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## **FLUID PROPERTIES WITH MEMORY – A CRITICAL REVIEW AND SOME ADDITIONS**

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### **ABSTRACT**

What is the effect of memory on fluids in porous media when predicting oil flow outcomes? Newtonian fluid flow equations have been considered the ideal models for making predictions. Even non-Newtonian models focus on what is immediately present and tangible in regard to fluid properties. This paper argues that the intangible dimension of time and other fluid and media properties have not been fully considered in existing models of fluid flow. The memory of fluid is the most important and most neglected feature in considering fluid flow models, since it represents the history of the fluid and how it will behave in the future. Models do not yet exist for true memory consideration. This paper reviews existing fluid flow models with memory, addresses the intangible problems of memory, and identifies the effects of considering memory and other properties such as stress, viscosity, surface tension, temperature etc. This review of the literature on memory as an important variable in predicting oil flow, critically reviews the existing models and introduces a relation of fluid viscosity, memory and permeability of media with stress to describe the behavior of a fluid in porous media. This idea can be used in the understanding of visco-elastic fluid flow behavior in the reservoir.

**KEY WORDS:** Fluid memory, porous media, Newtonian and non-Newtonian fluid, reservoir simulation

### **INTRODUCTION**

In literature notions of human memory are often associated with the flow of time. In nature groundwater, oil, gas and other naturally occurring fluids pick up and drop off all manner of various materials. The conventional approach in reservoir engineering has focused on the permeability of solid and semi-solid structures encountered by the flow of the fluid. What it doesn't do is follow these pathways. Fluid memory is an approach to factor this back in by switching the frame of reference from the external observation of flow to that of matter-molecules-within the flow. Some nonlinear, incompressible and viscous fluids possess some peculiar characteristics that suggest there are other properties in viscous fluids. The phenomenon that describes these special characteristics with time is stated as memory in fluid. There are very limited studies in the literature that describe this phenomenon clearly. Several non-Newtonian fluids behave chaotically at the time of flow through porous media especially in some geothermal areas. As time passes, this chaotic behavior results in

some precipitation of fluid minerals in the pore space thus squeezing the flow path in the reservoir. However, some fluids may react chemically with the medium, enlarging the pores. Some fluids carry solid particles that may obstruct some of the pores. Pore size may also be changed by the minerals precipitated by the fluid and, finally, by temperature variations induced by the flux. These phenomena create a spatially variable pattern of mineralization and permeability changes that can occur locally. Local permeability changes are of particular interest in geothermal studies. If it is considered that permeability diminishes with time, it is clear that the effect of fluid pressure at the boundary on the flow of fluid through the medium is delayed, and that the flow occurs as if the medium has a memory as well. In fluid memory, the filtering properties of the diffusion equation are altered relative to the corresponding equation based strictly on Darcy's law. The memory formalism then acts as a filter on the spectral properties of the fluid mass flow; the filter increases the low-frequency content and decreases the high-frequency content. The effect of the filter is increasingly more severe with larger values of relaxation time. It may be concluded that although other causes such as heterogeneity, anisotropy and inelasticity of the matrix, may be invoked to interpret certain phenomena, the memory mechanism could help in interpreting part of the phenomenology. Due to the inadequacy of current theories in accounting for memory, some authors (Hu and Cushman, 1994) also developed non-local flow theories using general principles of statistical physics under appropriate limiting conditions from which the classical Darcy's law is derived for saturated flow. This paper reviews existing fluid flow models, addresses the problem of memory and identifies the impact of considering memory on stress-strain relation.

## **CRITICAL LITERATURE REVIEW**

The critical review is based on how the memory of a fluid corresponds to a property and how this property is correlated with other more conventionally understood properties of the fluid and media. In this review, the media encompassing fluids are all porous and none of the models represent the true behavior and relationship of fluid and media properties with memory. As Table 1 indicates, research has tended to linearize the assumptions of available models.

Slattery (1967) studied viscoelastic fluid behavior with the Buckingham-pi theorem using the Ellis model fluid, power model fluid, Noll simple fluid and a Newtonian fluid. He pointed out that memory effects i.e. normal stress effects, represent the rate of deformation tensor as a function of the extra stress. In porous media, the memory effects are described in terms of permeability a change which is a function of the characteristic length of the system, the magnitude of the characteristic velocity, and the material parameters. He only showed these parameters as permeability functions. The material parameters are assumed to depend only upon the local thermodynamic state. These are normally viscosity, stress and diffusivity of the fluid. However he did not present any model which represents the whole scenario of the behavior.

