Problem 1

Subject Type

Quantum Mechanics → *Momentum Operator*

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}\psi \Rightarrow p\psi = \hbar k \psi \tag{1}$$

Problem 2

Subject Type

Atomic → *Bragg Diffraction*

Recall the Bragg Diffraction dispersion relation,

$$\lambda = 2d\sin\theta, \tag{2}$$

thus the maximal wavelength λ would be $2d$, choice (D). (One can derive that even if one does not remember the formula. Consider two lattice planes. View them from the side so that they appear as two parallel lines. A wave would hit the both planes at, say, an angle θ from the normal. The wave that reflects off the bottom lattice will have to travel an extra distance, relative to the wave hitting the top plane, equal to $2d\sin\theta$.)

Problem 3

Subject Type

Quantum Mechanics → *Bohr Theory*

Recall the Bohr Equation, $E_n = Z^2/n^2 E_1$, which applies to both the Hydrogen atom and hydrogen-like atoms. One can find the characteristic X-rays from that equation (since energy is related to wavelength and frequency of the X-ray by $E = \lambda f$).

The ratio of energies is thus $E(Z = 6)/E(Z = 12) = 6^2/12^2 = 1/4$, as in choice (A).

Problem 4

Subject Type

Mechanics → *Gravitational Law*

Recall the famous inverse square law determined almost half a millennium ago,

$$F = \frac{k}{r^2}, \tag{3}$$

where $k = GMm$.

The ratio of two inverse-square forces ($r > R$, where R is the radius of the planet or huge heavy object) would be

$$\frac{F(r_1)}{F(r_2)} = \frac{4r_2^2}{r_1^2}. \tag{4}$$

Thus, $\frac{F(R)}{F(2R)} = \frac{4R^2}{R^2} = 4$, which is choice (C).

Problem 5

Subject Type

Mechanics → *Gauss Law*

The inverse-square law doesn't hold inside the Earth, just like how Coulomb's law doesn't hold inside a solid sphere of uniform charge density. In electrostatics, one can use Gauss Law to determine the electric field inside a uniformly charged sphere. The gravitational version of Gauss Law works similarly in this mechanics question since $\nabla \cdot \vec{E} = \rho_e \Rightarrow \nabla \cdot \vec{g} = \rho_M$, where ρ_M is the mass density of M. In short, the gravitational field $\vec{g}$ plays the analogous role here as that of $\vec{E}$ Thus, $\int \vec{g} \cdot d\vec{a} = \int \rho dV$.

So, for $r < R$, $g(4\pi r^2) = \rho \frac{4}{3}\pi r^3 \Rightarrow g = r\frac{\rho}{3}$, where one assumes ρ is constant.

To express the usual inverse-square law in terms of ρ, one can apply the gravitational Gauss Law again for $r > R$, $g(4\pi r^2) = \rho \frac{4}{3}\pi R^3 \Rightarrow g = \frac{R^3}{r^2}\frac{\rho}{3}$.

Since $\vec{F} = m\vec{g}$ Therefore,

$$\frac{F(R)}{F(R/2)} = 2R. \tag{5}$$

Problem 6

Subject Type

Mechanics → *Method of Sections*

By symmetry, one can analyze this problem by considering only *one* triangular wedge. The normal force on one wedge is just $N = (m + M/2)g$, since by symmetry, the wedge (m) carries half the weight of the cube (M). The frictional force is given by $f = \mu N = \mu(m + M/2)g$.

Sum of the forces in the horizontal-direction yields $F_x = 0 \leq f - N_M/\sqrt{2} = \mu(m+M/2)g - Mg/2$ for static equilibrium to remain valid. (Note that the normal force of the cube is given by $N_M = Mg/\sqrt{2}$ since, summing up the forces perpendicular to the plane for M, one has, $N_M \sin(\pi/4) = Mg/2$. Also, note that it acts at a 45 degree angle to the wedge.)

Solving, one has $\mu(m + M/2)g \geq Mg/2 \Rightarrow M \leq \frac{2\mu m}{1-\mu}$.

(In a typical mechanical engineering course, this elegant method by symmetry is called the *method of sections*.)

Problem 7

Subject Type

Mechanics → *Normal Modes*

For normal mode oscillations, there is *always* a symmetric mode where the masses move together as if just one mass.

There are three degrees of freedom in this system, and ETS is nice enough to supply the test-taker with two of them. Since the symmetric mode frequency is not listed, choose choice it!—as in (A).

Problem 8

Subject Type

Mechanics → *Torque*

The problem wants a negative z component for τ. Recall that $r \times F = 0$ whenever r and F are parallel (or antiparallel). Thus, choices (A), (B), (E) are immediately eliminated. One can work out the cross-product to find that (D) yields a positive τ_z, thus (C) must be it.

Problem 9

Subject Type

Electromagnetism → *Current Directions*

The opposite currents cancel each other, and thus the induction (and field) outside is 0.

Problem 10

Subject Type

Electromagnetism → *Image Charges*

The conductor induces image charges -q and -2q since it is grounded at $x = 0$. Since these are (mirror) image charges, each charge induced is the same distance from the conducting plane as its positive component.

The net force on q is just the *magnitude* sum of the positive charge 2q and the two induced charges, $\sum F = \frac{q^2}{4\pi\epsilon_0}\left(1/(a^2) + 2/(2a^2) + 2/(a^2)\right) = \frac{q^2}{4\pi\epsilon_0 a^2}(1 + 1/2 + 2) = \frac{q^2}{4\pi\epsilon_0 a^2}\frac{7}{2}$, as in choice (E).

Problem 11

Subject Type

Electromagnetism → *RC Circuit*

The energy of a capacitor C with voltage V across it is given by $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$. ($Q = CV$ derives the other variations of the energy.)

From Ohm's Law, one arrives at the relation between charge and time, $Q/C + \dot{Q}R = 0 \Rightarrow \frac{Q}{RC} = -\frac{dQ}{dt} \Rightarrow -\frac{dt}{RC} = \frac{dQ}{Q}$. Integrating both sides, one finds that $Q(t) = Q_0 e^{-t/(RC)}$.

Plugging this into the energy equation above, one has $U \propto Q(t)^2 \propto e^{-2t/(RC)}$. Twice time required for the energy to dissipate by 2 is thus given by $1/2 = e^{-t/(RC)} \Rightarrow t_{1/2} = RCln(2$. Divide it by 2 to get choice (E).

Problem 12

Subject Type

Electromagnetism → *Potential*

A Potential V is related to the electric field E by $\vec{E} = \nabla V$.

Since the problem supplies the approximation tool that the planes are quite large, one can assume the field is approximately constant. The remaining parameter that can't be thrown out by this approximation is the angle, and thus the only choice that yields $\frac{d}{d\phi}V = constant$ is choice (B).

Problem 13

Subject Type

Electromagnetism → *Maxwell's Equations*

Magnetic monopoles remain a likeable (even lovable) theoretical construct because of their ability to perfectly symmetrize Maxwell's equations. Since the curl term has an electric current, the other curl term should have a magnetic current. ($\nabla \cdot B \neq 0$ is taken to be obvious in presence of magnetic charge.) The answer is thus (D), and the revised equations are,

$$\nabla \times H = J_e + \frac{\partial D}{\partial t} \tag{6}$$

$$\nabla \times E = -J_m - \frac{\partial B}{\partial t} \tag{7}$$

$$\nabla \cdot D = \rho_e \tag{8}$$

$$\nabla \cdot B = \rho_m. \tag{9}$$

Problem 14

Subject Type

Statistical Mechanics → *Blackbody Radiation Formula*

Recall

$$P = ut \propto T^4, \tag{10}$$

where P is the power and u the energy and T the temperature.

