Static of fluids

Pressure distribution (force and momentum)

Buoyancy

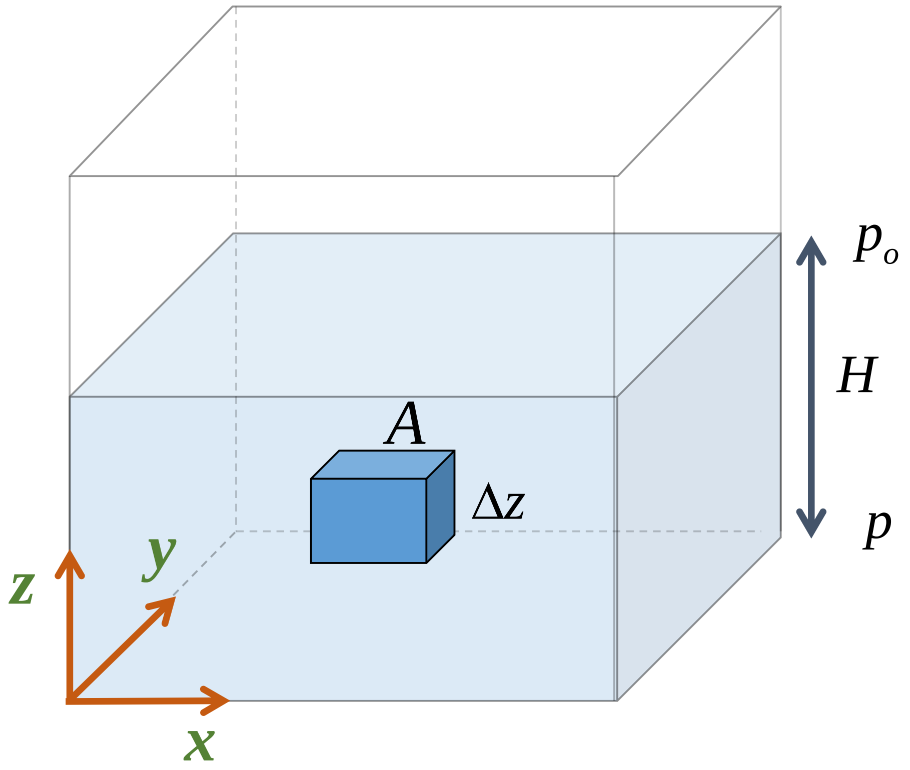
Stability

Capillary

Static of fluids: Fluids at rest, or at rest respect to a frame of reference, or a frame standing in one fixed point in the container or vessel of the fluid.

Objective: Calculate the pressure or forces (or stresses) at given point in space. The fluid may be at rest, or at zero velocity respect to a point in the container or reservoir.

1. Determine the pressure in a parallelepiped shaped container, at a depth *H*



System is at rest

$$p\,A\Big|_{z} - p\,A\Big|_{z+\Delta z} - \rho\, g\, \Delta z\, A = 0$$

$$\lim_{\Delta z \to 0} \frac{p\,A|_{z+\Delta z} - p\,A|_{z}}{\Delta z} = -\rho\, g$$

$$\frac{dp}{dz} = -\rho\, g \qquad \frac{dp}{dx} = 0 \qquad \frac{dp}{dy} = 0$$

$$\underline{\nabla} p = \rho\, \underline{g} \qquad \underline{g} = 0\,\hat{\imath} + 0\,\hat{\jmath} - g\,\hat{k}$$

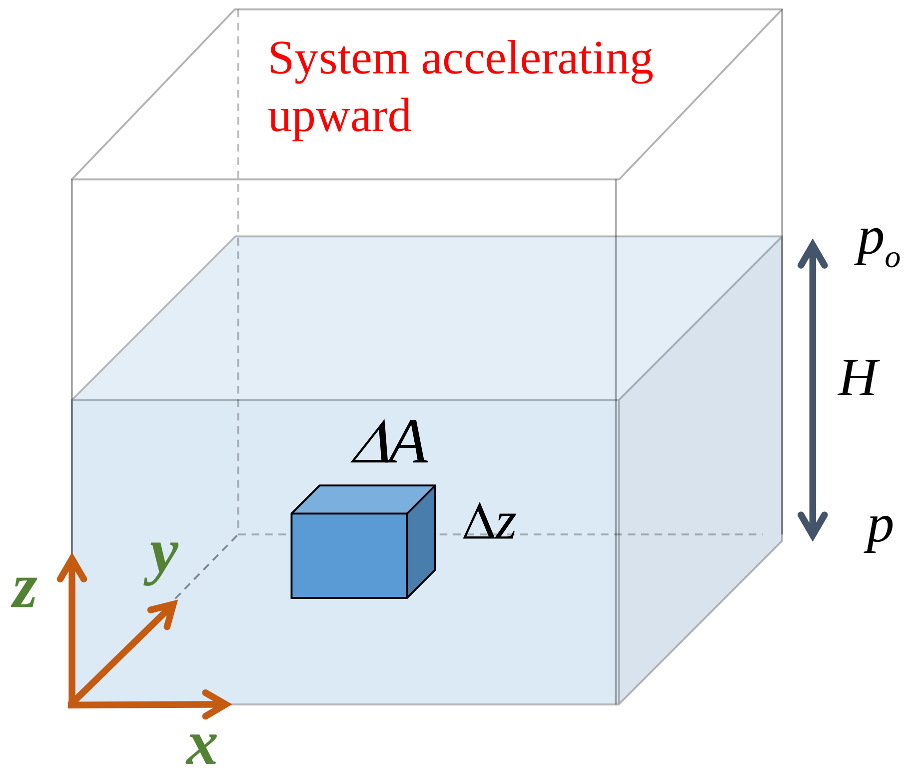
$$\int_{p}^{p_o} dp = -\int_{0}^{H} \rho\, g\, dz \qquad p_o - p = -\rho\, g\, H$$

$$\boxed{p = p_o + \rho\, g\, H}$$

Then pressure is calculated with the pressure at the surface and you add the hydraulic head, this is the product of density of fluid, gravity and depth.

Inertial frame of reference

2. If the container is under acceleration, then is not an inertial frame of reference, but using an inertial frame of reference to quantify the kinematics, we can re write the force balance. **Determine the pressure in a parallelepiped shaped container, at a depth *H***



$$p\,\Delta A\Big|_{z} - p\,\Delta A\Big|_{z+\Delta z} - \rho\, g\,\Delta z\,\Delta A = \frac{d}{dt}[\rho\, v_z \Delta z\,\Delta A]$$

$$-\left[\lim_{\Delta z \to 0} \frac{p\,A|_{z+\Delta z} - p\,A|_{z}}{\Delta z}\right] - \rho\, g = \rho\, a_z$$

$$-\frac{dp}{dz} - \rho\, g = \rho\, a_z \qquad g_z = -|g|$$

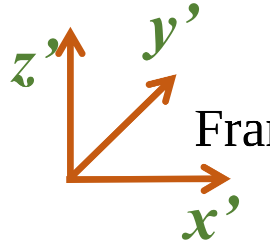
$$-\frac{dp}{dz} + \rho g_z = \rho\, a_z$$

Extending for all the axes

$$-\frac{dp}{dx}\hat{\imath} - \frac{dp}{dy}\hat{\jmath} - \frac{dp}{dz}\hat{k} + \rho\, g_x\hat{\imath} + \rho\, g_y\hat{\jmath} + \rho\, g_z\hat{k} = \rho\, a_x\hat{\imath} + \rho\, a_y\hat{\jmath} + \rho\, a_z\hat{k}$$

$$\rho\underline{a} = -\underline{\nabla}p + \rho\,\underline{g} \qquad \underline{\nabla}p = \rho\left[\underline{g} - \underline{a}\right]$$

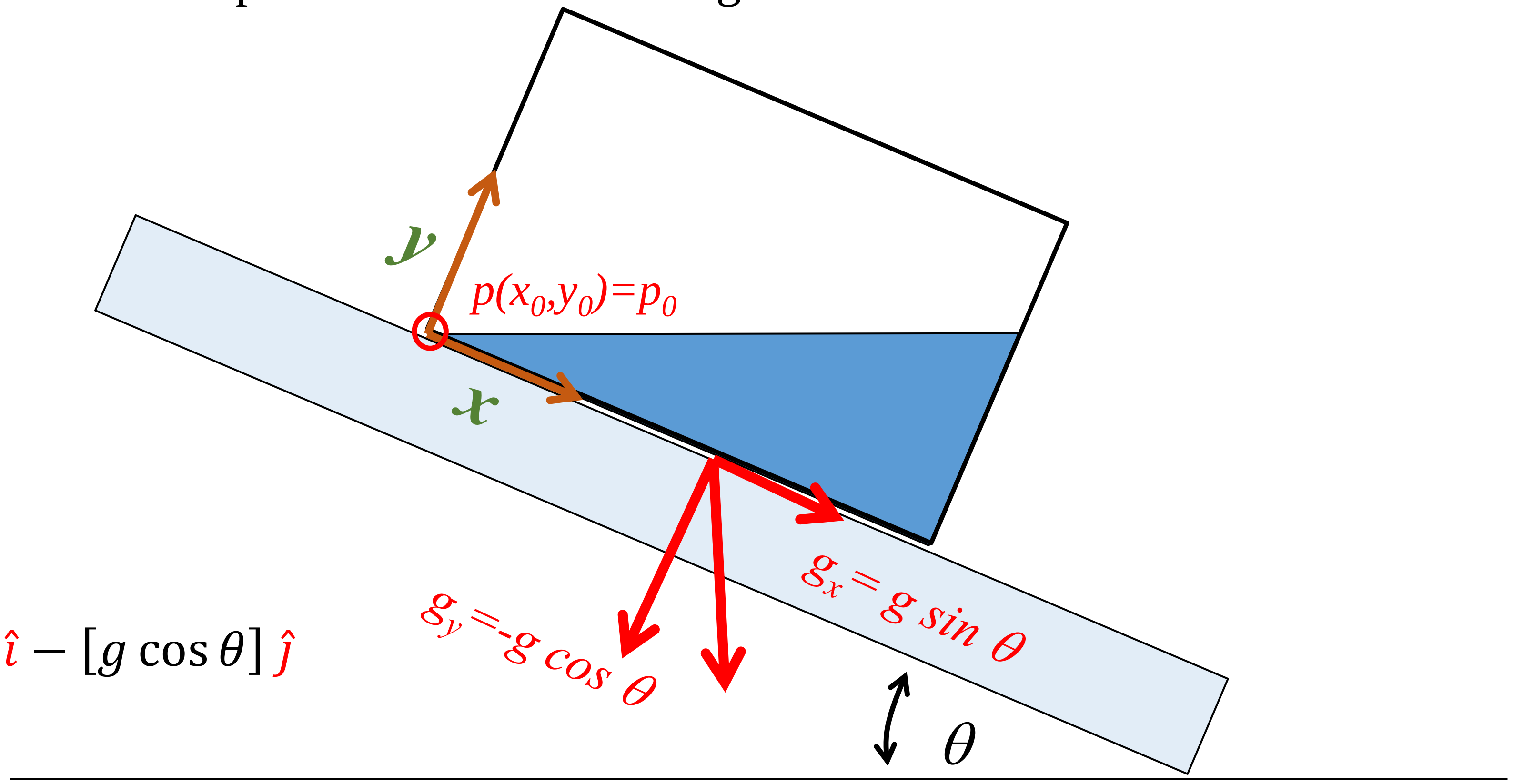
$$p = p_0 + \rho\, H\,(a_z + g)$$



Frame of reference is at rest

Inertial frame of reference

3. Determine the pressure in a parallelepiped shaped container, at any position. The container rests in a inclined plane as shown in the figure.



$$\underline{g} = [g \sin\theta]\,\hat{\imath} - [g\cos\theta]\,\hat{\jmath}$$

$$\underline{\nabla} p = \rho\,[g \sin\theta]\,\hat{\imath} - \rho\,[g\cos\theta]\,\hat{\jmath}$$

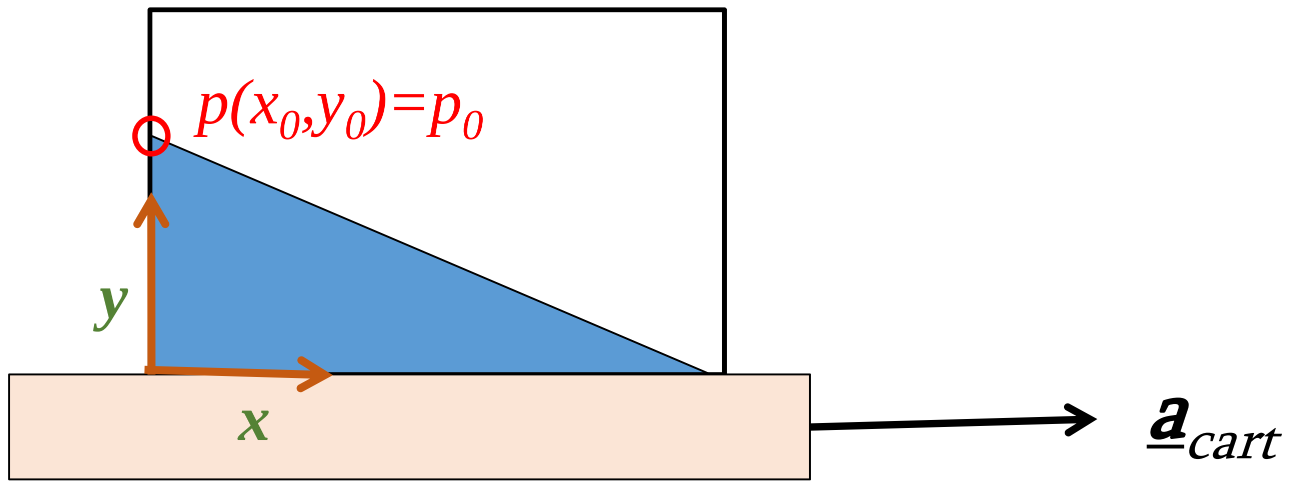
$$dp = \frac{dp}{dx}dx + \frac{dp}{dy}dy = \rho\,[g\sin\theta]\,dx - \rho\,[g\cos\theta]\,dy$$

$$\boxed{p - p_0 = \rho\,[g\sin\theta]\,[x - x_0] - \rho\,[g\cos\theta]\,[y - y_0]}$$

System is at rest

Inertial frame of reference

4. Determine the pressure in a parallelepiped shaped container, at any position. The container rests accelerating cart in *x*-positive direction



If the frame of reference is selected moving with the cart, then the acceleration field has two components (This is not an inertial frame of reference, then a fictitious force is needs, which must be included if that approach is used, otherwise use the equation of motion)

$\underline{b}$ = any external or fictitious body force field per unit mass

$$\underline{b} = -a_{cart}\hat{\imath} - g\,\hat{\jmath}$$

$$\underline{\nabla} p = \rho\,\underline{b}$$

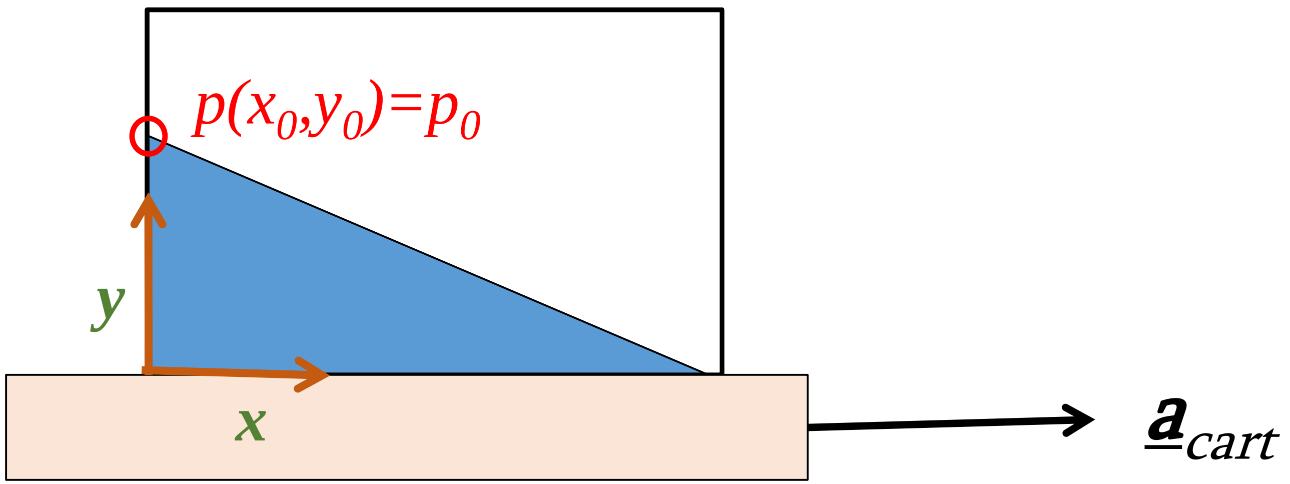
$$\underline{\nabla} p = -\rho\, a_{cart}\hat{\imath} - \rho\, g\,\hat{\jmath}$$

$$dp = -\rho\, a_{cart}dx - \rho\, g\, dy$$

$$\boxed{p - p_0 = -\rho\, a_{cart}[x - x_0] - \rho\, g\,[y - y_0]}$$

Non-inertial frame of reference

Determine the pressure in a parallelepiped shaped container, at any position. The container rests accelerating cart in *x*-positive direction



If different approach is used, i.e. The Euler equation of motion

$$\rho\, \underline{a} = -\underline{\nabla} p + \rho \underline{g}$$

$$\underline{a} = |a_{cart}|\,\hat{\imath} + 0\,\hat{\jmath} + 0\,\hat{k}$$

$$\underline{g} = 0\,\hat{\imath} - |g|\,\hat{\jmath} + 0\,\hat{k}$$

$$\underline{\nabla} p = \rho\left[\underline{g} - \underline{a}\right]$$

$$\frac{dp}{dx}\hat{\imath} + \frac{dp}{dy}\hat{\jmath} + \frac{dp}{dz}\hat{k} = -\rho|a_{cart}|\,\hat{\imath} - \rho|g|\,\hat{\jmath} + 0\,\hat{k}$$

$$dp = \frac{dp}{dx}dx + \frac{dp}{dy}dy + \frac{dp}{dz}dz = -\rho|a_{cart}|dx - \rho|g|\,dy + 0\,dz$$

$$p - p_0 = -\rho\, a_{cart}[x - x_0] - \rho\, g\,[y - y_0]$$

Non-inertial frame of reference

5. Determine the pressure in cylindrical container, at any position. The cylinder spins at constant angular speed on its own axis of symmetry. The axis of symmetry is parallel to the gravitational field.

New nomenclature

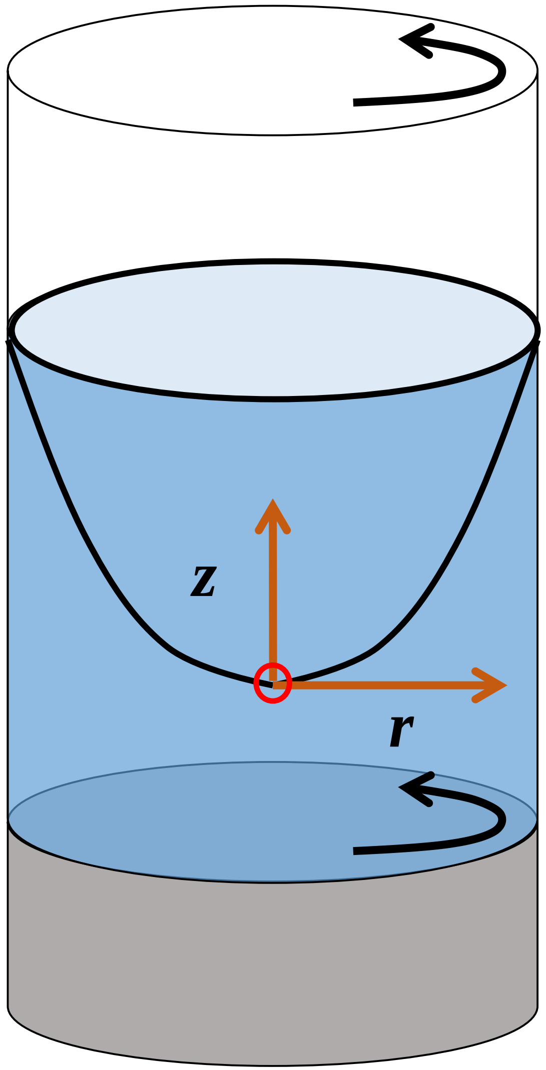
External force

$$\underline{F}_{ext} = m\,\underline{g}$$

External force field

Or external force per unit mass

$$\frac{\underline{F}_{ext}}{m} = \underline{f}_m = \underline{g}$$



$$\underline{\nabla} p = \rho\,\underline{g} = \rho\,\underline{b} = \rho\,\underline{f}_m$$

The gravitational field, is an external force field, lets call this force field $\underline{b}$ (i.e. f_m force per unit mass, or external force field)

$$\underrightarrow{\nabla} p = \rho b_r\ \hat{e}_r + \rho b_z\ \hat{e}_z + \rho b_\theta\ \hat{e}_\theta$$

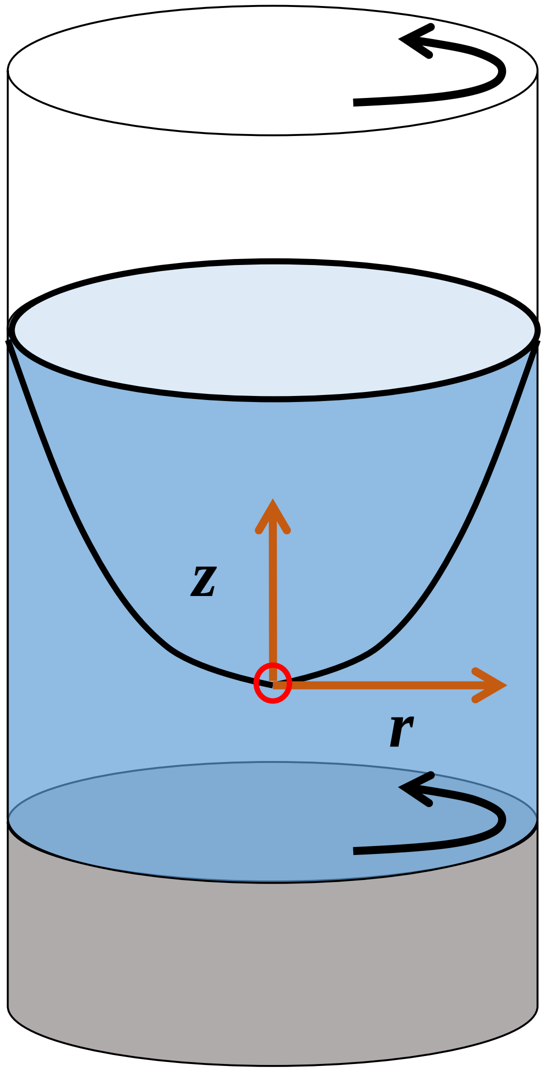
$$\underline{b} = \omega^2\, r\,\hat{e}_r - |g|\ \hat{e}_z + (0)\,\hat{e}_\theta = \omega^2\, r\,\hat{e}_r - |g|\ \hat{k}$$

$$dp = \rho\left[\omega^2\, r\, dr - |g|\ dz\right]$$

$$\boxed{\frac{p - p_0}{\rho} = \omega^2 \left(\frac{r^2 - r_0^2}{2}\right) - |g|\,(z\text{-}z_0)}$$

Non-inertial frame of reference

Determine the pressure in cylindrical container, at any position. The cylinder spins at constant angular velocity on its own axis of symmetry. The axis of symmetry is parallel to the gravitational field.



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$$\vec{\nabla} p = \rho b_r\ \hat{e}_r + \rho b_z\ \hat{e}_z + \rho b_\theta\ \hat{e}_\theta$$

$$\underline{b} = \omega^2\, r\,\hat{e}_r - |g|\ \hat{e}_z + (0)\,\hat{e}_\theta = \omega^2\, r\,\hat{e}_r - |g|\ \hat{k}$$

$$\frac{dp}{\rho} = \omega^2\, r\, dr - |g|\ dz$$

$$\boxed{p - p_0 = \rho\,\omega^2\left(\frac{r^2 - r_0^2}{2}\right) - \rho\,|g|\,(z - z_0)}$$

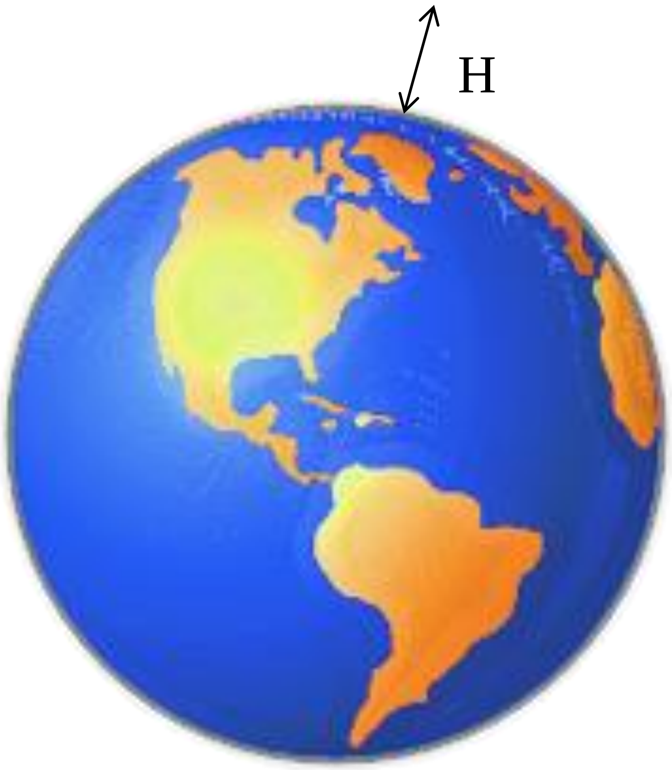
Non-inertial frame of reference

Hydrostatic pressure equation for rotating systems

$$\frac{p}{\rho} + gz - \frac{\Omega^2 r^2}{2} = \frac{p_0}{\rho} + gz_0 - \frac{\Omega^2 r_0^2}{2}$$

6. Determine the pressure over the surface of the earth, at any position over the sea level (when temperature variations with height are not important).

$$\frac{dp}{dz} = -\rho g$$



Assuming ideal gas

$$\frac{dp}{dz} = -\frac{p(M)}{RT} g$$

If the temperature is constant

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_0^H \frac{(M)}{RT} g dz$$

$$\boxed{\ln\left(\frac{p}{p_0}\right) = -\int_0^H \frac{(M)}{RT} g dz = -\frac{(M) g H}{RT}}$$

g=9.80665m/s^2, M=28.84 kg/kmol, R=8314.34 Pa-m^3/kmol-K, T=298.15 K

7. Determine the pressure over the surface of the earth, at any position over the sea level. (when temperature variations with height are important).

$$\frac{dp}{dz} = -\rho g$$

Γ is know as lapse rate constant

Assuming Halley's law to take into consideration temperature variations with height, and also considering ideal gas law

$$T = T_o - \Gamma\, z \qquad \Gamma = 6.5 \times 10^{-3}\ \text{K/m} \qquad \frac{dp}{dz} = -\frac{p(M)}{RT} g$$

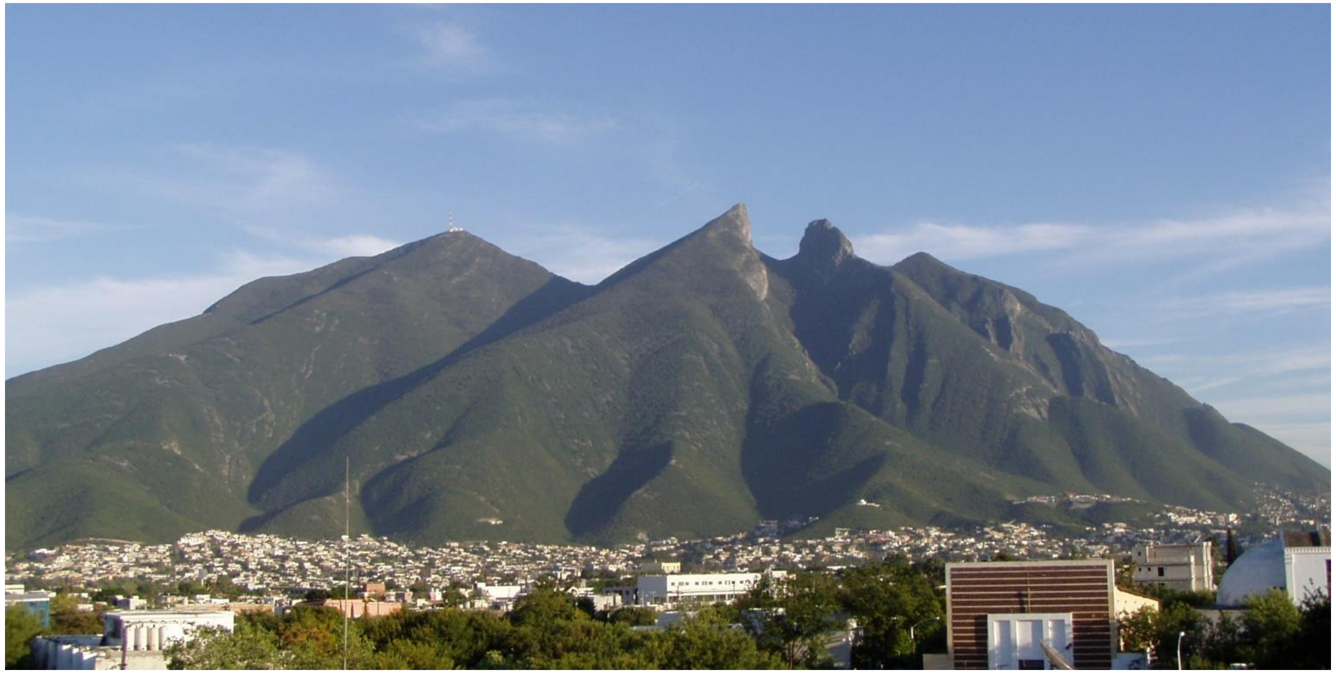
$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{0}^{H} \frac{M\,g}{R(T_o - \Gamma\, z)} dz \qquad \ln\left(\frac{p}{p_0}\right) = \frac{M\,g}{R\Gamma} \ln\left(\frac{T_o - \Gamma\, z}{T_o - \Gamma z_o}\right)$$

$$\boxed{\frac{p}{p_0} = \left[\frac{T_o - \Gamma\, z}{T_o - \Gamma z_o}\right]^{\frac{M\,g}{R\Gamma}}}$$

g=9.80665m/s^2, M=28.84 kg/kmol, R=8314.34 Pa-m^3/kmol-K, T=298.15 K, Γ= 6.5×10^{-3}(K/m)

Gravity and temperature are function of height (altitude).

$$\ln\left(\frac{p}{p_0}\right) = -\int_0^H \frac{(M)}{RT_o(a+bz)} g_0 \left(\frac{r_0}{r_0+z}\right)^2 dz$$



The pressure in Monterrey (25°40′N 100°18′W, 540 m over the sea level) is 720 mmHg estimate the pressure at highest peak of the Saddle Mountain located 1820 m over the sea level.

Solution

Pressure assuming constant density for air

$$p_o = p_B + \rho_{Air} \cdot g \cdot (z - z_o)$$

Pressure assuming constant temperature

$$\ln\left(p_A/p_o\right) = -M \cdot g \cdot \frac{z - z_o}{R \cdot T}$$

Pressure assuming linear variation of temperature respect to height

$$p_C/p_o = \left(\frac{T - \alpha \cdot z}{T - \alpha \cdot z_o}\right)^{M \cdot \frac{g}{\alpha \cdot R}}$$

$\alpha = 0.0065$ [K/m]
$g = 9.807$ [m/s^2]
$M = 28.84$ [kg/kmol]
$p_{A,Torr} = 619$ [torr]
$p_{B,Torr} = 611.2$ [torr]
$p_{C,Torr} = 616.5$ [torr]
$p_{D,Torr} = 616.5$ [torr]
$p_{o,Torr} = 720$ [torr]
$\rho_{Air} = 1.156$ [kg/m^3]
$T = 288.2$ [K]
$\zeta = 1820$ [m]

$\beta = 0.00002256$ [1/m]
$\gamma = 1.570 \times 10^{-7}$ [1/m]
$p_A = 82530$ [Pa]
$p_B = 81487$ [Pa]
$p_C = 82189$ [Pa]
$p_D = 82193$ [Pa]
$p_o = 95992$ [Pa]
$R = 8314$ [Pa-m^3/kmol-K]
$r_o = 6.371 \times 10^6$ [m]
$z = 1820$ [m]
$z_o = 540$ [m]

Pressure assuming linear variation of temperature respect to height, and variable gravity

$$\ln\left(p_D/p_o\right) = \left(M \cdot \frac{g}{R \cdot T}\right) \cdot \int_{z_o}^{z} \left[-\frac{1}{(1 - \beta \cdot \zeta) \cdot (1 + \gamma \cdot \zeta)^2}\right] \mathrm{d}\zeta$$

The problem can be solved using different assumptions, all of them will be listed and comparison will be made.

Reflect about the following results:

Nort Peak saddle Mountain (Monterrey) Elev=1820 m Reference Monterrey Elevation = 540 m, p = 720 mmHg	
Model	Pressure (mmHg)
I	611.2
II	619
III	616.5
IV	616.5

Cerro de la Silla

Small Mole Mountain (Monterrey) Elev=1000 m Reference Monterrey Elevation = 540 m, p = 720 mmHg	
Model	Pressure (mmHg)
I	681.9
II	680.1
III	681.3
IV	681.3

Cerro del Topo Chico

Mount Everest (Himalayas) Elev=8848 m Reference Patna India= 53 m, p = 755 mmHg	
Model	Pressure (mmHg)
I	-29.08
II	269.1
III	238.5
IV	238.9

Himalayas

Model	Assumptions
I	Incompressible fluid, constant gravity
II	Ideal Gas, Isothermal, constant gravity
III	Ideal Gas, Linear temperature variations, constant gravity
IV	Ideal gas, Linear temperature dependence

In a nutshell (On Earth)

- When changes in height are less than half of kilometer, changes in density can be neglected to estimate pressure variations.

- When changes in height are less than a kilometer, variations in temperature must be taken into consideration.

- When changes in height are greater than 10 km, gravity variations should be considered.

Why the temperature decrease with height ?

$$ds = c_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp$$

Entropy changes can be written in terms of temperature and pressure changes

If we assume that the process of exchange of air mass within layers is adiabatic and reversible or gradual, the process must approximate to an isentropic one, then entropy remains constant.

$$c_p \frac{dT}{T} = \left(\frac{\partial v}{\partial T}\right)_p dp$$

If air behaves as an ideal gas, then we have ideal gas equation of state.

$$p\,v = R\,T \qquad \left(\frac{\partial v}{\partial T}\right)_p = R/p \qquad p\,M = \rho\,R\,T$$

$$c_p \frac{dT}{T} = \frac{R}{p} dp = -\frac{R}{p} \rho\,g\,dz = -\frac{M}{T} g\,dz$$

Then pressure and temperature are related, and if this equation is coupled with the pressure gradient equation (Euler equation of fluid dynamics), results in the lapse rate equation.

$$\frac{dT}{dz} = -\frac{M}{c_p} g = -\Gamma$$

This expression is known as a lapse rate.

$$\frac{dT}{dz} = -\frac{M}{c_p} g = -\Gamma$$

The air can be considered as an ideal gas, and can be assumed as diatomic molecule (made up of diatomic molecules N_2 and O_2). Under this assumption and from molecular theory we have

$$c_p = \frac{7}{2} R$$

$$\Gamma = \frac{2\,M\,g}{7\,R} \approx \frac{2\,(29\ \mathrm{kg/kmol})(9.81\ \mathrm{m/s^2})}{7\,(8314.34\ \mathrm{Pa - m^3/kmol - K})} \approx 9.8 \times 10^{-3}\ \mathrm{K/m}$$

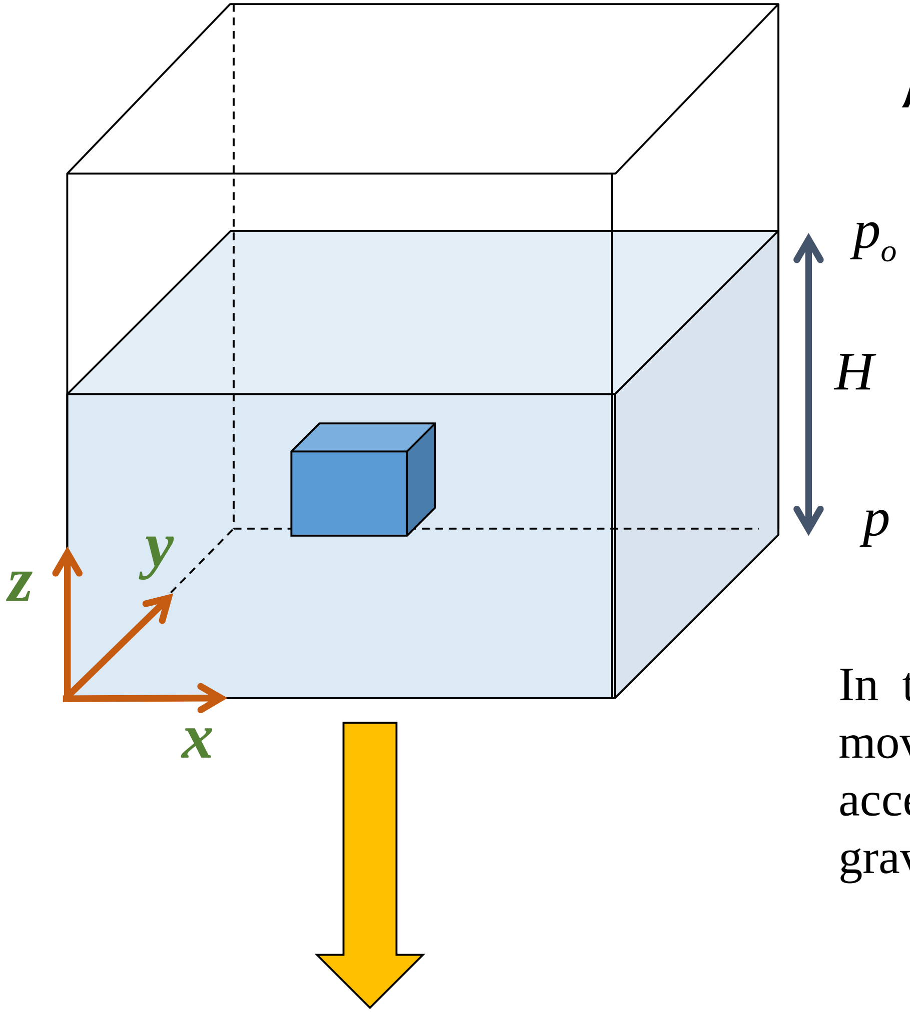
This value is known as a lapse rate, and is 9.8 K/km for completely dry air. It takes the value of 6.5 K/km if water vapor in the air does not condense to liquid ascending , while the value of 5.5 K/km is used when condensation takes place. (*)

Fang Chung. *An Introduction to Fluid Mechanics*. Sprinegr, 2019

λ= heat of vaporization of water, ϕ = relative humidity, M_w = molar mass of water.

$$\Gamma = \frac{M\,g(1+\beta\,r)}{R\left(c_p/R + r(\beta\,\epsilon)^2\right)} \qquad \beta = \frac{\lambda\,M}{R\,T} \qquad \epsilon = \frac{M_w}{M} \qquad r = \frac{\epsilon\,\varphi}{1-\varphi}$$

What if …the system is moving downward at accelerating of 9.80665 m/s^2



$$\rho \underline{a} = -\underline{\nabla}\, p + \rho \underline{g} \qquad \underline{a} = -|g|\hat{k} \quad \underline{g} = -|g|\hat{k}$$

$$\frac{dp}{dz} = \rho\left(-|g| - \left(-|g|\right)\right)$$

$$\frac{dp}{dz} = \rho\left(-|g| + |g|\right) = 0$$

In this case the frame of reference will move with the parallelepiped, then the acceleration component will cancel with gravity. Then there is no pressure gradient

System is moving downward at accelerating of 9.80665 m/s^2

Rigid Body motion

$$\underset{\rightarrow}{\nabla}\, p = \rho\left(\underset{\rightarrow}{g} - \underset{\rightarrow}{a} \right)$$

Hint: Signs depend on the frame of reference, and the definition of the system.

