

    G1_Sh170oC  G1_Sh190oC  G1_Sh210oC  G1_Sh230oC  G1_Sh250oC  G2_Sh170oC  \
0     343000.0    298000.0   288545.00   251301.00   218582.00    175000.0
1     279000.0    241000.0   231412.00   199744.00   179203.00    155000.0
2     226000.0    192000.0   182623.00   156375.00   146499.00    136000.0
3     179000.0    151000.0   141545.00   120746.00   118516.00    118000.0
4     139000.0    116000.0   108073.00    91516.80    94743.10    101000.0
5     107000.0     87900.0    81183.40    68305.90    74972.20     83700.0
6      80300.0     65500.0    60144.90    50295.80    58668.40     68100.0
7      59600.0     48100.0    43975.30    36564.70    45439.30     54500.0
8      43600.0     34800.0    31760.00    26301.90    34878.80     42700.0
9      31500.0     24900.0    22724.80    18728.20    26547.30     33000.0
10     22600.0     17600.0    16066.50    13228.90    20058.60     25000.0
11     16000.0     12400.0    11355.50     9269.49    15067.60     18700.0
12     11200.0      8590.0     7857.97     6441.07    11244.90     13900.0
13      7840.0      5920.0     5467.97     4457.20     8336.15     10100.0
14      5450.0      4060.0         NaN     3065.34     6159.25      7350.0
15         NaN      2760.0         NaN     2134.04     4534.14         NaN
16         NaN      1880.0         NaN     1377.14     3324.96         NaN
17         NaN      1320.0         NaN         NaN     2418.17         NaN
18         NaN       944.0         NaN         NaN     1766.42         NaN

    G2_Sh190oC  G2_Sh210oC  G2_Sh230oC  G2_Sh250oC  frecuencia_Sh170oC  \
0     163000.0   169359.00   150515.00   137616.00             500.000
1     142000.0   146049.00   128738.00   120310.00             281.000
2     123000.0   124870.00   109503.00   105154.00             158.000
3     105000.0   105000.00    91613.60    91057.70              88.900
4      88500.0    86653.80    75213.80    77870.90              50.000
5      72700.0    70188.50    60615.80    65873.40              28.100
6      58500.0    55798.10    47883.30    54882.00              15.800
7      46200.0    43587.80    37176.90    45151.80               8.890
8      35800.0    33575.10    28394.40    36729.00               5.000
9      27400.0    25403.80    21383.50    29542.20               2.810
10     20600.0    18996.10    15923.30    23515.30               1.580
```

| | | | | | |
|---|---|---|---|---|---|
| 11 | 15300.0 | 14030.40 | 11700.70 | 18530.00 | 0.889 |
| 12 | 11200.0 | 10253.40 | 8512.25 | 14497.10 | 0.500 |
| 13 | 8110.0 | 7445.87 | 6158.84 | 11208.90 | 0.281 |
| 14 | 5840.0 | NaN | 4399.15 | 8641.05 | 0.158 |
| 15 | 4160.0 | NaN | 3093.10 | 6595.03 | NaN |
| 16 | 2950.0 | NaN | 2211.87 | 5019.15 | NaN |
| 17 | 2100.0 | NaN | NaN | 3805.66 | NaN |
| 18 | 1450.0 | NaN | NaN | 2858.24 | NaN |

| | frecuencia_Sh190oC | frecuencia_Sh210oC | frecuencia_Sh230oC | \ |
|---|---|---|---|---|
| 0 | 500.0000 | 500.000000 | 500.000000 | |
| 1 | 281.0000 | 281.172000 | 281.172000 | |
| 2 | 158.0000 | 158.117000 | 158.117000 | |
| 3 | 88.9000 | 88.916000 | 88.916000 | |
| 4 | 50.0000 | 50.002000 | 50.002000 | |
| 5 | 28.1000 | 28.118700 | 28.118700 | |
| 6 | 15.8000 | 15.812500 | 15.812500 | |
| 7 | 8.8900 | 8.892090 | 8.892090 | |
| 8 | 5.0000 | 5.000490 | 5.000490 | |
| 9 | 2.8100 | 2.812010 | 2.812010 | |
| 10 | 1.5800 | 1.581330 | 1.581330 | |
| 11 | 0.8890 | 0.889252 | 0.889252 | |
| 12 | 0.5000 | 0.500061 | 0.500061 | |
| 13 | 0.2810 | 0.281212 | 0.281212 | |
| 14 | 0.1580 | NaN | 0.158138 | |
| 15 | 0.0889 | NaN | 0.088928 | |
| 16 | 0.0500 | NaN | 0.050009 | |
| 17 | 0.0281 | NaN | NaN | |
| 18 | 0.0158 | NaN | NaN | |

| | frecuencia_Sh250oC | tand_Sh170oC | tand_Sh190oC | tand_Sh210oC | \ |
|---|---|---|---|---|---|
| 0 | 500.000000 | 0.510204 | 0.546980 | 0.586941 | |
| 1 | 315.477000 | 0.555556 | 0.589212 | 0.631121 | |
| 2 | 199.051000 | 0.601770 | 0.640625 | 0.683758 | |
| 3 | 125.592000 | 0.659218 | 0.695364 | 0.741814 | |
| 4 | 79.242200 | 0.726619 | 0.762931 | 0.801808 | |
| 5 | 49.998000 | 0.782243 | 0.827076 | 0.864567 | |
| 6 | 31.546400 | 0.848070 | 0.893130 | 0.927728 | |
| 7 | 19.904300 | 0.914430 | 0.960499 | 0.991188 | |
| 8 | 12.558600 | 0.979358 | 1.028736 | 1.057151 | |
| 9 | 7.923950 | 1.047619 | 1.100402 | 1.117889 | |
| 10 | 4.999630 | 1.106195 | 1.170455 | 1.182342 | |
| 11 | 3.154540 | 1.168750 | 1.233871 | 1.235560 | |
| 12 | 1.990390 | 1.241071 | 1.303842 | 1.304841 | |
| 13 | 1.255830 | 1.288265 | 1.369932 | 1.361725 | |
| 14 | 0.792374 | 1.348624 | 1.438424 | NaN | |
| 15 | 0.499954 | NaN | 1.507246 | NaN | |
| 16 | 0.315445 | NaN | 1.569149 | NaN | |

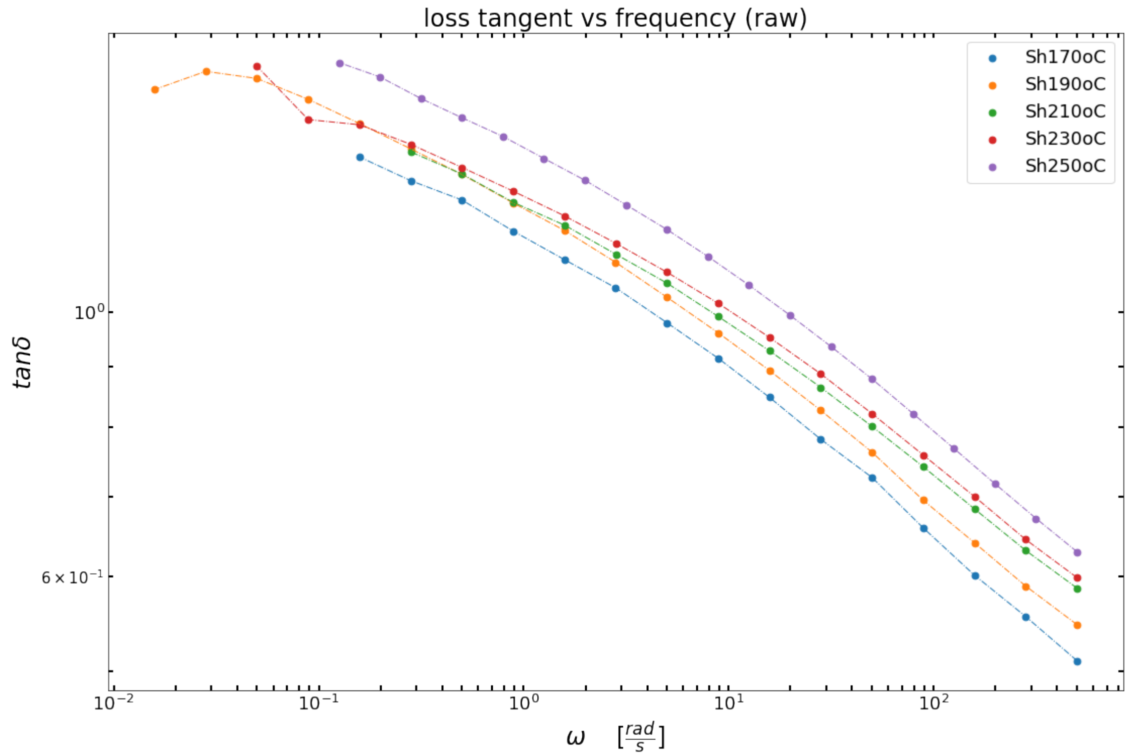
```
17             0.199032           NaN      1.590909            NaN
18             0.125580           NaN      1.536017            NaN

    tand_Sh230oC  tand_Sh250oC
0       0.598943      0.629585
1       0.644515      0.671362
2       0.700259      0.717780
3       0.758730      0.768316
4       0.821858      0.821916
5       0.887417      0.878638
6       0.952034      0.935461
7       1.016743      0.993673
8       1.079557      1.053047
9       1.141781      1.112814
10      1.203675      1.172330
11      1.262281      1.229791
12      1.321558      1.289216
13      1.381773      1.344614
14      1.435126      1.402939
15      1.449411      1.454527
16      1.606133      1.509537
17           NaN      1.573777
18           NaN      1.618098
```

---

### 6.2 Plot $tan\delta$ vs. frequency for each $T$

[4]:
```
# Set plot size and axis labels' font size
pltname = "loss tangent vs frequency (raw)";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();

# Iterate the data per sample
sample_id = '';
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Define x axis as the date axis
        x_str = 'frecuencia_' + sample_id; x_units = r'$[\frac{rad}{s}]$';
        y_str = 'tand_' + sample_id;       y_units = '';

        # Remove NANs from interesting x,y data
```

```
        df_fil = pd.DataFrame(df);
        df_fil = df_fil.dropna(subset=[x_str, y_str]);

        # Stablish the plot area
        ax0 = plt.gca();

        # Extract data from a specific country
        x  = df_fil.iloc[:][x_str];
        y = df_fil.iloc[:][y_str];

        # Scatter the data and plot a curve to join the points
        plt.scatter(x, y, s=45, marker='o', label=sample_id);
        plt.plot(x, y, linewidth=1, linestyle='-.');

# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));

# fig.autofmt_xdate();
ax0.set_xlabel(r'$\omega$' + '    ' + x_units, fontsize=24);
ax0.set_ylabel(r'$tan \delta$', fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,␣
 ↪top=True, left=True, right=True);

# Display main plot
plt.yscale('log');
plt.xscale('log');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```




---

### 6.3 Manually shift the curves to obtain the horizontal shift factor $a_T$

[5]:
```
# a_T was manually tuned until TTS
a_T = pd.Series([1.90, 1.20, 1.00, 0.85, 0.50]);

# Iterate the data per sample
sample_id = '';
sample_cnt = 0;
for i in range(len(df.columns)):
    if ("frecuencia_" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Compute tan d
        df['aT_frequency_' + sample_id] = df['frecuencia_' + sample_id] *␣
 ↪a_T[sample_cnt];

        sample_cnt = sample_cnt + 1;

df = df.reindex(sorted(df.columns), axis=1);
print("./data.csv"); display(df);
```