So, initially, the blackbody radiation emits $P_1 = kT^4$. When its temperature is doubled, it emits $P_2 = k(2T)^4 = 16kT^4$.

Recall that water heats according to $Q = mc\Delta T = \kappa\Delta T$. So, initially, the heat gain in the water is $Q_1 = \kappa(0.5^\circ)$. Finally, $Q_2 = \kappa x$, where x is the unknown change in temperature.

Conservation of energy in each step requires that $kT^4 t = \kappa/2$ and $16kT^4 t = \kappa x$, i.e., that $P_i t = Q_i$. Divide the two to get $\frac{1}{16} = \frac{2}{x} \Rightarrow x = \Delta T = 8^\circ$. Assuming the experiment is repeated from the same initial temperature, this would bring the initial 20° to 28°, as in choice (C).

Problem 15

Subject Type

Statistical Mechanics → *Heat Capacity*

Note that this problem wants the regime of high temperatures, and thus the answer is *not* $\frac{5}{2}R$ from classical thermodynamics, but rather $\frac{7}{2}R$.

The problem suggests that a quantized linear oscillator is used. From the energy relation $\epsilon = \left(j + \frac{1}{2}\right)\hbar\nu$, one can write a partition function and do the usual Stat Mech jig. Since one is probably too lazy to calculate entropy, one can find the specific heat (at constant volume) from $c_v = \frac{\partial U}{\partial T}\big|_v$, where $U = NkT^2\left(\frac{\partial Z}{\partial T}\right)_V$, where N is the number of particles, k is the Boltzmann constant.

There are actually three contributions to the specific heat at constant volume. $c_v = c_{translational} + c_{rotational} + c_{vibrational}$. Chunk out the math and take the limit of high temperature to find that $c_v = \frac{7}{2}R$.

Problem 16

Subject Type

Thermodynamics → *Carnot Engine*

Recall the common-sense definition of the efficiency e of an engine,

$$e = \frac{W_{accomplished}}{Q_{input}}, \tag{11}$$

where one can deduce from the requirements of a Carnot process (i.e., two adiabats and two isotherms), that it simplifies to

$$e = 1 - \frac{T_{low}}{T_{high}} \tag{12}$$

for Carnot engines, i.e., engines of maximum possible efficiency. (Q_{input} is heat put into the system to get stuff going, $W$is work done by the system and T_{low} (T_{high}) is the isotherm of the Carnot cycle at lower (higher) temperature.)

The efficiency of the Carnot engine is thus $e = 1 - \frac{800}{1000} = 0.2$, where one needs to convert the given temperatures to Kelvin units. (As a general rule, most engines have efficiencies lower than this.) The heat input in the system is $Q_{input} = 2000J$, and thus $W_{accomplished} = 400J$, as in choice (A).

Problem 17

Subject Type

Lab Methods → *Oscilloscope*

This problem can be solved by elimination. Since one is given two waves, one with twice the frequency of the other, one can approximate the superposed wave (which shows up on the oscilloscope) as $\sin(\omega t) + \sin(2\omega t)$.

The summed wave no longer looks like a sine wave. Instead, it looks like a series of larger amplitude humps alternating with regions of smaller amplitudes.

However, since one is not supplied with a graphing calculator on the test, one can qualitatively eliminate the other choices based on the equation above. It is obviously not choices (D) and (E) since the superposition is still a one-to-one function. It isn't choice (C) or (B) since those are just sin waves (cosine waves are just off by a phase), and one knows that the superposed wave would look more complicated than that. Thus, one arrives at choice (A), which is a zoomed-in-view of the superposition above.

Problem 18

Subject Type

Lab Methods → *Coax Cable*

Elimination time. The first-pass question to answer is *why is it important that a coax cable be terminated at an end*: (A) Perhaps...

(B) Probably not. Terminating the cable at an end would not help heat dissipation and thus should not prevent overheating.

(C) Perhaps...

(D) Probably not, since termination should attenuate the signal rather than to prevent it.

(E) Probably not, since image currents should be canceled by the outer sheath.

Choices (A) and (C) remain. Now, use the second fact supplied by ETS. The cable should be terminated by its characteristic impedance. Characteristic impedance has to do with resonance. Thus, it should prevent reflection of the signal.

Problem 19

Subject Type

Mechanics → *Mass of Earth*

If one does not remember the mass of the earth to be on the order of 10^24kg, one might remember the mass of the sun to be 10^30kg. Since the earth weighs much less than that, the answer would have to be either (A) or (B). The problem gives the radius of the earth, and one can assume that the density of the earth is a few thousand kg/m^3 and deduce an approximate mass from $m = \rho V$. The answer comes out to about 10^22, which implies that the earth is probably a bit more dense than one's original assumption. In either case, the earth *can't* be, on average, uniformly $10^9 kg/m^3$ dense. Thus (A) is the best (and correct) answer.

Problem 20

Subject Type

Optics → *Missing Fringes*

Missing fringes in a double-slit interference experiment results when diffraction minima cancel interference maxima.

From a bit of phasor analysis, one can derive the diffraction factor $\beta/2 = \pi w/\lambda \sin\theta$ and the interference factor $\delta/2 = \pi d/\lambda \sin\theta$, where w is the width of the slits and d is the separation (taken from slit centers). The angles belong in the intensity equation given by $I \propto \sin(\beta/2)^2 \cos(\delta/2)^2$.

Thus, the condition for a double-slit diffraction minimum is given by $\delta/2 = m_d\pi = \pi w/\lambda \sin\theta \Rightarrow m_d\lambda = w\sin\theta$.

Also, the condition for interference maximum is given by $\beta/2 = m_i\pi = \pi d/\lambda \sin\theta \Rightarrow m_i\lambda = d\sin\theta$.

Now, one needs to find the choice that allows for an integer m_d. This immediately eliminates choices (A) and (B). But, this leaves choices (C), (D), and (E). Among the remaining choices, there is only one choice that allows for slits that are smaller than the separation. This is choice (D). Take it.

Problem 21

Subject Type

Optics → *Thin Film*

Elimination time.

I. Can't be this, since one knows from basic thin-film theory that choice IV is right. (None of the letter choices allow for both choices I and IV.)

II. Thin film theory has $2t = \lambda/2$ for constructive interference and $2t = \lambda$ for destructive interference. Thus, the thickness of the film is smaller than that of the light. (Search on the homepage of this site for more on thin film theory—it is explained in the context of other problems.)

III. This phase change allows for the half-integer constructive interference.

IV. Phase change only occurs when light travels from a medium with lower index of refraction to a medium with higher index of refraction. Since at the back surface, the light would be going from higher to lower index of refraction, there is no phase change.

Thus, choice (E).

Problem 22

Subject Type

Optics → *Telescope*

The magnification for a telescope is related to the focal length for the eyepiece and objective by $M = f_o/f_e$. (Note that it is the eye-piece that magnifies it. The objective merely sends an image that's within view of the eye-piece. However, magnification is inversely related to focal length.)