Mifflin and Schowalter (1986) presented a technique to solve three-dimensional steady flows of memory integral fluids in enclosed or open flow systems. They consider the flow of a corotational Jeffrey's fluid as a sphere. They consider force and torque free laminar flow. They broke down the integral part of memory into velocity gradient which does not represent the true memory. They continued the calculation until the memory of the fluid decomposed sufficiently whereas the rest of the integral can either be neglected or set to a small constant value. They concluded that the noncirculating fluid having memory tends to remain farther from the sphere surface than in the Newtonian case. This model represents the relationship between fluid viscosity and stress tensor with time only.

Ciarletta and Scarpetta (1989) were concerned about the linearized progress for an incompressible fluid flow equation whose viscosity displays a fading memory of the past motions. They linearized by neglecting the non-linear convective term of the model. They considered a symmetric velocity gradient at every past instant to see the present status of stress. They only consider viscosity of a fluid as a function of stress with memory.

Eringen (1991) developed a nonlocal theory of memory dependent micropolar fluids with orientational effects. Orientational and nonlocal effects near the walls change viscosity drastically if polymeric fluids are squeezed in microscopic sizes. When channel surfaces are adsorbed with polymer layers, viscosity becomes a function of the channel gap. He pointed out that all fluids have internal structures with some internal characteristic length (such as radius of gyration) in microscopic scale. This length becomes comparable with the external characteristic length (such as channel depth) which leads to the concept of memory of a fluid. Nonlocal memory-dependent micropolar fluids have essential rotational degrees of freedom and twist inertia. These fluids are affected by couple stress, body couples and long-range interactions. He concluded that the memory effects become significant when the external characteristic length becomes small enough to compare with the average radius of the gyration of molecular elements of fluids. Such a situation arises in the case of thin film lubrications. He included the memory with stress and viscosity of fluid only.

Nibbi (1994) introduced a relationship with free energies relating to viscous fluids with memory. He also considered the quasi-static problem associated with viscous fluids with memory. He pointed out recent investigations performed on the determination of free energies for linear viscoelastic fluid. However, models characterizing viscoelastic fluids with memory are still unknown. For a linear isotropic, homogeneous, incompressible viscoelastic fluid, the constitutive equation is characterized by the symmetric stress tensor equation to represent fluid memory. It is unrealistic to consider such fluids. He mentions nothing about media as well, the real feature of memory

Broszeit (1997) dealt with the numerical simulation of steady state isothermal flow for liquids with memory in a Newtonian fluid. He applied a single-integral constitutive law, assuming that the fluid kinematics are known. He showed how the deformation histories of the fluid pathway play a role in the behavior of simulation. He tried only to describe and solve the memory of a fluid problem with stress. Practically memory is function fluid and media properties with time.

The memory of a fluid can be defined as a derivative of fractional order simulating the effect of a decrease of the permeability in time (Caputo, 1999). He briefly discussed the contribution of different researchers to the basic equations used to study fluid diffusion in porous media. These authors contributed various forms for setting equations that rigorously represent the interaction between an elastic porous medium and the flow of fluid through it, and obtaining solutions for the equations in many interesting cases. All the above-mentioned types of fluid flow imply that the permeability of the medium varies with time. The author investigated some geothermal areas where the fluids may precipitate minerals in the pores of the medium, thus diminishing their size. To study the flow of these fluids he modified the Darcy's law by introducing a memory formalism represented by a derivative of fractional order simulating the effect of a decrease of the permeability in time. He did not relate the fluid memory with other properties of fluid and did not show how this property plays a role in porous media. However, Caputo (1999) acknowledged that the memory of fluid in porous media can be described more extensively and accurately if the Darcy model is replaced by other appealing models which describe the media and fluid accurately.

Li et al. (2001) investigated some characteristics in non-Newtonian fluids. They identified the interaction and coalescence due to stress and their relaxation due to fluid memory. Their results suggest clearly that a new mechanism should be discovered for interactions and coalescence in non-Newtonian fluids. The memory effect of residual stresses holds the shear-thinning process during a certain time so that the local viscosity decreases. The memory effect irritates either interactions, through pure acceleration of rise in velocity, or coalescence at shorter injection periods between bubbles. Their model does not clearly describe or visualize the true feature of fluid memory. They did not include the properties of fluid and media with memory.