./data.csv

```
    G1_Sh170oC  G1_Sh190oC  G1_Sh210oC  G1_Sh230oC  G1_Sh250oC  G2_Sh170oC  \
0     343000.0    298000.0   288545.00   251301.00   218582.00    175000.0
1     279000.0    241000.0   231412.00   199744.00   179203.00    155000.0
2     226000.0    192000.0   182623.00   156375.00   146499.00    136000.0
3     179000.0    151000.0   141545.00   120746.00   118516.00    118000.0
4     139000.0    116000.0   108073.00    91516.80    94743.10    101000.0
5     107000.0     87900.0    81183.40    68305.90    74972.20     83700.0
6      80300.0     65500.0    60144.90    50295.80    58668.40     68100.0
7      59600.0     48100.0    43975.30    36564.70    45439.30     54500.0
8      43600.0     34800.0    31760.00    26301.90    34878.80     42700.0
9      31500.0     24900.0    22724.80    18728.20    26547.30     33000.0
10     22600.0     17600.0    16066.50    13228.90    20058.60     25000.0
11     16000.0     12400.0    11355.50     9269.49    15067.60     18700.0
12     11200.0      8590.0     7857.97     6441.07    11244.90     13900.0
13      7840.0      5920.0     5467.97     4457.20     8336.15     10100.0
14      5450.0      4060.0         NaN     3065.34     6159.25      7350.0
15         NaN      2760.0         NaN     2134.04     4534.14         NaN
16         NaN      1880.0         NaN     1377.14     3324.96         NaN
17         NaN      1320.0         NaN         NaN     2418.17         NaN
18         NaN       944.0         NaN         NaN     1766.42         NaN

    G2_Sh190oC  G2_Sh210oC  G2_Sh230oC  G2_Sh250oC  aT_frequency_Sh170oC  \
0     163000.0   169359.00   150515.00   137616.00              950.0000
1     142000.0   146049.00   128738.00   120310.00              533.9000
2     123000.0   124870.00   109503.00   105154.00              300.2000
3     105000.0   105000.00    91613.60    91057.70              168.9100
4      88500.0    86653.80    75213.80    77870.90               95.0000
5      72700.0    70188.50    60615.80    65873.40               53.3900
6      58500.0    55798.10    47883.30    54882.00               30.0200
7      46200.0    43587.80    37176.90    45151.80               16.8910
8      35800.0    33575.10    28394.40    36729.00                9.5000
9      27400.0    25403.80    21383.50    29542.20                5.3390
10     20600.0    18996.10    15923.30    23515.30                3.0020
11     15300.0    14030.40    11700.70    18530.00                1.6891
12     11200.0    10253.40     8512.25    14497.10                0.9500
13      8110.0     7445.87     6158.84    11208.90                0.5339
14      5840.0         NaN     4399.15     8641.05                0.3002
15      4160.0         NaN     3093.10     6595.03                   NaN
16      2950.0         NaN     2211.87     5019.15                   NaN
17      2100.0         NaN         NaN     3805.66                   NaN
18      1450.0         NaN         NaN     2858.24                   NaN

    aT_frequency_Sh190oC  aT_frequency_Sh210oC  aT_frequency_Sh230oC  \
0              600.00000            500.000000            425.000000
1              337.20000            281.172000            238.996200
2              189.60000            158.117000            134.399450
```

| | | | |
|---|---|---|---|
| 3 | 106.68000 | 88.916000 | 75.578600 |
| 4 | 60.00000 | 50.002000 | 42.501700 |
| 5 | 33.72000 | 28.118700 | 23.900895 |
| 6 | 18.96000 | 15.812500 | 13.440625 |
| 7 | 10.66800 | 8.892090 | 7.558276 |
| 8 | 6.00000 | 5.000490 | 4.250417 |
| 9 | 3.37200 | 2.812010 | 2.390208 |
| 10 | 1.89600 | 1.581330 | 1.344130 |
| 11 | 1.06680 | 0.889252 | 0.755864 |
| 12 | 0.60000 | 0.500061 | 0.425052 |
| 13 | 0.33720 | 0.281212 | 0.239030 |
| 14 | 0.18960 | NaN | 0.134417 |
| 15 | 0.10668 | NaN | 0.075589 |
| 16 | 0.06000 | NaN | 0.042507 |
| 17 | 0.03372 | NaN | NaN |
| 18 | 0.01896 | NaN | NaN |

| | aT_frequency_Sh250oC | frecuencia_Sh170oC | frecuencia_Sh190oC | \ |
|---|---|---|---|---|
| 0 | 250.000000 | 500.000 | 500.0000 | |
| 1 | 157.738500 | 281.000 | 281.0000 | |
| 2 | 99.525500 | 158.000 | 158.0000 | |
| 3 | 62.796000 | 88.900 | 88.9000 | |
| 4 | 39.621100 | 50.000 | 50.0000 | |
| 5 | 24.999000 | 28.100 | 28.1000 | |
| 6 | 15.773200 | 15.800 | 15.8000 | |
| 7 | 9.952150 | 8.890 | 8.8900 | |
| 8 | 6.279300 | 5.000 | 5.0000 | |
| 9 | 3.961975 | 2.810 | 2.8100 | |
| 10 | 2.499815 | 1.580 | 1.5800 | |
| 11 | 1.577270 | 0.889 | 0.8890 | |
| 12 | 0.995195 | 0.500 | 0.5000 | |
| 13 | 0.627915 | 0.281 | 0.2810 | |
| 14 | 0.396187 | 0.158 | 0.1580 | |
| 15 | 0.249977 | NaN | 0.0889 | |
| 16 | 0.157722 | NaN | 0.0500 | |
| 17 | 0.099516 | NaN | 0.0281 | |
| 18 | 0.062790 | NaN | 0.0158 | |

| | frecuencia_Sh210oC | frecuencia_Sh230oC | frecuencia_Sh250oC | tand_Sh170oC | \ |
|---|---|---|---|---|---|
| 0 | 500.000000 | 500.000000 | 500.000000 | 0.510204 | |
| 1 | 281.172000 | 281.172000 | 315.477000 | 0.555556 | |
| 2 | 158.117000 | 158.117000 | 199.051000 | 0.601770 | |
| 3 | 88.916000 | 88.916000 | 125.592000 | 0.659218 | |
| 4 | 50.002000 | 50.002000 | 79.242200 | 0.726619 | |
| 5 | 28.118700 | 28.118700 | 49.998000 | 0.782243 | |
| 6 | 15.812500 | 15.812500 | 31.546400 | 0.848070 | |
| 7 | 8.892090 | 8.892090 | 19.904300 | 0.914430 | |
| 8 | 5.000490 | 5.000490 | 12.558600 | 0.979358 | |

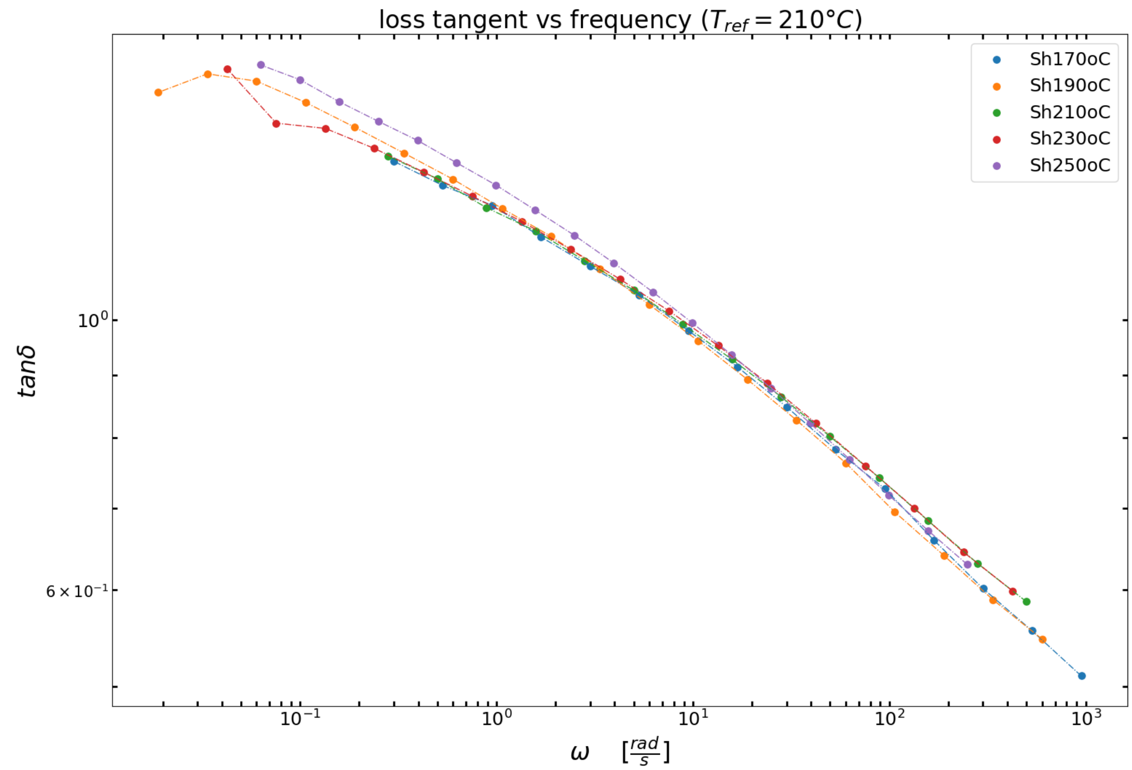
```
9          2.812010         2.812010         7.923950      1.047619
10         1.581330         1.581330         4.999630      1.106195
11         0.889252         0.889252         3.154540      1.168750
12         0.500061         0.500061         1.990390      1.241071
13         0.281212         0.281212         1.255830      1.288265
14              NaN         0.158138         0.792374      1.348624
15              NaN         0.088928         0.499954           NaN
16              NaN         0.050009         0.315445           NaN
17              NaN              NaN         0.199032           NaN
18              NaN              NaN         0.125580           NaN

    tand_Sh190oC  tand_Sh210oC  tand_Sh230oC  tand_Sh250oC
0       0.546980      0.586941      0.598943      0.629585
1       0.589212      0.631121      0.644515      0.671362
2       0.640625      0.683758      0.700259      0.717780
3       0.695364      0.741814      0.758730      0.768316
4       0.762931      0.801808      0.821858      0.821916
5       0.827076      0.864567      0.887417      0.878638
6       0.893130      0.927728      0.952034      0.935461
7       0.960499      0.991188      1.016743      0.993673
8       1.028736      1.057151      1.079557      1.053047
9       1.100402      1.117889      1.141781      1.112814
10      1.170455      1.182342      1.203675      1.172330
11      1.233871      1.235560      1.262281      1.229791
12      1.303842      1.304841      1.321558      1.289216
13      1.369932      1.361725      1.381773      1.344614
14      1.438424           NaN      1.435126      1.402939
15      1.507246           NaN      1.449411      1.454527
16      1.569149           NaN      1.606133      1.509537
17      1.590909           NaN           NaN      1.573777
18      1.536017           NaN           NaN      1.618098
```

---

### 6.4 Plot $\tan\delta$ vs. frequency for each $T$

[6]:
```python
# Set plot size and axis labels' font size
pltname = "loss tangent vs frequency (" + r'$T_{ref} = 210°C$' + ")";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();

# Iterate the data per sample
sample_id = '';
for i in range(len(df.columns)):
```

```python
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Define x axis as the date axis
        x_str = 'aT_frequency_' + sample_id; x_units = r'$[\frac{rad}{s}]$';
        y_str = 'tand_' + sample_id;       y_units = '';

        # Remove NANs from interesting x,y data
        df_fil = pd.DataFrame(df);
        df_fil = df_fil.dropna(subset=[x_str, y_str]);

        # Stablish the plot area
        ax0 = plt.gca();

        # Extract data from a specific country
        x  = df_fil.iloc[:][x_str];
        y = df_fil.iloc[:][y_str];

        # Scatter the data and plot a curve to join the points
        plt.scatter(x, y, s=45, marker='o', label=sample_id);
        plt.plot(x, y, linewidth=1, linestyle='-.');

# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));

# fig.autofmt_xdate();
ax0.set_xlabel(r'$\omega$' + '     ' + x_units, fontsize=24);
ax0.set_ylabel(r'$tan \delta$', fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,␣
 ↪top=True, left=True, right=True);

# Display main plot
plt.yscale('log');
plt.xscale('log');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```




---

### 6.5

$$G^*(\omega) = \left[G''(\omega)^2 + G'(\omega)^2\right]^{0.5}$$

[7]:
```
def _Gc(G1, G2):
    Gc = np.sqrt(G2**2 + G1**2);
    return Gc;