The problem gives angular magnification to be $M = 10 = f_o/f_e \Rightarrow f_e = f_o/10 = .1m$. The distance between the objective and eyepiece is the sum of the focal lengths (since the light comes from infinity). $d = f_o + f_e = 1.1$m as in choice (D).

Problem 23

Subject Type

Statistical Mechanics → *Fermi Temperature*

(Much of the stuff I classified as Stat Mech might also be considered Condensed Matter or Solid State Physics. They are classified as thus because the Stat Mech book I mentioned in the booklist on the site $http://grephysics.yosunism.com$ is perhaps the best intro to all this.)

The Fermi velocity is related by $\epsilon_F = kT_F = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2kT_F}{m}}$, where ϵ_F is the fermi energy, and T_F is the Fermi temperature.

One should know by heart the following quantities, $k = 1.381E-23$ and $m = 9.11E-31$ (but then again, they are also given in the table of constants included with the exam). Plug these numbers into the expression above to find v,

$v = \sqrt{\frac{2kT_F}{m}} \approx \sqrt{\frac{2\times 1.4E-23\times 8E5}{9E-31}} = \sqrt{\frac{2.8E-23\times 8E5}{9E-31}} \approx \sqrt{0.3E8\times 8E5} = \sqrt{24/10E13} \approx \sqrt{10^{12}} = 10^6$, the choice that comes closest to this order is choice (E).

Note that the hardest part of this problem is the approximation bit. No calculators allowed. Sadness.

Problem 24

Subject Type

Atomic → *Bonding*

Solid Argon is a Nobel gas. It has a full shell of outer electrons, and thus it cannot bond in anything but van der Walls bonding, which isn't really bonding, but more weak like charge-attraction.

One can arrive at this choice by elimination: (A) Ionic bonding occurs when one atom is a positive ion and the other the compensating negative ion. Since solid Argon isn't an ion, it can't do this.

(B) Covalent bonding occurs when electrons are shared between atoms. This only happens when the atom has unfilled orbitals. (Incidentally, it only occurs when two electrons are of opposite spins due to the Pauli Exclusion Principle. That is, they must have different quantum numbers so that they can both remain stable in a low energy state.)

(C) No partial charge-analysis needed.

(D) Argon isn't a metal.

(E) This is the one that remains.

Problem 25

Subject Type

Advanced Topics → *Particle Physics*

Choice (A) and (C) involve atoms, which are quite massive. Choice (B) involves protons, which are also pretty massive.

Problem 26

Subject Type

Lab Methods → *Log-Log graph*

Since initially, the counts per minute is $6E4$, the half-count amount would be $3E4$. This occurs between 5 and 10 minutes. Choice (B) seems a good interpolation.

Problem 27

Subject Type

Quantum Mechanics → *Uncertainty*

This problem looks much more complicated than it actually is. Since k and x are fourier variables, their localization would vary inversely, as in choice (B).

Problem 28

Subject Type

Quantum Mechanics → *Probability*

One doesn't actually need to know much (if anything) about spherical harmonics to solve this problem. One needs only the relation $P = \sum_i |\langle Y_i^3 | \psi(\theta, \phi)\rangle|^2$. Since the problem asks for states where $m = 3$, and it gives the form of spherical harmonics employed as $Y_l{}^m$, one can eliminate the third term after the dot-product.

So, the given wave function $\psi(\theta, \psi) = \frac{1}{\sqrt{30}}\left(5Y_4^3 + Y_6^3 - 2Y_6^0\right)$ gets dot-product'ed like $|\langle Y_i^3 | \psi(\theta, \phi)\rangle|^2 \left(\frac{1}{\sqrt{30}}\left(5Y_4^3 + Y_6^3\right)\right.$
$\frac{25+1}{30} = \frac{13}{15}$, as in choice (E).

Problem 29

Subject Type

Quantum Mechanics → *Bound State*

Tunneling should show exponential decay for a finite-potential well, and thus choice (E) is eliminated. Choice (C) is eliminated because the wave function is not continuous. One eliminates choice (D) because the bound-state wave functions of a finite well isn't linear. The wave function for a bound state should look similar to that of an infinite potential well, except because of tunneling, the well appears larger—thus the energy levels should be lower and the wave functions should look more spread out. Choice (B) shows a more-spread-out version of a wave function from the infinite potential well.

Problem 30

Subject Type

Quantum Mechanics → *Bohr Theory*

The ground state binding energy of positronium is half of that of Hydrogen. This is so because the energy is proportional to the reduced mass, and that of the positronium has a reduced mass of half that of Hydrogen.

Thus, from the Bohr formula, one has $E = Z^2 E_1/n^2$, where $E_1 = E_0/2$ and E_0 is the ground state energy fo Hydrogen.

Since $Z = 2$, then for $n = 2$, the energy is $E_1/4 = 3_0/8$, as in choice (E).

Problem 31

Subject Type

Atomic → *Spectroscopic Notations*

Spectroscopic notation is given by ${}^{2s+1}L_j$, and it's actually quite useful when one is dealing with multiple particles. $L \in (S, P, D, F)$, respectively, for orbital angular momentum values of $0, 1, 2, 3$. $s = 1/2$ for electrons. j is the total angular momentum.

Knowing the convention, one can plug in numbers to solve $3 = 2s + 1 \Rightarrow s = 1$. Since the main-script is a S, $l = 0$. The total angular momentum is $j = s + l = 1$.

Problem 32

Subject Type

Electromagnetism → *Circuits*

Power is related to current and resistance by $P = I^2 R$. The resistor that has the most current would be R_1 and R_{eq} (the equivalent resistance of all the resistors except for R_1), since all the other resistors share a current that is split from the main current running from the battery to R_1. Since $R_{eq} < R_1$, the most power is thus dissipated through R_1, as in choice (A).

Problem 33

Subject Type

Electromagnetism → *Circuits*

One can find the voltage across R_4 quite easily. The net resistance of all resistors except R_1 is $R_{eq} = ((1/R_3 + 1/R_4)^{-1} + R^5)^{-1} + 1/R_2 = 25\Omega$. Kirchhoff's Loop Law then gives $V = I(R_1 + R_{eq}) \Rightarrow I = 3/75A$.

Now that one knows the current, one trivially finds the voltage across R_2 to be $IR_2 = 1$ V. $I^{'}(R_{34} + R_5) = 1$, since the resistors are in parallel.

Since $R_{34} = 1/R_3 + 1/R_4 = 1/60 + 1/30 = 20\Omega$, the current $I^{'} = 1/(R_{34} + R_5) = 1/50$. The voltage across either R_3 or R_4 is just $1 - I^{'} R_5 = 1 - 30/50 = 0.4$, as in choice (A).

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GR9277 Solution

Solutions
Electromagnetism⇒}TEM Waves

The full formalism of a conducting cavity can be solved via TEM (transverse electromagnetic) wave guides. However, to solve this problem, one needs only the two boundary conditions from the reflection at a conducting surface, $\Delta E_{||}=0$and $\Delta B_{\perp}=0$.

The electric field parallel to the cavity is the transverse field, and thus one has choice (D), exactly the conditions above.

Optics⇒}Diffraction Grating

Diffraction gratings have the same formula as 2-slit interference, except each slit is (obviously) much smaller. The condition for maximum is given by $d\sin\theta=m\lambda$, relating the width of the slit to the wavelength and angle and order m.