Arenzon et al. (2003) studied a nonlinear diffusion model which describes density relaxation of densely packed particles under gravity and thermal vibration. They found a jamming transition line between a low-density fluid phase and a high-density glassy regime, characterized by diverging relaxation time and logarithmic or power-law compaction according to the specific form of the

diffusion coefficient. They showed history-dependent properties such as quasi-reversible–irreversible cycle and memory effects. They mentioned that memory phenomenon is a history dependent phenomenon. They included the perturbation to describe the memory of a fluid. They also pointed out that a memory is related to perturbations at early times. They concluded that the memory effects are simpler than their glassy counterpart and can be described in terms of the density profile properties. They did not consider other properties of fluid except density to describe the notion of memory based on a quasi-static flow regime. However, their model does not represent whole scenario of fluid memory.

Shin et al. (2003) studied the non-equilibrium mechanism in the transport of inertia-dominated particles. They explained the problem of particle deposition inside a turbulent boundary layer. They pointed out that a turbulent boundary layer is seriously affected by a non-equilibrium memory effect due to the inertia of particles and mean shearing of the carrier flows. While maintaining a partial memory of their earlier motion, part of the mean and fluctuating velocities at previous times are activated. This is called the non-equilibrium memory effect. The memory effect is sensitively dependent on the intermediate diffusion time scale and this has to be chosen depending on the characteristic time scale of the mechanism of interest. This model is for homogeneous surrounding media and is not sufficient to describe the full impact of fluid memory on flow behavior and in media.

Zhang (2003) studied the linkage between microscopic car-following and macroscopic fluid-like behavior of traffic flow. He found that driver memory in car-following guide the viscous effects in continuum traffic flow dynamics. The phenomena generated by traffic flow is established when he developed a second-order continuum viscosity model with memory. His models that attempt to describe the system have a role in explaining certain aspects of the system. The model contains traffic viscosity and is linked to driver memory. Memory is a function of time and space and forward time events depend on previous time events. Road car traffic model is the basis of his model. His model is fictitiously compared with road car traffic model and the true nature of fluid memory in a media.

Lu and Hanyga (2005) studied Biot's theory and the Johnson–Koplik–Dashen dynamic permeability model in wave field simulation of a heterogeneous porous medium. They addressed the time domain drag force expression of the model to express in terms of the shifted fractional derivative of the relative fluid velocity. The governing equations for the two-dimensional porous medium are reduced to a system of first-order differential equations for velocities, stresses, pore pressure and the quadrature variables associated with the drag forces. These variables represent the memory variables satisfying first-order relaxation differential equations. They did not show how the memory is likened with these properties. They tried to correlate the memory effect with ultrasound and seismic wave propagation with drag force. They concluded that the memory effect for the drag force should be expressed in terms of a time convolution with a singularity  $t^{-1/2}$  for  $t \rightarrow 0$ . However, they tried to establish evidence of experimental and theoretical findings at  $t \rightarrow 0$  which is not real.

Mobilization and subsequent flow in a porous medium of a fluid with a yield stress can be explained well when the notion of memory is introduced (Chen et al., 2005). Here the modeled fluid behavior is a Bingham plastic using single-capillary expressions for the mobilization and flow in a pore-throat. To incorporate dynamic effects due to the viscous friction of mobilization, researchers introduced the concept of invasion percolation with memory (IPM). This concept explains the macroscopic threshold (minimum pressure gradient) which directly follows from the geometry of the path, along which mobilization first occurs. The minimum threshold path (MTP) is connected through nearest neighboring paths between two given boundaries (or points), along which the sum of thresholds is the minimum possible. Fundamental to this concept is the notion that specific, local thresholds must be exceeded across a given pore throat. Within this threshold, the fluid is to be mobilized, and these thresholds are distributed in the network. IPM addressed static properties of various problems with yield stress. These are the onset of the mobilization of a single-phase Bingham fluid in a porous medium, or foam formation and propagation in porous media in the absence of flow effects. However, it did not account for dynamic (viscous flow) effects of fluid by which mobilization occurs. In their

calculations, flow in an open path did not affect the distribution of pressure, so the identification of higher-energy paths was strictly a static (quasi-thermodynamic) process. In the case of Bingham fluids, this would correspond to a vanishing plastic viscosity. They explained how IPM works, however they did not construct a model which represents the notion of fluid memory.

Gatti and Vuk (2006) studied the motion of a linear viscoelastic fluid in a two dimensional domain with periodic boundary conditions for the asymptotic behavior. They consider an isotropic homogeneous incompressible fluid of Jeffrey's type where the Reynolds number is equal to one. They also assumed that density is independent of time. They calculated the memory effect by also assuming that pressure and velocity are independent of time. These assumptions follow the conventional models.

