isothermal spinning of a Newtonian fluid and predicted the critical draw ratio of 20.210. Below the critical draw ratio, any disturbance along the filament is dampened out and the filament is stable. Above the critical draw ratio, any disturbance propagates along the filament as a wave, causing draw resonance. This prediction was supported by an experiment by Donnelly and Weinberger [62], using a Newtonian silicone oil. Hyun [63], using a kinematic wave theory for the throughput rate, predicted the critical draw ratio of 19.744 for isothermal Newtonian fluids. Mathematical analyses were extended to isothermal and non-isothermal power-law fluids [64, 65]. While some investigators suggested that the elasticity of a polymer melt would increase the critical draw ratio, Han and Kim [66] found in their experiment that the residual elastic stress reduced the critical draw ratio.

3.5.7 Flow Through Simple Dies

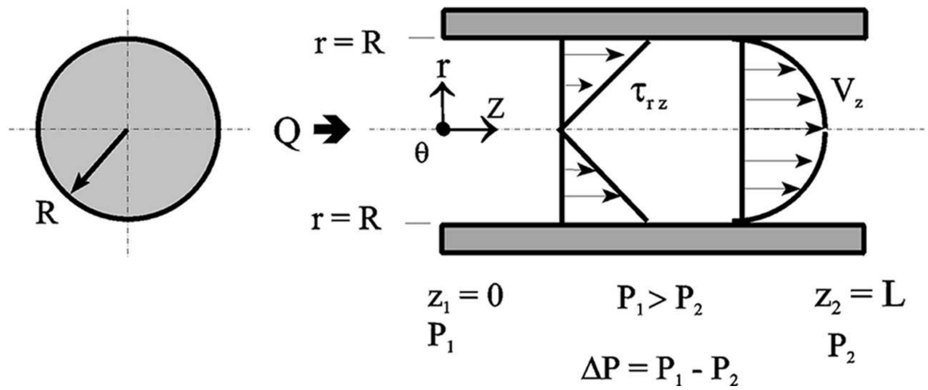
Analytical solutions of polymer melt flow can be obtained only for isothermal, steady flow through very simple dies. Transient flow cannot be analyzed because of the time dependence of viscosity. Analytical solutions for complex dies cannot be obtained because of the shear and time dependence of viscosity. However, computer programs based on finite element methods for complex dies are commercially available.

Analytical solutions for isothermal, steady flow through a circular tube and a slit (i.e., a wide flat opening without the ends) are presented below. These solutions have broad applications because the flow path in a small segment of many extrusion dies and adaptors can be approximated by a circular tube or a slit for the purpose of calculating pressure drop and flow rate. Dies for cast film and sheet are slits neglecting the two ends. Annular dies for blown film and blow molding are slits neglecting the curvature. A rectangular cross-section may be approximated by a circle. Although calculations based on such approximations will not give accurate predictions, the calculated values can serve as rough estimates.

3.5.7.1 Cir ular Tube

The isothermal, steady flow through a circular tube (or capillary) is analyzed in Section 3.5.3.1. Table 3.4 summarizes the analytical equations for isothermal, steady flow of Newtonian fluids and power-law fluids through a circular tube.

Table 3.4 Isothermal Flow Through Cir ular Tube



	Newtonian Fluid	Power- aw-Fluid
Viscosity	$\tau_w = \frac{\Delta P \cdot R}{2L}$	$\eta = \eta^o \cdot \left(\frac{\dot{\gamma}}{\dot{\gamma}^o}\right)^{(n-1)}$ with $\dot{\gamma}^o = 1.0\ s^{-1}$
Shear stress at the wall	$\tau_w = \frac{\Delta P \cdot R}{2L}$	$\tau_w = \frac{\Delta P \cdot R}{2L}$
Shear rate at the wall	$\dot{\gamma}_a = \frac{4Q}{\pi R^3}$	$\dot{\gamma}_w = \frac{\dot{\gamma}_a}{4} \cdot \left(3 + \frac{1}{n}\right)$
Maximum velocity at the tube center	$V_o = \frac{R^2 \cdot \Delta P}{4 \mu L}$	$V_o = \left(\frac{R \cdot n \cdot \dot{\gamma}^o}{n+1}\right) \cdot \left(\frac{R \cdot \Delta P}{2\eta^o \cdot \dot{\gamma}^o \cdot L}\right)^{1/n}$
Average velocity	$V_{avg} = \frac{1}{2} V_o$	$V_{avg} = \left(\frac{n+1}{3n+1}\right) \cdot V_o$
Volumetric flow rate	$Q = \frac{\pi R^4 \cdot \Delta P}{8 \mu L}$	$Q = \pi R^2 \cdot V_{avg}$
Pressure drop	$\Delta P = \frac{8 \mu L \cdot Q}{\pi R^4}$	$\Delta P = \left[\frac{Q}{\pi R^3} \cdot \left(\frac{3n+1}{n \cdot \dot{\gamma}^o}\right)\right]^n \cdot \left(\frac{2\eta^o \cdot \dot{\gamma}^o \cdot L}{R}\right)$
Velocity profile	$V_z(r) = V_o \cdot \left[1 - \left(\frac{r}{R}\right)^2\right]$	$V_z(r) = V_o \cdot \left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right]$
Shear rate profile	$\dot{\gamma}(r) = \left(\frac{2V_o}{R}\right) \cdot \left(\frac{r}{R}\right)$	$\dot{\gamma}(r) = \left(\frac{V_o}{R}\right) \cdot \left(\frac{n+1}{n}\right) \cdot \left(\frac{r}{R}\right)^{1/n}$