The width of each slit is given by the grating $d=(2000 lines/cm \times 100 cm/m)^{-1}=0.5E-5m$. Thus, plugging in the wavelength one has $\sin\theta=\lambda/d=5200E-10/0.5E-5\approx 10000E-5=1E-1$.

Now, the approximations to get rid of the trig function. Since $\theta<<1$, one can approximate $\sin\theta\approx\theta$, where the angle is in *radians*. Now, convert the angle from radians to degrees. $1E-1\times 180^{\circ}/\pi=18/\pi\approx 18/3=6^{\circ}$, as in choice (B).

Electromagnetism⇒}Boundary Conditions
The conductor perfectly reflects the incoming wave, and none is transmitted. The electric field is thus reversed. However, since E and B are perpendicular (related to each other by the Poyting Vector where the direction of propagation is given by the direction of $\vec{E}\times\vec{B}$), the magnitude of B is increased by 2, but its direction stays the same.

Search on the GRE Physics Solutions homepage with keyword conductors for more on this.

Special Relativity⇒}Maximal Velocity
The maximal velocity of any object, even light itself, is the speed of light. Moreover, light always travels at light speed (c). This is true in all frames, and in fact, it is one of the two postulates of Special Relativity (the other being the equivalence of inertia frames).

There's no need to chunk out the addition of velocity formula for this. The only possibilities are choices (A) and (D). Since $\gamma_2$is emitted backwards, according to the coordinate system in the diagram, its velocity would be $-c\hat{k}$, as in choice (A).

Special Relativity⇒}Time Dilation Formula

The time dilation formula is given by $t=\gamma t_0=\frac{\Delta t_0}{\sqrt{1-\beta_{ij}^2}}$, where time is dilated (lengthened) in all but the frame at rest (proper-time t_0). Note that $\beta_{ij}=v_{ij}/c$.

So, from that alone, one can deduce the following relations (without looking at the choices yet):

$$\Delta t_2=\frac{\Delta t_1}{\sqrt{1-\beta_{12}^2}}$$

$$\Delta t_3=\frac{\Delta t_1}{\sqrt{1-\beta_{13}^2}}$$

The latter deduction is just choice (B).

Advanced Topics⇒}Fourier Series
There's no need to go through the formalism of integrating out the coefficients.

One can tell by inspection that the function is odd. Thus, one would use the Fourier sine series. This leaves choices (B) and (A).

Choice (A) is trivially zero since for all integer n, $\sin(n\omega t)=0$. Choice (B) remains.

Mechanics⇒}Centripetal Force
There is no tangential acceleration (since otherwise it would slide and not roll---the frictional force balances the forward acceleration force). However, there is a centripetal acceleration that pulls the particles back in a circle, as in choice (C). This acceleration propels the tangential velocity to continue spinning in a circle.

Mechanics⇒}Energy

The kinetic energy is related to the inertia I and angular velocity ω by $K=\frac{1}{2}I\omega^2$.

The problem supplies $I=4kgm^2$ so one needs not calculate the moment of inertia. The angular velocity starts at $80rad/s$ and ends at $40rad/s$.

Thus, the kinetic energy lost $\Delta K=\frac{1}{2}I(\omega_f^2-\omega_0^2)=\frac{1}{2}(4)(40^2-80^2)=2(1600-6400)=-9600J$, as in choice (D).

Mechanics⇒}Angular Kinematics
Kinematics with angular quantities is exactly like linear kinematics with

$x\rightarrow\theta$ (length to angle)

$a\rightarrow\alpha$ (linear acceleration to angular acceleration)

$v\rightarrow\omega$ (linear velocity to angular velocity)

$m\rightarrow I$ (mass to moment of inertia)

$F\rightarrow\tau$ (force to torque).

Thus, one transforms $v=v_0+at\Rightarrow\omega=\omega_0+\alpha t$.

Plugging in the given quantities, one gets $\alpha=\frac{\omega-\omega_0}{t}=\frac{40-80}{10}=-4rad/s^2$.

The torque is given by $\tau=I\alpha=-16Nm$, whose magnitude is given by choice (D).

Mechanics⇒}Lagrangians
Recall the Lagrangian equations of motion $\frac{\partial L}{\partial q}=\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}$.

If $\frac{\partial L}{\partial q}=0$ then $\frac{\partial L}{\partial \dot{q}}=constant$, since its time-derivative is 0.

One can relate energy to momentum from elementary considerations by $\frac{\partial L}{\partial \dot{x}}=m\dot{x}$, where L is the kinetic energy $0.5m\dot{x}^2$. Thus, the generalized momentum defined for a generalized coordinate is just $p_n=\frac{\partial L}{\partial \dot{x}}$.

From the above deductions, the generalized momentum is constant, as in choice (B).

(Incidentally, the ignorable or cyclic coordinate would be q_n and not p_n since it does not appear in the

Lagrangian.)

Mechanics⇒}Lagrangians

The kinetic energy, in general, is given by $T=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)$. The potential energy is just $V=mgy$. The Lagrangian is given by $L=T-V=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)-mgy$.

Now, given the constraint $y=ax^2$, one can differentiate it and plug it into the Lagrangian above to reexpress the Lagrangian in terms of just y, for example.

Differentiating, one has $\frac{d}{dt}(y=ax^2)\Rightarrow\dot{y}=a2x\dot{x}\Rightarrow\dot{x}=\dot{y}/(2ax)$. Square that to get $\dot{x}^2=\frac{\dot{y}^2}{4a^2x^2}=\frac{\dot{y}^2}{4ay}$, where one replaces the x^2through the given relation $y=ax^2$.

Plug that back into the Lagrangian above to get exactly choice (A).

Mechanics⇒}Conservation of Energy

Conservation of energy gives $mgh=\frac{1}{2}mv_0^2$, where $v_0$is the velocity of the ball before it strikes the ground. Thus, $v_0^2=2gh$.

Afterward, the ball bounces back up with $v'=0.8v$. Apply conservation of energy again to get $mgh'=\frac{1}{2}mv'^2\Rightarrow v'^2=2gh'\Rightarrow h'=v'^2/(2g)$.

Plugging in $v'^2=0.8^2v_0^2$, one has $h'=0.64h$, which is choice (D).

Prob 46 Solution

Thermodynamics⇒}Critical Isotherm

The critical isotherm is the (constant temperature) line that just touches the critical liquid-vapor region, explained in the next question. The condition for the critical isotherm is $\left(\frac{dP}{dV}\right)_c=0$ and $\left(\frac{d^2P}{dV^2}\right)_c=0$, where c denotes the critical point.

Prob 47 Solution

Thermodynamics⇒}Liquid-Vapor Equilibrium

The liquid-vapor region is where the substance can coexist as both a liquid and vapor. (A gas is just a vapor at normal temperatures.)

In this region, the liquid and vapor are in equilibrium, hence their coexistence. Equilibrium occurs when $P_v=P_l$ and $\mu_v=\mu_l$, i.e., when the pressure and chemical potential of the liquid and vapor are equal to each other.

Since region B shows a constant pressure behavior, despite the volume-decrease, it is the region of liquid-vapor equilibrium.