Figure 1 shows the effects on viscosity of PS – Cyclohexane solution (after PS – polystyrene had absorbed on the mica surfaces of the channel) when memory is considered (Eringen, 1991). He matches his theoretical data with experimental results. He mentioned that the further interest to observe his theory accounts for the orientational changes of the molecular alignments with time. This means that the evaluation of the anisotropy in the flow with micro motions can be determined by solving field equations.

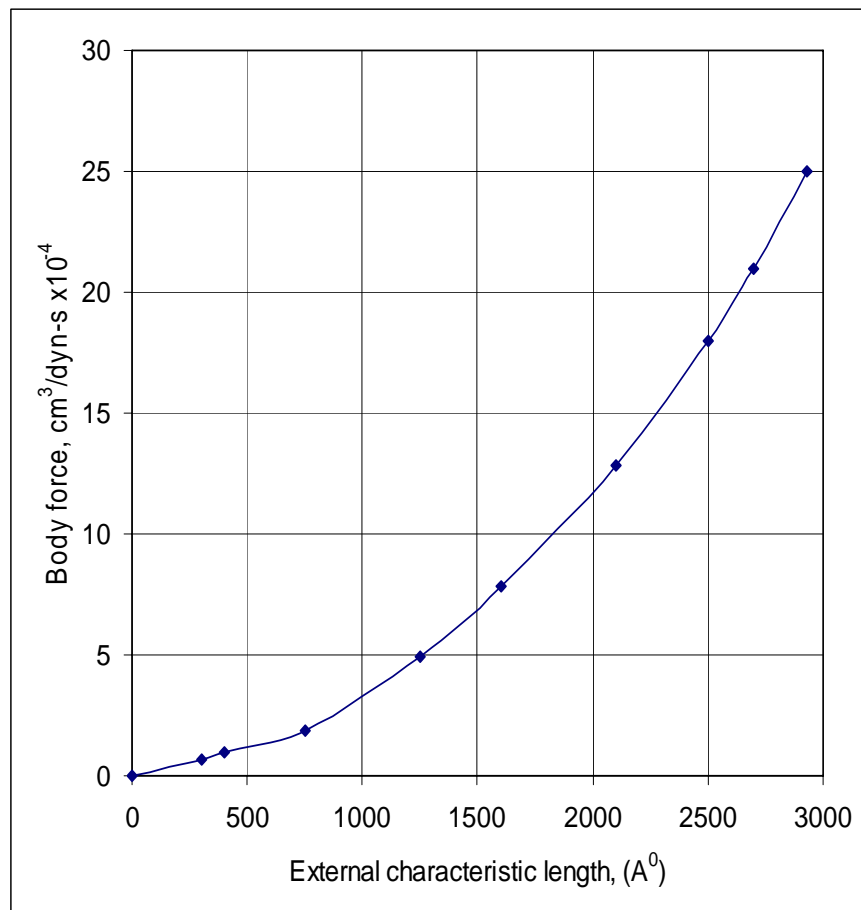


Figure 1: variation of body force with external characteristic length when memory is considered (redrawn from Eringen, 1991)

Table 1 shows the different available existing models that have critically reviewed with their assumptions.

Table 1: Comparison of memory models

Model	References	Assumptions
$\bar{K} = f(L, \bar{v}, t, \mu_0, P, \bar{\tau}, \alpha)$ $\bar{K}$ = permeability of the system $\bar{v}$ = velocity $t$ = Time $\mu_0$ = viscosity $P$ = Pressure $\bar{\tau}$ = stress tensor $\alpha$ = diffusivity	Slattery, (1967)	1) Steady state incompressible flow 2) Isotropic porous media 3) Neglect inertia effects 4) The parameters depend only upon local thermodynamic state
$\tau = \int \frac{2\eta_0}{\lambda_1} \left[ \left( 1 - \frac{\lambda_2}{\lambda_1} \right) e^{-(t-t')/\lambda_1} + \lambda_2 \delta(t) \right] \times \dot{\Gamma}(t, t') dt'$ $\tau$ = stress $\eta_0$ = zero shear rate viscosity $t$ = present time $t'$ = some past time $\lambda_1$ = relaxation time $\lambda_2$ = retardation time	Mifflin and Schowalter (1986)	1) Steady state incompressible flow 2) Homogeneous (spherical shape) fluid particles
$\bar{T}(x, t) = -p(x, t)I + \int_R 2\mu(\tau) \bar{D}(x, t - \tau) d\tau$ $(x, t) \in \Omega_T \equiv \Omega \times (0, T)$ $\bar{T}$ = stress $\bar{D}$ = velocity gradient $t$ = Time $\Omega$ = domain of physical space (i.e. $\equiv R^3$ ) filled by moving fluid $T$ = fixed positive number, ( $\leq +\infty$ ) $P$ = Pressure $\mu = \mu(\tau)$ = relaxation modulus of the viscosity	Ciarletta and Scarpetta, (1989)	1) Neglect non-linear convective term 2) Homogeneous, incompressible and viscous fluid 3) Smooth and bounded domain of physical space 4) Based on simple solid material
$t_{kl} = -\pi \delta_{kl} + T_{kl},$ $T_{kl} = 2 \int_{-\infty}^t d\tau \int_{\mathcal{D}-\sigma} dv' \sum_{klmn}^1 (s=t) \frac{\partial C'_{mn}}{\partial \tau}$ $\frac{\partial C_{mn}}{\partial \tau} = 2d_{ij}(\tau) \frac{\partial x_i(\tau) \partial x_j(\tau)}{\partial x_m(t) \partial x_n(t)}$ $2 \sum (s=t) = (\lambda_0 + \lambda_1 j) \delta_{kl} \delta_{mn} + (\mu_0 + \mu_1)$ $\times (\delta_{kl} \delta_{ln} + \delta_{kn} \delta_{lm})$	Eringen, (1991)	1) Non heat conducting fluid 2) Nonlocal effects are negligible 3) Homogeneous (spherical) molecules

