

Please turn in the exam Tuesday, October 8th, before 6:00pm.

1. An ideal gas is originally confined to a volume V_1 in an insulated container of volume $V_1 + V_2$. The remainder of the container is evacuated. The partition is then removed and the gas expands to fill the entire container. If the initial temperature of the gas was T , what is the final temperature? What is the change in entropy? Justify your answer.
2. Demonstrate the following thermodynamic relations:

$$(a) \left(\frac{\partial T}{\partial P} \right)_S = \frac{\alpha VT}{C_P}$$

$$(b) \left(\frac{\partial T}{\partial P} \right)_H = -\frac{V}{C_P} (1 - \alpha T)$$

$$(c) \left(\frac{\partial H}{\partial S} \right)_V = T \left[1 + \frac{\alpha V}{\kappa_T C_V} \right]$$

3. A certain system is found to have a Gibbs free energy given by

$$\overline{G} = RT \ln \left[\frac{aP}{(RT)^{5/2}} \right]$$

where a and R are constants. Find the specific heat at constant pressure, \overline{C}_P .

4. For a system whose internal energy is given by:

$$U = k \frac{S^3}{V}$$

where k is a constant.

- (a) What are the units of k ?
 - (b) Find the equation of the adiabats in the P vs. V plane.
 - (c) Find the derived expressions for P and T .
 - (d) Write the equation of state for the system in terms of P , V and T .
5. The heat of melting of ice at 1 atm and 0°C is 6.01 kJ/mol . The density of ice under these conditions is 0.917 g/cm^3 and the density of water is 0.9998 g/cm^3 . If 1 mole of ice is melted under these conditions, what will be
 - (a) the work done?
 - (b) the change in internal energy?
 - (c) the change in entropy?

Please turn in the exam Tuesday, October 9th, before 4:00pm.

6. The state equation of a new material is

$$P = \frac{aT^3}{V}$$

where a is a constant. The internal energy of the matter is

$$U = BT^n \ln \frac{V}{V_o} + f(T)$$

where B , n and V_o are all constants, $f(T)$ only depends on the temperature. Find B and n .

7. The molar entropy of an gas is given by

$$\bar{S} = \frac{1}{2} \left[\sigma + 5R \ln \bar{U} + 2R \ln \bar{V} \right]$$

(a) Find \bar{C}_V and \bar{C}_P .

(b) Find α and κ_T .

8. A paramagnetic system in a uniform magnetic field \mathcal{H} is thermally insulated from the surroundings. It has an induced magnetization $M = a\mathcal{H}/T$ and a heat capacity $C_{\mathcal{H}} = b/T^2$ at constant \mathcal{H} , where a and b are constants and T is the temperature. How will the temperature of the system change when \mathcal{H} is quasi-statically reduced to zero? In order to have the final temperature change by a factor of 2 from the initial temperature, how strong should be the initial \mathcal{H} ?

9. Demonstrate that

$$\bar{C}_P - \bar{C}_V = \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_P \quad \text{and that} \quad P + \left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

and calculate $\bar{C}_P - \bar{C}_V$ for a van der Waals gas. Compare how much these values differ for He and CO₂ at $T = 200$ K and $\bar{V} = 0.2$ L mol⁻¹.

gas	$a/\text{atm L}^2 \text{mol}^{-2}$	$b/\text{L mol}^{-1}$
He	0.0346	0.0238
CO ₂	3.64	0.0427

10. A thermally conducting, uniform and homogeneous bar of length L , cross section A , density ρ and specific heat at constant pressure c_P is brought to a nonuniform temperature distribution by contact at one end with a hot reservoir at a temperature T_h and at the other end with a cold reservoir at a temperature T_c . The bar is removed from the reservoirs, thermally insulated and kept at constant pressure.

(a) Determine the final temperature of the bar.

(b) Show that the change in entropy of the bar is

$$\Delta S = c_P \rho A L \left[1 + \ln T_f + \frac{T_c}{T_h - T_c} \ln T_c - \frac{T_h}{T_h - T_c} \ln T_h \right]$$