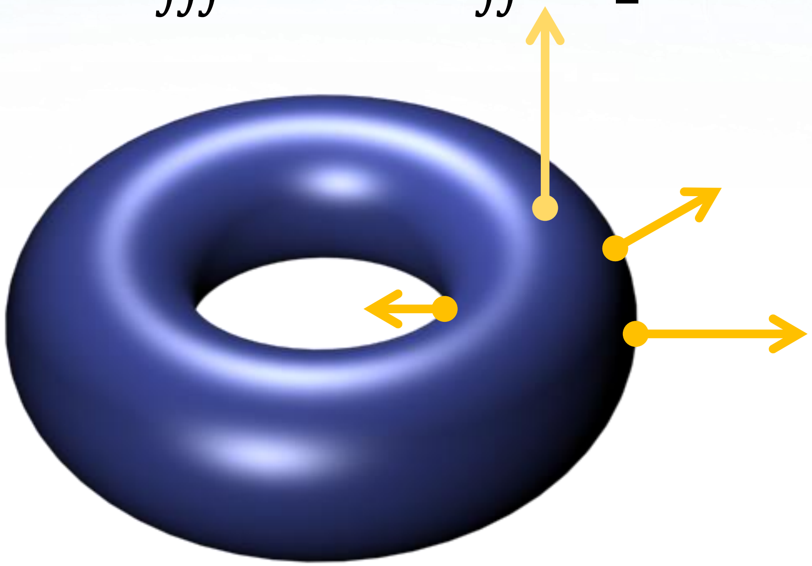
For a frame of reference where “z” is measured against the gravitational field.

$$\underset{\rightarrow}{g} = (0)\underset{\rightarrow}{i} + (0)\underset{\rightarrow}{j} - |g|\underset{\rightarrow}{k}$$

What happens with pressure surface forces once an object is completely immersed in a stagnant fluid, or at rest respect to the immersed object

$$\rho \underline{a} = \rho\, \underline{g} - \underline{\nabla} p \qquad\qquad 0 = \rho\, \underline{g} - \underline{\nabla} \cdot \left[p\, \underline{\underline{I}} \right]$$

$$\frac{d}{dt}[m_s \underline{v}] = \iiint \rho_s \underline{g}\, dV - \oiint \underline{n} \cdot \underline{\underline{I}}\, p\, dA$$



$$\frac{d}{dt}[m_s \underline{v}] = \iiint \rho_s \underline{g}\, dV - \iiint \underline{\nabla} \cdot \left[p\, \underline{\underline{I}} \right] dV$$

$$\frac{d}{dt}[m_s \underline{v}] = \iiint [\rho_s - \rho] \underline{g}\, dV$$

Weight contribution

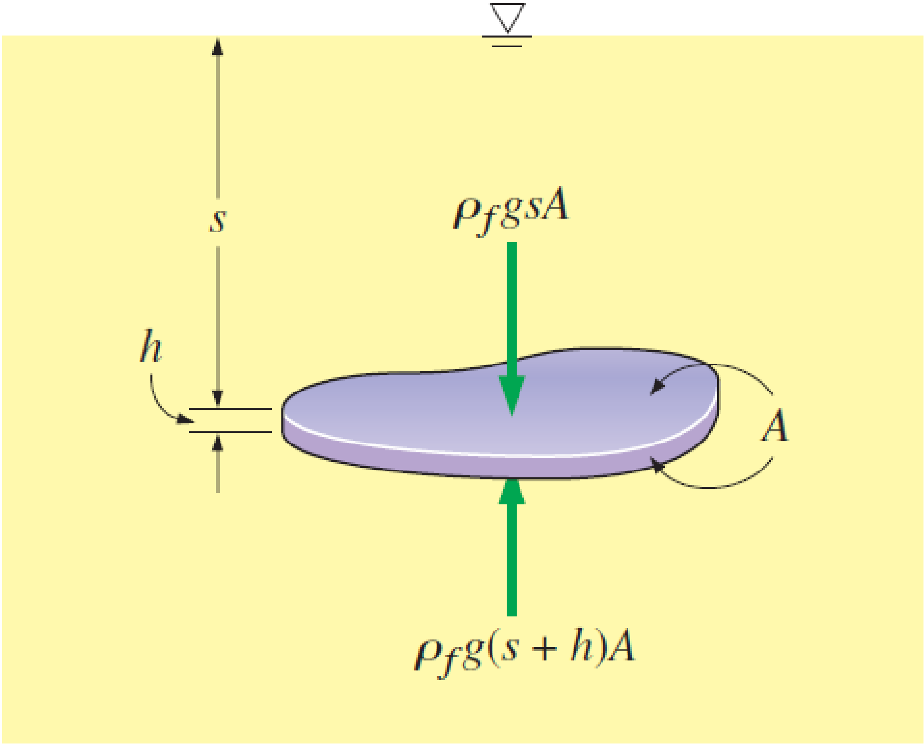
Buoyancy contribution

If the fluid is at rest, and viscous effects negligible

Turns out that for this scenario pressure forces may be replaced by the so called buoyancy, an is valid as long as the system is at rest.

Buoyancy And Stability

Buoyant force: The upward force a fluid exerts on a body immersed in it. The buoyant force is caused by the increase of pressure with depth in a fluid.



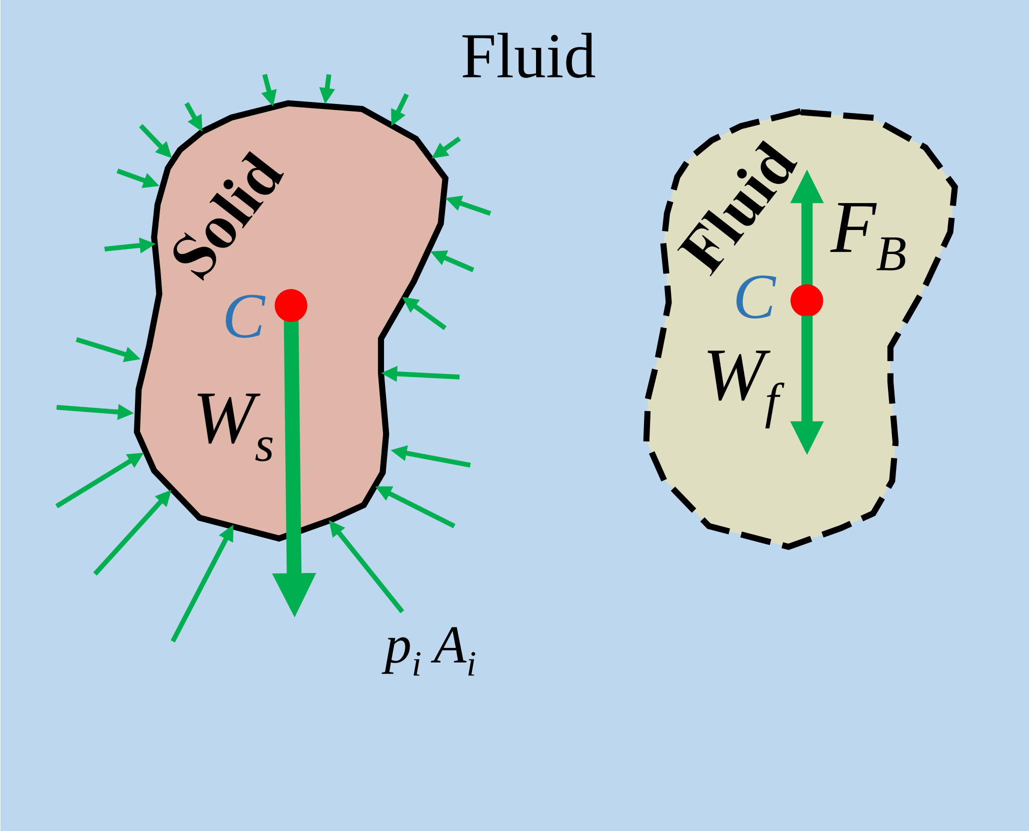
The buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.

For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface.

It is also independent of the density of the solid body.

A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface.

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f g s A = \rho_f g h A = \rho_f g V$$

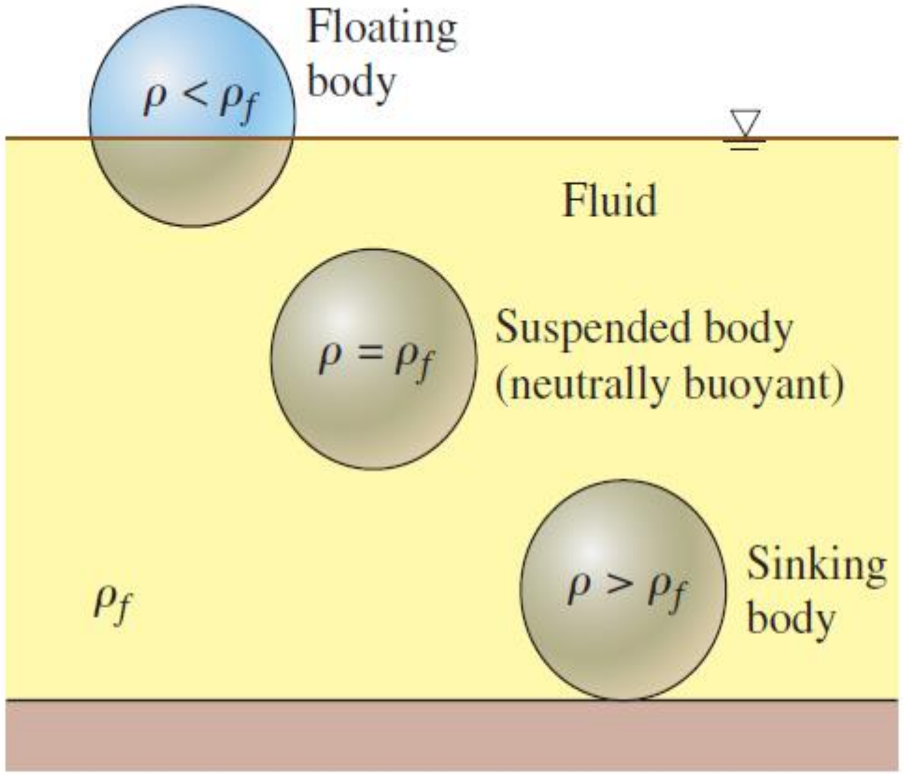


The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The buoyant force F_B acts upward through the centroid C of the displaced volume and is equal in magnitude to the weight W_f of the displaced fluid, but is opposite in direction. For a solid of uniform density, its weight W_s also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces. (Here $W_s > W$ and thus $W_s > F_B$; this solid body would sink.). The so called buoyancy is not more than the resulting force of the surface force caused by the pressure times the area of each differential of surface.

Archimedes' principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body when capillary effect is negligible:

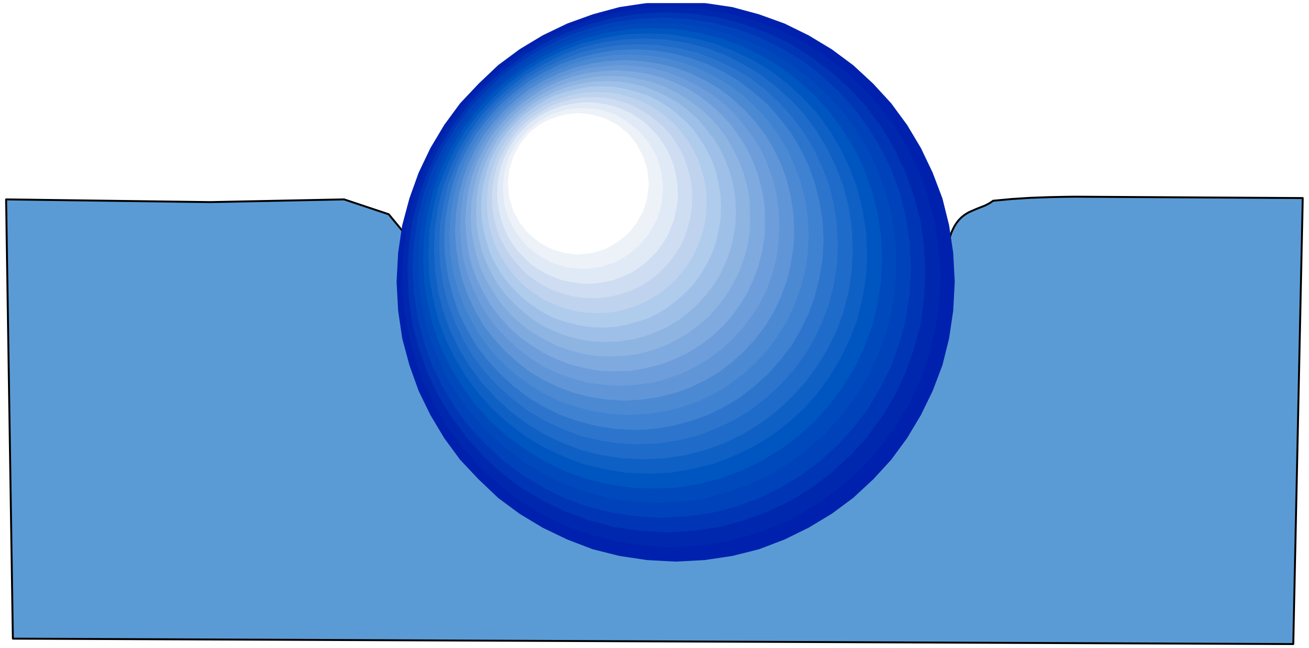
$$F_B = W \qquad \rho_f\, g\, V_{\text{sub}} = \rho_{\text{avg,body}}\, g\, V_{\text{total}} \qquad \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{avg,body}}}{\rho_f}$$



A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid if and only if capillary effect is negligible.

Warning: Moving solids, moving fluids, non uniform fluid density, and surface tension effects must be considered with different approach, and the concept of buoyancy may not be useful in theses cases.

Lets see the effect of Surface Tension.

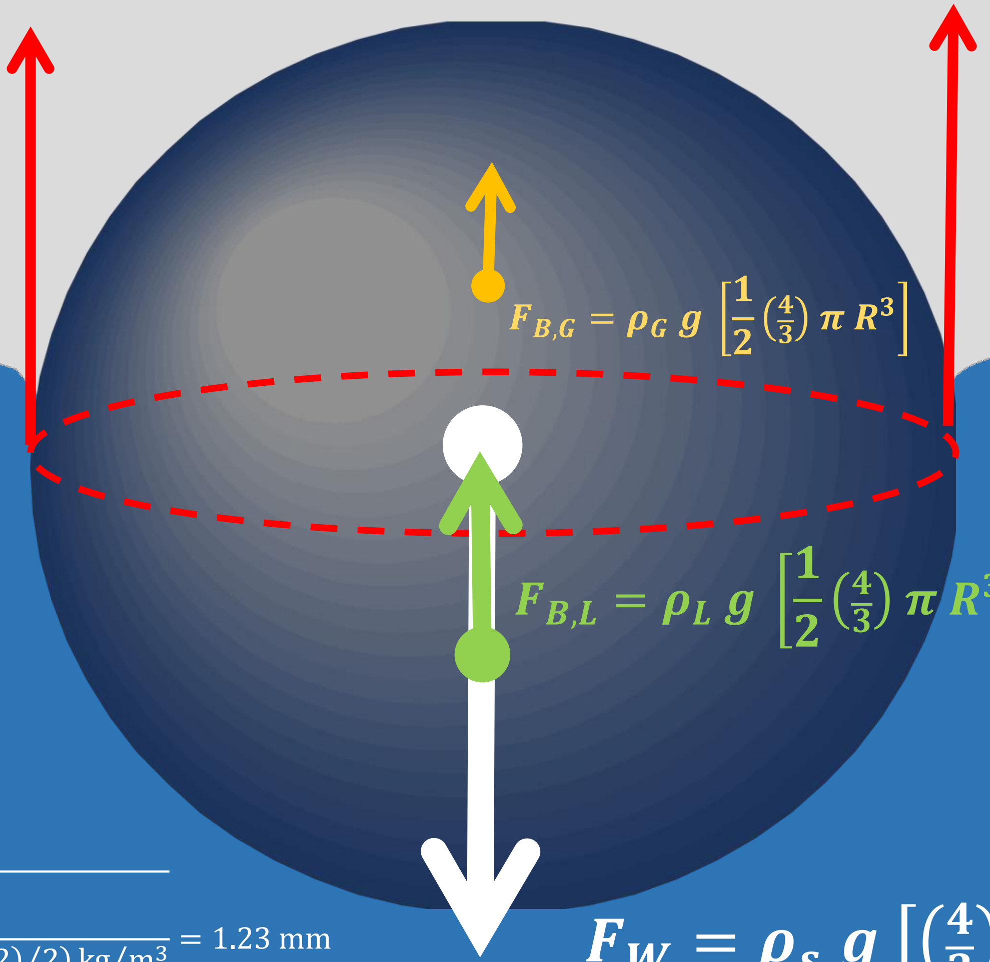


Calculate the maximum size of a steel sphere that will float in water if the surface is coated with a hydrophobic material. (i.e. contact angle 180°)

Assumption: Line force acts at the center of the sphere

$$F_s = \sigma\,[2\,\pi\,R]$$

$$R = \sqrt{\frac{3\,\sigma}{2\,g[\rho_s - (\rho_L + \rho_G)/2]}}$$



$$F_{B,G} = \rho_G\,g\,\left[\frac{1}{2}\left(\frac{4}{3}\right)\pi\,R^3\right]$$

$$F_{B,L} = \rho_L\,g\,\left[\frac{1}{2}\left(\frac{4}{3}\right)\pi\,R^3\right]$$

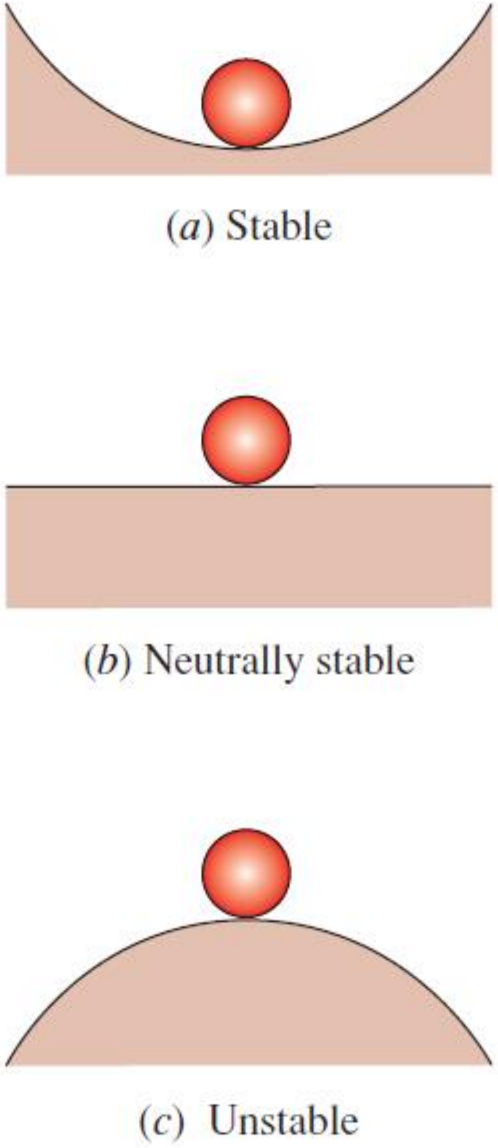
$$R = \sqrt{\frac{3\,(72.8\times 10^{-3})\,\frac{\mathrm{N}}{\mathrm{m}}}{2\,(9.81\ \mathrm{m/s^2})\,(7850 - (1000 + 1.2)/2)\,\mathrm{kg/m^3}}} = 1.23\ \mathrm{mm}$$

$$F_W = \rho_s\,g\,\left[\left(\frac{4}{3}\right)\pi\,R^3\right]$$

Stability of Immersed and Floating Bodies

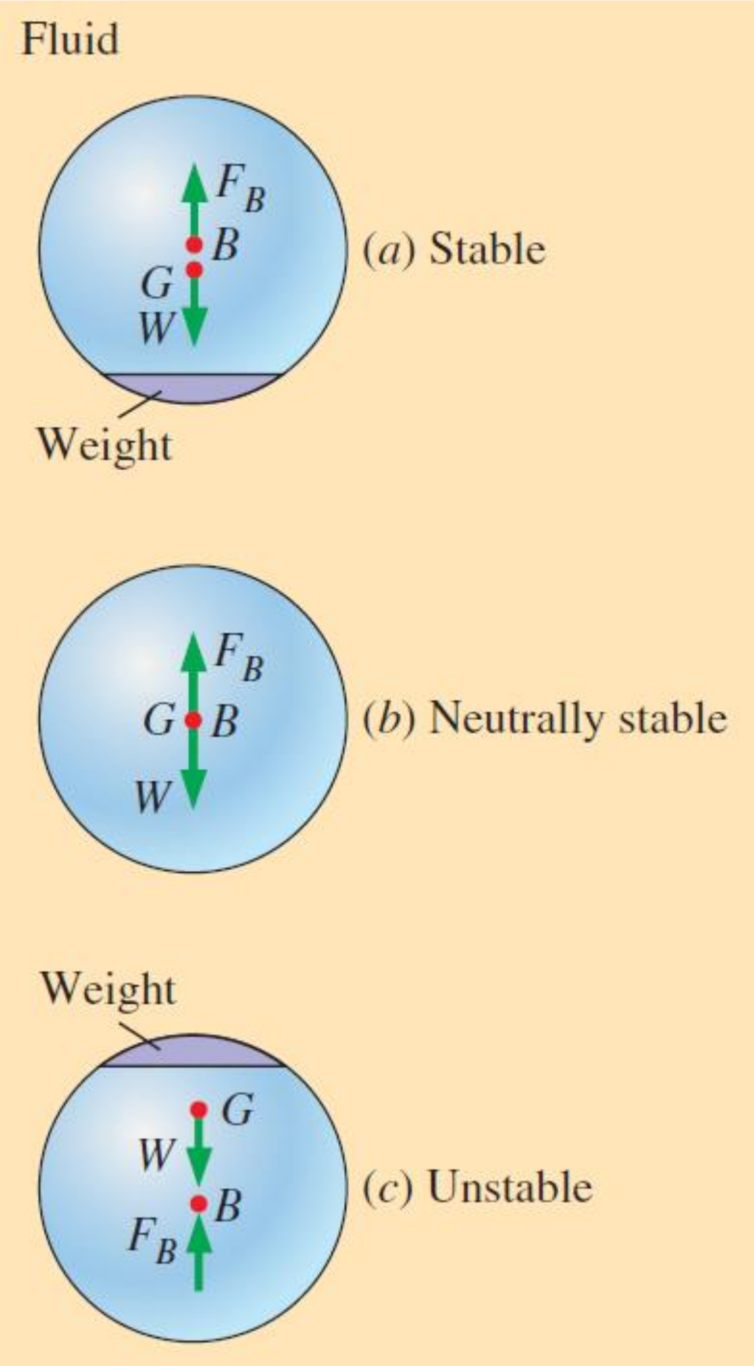
For floating bodies such as ships, stability is an important consideration for safety.

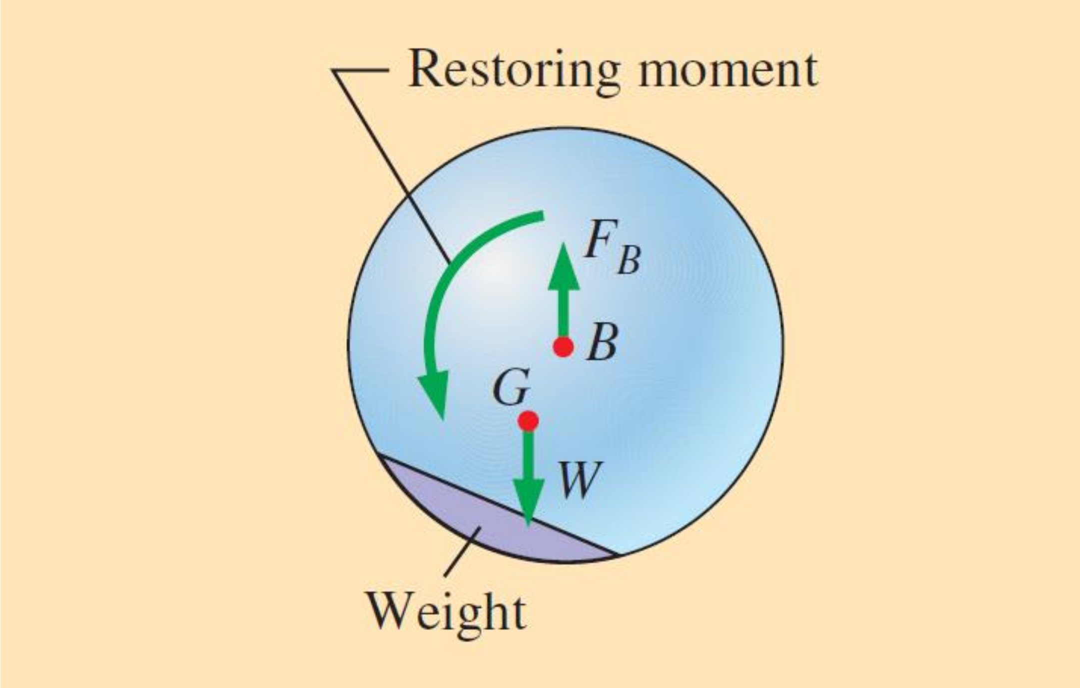
Stability is easily understood by analyzing a ball on the floor.

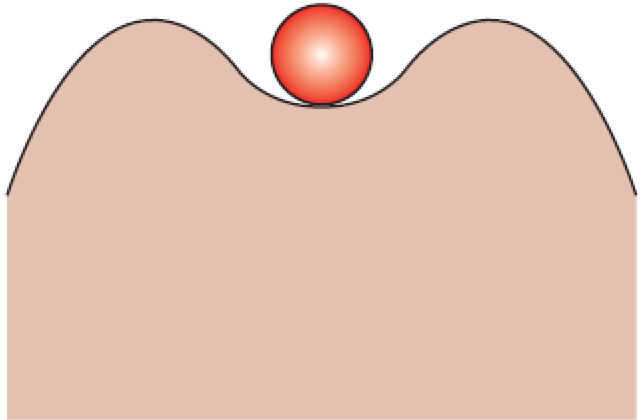


A floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.

An immersed neutrally buoyant body is (*a*) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body, (*b*) neutrally stable if *G* and *B* are coincident, and (*c*) unstable if *G* is directly above *B*.

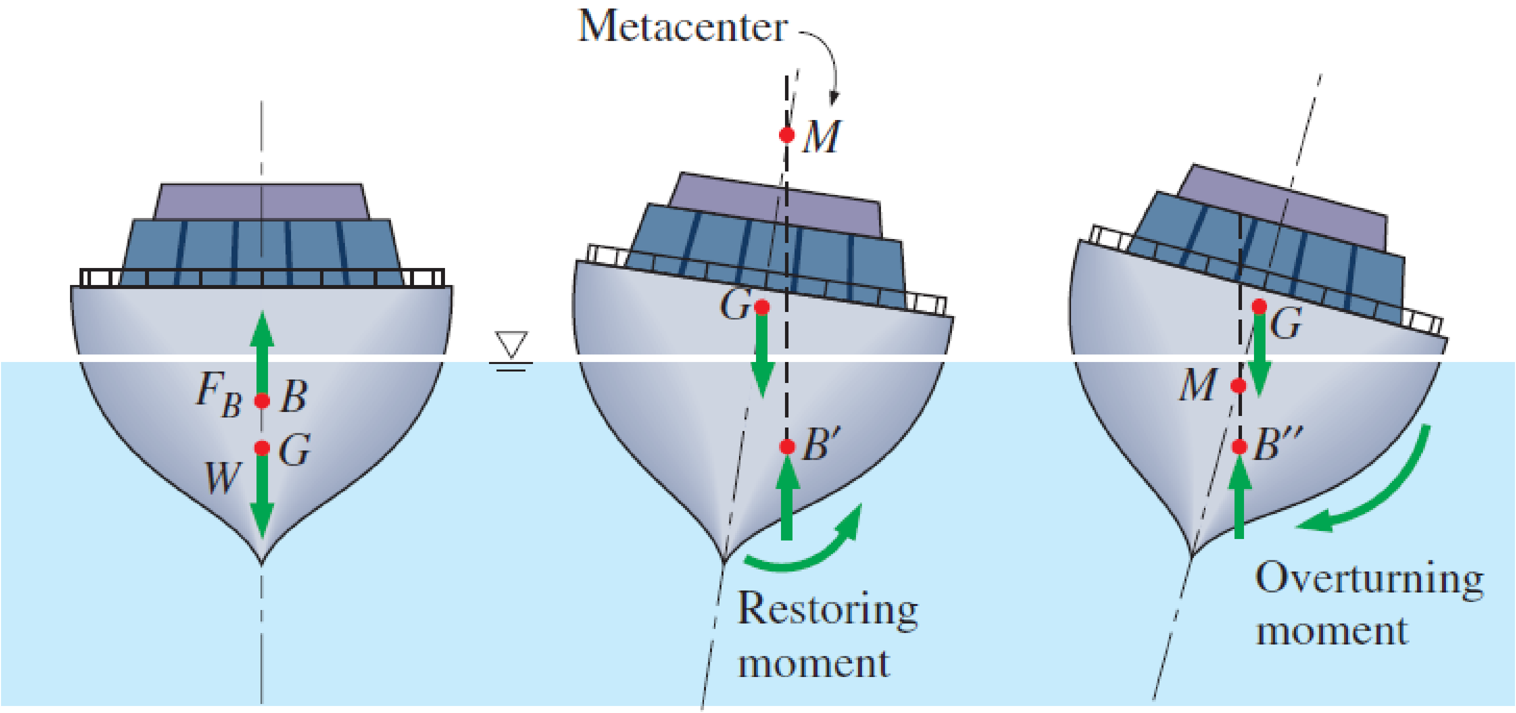






When the center of gravity *G* of an immersed neutrally buoyant body is not vertically aligned with the center of buoyancy *B* of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance.

A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.



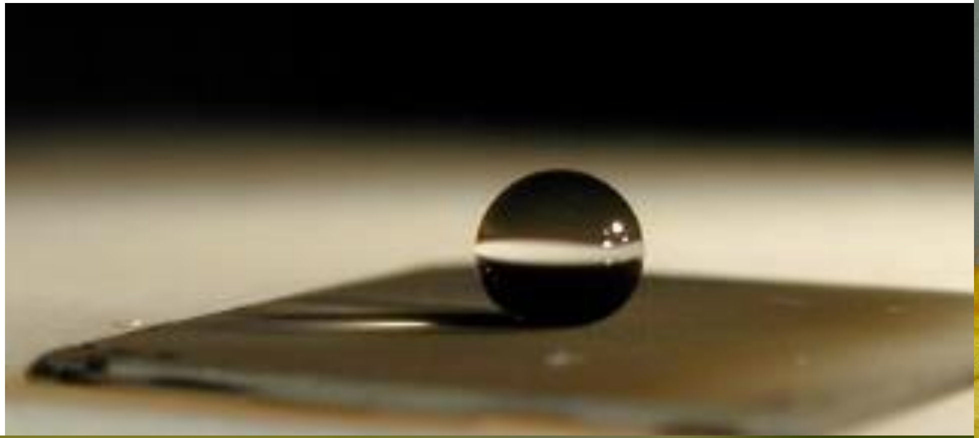
(a) Stable (b) Stable (c) Unstable

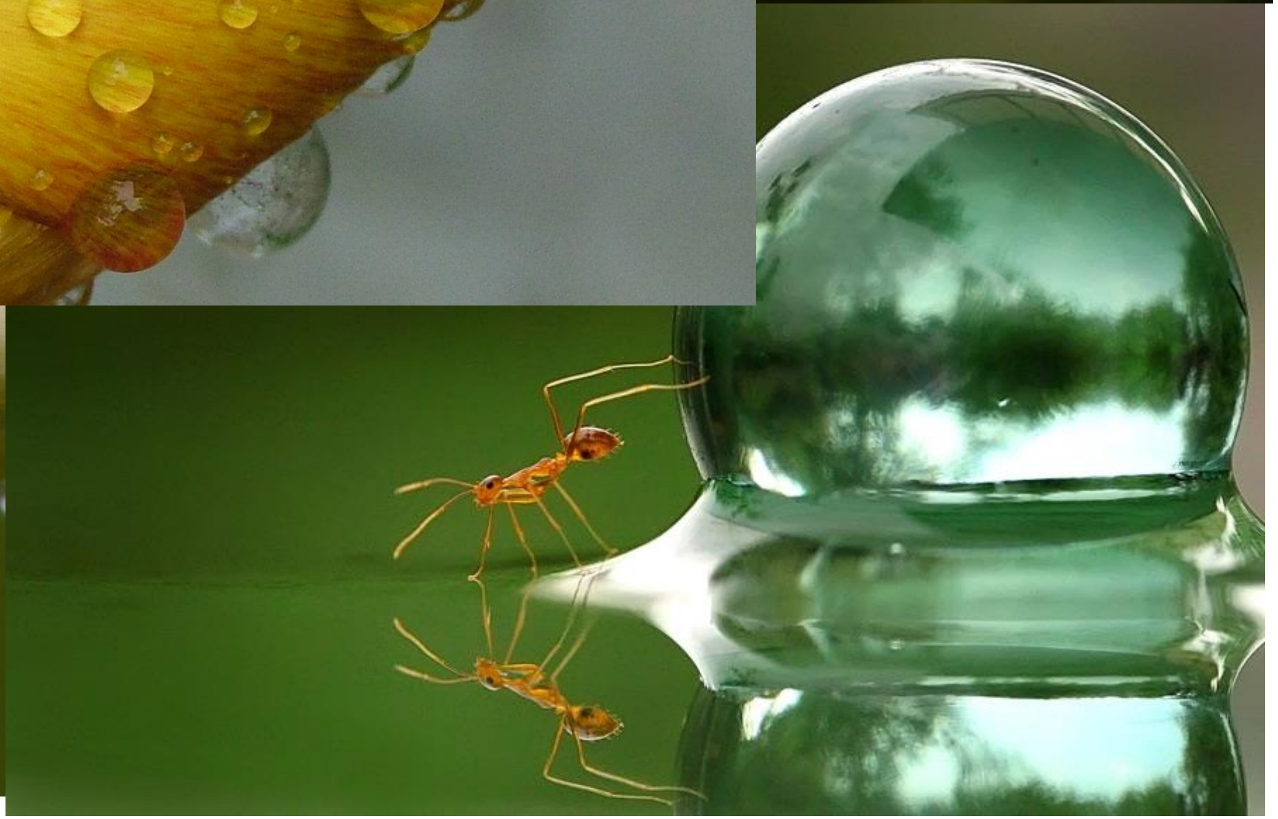
A floating body is *stable* if the body is bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or if the metacenter *M* is above point *G*. However, the body is *unstable* if point *M* is below point *G*.

Metacentric height *GM*: The distance between the center of gravity *G* and the metacenter *M*—the intersection point of the lines of action of the buoyant force through the body before and after rotation.

The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

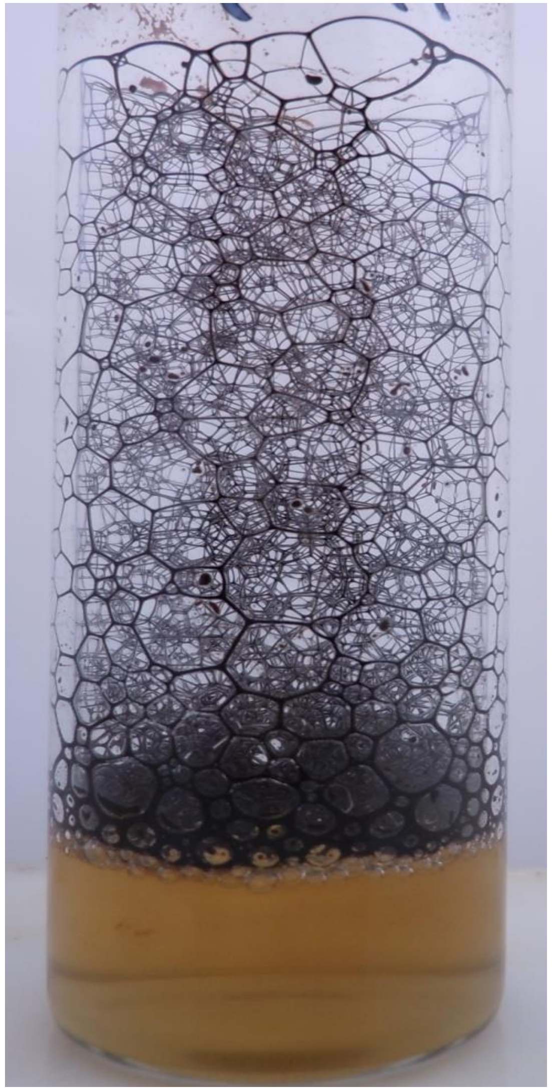
Surface Tension, Interfacial Tension, contact angle



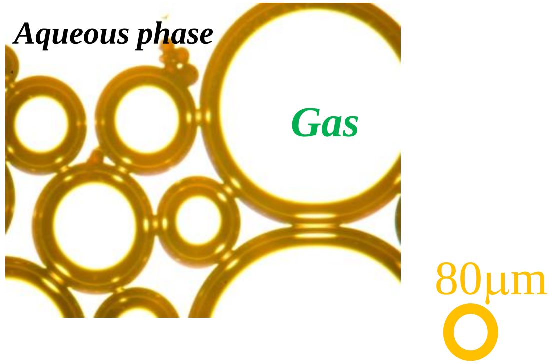


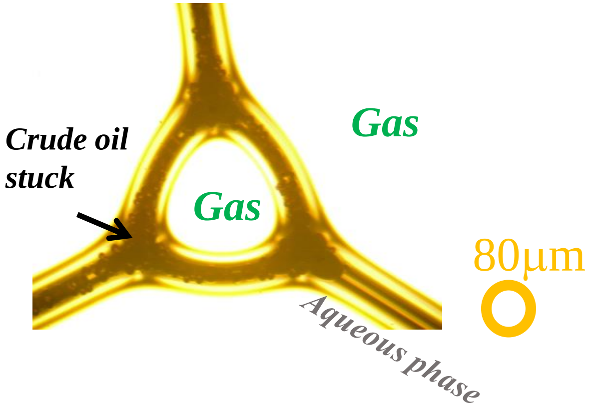
Foam in the presence of oil under the microscope at room temperature

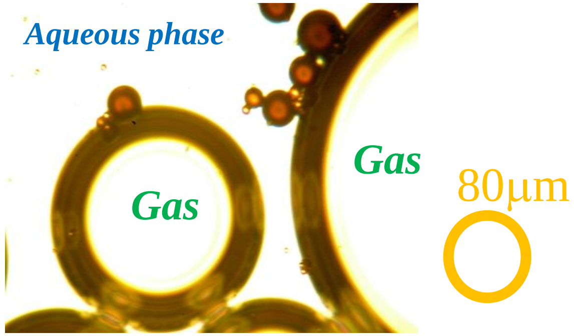
Foam sampled from shaking ~10 ml of 1% solution with 1 cc of synthetic oil.