# Iterate the data per sample
sample_id = '';
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Compute Complex Viscosity
        df['Gc_' + sample_id] = _Gc(df['G1_' + sample_id], df['G2_' +␣
↪sample_id]);

df = df.reindex(sorted(df.columns), axis=1);
print("./data.csv"); display(df);
```

./data.csv

| | G1_Sh170oC | G1_Sh190oC | G1_Sh210oC | G1_Sh230oC | G1_Sh250oC | G2_Sh170oC | \ |
|---|---|---|---|---|---|---|---|
| 0 | 343000.0 | 298000.0 | 288545.00 | 251301.00 | 218582.00 | 175000.0 | |
| 1 | 279000.0 | 241000.0 | 231412.00 | 199744.00 | 179203.00 | 155000.0 | |
| 2 | 226000.0 | 192000.0 | 182623.00 | 156375.00 | 146499.00 | 136000.0 | |
| 3 | 179000.0 | 151000.0 | 141545.00 | 120746.00 | 118516.00 | 118000.0 | |
| 4 | 139000.0 | 116000.0 | 108073.00 | 91516.80 | 94743.10 | 101000.0 | |
| 5 | 107000.0 | 87900.0 | 81183.40 | 68305.90 | 74972.20 | 83700.0 | |
| 6 | 80300.0 | 65500.0 | 60144.90 | 50295.80 | 58668.40 | 68100.0 | |
| 7 | 59600.0 | 48100.0 | 43975.30 | 36564.70 | 45439.30 | 54500.0 | |
| 8 | 43600.0 | 34800.0 | 31760.00 | 26301.90 | 34878.80 | 42700.0 | |
| 9 | 31500.0 | 24900.0 | 22724.80 | 18728.20 | 26547.30 | 33000.0 | |
| 10 | 22600.0 | 17600.0 | 16066.50 | 13228.90 | 20058.60 | 25000.0 | |
| 11 | 16000.0 | 12400.0 | 11355.50 | 9269.49 | 15067.60 | 18700.0 | |
| 12 | 11200.0 | 8590.0 | 7857.97 | 6441.07 | 11244.90 | 13900.0 | |
| 13 | 7840.0 | 5920.0 | 5467.97 | 4457.20 | 8336.15 | 10100.0 | |
| 14 | 5450.0 | 4060.0 | NaN | 3065.34 | 6159.25 | 7350.0 | |
| 15 | NaN | 2760.0 | NaN | 2134.04 | 4534.14 | NaN | |
| 16 | NaN | 1880.0 | NaN | 1377.14 | 3324.96 | NaN | |
| 17 | NaN | 1320.0 | NaN | NaN | 2418.17 | NaN | |
| 18 | NaN | 944.0 | NaN | NaN | 1766.42 | NaN | |

| | G2_Sh190oC | G2_Sh210oC | G2_Sh230oC | G2_Sh250oC | Gc_Sh170oC | \ |
|---|---|---|---|---|---|---|
| 0 | 163000.0 | 169359.00 | 150515.00 | 137616.00 | 385063.631105 | |
| 1 | 142000.0 | 146049.00 | 128738.00 | 120310.00 | 319164.534371 | |
| 2 | 123000.0 | 124870.00 | 109503.00 | 105154.00 | 263765.046964 | |
| 3 | 105000.0 | 105000.00 | 91613.60 | 91057.70 | 214394.496198 | |
| 4 | 88500.0 | 86653.80 | 75213.80 | 77870.90 | 171819.672913 | |
| 5 | 72700.0 | 70188.50 | 60615.80 | 65873.40 | 135848.040104 | |
| 6 | 58500.0 | 55798.10 | 47883.30 | 54882.00 | 105288.650860 | |
| 7 | 46200.0 | 43587.80 | 37176.90 | 45151.80 | 80761.438818 | |
| 8 | 35800.0 | 33575.10 | 28394.40 | 36729.00 | 61026.633530 | |
| 9 | 27400.0 | 25403.80 | 21383.50 | 29542.20 | 45620.718977 | |
| 10 | 20600.0 | 18996.10 | 15923.30 | 23515.30 | 33701.038560 | |
| 11 | 15300.0 | 14030.40 | 11700.70 | 18530.00 | 24610.770000 | |
| 12 | 11200.0 | 10253.40 | 8512.25 | 14497.10 | 17850.770292 | |
| 13 | 8110.0 | 7445.87 | 6158.84 | 11208.90 | 12785.757701 | |
| 14 | 5840.0 | NaN | 4399.15 | 8641.05 | 9150.136611 | |
| 15 | 4160.0 | NaN | 3093.10 | 6595.03 | NaN | |
| 16 | 2950.0 | NaN | 2211.87 | 5019.15 | NaN | |
| 17 | 2100.0 | NaN | NaN | 3805.66 | NaN | |
| 18 | 1450.0 | NaN | NaN | 2858.24 | NaN | |

| | Gc_Sh190oC | Gc_Sh210oC | Gc_Sh230oC | Gc_Sh250oC | \ |
|---|---|---|---|---|---|
| 0 | 339666.012430 | 334575.384489 | 292928.246890 | 258294.897704 | |
| 1 | 279723.077346 | 273645.435089 | 237636.567430 | 215843.024694 | |
| 2 | 228019.735988 | 221232.179009 | 190903.241549 | 180331.141839 | |

| | | | | |
|---|---|---|---|---|
| 3 | 183918.460194 | 176238.437990 | 151567.305910 | 149457.508963 |
| 4 | 145904.934803 | 138523.118588 | 118458.602021 | 122638.216166 |
| 5 | 114068.838865 | 107318.078476 | 91323.442688 | 99800.478959 |
| 6 | 87820.840351 | 82041.678186 | 69444.063220 | 80336.884944 |
| 7 | 66693.702851 | 61917.068074 | 52144.982306 | 64057.903710 |
| 8 | 49926.746339 | 46216.717106 | 38704.416996 | 50651.259910 |
| 9 | 37023.911193 | 34084.741300 | 28425.332847 | 39717.763257 |
| 10 | 27094.648918 | 24879.393832 | 20701.576706 | 30908.199042 |
| 11 | 19693.907687 | 18049.917019 | 14927.485567 | 23882.911668 |
| 12 | 14114.818454 | 12918.200497 | 10674.538997 | 18347.034758 |
| 13 | 10040.841598 | 9237.947606 | 7602.495773 | 13968.923940 |
| 14 | 7112.608523 | NaN | 5361.793547 | 10611.508171 |
| 15 | 4992.314093 | NaN | 3757.844373 | 8003.302208 |
| 16 | 3498.128071 | NaN | 2605.548594 | 6020.566894 |
| 17 | 2480.403193 | NaN | NaN | 4508.946017 |
| 18 | 1730.212704 | NaN | NaN | 3360.026118 |

| | aT_frequency_Sh170oC | aT_frequency_Sh190oC | aT_frequency_Sh210oC | \ |
|---|---|---|---|---|
| 0 | 950.0000 | 600.00000 | 500.000000 | |
| 1 | 533.9000 | 337.20000 | 281.172000 | |
| 2 | 300.2000 | 189.60000 | 158.117000 | |
| 3 | 168.9100 | 106.68000 | 88.916000 | |
| 4 | 95.0000 | 60.00000 | 50.002000 | |
| 5 | 53.3900 | 33.72000 | 28.118700 | |
| 6 | 30.0200 | 18.96000 | 15.812500 | |
| 7 | 16.8910 | 10.66800 | 8.892090 | |
| 8 | 9.5000 | 6.00000 | 5.000490 | |
| 9 | 5.3390 | 3.37200 | 2.812010 | |
| 10 | 3.0020 | 1.89600 | 1.581330 | |
| 11 | 1.6891 | 1.06680 | 0.889252 | |
| 12 | 0.9500 | 0.60000 | 0.500061 | |
| 13 | 0.5339 | 0.33720 | 0.281212 | |
| 14 | 0.3002 | 0.18960 | NaN | |
| 15 | NaN | 0.10668 | NaN | |
| 16 | NaN | 0.06000 | NaN | |
| 17 | NaN | 0.03372 | NaN | |
| 18 | NaN | 0.01896 | NaN | |

| | aT_frequency_Sh230oC | aT_frequency_Sh250oC | frecuencia_Sh170oC | \ |
|---|---|---|---|---|
| 0 | 425.000000 | 250.000000 | 500.000 | |
| 1 | 238.996200 | 157.738500 | 281.000 | |
| 2 | 134.399450 | 99.525500 | 158.000 | |
| 3 | 75.578600 | 62.796000 | 88.900 | |
| 4 | 42.501700 | 39.621100 | 50.000 | |
| 5 | 23.900895 | 24.999000 | 28.100 | |
| 6 | 13.440625 | 15.773200 | 15.800 | |
| 7 | 7.558276 | 9.952150 | 8.890 | |
| 8 | 4.250417 | 6.279300 | 5.000 | |

| | | | |
|---|---|---|---|
| 9 | 2.390208 | 3.961975 | 2.810 |
| 10 | 1.344130 | 2.499815 | 1.580 |
| 11 | 0.755864 | 1.577270 | 0.889 |
| 12 | 0.425052 | 0.995195 | 0.500 |
| 13 | 0.239030 | 0.627915 | 0.281 |
| 14 | 0.134417 | 0.396187 | 0.158 |
| 15 | 0.075589 | 0.249977 | NaN |
| 16 | 0.042507 | 0.157722 | NaN |
| 17 | NaN | 0.099516 | NaN |
| 18 | NaN | 0.062790 | NaN |

| | frecuencia_Sh190oC | frecuencia_Sh210oC | frecuencia_Sh230oC | \ |
|---|---|---|---|---|
| 0 | 500.0000 | 500.000000 | 500.000000 | |
| 1 | 281.0000 | 281.172000 | 281.172000 | |
| 2 | 158.0000 | 158.117000 | 158.117000 | |
| 3 | 88.9000 | 88.916000 | 88.916000 | |
| 4 | 50.0000 | 50.002000 | 50.002000 | |
| 5 | 28.1000 | 28.118700 | 28.118700 | |
| 6 | 15.8000 | 15.812500 | 15.812500 | |
| 7 | 8.8900 | 8.892090 | 8.892090 | |
| 8 | 5.0000 | 5.000490 | 5.000490 | |
| 9 | 2.8100 | 2.812010 | 2.812010 | |
| 10 | 1.5800 | 1.581330 | 1.581330 | |
| 11 | 0.8890 | 0.889252 | 0.889252 | |
| 12 | 0.5000 | 0.500061 | 0.500061 | |
| 13 | 0.2810 | 0.281212 | 0.281212 | |
| 14 | 0.1580 | NaN | 0.158138 | |
| 15 | 0.0889 | NaN | 0.088928 | |
| 16 | 0.0500 | NaN | 0.050009 | |
| 17 | 0.0281 | NaN | NaN | |
| 18 | 0.0158 | NaN | NaN | |

| | frecuencia_Sh250oC | tand_Sh170oC | tand_Sh190oC | tand_Sh210oC | \ |
|---|---|---|---|---|---|
| 0 | 500.000000 | 0.510204 | 0.546980 | 0.586941 | |
| 1 | 315.477000 | 0.555556 | 0.589212 | 0.631121 | |
| 2 | 199.051000 | 0.601770 | 0.640625 | 0.683758 | |
| 3 | 125.592000 | 0.659218 | 0.695364 | 0.741814 | |
| 4 | 79.242200 | 0.726619 | 0.762931 | 0.801808 | |
| 5 | 49.998000 | 0.782243 | 0.827076 | 0.864567 | |
| 6 | 31.546400 | 0.848070 | 0.893130 | 0.927728 | |
| 7 | 19.904300 | 0.914430 | 0.960499 | 0.991188 | |
| 8 | 12.558600 | 0.979358 | 1.028736 | 1.057151 | |
| 9 | 7.923950 | 1.047619 | 1.100402 | 1.117889 | |
| 10 | 4.999630 | 1.106195 | 1.170455 | 1.182342 | |
| 11 | 3.154540 | 1.168750 | 1.233871 | 1.235560 | |
| 12 | 1.990390 | 1.241071 | 1.303842 | 1.304841 | |
| 13 | 1.255830 | 1.288265 | 1.369932 | 1.361725 | |
| 14 | 0.792374 | 1.348624 | 1.438424 | NaN | |

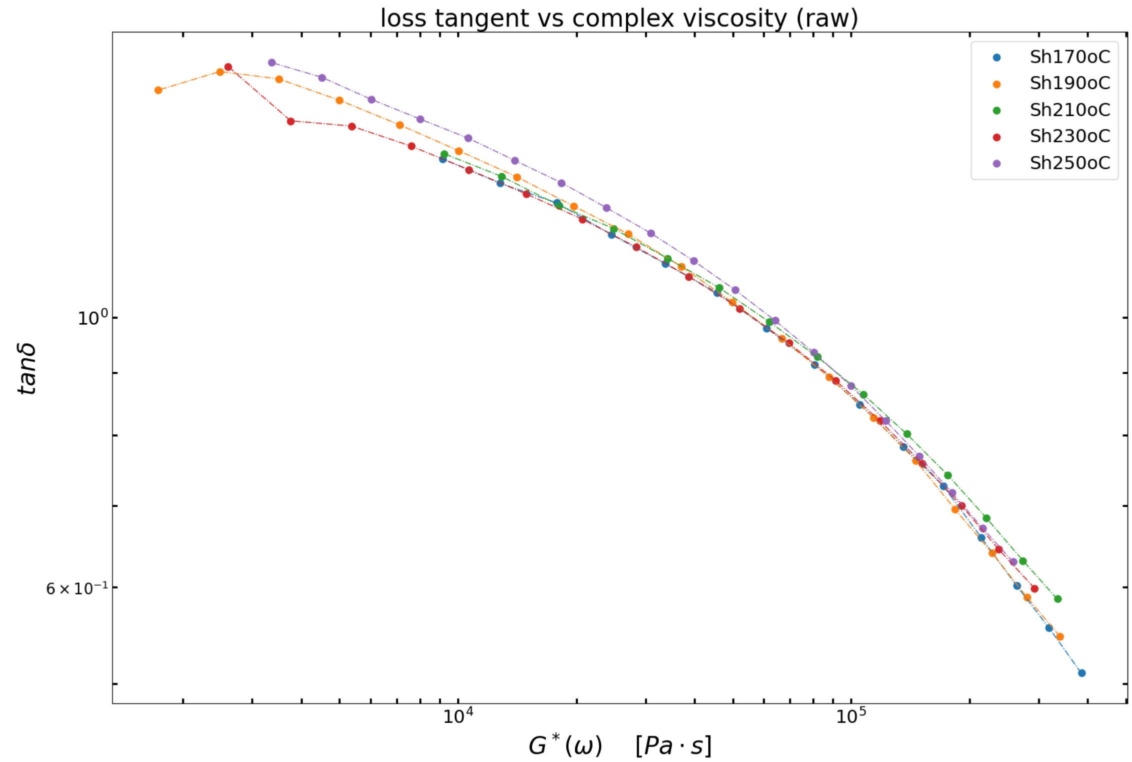
```
15          0.499954          NaN      1.507246            NaN
16          0.315445          NaN      1.569149            NaN
17          0.199032          NaN      1.590909            NaN
18          0.125580          NaN      1.536017            NaN

