Mathematica Problem Sheet 01

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ITESM Campus Monterrey
Mathematical Physical Modelling F4005
HW4: Linear transformations I
Due Date: February 17-2019, 23:59 hrs.
Professor: Ph.D Daniel López Aguayo

Full names of team members: ____________________

Instructions: Please write neatly on each page of your homework and send it in pdf format to dlopez.aguayo@tec.mx. Typed solutions in LaTeX (only) will be given extra credit; no late homework will be accepted. Each team should consist (of at most) 5 students.

$\boxed{1}$ Consider the map given by $T : \mathbb{R}^3 \to \mathbb{R}$ given by $T(x, y, z) = \frac{1}{(x-2)^2+(z-2)^2+(y-2)^2}$.

(a) Find the domain of T and plot the subset of $\mathbb{R}^3$ that represents the domain.

(b) Is this a linear transformation? justify carefully your answer.

$\boxed{2}$ Consider the map $W : \mathbb{R}^2 \to \mathbb{R}$ given by $W(a, b) = \frac{1}{sin(\frac{a}{2})} + \frac{22222}{\sqrt{b-1}}$.

(a) Find the domain of W.

(b) Is this a linear transformation? justify carefully your answer.

$\boxed{3}$ Consider the map $T : \mathbb{R} \to \mathbb{R}^2$ given by $T(x) = (\frac{sinx}{\pi \cdot e}, \frac{cosx}{\pi \cdot e})$. Prove (mathematically) that the range of T is a circle and find its radius and center.

$\boxed{4}$ Let $N : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $N(a, b) = (a - b, 3b - 3a)$. Compute, mathematically, the range of N and plot it.

$\boxed{5}$ Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x, -y)$.

(a) Find the domain of T.

(b) Prove that T is a linear transformation (**verify both properties**).

(c) Plot some points and deduce the range of T.

(d) What is the geometric interpretation of T? Is it any reflection? What kind?

(e) **Optional**. How can you infer the range of T by using Mathematica? *Hint*: Make use of the *ListPlot* and the *Table* commands, together with a list with two parameters.

$\boxed{6}$ Is the map $P : \mathbb{R} \to \mathbb{R}^2$ given by $P(y) = (y, 0)$ linear? prove in detail your answer.

$\boxed{7}$ Is the map $M : \mathbb{R}^2 \to \mathbb{R}$ given by $M(x, y) = x + y + 2$ linear? prove in detail your answer.

$\boxed{8}$ Is the map $Q : \mathbb{R}^2 \to \mathbb{R}^3$ given by $Q(x, y) = (x, y, \sqrt{2} + \sqrt{31})$ linear? prove in detail your answer.

$\boxed{9}$ Use the following theorem (which I proved and was motivated by a great question by Luis Alejandro Garza Soto!) to answer the questions below it; simply state if the range is a line through the origin; one of the coordinate axes; or the entire plane.

Theorem. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y) = (ax + by, cx + dy)$ where a, b, c, d are arbitrary real numbers.

(a) If $a = b = c = d = 0$, then the range of T is simply the origin in $\mathbb{R}^2$.

(b) If $ad - bc \neq 0$, then the range of T is the whole plane $\mathbb{R}^2$.

(c) If $ad - bc = 0$, and if at least one of the constants a, b, c, d is non-zero, then the range of T is a line through the origin (either a diagonal line, or the y-axis or x-axis).

(i) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (4x - y, 4x + y)$.

(ii) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x, 0)$. What is the geometric interpretation of T?

(iii) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (0, y)$. What is the geometric interpretation of T?

(iv) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (91x - y, 91x - y)$? What is the graph of T? Hint: it should be familiar to you!

$\boxed{10}$ Suppose x is your grade corresponding to the first partial period; y is the grade corresponding to the second partial period, and z to the final period. Recall that the weighing formula for the final grade of the course is as follows: 30% first partial period, 30% second partial period and 40% final period.

(a) Construct a transformation $T : \mathbb{R}^3 \to \mathbb{R}$ whose image is precisely the final grade of the course.