3.5.7.2 Slit

Table 3.5 summarizes the analytical equations for isothermal, steady flow of Newtonian fluids and power-law fluids through a slit.

Example 3.9 Pressure Drop Through a Circular Die

A high density polyethylene (HDPE) with 0.950 g/cm^3 density, used in Example 3.5, is extruded through a circular die with 10 mm diameter and 100 mm length at the melt temperature of 190 °C to produce a rod. The viscosity data of the HDPE at 190 °C are given in the figure of Example 3.5 in Section 3.5.2.2. The density of HDPE at 190 °C is 0.7612 g/cm^3.

Calculate the pressure drop through the die and the shear rate at the die wall for the two production rates given below.

1. Production rate = 100 kg/h
2. Production rate = 200 kg/h

Discuss factors that might affect the accuracy of the calculated results.

Solution:

The equations in Table 3.4 for the flow through a circular tube are used in the following calculations.

1. Production rate = 100 kg/h

The flow rate is

$$Q = 100{,}000\ \frac{\text{g}}{\text{h}} \times \frac{1\ \text{h}}{3{,}600\ \text{s}} \times \frac{1\ \text{cm}^3}{0.7612\ \text{g}} = 36.49\ \text{cm}^3/\text{s}$$

Newto ia fluid method:

The apparent shear rate at the die wall is

$$\dot{\gamma}_a = \frac{4Q}{\pi R^3} = \frac{4 \times 36.49\ \text{cm}^3/\text{s}}{3.14 \times (0.5\ \text{cm})^3} = 371.9\ \text{s}^{-1}$$

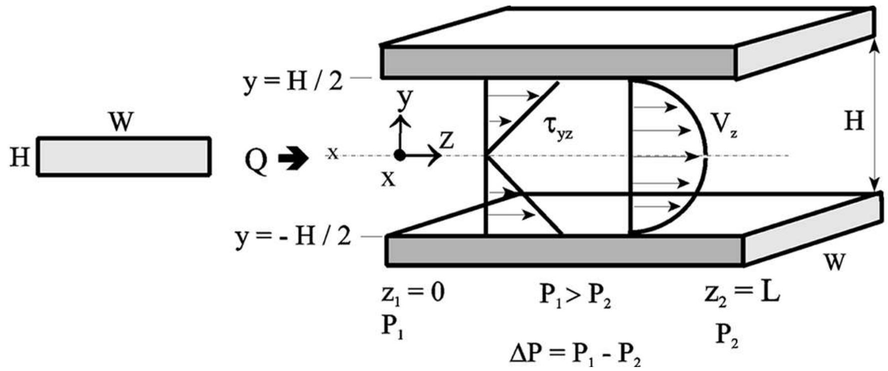
The viscosity at 371.9 s^{-1} is read from the figure in Example 3.5.

= $\mu \approx 400$ Pa-s

The pressure drop according to the Newtonian fluid equation is

$$\Delta P = \frac{8\mu L \cdot Q}{\pi R^4} = \frac{8 \times 400\ \text{Pa-s} \times 10\ \text{cm} \times 36.49\ \text{cm}^3/\text{s}}{3.14 \times (0.5\ \text{cm})^4} = 5{,}950\ \text{kPa (862 psi)}$$