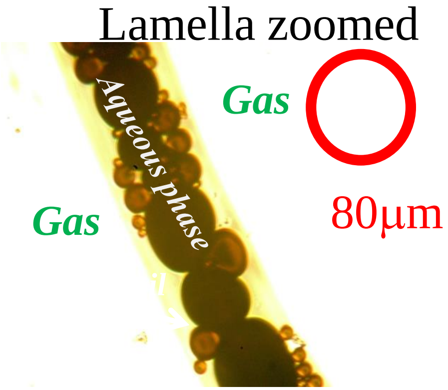


Foam form a EOR Process







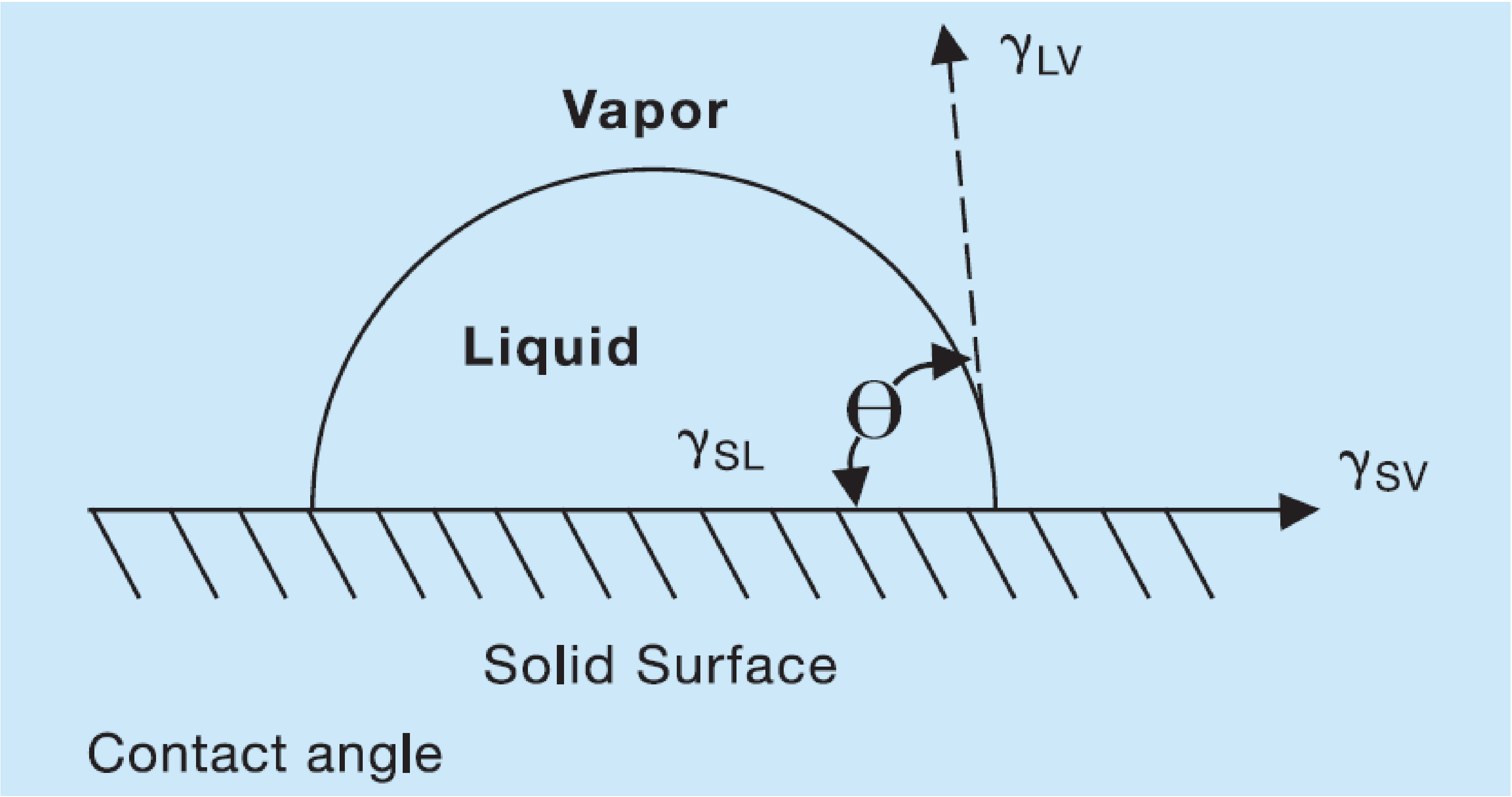


Effect of oil on the foam in SW, The same trend is observed for the system Zwitterionic + Anionic (2-1) foam in Sea water 94°C

Surface properties are important in multiphase flow

Contact Angle: Is a parameter that affects the capillary effects over surfaces, depends on multiple factors, like nature of the solid, the involved fluids, the roughness of solid, the ageing of the fluids in contact with the solid, and the chemistry of the media (pH, redox, hardness, speciation, etc.), in a nutshell, contact angle is the property that tells what is the direction of the line force.

Contact angle and wettability.



Water-Air contact angles over different solid surfaces

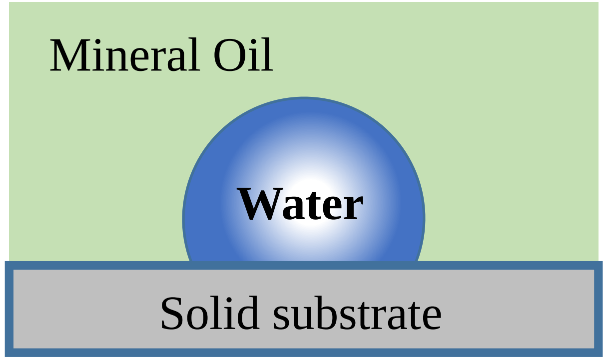
	θ		
heptadecafluorodecyltrimethoxysilane*	115°	diamond	87°
poly(tetrafluoroethylene)	108-112°	graphite	86°
poly(propylene)	108°	silicon (etched)	86-88°
octadecyldimethylchlorosilane*	110°	talc	50-55°
octadecyltrichlorosilane*	102-109°	chitosan	80-81°
tris(trimethylsiloxy)silylethyl-dimethylchlorosilane	104°	steel	70-75°
octyldimethylchlorosilane*	104°	gold, typical (see gold, clean)	66°
dimethyldichlorosilane*	95-105°	intestinal mucosa	50-60°
butyldimethylchlorosilane*	100°	kaolin	42-46°
trimethylchlorosilane*	90-100°	platinum	40°
poly(ethylene)	88-103°	silicon nitride	28-30°
poly(styrene)	94°	silver iodide	17°
poly(chlorotrifluoroethylene)	90°	soda-lime glass	<15°
human skin	75-90°	gold, clean	<10°

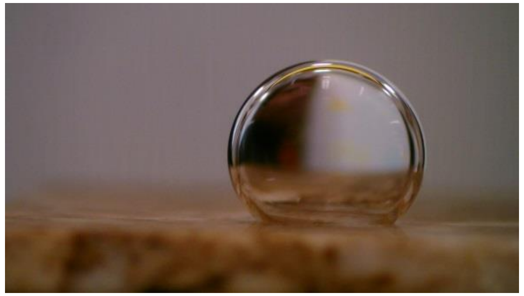
*Note: Contact angles for silanes refer to smooth treated surfaces.

Water-oil contact angles over different solid surfaces

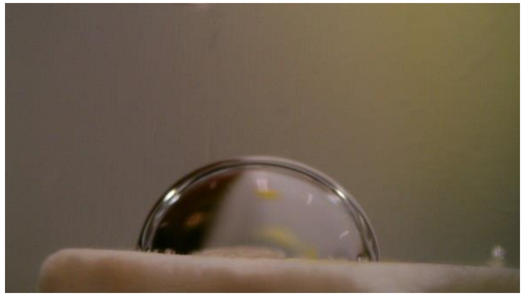
Water-Oil contact angles

Contact angle in liquid-liquid-solid system





Marble aged in crude oil



Marble

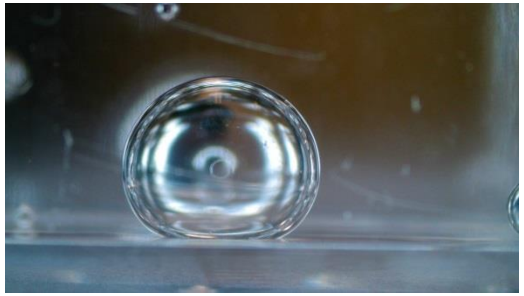
$CaMg(CO_3)_2$ $\theta \approx 69°$

Quartz

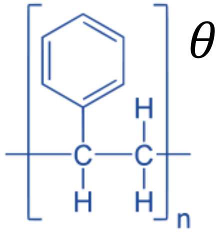
SiO_2

Selenite

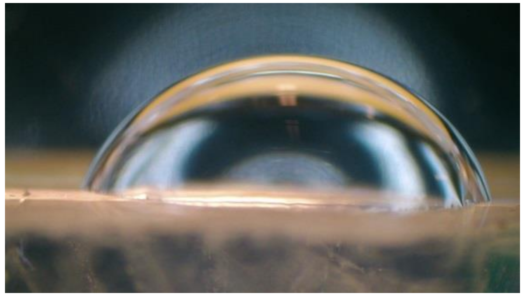
$Ca\,SO_4 \cdot 2\,H_2O$ $\theta < 9°$



Polyestyrene



$\theta \approx 155°$



Icelandspar

$Ca\,CO_3$ $\theta \approx 65°$

Surrounding fluid: Mineral Oil
Drop fluid: water
Solid surface: Indicated on each picture

CHEMICALS MEASURED

NAME	CAS RN	CHEMICAL FORMULA
acetic acid	64-19-7	$C_2H_4O_2$
acetone	67-64-1	C_3H_6O
aniline	62-53-3	C_6H_7N
benzene	71-43-2	C_6H_6
carbon tetrachloride	56-23-5	CCl_4
chlorobenzene	108-90-7	C_6H_5Cl
chloroform	67-66-3	$CHCl_3$
cyclohexane	110-82-7	C_6H_{12}
1, 2-dichlorobenzene	95-50-1	$C_6H_4Cl_2$ (ORTHO)
1, 4-dichlorobenzene	106-46-7	$C_6H_4Cl_2$ (PARA)
ethanol 95%(denatured)	64-17-5	C_2H_6O
ethylbenzene	100-41-4	C_8H_{10}
ethylene glycol	107-21-1	$C_2H_6O_2$
mercury	7439-97-6	Hg
methanol	67-56-1	CH_4O
nitrobenzene	98-95-3	$C_6H_5N0_2$
toluene	108-88-3	C_7H_8
1, 2, 4-trichlorobenzene (TCB)	120-82-1	$C_6H_3Cl_3$
water	7732-18-5	H_2O
m-xylene	108-38-3	C_8H_{10} (META)
o-xylene	95-47-6	C_8H_{10} (ORTHO)
p-xylene	106-42-3	C_8H_{10} (PARA)

CAS RN refers to the Chemical Abstracts Service Registry Number.

Liquid-air contact angles over Quartz

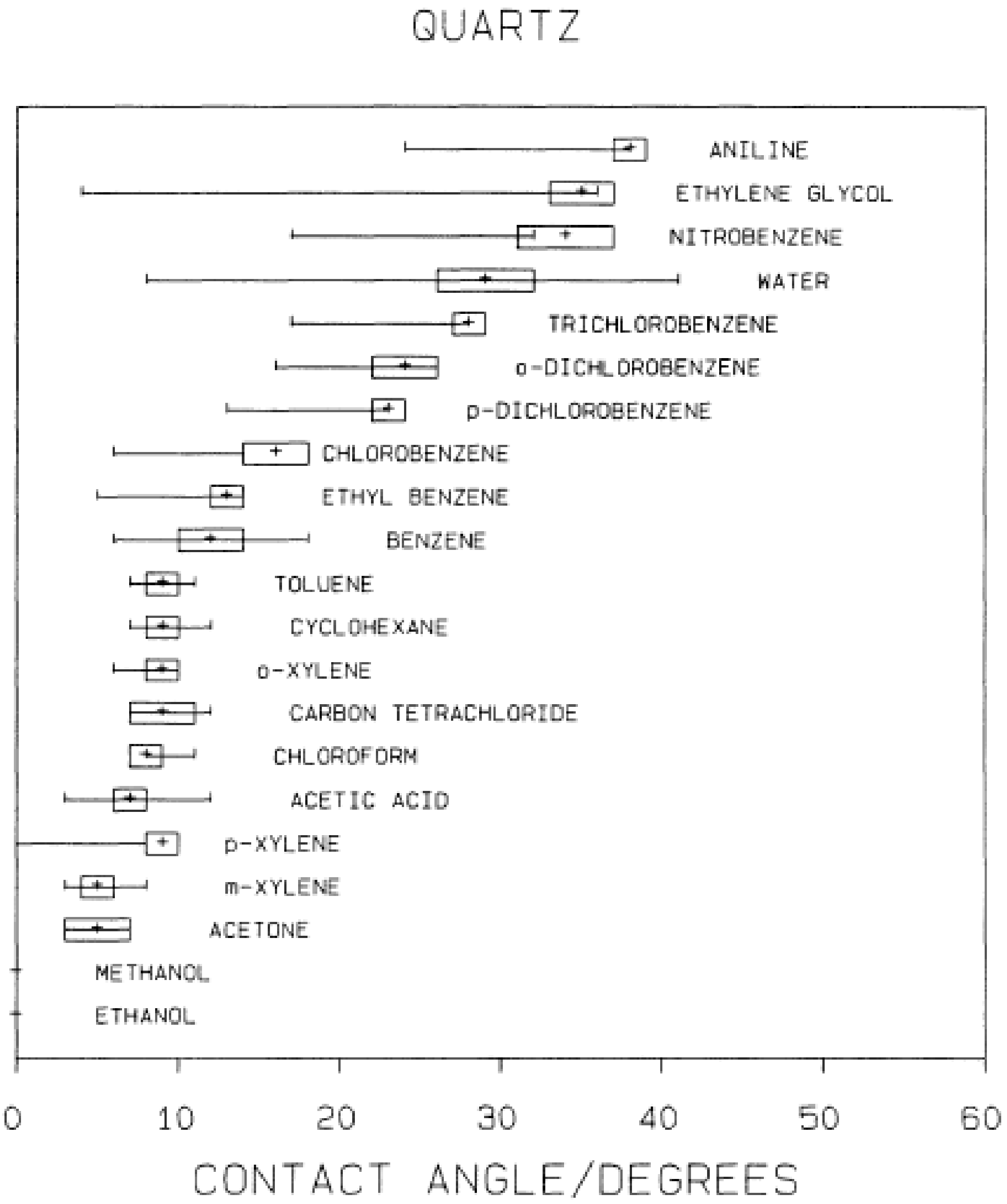
Quartz SiO_2

USGS-OFR-90-409

Edgar F. Ethington[1]

July 1990

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY



Mercury-air contact angles over different rocks

Quartz SiO_2

Calcite $CaCO_3$

Mica (Biotite) $K(Mg,Fe)_3AlSi_3O_{10}(F,OH)_2$

Clay (Montmorillonite) $(Na,Ca)_{0.33}(Al,Mg)_2(Si_4O_{10})(OH)_2 \cdot nH_2O$

Contact angle of mercury- air system over a rock

$136° < \theta < 138°$ $\Delta\theta = \pm 3°$

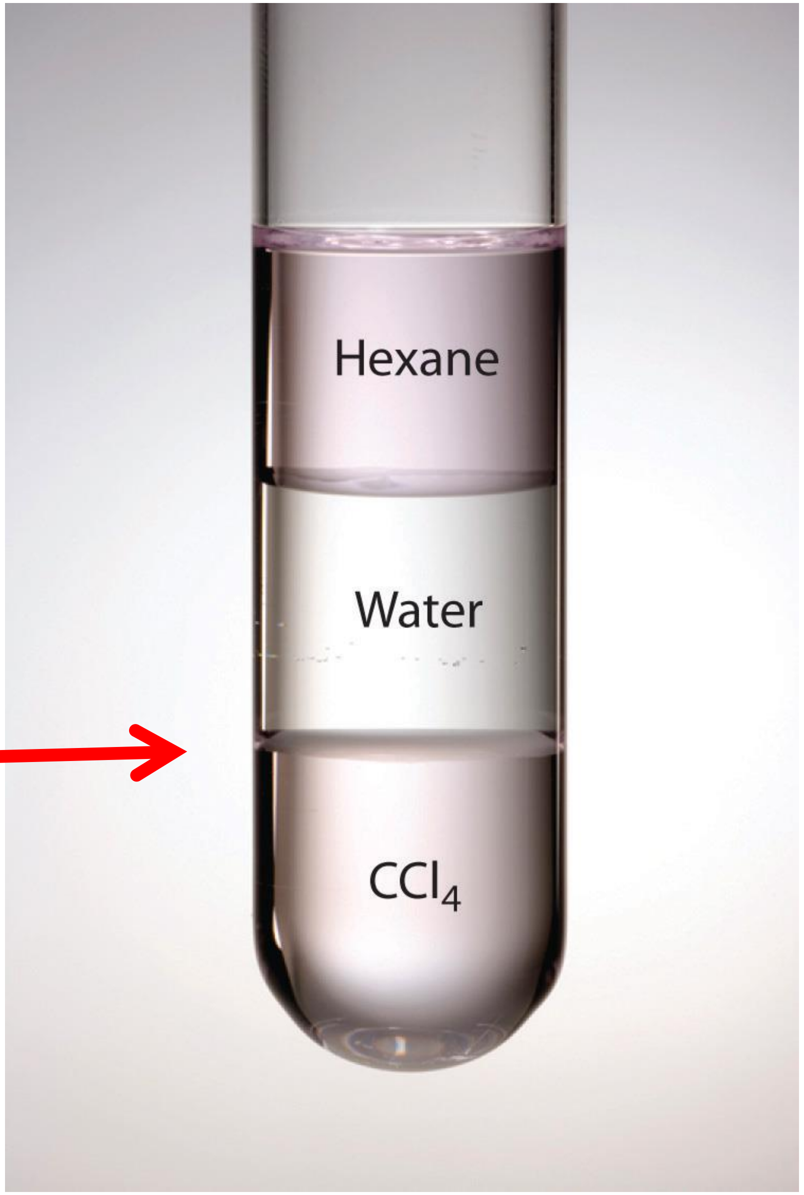
$133° < \theta < 136°$ $\Delta\theta = \pm 3°$

$\theta \approx 138°$ $\Delta\theta = \pm 4°$

$\theta \approx 158°$ $\Delta\theta = \pm 6°$

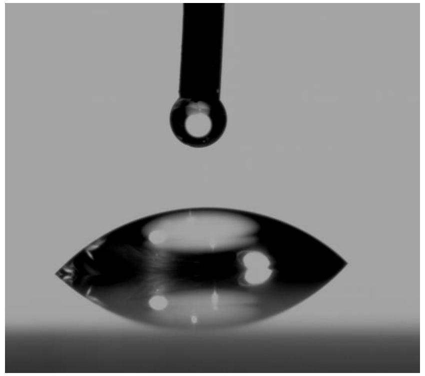
Interfacial tension between water and immiscible organic liquids

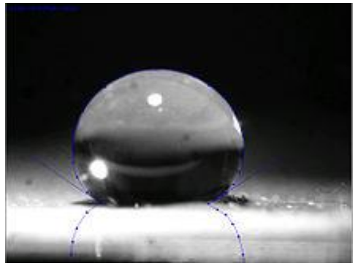
Organic Liquid	Interfacial Tension (mN/m)
Aniline	5.8
Benzaldehyde	15.5
Benzene	35.0
Bromobenzene	38.1
Carbon disulfide	48.4
Carbon tetrachloride	45.0
Chlorobenzene	37.4
Chloroform	31.6
Cyclohexane	50.2



Interfacial tension between water and immiscible organic liquids

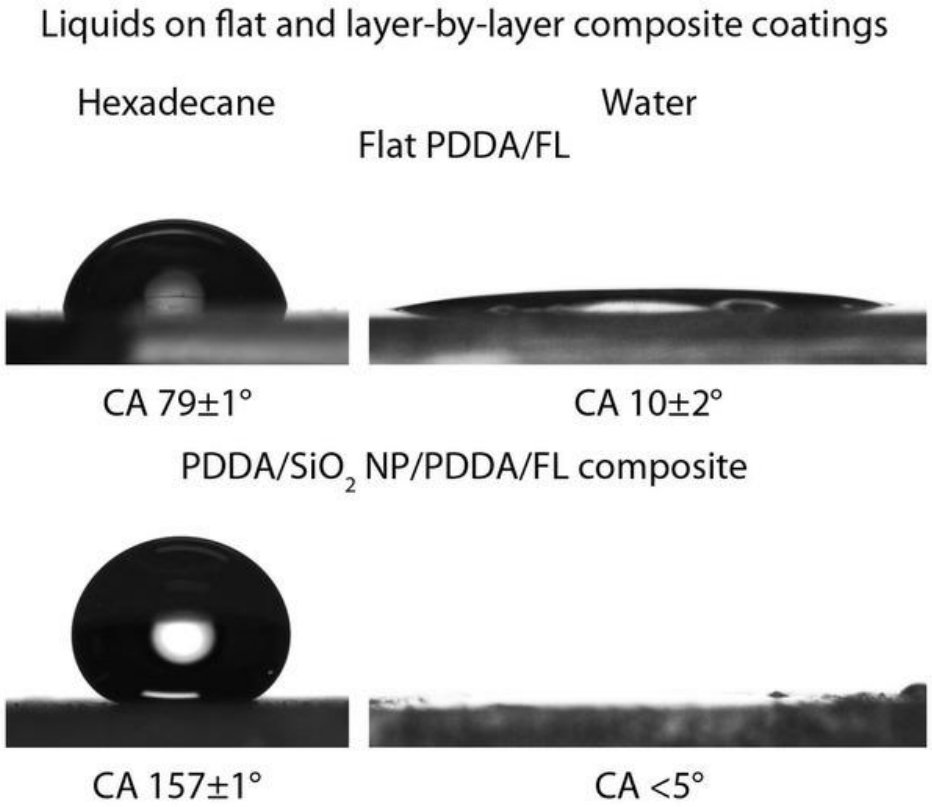
Organic Liquid	Interfacial Tension (mN/m)	Organic Liquid	Interfacial Tension (mN/m)
		n-Amyl alcohol	4.4
Cyclohexanol	3.9	*n*-Butyl acetate	14.5
Decalin	51.4	*n*-Butyl alcohol	1.8
Dichloromethane	28.3	*n*-Decane	51.2
Ethyl acetate	6.8	*n*-Heptane	50.2
Ethyl bromide	31.2	*n*-Hexane	51.1
Iodobenzene	41.8	Nitrobenzene	25.7
Isoamyl alcohol	4.8	*n*-Octane	50.8
Isobutyl alcohol	2.0	*n*-Pentane	49.0
Isopentane	48.7	Octanoic acid	8.5
Mesitylene	38.7	*o*-Nitrotoluene	27.2
m-Nitrotoluene	27.7	*o*-Xylene	36.1
m-Xylene	37.9	*p*-Xylene	37.8
		Tetrachloroethylene	47.5
		Toluene	36.1

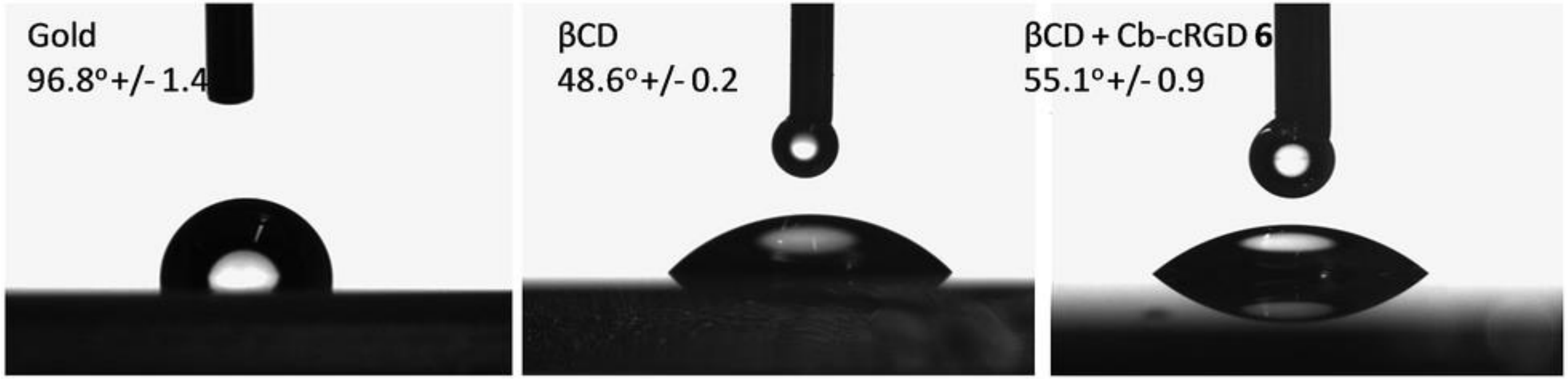




Liquid	Solid	Contact angle
water ethanol diethyl ether carbon tetrachloride glycerol acetic acid	soda-lime glass lead glass fused quartz	0°
water	paraffin wax	107°
	silver	90°
methyl iodide	soda-lime glass	29°
	lead glass	30°
	fused quartz	33°
mercury	soda-lime glass	140°
galinstan	glass	121.5° (receding) - 146°(advancing)

Some liquid-solid contact angles


Liquids on flat and layer-by-layer composite coatings
Hexadecane
Water
Flat PDDA/FL
CA 79±1°
CA 10±2°
PDDA/SiO_2 NP/PDDA/FL composite
CA 157±1°
CA <5°


Gold
96.8°+/- 1.4
βCD
48.6°+/- 0.2
βCD + Cb-cRGD **6**
55.1°+/- 0.9

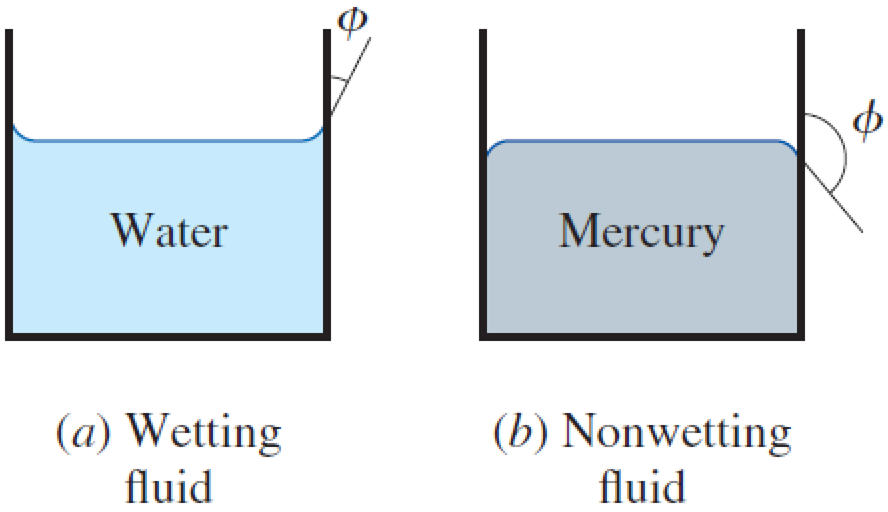
Surface tension of various liquids in dyn/cm against air Mixture compositions denoted "%" are by mass dyn/cm is equivalent to the SI units of mN/m (millinewton per meter)		
Liquid	Temperature °C	Surface tension, γ
Acetic acid	20	27.60
Acetic acid (40.1%) + Water	30	40.68
Acetic acid (10.0%) + Water	30	54.56
Acetone	20	23.70
Diethyl ether	20	17.00
Ethanol	20	22.27
Ethanol (40%) + Water	25	29.63
Ethanol (11.1%) + Water	25	46.03
Glycerol	20	63.00
n-Hexane	20	18.40
Hydrochloric acid 17.7M aqueous solution	20	65.95
Isopropanol	20	21.70
Liquid helium II	−273	0.37
Liquid nitrogen	−196	8.85
Mercury	15	487.00
Galinstan (Ga 68.5%, In 21.5%, Sn 10%)	20	718
Methanol	20	22.60
n-Octane	20	21.80
Sodium chloride 6.0M aqueous solution	20	82.55
Sucrose (55%) + water	20	76.45
Water	0	75.64
Water	25	71.97
Water	50	67.91
Water	100	58.85
Toluene	25	27.73

Capillary effect: The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

Capillaries: Such narrow tubes or confined flow channels. The capillary effect is partially responsible for the rise of water to the top of tall trees.

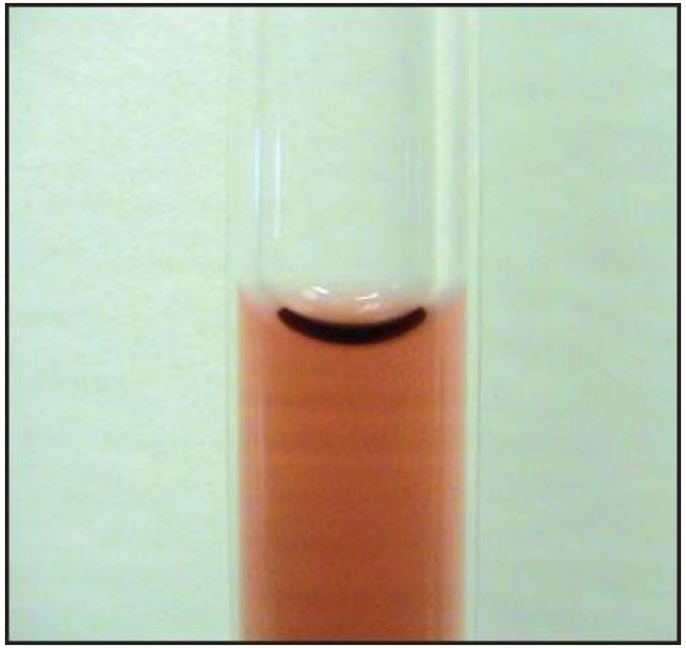
Meniscus: The curved free surface of a liquid in a capillary tube.

The strength of the capillary effect is quantified by the **contact** (or **wetting**) **angle**, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

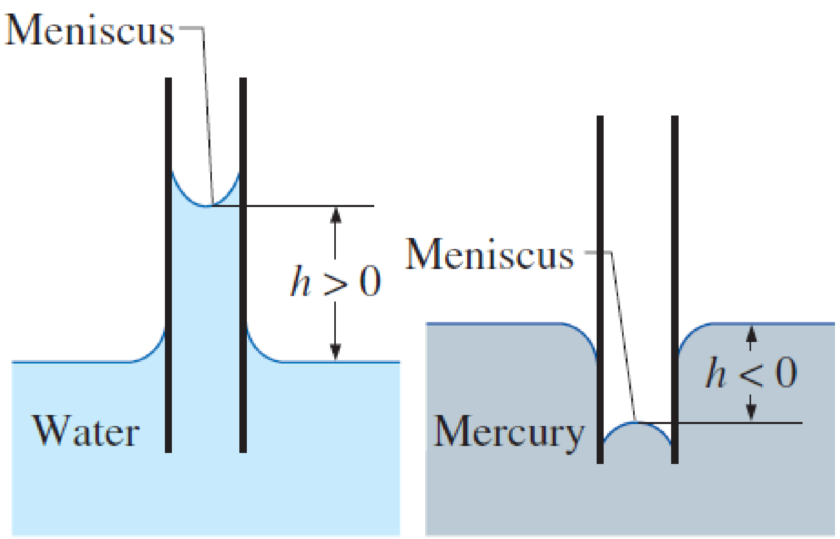


The contact angle for wetting and non-wetting fluids.

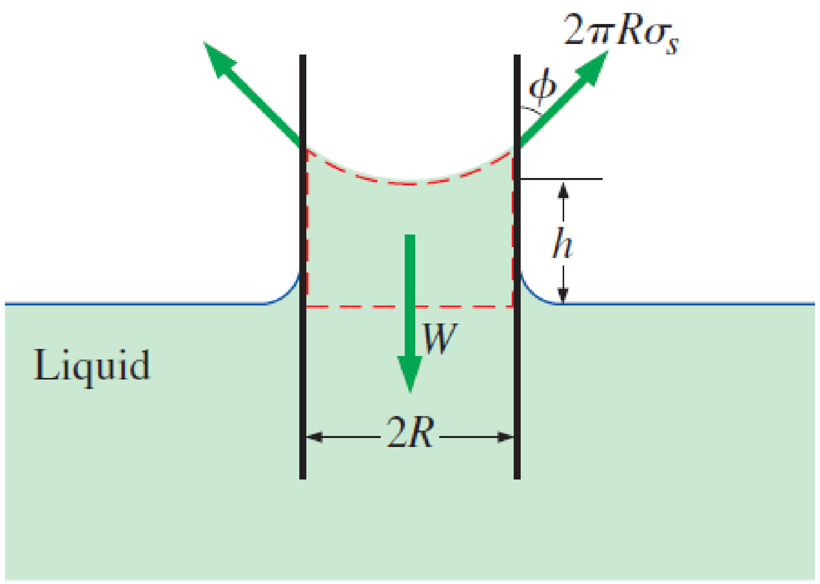
Capillary Effect



The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.



The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.



The forces acting on a liquid column that has risen in a tube due to the capillary effect.

$$h = \frac{2\,\sigma\cos\phi}{\Delta\rho\ g\ R}$$

- Capillary rise is inversely proportional to the radius of the tube and density between the liquid and gas.

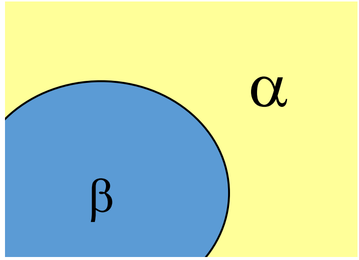
Young-Laplace (physics approach)

$$\left(p^{\beta}-p^{\alpha}\right)=\Delta p=-\sigma\,\underset{\rightarrow}{\nabla}\cdot\underset{\rightarrow}{n}=2\,\sigma\,H=\sigma_{BA}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

Convex Surface r>0
Concave Surface r>0

Where does Yong-Laplace equation comes from ?

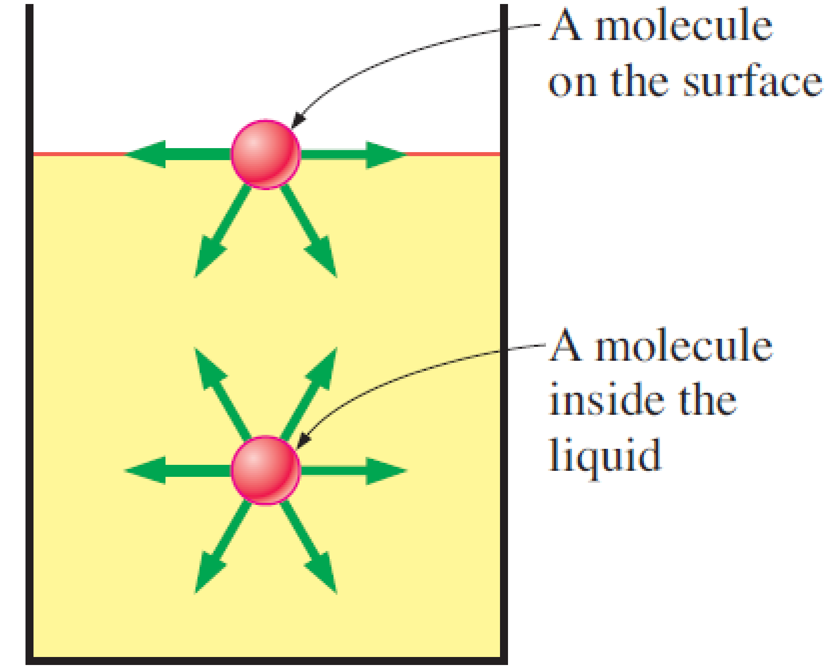
Energy approach (thermodynamics approach)



$$dU=TdS-p^{\beta}\,dV^{\beta}-p^{\alpha}dV^{\alpha}+\sum_{j=1}^{n}\overline{\mu}_j\,\mathbf{dN_j}+(\sigma)dA$$

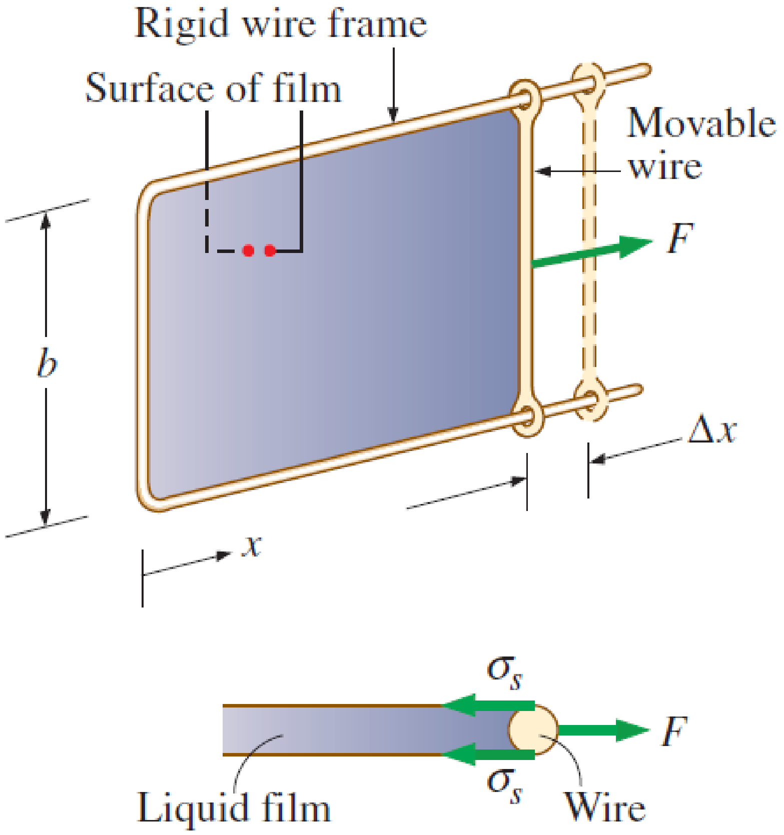
$$\sigma=\left(\frac{\partial U}{\partial A}\right)_{S,N_i,V}$$

(molecular approach)



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.

(force approach)



$$\sigma_s = \frac{F}{2b}$$

Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length *b*.

$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

Surface tension: The work done per unit increase in the surface area of the liquid.

Forces

a) Body forces
b) Surface forces
c) Line forces or surface tension

a) Body forces
b) Surface forces
c) Line forces or surface tension

Body forces : this type of force acts upon the whole material volume at a distance without contact, e.g. gravitational force

$$\underrightarrow{f_v} = force/volume \qquad \underrightarrow{f_m} = force/mass$$

$$d\,\underrightarrow{F_V} = \underrightarrow{f_V}\,dV = \underrightarrow{f_m}\,dm = \rho\,\underrightarrow{f_m}\,dV$$

$$\underrightarrow{f_v} = \rho\,\underrightarrow{g}$$ gravity

$$\underrightarrow{f_v} = \rho_e \underrightarrow{E} + \underrightarrow{J} \times \underrightarrow{B}$$ electromagnetic

$$\boxed{\text{Lorentz force: } \rho_e \underline{E}}$$

$$\underline{f_v} = -\rho\left[\underline{a_0} + \underline{\dot{\Omega}} \times \underline{r} + \underline{\Omega} \times \left(\underline{\Omega} \times \underline{r}\right) + 2\underline{\Omega} \times \underline{v}\right]$$ Inertial forces

$$\underline{a} = \underline{a}_0 + \underline{\dot{\Omega}} \times \underline{r} + \underline{\Omega} \times \left(\underline{\Omega} \times \underline{r}\right) + 2\underline{\Omega} \times \underline{v}$$

Origin acceleration

Angular acceleration

Centripetal acceleration

Coriolis acceleration

$$\underline{f_v} = -\rho\left[\underline{a}_0 + \underline{\dot{\Omega}} \times \underline{r} + \underline{\Omega} \times \left(\underline{\Omega} \times \underline{r}\right) + 2\underline{\Omega} \times \underline{v}\right]$$

$$\underline{f_m} = -\left[\underline{a}_0 + \underline{\dot{\Omega}} \times \underline{r} + \underline{\Omega} \times \left(\underline{\Omega} \times \underline{r}\right) + 2\underline{\Omega} \times \underline{v}\right]$$

$$\underrightarrow{F_V} = \int_V \underrightarrow{f_V}\, dV = \int_V \rho\, \underrightarrow{f_m} dV$$

a) Body forces
b) Surface forces
c) Line forces or surface tension

Surface forces : forces act upon the surface of the fluid particle or upon the surface of the considered fluid domain.(e.g. pressure , friction, etc)

$$\underrightarrow{f_s} = force / surface$$

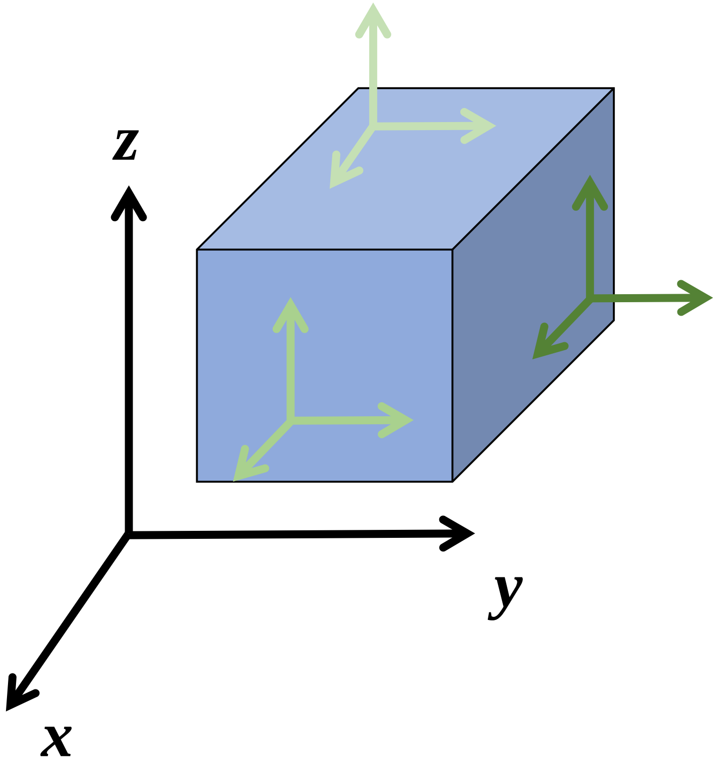
Forces are computed from stress

$$d\underrightarrow{F_S} = \underrightarrow{f_S}\, dA \qquad \underrightarrow{F_S} = \int_A \underrightarrow{f_S}\, dA$$

$$\underrightarrow{f_S} = -p\,\underrightarrow{n} = -p\underrightarrow{n} \cdot \underline{\underline{I}}$$

pressure

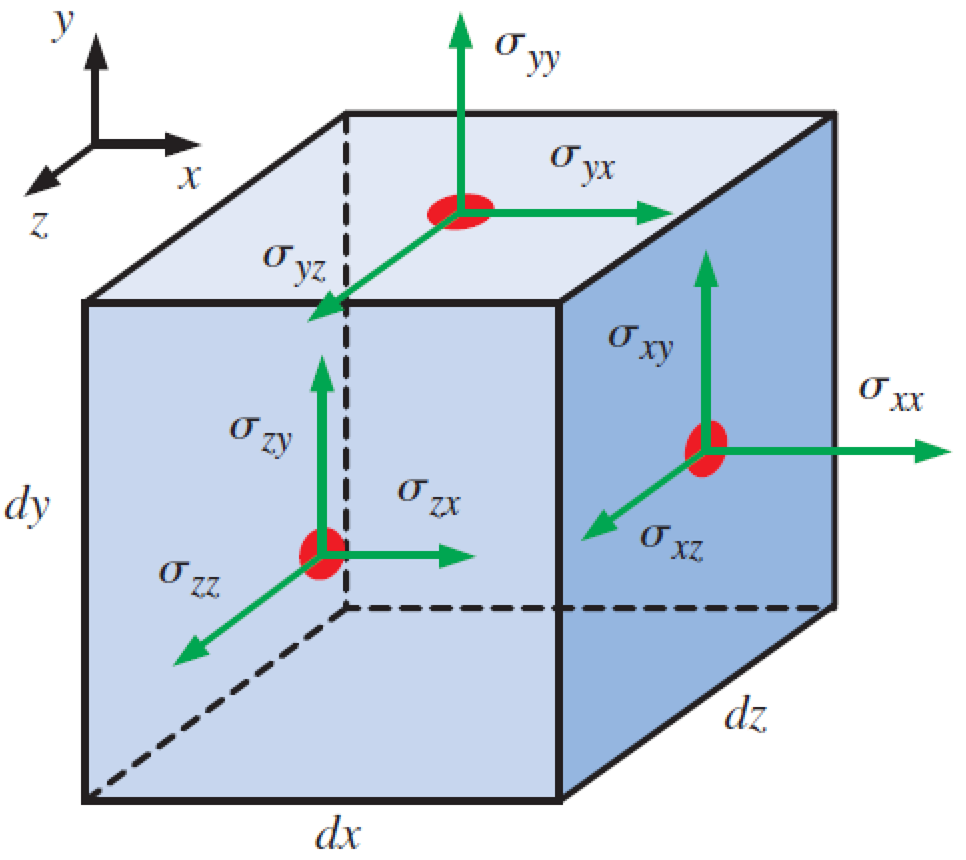
Stress tensor



$$\underline{\underline{\tau}} = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix}$$

$$\underrightarrow{f_S} = \underrightarrow{n} \cdot \underline{\underline{\tau}}$$

$$\underrightarrow{f_S} = -p\underrightarrow{n} \cdot \underline{\underline{I}}$$



Components of the stress tensor in Cartesian coordinates on the right, top, and front faces.