    tand_Sh230oC  tand_Sh250oC
0       0.598943      0.629585
1       0.644515      0.671362
2       0.700259      0.717780
3       0.758730      0.768316
4       0.821858      0.821916
5       0.887417      0.878638
6       0.952034      0.935461
7       1.016743      0.993673
8       1.079557      1.053047
9       1.141781      1.112814
10      1.203675      1.172330
11      1.262281      1.229791
12      1.321558      1.289216
13      1.381773      1.344614
14      1.435126      1.402939
15      1.449411      1.454527
16      1.606133      1.509537
17           NaN      1.573777
18           NaN      1.618098
```

---

### 6.6 Plot $\tan\delta$ vs. $G^*(\omega)$ for each $T$

[8]:
```
# Set plot size and axis labels' font size
pltname = "loss tangent vs complex viscosity (raw)";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();

# Iterate the data per sample
sample_id = '';
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Define x axis as the date axis
        x_str = 'Gc_' + sample_id; x_units = r'$[Pa \cdot s]$';
        y_str = 'tand_' + sample_id;          y_units = '';
```

```
        # Remove NANs from interesting x,y data
        df_fil = pd.DataFrame(df);
        df_fil = df_fil.dropna(subset=[x_str, y_str]);

        # Stablish the plot area
        ax0 = plt.gca();

        # Extract data from a specific country
        x  = df_fil.iloc[:][x_str];
        y = df_fil.iloc[:][y_str];

        # Scatter the data and plot a curve to join the points
        plt.scatter(x, y, s=45, marker='o', label=sample_id);
        plt.plot(x, y, linewidth=1, linestyle='-.');

# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));

# fig.autofmt_xdate();
ax0.set_xlabel(r'$G^*(\omega)$' + '    ' + x_units, fontsize=24);
ax0.set_ylabel(r'$tan \delta$', fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,␣
 ↪top=True, left=True, right=True);

# Display main plot
plt.yscale('log');
plt.xscale('log');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```




---

### 6.7 Manually shift the curves to obtain the vertical shift factor $b_T$

[9]:
```
# a_T was manually tuned until TTS
b_T = pd.Series([1.1, 1.05, 1, 1.1, 1.08]);

# Iterate the data per sample
sample_id = '';
sample_cnt = 0;
for i in range(len(df.columns)):
    if ("frecuencia_" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Compute tan d
        df['bT_Gc_' + sample_id] = df['Gc_' + sample_id] * b_T[sample_cnt];

        sample_cnt = sample_cnt + 1;

df = df.reindex(sorted(df.columns), axis=1);
print("./data.csv"); display(df);
```

```
./data.csv
```

| | G1_Sh170oC | G1_Sh190oC | G1_Sh210oC | G1_Sh230oC | G1_Sh250oC | G2_Sh170oC | \ |
|---|---|---|---|---|---|---|---|
| 0 | 343000.0 | 298000.0 | 288545.00 | 251301.00 | 218582.00 | 175000.0 | |
| 1 | 279000.0 | 241000.0 | 231412.00 | 199744.00 | 179203.00 | 155000.0 | |
| 2 | 226000.0 | 192000.0 | 182623.00 | 156375.00 | 146499.00 | 136000.0 | |
| 3 | 179000.0 | 151000.0 | 141545.00 | 120746.00 | 118516.00 | 118000.0 | |
| 4 | 139000.0 | 116000.0 | 108073.00 | 91516.80 | 94743.10 | 101000.0 | |
| 5 | 107000.0 | 87900.0 | 81183.40 | 68305.90 | 74972.20 | 83700.0 | |
| 6 | 80300.0 | 65500.0 | 60144.90 | 50295.80 | 58668.40 | 68100.0 | |
| 7 | 59600.0 | 48100.0 | 43975.30 | 36564.70 | 45439.30 | 54500.0 | |
| 8 | 43600.0 | 34800.0 | 31760.00 | 26301.90 | 34878.80 | 42700.0 | |
| 9 | 31500.0 | 24900.0 | 22724.80 | 18728.20 | 26547.30 | 33000.0 | |
| 10 | 22600.0 | 17600.0 | 16066.50 | 13228.90 | 20058.60 | 25000.0 | |
| 11 | 16000.0 | 12400.0 | 11355.50 | 9269.49 | 15067.60 | 18700.0 | |
| 12 | 11200.0 | 8590.0 | 7857.97 | 6441.07 | 11244.90 | 13900.0 | |
| 13 | 7840.0 | 5920.0 | 5467.97 | 4457.20 | 8336.15 | 10100.0 | |
| 14 | 5450.0 | 4060.0 | NaN | 3065.34 | 6159.25 | 7350.0 | |
| 15 | NaN | 2760.0 | NaN | 2134.04 | 4534.14 | NaN | |
| 16 | NaN | 1880.0 | NaN | 1377.14 | 3324.96 | NaN | |
| 17 | NaN | 1320.0 | NaN | NaN | 2418.17 | NaN | |
| 18 | NaN | 944.0 | NaN | NaN | 1766.42 | NaN | |

| | G2_Sh190oC | G2_Sh210oC | G2_Sh230oC | G2_Sh250oC | Gc_Sh170oC | \ |
|---|---|---|---|---|---|---|
| 0 | 163000.0 | 169359.00 | 150515.00 | 137616.00 | 385063.631105 | |
| 1 | 142000.0 | 146049.00 | 128738.00 | 120310.00 | 319164.534371 | |
| 2 | 123000.0 | 124870.00 | 109503.00 | 105154.00 | 263765.046964 | |
| 3 | 105000.0 | 105000.00 | 91613.60 | 91057.70 | 214394.496198 | |
| 4 | 88500.0 | 86653.80 | 75213.80 | 77870.90 | 171819.672913 | |
| 5 | 72700.0 | 70188.50 | 60615.80 | 65873.40 | 135848.040104 | |
| 6 | 58500.0 | 55798.10 | 47883.30 | 54882.00 | 105288.650860 | |
| 7 | 46200.0 | 43587.80 | 37176.90 | 45151.80 | 80761.438818 | |
| 8 | 35800.0 | 33575.10 | 28394.40 | 36729.00 | 61026.633530 | |
| 9 | 27400.0 | 25403.80 | 21383.50 | 29542.20 | 45620.718977 | |
| 10 | 20600.0 | 18996.10 | 15923.30 | 23515.30 | 33701.038560 | |
| 11 | 15300.0 | 14030.40 | 11700.70 | 18530.00 | 24610.770000 | |
| 12 | 11200.0 | 10253.40 | 8512.25 | 14497.10 | 17850.770292 | |
| 13 | 8110.0 | 7445.87 | 6158.84 | 11208.90 | 12785.757701 | |
| 14 | 5840.0 | NaN | 4399.15 | 8641.05 | 9150.136611 | |
| 15 | 4160.0 | NaN | 3093.10 | 6595.03 | NaN | |
| 16 | 2950.0 | NaN | 2211.87 | 5019.15 | NaN | |
| 17 | 2100.0 | NaN | NaN | 3805.66 | NaN | |
| 18 | 1450.0 | NaN | NaN | 2858.24 | NaN | |

| | Gc_Sh190oC | Gc_Sh210oC | Gc_Sh230oC | Gc_Sh250oC | \ |
|---|---|---|---|---|---|
| 0 | 339666.012430 | 334575.384489 | 292928.246890 | 258294.897704 | |
| 1 | 279723.077346 | 273645.435089 | 237636.567430 | 215843.024694 | |
| 2 | 228019.735988 | 221232.179009 | 190903.241549 | 180331.141839 | |
| 3 | 183918.460194 | 176238.437990 | 151567.305910 | 149457.508963 | |
| 4 | 145904.934803 | 138523.118588 | 118458.602021 | 122638.216166 | |