	Lab Methods⇒}Uncertainty The general equation for uncertainty is given by $\delta f/f=\sqrt{\sum_i(\frac{\delta x_i}{x_i})^2}$, where $f=\Pi_i x_i$ and δx_i is the generalized standard deviation of quantity x_i. So, for this case, one has $f=ma$ and thus its uncertainty is given by $\delta f/f=\sqrt{(\frac{\delta m}{m})^2+(\frac{\delta a}{a})^2}$. (Basically, one has $x_1=m$ and $x_2=a$.) This is choice (C).
	Advanced Topics⇒}Scintillator The maximal speed the muons can travel at is slightly less than c. Thus, since the distance is $x=3m$, the time required would be $c=x/t\Rightarrow t=x/c=1E-8$. The largest scintillator time is the one closest to this, which is 1 ns, as in choice (B).
	Quantum Mechanics⇒}Simultaneous Eigenstates QM in verse... Two operators, both alike in state functions, In fair bases, where we lay our scene, From ancient grudge break new mutiny... Two operators unlike in eigenvalues Yet star-crossed lovers commute. So anyway, the problem gives $A\|a\rangle=a\|a\rangle$ and $B\|a\rangle=\beta\|a\rangle$. That is, both A and B share the same eigenstate $\|a\rangle$. Consider . But, the scalar term is 0. This implies that in order for both operators to have the same eigenstate, $[A,B]=0$. (Also, one knows that $\langle a'\|a''\rangle=\delta_{','}$ by definition of orthonormal eigenfunctions.)
	Quantum Mechanics⇒}Momentum The momentum operator in position space is given by $p=\hbar/i\frac{\partial}{\partial x}$. Thus, given the wave function, one can calculate the expectation value as , since sine's and cosine's are orthogonal over a whole period.

The answer is thus (A).

Quantum Mechanics⇒}Orthonormality
$\langle\psi_m|\psi_n\rangle=\delta_{nm}$

This is the definition of orthonormality, i.e., something that is both orthogonal (self dot others = 0) and normal (self dot self = 1).

Quantum Mechanics⇒}Energy
If one forgets the energy of an infinite well, one can quickly derive it from the time-independent Schrodinger's Equation $-\frac{\hbar}{2m}\psi''+V\psi=E\psi$. However, since $V=0$ inside, one has $-\frac{\hbar}{2m}\psi''=E\psi$.

Plug in the ground-state wave function $\psi=A\sin(kx)$, where $k=n\pi/a$. Chunk out the second derivative to get $E=\frac{k^2\hbar^2}{2m}$. Plug in k to get $E=\frac{\hbar^2 n^2\pi^2}{2ma^2}$.

Note that $k=n\pi/a$ can be deduced from boundary conditions, i.e., the wave function vanishes at both ends ($\psi(0)=0$ and $\psi(a)=0$). The second boundary condition forces the n's to be integers. Since one can't have a trivial wave function, $|n|\geq 1$, and thus $n^2\geq 1$. One finds that $E\geq\frac{\pi^2\hbar^2}{2ma^2}$, since $n=1,2,3...$, as in choice (E).

Electromagnetism⇒}Faraday Law

The induced current would act, according to Lenz Law, to oppose the change. In this case, since the field is decreasing (the wire is being pulled away from the field), the induced current would act to increase the field. On the side closest to the long wire, it would thus point in the same direction as the current from the long-wire. This eliminates all but choices (D) and (E).

Now, since the rectangular loop wire cannot induce a force on itself, the force is due to the field from the long wire. To the left of the loop, the long wire has a field pointing into the page, and thus the force there is left-wards. One can check again that choice (E) is right by right-hand-ruling the field on the right side of the loop. Since the field due to the long wire is again into the page, the force here is towards the right (since the current runs down the page on the right side of the loop).

Electromagnetism⇒}Magnetic Force
The magnetic force of a wire is given by $\vec{F}=I\vec{l}\times\vec{B}$, where I is the current of the wire and l its length.

The field that produces the force on the loop is given by the long wire (see the previous problem for why). The field of that wire is given trivially by Ampere's Law to be $\vec{B}=\frac{\mu_0 I}{2\pi r}$, where r is the radial distance away from its center.

Only two wires from the loop contribute to the force, since the cross-product yields 0 force for the two horizontal components. Thus, the net force on the loop with current i with vertical components of length b is $|F_{left}+F_{right}|=|ib\frac{\mu_0 I}{2\pi}\left(\frac{1}{r}-\frac{1}{r+a}\right)|$. Combine the fraction to get choice (D).

Quantum Mechanics⇒}Simple Harmonic Oscillator
The energy of a simple harmonic oscillator is given by $E_n = (n+\frac{1}{2})h\nu$.

Thus, the ground state energy is simply $E_0 = h\nu/2$, as in choice (C).

Electromagnetism⇒}Faraday Law
Faraday's Law has the induced voltage is given by the change in magnetic flux, as $V = -\frac{dB \cdot A}{dt}$. (The minus sign shows that the induced voltage opposes the change.)

Since the induced voltage has to be periodic (as the half-circle rotates around A), choices (D) and (E) are immediately eliminated.

The voltage changes from positive to negative in regions where the change in flux is slowing down, goes to 0, then speeds up again. Thus, choice (C) is out.

The change in flux is constantly increasing as the loop spins into the field, and it is constantly decreasing as it spins out of the field. This is choice (A).

Atomic⇒}Orbitals
Eliminate (E) immediately since the superscripts do not add to 11. Each superscript stands for an electron.

Eliminate (B) because the s orbital can only carry 2 electrons.

Ground state means none of the electrons are promoted, and there are no states with unfilled gaps in them.

Eliminate (A) since it promotes the 2p electron to 3s, leaving a unfilled orbital of lower energy.

Eliminate (D) since it promotes the 3s electron to 3p, leaving an empty orbital of lower energy.

Choice (C) is it.

Atomic⇒}Orbital

The ground state of Helium has $1s^2$ which is $l=0\ n=1$.

However, because both electrons are in the same l and n state, the Pauli Exclusion Principle (no two electrons can have exactly the same quantum number) requires that one have $s=1/2$ and the other has $s=-1/2$ for a combined total spin of $s=0$, as in a spin singlet.

Thanks to user cakedamber for pointing this out.

(Compare with things in the p orbitals, which have $l=1$, allowing for $m_l=-1,0,1$.)

Electromagnetism⇒}Cyclotron Frequency
The cyclotron frequency is given by $F=qvB=mv^2/r\Rightarrow qB=mv/r=m\omega$, where one merely equates the Lorentz Force with the centripetal force using $v=r\omega$ to relate angular velocity with velocity.

So, $\omega=qB/m$. Plug in the quantities to get choice (D).

Mechanics⇒}Small Oscillations
One can derive the frequency of small oscillation for a rigid body in general by using the torque form of Newton's Laws: $\tau=I\ddot{\theta}=\vec{r}\times\vec{F}$. (I is moment of inertia, r is moment arm)

In this case, one has a constant downwards force $F=mg$, which acts at a moment arm angle θ. Thus, $I\ddot{\theta}=-rmg\sin\theta\approx-rmg\theta$, where the approximation works if $\theta<<1$.

The equation of motion for small angles is thus $\ddot{\theta}=-(mgr/I)\theta$. This is similar in form to that of a simple harmonic oscillator with the angular frequency being $\omega=\sqrt{mgr/I}$.

Now, the problem is to find the angular frequency for each system.