σ_{ij} =Stress in the cross sectional area perpendicular to “i” axis acting in “j” direction (The normal vector is in “i” direction)

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$\underline{s}$ ($\underline{n}$)= Stress exerted by fluid on a particular side of the surface

$$\underline{s}(\underline{n}) = \underline{n} \cdot \underline{\underline{\sigma}}$$

$$\underline{s}(\underline{n}) = s(e_j) = \sigma_{yx}\underline{e_x} + \sigma_{yy}\underline{e_y} + \sigma_{yz}\underline{e_z}$$

σ_{yx}=Is seen to be the force per unit area on a plane perpendicular to the y axis, acting in the ***x*** direction and exerted by the fluid at greater ***y***.

σ_{yx} has three parts, reference plane, a direction and a sign convention. The orientation of the reference plane is specified by the first subscript, and the direction of the force is indicated by the second subscript. Positive values of corresponds to transfer of ***x*** momentum in the –***y*** direction

Caution !

$$\underline{s}(\underline{n}) = \underline{n} \cdot \underline{\underline{\sigma}}$$

Other authors define the stress tensor using subscripts in reverse order, on that case the stress vector is:

$$\underline{s}(\underline{n}) = \underline{\underline{\sigma}} \cdot \underline{n}$$

Some other authors define the stress tensor inverting the sign to extend the analogy of heat and mass transfer. (i.e. Fick’s and Fourier)

a) Body forces
b) Surface forces
c) Line forces or surface tension

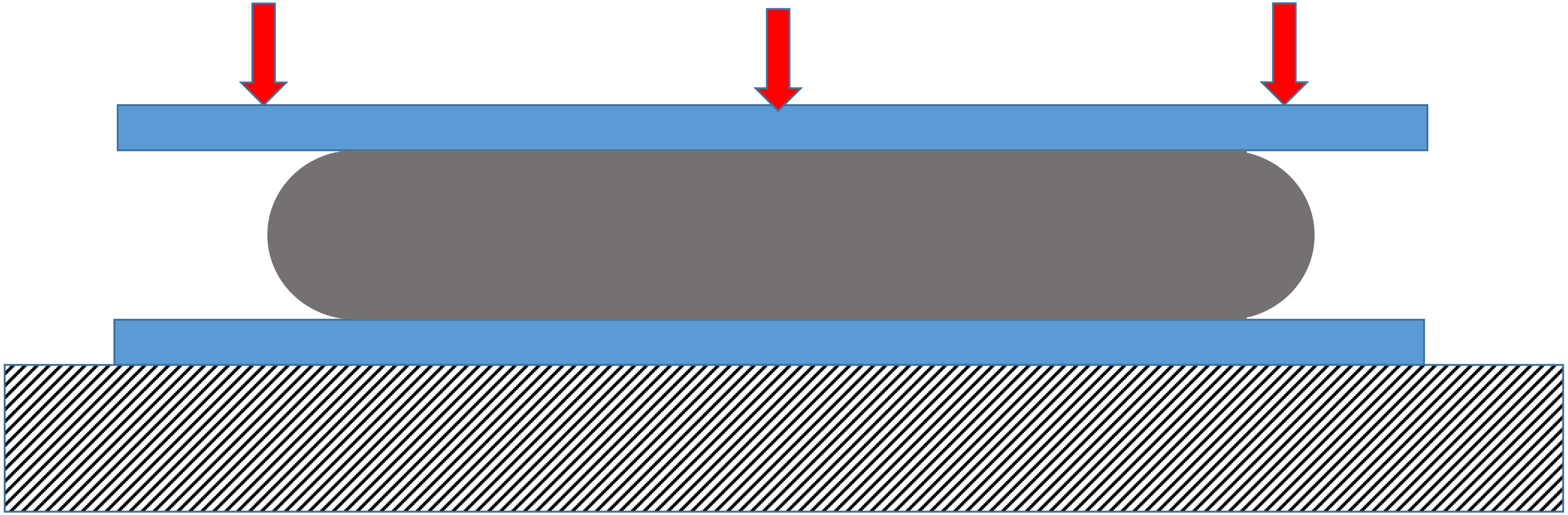
Line forces: forces acting in the three phases contact line

$$\underrightarrow{f_s} = force/line$$

$$d\underrightarrow{F_L} = \underrightarrow{f_L}\, dL$$

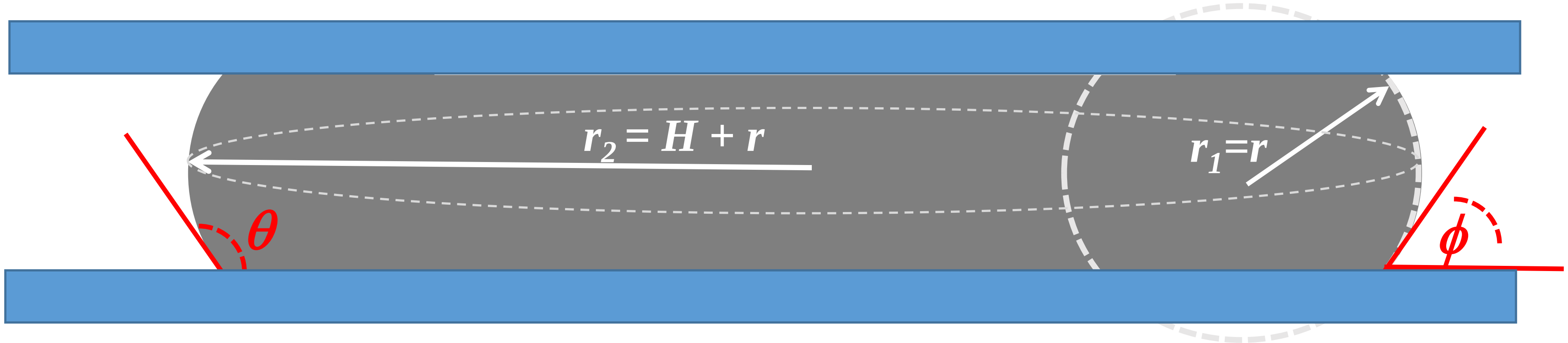
$$\underrightarrow{F_L} = \int_L \underrightarrow{f_L}\, dL$$

Problem 1



a) Two 10-cm ×10-cm 6mm-thick glass plates are used to compress galinstan (6440 kg/m^3), a) If the original volume of the metal liquid is 10 cm^3, calculate the separation between plates, at equilibrium conditions. b) How much force is required to reduce separation between plates to half the equilibrium position. c) Give the dimensions and geometry of the puddle formed just before placing the upper plate over the fluid.

Note: as a first approach you can assume the contact angle to be 180°, and neglect gravity effects.

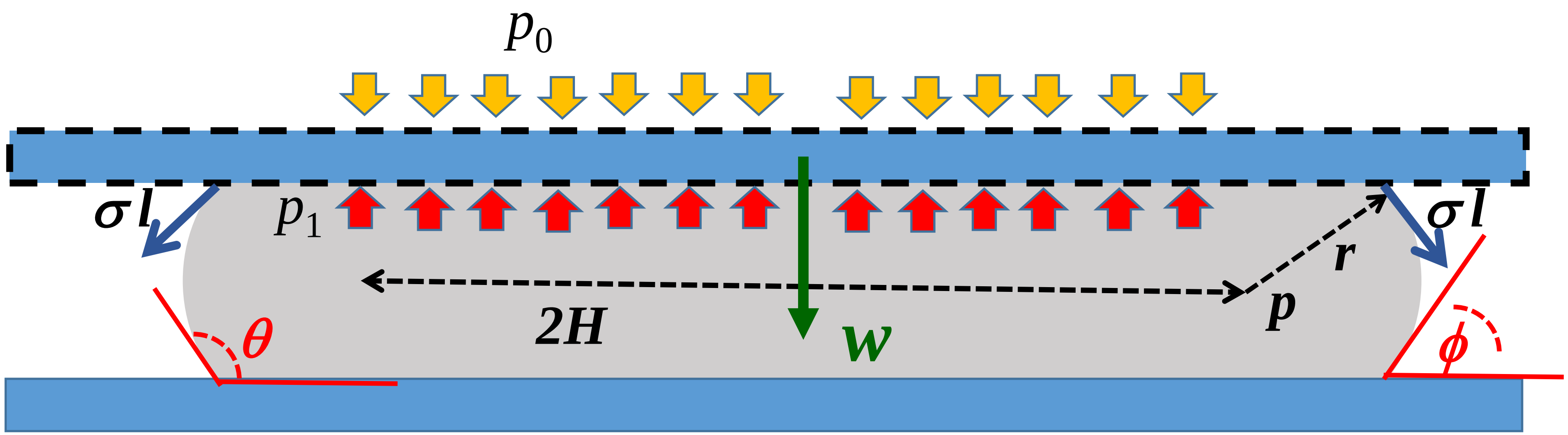


This equation gives the volume of the trapped liquid:

$$\frac{V}{2\pi} = (H^2 + r^2)rCos\varphi - \frac{r^3Cos^3\varphi}{3} + 2H\left[\frac{r^2Sin(2\varphi)}{4} + \frac{r^2}{2}\left(\frac{\pi}{2} - \varphi\right)\right]$$

Problem 1

Forces acting over the upper plate (Subsystem 1: upper plate)

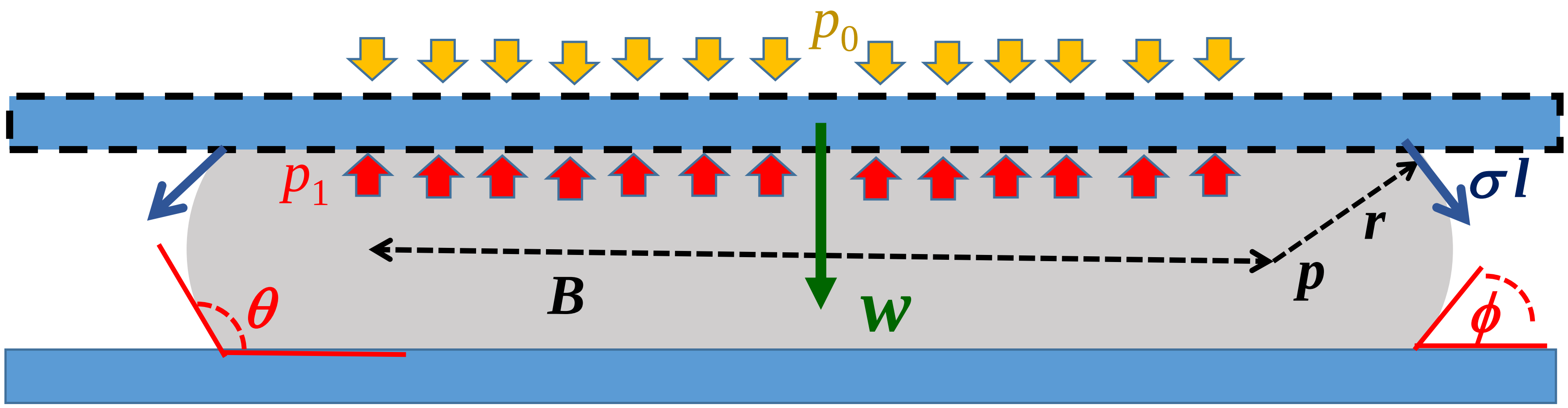


Assumptions: Gravity effects over the fluid are negligible compared with capillary forces, so We can assume symmetry in geometric shape of the liquid interface between plates

Problem 1

Forces acting over the upper plate (**Subsystem 1**: upper plate)

$$p_1\, A - p_o\, A - w - \sigma\, l\, sin\varphi = 0$$



Assumptions: Gravity effects over the fluid are negligible compared with capillary forces, so We can assume symmetry in geometric shape of the liquid interface between plates

Subsystem 2: fluid between plates

$$p - p_o = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Pressure difference across the meniscus

$$p - p_1 = \Delta\rho\, g\, r\, cos\varphi$$

Pressure difference between center and upper plate

Equations

Plate length and width

$L = 0.1 \;\; [\mathrm{m}]$

$\Delta x = 6 \times 10^{-3} \;\; [\mathrm{m}]$

$m_{plate} = V_{plate} \cdot \rho_{plate}$

$\rho_{plate} = 2480 \;\; [\mathrm{kg/m^3}]$

$V_{plate} = \Delta x \cdot L^2$

Volume of liquid

$V = 10 \;\; [\mathrm{cm^3}] \cdot \left| 1 \times 10^{-6} \; \frac{\mathrm{m^3}}{\mathrm{cm^3}} \right|$

First estimation of vlomume

$$V_2 = \pi \cdot H^2 \cdot (2 \cdot y)$$

Volume of liquid trapped between plates

$$\frac{V}{2 \cdot \pi} = (H^2 + r^2) \cdot r \cdot \mathrm{Cos}(\phi) - \frac{(r \cdot \mathrm{Cos}(\phi))^3}{3} + 2 \cdot H \cdot \left(r^2 \cdot \frac{\sin(2 \cdot \phi)}{4} + \left(\frac{r^2}{2}\right) \cdot (\pi/2 - \phi) \right)$$

Surface tension

$$\sigma = 718 \times 10^{-3} \ \ [\mathrm{N/m}]$$

Capillary pressure

$$p - p_o = \sigma \cdot \left(1/r + \frac{1}{H + r} \right)$$

$$r = \frac{y}{\mathrm{Cos}(\phi)}$$

contact angle

$$\theta = (146/180) \cdot \pi$$

suplementary to the contact angle

$$\phi + \theta = \pi$$

$$p_o = 101325 \; [\mathrm{Pa}]$$

$$g = 9.80665 \; [\mathrm{m/s^2}]$$

Force balance in y-axis

$$(p_1 - p_o) \cdot A - \sigma \cdot \sin(\phi) \cdot l_{contact} = m_{plate} \cdot g$$

Contact line glass-liquid-air

$$l_{contact} = 2 \cdot \pi \cdot (H + r \cdot \sin(\phi))$$

Area of the solid in contact with the fluid

$$A = \pi \cdot (H + r \cdot \sin(\phi))^2$$

$$\rho_f = 6440 \; [\mathrm{kg/m^3}]$$

$$p = p_1 + \rho_f \cdot g \cdot r \cdot \mathrm{Cos}(\phi)$$

Scenario 2, recalculate but adding a force to reduce the gap between plate to half the equilibrium value

$r_2 = r/2$

$$\frac{V}{2\cdot\pi} = \left(H_2^2 + r_2^2\right)\cdot r_2\cdot \mathrm{Cos}(\phi) - \frac{\left(r_2\cdot \mathrm{Cos}(\phi)\right)^3}{3} + 2\cdot H_2\cdot\left(r_2^2\cdot\frac{\sin\left(2\cdot\phi\right)}{4} + \left(\frac{r_2^2}{2}\right)\cdot\left(\pi/2-\phi\right)\right)$$

$$p_{c2} - p_o = \sigma\cdot\left(1/r_2 + \frac{1}{H_2 + r_2}\right)$$

$$A_2 = \pi\cdot\left(H_2 + r_2\cdot\sin\left(\phi\right)\right)^2$$

$$l_{contact,2} = 2\cdot\pi\cdot\left(H_2 + r_2\cdot\sin\left(\phi\right)\right)$$

$$p_{c2} = p_{1,2} + \rho_f\cdot g\cdot r_2\cdot \mathrm{Cos}(\phi)$$

$$\left(p_{1,2} - p_o\right)\cdot A_2 - \sigma\cdot\sin\left(\phi\right)\cdot l_{contact,2} = m_{plate}\cdot g + F$$

Bond Number to verify effects on gravity

$$Bo = \rho_f \cdot g \cdot \frac{r^2}{\sigma}$$

$$Bo_2 = \rho_f \cdot g \cdot \frac{r_2^2}{\sigma}$$

If no Glass is placed in the top of the fluid what are the dimensions and geometry of the puddle

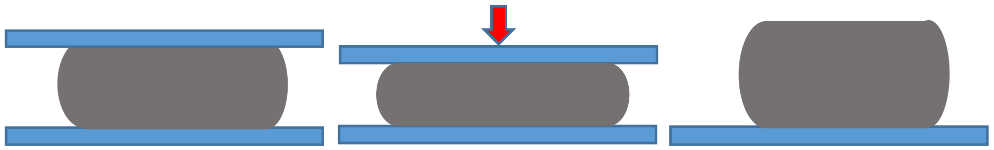
$$r_{no,Glass} = \sqrt{\frac{\sigma}{2 \cdot \rho_f \cdot g}}$$

$$V = 2 \cdot r_{no,Glass} \cdot \pi \cdot H_{no,Glass}^2$$

Solution

$A = 0.003812$ [m^2]
$A_2 = 0.007759$ [m^2]
$Bo = 0.2085$
$Bo_2 = 0.05214$
$\Delta x = 0.006$ [m]
$F = 5.45$ [N]
$g = 9.807$ [m/s^2]
$H = 0.03397$ [m]
$H_2 = 0.04927$ [m]
$H_{no,Glass} = 0.02584$ [m]
$L = 0.1$ [m]
$l_{contact} = 0.2189$ [m]
$l_{contact,2} = 0.3123$ [m]
$m_{plate} = 0.1488$ [kg]
$p = 101812$ [Pa]
$\phi = 0.5934$
$p_1 = 101731$ [Pa]
$p_{1,2} = 102232$ [Pa]
$p_{c2} = 102272$ [Pa]
$p_o = 101325$ [Pa]
$r = 0.00154$ [m]
$\rho_f = 6440$ [kg/m^3]
$\rho_{plate} = 2480$ [kg/m^3]
$r_2 = 0.0007699$ [m]
$r_{no,Glass} = 0.002384$ [m]
$\sigma = 0.718$ [N/m]
$\theta = 2.548$
$V = 0.00001$ [m^3]
$V_2 = 0.000009256$ [m^3]
$V_{plate} = 0.00006$ [m^3]
$y = 0.001277$ [m]

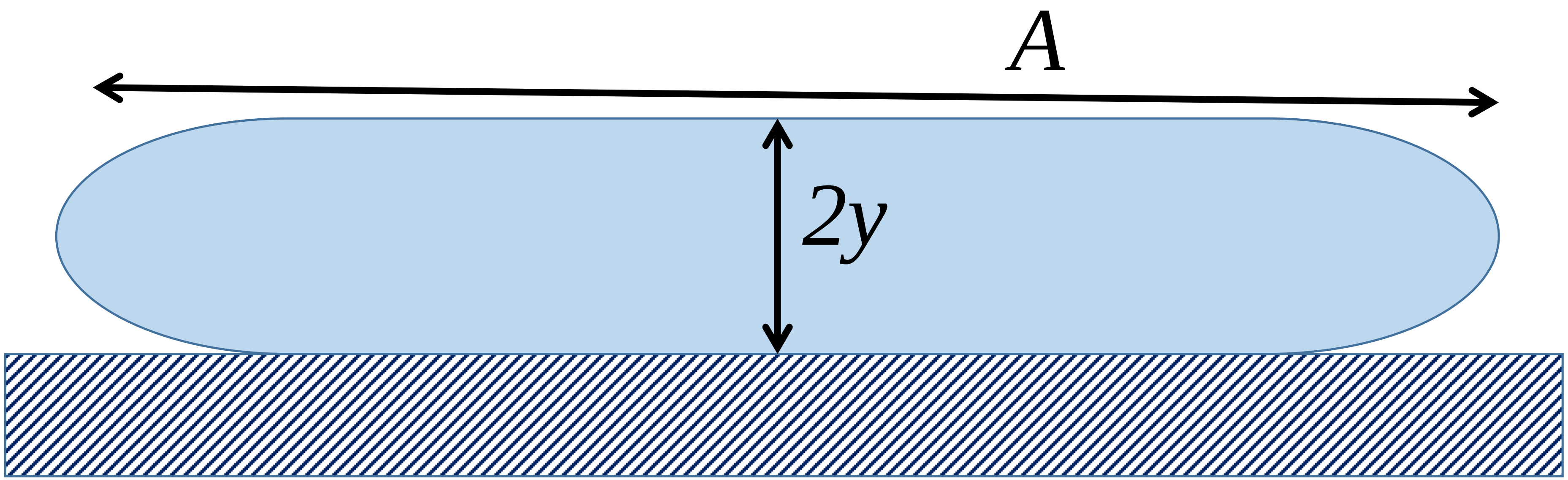
$\theta = 146°$



Variable	Glass cover	Glass cover and Force	No glass on the cover
H (cm)	3.39	4.93	2.58
r (mm)	1.54	0.77	-
y (mm)	1.27	0.64	2.38
F (N)	0	5.45	0

$\theta = 180°$

Variable	Glass cover	Glass cover and Force
H (cm)	3.25	4.69
r (mm)	1.41	0.71
F (N)	0	5.38



Total Energy	Potential Energy	Surface Energy

$$E = \rho\, V\, g \left(\frac{2y}{2}\right) + \sigma\, A$$

Volume of the puddle

$$V = A\,(2\,y)$$

Divide by mass to obtain energy per unit mass, and by coupling energy equation and volume, we have an equation of specific energy in terms of the height of the puddle

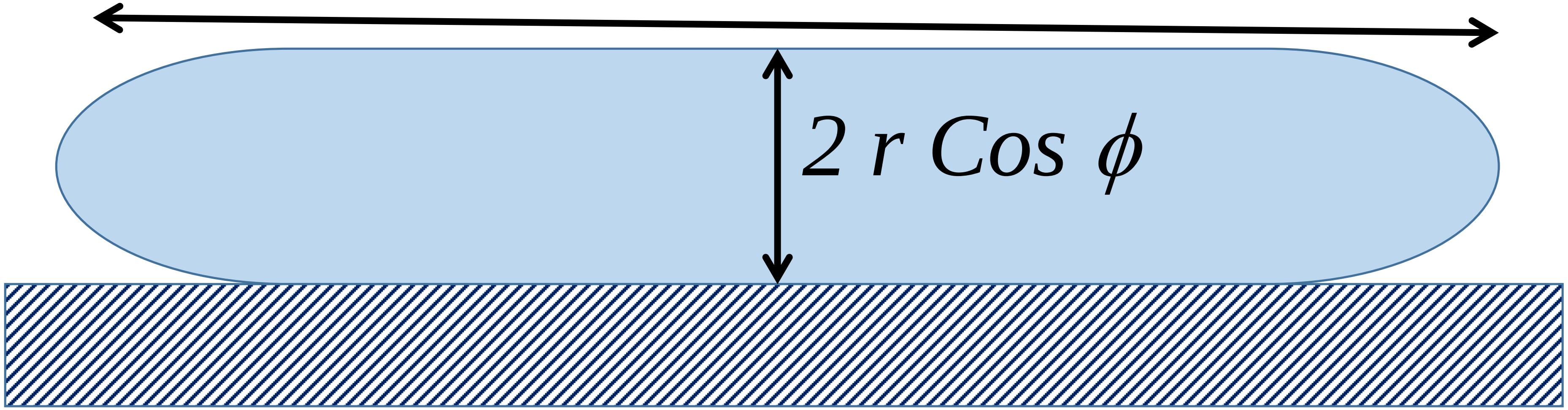
$$\hat{E} = \frac{E}{\rho\, V} = g\,\left(2\frac{y}{2}\right) + \sigma\,\frac{A}{\rho\, V} = g\,\left(2\frac{y}{2}\right) + \sigma\,\frac{1}{2\rho\, y}$$

The equation is differentiated respect to height to minimize the energy

$$\frac{d\hat{E}}{dy} = g\, - \sigma\,\frac{1}{2\rho\, y^2} = 0$$

$$2\, y_{opt} = \sqrt{\frac{2\sigma}{\rho\, g}} \qquad V = 2\, A_{opt} y_{opt}$$

Energy Approach: Minimize the energy (This allows you to use a more robust model)



Total Energy | Potential Energy | Surface Energy

$$E = \rho\, V\, g\, (r\, Cos\varphi) + \sigma\, A$$

Objective function to minimize

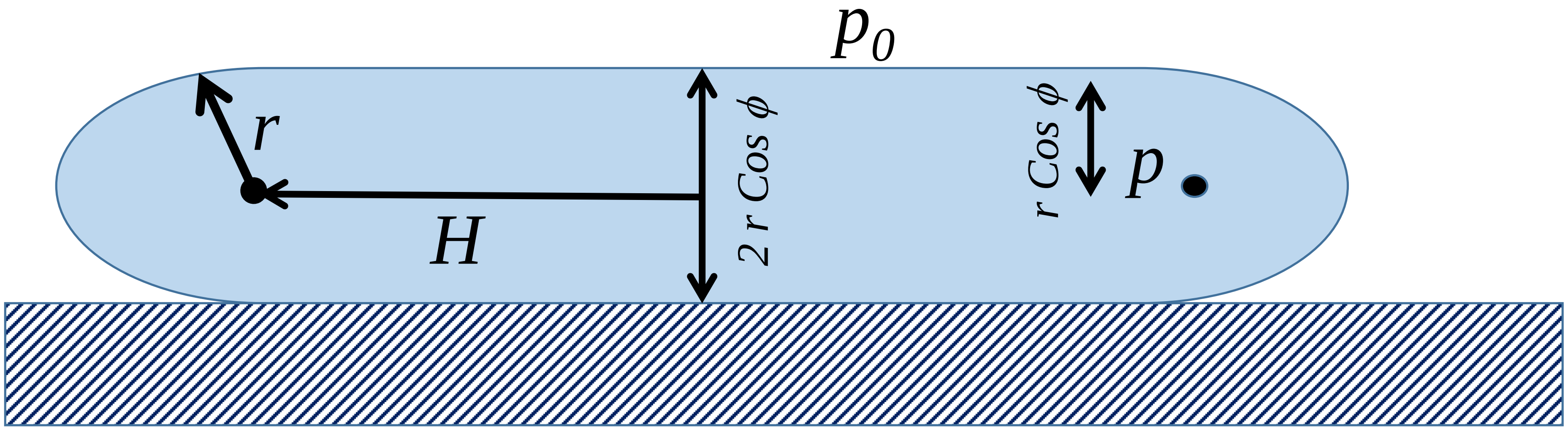
$$A \approx \pi\big(H + r\, Sin(\varphi)\big)^2 + 2\pi H(2r)\left[\frac{\pi}{2} - \varphi\right]$$

Area of the interface liquid-gas

$$\frac{V}{2\pi} = (H^2 + r^2) r Cos\varphi - \frac{r^3 Cos^3\varphi}{3} + 2H\left[\frac{r^2 Sin(2\varphi)}{4} + \frac{r^2}{2}\left(\frac{\pi}{2} - \varphi\right)\right]$$

Constraint: The volume is fixed

Pressure Approach: Calculate the pressure at the center)



$$p = p_0 + \Delta\rho\, g(r\, Cos\varphi)$$ Hydrostatic pressure

$$p - p_0 = \sigma\left(\frac{1}{r} + \frac{1}{(r+H)}\right)$$ Capillary Pressure

$$V \approx 2\pi\big(H + r\, Sin(\varphi)\big)^2 rCos\varphi + 2\pi H\left[r^2\left(\frac{\pi}{2} - \varphi\right) - r^2 Sin(\varphi) Cos(\varphi)\right]$$ Constraint: The volume is fixed

Equations

Code to estimate the dimensions of the puddle formed by a fluid

Density of Galinstan

$$\rho_l = 6440 \ \left[\mathrm{kg/m^3}\right]$$

Air Density

$$\rho_{Air} = 1.14 \ \left[\mathrm{kg/m^3}\right]$$

Density Difference

$$\Delta\rho = \rho_l - \rho_{Air}$$

Volume of the puddle

$$V = 10 \ \left[\mathrm{cm^3}\right] \cdot \left| 1 \times 10^{-6} \ \frac{\mathrm{m^3}}{\mathrm{cm^3}} \right|$$

Gravitational field

$$g = g\#$$

Contact angle, in radians

$$\theta = (146/180) \cdot \pi$$

Suplementary angles

$$\phi + \theta = \pi$$

Pressure difference, between the top of the puddle and its center

Hydrostatic pressure

$$\Delta p = \Delta\rho \cdot g \cdot r \cdot \mathrm{Cos}(\phi)$$

Surface tension of Galinstan

$$\sigma = 718 \times 10^{-3} \ \ [\mathrm{N/m}]$$

Capullary pressure, via Young-Laplace

$$\Delta p = \sigma \cdot \left(1/r + \frac{1}{H + r}\right)$$

Approximate volume of the puddle, you can use any other approximation

or the actual volume of the puddle

$$V = 2 \cdot \pi \cdot r \cdot \mathrm{Cos}(\phi) \cdot (H + r \cdot \sin(\phi))^2 + 2 \cdot \pi \cdot H \cdot r^2 \cdot (\pi/2 - \phi - \sin(\phi) \cdot \mathrm{Cos}(\phi))$$

depth of the puddle

$$\Delta y = 2 \cdot r \cdot \mathrm{Cos}(\phi)$$

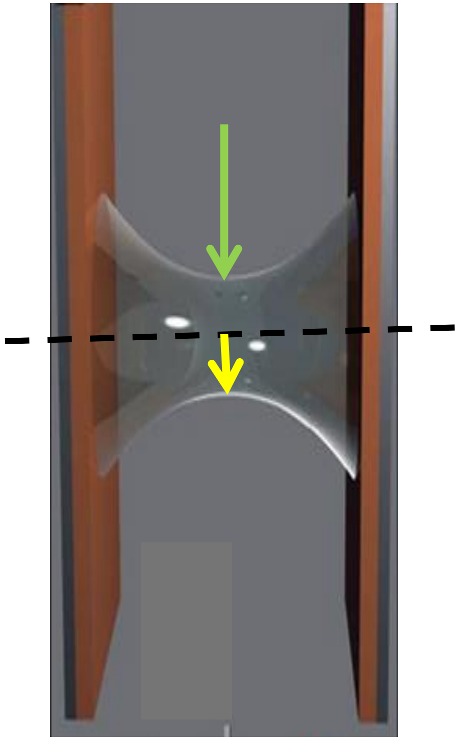
Solution

$$\Delta p = 210.5\ [\mathrm{Pa}] \qquad \Delta\rho = 6439\ [\mathrm{kg/m^3}]$$
$$\Delta y = 0.006666\ [\mathrm{m}] \qquad g = 9.807\ [\mathrm{m/s^2}]$$
$$H = 0.01852\ [\mathrm{m}] \qquad \phi = 0.5934$$
$$r = 0.00402\ [\mathrm{m}] \qquad \rho_{Air} = 1.14\ [\mathrm{kg/m^3}]$$
$$\rho_l = 6440\ [\mathrm{kg/m^3}] \qquad \sigma = 0.718\ [\mathrm{N/m}]$$
$$\theta = 2.548 \qquad V = 0.00001\ [\mathrm{m^3}]$$

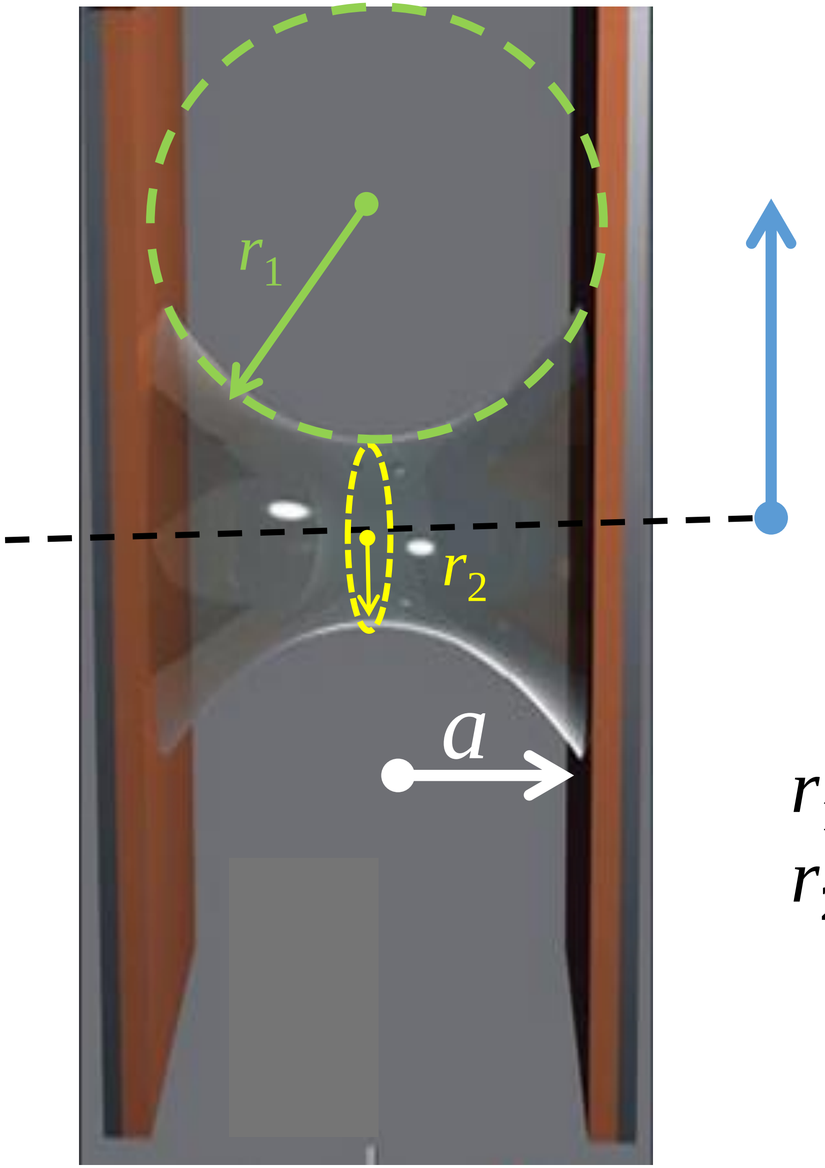
$\Delta y = y-(-y)=2y$

Problem 2

2. Two glass plates are held together by a thin film of water (1 mm), calculate the force required to take them apart if during all the process they are kept parallel. The glass was treated to be completely hydrophilic. Calculate the work as a function of the distance, and the power if the velocity to of the process is 1 cm/min(Plates dimensions: L_1=30 cm, L_2=30 cm, L_3=3 mm)

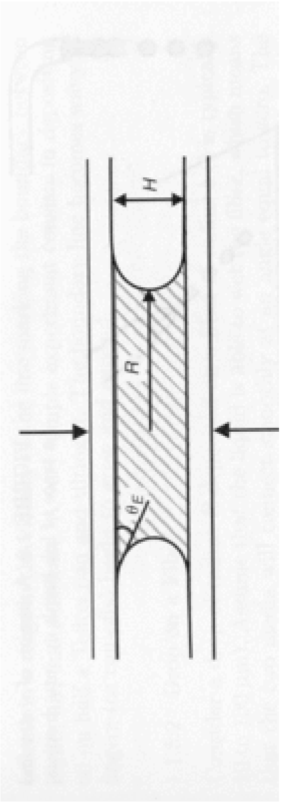


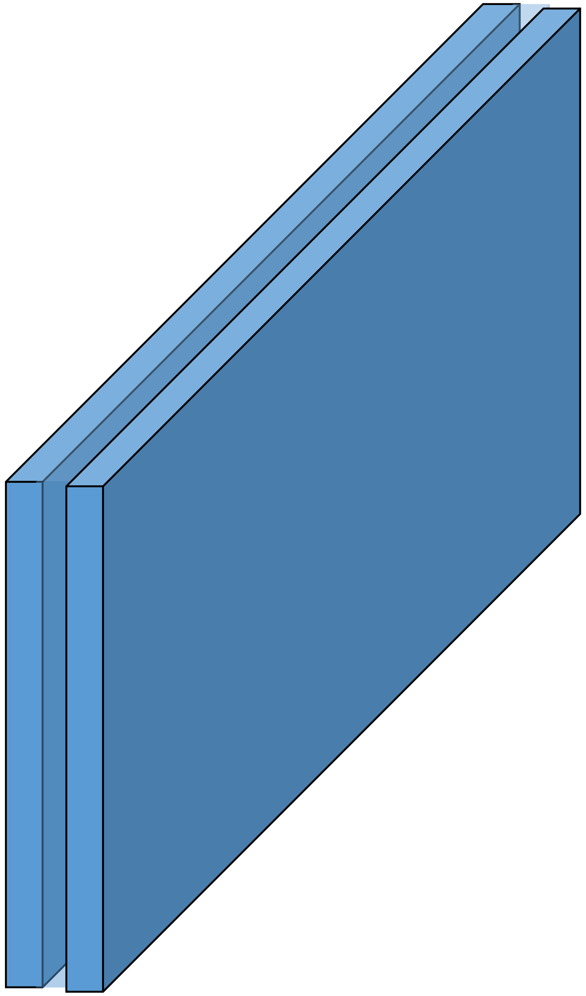
$$V = 2\pi\left[aR^2 + \frac{2}{3}a^3 - \frac{\pi}{4}Ra^2\right]$$



$r_1 < 0$

$r_2 > 0$





$$\Delta p = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$\sigma = 72\text{x}10^{-3}$ N/m

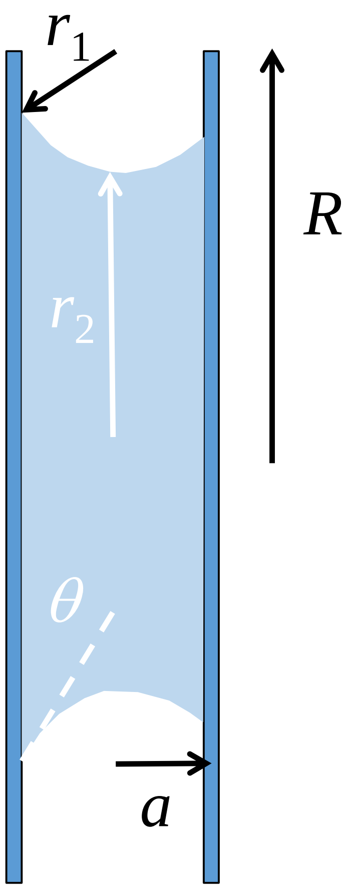
$|r_2| >>> |r_1|$

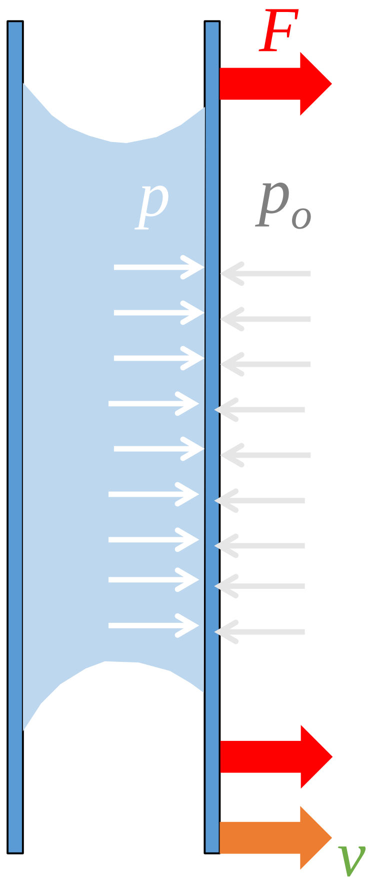
$r_1 = 0.5\text{x}10^{-3}$m

Mathematical Model

Physics:
Young Laplace equation
For capillary pressure

$$\Delta p = p - p_o = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$



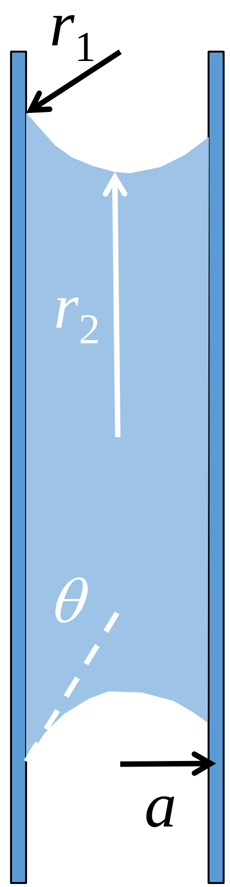


Physics:
Force balance

$(p\text{-}p_o)A+F=0$

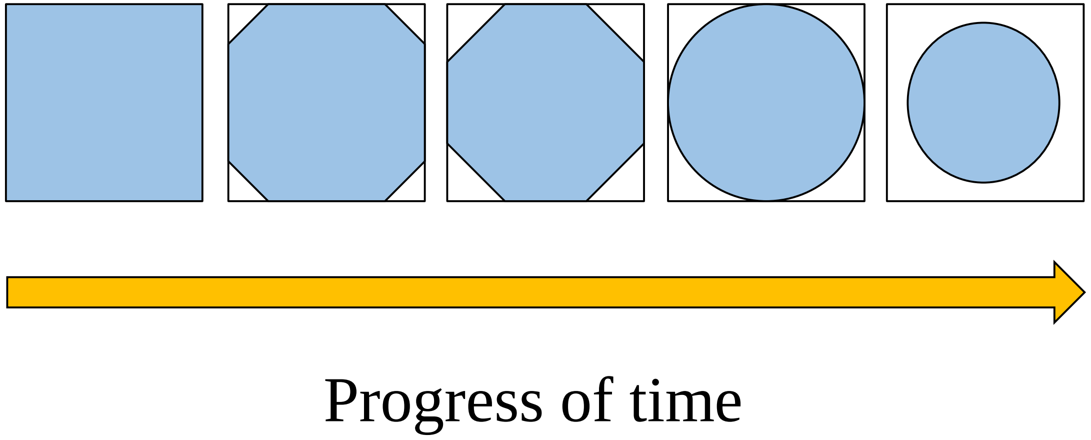
Geometry:
Volume of the bridge

$$V = 2\pi\left[aR^2 + \frac{2}{3}a^3 - \frac{\pi}{4}Ra^2\right]$$



Assumptions:

- Speed is so slow that viscous effects are negligible
- Gravity forces are negligible during the process
- Contact angle is zero, so the water is wetting completely
- At the beginning of the process the contact line is a square
- The contact line turns into a polygon
- At critical separation the contact line turns into a circle, at this point the contact line detach form the edges.