| | | | | |
|---|---|---|---|---|
| 5 | 114068.838865 | 107318.078476 | 91323.442688 | 99800.478959 |
| 6 | 87820.840351 | 82041.678186 | 69444.063220 | 80336.884944 |
| 7 | 66693.702851 | 61917.068074 | 52144.982306 | 64057.903710 |
| 8 | 49926.746339 | 46216.717106 | 38704.416996 | 50651.259910 |
| 9 | 37023.911193 | 34084.741300 | 28425.332847 | 39717.763257 |
| 10 | 27094.648918 | 24879.393832 | 20701.576706 | 30908.199042 |
| 11 | 19693.907687 | 18049.917019 | 14927.485567 | 23882.911668 |
| 12 | 14114.818454 | 12918.200497 | 10674.538997 | 18347.034758 |
| 13 | 10040.841598 | 9237.947606 | 7602.495773 | 13968.923940 |
| 14 | 7112.608523 | NaN | 5361.793547 | 10611.508171 |
| 15 | 4992.314093 | NaN | 3757.844373 | 8003.302208 |
| 16 | 3498.128071 | NaN | 2605.548594 | 6020.566894 |
| 17 | 2480.403193 | NaN | NaN | 4508.946017 |
| 18 | 1730.212704 | NaN | NaN | 3360.026118 |

| | aT_frequency_Sh170oC | aT_frequency_Sh190oC | aT_frequency_Sh210oC | \ |
|---|---|---|---|---|
| 0 | 950.0000 | 600.00000 | 500.000000 | |
| 1 | 533.9000 | 337.20000 | 281.172000 | |
| 2 | 300.2000 | 189.60000 | 158.117000 | |
| 3 | 168.9100 | 106.68000 | 88.916000 | |
| 4 | 95.0000 | 60.00000 | 50.002000 | |
| 5 | 53.3900 | 33.72000 | 28.118700 | |
| 6 | 30.0200 | 18.96000 | 15.812500 | |
| 7 | 16.8910 | 10.66800 | 8.892090 | |
| 8 | 9.5000 | 6.00000 | 5.000490 | |
| 9 | 5.3390 | 3.37200 | 2.812010 | |
| 10 | 3.0020 | 1.89600 | 1.581330 | |
| 11 | 1.6891 | 1.06680 | 0.889252 | |
| 12 | 0.9500 | 0.60000 | 0.500061 | |
| 13 | 0.5339 | 0.33720 | 0.281212 | |
| 14 | 0.3002 | 0.18960 | NaN | |
| 15 | NaN | 0.10668 | NaN | |
| 16 | NaN | 0.06000 | NaN | |
| 17 | NaN | 0.03372 | NaN | |
| 18 | NaN | 0.01896 | NaN | |

| | aT_frequency_Sh230oC | aT_frequency_Sh250oC | bT_Gc_Sh170oC | bT_Gc_Sh190oC | \ |
|---|---|---|---|---|---|
| 0 | 425.000000 | 250.000000 | 423569.994216 | 356649.313051 | |
| 1 | 238.996200 | 157.738500 | 351080.987808 | 293709.231213 | |
| 2 | 134.399450 | 99.525500 | 290141.551661 | 239420.722787 | |
| 3 | 75.578600 | 62.796000 | 235833.945818 | 193114.383203 | |
| 4 | 42.501700 | 39.621100 | 189001.640205 | 153200.181544 | |
| 5 | 23.900895 | 24.999000 | 149432.844114 | 119772.280808 | |
| 6 | 13.440625 | 15.773200 | 115817.515946 | 92211.882369 | |
| 7 | 7.558276 | 9.952150 | 88837.582700 | 70028.387994 | |
| 8 | 4.250417 | 6.279300 | 67129.296883 | 52423.083656 | |
| 9 | 2.390208 | 3.961975 | 50182.790875 | 38875.106752 | |
| 10 | 1.344130 | 2.499815 | 37071.142416 | 28449.381364 | |

| | | | | |
|---|---|---|---|---|
| 11 | 0.755864 | 1.577270 | 27071.847000 | 20678.603072 |
| 12 | 0.425052 | 0.995195 | 19635.847321 | 14820.559377 |
| 13 | 0.239030 | 0.627915 | 14064.333472 | 10542.883678 |
| 14 | 0.134417 | 0.396187 | 10065.150272 | 7468.238949 |
| 15 | 0.075589 | 0.249977 | NaN | 5241.929797 |
| 16 | 0.042507 | 0.157722 | NaN | 3673.034474 |
| 17 | NaN | 0.099516 | NaN | 2604.423353 |
| 18 | NaN | 0.062790 | NaN | 1816.723339 |

| | bT_Gc_Sh210oC | bT_Gc_Sh230oC | bT_Gc_Sh250oC | frecuencia_Sh170oC | \ |
|---|---|---|---|---|---|
| 0 | 334575.384489 | 322221.071579 | 278958.489520 | 500.000 | |
| 1 | 273645.435089 | 261400.224173 | 233110.466669 | 281.000 | |
| 2 | 221232.179009 | 209993.565704 | 194757.633186 | 158.000 | |
| 3 | 176238.437990 | 166724.036502 | 161414.109680 | 88.900 | |
| 4 | 138523.118588 | 130304.462223 | 132449.273459 | 50.000 | |
| 5 | 107318.078476 | 100455.786957 | 107784.517275 | 28.100 | |
| 6 | 82041.678186 | 76388.469542 | 86763.835740 | 15.800 | |
| 7 | 61917.068074 | 57359.480537 | 69182.536007 | 8.890 | |
| 8 | 46216.717106 | 42574.858695 | 54703.360702 | 5.000 | |
| 9 | 34084.741300 | 31267.866132 | 42895.184317 | 2.810 | |
| 10 | 24879.393832 | 22771.734376 | 33380.854966 | 1.580 | |
| 11 | 18049.917019 | 16420.234124 | 25793.544602 | 0.889 | |
| 12 | 12918.200497 | 11741.992897 | 19814.797539 | 0.500 | |
| 13 | 9237.947606 | 8362.745351 | 15086.437855 | 0.281 | |
| 14 | NaN | 5897.972901 | 11460.428825 | 0.158 | |
| 15 | NaN | 4133.628810 | 8643.566385 | NaN | |
| 16 | NaN | 2866.103454 | 6502.212245 | NaN | |
| 17 | NaN | NaN | 4869.661698 | NaN | |
| 18 | NaN | NaN | 3628.828207 | NaN | |

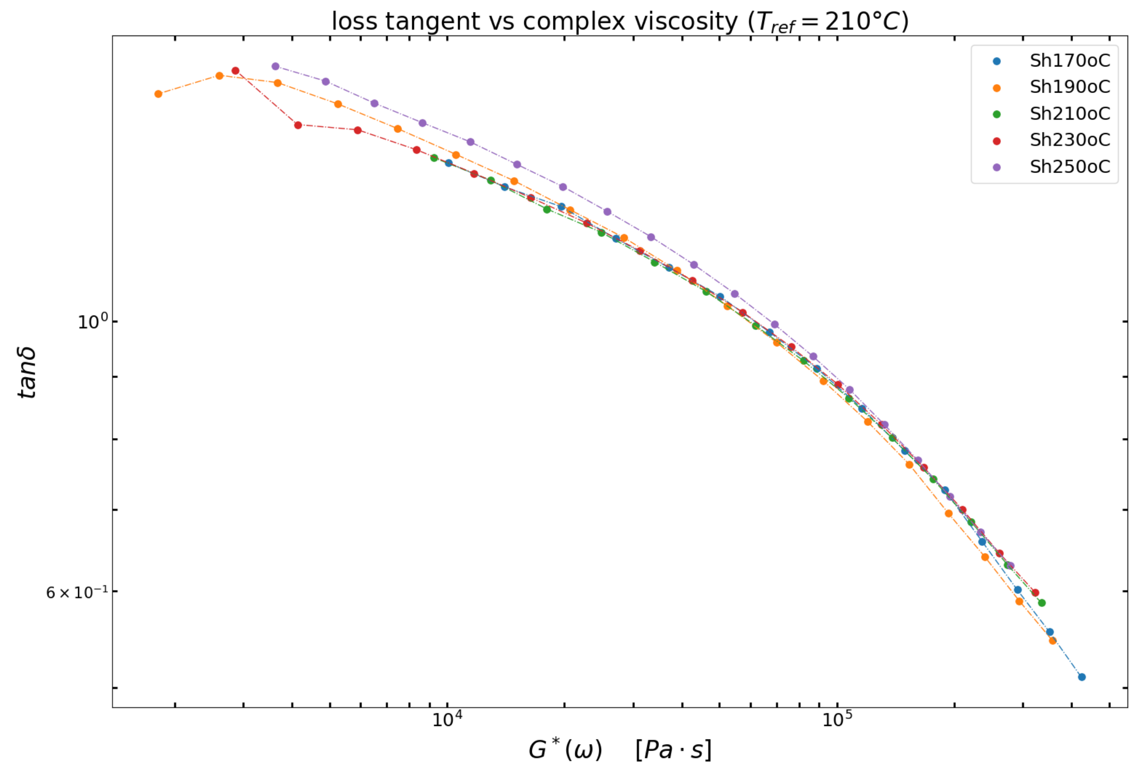
| | frecuencia_Sh190oC | frecuencia_Sh210oC | frecuencia_Sh230oC | \ |
|---|---|---|---|---|
| 0 | 500.0000 | 500.000000 | 500.000000 | |
| 1 | 281.0000 | 281.172000 | 281.172000 | |
| 2 | 158.0000 | 158.117000 | 158.117000 | |
| 3 | 88.9000 | 88.916000 | 88.916000 | |
| 4 | 50.0000 | 50.002000 | 50.002000 | |
| 5 | 28.1000 | 28.118700 | 28.118700 | |
| 6 | 15.8000 | 15.812500 | 15.812500 | |
| 7 | 8.8900 | 8.892090 | 8.892090 | |
| 8 | 5.0000 | 5.000490 | 5.000490 | |
| 9 | 2.8100 | 2.812010 | 2.812010 | |
| 10 | 1.5800 | 1.581330 | 1.581330 | |
| 11 | 0.8890 | 0.889252 | 0.889252 | |
| 12 | 0.5000 | 0.500061 | 0.500061 | |
| 13 | 0.2810 | 0.281212 | 0.281212 | |
| 14 | 0.1580 | NaN | 0.158138 | |
| 15 | 0.0889 | NaN | 0.088928 | |
| 16 | 0.0500 | NaN | 0.050009 | |

```
17              0.0281                 NaN                  NaN
18              0.0158                 NaN                  NaN

    frecuencia_Sh250oC  tand_Sh170oC  tand_Sh190oC  tand_Sh210oC  \
0           500.000000      0.510204      0.546980      0.586941
1           315.477000      0.555556      0.589212      0.631121
2           199.051000      0.601770      0.640625      0.683758
3           125.592000      0.659218      0.695364      0.741814
4            79.242200      0.726619      0.762931      0.801808
5            49.998000      0.782243      0.827076      0.864567
6            31.546400      0.848070      0.893130      0.927728
7            19.904300      0.914430      0.960499      0.991188
8            12.558600      0.979358      1.028736      1.057151
9             7.923950      1.047619      1.100402      1.117889
10            4.999630      1.106195      1.170455      1.182342
11            3.154540      1.168750      1.233871      1.235560
12            1.990390      1.241071      1.303842      1.304841
13            1.255830      1.288265      1.369932      1.361725
14            0.792374      1.348624      1.438424           NaN
15            0.499954           NaN      1.507246           NaN
16            0.315445           NaN      1.569149           NaN
17            0.199032           NaN      1.590909           NaN
18            0.125580           NaN      1.536017           NaN

    tand_Sh230oC  tand_Sh250oC
0       0.598943      0.629585
1       0.644515      0.671362
2       0.700259      0.717780
3       0.758730      0.768316
4       0.821858      0.821916
5       0.887417      0.878638
6       0.952034      0.935461
7       1.016743      0.993673
8       1.079557      1.053047
9       1.141781      1.112814
10      1.203675      1.172330
11      1.262281      1.229791
12      1.321558      1.289216
13      1.381773      1.344614
14      1.435126      1.402939
15      1.449411      1.454527
16      1.606133      1.509537
17           NaN      1.573777
18           NaN      1.618098
```