Table 1: Comparison of memory models cont'd.....

Model	References	Assumptions
$t_{kl}$ = stress tensor $\lambda_0, \lambda_1, \mu_0, \mu_1$ = viscosity moduli $t$ = Time $\sigma$ = spin density $\delta$ = orthogonal tensor $\tau$ = dummy time variable		
$\bar{T}(t) = -p(t)I + 2 \int_0^{+\infty} \mu(s) \bar{D}^t(s) ds,$ $\bar{T}$ = symmetric stress tensor $\bar{D}$ = infinitesimal rate strain tensor $t$ = Time $\bar{D}^t$ = history of $\bar{D}$ up to time, $t$ $= \bar{D}^t(s) = \bar{D}(t-s)$ $P$ = Pressure $\mu = \mu(\tau)$ = relaxation modulus of the viscosity	Nibbi, (1994)	1) Linear isotropic, homogeneous, incompressible and viscous fluid 2) The relaxation function has to be satisfy; $\mu \in L^1(0, +\infty)$ , $\dot{\mu} \in L^1(0, +\infty) \cap L^2(0, +\infty)$
$q = -\eta \rho_0 (\partial^\alpha / \partial t^\alpha) (\partial p / \partial y)$ $\partial^\alpha p(y, t) / \partial t^\alpha = [1/\Gamma(1-\alpha)] \int_0^t (t-u)^{-\alpha} [\partial p(y,$ $, \text{ Where, } 0 \leq \alpha < 1$ $p(y, t)$ = Fluid pressure with time $\rho_0$ = density of the fluid in the undisturbed condition $\eta$ = Ratio of the pseudo-permeability of the medium with memory to fluid viscosity $\alpha$ = fractional order of differentiation $t$ = Time $u$ = fluid velocity in the plane of the integral $z = (1-\alpha)$ = Definition to simplify the computations $K$ = Permeability of the system $P$ = Pressure $q$ = fluid mass flow rate per unit area	Caputo, (1999)	1) Linear isotropic, homogeneous, incompressible and viscous fluid 2) Permeability diminishes with time only
$\frac{d\tau_m}{dt} = -\alpha\tau_m + \beta\dot{\gamma}_B$ $\tau_m$ = mean stress in a cell, pa $\dot{\gamma}_m$ = shear rate due to residual stresses, 1/s $\dot{\gamma}_B$ = shear rate due to the passage of bubbles, 1/s $\alpha, \beta$ = Constant determined by the rheological simulation under different conditions of fluid and bubble volume	Li et al., (2001)	1) Homogeneous (spherical shape) bubble 2) Homogeneous stresses and composition 3) A constant formation frequency



Table 1: Comparison of memory models cont'd.....