Equations

$L = 0.3 \ \ [\mathrm{m}]$

Length and height

$\Delta x = 1 \times 10^{-3} \ \ [\mathrm{m}]$

Initial gap between parallel plates

$V = L^2 \cdot \Delta x$

Volume of liquid between plates

Volume of the bridge in for a circular contact line at critical point

$$V = 2 \cdot \pi \cdot \left(a_o \cdot R_o^2 + 2 \cdot \frac{a_o^3}{3} - \pi \cdot R_o \cdot \frac{a_o^2}{4} \right)$$

Radius of the contact line, when the contact line is a circle

$R_o = L/2$

a_o is half of the separation between plates at critical point

Critical point is when polyhedral contact line turns into a circular contact line

$$\Delta x_c = 2 \cdot a_o$$

$$V = L^2 \cdot \Delta x_c - 2 \cdot L_{c,2}^2 \cdot \Delta x_c$$

Volume of the polyhedral

$$A_{c,p} = L^2 - 2 \cdot L_{c,2}^2$$

Area of contact water-glass interface of one side

$$A_1 = L^2$$

One side contact area of water-glass inteface

$$\sigma = 72 \times 10^{-3} \ [\mathrm{N/m}]$$

Surface tension

$$r_{1,0} = -\frac{\Delta x}{2}$$

Radius of curvature of meniscus at the beginning of the process

$$r_{2,0} = 1000 \cdot r_{1,0}$$

$$\Delta p_1 = \sigma \cdot (1/r_{1,0} + 1/r_{2,0})$$

Capillary pressure at the beginning of the process

$$F_1 = \Delta p_1 \cdot A_1$$

$$\Delta p_{2,c} = \sigma \cdot (1/r_{1,c} + 1/r_{2,c})$$

Capillary pressure when the contact line is at critical point as a circle

$$r_{2,c} = R_o$$

$$r_{1,c} = -a_o$$

$$F_2 = \Delta p_{2,c} \cdot \pi \cdot R_o^2$$

$$\Delta p_{2,p} = \sigma \cdot (1/r_{1,c} + 1/r_{2,0})$$

Capillary pressure when the contact line is at critical point polyhedral

Capillary pressure as polyhedral contact line

x is the dummy variable while contact line is a polyhedral, position of the second plate in the horizontal axis

$$V = L^2 \cdot x - 2 \cdot L_i^2 \cdot x$$

$$A_p = L^2 - 2 \cdot L_i^2$$

$$r_{1,p} = -x/2$$

$$\Delta p_p = \sigma \cdot (1/r_{1,p} + 1/r_{2,0})$$

$$F_p = A_p \cdot \Delta p_p$$

$$W_p = \int_{\Delta x}^{\Delta x_c} F_p \, \mathrm{d}x$$

Work, while being a polyhedral

Capillary pressure as circular contact line

Volume of the bridge in for a circular contact line

$$R = (a/2) \cdot \left(\pi/4 + \sqrt{abs\left((\pi/4)^2 + 4 \cdot \left(\frac{V}{2 \cdot \pi \cdot a^3} - 2/3 \right) \right)} \right)$$

$$a = 2 \cdot \chi$$

$$r_f = -a$$

$$\Delta p_c = \sigma \cdot \left(\frac{1}{R + r_f} + 1/r_f \right)$$

$$F_c = \Delta p_c \cdot A_c$$

$$A_c = \pi \cdot R^2$$

$$W_c = \int_{\Delta x_c}^{2 \cdot \Delta x_c} F_c \, \mathrm{d}\chi$$

Work, while being a circle

Work, while being a circle

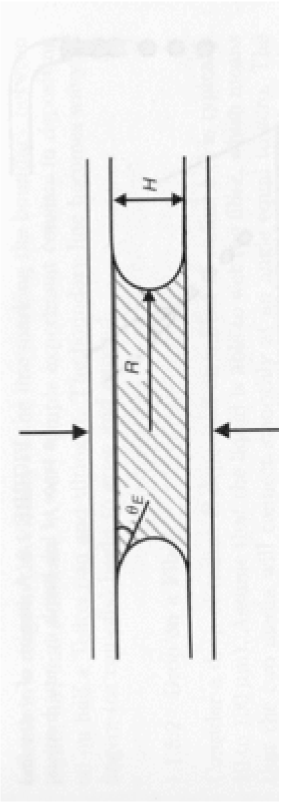
$$vel = 1\ \ [\mathrm{cm/min}] \cdot \left| 1.66667 \times 10^{-4}\ \frac{\mathrm{m/s}}{\mathrm{cm/min}} \right|$$

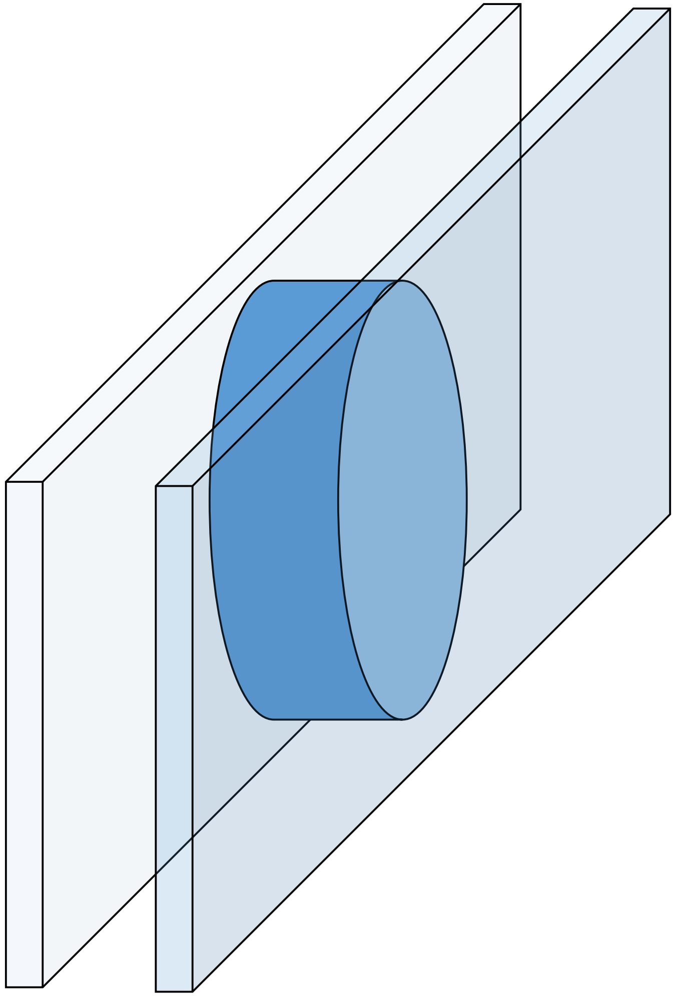
$$\dot{W}_p = F_p \cdot vel$$

$$\dot{W}_c = F_c \cdot vel$$

Solution

$a = 0.00511$ [m]	$A_1 = 0.09$ [m^2]
$A_c = 0.009443$ [m^2]	$A_{c,p} = 0.07045$ [m^2]
$a_o = 0.0006387$ [m]	$A_p = 0.07045$ [m^2]
$\chi = 0.002555$ [m]	$\Delta p_1 = -144.1$ [Pa]
$\Delta p_{2,c} = -112.2$ [Pa]	$\Delta p_{2,p} = -112.9$ [Pa]
$\Delta p_c = -12.64$ [Pa]	$\Delta p_p = -112.9$ [Pa]
$\Delta x = 0.001$ [m]	$\Delta x_c = 0.001277$ [m]
$F_1 = -12.97$ [N]	$F_2 = -7.934$ [N]
$F_c = -0.1194$ [N]	$F_p = -7.951$ [N]
$L = 0.3$ [m]	$L_{c,2} = 0.09887$ [m]
$L_i = 0.09887$ [m]	$R = 0.05482$ [m]
$r_{1,0} = -0.0005$ [m]	$r_{1,c} = -0.0006387$ [m]
$r_{1,p} = -0.0006387$ [m]	$r_{2,0} = -0.5$ [m]
$r_{2,c} = 0.15$ [m]	$r_f = -0.00511$ [m]
$R_o = 0.15$ [m]	$\sigma = 0.072$ [N/m]
$V = 0.00009$ [m^3]	$vel = 0.0001667$ [m/s]
$W_c = -0.0003112$ [J]	$\dot{W}_c = -0.0000199$ [W]
$\dot{W}_p = -0.001325$ [W]	$W_p = -0.002818$ [J]
$x = 0.001277$ [m]	



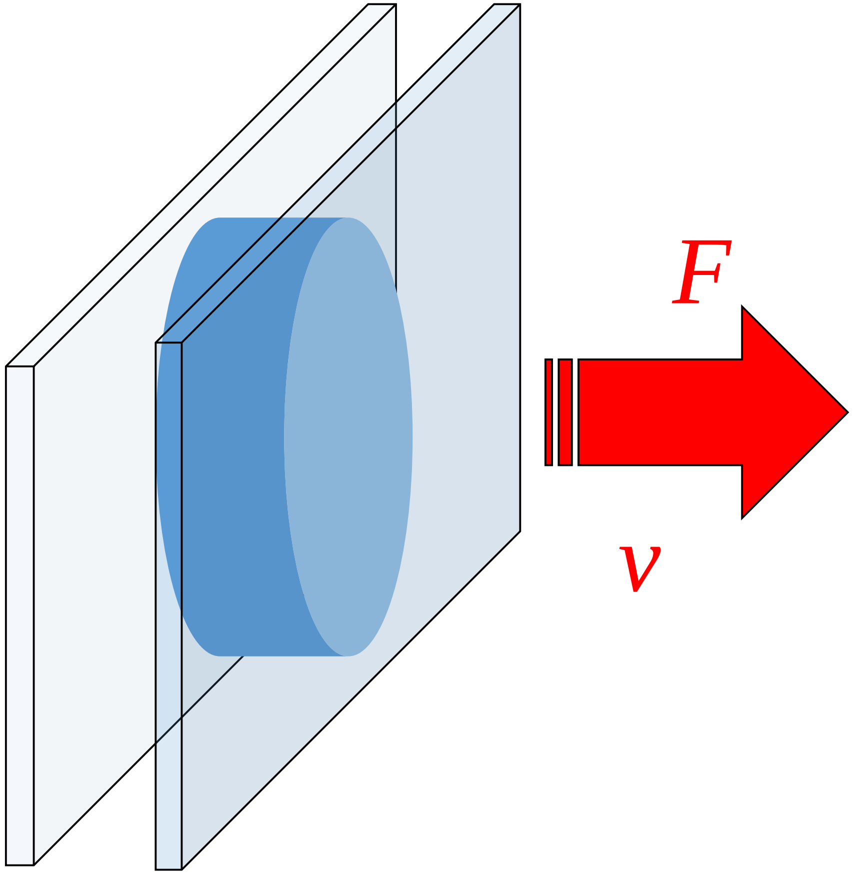


$$\Delta p = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

$\sigma = 72\text{x}10^{-3}$ N/m

$|r_2| >>> |r_1|$

$r_1 = 0.5\text{x}10^{-3}$m


F
v

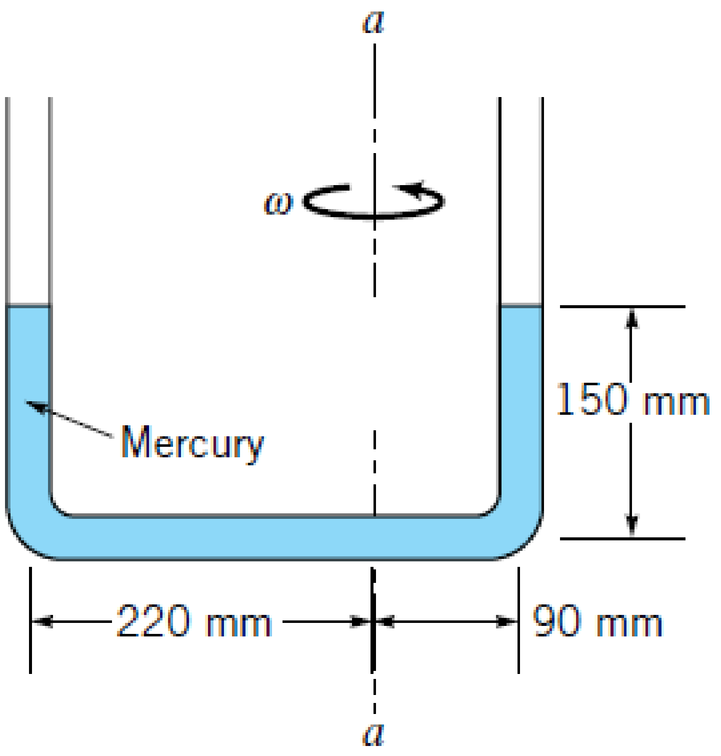
$$\underset{\rightarrow}{\nabla} p = \rho \underset{\longrightarrow}{f_m} = \underset{\rightarrow}{f_v}$$

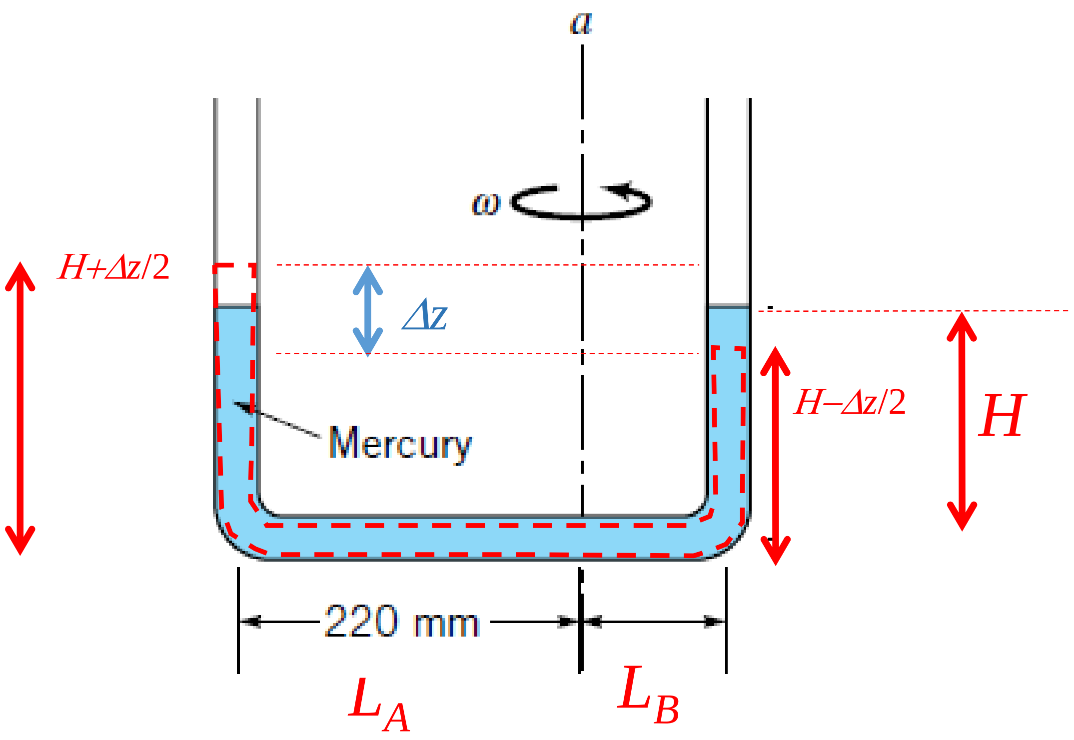
$$\underset{\rightarrow}{\nabla} p = \rho\left(\underset{\rightarrow}{g} - \underset{\rightarrow}{a} \right)$$

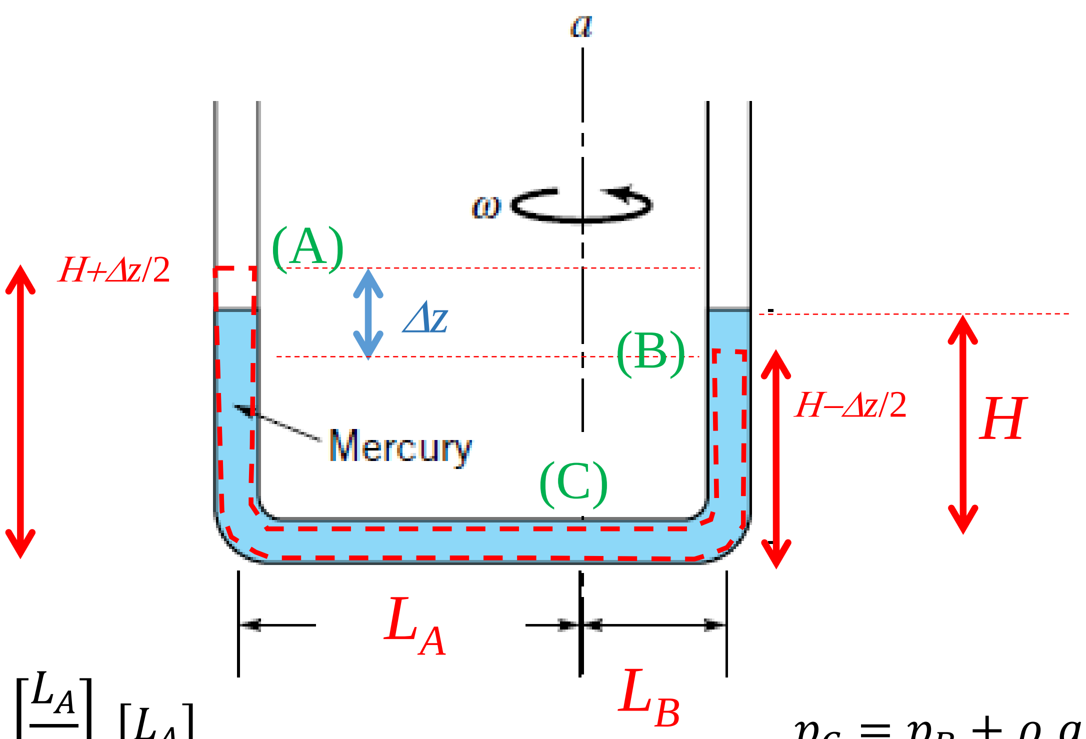
Divergence theorem

$$\oint_A \underset{\longrightarrow}{\Psi} \cdot \underset{\rightarrow}{n} dA = \int_V \underset{\rightarrow}{\nabla} \cdot \underset{\longrightarrow}{\Psi} dV$$

2.100 The U-tube of Fig. P2.100 contains mercury and rotates about the off-center axis *a–a*. At rest, the depth of mercury in each leg is 150 mm as illustrated. Determine the angular velocity for which the difference in heights between the two legs is 75 mm.







$$p_C = p_A + \rho\, g\, [H + \Delta z/2] - \rho\, \omega^2 \left[\frac{L_A}{2}\right] [L_A]$$

$$p_C = p_B + \rho\, g\, [H - \Delta z/2] - \rho\, \omega^2 \left[\frac{L_B}{2}\right] [L_B]$$

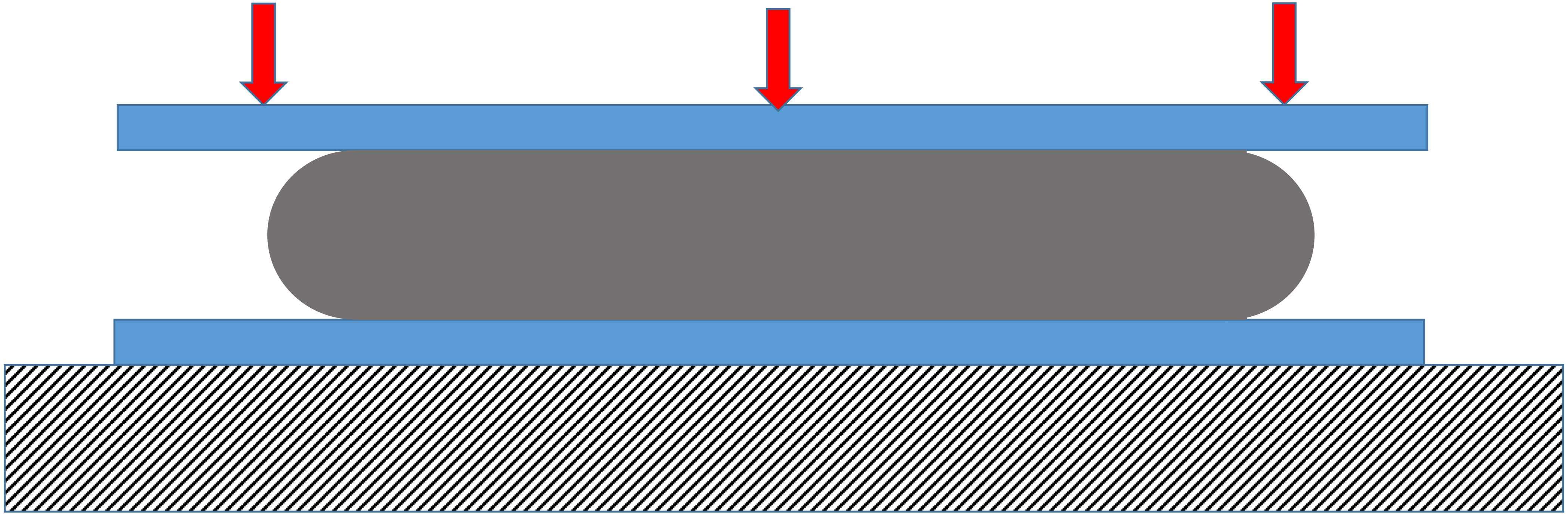
$$p_{atm} = p_A = p_B$$

$$p_A + \rho\, g\, [H + \Delta z/2] - \rho\, \omega^2 \left[\frac{L_A}{2}\right] [L_A] = p_B + \rho\, g\, [H - \Delta z/2] - \rho\, \omega^2 \left[\frac{L_B}{2}\right] [L_B]$$

$$\omega = \sqrt{\frac{2\, g\, \Delta z}{L_A^2 - L_B^2}}$$

$$\omega = \sqrt{\frac{2\,(9.81\ \mathrm{m/s^2})(0.075\ \mathrm{m})}{(0.22^2 - 0.09^2)\ \mathrm{m^2}}} = 5.92\ \mathrm{s^{-1}} = 56.54\ \mathrm{RPM}$$

Problem 3

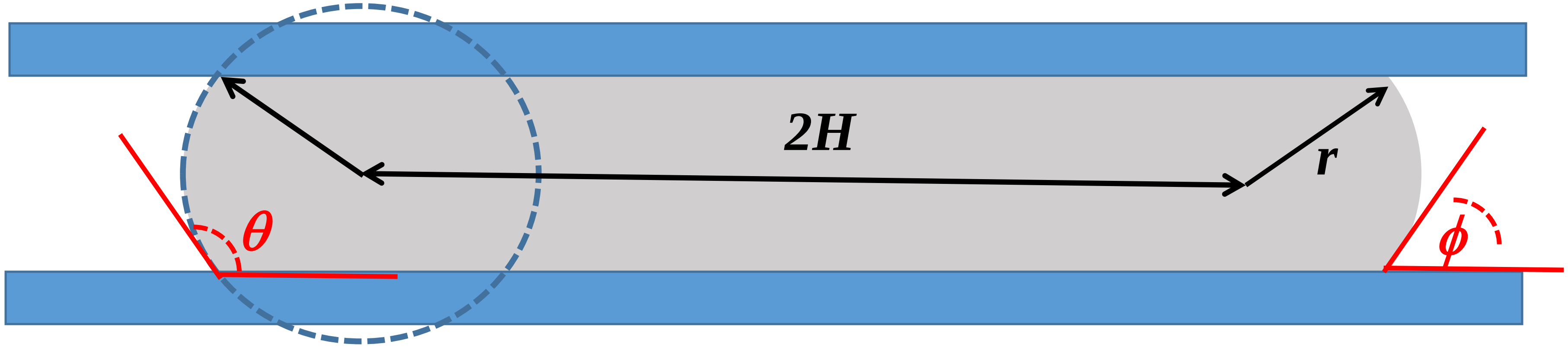


Two 10-cm ×10-cm × 6-mm-thick glass plates are used to compress galinstan (density 6440 kg/m^3 , surface tension 0.535 N/m), a) If the original volume of the liquid metal is 10 cm^3, calculate the separation between plates, at equilibrium conditions. b) How much force is required to reduce separation between plates to half the equilibrium position.

Note: as a first approach you can assume the contact angle to be 180°, and neglect gravity effects over the fluid. (Simplify the geometry as much as possible)

Fluid Mechanics

Problem 3b. If you are willing to consider that the contact angle is 146° (σ = 718 mN/m), and the volume of the trapped liquid given by the equation given below, recalculate and compare your solution with the previous one.



$$\frac{V}{2\pi} = (H^2 + r^2) r Cos\varphi - \frac{r^3 Cos^3\varphi}{3} + 2H\left[\frac{r^2 Sin(2\varphi)}{4} + \frac{r^2}{2}\left(\frac{\pi}{2} - \varphi\right)\right]$$

1. A rubber balloon has a mass of 5.1±0.2 g, if you fill a balloon with gas up to a diameter of 30 cm, calculate the acceleration when it is released on air if:

a) You fill the balloon with air (M=29 kg/kmol)
b) You fill the balloon with Helium (M=4 kg/kmol)
c) You fill the balloon with sulfur hexafluoride (M=146.07 kg/kmol)

2. Calculate the terminal velocity in all the cases if the friction force can be estimated Stokes' law:

$$F_{fricton} = 6\,\pi\,\mu\,v\,R$$

μ=Air viscosity =1.98x10^{-5} Pa-s

For the physics of the balloon you can read: http://arxiv.org/pdf/1103.2126v1.pdf

Trajektorien von Gummiballons in Ballonwettbewerben: Theorie und Anwendung Patrick Glaschke

Pressure inside the balloon:

$$\Delta p(R) = \frac{4}{3}\frac{V_{\mathrm{Gummi}}}{V_0}C_1\left(\frac{R_0}{R}-\frac{R_0^7}{R^7}\right)\left(1+\frac{C_{-1}}{C_1}\frac{R^2}{R_0^2}\right) \qquad V_0 = \frac{4\pi}{3}R_0^3$$

$C_1 \approx 0{,}17$ MPa $\qquad C_{-1}/C_1 \approx 0,1 \qquad V_{\mathrm{Gummi}} = 5\,\mathrm{cm}^3,\ V_0 = 150$ ml

1. A rubber balloon has a mass of 5.1±0.2 g, if you fill a balloon with gas up to a diameter of 30 cm, calculate the acceleration when it is released on air if:

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b) You fill the balloon with Helium (M=4 kg/kmol)
c) You fill the balloon with sulfur hexafluoride (M=146.07 kg/kmol)

2. Calculate the terminal velocity in all the cases if the friction force can be estimated as:

$$F_{fricton} = 0.46\left[\frac{\pi D^2}{4}\right]\left(\frac{1}{2}\rho v^2\right)$$

μ=Air viscosity =1.98x10^{-5} Pa-s

For the physics of the balloon you can read: http://arxiv.org/pdf/1103.2126v1.pdf

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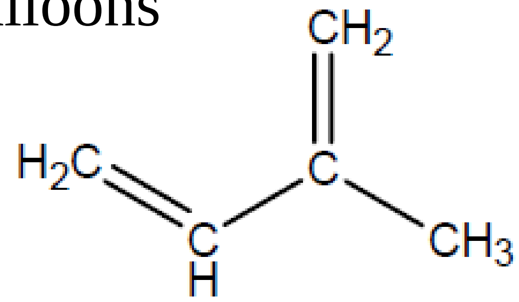
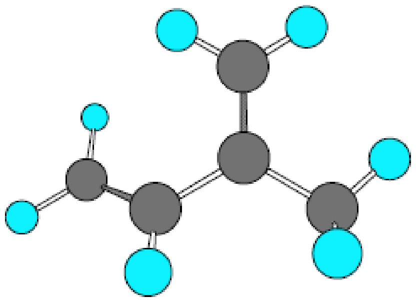
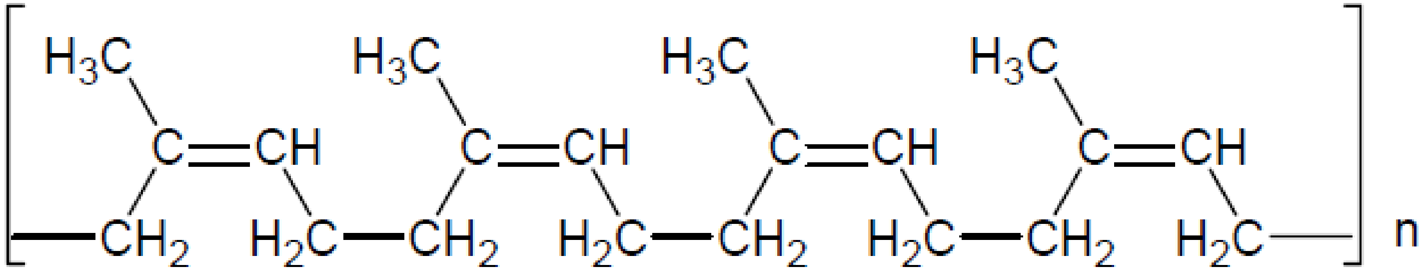
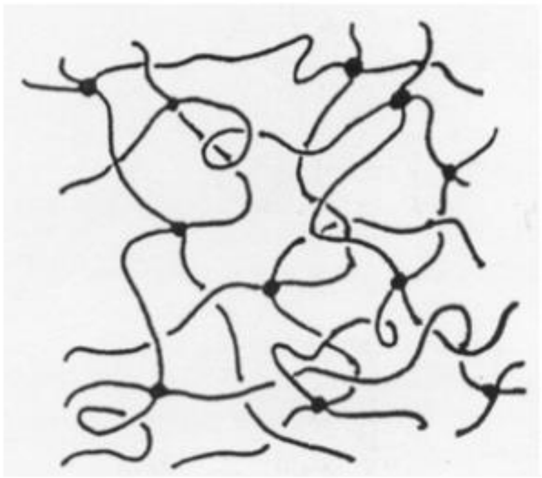
Chemistry of balloons







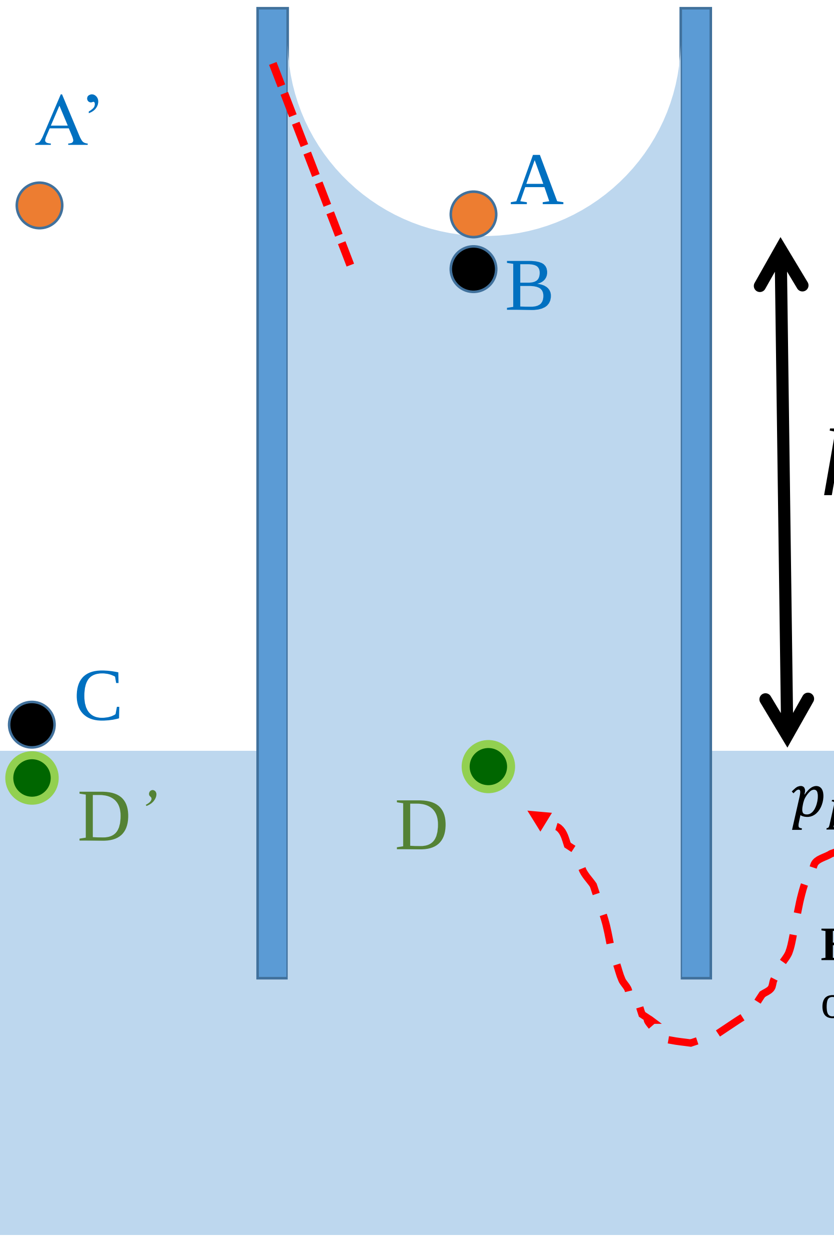


Problem 3 Calculate the capillary rise of water in a 50 mm diameter glass tube.

Pressure difference across a curved interfaces is calculated using Young-Laplace, this can be used to predict the capillary rise, with aid of hydrostatic pressure we have learn so far.