### 6.8 Plot $\tan\delta$ vs. $G^*(\omega)$ for each $T$

[10]:
```
# Set plot size and axis labels' font size
pltname = "loss tangent vs complex viscosity (" + r'$T_{ref} = 210°C$' + ")";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();

# Iterate the data per sample
sample_id = '';
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Define x axis as the date axis
        x_str = 'bT_Gc_' + sample_id; x_units = r'$[Pa \cdot s]$';
        y_str = 'tand_' + sample_id;         y_units = '';

        # Remove NANs from interesting x,y data
        df_fil = pd.DataFrame(df);
        df_fil = df_fil.dropna(subset=[x_str, y_str]);

        # Stablish the plot area
        ax0 = plt.gca();

        # Extract data from a specific country
        x  = df_fil.iloc[:][x_str];
        y = df_fil.iloc[:][y_str];

        # Scatter the data and plot a curve to join the points
        plt.scatter(x, y, s=45, marker='o', label=sample_id);
        plt.plot(x, y, linewidth=1, linestyle='-.');

# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));

# fig.autofmt_xdate();
ax0.set_xlabel(r'$G^*(\omega)$' + '     ' + x_units, fontsize=24);
ax0.set_ylabel(r'$tan \delta$', fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,␣
 ↪top=True, left=True, right=True);
```

```
# Display main plot
plt.yscale('log');
plt.xscale('log');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```




---

## 6.9 Get the activation energy $E_H$ and $E_V$

## 6.10

$$\log a_T \ vs. \ \left(\frac{1}{T} - \frac{1}{T_0}\right)$$

[11]:
```
def _invT(T, T0):
    invT = (1/T) - (1/T0);
    return invT;

# Declare sample temperatures in Celsius and Kelvin
```

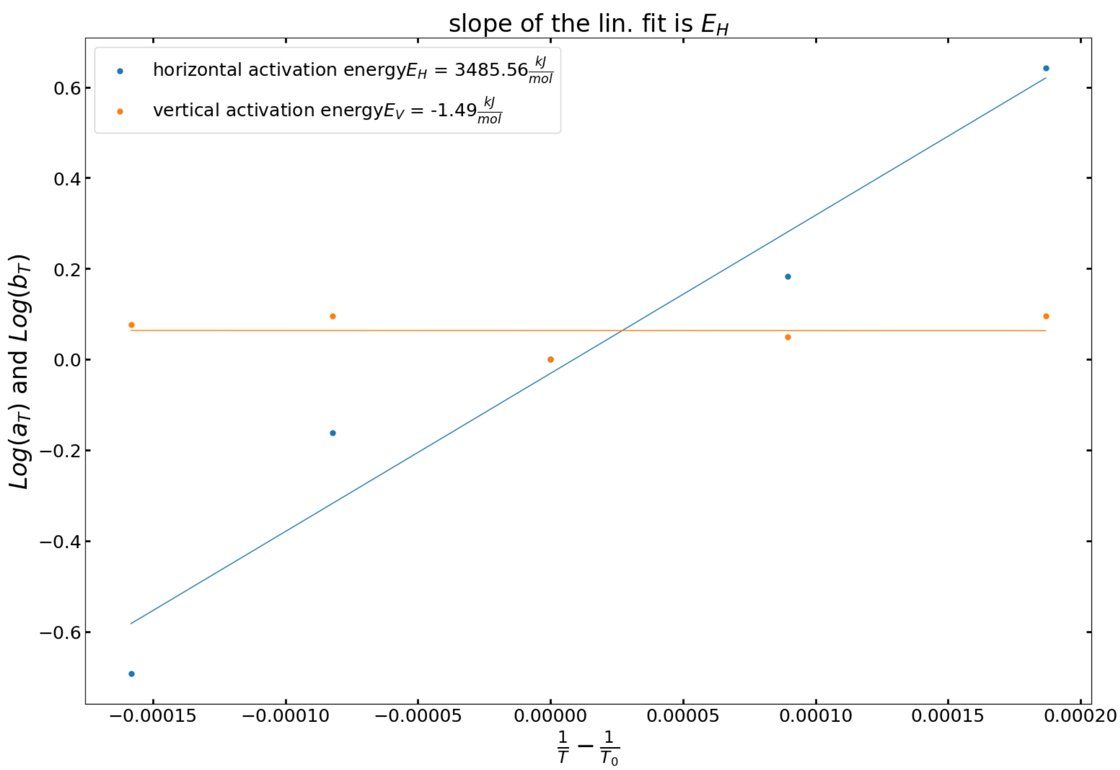
```
T_C    = pd.Series([170, 190, 210, 230, 250, 180, 225]);
T_K    = T_C + 273.15;
T_Cref = T_C[2];
T_Kref = T_K[2];
```

---

### 6.11 Plot $\log a_T$ and $\log b_T$ vs. $\left(\frac{1}{T} - \frac{1}{T_0}\right)$ and fit a linear regression to get $E_H$ and $E_V$

[12]:
```
# Set plot size and axis labels' font size
pltname = "slope of the lin. fit is $E_H$";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();
ax0 = plt.gca();

# calculate varibles
x = _invT(T_K[:-2], T_Kref);

# HORIZONTAL ACTIVATION ENERGY EH

# perform a linear regression
y = np.log(a_T);
model = LinearRegression().fit(np.array(x).reshape((-1, 1)), np.array(y));
# get the slope
E_H = model.coef_[0];

# plot variables and lin. fit
plt.scatter(x, y, s=25, label='horizontal activation energy ' + r'$E_H$' + " =␣
↪" + str(round(E_H, 2)) + r'$\frac{kJ}{mol}$');
plt.plot(x, model.predict(np.array(x).reshape((-1, 1))), linewidth=1);

# VERTICAL ACTIVATION ENERGY EV

# perform a linear regression
y = np.log(b_T);
model = LinearRegression().fit(np.array(x).reshape((-1, 1)), np.array(y));
# get the slope
E_V = model.coef_[0];

# plot variables and lin. fit
plt.scatter(x, y, s=25, label='vertical activation energy ' + r'$E_V$' + " = "␣
↪+ str(round(E_V, 2)) + r'$\frac{kJ}{mol}$');
plt.plot(x, model.predict(np.array(x).reshape((-1, 1))), linewidth=1);
```

```
# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));

# fig.autofmt_xdate();
ax0.set_xlabel(r'$\frac{1}{T} - \frac{1}{T_0}$', fontsize=24);
ax0.set_ylabel(r'$Log(a_T)$' + ' and ' + r'$Log(b_T)$', fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,␣
↪top=True, left=True, right=True);

# Display main plot
plt.yscale('linear');
plt.xscale('linear');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```



### 6.12 Compute the shift factor $a_T$ for arbitrary temperatures

$$a_T = \exp\left[E_H\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$
$$b_T = \exp\left[E_V\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

[13]:
```
at1 = np.exp(E_H*_invT(T_K[5], T_Kref));
at2 = np.exp(E_H*_invT(T_K[6], T_Kref));

bt1 = np.exp(E_V*_invT(T_K[5], T_Kref));
bt2 = np.exp(E_V*_invT(T_K[6], T_Kref));

a_T = a_T.append(pd.Series([at1, at2]));
b_T = b_T.append(pd.Series([bt1, bt2]));

display('a_T'); display(a_T);
display('b_T'); display(b_T);
display('T_C'); display(T_C);
```

```
'a_T'


0    1.900000
1    1.200000
2    1.000000
3    0.850000
4    0.500000
0    1.612211
1    0.804744
dtype: float64


'b_T'


0    1.100000
1    1.050000
2    1.000000
3    1.100000
4    1.080000
0    0.999795
1    1.000093
dtype: float64


'T_C'
```

```
0     170
1     190
2     210
3     230
4     250
5     180
6     225
dtype: int64
```

---

### 6.13

$$\eta^*(\omega) = \frac{G^*(\omega)}{\omega}$$

[14]:

```
def _etac(G2, w):
    etac = G2/w;
    return etac;

# Iterate the data per sample
sample_id = '';
etacmaster_Sh210oC = pd.Series([]);
freqmaster_Sh210oC = pd.Series([]);
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Compute Complex Viscosity
        df['aT_etac_' + sample_id] = _etac(df['bT_Gc_' + sample_id],␣
 ↪df['aT_frequency_' + sample_id]);

        # Store master curve
        etacmaster_Sh210oC = etacmaster_Sh210oC.append(df['aT_etac_' +␣
 ↪sample_id]);
        freqmaster_Sh210oC = freqmaster_Sh210oC.append(df['aT_frequency_' +␣
 ↪sample_id]);

# Sort data
freqmaster_Sh210oC = freqmaster_Sh210oC.reset_index(drop=True)
etacmaster_Sh210oC = etacmaster_Sh210oC.reset_index(drop=True)
mastercurve = pd.DataFrame()
mastercurve['freq'] = pd.Series(freqmaster_Sh210oC)
mastercurve['eta']  = pd.Series(etacmaster_Sh210oC)
mastercurve = mastercurve.sort_values('freq')

df = df.reindex(sorted(df.columns), axis=1);
print("./data.csv"); display(df);
```

```
./data.csv

    G1_Sh170oC  G1_Sh190oC  G1_Sh210oC  G1_Sh230oC  G1_Sh250oC  G2_Sh170oC  \
0     343000.0    298000.0   288545.00   251301.00   218582.00    175000.0
1     279000.0    241000.0   231412.00   199744.00   179203.00    155000.0
2     226000.0    192000.0   182623.00   156375.00   146499.00    136000.0
3     179000.0    151000.0   141545.00   120746.00   118516.00    118000.0
4     139000.0    116000.0   108073.00    91516.80    94743.10    101000.0
5     107000.0     87900.0    81183.40    68305.90    74972.20     83700.0
6      80300.0     65500.0    60144.90    50295.80    58668.40     68100.0
7      59600.0     48100.0    43975.30    36564.70    45439.30     54500.0
8      43600.0     34800.0    31760.00    26301.90    34878.80     42700.0
9      31500.0     24900.0    22724.80    18728.20    26547.30     33000.0
10     22600.0     17600.0    16066.50    13228.90    20058.60     25000.0
11     16000.0     12400.0    11355.50     9269.49    15067.60     18700.0
```

| | | | | | | |
|---|---|---|---|---|---|---|
| 12 | 11200.0 | 8590.0 | 7857.97 | 6441.07 | 11244.90 | 13900.0 |
| 13 | 7840.0 | 5920.0 | 5467.97 | 4457.20 | 8336.15 | 10100.0 |
| 14 | 5450.0 | 4060.0 | NaN | 3065.34 | 6159.25 | 7350.0 |
| 15 | NaN | 2760.0 | NaN | 2134.04 | 4534.14 | NaN |
| 16 | NaN | 1880.0 | NaN | 1377.14 | 3324.96 | NaN |
| 17 | NaN | 1320.0 | NaN | NaN | 2418.17 | NaN |
| 18 | NaN | 944.0 | NaN | NaN | 1766.42 | NaN |

| | G2_Sh190oC | G2_Sh210oC | G2_Sh230oC | G2_Sh250oC | Gc_Sh170oC | \ |
|---|---|---|---|---|---|---|
| 0 | 163000.0 | 169359.00 | 150515.00 | 137616.00 | 385063.631105 | |
| 1 | 142000.0 | 146049.00 | 128738.00 | 120310.00 | 319164.534371 | |
| 2 | 123000.0 | 124870.00 | 109503.00 | 105154.00 | 263765.046964 | |
| 3 | 105000.0 | 105000.00 | 91613.60 | 91057.70 | 214394.496198 | |
| 4 | 88500.0 | 86653.80 | 75213.80 | 77870.90 | 171819.672913 | |
| 5 | 72700.0 | 70188.50 | 60615.80 | 65873.40 | 135848.040104 | |
| 6 | 58500.0 | 55798.10 | 47883.30 | 54882.00 | 105288.650860 | |
| 7 | 46200.0 | 43587.80 | 37176.90 | 45151.80 | 80761.438818 | |
| 8 | 35800.0 | 33575.10 | 28394.40 | 36729.00 | 61026.633530 | |
| 9 | 27400.0 | 25403.80 | 21383.50 | 29542.20 | 45620.718977 | |
| 10 | 20600.0 | 18996.10 | 15923.30 | 23515.30 | 33701.038560 | |
| 11 | 15300.0 | 14030.40 | 11700.70 | 18530.00 | 24610.770000 | |
| 12 | 11200.0 | 10253.40 | 8512.25 | 14497.10 | 17850.770292 | |
| 13 | 8110.0 | 7445.87 | 6158.84 | 11208.90 | 12785.757701 | |
| 14 | 5840.0 | NaN | 4399.15 | 8641.05 | 9150.136611 | |
| 15 | 4160.0 | NaN | 3093.10 | 6595.03 | NaN | |
| 16 | 2950.0 | NaN | 2211.87 | 5019.15 | NaN | |
| 17 | 2100.0 | NaN | NaN | 3805.66 | NaN | |
| 18 | 1450.0 | NaN | NaN | 2858.24 | NaN | |