Model	References	Assumptions
$v_t^+ = v_{t,eq}^+ + \Delta v_t^+$ $v_{t,eq}^+ = -\frac{24}{R_{ep}} \frac{1}{C_D} \tau_p \frac{d}{dy} \left( \zeta_{yy} - D_{yy} \frac{d\bar{v}_y}{dy} \right) \frac{1}{u^*}$ $\Delta v_t^+ = \frac{24}{R_{ep}} \frac{1}{C_D} \tau_p \frac{d}{dy} \left( \tau_\beta \frac{d\zeta_{yy}}{dy} \right) \frac{1}{u^*}$ <p> <math>v_{t,eq}^+</math> = equilibrium turbophoretic velocity  <math>\Delta v_t^+</math> = non-equilibrium turbophoretic velocity with memory  <math>R_{ep}</math> = Reynolds number  <math>C_D</math> = drag force  <math>\tau</math> = relaxation time  <math>\tau_\beta</math> = relaxation time scale required to reach a local equilibrium state of the particle Reynolds stress  <math>\zeta_{yy}</math> = Maxwell distribution parameter  <math>D_{yy}</math> = coefficient  <math>u^*</math> = friction velocity </p>	<p>Shin et al., (2003)</p>	<ol style="list-style-type: none"> <li>1) Stokes drag force particle motion</li> <li>2) The variation of mean shear rates of both the phases is unrelated</li> <li>3) Fluctuating velocities of particles is Gaussian</li> <li>4) The long-time diffusivity of turbulent particle is independent of mean shearing of the carrier flow field</li> <li>5) Homogeneous medium</li> <li>6) Independent of the mean shear rate of the flow.</li> </ol>
$\mu(\rho) = 2\beta\tau_\epsilon c^2(\rho) = 2\beta\tau(\rho V'(\rho)_*)^2$ $v_t + \{v + c(\rho)\}v_x = \mu(\rho)v_{xx}$ <p> <math>\rho</math> = traffic density  <math>\beta</math> = a parameter that describe memory  <math>\tau</math> = relaxation time  <math>v</math> = traffic velocity  <math>\mu</math> = viscosity of traffic fluid  <math>c</math> = concentration of fluid </p>	<p>Zhang, (2003)</p>	<ol style="list-style-type: none"> <li>1) Isotropic media</li> <li>2) Taylor series expansion is considered</li> <li>3) Road car traffic model is the basis</li> <li>4) The generic, monotonic function, <math>G_*</math> is assumed as linear function</li> </ol>

## STRESS RELATION WITH MEMORY

As the preceding literature review suggests, attempts to account for fluid memory is a relatively intangible formulation of the observable effects of some underlying physical process. In terms of other well-understood, more or less tangible physical phenomena associated with fluid flow, researchers attempting to broaden these boundaries have encountered many difficulties. Recently Hossain et al. (2006) have theorised a relationship between fluid properties and media properties, incorporating fluid memory. This theory describes the role of stress with strain when the effects of fluid properties such as viscosity, density, diffusivity and compressibility of fluid with fluid memory are added. The media properties such as surface tension, porosity and permeability are also considered to describe the combined effects of both fluid and media properties with the notion of memory of a fluid. Finally incorporating temperature and pressure effects, the stress strain relation shows such Equation 1 which is presented below. The shear stress and shear rate of strain becomes:

$$\tau_T = \frac{k\Delta p A_{xz} \Gamma(1-\alpha)}{\mu_0^2 \eta \rho_o \phi y \int_0^t \frac{(t-\xi)^{-\alpha}}{k} \left( \frac{\partial c}{\partial \xi} \frac{\partial p}{\partial \xi} - \frac{c}{k} \frac{\partial k}{\partial \xi} \frac{\partial p}{\partial \xi} + c \frac{\partial^2 p}{\partial \xi^2} \right) d\xi} \times \left[ \left( \frac{\partial \sigma}{\partial T} \frac{\Delta T}{\alpha_D M_a} \right) \times e^{\left( \frac{E}{RT} \right)} \right] \frac{du_x}{dy} \quad (1)$$

The above mathematical model implies the almost all possible fluid and fluid media properties. If we consider permeability of media and compressibility of fluid does not change with time, Equation 1 can be written as:

$$\tau_T = \frac{k^2 \Delta p A_{xz} \Gamma(1-\alpha)}{\mu_0^2 \eta \rho_o \phi y c \int_0^t (t-\xi)^{-\alpha} \left( \frac{\partial^2 p}{\partial \xi^2} \right) d\xi} \times \left[ \left( \frac{\partial \sigma}{\partial T} \frac{\Delta T}{\alpha_D M_a} \right) \times e^{\left( \frac{E}{RT} \right)} \right] \frac{du_x}{dy}$$

$$\text{Let, } I = \int_0^t (t-\xi)^{-\alpha} \left( \frac{\partial^2 p}{\partial \xi^2} \right) d\xi$$

The above equation reduces to:

$$\tau_T = \frac{k^2 \Delta p A_{xz} \Gamma(1-\alpha)}{\mu_0^2 \eta \rho_o \phi y c I} \times \left[ \left( \frac{\partial \sigma}{\partial T} \frac{\Delta T}{\alpha_D M_a} \right) \times e^{\left( \frac{E}{RT} \right)} \right] \frac{du_x}{dy} \quad (2)$$

Equation (2) is solved to investigate the combined effects on stress strain.

