Homework 03 - Matrix Inversion

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ITESM Campus Monterrey
Mathematical Physical Modelling F4005
Mathematica problem sheet 1
Due Date: February 5-2019, 23:59.
Professor: Ph.D Daniel López Aguayo

Name and ID: ______________________________

Instructions: this is an individual assignment. The solutions to this assignment must be typed and must include all the Mathematica input or output, you must send them to dlopez.aguayo@tec.mx. You are not allowed to ask questions, but feel free to use the web to read any documentation. The purpose of this activity is to **develop your research skills and motivate you to persevere**. We will discuss the solutions next week (after the deadline). No late homework will be accepted.

$\boxed{1}$ Construct a matrix A of size 10×10 whose first row consists of all the integers in $[1, 10]$; the second row consists of all the integers in $[11, 20]$, and so on. For this exercise you will need to read carefully about the commands *Table* and *Range*. Then, use Mathematica to find $a_{4,9}$.

$\boxed{2}$ Use Mathematica to compute the inverse of A, where A is the matrix of the previous exercise. In case A is not invertible explain how can you prove it mathematically without many computations.

$\boxed{3}$ Use Mathematica to create the following matrix B of size 30×30

$$\begin{bmatrix} \pi & 0 & 0 & \cdots & 0 \\ 0 & \pi & 0 & \cdots & 0 \\ 0 & 0 & \pi & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & \cdots & \pi \end{bmatrix}$$

(a) Is this an invertible matrix? Verify with Mathematica and also explain your answer mathematically.
(b) Use Mathematica to compute the inverse of B and also explain how can you compute the inverse of B by hand.
(c) Compute the rank of B.

$\boxed{4}$ Use Mathematica to create the following matrix C of size 16×16

$$\begin{bmatrix} \pi & 0 & 0 & \cdots & 0 & 0 \\ 0 & \pi+1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \pi+2 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \pi+3 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \pi+14 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \pi+15 \end{bmatrix}$$

(a) Is this an invertible matrix? Verify with Mathematica and also explain your answer mathematically.
(b) Use Mathematica to compute the inverse of C and also explain how you can compute the inverse of C by hand.
(c) Without any computations, how would you find the reduced row echelon form of C? Once done this, compute the rank of C and verify both answers with Mathematica.

$\boxed{5}$ Mathematica is also capable of computing formulas, not only specific calculations. For instance, suppose a user wants to compute the inverse of

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(a) Save this matrix and compute its inverse.
(b) What is the error behind the output given by Mathematica? Try to see if an inverse exists when $a = 2, d = 2, b = 1, c = 4$.

The point of this exercise is that you have to be careful with the output given by computers, sometimes they made additional assumptions.

$\boxed{6}$ Suppose we want to solve the system

$$\begin{cases} x - y = 3 \\ \pi \cdot x - \pi \cdot y = 3\pi \end{cases}$$

(a) Use the *Solve* command in Mathematica and analyze the output.
(b) Read about the *Plot* command and graph both equations.
(c) Using the above plot, deduce the number of solutions.
(d) Use matrix inversion and the theorem we saw in class to analyze the number of solutions.

$\boxed{7}$ Suppose we want to solve the system

$$\begin{cases} x + y + z = 10 \\ \frac{2}{3}x + \frac{2}{3}y + \frac{2}{3}z = 11 \\ \frac{5}{9}x + \frac{5}{9}y + \frac{5}{9}z = 12 \end{cases}$$

(a) Read about the *Plot3D* command and plot the above system in Mathematica.
(b) Read about the term **normal vector** to a plane and include the definition. Using this concept, how can we infer that the system is inconsistent?
(c) Use Mathematica to find the solution of the system.
(d) Let A be the coefficient matrix and let $[A|B]$ be the augmented matrix. Using only your logic, how can we know the reduced row echelon form of A without any computations? Find the rank of A.
(e) Verify with Mathematica the above answer.
(f) What is the reduced row echelon form of $[A|B]$? Compute the rank of $[A|B]$.
(g) Apply the theorem given in class to determine whether the system is consistent or inconsistent (i.e compare ranks).

1 Answer to Problem I

$\mathrm{SQRmatrixSize} = 10;$

$\mathrm{matrixAentries} = \mathrm{Range}[1, \mathrm{SQRmatrixSize} * \mathrm{SQRmatrixSize}];$

$\mathrm{matrixAentries} = \mathrm{ArrayReshape}[\mathrm{matrixAentries}, \{\mathrm{SQRmatrixSize}, \mathrm{SQRmatrixSize}\}];$

$\mathrm{matrixA} = \mathrm{Table}[\mathrm{matrixAentries}[[i, j]], \{i, 1, \mathrm{SQRmatrixSize}\}, \{j, 1, \mathrm{SQRmatrixSize}\}];$

$\mathrm{MatrixForm}[\mathrm{matrixA}]$

$$\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\
21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 \\
31 & 32 & 33 & 34 & 35 & 36 & 37 & 38 & 39 & 40 \\
41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 & 49 & 50 \\
51 & 52 & 53 & 54 & 55 & 56 & 57 & 58 & 59 & 60 \\
61 & 62 & 63 & 64 & 65 & 66 & 67 & 68 & 69 & 70 \\
71 & 72 & 73 & 74 & 75 & 76 & 77 & 78 & 79 & 80 \\
81 & 82 & 83 & 84 & 85 & 86 & 87 & 88 & 89 & 90 \\
91 & 92 & 93 & 94 & 95 & 96 & 97 & 98 & 99 & 100
\end{pmatrix}$$

2 Answer to Problem II

matrixA is singular, therefore Inverse[matrixA] does not exist.