| | Gc_Sh190oC | Gc_Sh210oC | Gc_Sh230oC | Gc_Sh250oC | \ |
|---|---|---|---|---|---|
| 0 | 339666.012430 | 334575.384489 | 292928.246890 | 258294.897704 | |
| 1 | 279723.077346 | 273645.435089 | 237636.567430 | 215843.024694 | |
| 2 | 228019.735988 | 221232.179009 | 190903.241549 | 180331.141839 | |
| 3 | 183918.460194 | 176238.437990 | 151567.305910 | 149457.508963 | |
| 4 | 145904.934803 | 138523.118588 | 118458.602021 | 122638.216166 | |
| 5 | 114068.838865 | 107318.078476 | 91323.442688 | 99800.478959 | |
| 6 | 87820.840351 | 82041.678186 | 69444.063220 | 80336.884944 | |
| 7 | 66693.702851 | 61917.068074 | 52144.982306 | 64057.903710 | |
| 8 | 49926.746339 | 46216.717106 | 38704.416996 | 50651.259910 | |
| 9 | 37023.911193 | 34084.741300 | 28425.332847 | 39717.763257 | |
| 10 | 27094.648918 | 24879.393832 | 20701.576706 | 30908.199042 | |
| 11 | 19693.907687 | 18049.917019 | 14927.485567 | 23882.911668 | |
| 12 | 14114.818454 | 12918.200497 | 10674.538997 | 18347.034758 | |
| 13 | 10040.841598 | 9237.947606 | 7602.495773 | 13968.923940 | |
| 14 | 7112.608523 | NaN | 5361.793547 | 10611.508171 | |
| 15 | 4992.314093 | NaN | 3757.844373 | 8003.302208 | |
| 16 | 3498.128071 | NaN | 2605.548594 | 6020.566894 | |
| 17 | 2480.403193 | NaN | NaN | 4508.946017 | |

| | | | | |
|---|---|---|---|---|
| 18 | 1730.212704 | NaN | NaN | 3360.026118 |

| | aT_etac_Sh170oC | aT_etac_Sh190oC | aT_etac_Sh210oC | aT_etac_Sh230oC | \ |
|---|---|---|---|---|---|
| 0 | 445.863152 | 594.415522 | 669.150769 | 758.167227 | |
| 1 | 657.578175 | 871.023817 | 973.231457 | 1093.742177 | |
| 2 | 966.494176 | 1262.767525 | 1399.167572 | 1562.458520 | |
| 3 | 1396.210679 | 1810.221065 | 1982.077894 | 2205.968839 | |
| 4 | 1989.490950 | 2553.336359 | 2770.351558 | 3065.864712 | |
| 5 | 2798.892004 | 3551.965623 | 3816.608822 | 4203.013609 | |
| 6 | 3858.011857 | 4863.495906 | 5188.406526 | 5683.401593 | |
| 7 | 5259.462595 | 6564.340832 | 6963.162549 | 7588.962978 | |
| 8 | 7066.241777 | 8737.180609 | 9242.437662 | 10016.632181 | |
| 9 | 9399.286547 | 11528.797969 | 12121.130899 | 13081.647953 | |
| 10 | 12348.814929 | 15004.947977 | 15733.208016 | 16941.609744 | |
| 11 | 16027.379670 | 19383.767409 | 20297.864969 | 21723.788643 | |
| 12 | 20669.312969 | 24700.932295 | 25833.249338 | 27624.848350 | |
| 13 | 26342.636208 | 31265.965831 | 32850.474398 | 34986.145478 | |
| 14 | 33528.148808 | 39389.445934 | NaN | 43878.078948 | |
| 15 | NaN | 49136.949731 | NaN | 54685.608367 | |
| 16 | NaN | 61217.241240 | NaN | 67425.861368 | |
| 17 | NaN | 77236.754231 | NaN | NaN | |
| 18 | NaN | 95818.741502 | NaN | NaN | |

| | aT_etac_Sh250oC | aT_frequency_Sh170oC | aT_frequency_Sh190oC | \ |
|---|---|---|---|---|
| 0 | 1115.833958 | 950.0000 | 600.00000 | |
| 1 | 1477.828600 | 533.9000 | 337.20000 | |
| 2 | 1956.861640 | 300.2000 | 189.60000 | |
| 3 | 2570.452094 | 168.9100 | 106.68000 | |
| 4 | 3342.897432 | 95.0000 | 60.00000 | |
| 5 | 4311.553153 | 53.3900 | 33.72000 | |
| 6 | 5500.712331 | 30.0200 | 18.96000 | |
| 7 | 6951.516608 | 16.8910 | 10.66800 | |
| 8 | 8711.697276 | 9.5000 | 6.00000 | |
| 9 | 10826.717563 | 5.3390 | 3.37200 | |
| 10 | 13353.330133 | 3.0020 | 1.89600 | |
| 11 | 16353.284220 | 1.6891 | 1.06680 | |
| 12 | 19910.467334 | 0.9500 | 0.60000 | |
| 13 | 24026.242174 | 0.5339 | 0.33720 | |
| 14 | 28926.816944 | 0.3002 | 0.18960 | |
| 15 | 34577.446666 | NaN | 0.10668 | |
| 16 | 41225.647865 | NaN | 0.06000 | |
| 17 | 48933.454906 | NaN | 0.03372 | |
| 18 | 57793.091376 | NaN | 0.01896 | |

| | aT_frequency_Sh210oC | aT_frequency_Sh230oC | aT_frequency_Sh250oC | \ |
|---|---|---|---|---|
| 0 | 500.000000 | 425.000000 | 250.000000 | |
| 1 | 281.172000 | 238.996200 | 157.738500 | |
| 2 | 158.117000 | 134.399450 | 99.525500 | |

| | | | |
|---|---|---|---|
| 3 | 88.916000 | 75.578600 | 62.796000 |
| 4 | 50.002000 | 42.501700 | 39.621100 |
| 5 | 28.118700 | 23.900895 | 24.999000 |
| 6 | 15.812500 | 13.440625 | 15.773200 |
| 7 | 8.892090 | 7.558276 | 9.952150 |
| 8 | 5.000490 | 4.250417 | 6.279300 |
| 9 | 2.812010 | 2.390208 | 3.961975 |
| 10 | 1.581330 | 1.344130 | 2.499815 |
| 11 | 0.889252 | 0.755864 | 1.577270 |
| 12 | 0.500061 | 0.425052 | 0.995195 |
| 13 | 0.281212 | 0.239030 | 0.627915 |
| 14 | NaN | 0.134417 | 0.396187 |
| 15 | NaN | 0.075589 | 0.249977 |
| 16 | NaN | 0.042507 | 0.157722 |
| 17 | NaN | NaN | 0.099516 |
| 18 | NaN | NaN | 0.062790 |

| | bT_Gc_Sh170oC | bT_Gc_Sh190oC | bT_Gc_Sh210oC | bT_Gc_Sh230oC | bT_Gc_Sh250oC | \ |
|---|---|---|---|---|---|---|
| 0 | 423569.994216 | 356649.313051 | 334575.384489 | 322221.071579 | 278958.489520 | |
| 1 | 351080.987808 | 293709.231213 | 273645.435089 | 261400.224173 | 233110.466669 | |
| 2 | 290141.551661 | 239420.722787 | 221232.179009 | 209993.565704 | 194757.633186 | |
| 3 | 235833.945818 | 193114.383203 | 176238.437990 | 166724.036502 | 161414.109680 | |
| 4 | 189001.640205 | 153200.181544 | 138523.118588 | 130304.462223 | 132449.273459 | |
| 5 | 149432.844114 | 119772.280808 | 107318.078476 | 100455.786957 | 107784.517275 | |
| 6 | 115817.515946 | 92211.882369 | 82041.678186 | 76388.469542 | 86763.835740 | |
| 7 | 88837.582700 | 70028.387994 | 61917.068074 | 57359.480537 | 69182.536007 | |
| 8 | 67129.296883 | 52423.083656 | 46216.717106 | 42574.858695 | 54703.360702 | |
| 9 | 50182.790875 | 38875.106752 | 34084.741300 | 31267.866132 | 42895.184317 | |
| 10 | 37071.142416 | 28449.381364 | 24879.393832 | 22771.734376 | 33380.854966 | |
| 11 | 27071.847000 | 20678.603072 | 18049.917019 | 16420.234124 | 25793.544602 | |
| 12 | 19635.847321 | 14820.559377 | 12918.200497 | 11741.992897 | 19814.797539 | |
| 13 | 14064.333472 | 10542.883678 | 9237.947606 | 8362.745351 | 15086.437855 | |
| 14 | 10065.150272 | 7468.238949 | NaN | 5897.972901 | 11460.428825 | |
| 15 | NaN | 5241.929797 | NaN | 4133.628810 | 8643.566385 | |
| 16 | NaN | 3673.034474 | NaN | 2866.103454 | 6502.212245 | |
| 17 | NaN | 2604.423353 | NaN | NaN | 4869.661698 | |
| 18 | NaN | 1816.723339 | NaN | NaN | 3628.828207 | |

| | frecuencia_Sh170oC | frecuencia_Sh190oC | frecuencia_Sh210oC | \ |
|---|---|---|---|---|
| 0 | 500.000 | 500.0000 | 500.000000 | |
| 1 | 281.000 | 281.0000 | 281.172000 | |
| 2 | 158.000 | 158.0000 | 158.117000 | |
| 3 | 88.900 | 88.9000 | 88.916000 | |
| 4 | 50.000 | 50.0000 | 50.002000 | |
| 5 | 28.100 | 28.1000 | 28.118700 | |
| 6 | 15.800 | 15.8000 | 15.812500 | |
| 7 | 8.890 | 8.8900 | 8.892090 | |
| 8 | 5.000 | 5.0000 | 5.000490 | |

| | | | |
|---|---|---|---|
| 9 | 2.810 | 2.8100 | 2.812010 |
| 10 | 1.580 | 1.5800 | 1.581330 |
| 11 | 0.889 | 0.8890 | 0.889252 |
| 12 | 0.500 | 0.5000 | 0.500061 |
| 13 | 0.281 | 0.2810 | 0.281212 |
| 14 | 0.158 | 0.1580 | NaN |
| 15 | NaN | 0.0889 | NaN |
| 16 | NaN | 0.0500 | NaN |
| 17 | NaN | 0.0281 | NaN |
| 18 | NaN | 0.0158 | NaN |