One needs not worry about the rod, since it is massless and has no moment of inertia.

The moment of inertia of system I is just $I_I=2mr^2$. The radius of gyration r is just $r_I=2r$ (an r for each mass).

The moment of inertia of system II is $I_{II}=mr^2+m(r/2)^2=\frac{5}{4}mr^2$. The radius of gyration r is just $r_{II}=r/2+r=3r/2$.

Thus $\omega_{II}/\omega_I=\sqrt{(r_{II}I_I)/(r_I I_{II})}=\sqrt{2\times 3r/2/(2(5/4)\times r)}=\sqrt{6/5}$, as in choice (A).

62 Thermodynamics⇒}Work
The work done by a gas in an isothermal expansion is related to the log of the volumes. If one forgets this, one can quickly derive it from recalling the definition of work $W=\int PdV$ and the ideal gas law equation of state $PV=nRT \Rightarrow P=nRT/V$.

One has $W=\int nRTdV/V=nRT\ln(V_1/V_0)$. For 1 mole, one has $n=1$, which yields choice (E).

(And the condition for isothermality $P_1V_1=P_0V_0=nRT_1=nRT_0$ allows one to change the argument in the log.)

63 Statistical Mechanics⇒}Maximal Probability
According to statistical mechanics, maximal probability is the sate of highest entropy---it's the peak of a Gaussian curve, the average score on a normally-curved test.

Spontaneous change to lower probability thus does not occur since maximal probability is the most stable state--one of highest entropy. Boltzmann's constant never approaches 0, however in the third law of thermodynamics, one has the entropy approaching 0 for $T\rightarrow 0$.

Eliminating choices, one has choice (D).

Electromagnetism⇒}Gauss Law
Gauss Law gives $\nabla\cdot\vec{E}=\rho/\epsilon_0$. Since the divergence of E in Cartesian coordinates is non-zero, there is a charge density in the region. QED

Electromagnetism⇒}Small Oscillations
The force on the charge in the center due to the charges on both sides is $F=\frac{2Qq}{4\pi\epsilon_0 R^2}$.

Small oscillations have a form $\ddot{x}=-\omega_0^2 x$, which comes from $m\ddot{x}=-kx$.

Thus, the Coulomb Force above gives $m\ddot{y}=-\frac{2Qqy}{4\pi\epsilon_0 R^3}$. Note the compensating R on the denominator to account for the y.

Thus, the angular frequency is given by (E).

Mechanics⇒}Work

Work is defined by $W = \vec{F} \cdot d\vec{l}$.

The force here is just due to gravity, thus $F = \rho y g$, where $\rho = 2kg/m$ is the density of the chain. The chain is wound upwards, so work is $W = \int_0^{10} \rho g y dy = \frac{2}{2}(gx^2)_0^{10} = 10 \times 100 = 1000J$, as in choice (C). (The approximation $g \approx 10m/s^2$ is made.)

Optics⇒}Polarized light

A plane-polarized wave has intensity $I \propto \cos^2\theta$, where θ is the angle from the wave to the polarization axis. (This is also known as Malus' Law.)

An unpolarized wave has intensity $I = const$.

Since ETS is generous enough to supply the intensity, one can easily deduce choice (C).

Optics⇒}Aperture Formula

The formula that relates the angle of an angular aperture to the wavelength and diameter is $\theta = 1.22\lambda/d$. Thus, $d = 1.22\lambda/\theta$. Plug in numbers to get (C).

Optics⇒}Speed of Light

The speed of light is related to the index of refraction by $n = c/v$. Thus, the minimal velocity the particle must have is $v = c/n = 2/3c$, since $n = 3/2$.

Special Relativity⇒}Gamma

$E = \gamma mc^2 = 100mc^2$, where ETS supplies the total energy to be 100 times the rest energy. Thus, $p = \gamma mv = 100mv$, but since $\gamma = 100 = \frac{1}{1-\beta^2}$, where $\beta = v/c$, one has $v \rightarrow c$, as in choice (D).

71 Statistical Mechanics⇒}Distributions

The Fermi-Dirac distribution, in general, gives the number of states in E_i to be $N_{FD} = N_0 \frac{1}{1+e^{-E_i/kT}}$, where N_0 is the total number of states. (The Fermi-Dirac distribution is used since there are only two states.)

Define $E_1 = \epsilon$ and $E_2 = 2\epsilon$.

The number of states in 1 is just $N_1 = N_0 \frac{1}{1+e^{-E_1/kT}} = N_0 \frac{1}{1+e^{-\epsilon/kT}}$, which is choice (B).

72 Statistical Mechanics⇒}Heat Capacity

The heat capacity is just dU/dT, where ETS generously supplies U, the internal energy. Since E_1 and N_0 are constants, the first term is trivial.

The temperature-derivative of the second term is $N_0 \epsilon^2/(kT^2) e^{\epsilon/kT}/(1+e^{\epsilon/kT})^2 =$, as in choice (A).

(The temperature derivative is easily done if one applies the chain-rule $\frac{df}{dy}\frac{dy}{du}\frac{du}{dT}$ where $f=1/y$, $y=1+e^u$, $u=\epsilon/(kT)$.)

73 Statistical Mechanics⇒}Entropy

The third law of thermodynamics says that $S(T\rightarrow 0)\rightarrow 0$.

Also, the statistical definition of entropy is just $S=Nkln Z$, where Z is the partition function. For this problem, one has $Z=e^{-\epsilon/kT}+e^{-2\epsilon/kT}$. For high temperatures, one has $Z\rightarrow 1+1=2$, since $e^x\approx 1+x$ for small x (and then $1+x\approx 1$ for very small x).

Thus the entropy behaves as in choice (C).

Mechanics⇒}Small Oscillations

The small oscillations of the hoop has the same frequency as that of a simple pendulum. Thus, $\omega^2=\frac{g}{l}$. However, in this case, l is the distance from the center of mass to the oscillation point---which is just the radius of the loop.

Since $\omega=2\pi/T$, the period $T\propto\sqrt{\frac{r}{g}}\propto\sqrt{r}$ does not depend on mass.

Since $r_x=4r_y$. The ratio of periods is $T_X/T_Y T/\sqrt{r_y}T/\sqrt{r_x/4}2$. Thus, the period of Y is just $T/2$. (Note, the technique of leaving out constants requires that mimeTeX failed to render your expression's are used instead of $=$'s. Practice a few times with this technique, as this will save time on the actual exam.)

Advanced Topics⇒}Binding Energy

The binding energy for heavy atoms ($>200 nucelons$) is about $8MeV/nucleon$. The change in binding energy is the kinetic energy, thus the Helium atom has a kinetic energy of $(235-231)8MeV$. (The binding energy of He is ignored.) This is much larger than the kinetic energy of the He nucleus.

Atomic⇒}Orbitals

The total angular momentum is given by $j=l+s$ where l is the orbital angular momentum and s is the spin angular momentum. (Note that, to an extent, l and s can be viewed as magnitudes, while m_l and m_s as directions.)

The total orbital angular momentum is just $0+1+1$, since one should recall that $(s,p,d,f)\in(0,1,2,3)$.

The spin angular momentum is just $1/2+1/2+1/2$ because one has three electrons.(Electrons are fermions that have spin $1/2$.)

Thus, the total angular momentum is $j=2+3/2=7/2$, as in choice (A).