3 Answer to Problem III

```
SQRmatrixSize = 30;
matrixB = π * IdentityMatrix[SQRmatrixSize];
MatrixForm[matrixB]
```

$$
\left(\begin{matrix}
\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi
\end{matrix}\right.
$$

3.a & 3.b
Yes, "A triangular matrix (upper, lower or diagonal) is invertible if and only if no element on its main diagonal is 0."

"by hand" ...
Since, Inverse[A] equals diag($\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$), Inverse[matrixB] is equal to $\frac{1}{\pi}$IdentityMatrix[30]

Verification with Mathematica is shown below ...

Inverse[matrixB];

MatrixForm[%]

$$
\left(\begin{array}{cccccccccccccccccccccccccccc}
\frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.
$$

3.c

MatrixRank[matrixB]

30

4 Answer to Problem IV

$\mathrm{SQRmatrixSize} = 16;$

$\mathrm{matrixC} = \mathrm{Table}[0, \{i, 1, \mathrm{SQRmatrixSize}\}, \{j, 1, \mathrm{SQRmatrixSize}\}];$

$\mathrm{rows} = \mathrm{Dimensions}[\mathrm{matrixC}][[1]];$

$\mathrm{cols} = \mathrm{Dimensions}[\mathrm{matrixC}][[2]];$

$\mathrm{For}[i = 1, i \leq \mathrm{rows}, i{+}{+},$

$\mathrm{For}[j = 1, j \leq \mathrm{cols}, j{+}{+},$

$\mathrm{If}[i == j,$

$\mathrm{matrixC}[[i, j]] = (\pi + i) - 1,$

$\mathrm{matrixC}[[i, j]] = 0];$

$];$

$];$

$\mathrm{MatrixForm}[\mathrm{matrixC}]$

$$\left(\begin{array}{cccccccccccccc}
\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 7+\pi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8+\pi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9+\pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10+\pi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11+\pi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12+\pi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13+\pi \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

4.a & 4.b
Yes, "A triangular matrix (upper, lower or diagonal) is invertible if and only if no element on its main diagonal is 0."

"by hand" ...
Since, Inverse[A] equals diag($\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$), Inverse[matrixB] is equal to $\frac{1}{(\pi+i)-1}$IdentityMatrix[16]

Verification with Mathematica is shown below ...

Inverse[matrixC];

MatrixForm[%]

$$\begin{pmatrix}
\frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{1+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{3+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{4+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{5+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{7+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9+\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{10+\pi} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{11+\pi} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{12+\pi} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{13+\pi} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{14+\pi} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{15+\pi}
\end{pmatrix}$$

4.c
Multiplying matrixC times the inverse of matrixC. However, since we know that Inverse[matrixC] exists, the reduced echelon form is the identity matrix (of size 16x16).

MatrixRank[matrixC];

RowReduce[matrixC];

MatrixForm[%]

$$
\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}
$$

5 Answer to Problem V

5.a

$$\textbf{matrixD} = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix};$$

Inverse[matrixD];

MatrixForm[%]

$$\begin{pmatrix} \frac{d}{-bc+ad} & -\frac{b}{-bc+ad} \\ -\frac{c}{-bc+ad} & \frac{a}{-bc+ad} \end{pmatrix}$$

5.b

When a = 2; d = 2; b = 1; c = 4, Inverse[matrixD] does not exist. Mathematica outputs the following error:

$\cdots$ Inverse: Matrix {{2,1},{4,2}} is singular

6 Answer to Problem VI

6.a

```
Solve[{
x - y == 3,
x * π - y * π == 3 * π
}, {x, y}]
```

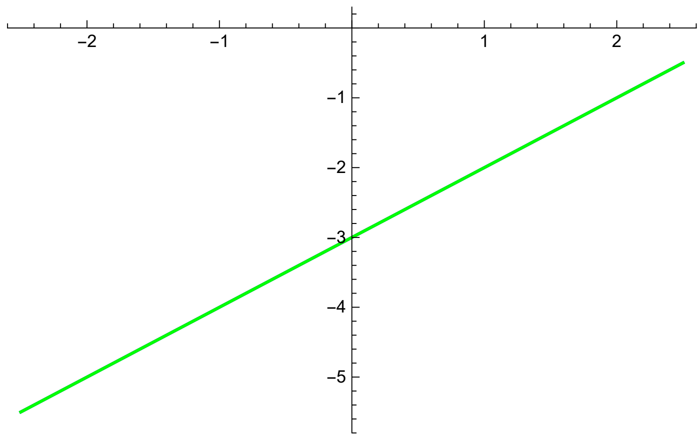
Solve : Equations may not give solutions for all “solve” variables.