Thermodynamics: Young-Laplace will help to calculate pressure difference across the curved interface

$$\Delta p = p_A - p_B = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$



If you have a continuous stagnate fluid located at the same energy level (in this case potential energy) its pressure remains. In this case (p_A=$p_{A'}$ and p_D=$p_{D'}$)

" *a* " = air over the liquid

$$p_C = p_A + \rho_a g h$$

$$p_D = p_C$$

Flat interfaces have equal pressure

$$p_D = p_B + \rho_f g h$$

Hydrostatics: Allows calculation of pressure as function of depth

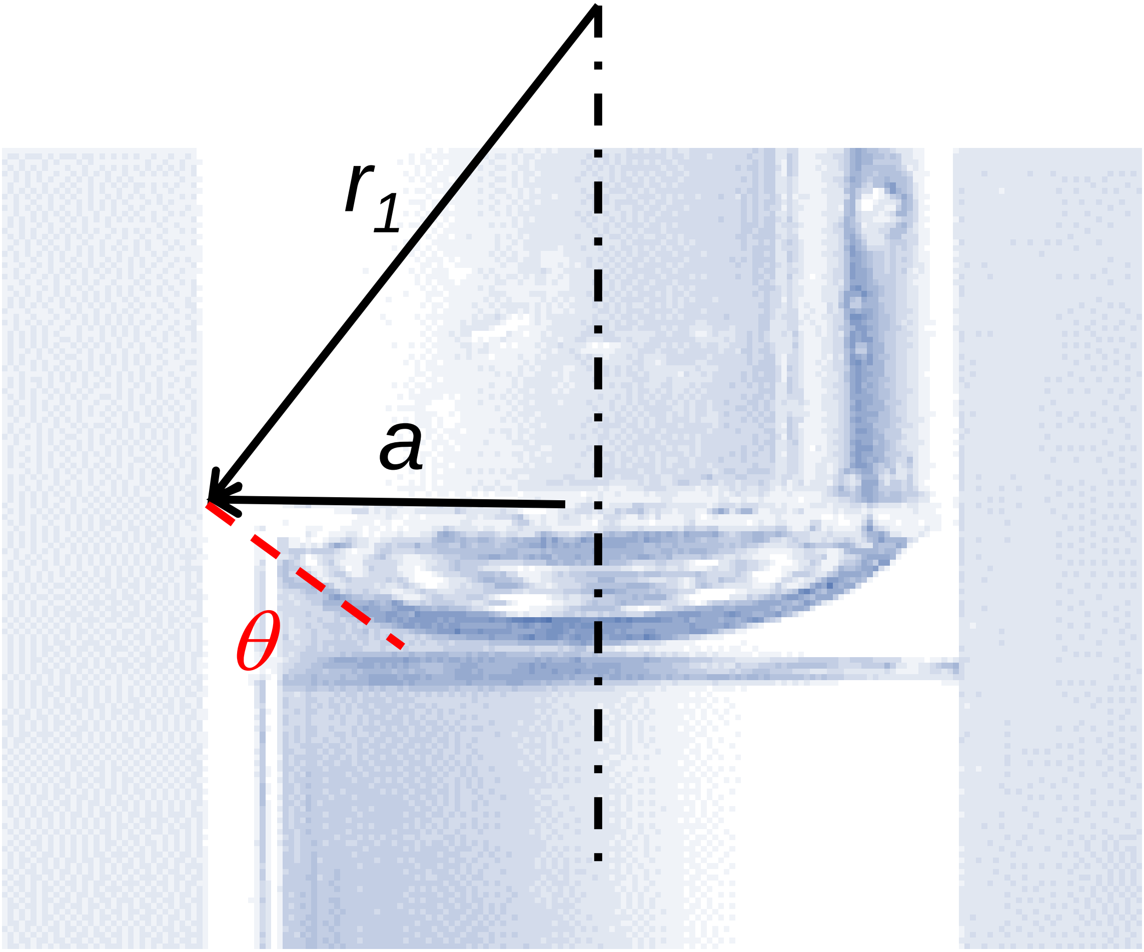
" *f* " = liquid fluid

Trigonometric relationships are used to relate curvature radius and the capillary's radius.

$$a = r_1 cos\theta$$

Capillary radius

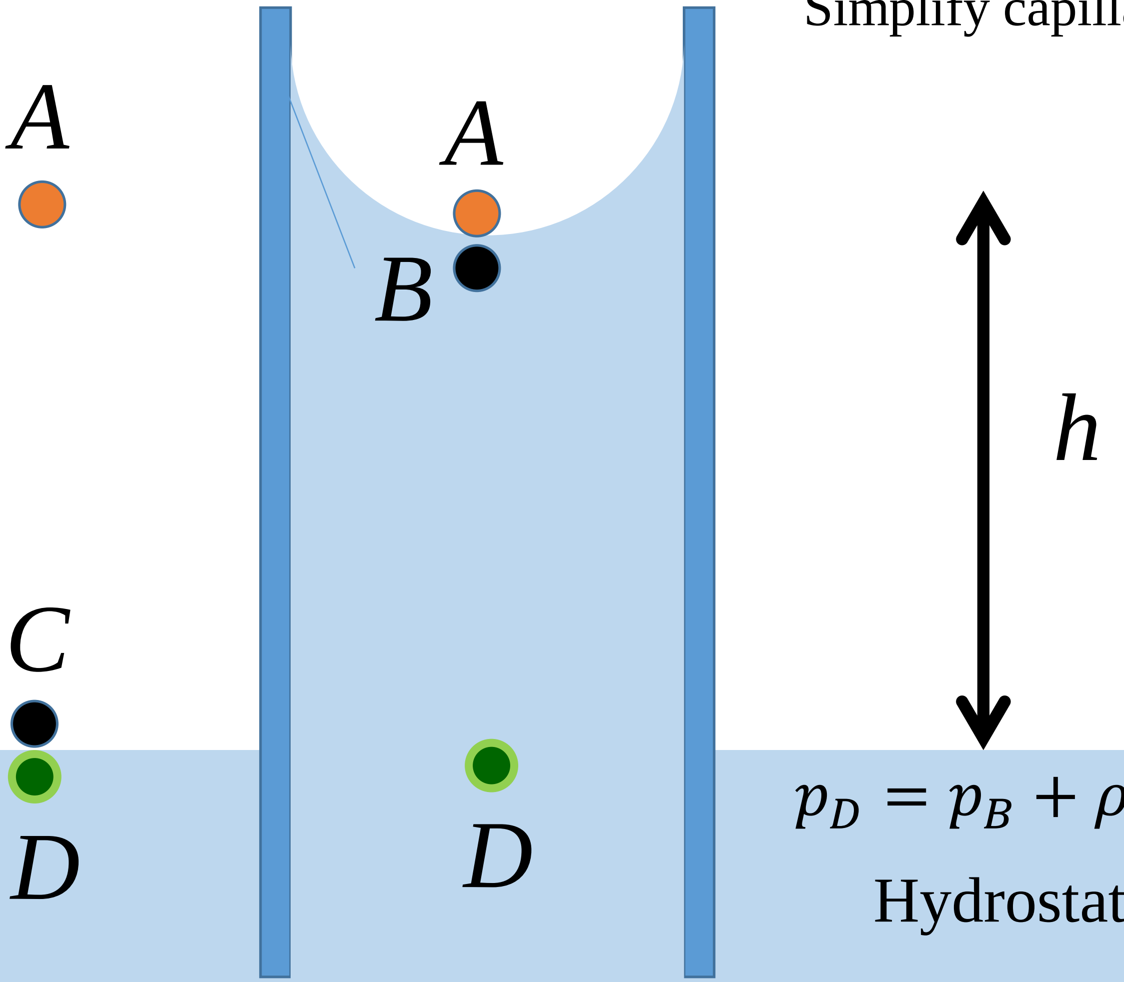
Both capillary radii are equal (r_1=r_2)



Assumption: The meniscus has a uniform radius of curvature in any direction (i.e. the meniscus interface can be represented as a portion of a sphere)

Simplify capillary rise in terms of curvature

$$p_A = p_B + \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$



$$p_D = p_A + \rho_a g h$$

$$p_D = p_B + \rho_f g h$$

Hydrostatics

$$p_D = p_A + \rho_a g h = p_B + \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \rho_a g h$$

$$p_D = p_B + \rho_f gh$$

Pressure at "D" in terms of pressure "B" across the liquid phase

$$p_D = p_B + \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \rho_a gh$$

Pressure at "D" in terms of pressure "B" traveling across the gas phase

Matching pressure D, the final equation takes the form:

$$p_D = \cancel{p_B} + \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \rho_a gh = \cancel{p_B} + \rho_f gh$$

Using the trigonometric relationships for both curvature radii:

$$a = r_1 cos\theta$$

$$\frac{2\sigma}{a} cos\theta = (\rho_f - \rho_a) gh$$

Collecting terms, the capillary rise as function of contact angle θ, surface tension σ, capillary radius a, density difference $\Delta\rho$ and gravity g has the form :

$$h = \frac{2\sigma}{a\Delta\rho g} cos\theta$$

Finally grouping in dimensionless numbers:

$$\left(\frac{h}{a}\right) = \frac{2\ cos\theta}{\left(\frac{a^2\Delta\rho g}{\sigma}\right)}$$

$$\left(\frac{h}{a}\right) = \frac{2\ cos\theta}{Bo}$$

$$Bo = \frac{a^2\Delta\rho g}{\sigma}$$

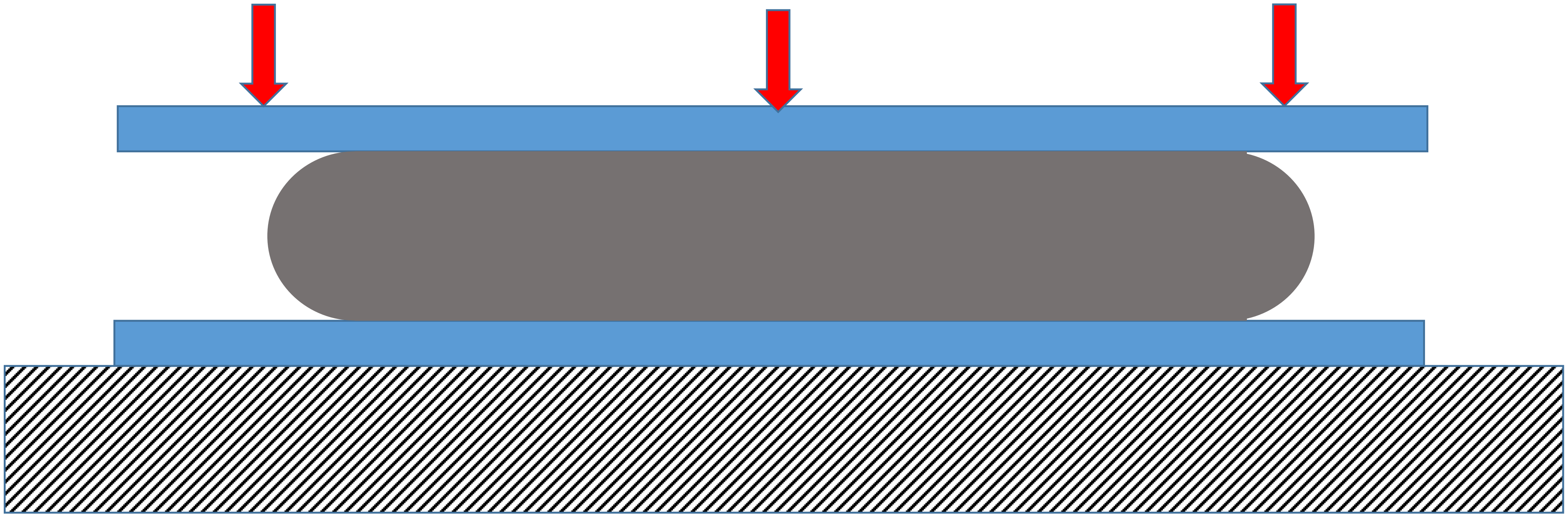
$$\left(\frac{h}{a}\right) = \frac{2\, cos\theta}{Bo} \qquad\qquad Bo = \frac{a^2 \Delta\rho g}{\sigma}$$

Then the ratio of capillary height rise and its radius is function of the wettability or contact angle, and the Bond number.

The Bond number (sometimes called Eötvös number) is the ratio between volume forces (gravitational forces or weight) and the line forces (interfacial or surface tension forces)

If the Bond number is grater than one ($Bo \gg 1$), gravity dominates, and the capillary rise is negligible. If the Bond number is less than one ($Bo \ll 1$), line forces dominate and the capillary rise/depression is important. For hydrophilic surfaces ($\theta < 90°$) capillary rise is positive, and for hydrophobic surfaces ($\theta > 90°$) there is a capillary depression.

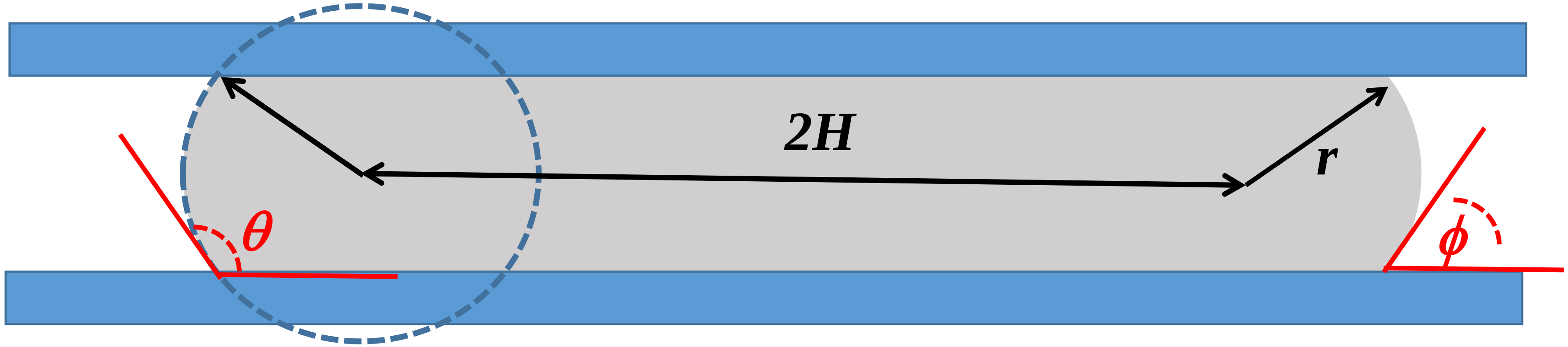
Problem 1



Two 10-cm ×10-cm × 6-mm-thick glass plates are used to compress galinstan (density 6440 kg/m^3 , surface tension 0.535 N/m), a) If the original volume of the liquid metal is 10 cm^3, calculate the separation between plates, at equilibrium conditions. b) How much force is required to reduce separation between plates to half the equilibrium position.

Note: as a first approach you can assume the contact angle to be 180°, and neglect gravity effects over the fluid.

Problem 1b. If you are willing to consider that the contact angle is 146° (σ = 718 mN/m), and the volume of the trapped liquid given by the equation given below, recalculate and compare your solution with the previous one.



$$\frac{V}{2\pi} = (H^2 + r^2)rCos\varphi - \frac{r^3Cos^3\varphi}{3} + 2H\left[\frac{r^2Sin(2\varphi)}{4} + \frac{r^2}{2}\left(\frac{\pi}{2} - \varphi\right)\right]$$

FAQs:

1. Is redundant to estimate the force caused by capillary pressure and the line force ?
2. What is the direction of the line force (e.g. forced caused by interfacial tension or surface tension)?
3. When you have two curvature radii, how to know their sign ?
4. What is the shape of any portion continuous fluid ?
5. Is equivalent to estimate buoyancy over a body immersed in fluid as the weight of displaced fluid, and the resultant of the hydrostatic pressure ?

3–155 A U-tube contains water in the right arm, and another liquid in the left arm. It is observed that when the U-tube rotates at 50 rpm about an axis that is 15 cm from the right arm and 5 cm from the left arm, the liquid levels in both arms become the same, and the fluids meet at the axis of rotation. Determine the density of the fluid in the left arm.

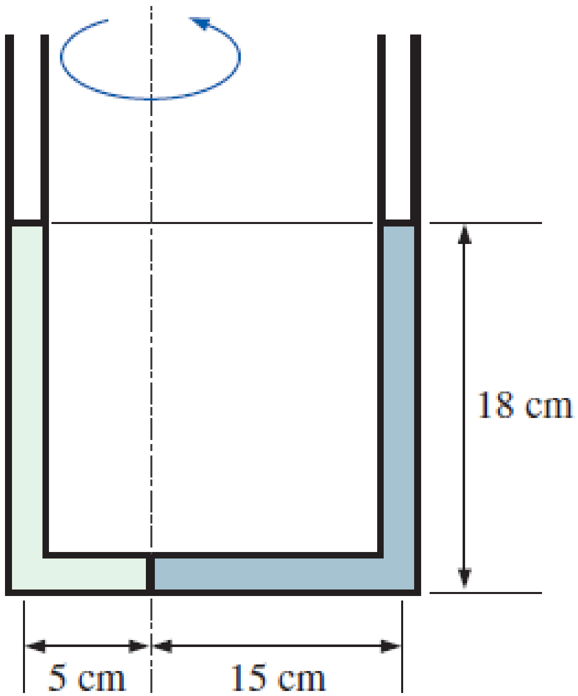
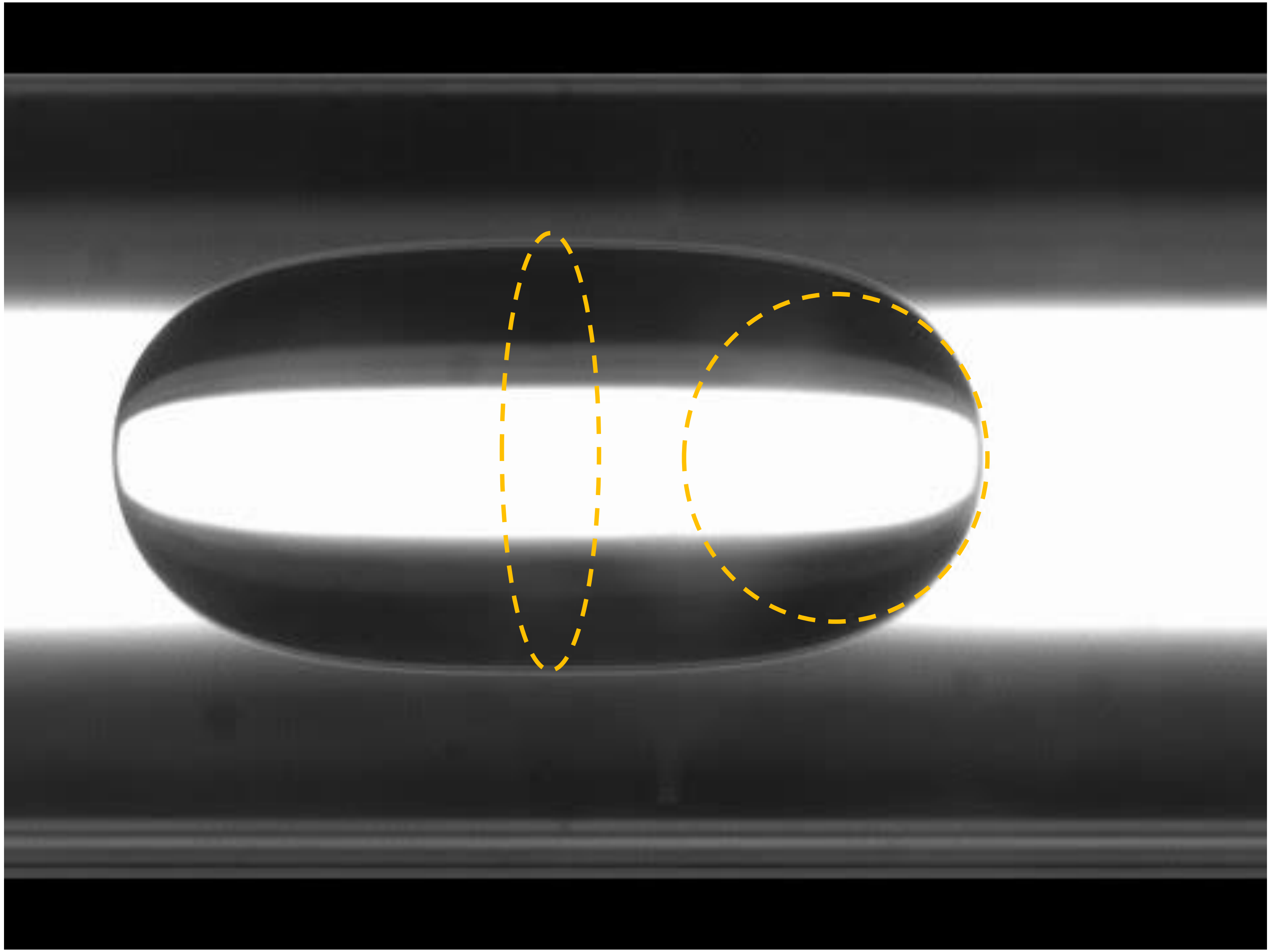


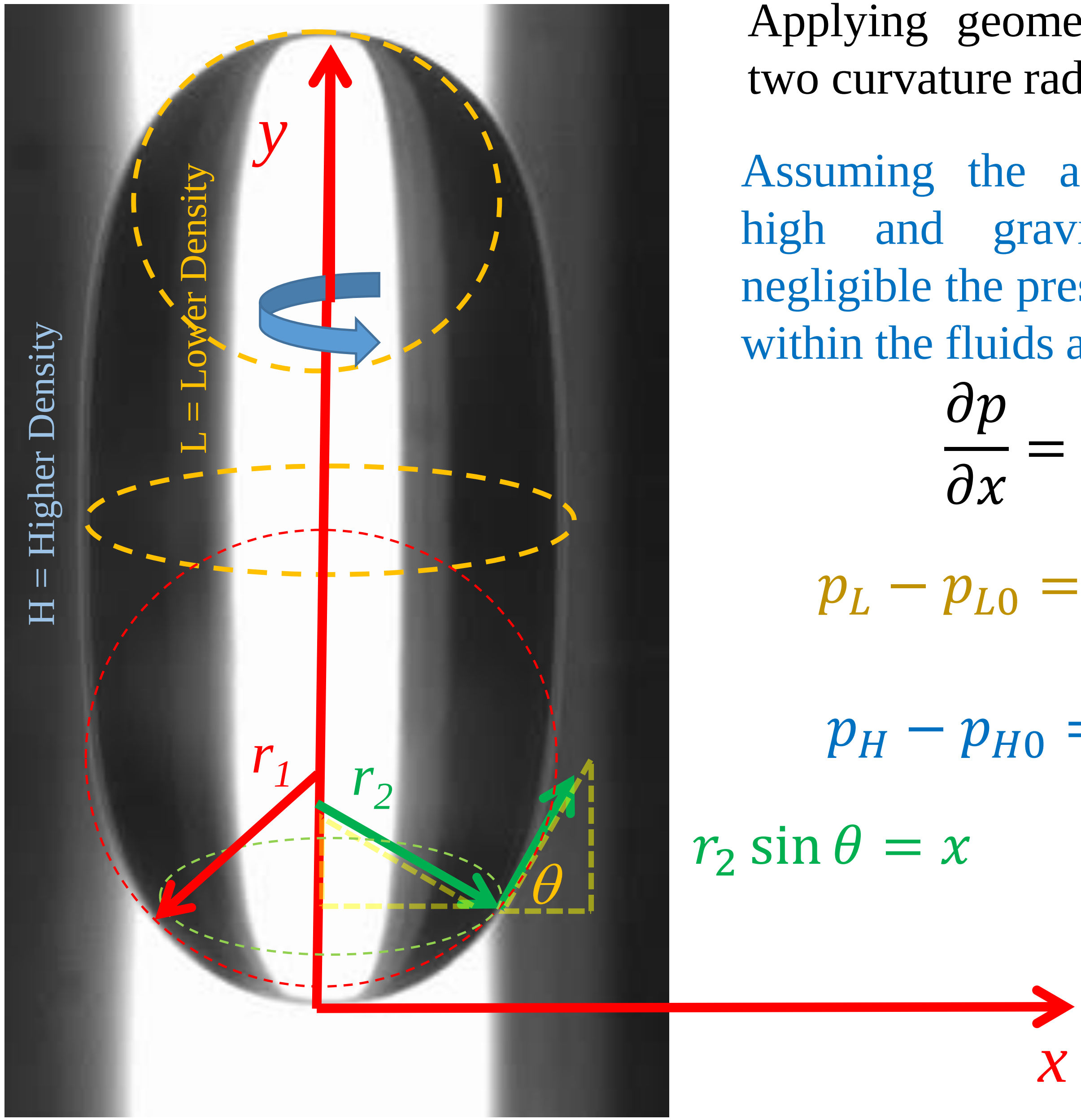
FIGURE P3–155

Estimate the shape of a drop of lightweight immersed in heavier fluid spinning



Estimate the shape of a drop of lightweight immersed in heavier fluid spinning

The drop will be rotated to simplify the modeling of the drop.



$$r_1 = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Applying geometric analysis the two curvature radii are (r_1 and r_2)

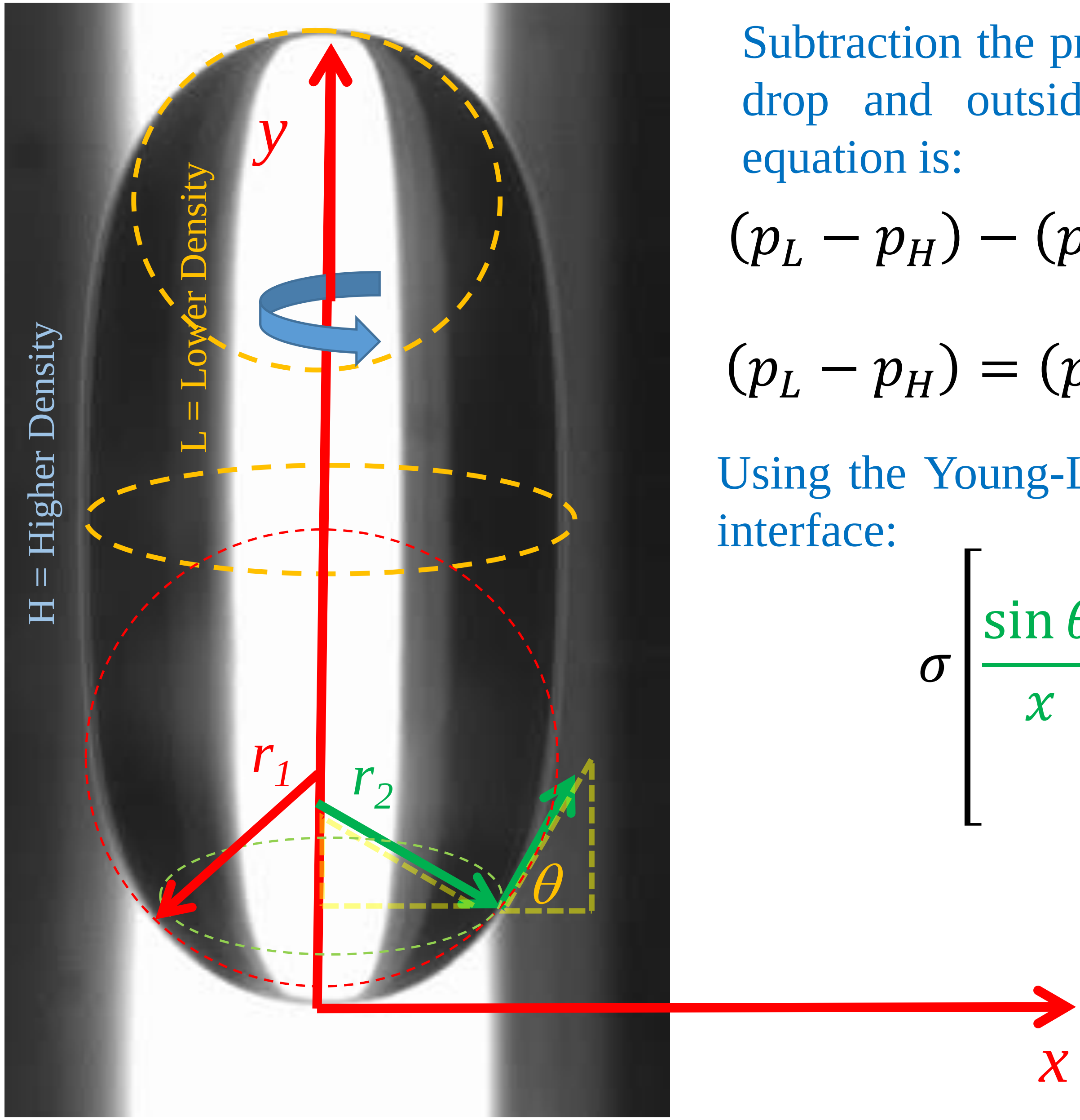
Assuming the angular speed is high and gravitational effects negligible the pressure distribution within the fluids are:

$$\frac{\partial p}{\partial x} = \rho\omega^2 x$$

$$p_L - p_{L0} = \frac{\rho_L \omega^2 x^2}{2}$$

$$p_H - p_{H0} = \frac{\rho_H \omega^2 x^2}{2}$$

$$r_2 \sin\theta = x$$



Subtraction the pressure inside the drop and outside the drop the equation is:

$$(p_L - p_H) - (p_{L0} - p_{H0}) = \frac{-(\rho_H - \rho_L)\omega^2 x^2}{2}$$

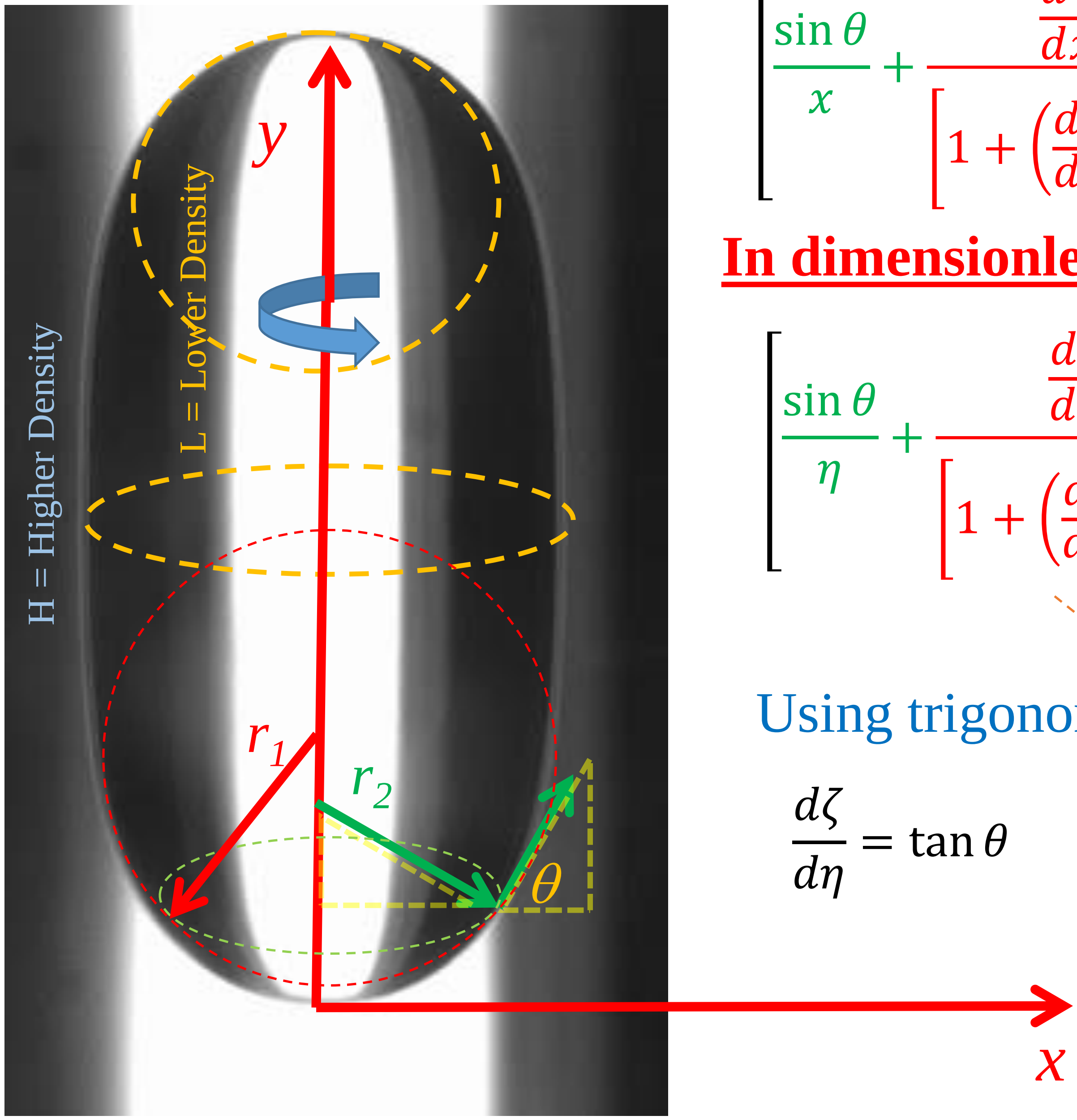
$$(p_L - p_H) = (p_{L0} - p_{H0}) - \frac{\Delta\rho\omega^2 x^2}{2}$$

Using the Young-Laplace equation to calculate pressure difference across the interface:

$$\sigma\left[\frac{\sin\theta}{x} + \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}\right] = (p_{L0} - p_{H0}) - \frac{\Delta\rho\omega^2 x^2}{2}$$

At the apex and nadir of the figure the both curvature radii are equal R_0:

$$(p_{L0} - p_{H0}) = \frac{2\sigma}{R_0}$$



$$\left[\frac{\sin\theta}{x} + \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}\right] = \frac{2}{R_0} - \frac{\Delta\rho\omega^2x^2}{2\sigma}$$

$$\alpha = \frac{\Delta\rho\omega^2{R_0}^3}{2\sigma}$$

$$\eta = \frac{x}{R_0} \qquad \zeta = \frac{y}{R_0}$$

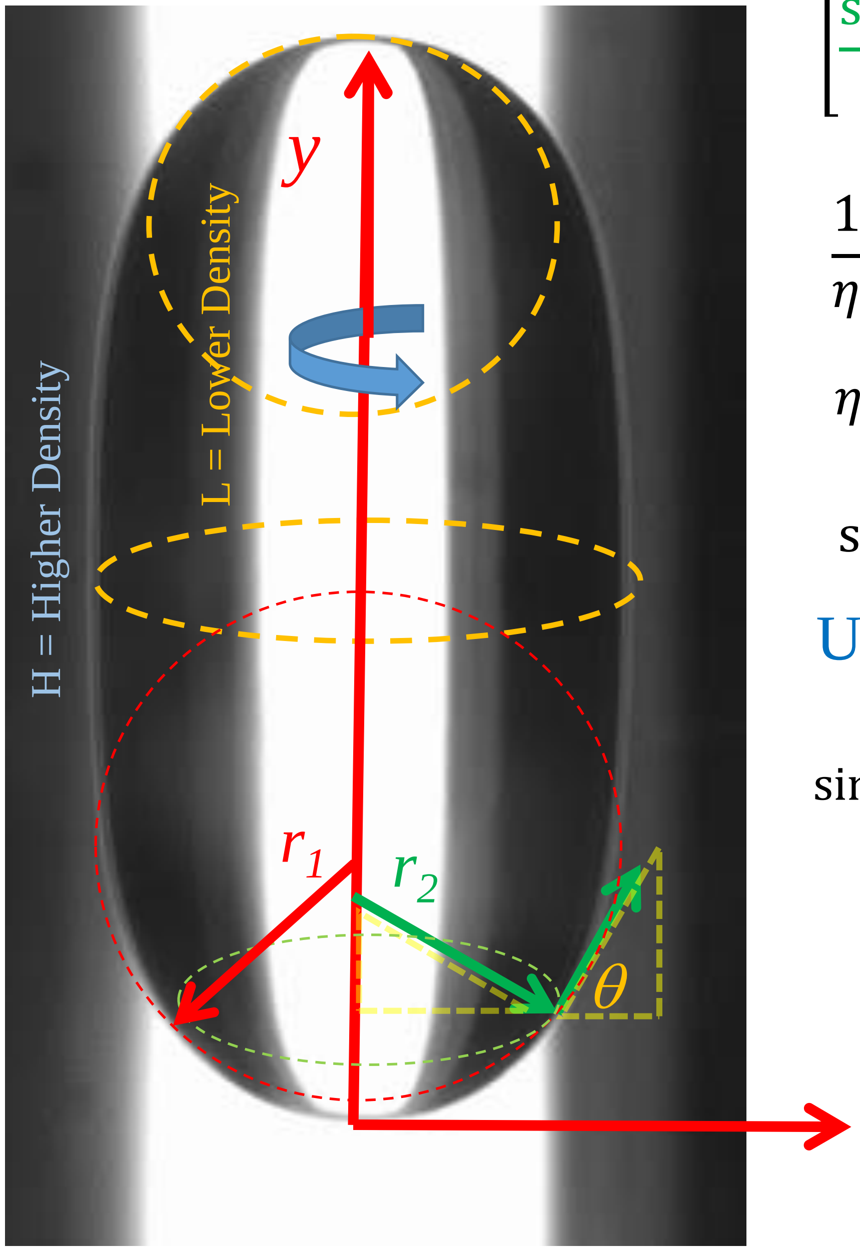
In dimensionless form

$$\left[\frac{\sin\theta}{\eta} + \frac{\frac{d^2\zeta}{d\eta^2}}{\left[1+\left(\frac{d\zeta}{d\eta}\right)^2\right]^{3/2}}\right] = 2 - \alpha\,\eta^2$$

Using trigonometric identities the equation can be re cased as:

$$\frac{d\zeta}{d\eta} = \tan\theta \qquad \left[1+\left(\frac{d\zeta}{d\eta}\right)^2\right]^{3/2} = sec^3\theta = \frac{1}{cos^3\theta} \qquad \frac{d}{d\eta}\frac{d\zeta}{d\eta} = sec^2\,\theta\,\frac{d\theta}{d\eta}$$

$$\frac{\frac{d^2\zeta}{d\eta^2}}{\left[1+\left(\frac{d\zeta}{d\eta}\right)^2\right]^{3/2}} = \frac{d\sin\theta}{d\eta}$$



$$\left[\frac{\sin\theta}{\eta} + \frac{\mathrm{d}\sin\theta}{d\,\eta}\right] = 2 - \alpha\,\eta^2$$

$$\frac{1}{\eta}\frac{d(\eta\sin\theta)}{d\,\eta} = 2 - \alpha\,\eta^2$$

$$\eta\sin\theta = \eta^2 - \frac{\alpha}{4}\,\eta^4 + \mathrm{c}$$

$$\sin\theta = \eta - \frac{\alpha}{4}\,\eta^3$$

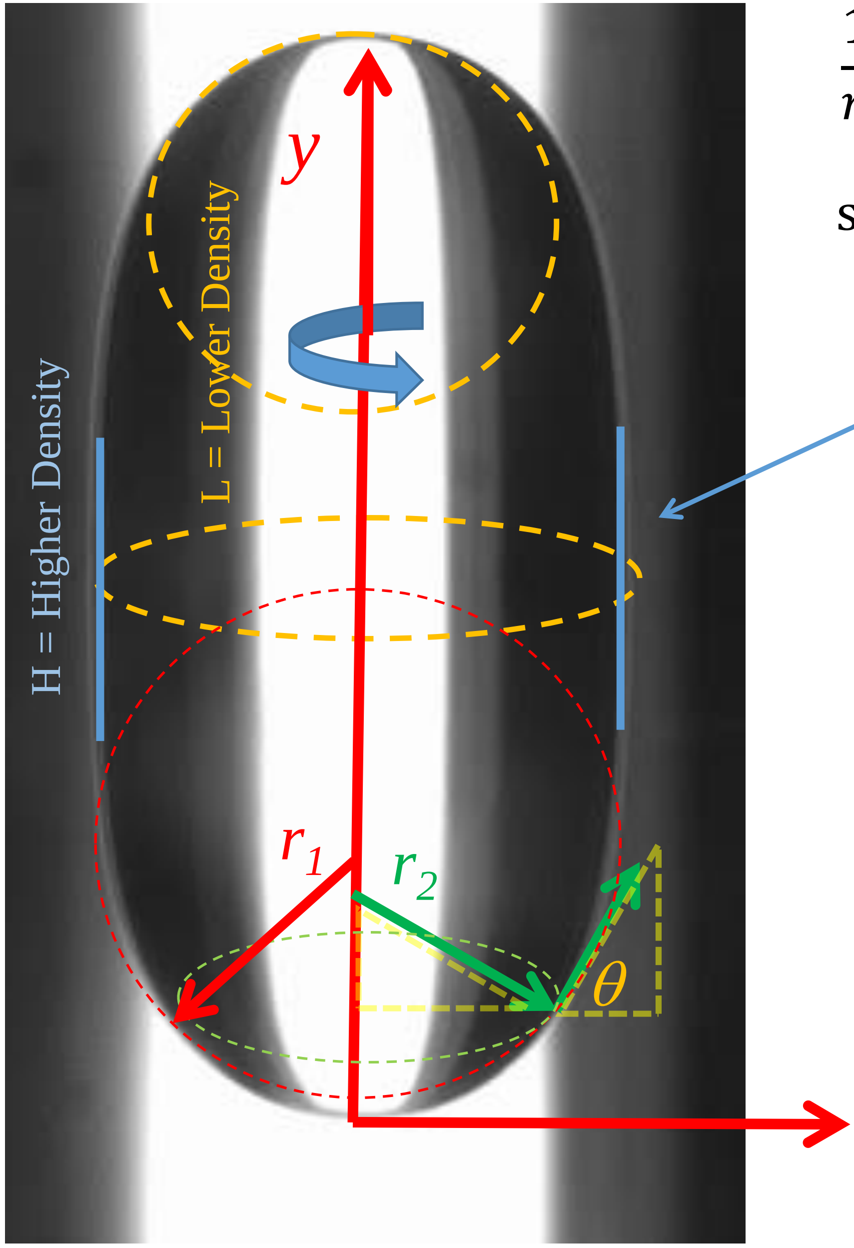
Using trigonometric identities the equation can be re cased as:

$$\sin\theta = \frac{\sin\theta\cos\theta}{\cos\theta} = \frac{\tan\theta}{\sec\theta} = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{\frac{d\zeta}{d\eta}}{\sqrt{1+\left(\frac{d\zeta}{d\eta}\right)^2}}$$

$$\frac{d\zeta}{d\eta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \qquad \frac{d\zeta}{d\eta} = \frac{\eta - \frac{\alpha}{4}\,\eta^3}{\sqrt{1-\left[\eta - \frac{\alpha}{4}\,\eta^3\right]^2}}$$

This can be integrated, but before doing so, some conclusions may be inferred:

What happen when drop is elongated?



$$\frac{1}{\eta}\frac{d(\eta \sin\theta)}{d\,\eta} = 2 - \alpha\,\eta^2$$

$$\sin\theta = \eta - \frac{\alpha}{4}\,\eta^3$$

At the middle section of the drop, takes cylindrical shape, and η has its maximum value, and $\theta = \pi/2$ ($\sin\theta = 1$) then

$$1 = \eta_m - \frac{\alpha}{4}\eta_m^3$$

At the middle section of the drop, the value of $\theta = \pi/2$ ($\sin\theta = 1$) remains constant then also

$$\frac{1}{\eta}\frac{d(\eta \sin\theta)}{d\,\eta} \approx \frac{1}{\eta}\frac{d(\eta)}{d\,\eta} \approx \frac{1}{\eta} \approx 2 - \alpha\,\eta^2$$

$$1 \approx 2\eta_m - \alpha\,\eta_m^3$$

Solving this two equations

$$\eta_m \approx \frac{3}{2} \qquad \alpha \approx \frac{16}{27} \qquad R_m \approx \frac{3}{2}R_0$$