(b) Is the above function a linear transformation? In case it is, prove it; otherwise explain why not.

$\boxed{11}$ Consider the map $T : \mathbb{R}^3 \to \mathbb{R}$ given by $T(x, y, z) = \frac{y}{x^2+z^2+4000}$. Find the domain of T and make a plot of the subset of $\mathbb{R}^3$ that represents the domain.

$\boxed{12}$ Give a concrete example of a transformation $T : \mathbb{R} \to \mathbb{R}^2$ that satisfies $T(0) = (0, 0)$ but such that T is **not** linear; justify why T is not linear.

1 Answer to Problem I

1.1

$T:\mathbf{R}^3 \rightarrow \mathbf{R}$ given by $T(x,y,z)=\frac{1}{(x-2)^2+(z-12)^2+(x-2)^2}$

$D_{(T)}=\mathbf{R}^3$

In[1]:=
```
T=1/((x-2)^2+(z-12)^2+(y-2)^2);
contourLimits=100;
membershipConditions=FunctionDomain[T,{x,y,z}];
domain=ImplicitRegion[membershipConditions,{x,y,z}];

Print["Domain:"];
RegionMember[domain,{x,y,z}]
myPlot=RegionPlot3D[domain,Axes→True]
Export["./media/problem1aDomain.png",myPlot];

Print["Domain subset Plot:"];
myPlot=Plot3D[T,{x,y,z}∈domain,Axes→True]
Export["./media/problem1aDomainSubset1.png",myPlot];
myPlot=DensityPlot3D[T,{x,y,z}∈domain,Axes→True]
Export["./media/problem1aDomainSubset2.png",myPlot];
myPlot=ContourPlot3D[T,{x,y,z}∈domain,Axes→True]
Export["./media/problem1aDomainSubset3.png",myPlot];
```

Domain:

Out[1]= $(x|y|z)\in \mathrm{R}\&\&-4\ x+x^2-4y+y^2-24z+z^2\neq-152$

Out[2]=

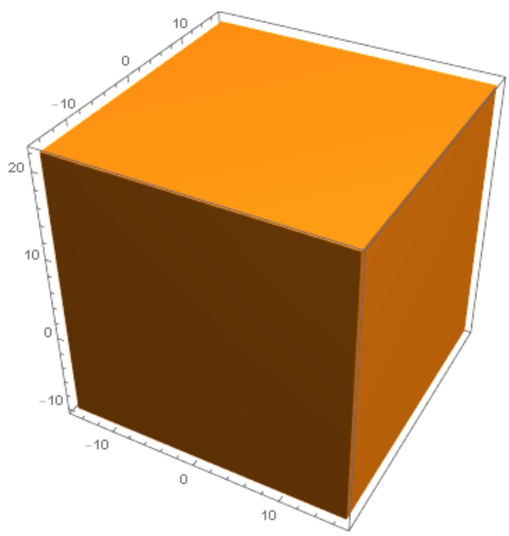

Figure 1: Domain of T.

Domain subset Plot:

Out[3]= `Plot3D[T,{x,y,z}∈domain,Axes→True]`

Out[4]=

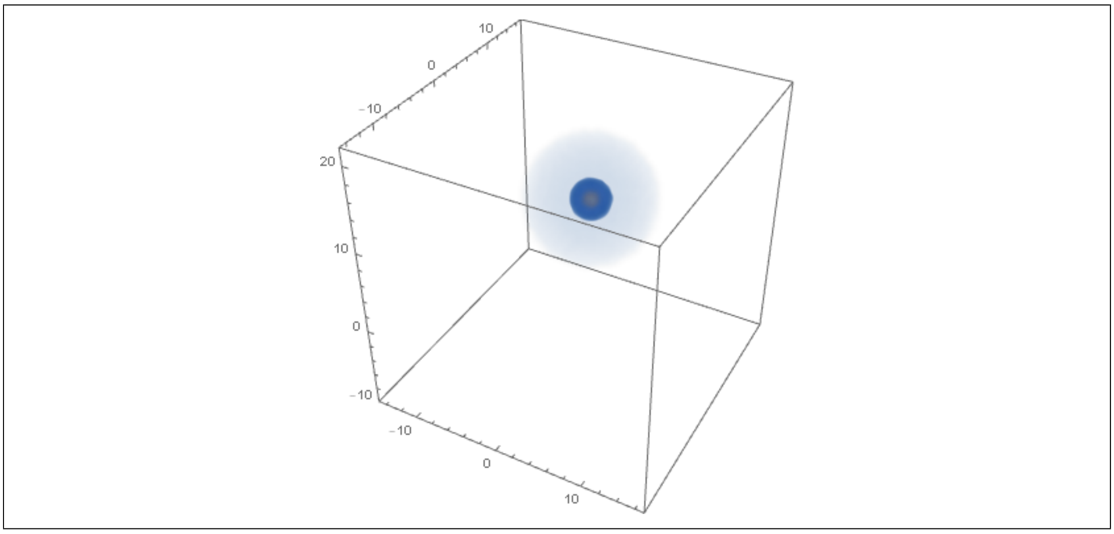

Figure 2: Subset that represents the domain.

Out[5]= `ContourPlot3D[T,{x,y,z}∈domain,Axes→True]`

1.2

a) Check T preserves sums

let u=(a,b,c), v=(d,e,f) be elements of $\mathbb{R}^3$

$T(u+v)=T((a,b,c)+(d,e,f))$

$=T(a+d,b+e,c+f)$

$$=\frac{1}{(a+d-2)^2+(c+f-12)^2+(b+e-2)^2}$$

$T(u)+T(v)=T(a,b,c)+T(d,e,f)$

$$=\frac{1}{(a-2)^2+(c-12)^2+(b-2)^2}+\frac{1}{(d-2)^2+(f-12)^2+(e-2)^2}$$

T is NOT a linear transformation since it does not preserve sums.

b) Check T preserves scalars
no need to check ...

2 Answer to Problem II

2.1

$$T:R^2 \to R \text{ given by } W(a,b)=\frac{1}{\mathrm{Sin}[\frac{a}{2}]+\frac{22222}{\sqrt{b-1}}}$$

$$D_{(T)}=R^2$$

In[6]:=
```
W=1/(Sin[a/2]+22222/Sqrt[b-1]);
contourLimits=100;
membershipConditions=FunctionDomain[W,{a,b}];
domain=ImplicitRegion[membershipConditions,{a,b}];

Print["Domain:"];
domainRegion=RegionMember[domain,{a,b}]
myPlot=RegionPlot[domainRegion,{a,0,15},{b,0,15},Axes→True]
Export["./media/problem2aDomain.png",myPlot];
```

Domain:

Out[6]= $(a|b)\in R \&\& b>1 \&\& \frac{22222}{\sqrt{-1+b}}+\mathrm{Sin}[\frac{a}{2}]\neq 0$

Out[7]=

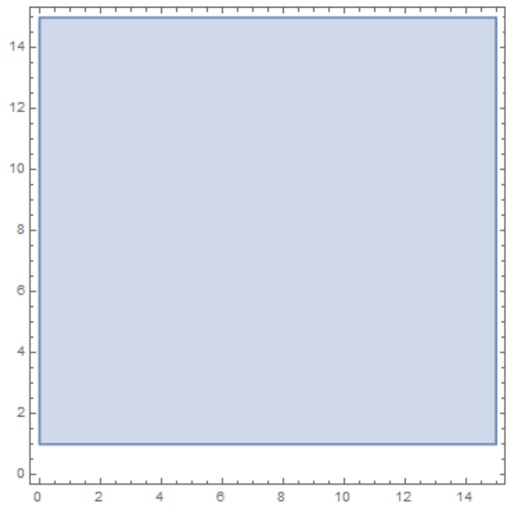

Figure 3: Subset that represents the domain.