## RESULTS AND DISCUSSION

In this section, we have used FORTRAN programming to solve the equation (2). For numerical simulation, API 28.8 gravity crude oil is used. To solve the equation,  $A_{xz} = 1.0 \text{ m}^2$ ;  $c = 1.2473 \times 10^{-9} \text{ 1/pa}$  ( $= 0.00000861/\text{psi}$ );  $E = 85.2 \text{ KJ/mol}$ ;  $h = 1.0 \text{ m}$ ;  $k = 30.0 \times 10^{-15} \text{ m}^2$ ;  $Ma = 3.98$ ;  $\Delta p = 150.0 \text{ N/m}^2$ ;  $R = 0.008314 \text{ kJ/mol-K}$ ;  $\Delta T = 75.0 \text{ }^\circ\text{K}$ ;  $0 \leq \alpha < 1$ ;  $y = 1.0 \text{ m}$ ;  $\alpha_D = 6.75 \times 10^{-7} \text{ m}^2/\text{s}$ ;  $\mu_0 = 87.4 \times 10^{-3} \text{ Pa-s}$  at  $298 \text{ }^\circ\text{K}$  ( $25 \text{ }^\circ\text{C}$ );  $\phi = 30\%$ ;  $du_x/dy = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$  and  $1.0$ ;  $\rho_o = 645.1 \text{ kg/m}^3$  at  $298 \text{ }^\circ\text{K}$  ( $25 \text{ }^\circ\text{C}$ );  $\eta = 0.343249$ ;  $\partial \sigma / \partial T = 0.165293$  have been considered. In equation (2) one of the fluid memory part in the denominator (the integral part) is varied ( $I = 0.5, 0.3, 0.1, 0.09, 0.08, 0.03$ ) to investigate how memory effect plays a role on fluid behavior. Similar data is also used by Lu et al. (2005) for their model.

Figure 2 shows the variation of stresses with rate of strain when memory is considered with other fluid and media properties. The stress-strain relation for Newton's law of viscosity has been presented. Initially stress increases gradually with the rate of strain. This trend follows up to a strain rate of 0.7. However, beyond this point, stress increases rapidly with a slide change of rate of strain. It is observed that, when the effect of memory on a fluid goes down, the shape of the curve is closer to the Newtonian type. This indicates that if the memory of fluid and media is ignored for any fluid or media, the behavior looks like Newtonian type of fluid except its nonlinearity. However, due to other effects on stress-strain, the shape of the curve is almost exponential or any power series polynomial type which indicates the nonlinearity of the behavior of fluid and porous media. The trend of the curve is almost similar with Figure 1. This indicates that the proposed model is a good agreement with Eringen model.