In terms of the radius at the apex

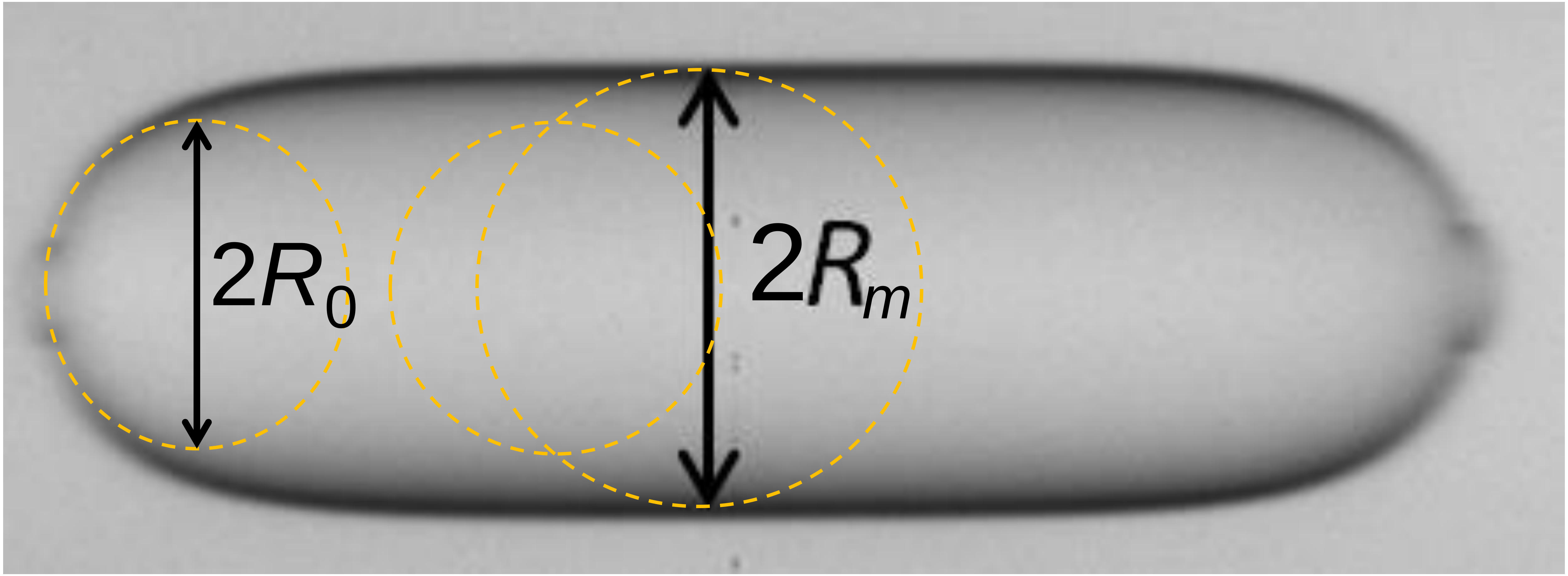
$$\alpha = \frac{\Delta\rho\omega^2 {R_0}^3}{2\sigma}$$

$$\alpha \approx \frac{16}{27}$$

$$R_m \approx \frac{3}{2} R_0$$

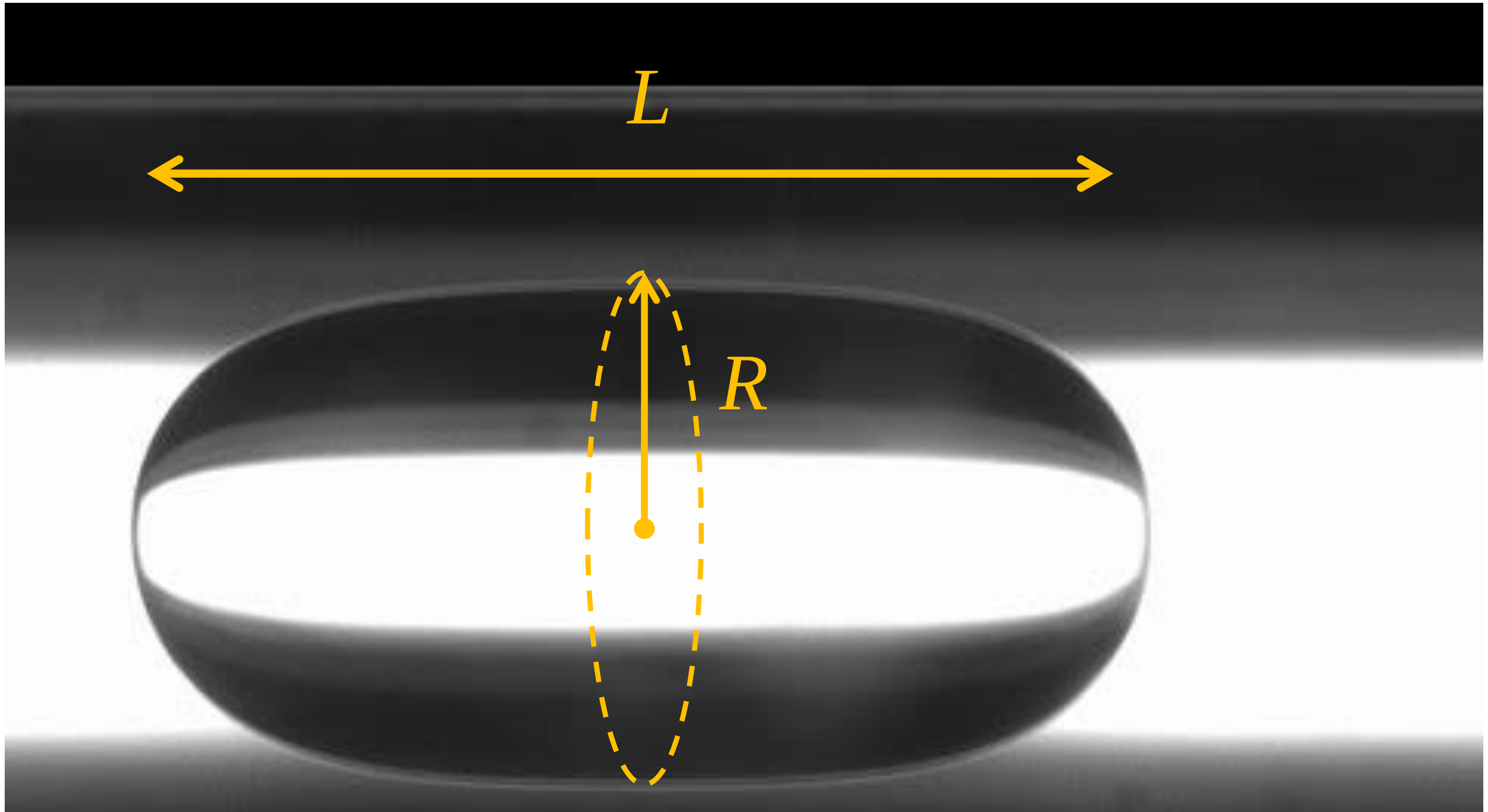
$$1 \approx \frac{\Delta\rho\omega^2 {R_m}^3}{4\sigma}$$

This equation can be used to estimate IFT for very elongated drops



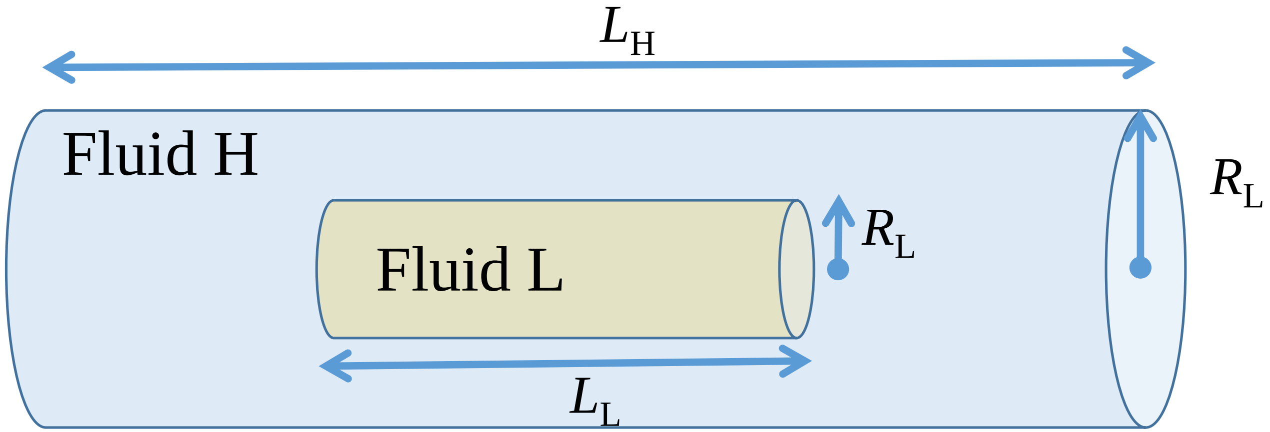
$$\alpha = \frac{4(\eta_m - 1)}{\eta_m^3}$$

$$0 < \alpha < \frac{16}{27}$$



When this drop is elongated enough, this is $L > 8\ R$, the Interfacial tension can be estimated knowing the angular speed and drop radius. Obtain this relationship when you minimize the energy of the system.

$$\Psi = \frac{p}{\rho} + gz + \frac{1}{2}v^2 - \frac{\Omega^2 r^2}{2} = \frac{p_0}{\rho} + gz_0 + \frac{1}{2}v_0^2 - \frac{\Omega^2 r_0^2}{2}$$



The so called Bernoulli equation measures energy per unit mass, to quantify the energy of a spinning cylinder the equation has to be integrated

$$E_{scyl} = -\int (2\pi r\ L)\ \rho\ \frac{\Omega^2 r^2}{2} dr = -\rho\pi L R^4 \frac{\Omega^2}{4}$$

$$E_{scyl} = -\int (2\pi r\ L)\ \rho\ \frac{\Omega^2 r^2}{2} dr = -\rho\pi L R^4 \frac{\Omega^2}{4}$$

$$E_{Total} \approx -\rho_H \pi \frac{\Omega^2}{4}[L_H R_H^4 - L_L R_L^4] - \rho_L \pi \frac{\Omega^2}{4}[L_L R_L^4] + 2\ \pi\ R_L L_L \sigma$$

Energy of the spinning heavier phase

Energy of the spinning lighter phase

Energy of the interface

Volumes of heavier fluid and the lighter one remain constant, but by changing angular speed the geometric characteristics of the drop (i.e. diameter and length) will change, then to minimize energy, the required constraint is to keep constant volume of the drop.

$$V_{Drop} = \pi\ R_L^2 L_L$$

$$E_{Total} \approx -\rho_H \pi \frac{\Omega^2}{4}[L_H R_H^4] + [\rho_H - \rho_L]\ \pi \frac{\Omega^2}{4}[L_L R_L^4] + 2\ \pi\ R_L L_L \sigma$$

$$V_{Drop} = \pi\, R_L^2 L_L$$

After collecting terms and replacing L_L with the volume of drop equation, the resulting equation is:

$$E_{Total} \approx -\rho_H \pi \frac{\Omega^2}{4} [L_H R_H^4] + V_{Drop} \left[[\rho_H - \rho_L]\, \frac{\Omega^2}{4} R_L^2 + \frac{2}{R_L}\, \sigma \right]$$

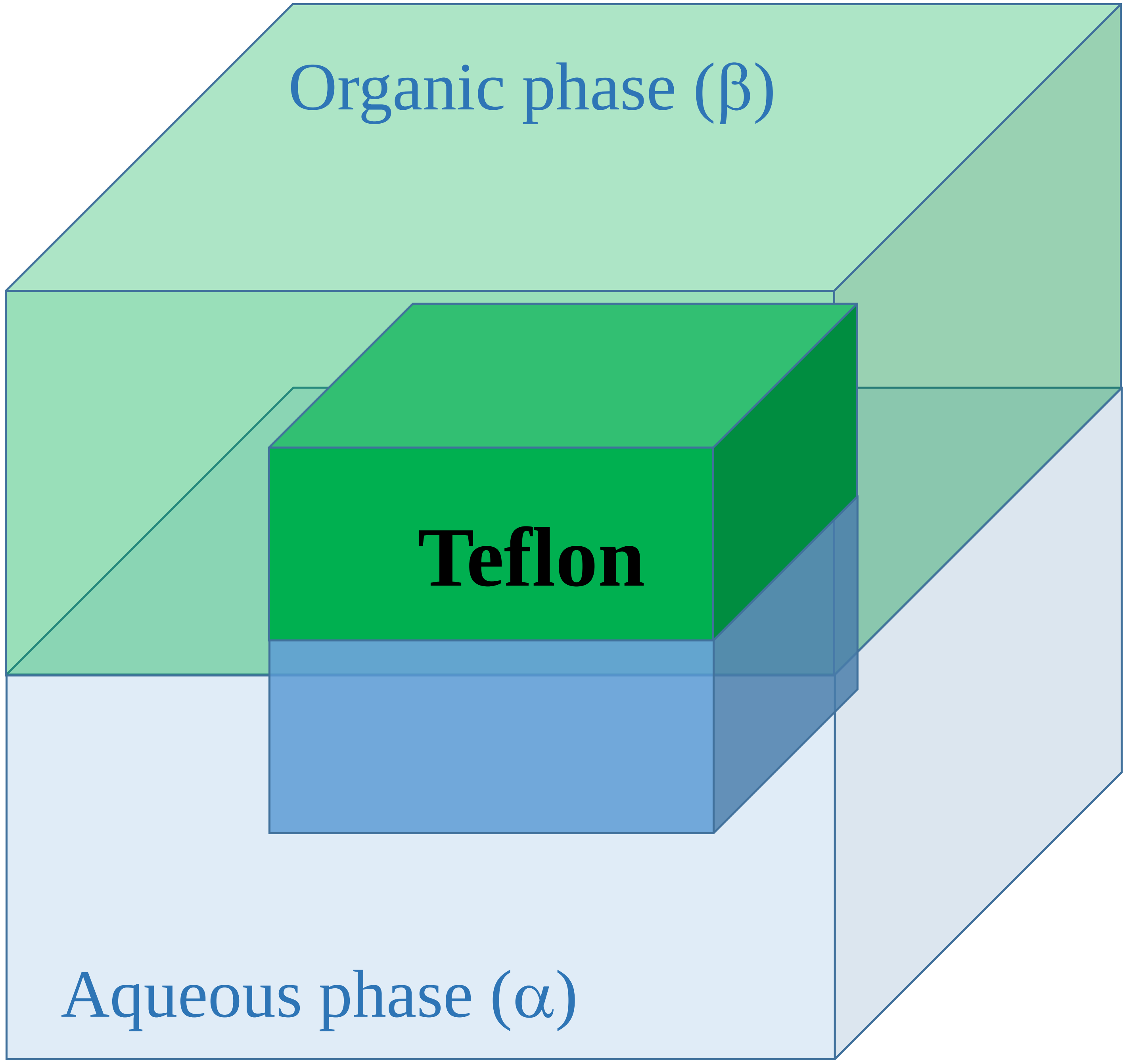
The next step is to find the value of the drop radius that minimizes energy, this is done with the differential of the energy with respect to the radius of the inner drop:

$$\frac{dE_{Total}}{dR_L} = V_{Drop} \left[2\, [\rho_H - \rho_L]\, \frac{\Omega^2}{4} R_L - \frac{2}{R_L^2}\, \sigma \right] = 0$$

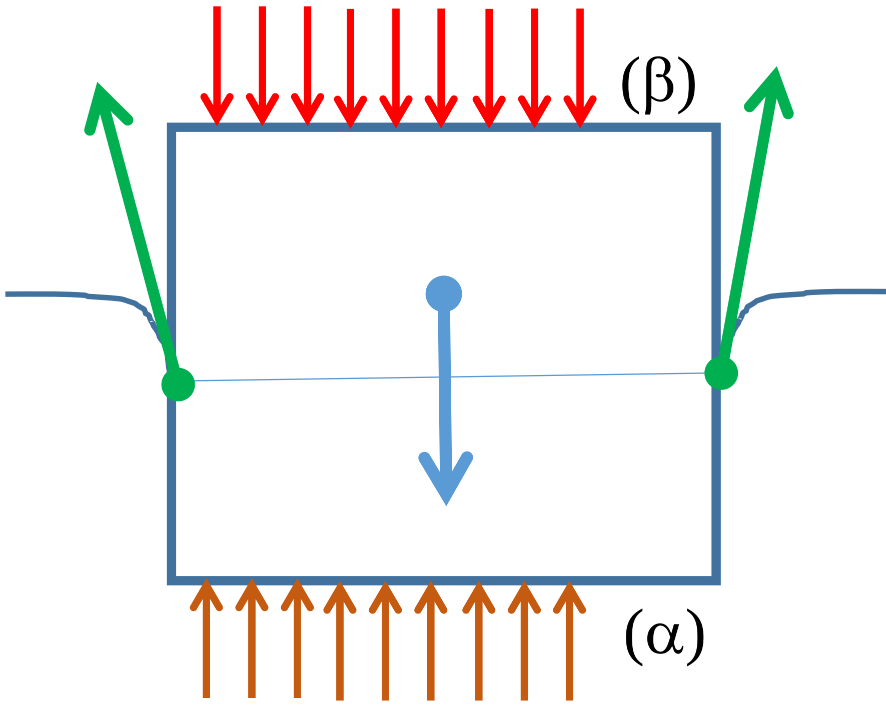
$$\sigma \approx\; [\rho_H - \rho_L]\, \frac{\Omega^2}{4} R_L^3$$

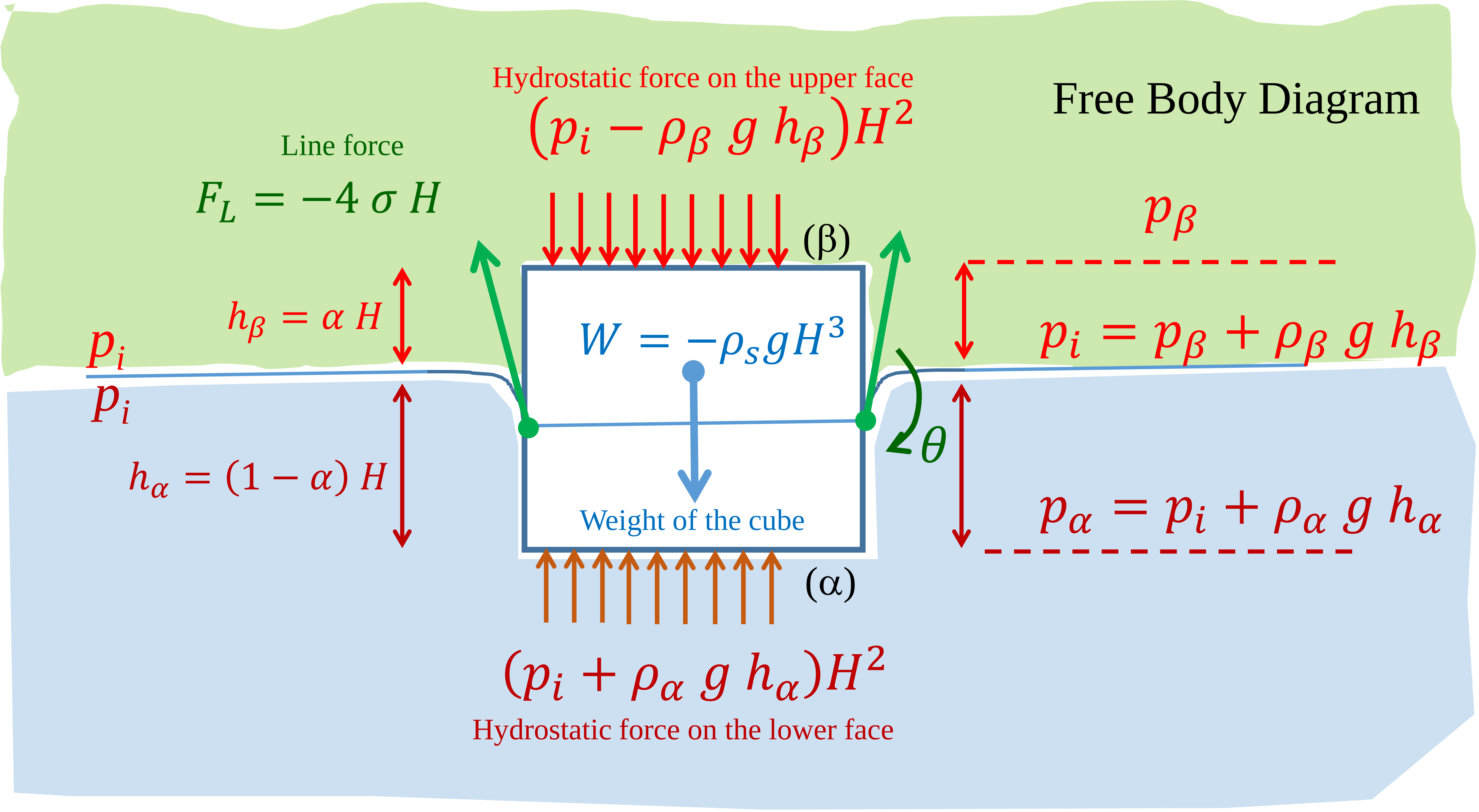
This equation coincides with the previous result:

With this analysis one can understand that the Surface tension not only measures pressure difference across curved interfaces, but the energy required to deform an interface.



A 3-mm Teflon cube floats between oil and water, calculate the portion of the cube in the oil phase.





Forces acting over the cube, pressure analysis is included as well

Forces balance in y-axis

$$(p_i + \rho_\alpha\, g\, h_\alpha)H^2 - \left(p_i - \rho_\beta\, g\, h_\beta\right)H^2 - 4\,\sigma\, H\cos\theta - \rho_s g H^3 = 0$$

$$-4\,\sigma\, H\cos\theta = (1-\alpha)(\rho_s - \rho_\alpha)gH^3 + (\alpha)\left(\rho_s - \rho_\beta\right)gH^3$$

$$-4\ \cos\theta = (1-\alpha)Bo_{s\alpha} + (\alpha)Bo_{s\beta}$$

$$\frac{-4\ \cos\theta - Bo_{s\alpha}}{Bo_{s\beta} - Bo_{s\alpha}} = \alpha$$

$$Bo = \frac{\Delta\rho g H^2}{\sigma}$$

Equations

$$\rho_w = 997 \ \left[\mathrm{kg/m^3}\right]$$

$$\rho_T = 2189 \ \left[\mathrm{kg/m^3}\right]$$

$$\rho_o = 800 \ \left[\mathrm{kg/m^3}\right]$$

$$\sigma = 30 \times 10^{-3} \ [\mathrm{N/m}]$$

$$g = 9.80665 \ \left[\mathrm{m/s^2}\right]$$

$$Bo_{sw} = (\rho_T - \rho_w) \cdot g \cdot \frac{H^2}{\sigma}$$

$$Bo_{so} = (\rho_T - \rho_o) \cdot g \cdot \frac{H^2}{\sigma}$$

$$H = 3.0 \times 10^{-3} \ [\mathrm{m}]$$

$$\theta = \pi$$

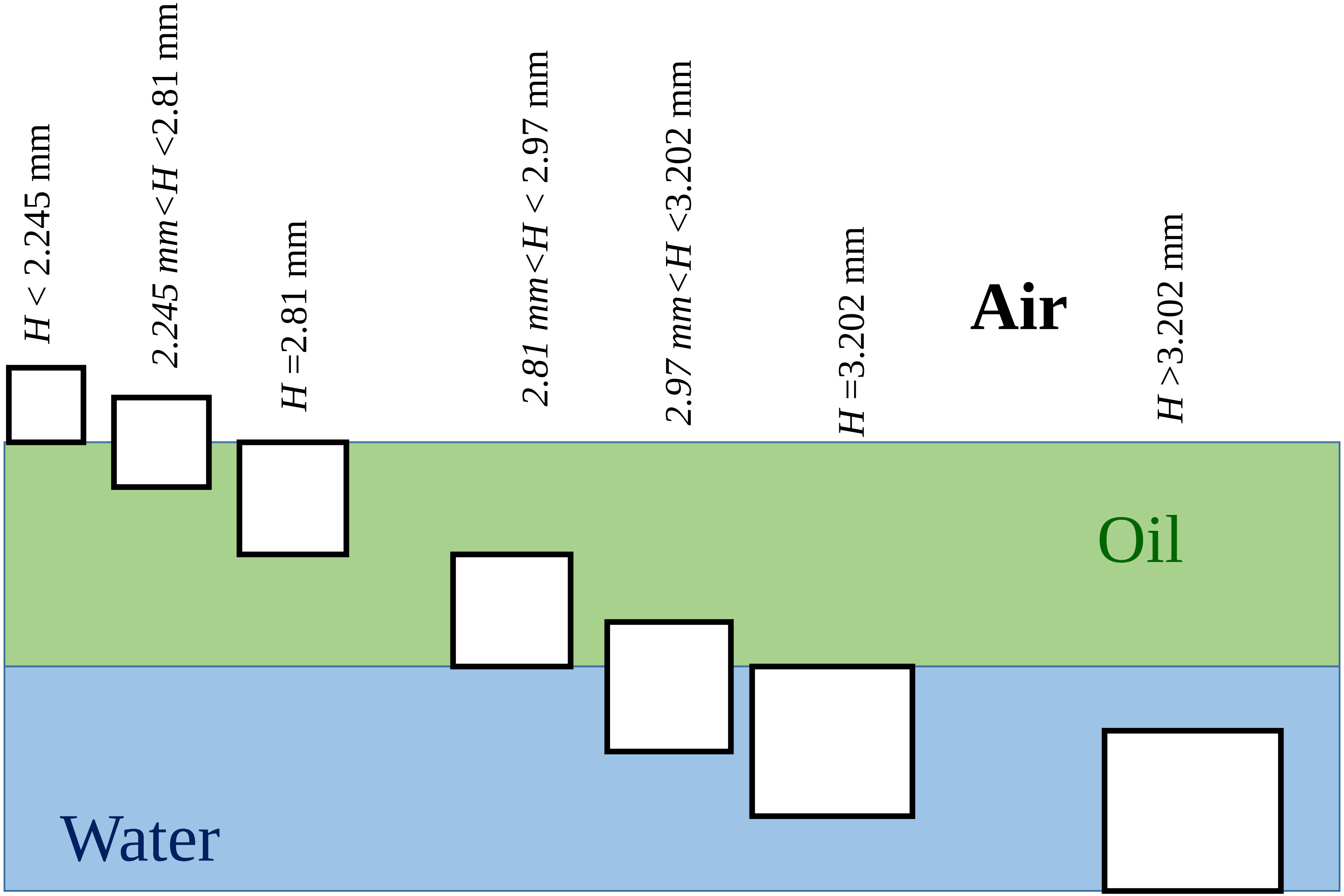
$$\alpha = \frac{-4 \cdot \mathrm{Cos}(\theta) - Bo_{sw}}{Bo_{so} - Bo_{sw}}$$

Solution

$$\alpha = 0.8509$$
$$Bo_{sw} = 3.507$$
$$H = 0.003 \ [\mathrm{m}]$$
$$\rho_T = 2189 \ \left[\mathrm{kg/m^3}\right]$$
$$\sigma = 0.03 \ [\mathrm{N/m}]$$
$$Bo_{so} = 4.086$$
$$g = 9.807 \ \left[\mathrm{m/s^2}\right]$$
$$\rho_o = 800 \ \left[\mathrm{kg/m^3}\right]$$
$$\rho_w = 997 \ \left[\mathrm{kg/m^3}\right]$$
$$\theta = 3.142$$

85% of the cube is immersed in oil, and the 15% in water. In order to remain at the interface the cube length should be between 2.97 mm to 3.202 mm, if bigger than 3.202 mm will sink. If smaller than 2.97 mm will be at the bottom of the oil phase. It can not float at the interface air oil, because is oil wet, but if the cube is oleophobic as well, it may float at the air-oil interface when its dimensions are between 2.245 mm and 2.81mm, assuming contact angle of 180°, and surface tension of oil of 27 dyne/cm .

In a nut shell, depending of the dimensions the cube may be located at different depths



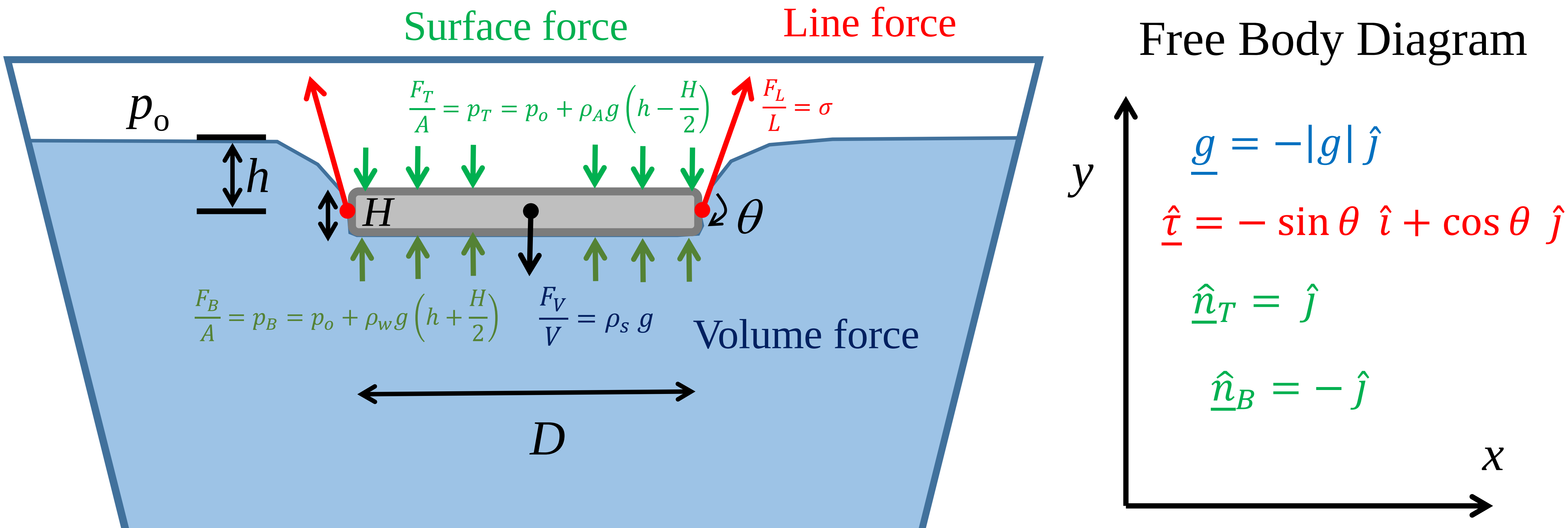
Objectives:

- Identify Three Different kind of forces (Volume, Surface and Line).
- Utilize vector notation.
- Recall concept of normal unit vector and unit tangent vector.

Under the right conditions, it is possible, due to surface tension, to have metal objects float on water. Consider placing a small Japanese coin (i.e. one yen, 20-mm diameter, 1.5-mm thickness, 2700-kg/m^3 density made of pure aluminum) on a surface of water. What is the depth, below the flat water surface at which the coin will rest in equilibrium without sinking ? Assume that the surface tension of the air water interface is 72 mN/m, and the contact angle is 180° (Contact angle is the angle between the aluminum-water interface respect to the water-air interface at the contact line, and the contact line is the line where the three phases meet)

D=20 mm

H =1.5 mm



Frame Of Reference

$$\frac{d(m\underline{v})}{dt} = \sum \underline{F}_i = \underbrace{\rho_S\, V_s\, \underline{g}}_{\text{Volume Forces}} - \underbrace{\sum_k p_k\, A_k\, \underline{\hat{n}}_k}_{\text{Surface Forces}} - \underbrace{\sigma\, L\, \underline{\hat{\tau}}}_{\text{Line Forces}}$$

The system is the coin (the solid)

$\underline{\hat{n}}$ The unit normal vector points outward the solid in contact with the fluid

$\underline{\hat{\tau}}$ The unit tangent vector is tangent to the interface between fluids and points toward the solid

Equations

$$D = 20\ [\mathrm{mm}] \cdot \left| 0.001\ \frac{\mathrm{m}}{\mathrm{mm}} \right|$$

$$H = 1.5\ [\mathrm{mm}] \cdot \left| 0.001\ \frac{\mathrm{m}}{\mathrm{mm}} \right|$$

$$\rho = 2700\ [\mathrm{kg/m^3}]$$

$$V = \pi \cdot (D/2)^2 \cdot H$$

$$m = \rho \cdot V$$

$$g = 9.80665\ [\mathrm{m/s^2}]$$

$$\rho_w = 997\ [\mathrm{kg/m^3}]$$

$$\rho_a = 1.18\ [\mathrm{kg/m^3}]$$

$$\sigma = 72 \times 10^{-3}\ [\mathrm{N/m}]$$

$$p = 2 \cdot \pi \cdot (D/2)$$

$$-m \cdot g + V \cdot \phi \cdot \rho_w \cdot g - \sigma \cdot \mathrm{Cos}(\theta) \cdot p = 0$$

Approach I (Buoyancy force)

$$\theta = 180\ [\mathrm{deg}]$$

Approach 2

$$p_0 = 101325\ [\mathrm{Pa}]$$

$$A = \pi \cdot \frac{D^2}{4}$$

$$p_T = p_0 + \rho_a \cdot g \cdot (h_{CG} - H/2)$$

$$p_B = p_0 + \rho_w \cdot g \cdot (h_{CG} + H/2)$$

$$L = p$$

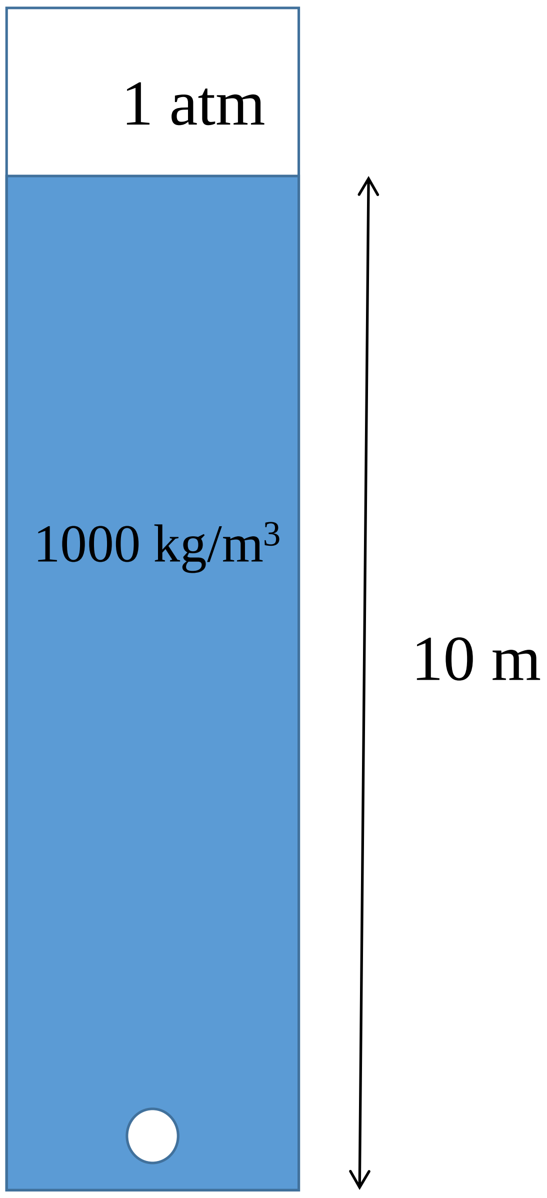
$$p_B \cdot A - p_T \cdot A - \rho \cdot g \cdot V - \sigma \cdot \mathrm{Cos}(\theta) \cdot L = 0$$

Approach II (Pressure forces)

$$\phi = h_{CG1}/H + 1/2$$

Solution

$$A = 0.0003142 \ [\mathrm{m^2}] \qquad D = 0.02 \ [\mathrm{m}]$$
$$g = 9.807 \ [\mathrm{m/s^2}] \qquad H = 0.0015 \ [\mathrm{m}]$$
$$\boxed{h_{CG} = 0.001841 \ [\mathrm{m}]} \qquad h_{CG1} = 0.001839 \ [\mathrm{m}]$$
$$L = 0.06283 \ [\mathrm{m}] \qquad m = 0.001272 \ [\mathrm{kg}]$$
$$p = 0.06283 \ [\mathrm{m}] \qquad \phi = 1.726$$
$$p_0 = 101325 \ [\mathrm{Pa}] \qquad p_B = 101350 \ [\mathrm{Pa}]$$
$$p_T = 101325 \ [\mathrm{Pa}] \qquad \rho = 2700 \ [\mathrm{kg/m^3}]$$
$$\rho_a = 1.18 \ [\mathrm{kg/m^3}] \qquad \rho_w = 997 \ [\mathrm{kg/m^3}]$$
$$\sigma = 0.072 \ [\mathrm{N/m}] \qquad \theta = 180 \ [\mathrm{deg}]$$
$$V = 4.712 \times 10^{-7} \ [\mathrm{m^3}]$$



Problem No.5 An air bubble at the bottom of a tank has a diameter of 0.5 mm, calculate:

a) The pressure inside the bubble at the bottom of the tank

b) The diameter of the spherical bubble at the center of the tank (depth 5m)

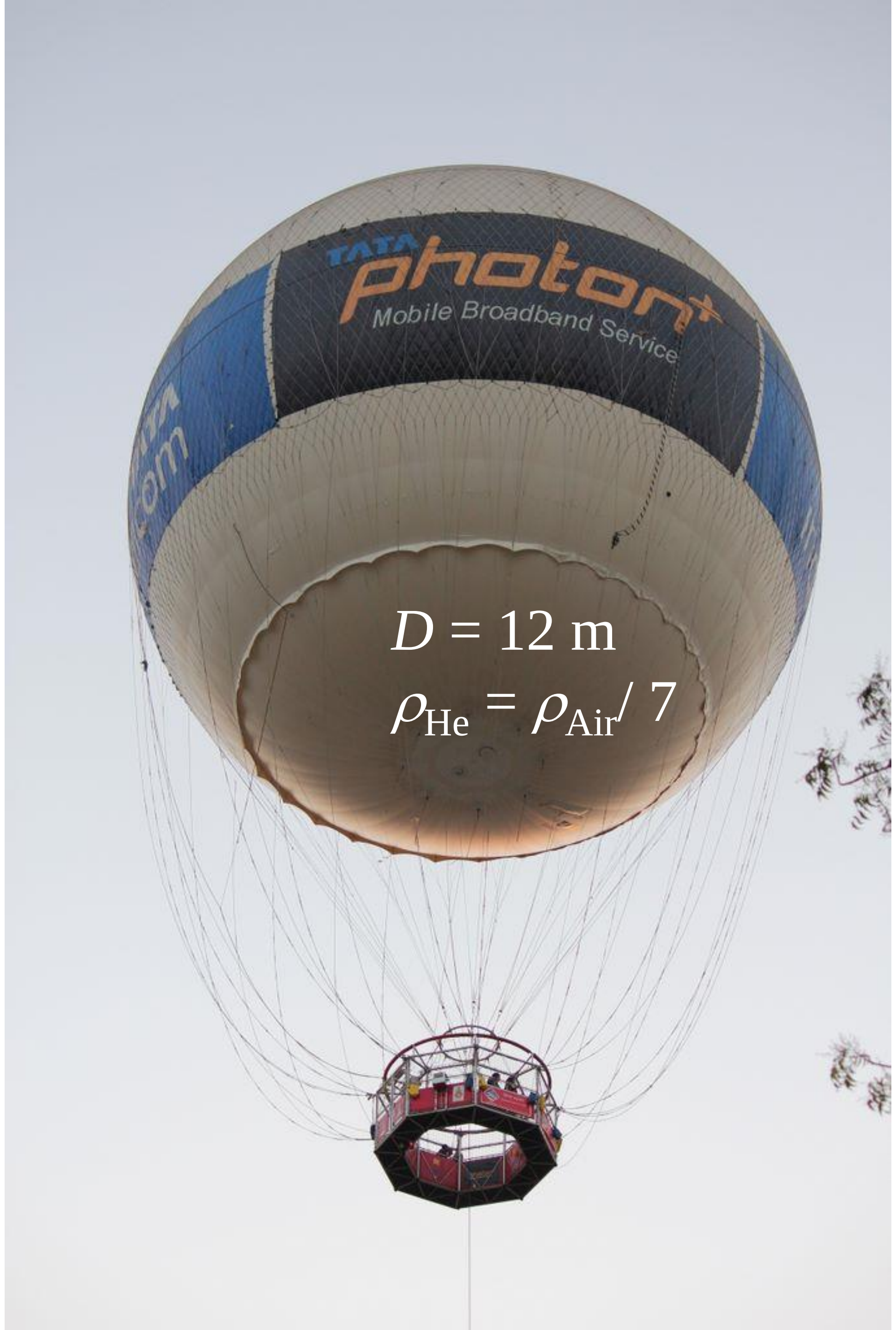
c) The pressure inside the bubble just before reaching the Surface.