Quantum Mechanics⇒}Gyromagnetic Ratio

The intrinsic magnetic moment is defined in terms of the gyromagnetic ratio and spin as $\vec{\mu}_s=\gamma\vec{S}$, where $\gamma=\frac{eg}{2m}$ (g is the Lande g-factor).

Thus, one sees that the magnetic moment is inversely related to mass.

The ratio of the magnetic moment of a nucleus to that of an electron is $\mu_n/\mu_e=m_e/m_n<<1$, as in choice (E). (One can cancel out the S since ETS is nice enough to have the nucleus have the same spin as the electron.).

Problem 78 Solution

Mechanics⇒}Multiple Particles

The angular momentum equation gives the angular frequency. $L=I\omega=m\vec{r}\times\vec{v}$, which relates the angular momentum to the moment of inertia, the angular velocity, the radius of gyration and the linear velocity.

The system spins about its center of mass, which is conserved. Since the pole is massless and the skaters are off the same mass, $r_{cm}=b/2$. The moment of inertia of the system is just $I=2mr_{cm}^2=2m(b/2)^2$.

Thus, the angular momentum equation gives $I\omega=m\vec{r}\times\vec{v}\Rightarrow 2m(b/2)^2\omega=mb/2(v+2v)\Rightarrow 2(b/2)\omega=3v\Rightarrow\omega=3v/b$, since the cross-products point in the same direction. Now that one has the angular velocity, one eliminates all but choices (B) and (C).

Now, take the time-derivative of x for choices (B) and (C), then evaluate it at $t=0$.

For B, one has $dx/dt=v+1.5v\cos(3vt/b)\rightarrow 2.5v$ for $t=0$.

For C, one has $dx/dt=0.5v+1.5v\cos(3vt/b)\rightarrow 2v$ for $t=0$.

Since the top skater is initially at $v(0)=2v$, only choice (C) has the right initial condition. Choose choice (C).

(FYI: The center of mass velocity is given by $v_{cm}=\frac{2mv-mv}{2m}=v/2$. One can also arrive at (C) by noting conservation of center of mass velocity, since there is no net force.)

Wave Phenomena⇒}Group Velocity

Recall that the group velocity is given by $v_g=\frac{d\omega}{dk}$ and the phase velocity is given by $v_p=\omega/k$.

In the region between k_1 and k_2, the derivative is a constant negative quantity (approximately just the derivative of a line with negative slope). However, ω/k is positive in this region. Thus, the phase and group velocity are traveling in opposite directions. Thus, choose choice (A).

Quantum Mechanics⇒}Planck Energy
The key equation is $E = hc/\lambda$. Since $hc = 1.24E-6eVm$and $E = 25E3eV = 2.5E4eV$, one can immediately plug the quantities in to solve for $\lambda = hc/E = 1.24E-6/2.5E4 = 0.5E-10$, which is just choice (B).

No knowledge of X-rays required, other than the elementary knowledge that it's a electromagnetic wave and allows one to write the Planck energy as $E = hf = hc/\lambda$.

Electromagnetism⇒}Resonant Frequency
The maximum steady-state amplitude (after transients die out) occurs at the resonant frequency, which is given by setting the impedance of the capacitor and inductor equal $X_C = X_L \Rightarrow \frac{1}{\omega C} = \omega L \Rightarrow \omega^2 = 1/(LC)$, as in choice (C).

Mechanics⇒}Torque
The problem gives $H = \int \tau dt = I\alpha t$, but $\omega = \alpha t$. Thus, $\omega = H/I$.

The moment of inertia of a plate about the z-axis is just $1/3Md^2$. Plug this into ω to get choice (D).

Electromagnetism⇒}Forces
Sum of the forces for one of the mass in the x (horizontal) and y (vertical) directions gives,
$\sum F_x = 0 = T\sin\theta - kq^2/d^2$
$\sum F y = 0 = T\cos\theta - mg
$
For small angles, $\cos\theta \approx 1 \Rightarrow T \approx mg$. From the geometry, one can deduce that $\sin\theta = (d/2)/L$.

Thus, the x equation yields $T(d/2)/L = kq^2/d^2 \Rightarrow d^3 = 2kq^2L/(mg)$ (since $T \approx mg$from the y equation for small angles). This is choice (A).

Electromagnetism⇒}Accelerating Charges
Elimination time:

(A) By the Larmor formula, one has $E \propto q^2 a^2$, where q is the charge and a is the acceleration. Since the charge is constant, this choice is true.

(B) This is also true by Larmor's formula.

(C) True. The energy radiated through a perpendicular unit area is given by the Poynting vector. $\vec{S} \propto \vec{E} \times \vec{B} \propto \vec{E} \times (\hat{r} \times \vec{E}) \propto E^2 \hat{r} - (\hat{r} \cdot \vec{E})\vec{E}$ and $E^2 \propto 1/r^2$ far away. Also, less rigorously, one can arrive at the same result from recalling the surface area of a sphere, $4\pi r^2$, and thus any term in $\vec{S}$ proportional to $1/r^2$ will yield a finite, thus acceptable answer. (Thanks to the user astro_allison for this pointer. See p460ff of Griffiths, Introduction to Electrodynamics, 3rd Edition for more details.)

(D) False. It's a minimum in the plane.

(E) True, since far from the electron the field behaves as plane waves, with E_z and B_z both 0.

Special Relativity⇒}Momentum
Given a total energy of $\gamma mc^2 = 1.5 MeV$ and the rest mass of the electron to be $m_e = .5 MeV/c^2$, one can figure out $\gamma = 3$.

The momentum is given by $p = \gamma m v = 3mv = 3v/2 (MeV/c^2)$.

Solve for the velocity from . Thus, the velocity is $v = 2c/3$.

Plugging this into the equation for momentum, one gets $p = 3/2 \times \sqrt{8/9} = \sqrt{2}$, and thus its momentum is about 1.4, as in choice (C).

Lab Methods⇒}Oscilloscopes
The discharge of the capacitor after it has been charged to V_0 is just $V(t) = V_0(1 - e^{-\omega t})$, where $\omega = 1/(RC)$. One can find C by knowing R and the sweep rate, which is related to t.
(Solution due to David Latchman.)

Mechanics⇒}Conservation of Energy
Do not immediately try applying the Virial Theorem to this one. Instead, consider conservation of energy. Coming in from far away, the particle has $E = V = 0$ the total energy equal to the potential energy equal to 0.

Alternatively, one has, for a circular orbit, the equality between centripetal force and the attractive force, $mv^2/r = K/r^3 \Rightarrow v^2 = K/(mr^2)$.

Thus, the kinetic energy is just $T=p^2/(2m)=K/(2r^2)$.

Since $V=-\int F dr=-K/(2r^2)$, where the extra negative sign for the potential energy is due to an attractive potential.

Thus, the total energy $E=T+V=0$, which is choice (C).

(Alternate solution is due to user crichigno.)

Electromagnetism⇒}Capacitors
From the problem and the basic relation for capacitors $Q=CV$, one immediately deduces that the initial charge is $Q_0=C_0V_0$ and the final charge is $Q_f=\kappa C_0V_0$, and thus choice (C) is out.

The potential is constant $V_f=V_0$, and thus choices (A) and (B) are out.