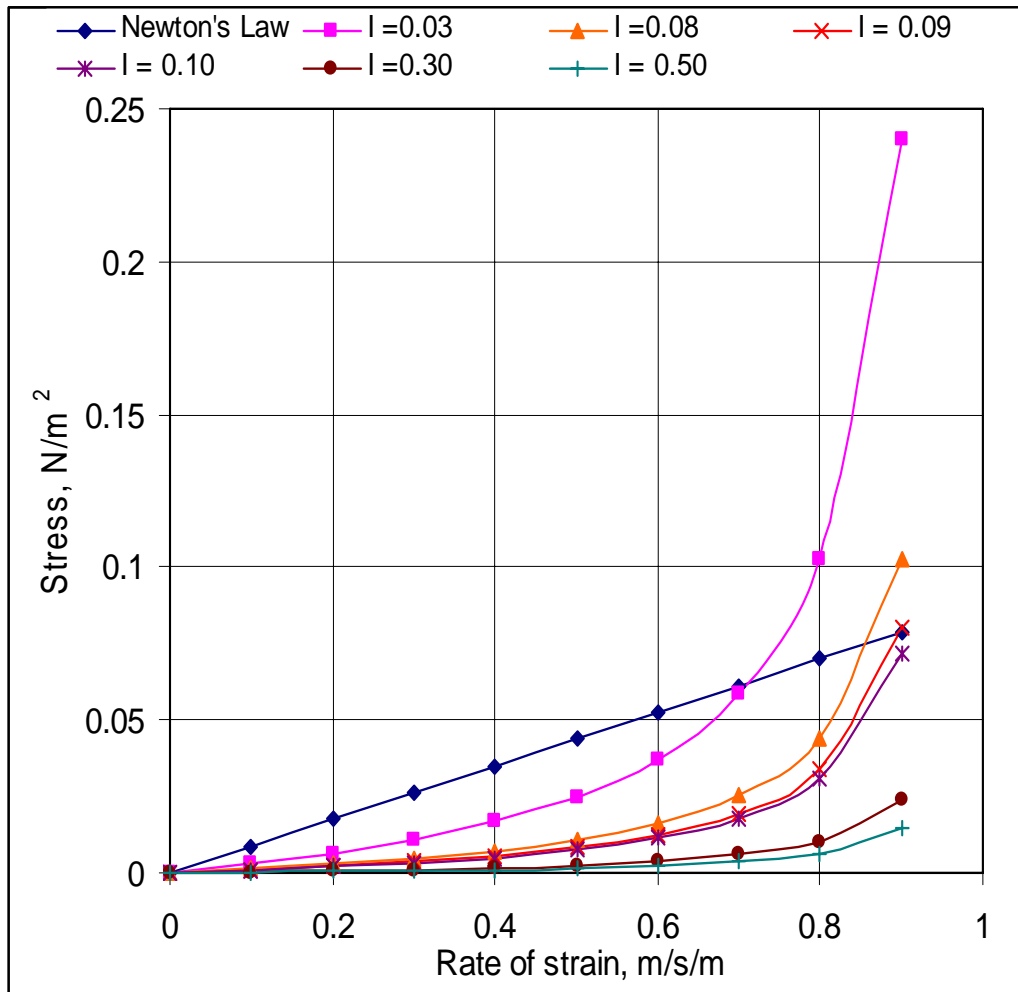


Figure 2: Stress variation with rate of strain with memory for a crude oil

## CONCLUSION

The literature to date has yet to conceptualize fluid memory in a comprehensive way. The particularity and uniqueness of memory – its definition varies with different combinations of any given fluid and its particular medium – has posed the greatest difficulty. Memory itself is a function of all possible properties of the given fluid and its medium over time. When memory, fluid and media properties are considered simultaneously, the stress-strain trend is nonlinear, rather than a linear function. Another outstanding challenge is the understanding and formulation of effects and behavior of a memory-induced fluid in porous media. As a starting-point, some dependence on complex phenomena of the fluid and some relationship to fluid viscosity and density may be hypothesized. In porous media, there are drastic effects on permeability with time. A comprehensive description of fluid behavior awaits fuller elaboration of new sets of relations between fluid viscosity and memory.

## ACKNOWLEDGEMENT

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## NOMENCLATURE

$A_{xz}$  = cross sectional area of rock perpendicular to the flow of heat,  $m^2$

$c$  = total compressibility of the system,  $1/pa$

$E$  = Activation energy for viscous flow of 28.8 API gravity oils,  $KJ/mol$

$h$  = Length in temperature gradient (i.e. Length between the two points along the y- direction),  $m$

$k$  = Permeability of the system,  $md$

$Ma$  = Marangoni number

$p(y,t)$  = Fluid pressure,  $N/m^2$

$\Delta p = p_T - p_0$  = Pressure difference,  $N/m^2$

$R$  = universal gas constant,  $kJ/mol-K$

$T$  = Temperature,  $^{\circ}K$

$\Delta T = T_T - T_0$ ,  $^{\circ}K$

$t$  = Time,  $sec$

$u_x$  = fluid velocity in the direction of  $x$ ,  $m/s$

$y$  = Distance from the boundary plan,  $m$

$\sigma$  = Surface tension,

$\alpha_D$  = Thermal diffusivity,  $m^2/s$

$\mu_0$  = fluid dynamic viscosity,  $cp$

$\alpha$  = fractional order of differentiation

$\phi$  = porosity of fluid media

$\tau_T$  = Shear stress at temperature  $T$ ,  $^{\circ}K$

$\frac{du_x}{dy}$  = Velocity gradient along y-direction,  $m/s/m$

$\xi$  = Dummy variable for time i.e. real part in the plane of the integral

$\rho_o$  = density of the fluid,  $kg/m^3$

$\eta$  = Ratio of the pseudo-permeability of the medium with memory to fluid viscosity

$|\partial\sigma/\partial T|$  = The derivative of surface tension  $\sigma$  with temperature and can be positive or negative depending on the substance

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