$\{\{y \to -3 + x\}\}$

6.b

```
contourLimits = 2.5;
(*
ContourPlot[{
x - y == 3,
x * π - y * π == 3 * π
}, {x, 0.5, 1.0}, {y, -2.0, -2.5}, ContourStyle → {Blue, Orange}]
*)
Show[{
Plot[x - 3, {x, -contourLimits, contourLimits}, PlotStyle → Blue],
Plot[x - 3, {x, -contourLimits, contourLimits}, PlotStyle → Green]
}, PlotRange → All, AxesOrigin → {0, 0}]
```

Out[ ]=



6.c
The system has infinitely many solutions.

6.d

$\text{matrixA} = \begin{pmatrix} 1 & -1 \\ \pi & -\pi \end{pmatrix};$

$\text{matrixB} = \begin{pmatrix} 3 \\ 3*\pi \end{pmatrix};$

```
(*rows = m = i*)(*cols = n = j*)
getNumOfSolutions[varA_, varB_]:=Module[{vA = varA, vB = varB, vAB, Arank, ABrank, n},
vAB = ArrayFlatten[{{vA, vB}}];
Arank = MatrixRank[vA];
ABrank = MatrixRank[vAB];
n = Dimensions[vA][[2]];
If[Arank < ABrank,
Print["The system has no solution"];,
If[Arank == ABrank&&Arank < n&&ABrank < n,
Print["The system has infinitely many solutions"];,
If[Arank == ABrank&&Arank == n,
Print["The system has a unique solution"];,
Print["[ERROR] The system has ? solution(s)\n
Beware, if the system is homogeneous (varB is a zero vector and n>m (more variables than
equations), then the system has infinitely many solutions)"];
];
];
];
];
getNumOfSolutions[matrixA, matrixB]
```

The system has infinitely many solutions

7 Answer to Problem VII

7.a

contourLimits = 5;

(*

ContourPlot3D[{

$\boldsymbol{x + y + z == 10,}$

$\boldsymbol{\frac{2}{3} * x + \frac{2}{3} * y + \frac{2}{3} * z == 11,}$

$\boldsymbol{\frac{5}{9} * x + \frac{5}{9} * y + \frac{5}{9} * z == 12}$

},

{x, −contourLimits, contourLimits},

{y, −contourLimits, contourLimits},

{z, −contourLimits, contourLimits}, Axes → True, PlotLegends → “Expressions”]

***)**

Show[{

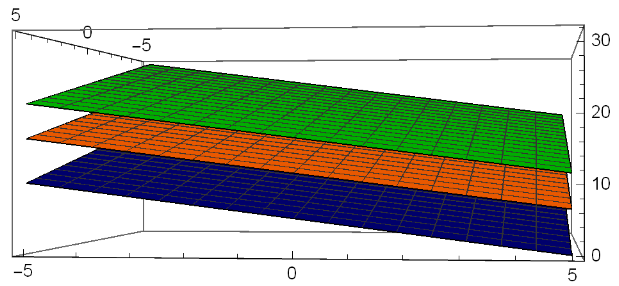
Plot3D[10 − x − y, {x, −contourLimits, contourLimits}, {y, −contourLimits, contourLimits}, PlotStyle → B

Plot3D $\left[\frac{1}{2}(33 - 2x - 2y)\right.$, {$x$, −contourLimits, contourLimits}, {y, −contourLimits, contourLimits}, PlotStyl

Plot3D $\left[\frac{1}{5}(108 - 5x - 5y)\right.$, {$x$, −contourLimits, contourLimits}, {y, −contourLimits, contourLimits}, PlotSty

}, PlotRange → All, AxesOrigin → {0, 0}]

Out[ ]=



7.b
7.b.1 **normal vector:** A nonzero vector that is perpendicular to the plane is called a normal vector to the plane.
7.b.2 The system is inconsistent because the three normal vectors point to the same direction.

7.c

Solve[{

$\boldsymbol{x + y + z == 10,}$

$\frac{2}{3}*x+\frac{2}{3}*y+\frac{2}{3}*z==11,$

$\frac{5}{9}*x+\frac{5}{9}*y+\frac{5}{9}*z==12$

}, {x, y, z}]

{}

7.d & 7.e

The reduced row echelon of A will be $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, since all the entries are the same within each row. MatrixRank[A]=1

Verification with Mathematica is shown below ...

$$\textbf{matrixA} = \begin{pmatrix} 1 & 1 & 1 \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{5}{9} & \frac{5}{9} & \frac{5}{9} \end{pmatrix};$$

$$\textbf{matrixB} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix};$$

matrixAB = ArrayFlatten[{{matrixA, matrixB}}];

RowReduce[matrixA]

MatrixForm[%]

MatrixRank[matrixA]

{{1, 1, 1}, {0, 0, 0}, {0, 0, 0}}

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1

7.f

RowReduce[matrixAB];

MatrixForm[%]

MatrixRank[matrixAB]

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2

7.g

getNumOfSolutions[matrixA, matrixB]

The system has no solution