The electric field for a parallel plate capacitor is given by $E=\sigma/\epsilon$. Since $E_0=\sigma_0/\epsilon_0$ and $E_f=\sigma_f/(\kappa\epsilon_0)$, the final field is the same as the initial field. (To wit: $\sigma=Q/A\Rightarrow\sigma\propto Q$.) Thus, (D) is false. (Thanks to the user whose alias is "poop" for pointing this out.)

From the definition of $D=\epsilon_0E_0+P=\epsilon E_0$, one has $D_0<D_f$, since $\epsilon_0<\kappa\epsilon_0$. Choice (E) is right.

Quantum Mechanics⇒}Symmetry
There is now a node in the middle of the well. By symmetry, the ground state will disappear $n=0$, as well all the even n states. Thus, the remaining states are the odd states, as in choice (E).

Quantum Mechanics⇒}Rotational Energy Level
Rotational energy is related to angular momentum by $E_{rot}=L^2/(2I)$. Quantum mechanics quantizes the angular momentum $L^2=\hbar^2 l(l+1)$. Thus, [illegible].

The approximate spacing between two levels is given by [illegible].

The moment of inertia of H_2 is just that of two point-masses rotating about a center-point, thus $I=2mr^2$, taking $r=0.5E-10m$ and $m=E-27kg$ (mass of proton). $I\approx E-47kgm^2$.
Now, $\hbar^2\approx(6E-34)^2/(6)^2=E-68$. Plug everything in to get the right answer. $\hbar^2/I\approx E-68/E-47=E-21J$.
Converting J to eV, one has $E-21J/E-19=E-3$ eV, as in choice (B).

Advanced Topics⇒}Strangeness
Elimination time:

(A) Only muons, neutrinos and electrons are leptons. Moreover, the pi-meson is a meson, which is a hadron with baryon number 0. (Hadrons interact with the strong nuclear force, while leptons interact with the weak nuclear force, em force, and possibly even the gravitation force.)

(B) The lambda has spin 1/2, as do most baryons. (The mesons have spin 0, but positive strangeness numbers.)

(C) Lepton number is already conserved, since none of the particles involved have non-zero lepton numbers. Thus, introducing a neutrino would violate (electron) lepton number conservation.

(D) No reason why...

(E) Only hadrons have non-zero strangeness (strangeness was proposed when strong particles interact as if weak particles---i.e., instead of having super-fast decay times characteristic of strong-force particles, their decay times appeared as if weak-force decays). Protons have 0 strangeness, as do pi-mesons, even though they are both hadrons. However, the lambda has -1 strangeness. Thus, strangeness is not conserved.

Electromagnetism⇒}Frequency
A three-pole magnet should produce three voltage peaks, and thus the frequency is 30 Hz. (Solution due to David Latchman.)

Mechanics⇒}Boundary Condition
Getting low on time, one should begin scoring points based more of testing strategy than sound rigorous physics. At the initial release point, the acceleration is due to gravity and the tension is 0 (no centripetal acceleration). The only choice that gives $a(\theta=0)=g$ is choice (E).

Mechanics⇒}Boundary Condition
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Special Relativity⇒}Lorentz Transformation

Lorentz transformations are given by

$$x'=\gamma(x-vt)$$

$$t'=\gamma(t-vx/c^2)$$

Factoring out the terms, choice (C) is $x=5/4(x-3/5t)$, and thus $\gamma=5/4$and $v=3/5$. Since the equation for t fits the form above, this is a valid Lorentz Transformation.

Advanced Topics⇒}Dimensional Analysis
The final units must be $cm^2/steradians$. One is given

$10^{12} protons/s$

$10^{20} nuclei/cm^2$

$10^2 protons/s$

$10^{-4} steradians$

The combination $(10^2/10^{20})(1/10^{20})(1/10^{-4})$gives the right units as well as answer choice (C).

Optics⇒}Interferometer
A (effective) path change of λ produces a fringe shift. Thus, the interferometer formula is similar to the interference formula at normal incident, $m=2\frac{\Delta L}{\lambda}$.

Thus, $m=2(dn/\lambda-d/\lambda)=2d/\lambda(n-1)$. Thus, $n=\frac{m\lambda}{2d}+1=1.0002$, as in choice (C).

See GR0177.100 on the same site for more info.

Advanced Topics⇒}Solid State Physics
This is a result one remembers by heart from a decent solid state physics course. It has to do with band gaps, which is basically the core of such a course.

Then again, one can easily derive it from scratch upon recalling some basic principles: $E=p^2/(2m)=\hbar^2k^2/(2m)$, $p=\hbar k=mv$, where k is the wave vector, E is the energy, m is the mass, and p is the momentum.

From the above, one has $dv/dt=\frac{1}{m}\frac{dp}{dt}=\frac{\hbar}{m}\frac{dk}{dt}$.

Set the two dv/dt's equal to get $\frac{\hbar}{m}\frac{dk}{dt}=\frac{1}{\hbar}\frac{d^2E}{dtdk}$. Cancel out the dt's to get $\frac{\hbar^2}{m}dk=\frac{dE}{dk}\Rightarrow m=\hbar^2/(\frac{dE}{dk})$, after differentiating with respect to k on both sides.

Alternatively, one can try it Kittel's way:

Start with $\hbar v_g=dE/dk$. Then, $dv_g/dt=\hbar^{-1}(d^2E/dt^2dk/dt)=\hbar^{-1}(d^2E/dt^2F/\hbar)$. Thus, the effective mass is defined by $F=\hbar^2/(d^2E/dk^2)dv_g/dt=mdv_g/dt$.

Quantum Mechanics⇒}Characteristic Equation
The characteristic equation of the matrix solves for the eigenvalues. It is $-\lambda(\lambda^2)+1=0$. Not all solutions are real, since $\lambda=e^{2i\pi/3}=\cos(2\pi/3)+i\sin(2\pi/3)$, where Euler's relation is used.

Quantum Mechanics⇒}Perturbation Theory
The perturbed Hamiltonian is given by $\Delta H=eEz=eEr\cos\theta$, where the last substitution is made for spherical coordinates.

The first-order energy-shift is given by $\langle\psi_0|\Delta H|\psi_0\rangle$, where $\psi_0\propto e^{-kr}$.

$dV=r^2drd\Omega=r^2\sin^2\theta d\phi d\theta dr$.

Thus, $E_0=\int_0^\infty\int_0^\pi\int_0^{2\pi}e^{-kr}\cos\theta\sin^2\theta d\theta d\phi=0$, since the integral of $\cos\theta\sin^2\theta$ over 0 to π is 0.

Search this site for keyword perturbation for more on this.

Mechanics⇒}Sum of Moments
Take the sum of the moments (or torque) about the triangular pivot fulcrum and set it to 0.

$\sum M=20gd+20gg-40gx=0$, where d is the distance from the fulcrum to the 20kg mass, x is the distance from the fulcrum to the 40kg mass and q is the distance from the fulcrum to the center of mass of the rod.

From conservation of length, one also has $d+x=10$ and $q+x=5$.

Plug everything into the moment equation. Shake and bake at 300 K. Solve for q to get choice (C).

Alternatively, one can solve this problem in one fell swoop. Taking q as the distance from the fulcrum to the center of mass of the rod, one sums the moment about the fulcrum to get $\sum M = 20gq+20g(5+q)-40(5-q)g=0$. Solve for q to get choice (C). (This is due to the user astro_allison.)