For water-air $\sigma = 72.8 \times 10^{-3}$ N/m

Problem No.7 Balloons are often filled with helium gas because it weights only about one-seventh of what air weights under identical conditions. The buoyancy force, which can be expressed as $\boldsymbol{F}_b = \rho_{air}\, g\, V_{balloon}$, will push the balloon upward. If the balloon has a diameter of 12 m and carries two people, 70 kg each, determine the acceleration of the balloon when it is first released. Assume the density of air is ρ = 1.16 kg/m^3, and neglect the weight of the ropes and cage.

b) Calculate the height if the balloon, if after 1 h the sensors are measuring a pressure of 680 Torr. (the balloon was released at sea level where the pressure is ca 760 Torr)

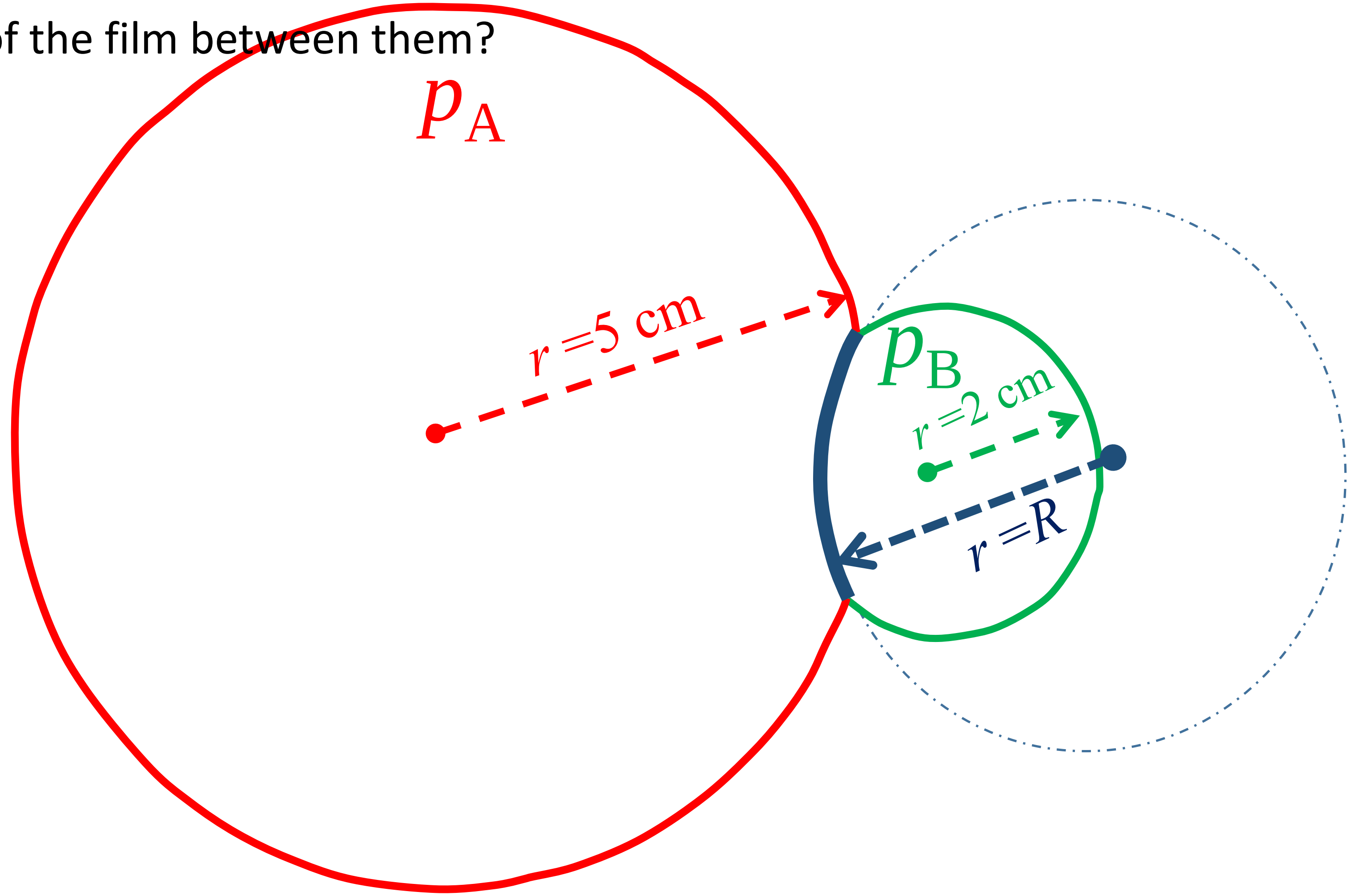
There is a World-Wide Shortage of Helium



Problem No.8 A Scientist is studying the lifetime of soap bubbles, and for this purpose, he needs to calculate the pressure inside them, and the radius of curvature of the film between them when two of them stick together. If the surface tension of an aqueous surfactant solution is $30x10^{-3}$ N/m and the radii are 5-cm and 2-cm for each bubble.

a) What is pressure inside each bubble?

b) What is radius of curvature of the film between them?

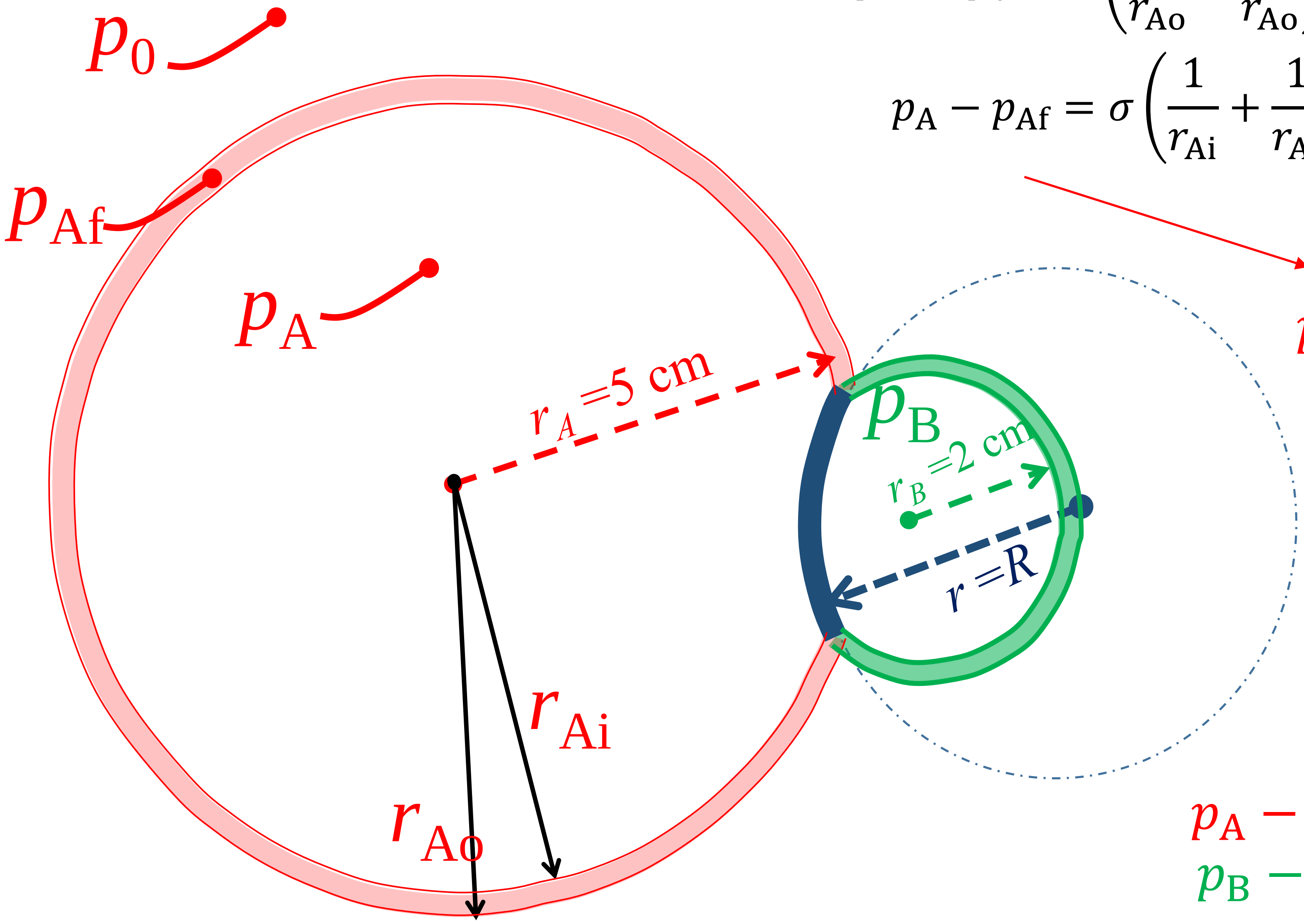


$$p_{\mathrm{Af}} - p_0 = \sigma\left(\frac{1}{r_{\mathrm{Ao}}} + \frac{1}{r_{\mathrm{Ao}}}\right)$$

Pressure difference between film of bubble A and atmosphere

$$p_{\mathrm{A}} - p_{\mathrm{Af}} = \sigma\left(\frac{1}{r_{\mathrm{Ai}}} + \frac{1}{r_{\mathrm{Ai}}}\right)$$

Pressure difference between gas trapped within bubble A and the film of bubble A



Adding both to cancel the pressure within the film

$$p_{\mathrm{A}} - p_0 = \sigma\left(\frac{2}{r_{\mathrm{Ai}}} + \frac{2}{r_{\mathrm{Ao}}}\right) \sim \frac{4\sigma}{r_{\mathrm{A}}}$$

$$p_{\mathrm{B}} - p_0 = \sigma\left(\frac{2}{r_{\mathrm{Bi}}} + \frac{2}{r_{\mathrm{Bo}}}\right) \sim \frac{4\sigma}{r_{\mathrm{B}}}$$

$$p_{\mathrm{B}} - p_{\mathrm{A}} = \sigma\left(\frac{2}{R_i} + \frac{2}{R_o}\right) \sim \frac{4\sigma}{R}$$

$p_{\mathrm{A}} - p_0 = 2.4$ Pa

$p_{\mathrm{B}} - p_0 = 6$ Pa

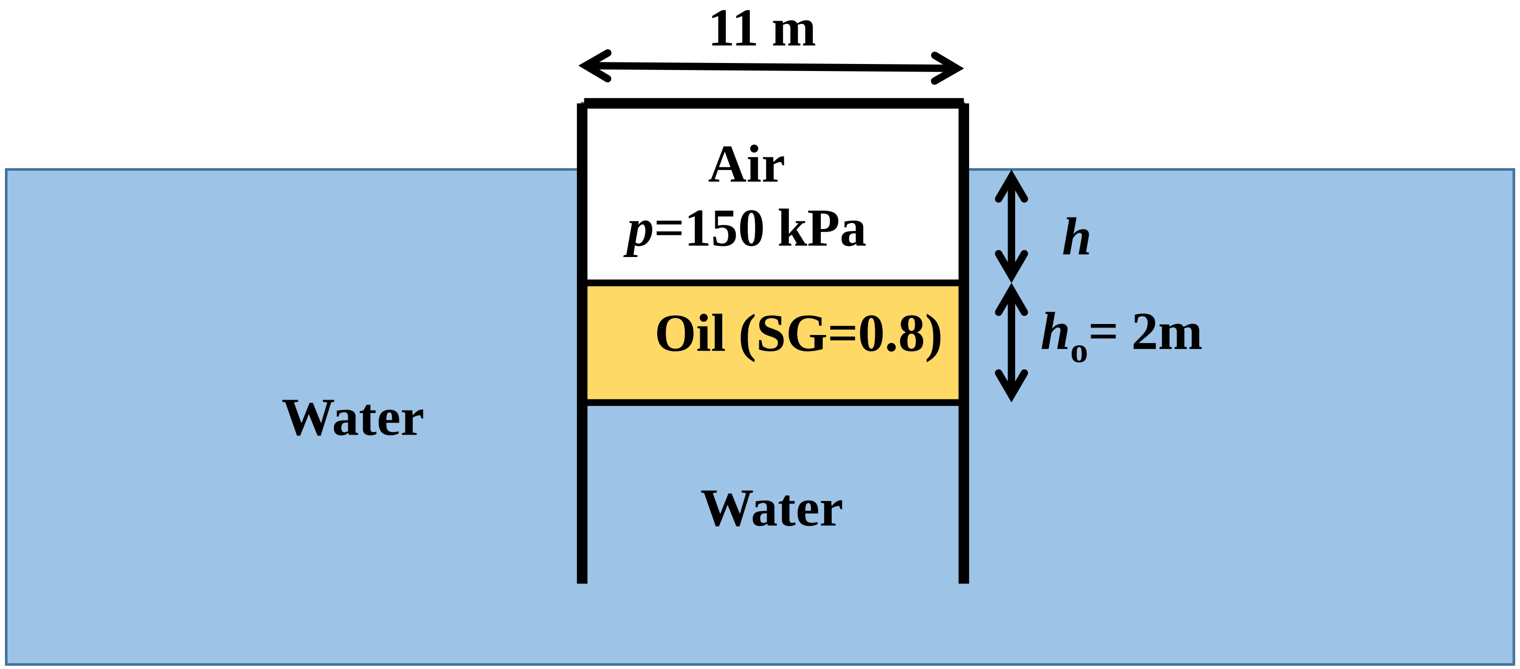
$p_{\mathrm{B}} - p_{\mathrm{A}} = 3.6$ Pa

R=3.33 cm

Problem No.10 A container with a square cross section measuring 11 m by 11m is floating in water as shown in the figure. Assume that tank wall has negligible thickness compared with dimensions shown. Calculate:

a) the distance *h* (m)

b) The weight of the tank (N)



A container with a square cross section measuring 11 m by 11m is floating in water as shown in the figure. Assume that tank wall has negligible thickness compared with dimensions shown. Calculate:

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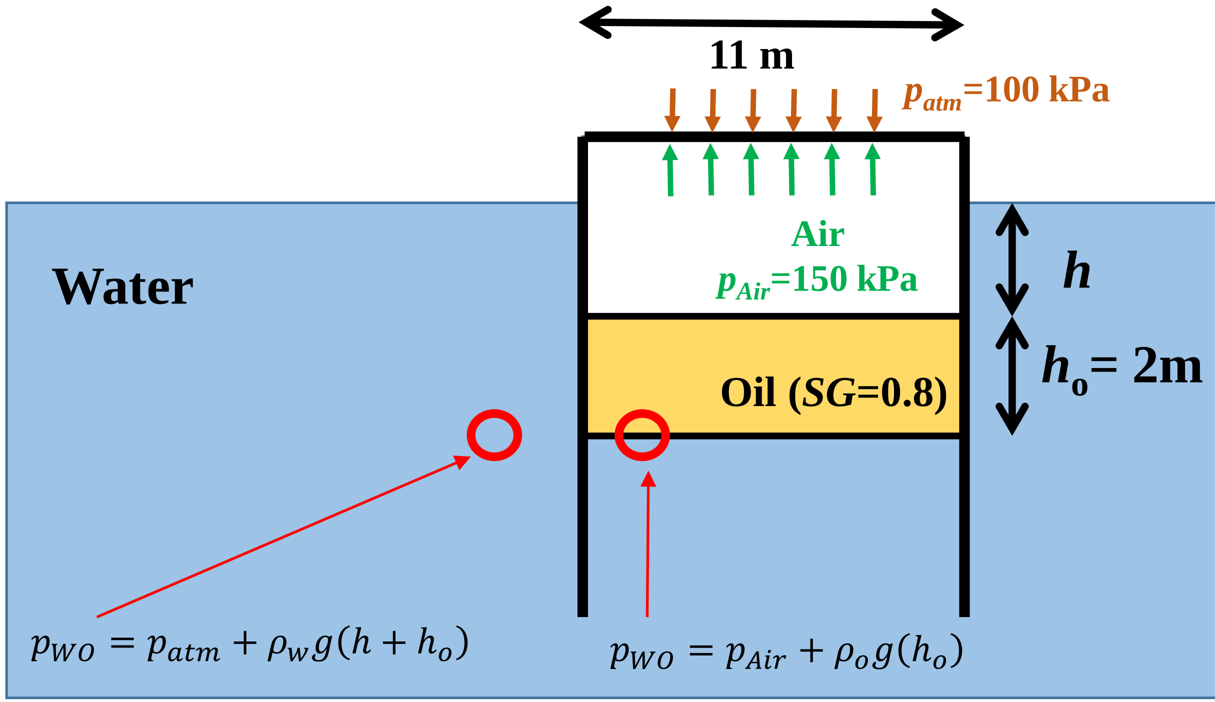


$$p_{Air} + \rho_o g(h_o) = p_{atm} + \rho_w g(h + h_o)$$

$$p_{Air} - p_{atm} = \rho_w g(h + h_o)\text{-}\rho_o g(h_o)$$

$$p_{Air} - p_{atm} - (\rho_w - \rho_o)\, g\, h_o = \rho_w\, g\, h$$

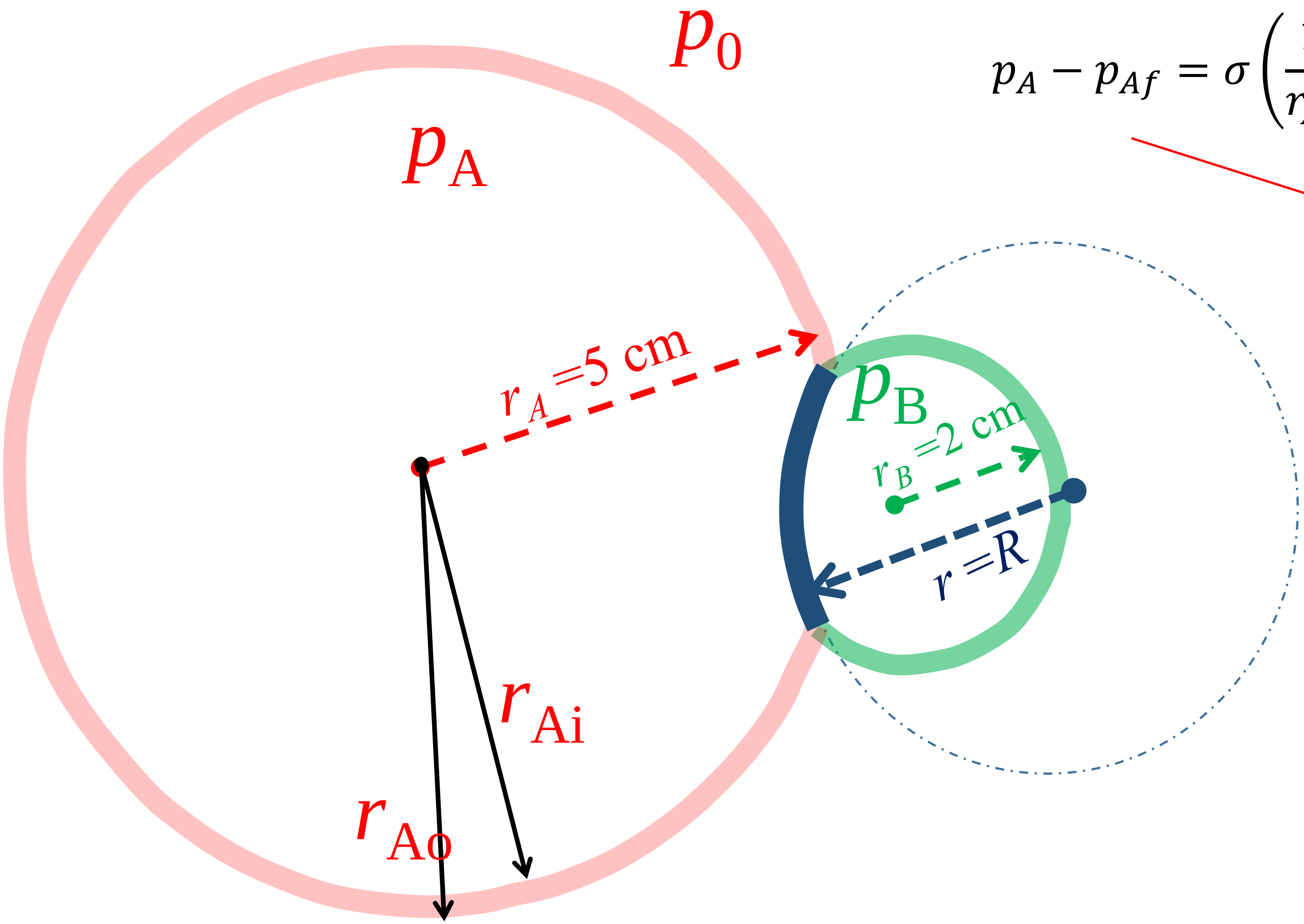
$$\frac{p_{Air} - p_{atm} - (\rho_w - \rho_o)\, g\, h_o}{\rho_w\, g} = h$$

$$\boxed{\frac{p_{Air} - p_{atm} - \rho_w(1 - SG)\, g\, h_o}{\rho_w\, g} = h}$$

h=4.697 m

Force balance in *y*-axis $(p_{Air} - p_{atm})A - W = 0$

$$W = (p_{Air} - p_{atm})A = (150000 - 100000)\text{Pa}\ (121\ \text{m}^2) = 6.05\ \text{MN}$$



$$p_{Af} - p_0 = \sigma\left(\frac{1}{r_{Ao}} + \frac{1}{r_{Ao}}\right)$$

Pressure difference between film of bubble A and atmosphere

$$p_A - p_{Af} = \sigma\left(\frac{1}{r_{Ai}} + \frac{1}{r_{Ai}}\right)$$

Pressure difference between gas trapped within bubble A and the film of bubble A

Adding both to cancel the pressure within the film

$$p_A - p_0 = \sigma\left(\frac{2}{r_{Ai}} + \frac{2}{r_{Ao}}\right) \sim \frac{4\sigma}{r_A}$$

$$p_B - p_0 = \sigma\left(\frac{2}{r_{Bi}} + \frac{2}{r_{Bo}}\right) \sim \frac{4\sigma}{r_B}$$

$$p_B - p_A = \sigma\left(\frac{2}{R_i} + \frac{2}{R_o}\right) \sim \frac{4\sigma}{R}$$

$$p_A - p_0 = 2.4 \text{ Pa}$$

$$p_B - p_0 = 6 \text{ Pa} \qquad R\text{=3.33 cm}$$

$$p_B - p_A = 3.6 \text{ Pa}$$

Forces

a) Body forces

b) Surface forces

c) Line forces or surface tension

$$\underline{F_w} = \rho\, V\, \underline{g}$$

$$\underline{F_p} = p\, A\, \underline{n} = \underline{n} \cdot \underline{\underline{I}}\, p\, A$$

$$\underline{F_L} = \sigma\, L\, \underline{\tau}$$

θ = contact angle

$$\underline{\hat{\tau}} = \sin\varphi\ \hat{\imath} + \cos\varphi\ \hat{\jmath}$$

ϕ is a function of contact angle, i.e. $\phi = \phi(\theta)$

Type of forces in fluids

Body Forces:

Gravity $\underline{F_w} = \rho V \underline{g}$

Electromagnetic $\underline{F_e} = \rho_e V \underline{E}$

Centrifugal $\underline{F_c} = \rho V [\underline{\Omega} \times \underline{R} \times \underline{\Omega}]$

Coriolis $\underline{F_c} = \rho V [2 \underline{v} \times \underline{\Omega}]$

Line Forces:

Interfacial tension

Surface tension

$$\underline{F_v} = \sigma L \underline{\tau}$$

Surface Forces:

$$\underline{F_v} = [\underline{n} \cdot \underline{\underline{\tau}}] A$$

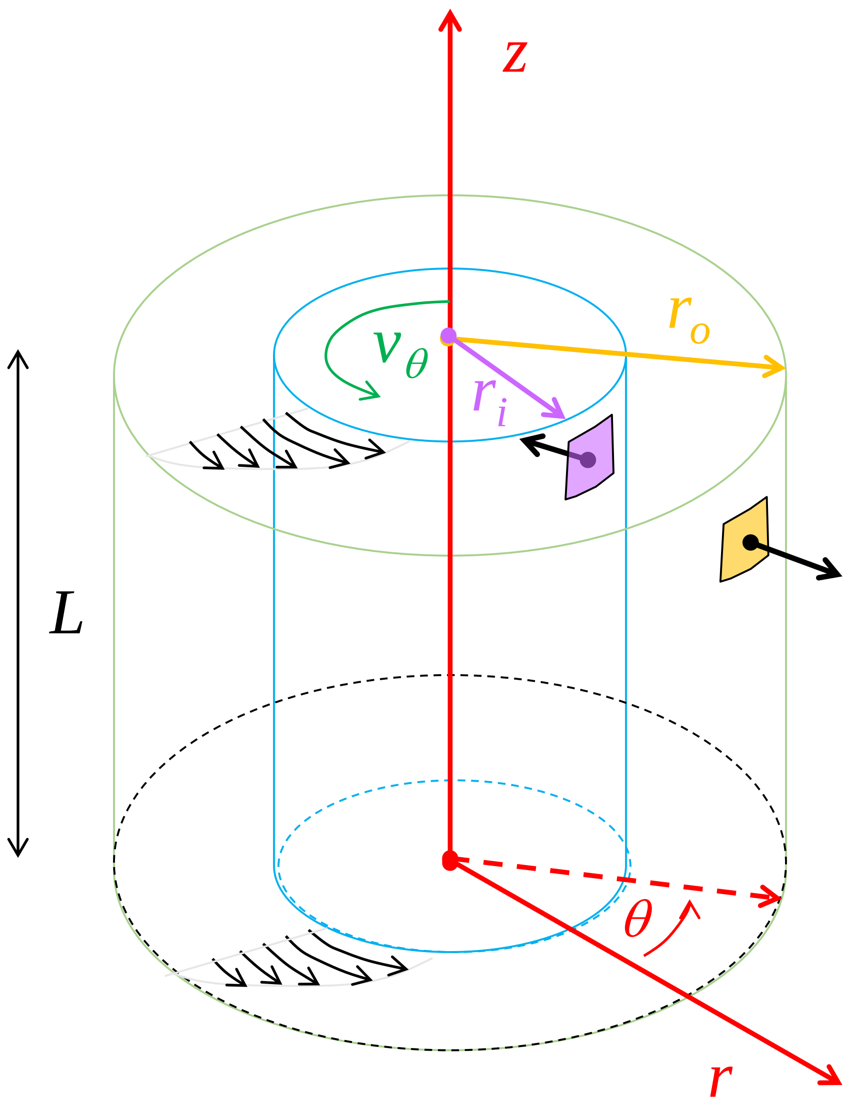
Viscous stresses

Pressure stresses $\underline{F_p} = p A \underline{n} = \underline{n} \cdot \underline{\underline{I}} p A$

Buoyancy (construct resulting from pressure stress)

Note: Some of these called body forces sometimes are called fictitious forces (for instance Coriolis), and some forces can be quantified as body forces, but they are surface forces like buoyancy.

Viscous forces over the wall of a spinning rod (ω) within a concentric fixed hollow cylinder filled with fluid



$$\underline{F_v} = \oint \underline{n} \cdot \mu\, 2\, \underline{\underline{\Gamma}}\, dA \qquad \underline{\underline{\Gamma}} = \frac{1}{2}\left[\underline{\nabla}\, \underline{V} + \left(\underline{\nabla}\, \underline{V}\right)^T\right]$$

$$\underline{S_v} = \underline{n} \cdot \mu\, \underline{\underline{\Gamma}}$$

Viscous Surface forces

$$\underline{F_v} = \underline{n} \cdot \underline{\underline{\tau}}\, A \qquad A = 2\,\pi\, r_i\, L$$

At the inner surface

$$\underline{n_i} = -\underline{e_r}$$

$$\underline{\underline{\tau}} = \tau_{r\theta} = \mu \frac{dv_\theta}{dr}\, \underline{e_r}\, \underline{e_\theta}$$

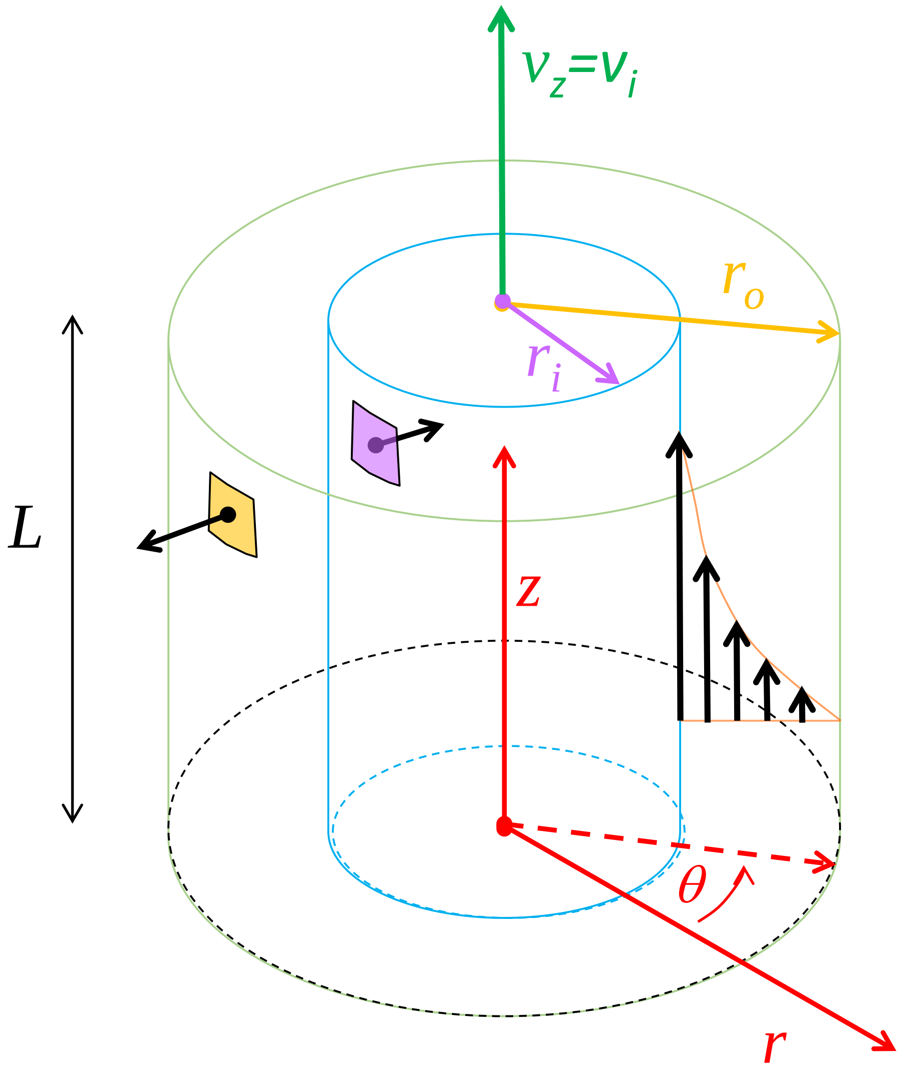
$$\underline{F_v} = \underline{n} \cdot \underline{\underline{\tau}}\, A = -2\,\pi\, r_i\, L \left[\mu \frac{dv_\theta}{dr}\right] \left[\underline{e_r} \cdot \underline{e_r}\, \underline{e_\theta}\right]$$

$$\underline{F_v} = -2\,\pi\, r_i\, L \left[\mu \frac{dv_\theta}{dr}\right] \left[\underline{e_\theta}\right] \approx -2\,\pi\, r_i\, L \mu \left[\frac{v_{\theta o} - v_{\theta i}}{r_o - r_i}\right] \left[\underline{e_\theta}\right]$$

$$\underline{F_v} \approx 2\,\pi\, r_i\, L \mu \left[\frac{\omega\, r_i}{r_o - r_i}\right] \left[\underline{e_\theta}\right]$$

Viscous Surface force at the inner boundary

Viscous forces over the wall of an axial displacing rod (ω) within a concentric fixed hollow cylinder filled with fluid



$$\underline{F_v} = \oint \underline{n} \cdot \mu\, 2\, \underline{\underline{\Gamma}}\, dA \qquad \underline{\underline{\Gamma}} = \frac{1}{2}\left[\underline{\nabla}\, \underline{V} + \left(\underline{\nabla}\, \underline{V}\right)^T\right]$$

$$\underline{S_v} = \underline{n} \cdot \mu\, \underline{\underline{\Gamma}}$$

Viscous Surface forces

$$\underline{F_v} = \underline{n} \cdot \underline{\underline{\tau}}\, A \qquad A = 2\,\pi\, r_i\, L$$

At the inner surface

$$\underline{n_i} = -\underline{e_r}$$

$$\underline{\underline{\tau}} = \tau_{rz} = \mu \frac{dv_z}{dr} \underline{e_r}\underline{e_z}$$

$$\underline{F_v} = \underline{n} \cdot \underline{\underline{\tau}}\, A = -2\,\pi\, r_i\, L \left[\mu \frac{dv_z}{dr}\right] \left[\underline{e_r} \cdot \underline{e_r}\underline{e_z}\right]$$

$$\underline{F_v} = -2\,\pi\, r_i\, L \left[\mu \frac{dv_z}{dr}\right] \left[\underline{e_z}\right] \approx -2\,\pi\, r_i\, L\mu \left[\frac{v_{zo} - v_{zi}}{r_o - r_i}\right] \left[\underline{e_z}\right]$$

$$\underline{F_v} \approx 2\,\pi\, r_i\, L\mu \left[\frac{v_i}{r_o - r_i}\right] \left[\underline{e_z}\right]$$

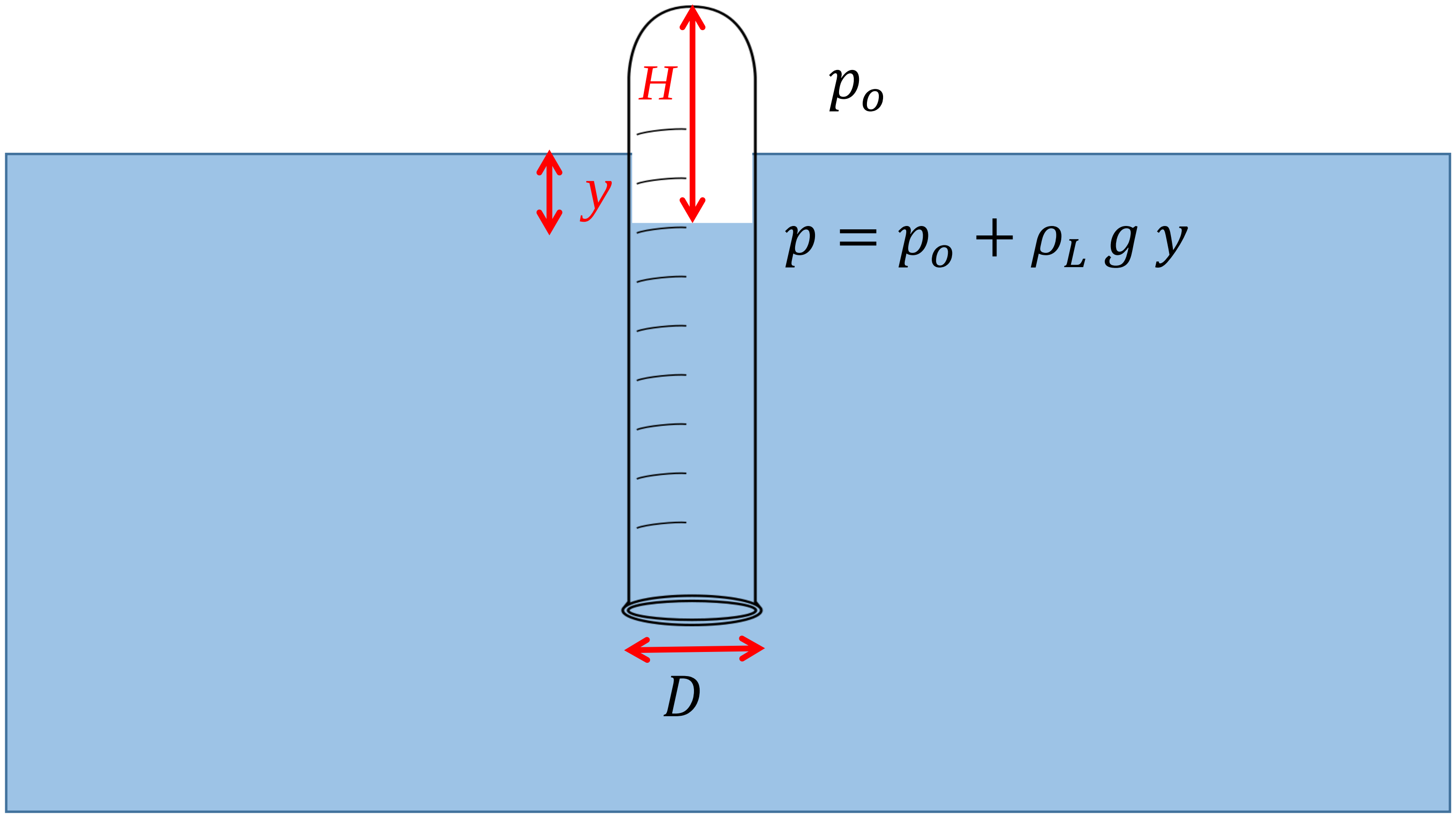
Viscous Surface force at the inner boundary

Quiz No.1

1. Make a sketch of one of the experiments conducted in class and label the different forces you did observe.

2. What are the units of force, linear momentum, work, power, torque and energy.

3. State with your own words the law of dimensional homogeneity.



m_T = mass of the test tube

ρ_L = density of the liquid

D= inner diameter of the test tube

p_o= pressure over the liquid surface

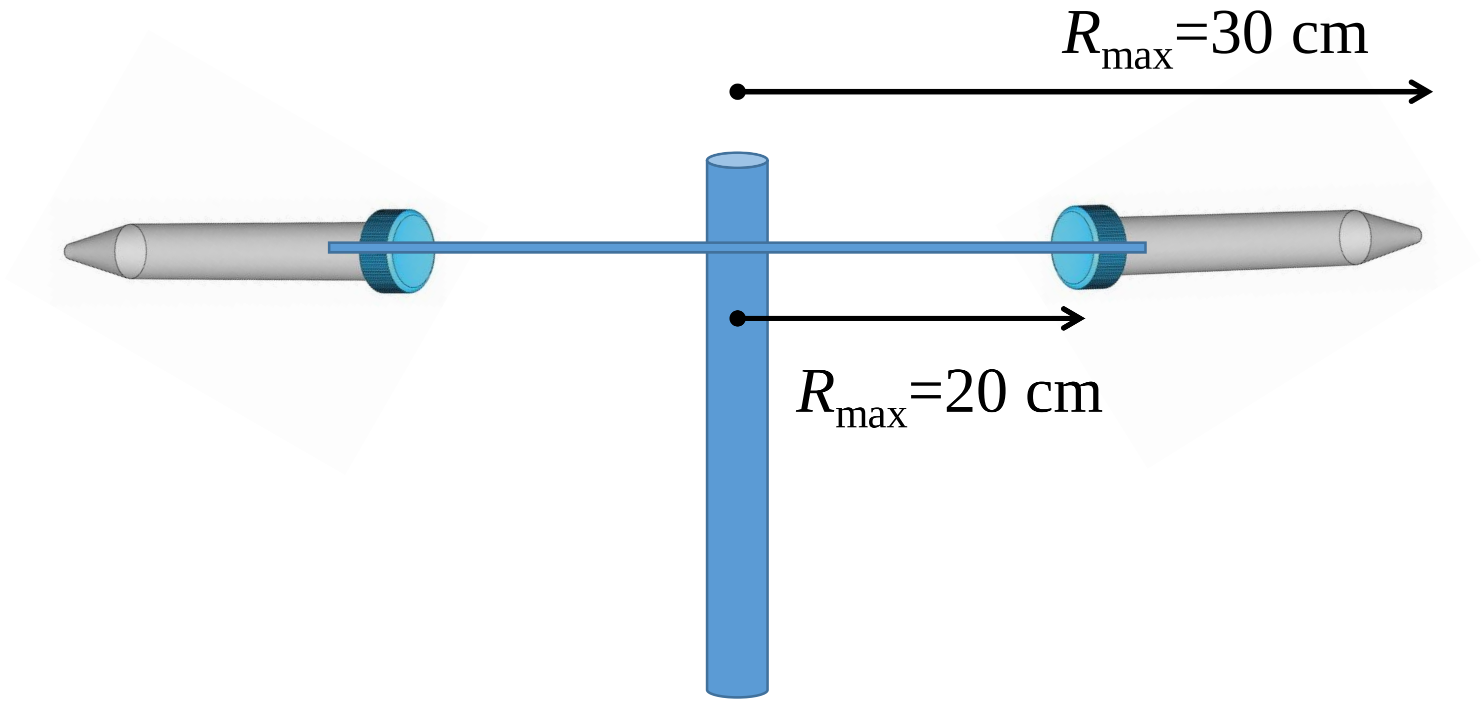
y= level of meniscus within test tube below the free surface of liquid

H = height of gas trapped within test tube

a) If all the information is known but the mass of the test tube, calculate the mass of the test tube.
b) If the pressure of the surface is changed, recalculate the value of H


wyle

The **ultracentrifuge** is a centrifuge optimized for spinning a rotor at very high speeds, capable of generating acceleration as high as 1 000 000 *g* (approx. 9 800 km/s²). If the angular velocity is 55 000 r.p.m. If the samples were placed at atmospheric pressure and well sealed, calculate the pressure that must stand the tubes.



By: Dr. José Luis López Salinas
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