| | frecuencia_Sh230oC | frecuencia_Sh250oC | tand_Sh170oC | tand_Sh190oC | \ |
|---|---|---|---|---|---|
| 0 | 500.000000 | 500.000000 | 0.510204 | 0.546980 | |
| 1 | 281.172000 | 315.477000 | 0.555556 | 0.589212 | |
| 2 | 158.117000 | 199.051000 | 0.601770 | 0.640625 | |
| 3 | 88.916000 | 125.592000 | 0.659218 | 0.695364 | |
| 4 | 50.002000 | 79.242200 | 0.726619 | 0.762931 | |
| 5 | 28.118700 | 49.998000 | 0.782243 | 0.827076 | |
| 6 | 15.812500 | 31.546400 | 0.848070 | 0.893130 | |
| 7 | 8.892090 | 19.904300 | 0.914430 | 0.960499 | |
| 8 | 5.000490 | 12.558600 | 0.979358 | 1.028736 | |
| 9 | 2.812010 | 7.923950 | 1.047619 | 1.100402 | |
| 10 | 1.581330 | 4.999630 | 1.106195 | 1.170455 | |
| 11 | 0.889252 | 3.154540 | 1.168750 | 1.233871 | |
| 12 | 0.500061 | 1.990390 | 1.241071 | 1.303842 | |
| 13 | 0.281212 | 1.255830 | 1.288265 | 1.369932 | |
| 14 | 0.158138 | 0.792374 | 1.348624 | 1.438424 | |
| 15 | 0.088928 | 0.499954 | NaN | 1.507246 | |
| 16 | 0.050009 | 0.315445 | NaN | 1.569149 | |
| 17 | NaN | 0.199032 | NaN | 1.590909 | |
| 18 | NaN | 0.125580 | NaN | 1.536017 | |

| | tand_Sh210oC | tand_Sh230oC | tand_Sh250oC |
|---|---|---|---|
| 0 | 0.586941 | 0.598943 | 0.629585 |
| 1 | 0.631121 | 0.644515 | 0.671362 |
| 2 | 0.683758 | 0.700259 | 0.717780 |
| 3 | 0.741814 | 0.758730 | 0.768316 |
| 4 | 0.801808 | 0.821858 | 0.821916 |
| 5 | 0.864567 | 0.887417 | 0.878638 |
| 6 | 0.927728 | 0.952034 | 0.935461 |
| 7 | 0.991188 | 1.016743 | 0.993673 |
| 8 | 1.057151 | 1.079557 | 1.053047 |
| 9 | 1.117889 | 1.141781 | 1.112814 |
| 10 | 1.182342 | 1.203675 | 1.172330 |
| 11 | 1.235560 | 1.262281 | 1.229791 |
| 12 | 1.304841 | 1.321558 | 1.289216 |
| 13 | 1.361725 | 1.381773 | 1.344614 |
| 14 | NaN | 1.435126 | 1.402939 |

```
15            NaN      1.449411      1.454527
16            NaN      1.606133      1.509537
17            NaN           NaN      1.573777
18            NaN           NaN      1.618098
```

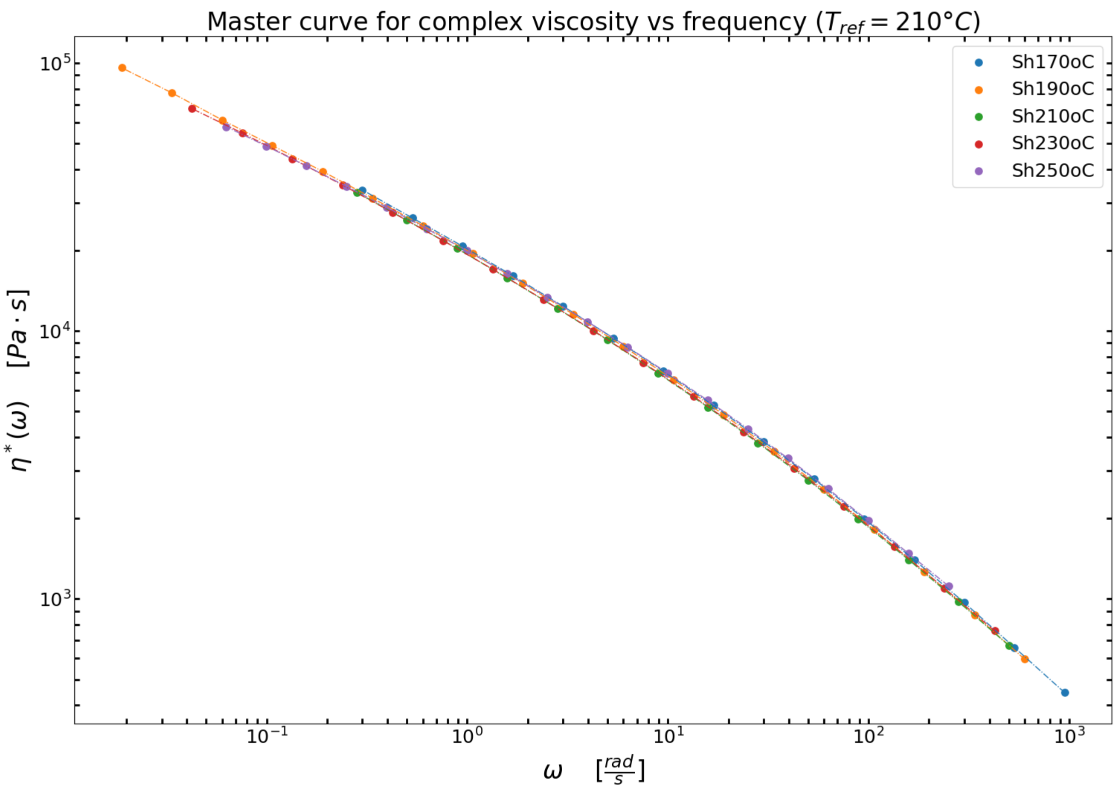
---

## 6.14 Plot master curve for complex viscosity vs frequency

[15]:

```python
# Set plot size and axis labels' font size
pltname = "Master curve for complex viscosity vs frequency (" + r'$T_{ref} =␣
↪210°C$' + ")";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();

# Iterate the data per sample

sample_id = '';
for i in range(len(df.columns)):
    if ("frecuencia" in df.columns[i]):
        sample_id = df.columns[i].split('_')[1];

        # Define x axis as the date axis
        x_str = 'aT_frequency_' + sample_id; x_units = r'$[\frac{rad}{s}]$';
        y_str = 'aT_etac_' + sample_id;      y_units = r'$[Pa \cdot s]$';

        # Remove NANs from interesting x,y data
        df_fil = pd.DataFrame(df);
        df_fil = df_fil.dropna(subset=[x_str, y_str]);

        # Stablish the plot area
        ax0 = plt.gca();

        # Extract data from a specific country
        x = df_fil.iloc[:][x_str];
        y = df_fil.iloc[:][y_str];

        # Scatter the data and plot a curve to join the points
        plt.scatter(x, y, s=45, marker='o', label=sample_id);
        plt.plot(x, y, linewidth=1, linestyle='-.');

# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));
```

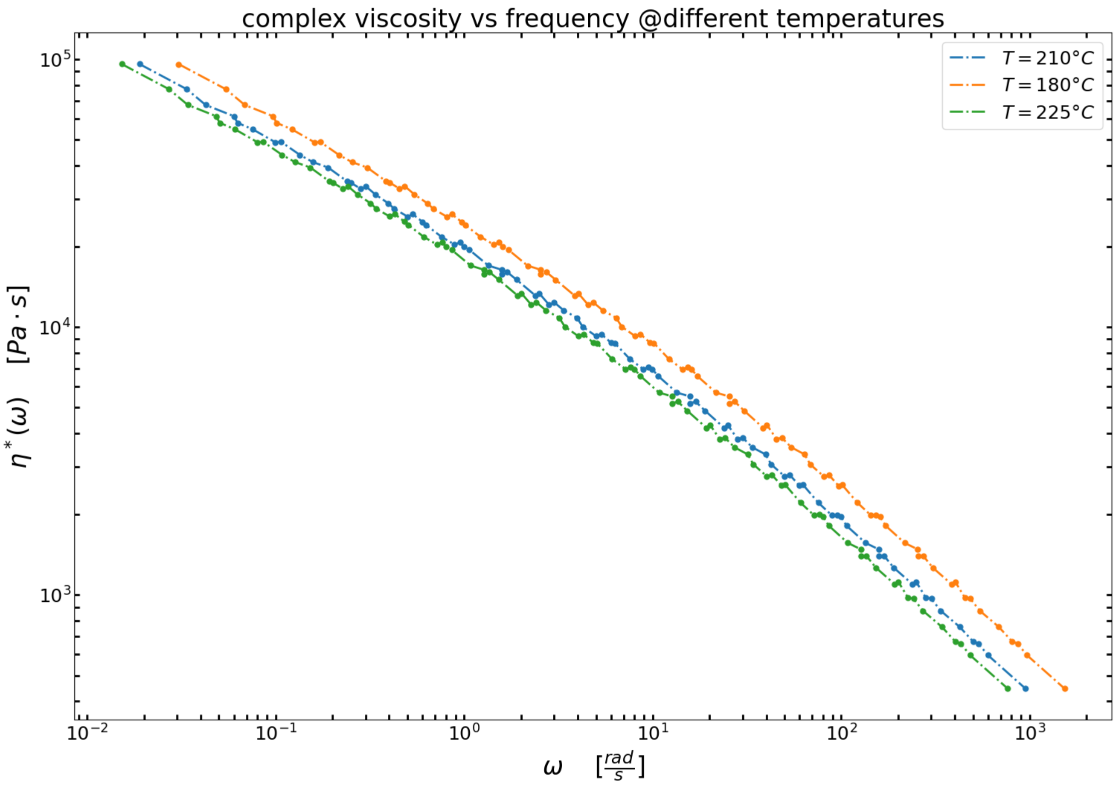
```python
# fig.autofmt_xdate();
ax0.set_xlabel(r'$\omega$' + '     ' + x_units, fontsize=24);
ax0.set_ylabel(r'$\eta^*(\omega)$' + '     ' + y_units, fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,
 ↪top=True, left=True, right=True);

# Display main plot
plt.yscale('log');
plt.xscale('log');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```



### 6.15 Plot complex viscosity vs frequency @different temperatures

[16]:
```
# Set plot size and axis labels' font size
pltname = "complex viscosity vs frequency @different temperatures";
scale   = 6;
fig     = plt.figure(figsize=(3*scale, 2*scale));
plt.rc('xtick', labelsize=15);
plt.rc('ytick', labelsize=15);
plt.tight_layout();

# Stablish the plot area
ax0 = plt.gca();

# Extract data from a specific country
x = mastercurve['freq'];
y = mastercurve['eta'];

# Scatter the data and plot a curve to join the points
print(np.array(T_C))
for T, a, b in zip(T_C, a_T, b_T):
    if T in [210, 180, 225]:
        plt.scatter(x*a, y*b, s=25, marker='o');
        plt.plot(x*a, y*b, linewidth=2, linestyle='-.', label=r'$T = $' +␣
 ↪str(T) + '$ °C$');

# Show the plot lengend to link colors and polymer names
handles, labels = ax0.get_legend_handles_labels();
lgd = dict(zip(labels, handles));

# fig.autofmt_xdate();
ax0.set_xlabel(r'$\omega$' + '     ' + x_units, fontsize=24);
ax0.set_ylabel(r'$\eta^*(\omega)$' + '     ' + y_units, fontsize=24);

for tick in ax0.xaxis.get_major_ticks(): tick.label.set_fontsize(18);
for tick in ax0.yaxis.get_major_ticks(): tick.label.set_fontsize(18);
ax0.tick_params(which='both', direction='in', length=5, width=2, bottom=True,␣
 ↪top=True, left=True, right=True);

# Display main plot
plt.yscale('log');
plt.xscale('log');
plt.legend(lgd.values(), lgd.keys(), prop={'size': 18}, loc="best");
plt.title(pltname, size=24);
plt.savefig(pltname + '.png', dpi=200, bbox_inches='tight');
plt.show();
mpl.rcParams.update(mpl.rcParamsDefault); # Recover matplotlib defaults
```

```
[170 190 210 230 250 180 225]
```




---

# 7 References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- A. Oseli, A. Aulova, M. Gergesova, I. Emri, Time-Temperature Superposition in Linear and Non-linear Domain, Mater. Today Proc. 3 (2016) 1118–1123. https://doi.org/10.1016/j.matpr.2016.03.059.
- H. Mavridis, R.N. Shroff, Temperature dependence of polyolefin melt rheology, Polym. Eng. Sci. 32 (1992) 1778–1791. https://doi.org/10.1002/pen.760322307.


```
[ ]:
```