Table 3.5 Isothermal Flow Through Slit



	Newtonian Fluid	Power- aw-Fluid
Viscosity	$\eta = \mu = \text{constant}$	$\eta = \eta^o \cdot \left(\frac{\dot{\gamma}}{\dot{\gamma}^o}\right)^{(n-1)}$ with $\dot{\gamma}^o = 1.0\ s^{-1}$
Shear stress at the wall	$\tau_w = \frac{\Delta P \cdot H}{2L}$	$\tau_w = \frac{\Delta P \cdot H}{2L}$
Shear rate at the wall	$\dot{\gamma}_a = \frac{6Q}{W \cdot H^2}$	$\dot{\gamma}_w = \frac{\dot{\gamma}_a}{3} \cdot \left(\frac{2n+1}{n}\right)$
Maximum velocity at the mid-plane	$V_o = \frac{H^2 \cdot \Delta P}{8 \mu L}$	$V_o = \frac{H}{2}\left(\frac{n \cdot \dot{\gamma}^o}{n+1}\right) \cdot \left(\frac{H \cdot \Delta P}{2\eta^o \cdot \dot{\gamma}^o \cdot L}\right)^{1/n}$
Average velocity	$V_{avg} = \frac{2}{3} V_o$	$V_{avg} = \left(\frac{n+1}{2n+1}\right) \cdot V_o$
Volumetric flow rate	$Q = \frac{W \cdot H^3 \cdot \Delta P}{12 \mu L}$	$Q = W \cdot H \cdot V_{avg}$
Pressure drop	$\Delta P = \frac{12 \mu L \cdot Q}{W \cdot H^3}$	$\Delta P = \left[\left(\frac{2Q}{W \cdot H^2}\right) \cdot \left(\frac{2n+1}{n \cdot \dot{\gamma}^o}\right)\right]^n \cdot \left(\frac{2\eta^o \cdot \dot{\gamma}^o \cdot L}{H}\right)$
Velocity profile	$V_z(y) = V_o \cdot \left[1 - \left\lvert\frac{2y}{H}\right\rvert^2\right]$	$V_z(y) = V_o \cdot \left[1 - \left\lvert\frac{2y}{H}\right\rvert^{(n+1)/n}\right]$
Shear rate profile	$\dot{\gamma}(y) = \left(\frac{4V_o}{H}\right) \cdot \left\lvert\frac{2y}{H}\right\rvert$	$\dot{\gamma}(y) = \left(\frac{2V_o}{H}\right) \cdot \left(\frac{n+1}{n}\right) \cdot \left\lvert\frac{2y}{H}\right\rvert^{1/n}$

Power-law fluid method:

The shear rate in the die is maximum at the die wall and decreases toward the center of the die, becoming zero at the center. The important shear rate in this calculation is the true shear rate at the die wall, which will be higher than the apparent shear rate of 371.9 s^{-1}. Calculation of the true shear rate requires the power-law exponent, , whose value depends on the range of shear rate. Referring to Example 3.5 in Section 3.5.2.2, the power-law for the HDPE covering the shear rate range of about 30–300 s^{-1} was found to be

$$\eta = \eta^{o} \cdot \dot{\gamma}^{(n-1)}, \quad \text{where } \eta^{o} = 17{,}589 \text{ Pa-s and } n = 0.378$$

The above power-law constants will be assumed to be applicable to the true shear rate at the die wall. The true shear rate at the die wall is

$$\dot{\gamma}_{w} = \frac{\dot{\gamma}_{a}}{4} \cdot \left(3 + \frac{1}{n}\right) = \frac{371.9 \text{ s}^{-1}}{4} \times \left(3 + \frac{1}{0.378}\right) = 524.9 \text{ s}^{-1}$$

The true shear rate is found to be 1.41 times the apparent shear rate because of the high shear sensitivity of the viscosity with a small value of . The power-law viscosity at 524.9 s^{-1} is

$$\eta = \eta^{o} \cdot \dot{\gamma}^{(n-1)} = 17{,}589 \text{ Pa-s} \times (598.3)^{(0.378-1)} = 329.8 \text{ Pa-s}$$

The pressure drop according to the power-law fluid equation is

$$\begin{aligned} Q &= \pi R^{2} \cdot V_{avg} = \pi R^{2} \cdot \left(\frac{n+1}{3n+1}\right) \cdot V_{o} \\ &= \pi R^{2} \cdot \left(\frac{n+1}{3n+1}\right) \cdot \left(\frac{n\,R \cdot \dot{\gamma}^{o}}{n+1}\right) \cdot \left(\frac{R \cdot \Delta P}{2\eta^{o} \cdot \dot{\gamma}^{o} \cdot L}\right)^{1/n} \end{aligned}$$

$$\begin{aligned} \Delta P &= \left[\frac{Q}{\pi R^{3}} \cdot \left(\frac{3n+1}{n \cdot \dot{\gamma}^{o}}\right)\right]^{n} \cdot \left(\frac{2\eta^{o} \cdot \dot{\gamma}^{o} \cdot L}{R}\right) \\ &= \left[\frac{36.49 \text{ cm}^{3}/\text{s}}{3.14 \times (0.5\,\text{cm})^{3}} \cdot \left(\frac{3 \times 0.378 + 1}{0.378 \times 1.0\ /\text{s}}\right)\right]^{0.378} \\ &\quad \cdot \left(\frac{2 \times 17{,}589 \text{ Pa-s} \times 1.0/\text{s} \times 10 \text{ cm}}{0.5\,\text{cm}}\right) = 7{,}507 \text{ kPa } (1{,}088 \text{ psi}) \end{aligned}$$