2.2

a) Check T preserves sums

let u=(a,b), v=(d,e) be elements of $\mathbb{R}^2$

T(u+v)=T((a,b)+(d,e))

$=T(a+d,b+e)$

$=\frac{1}{\mathrm{Sin}[\frac{a+d}{2}]+\frac{22222}{\sqrt{b+e-1}}}$

T(u)+T(v)=T(a,b)+T(d,e)

$=\frac{1}{\mathrm{Sin}[\frac{a}{2}]+\frac{22222}{\sqrt{b-1}}}+\frac{1}{\mathrm{Sin}[\frac{d}{2}]+\frac{22222}{\sqrt{e-1}}}$

$\boxed{\text{T is NOT a linear transformation since it does not preserve sums.}}$

b) Check T preserves scalars

no need to check ...

3 Answer to Problem III

3.1

A circle can be described as the set of points (x,y) that satisfy $x^2+y^2=r^2$. Hence, if $x=\frac{\text{Sin}[x]}{\pi e}$ and $y=\frac{\text{Cos}[x]}{\pi e}$ then $x^2+y^2=(\frac{\text{Sin}[x]}{\pi e})^2+(\frac{\text{Cos}[x]}{\pi e})^2=r^2$; where r is the radius.

3.2

In[8]:=
```
radius=FullSimplify[Sqrt[(Sin[x]/(π e))^2+(Cos[x]/(π e))^2]];
radius
N[%]
```

The circle equation is in the format $(x--h)^2+(y--k)^2=r^2$, with the center being at the point (h,k) and the radius being "r". Since h & k are equal to zero, then the center is at the origin.

Out[8]= $\frac{1}{e\ \pi}$

Out[9]= 0.1171

3.3

In[10]:=
```
ClearAll["Global`*"];
T={Sin[x]/(π*e), Cos[x]/(π*e)};

contourLimits=100;
membershipConditions=FunctionDomain[T,{x}];
domain=ImplicitRegion[membershipConditions,{x}];

Print["Range:"];
points=Table[{Sin[x]/(π*e), Cos[x]/(π*e)},{x,-150,150}];
ListPlot[points]
plots = Table[
    Show[Graphics[Point[points[[n]]]],
    PlotRange →{{-0.12,0.12},{-0.12,0.12}},
    Axes→Automatic],
    {n,Length[points]}
];
frames=FoldList[Show,plots];
myPlot=ListAnimate[frames,AnimationRate→60]
Export["./media/problem3aRange.swf",myPlot];
```

Out[10]= Range:

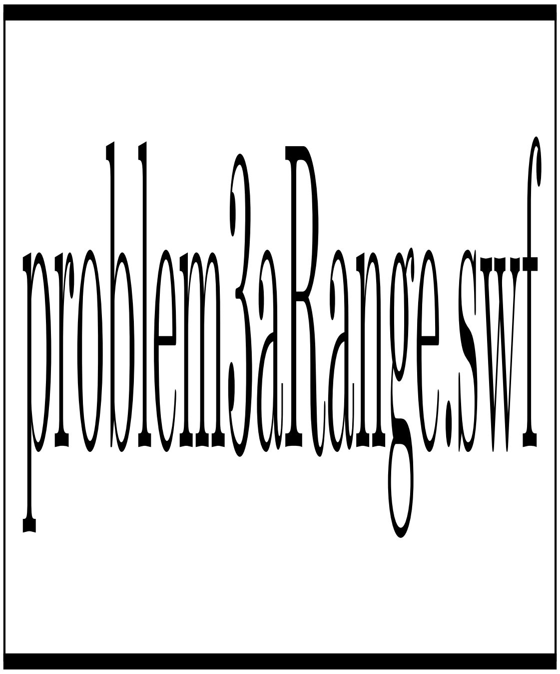

Figure 4: Range of T.

4 Answer to Problem IV

5 Answer to Problem V

6 Answer to Problem VI

7 Answer to Problem VII

8 Answer to Problem VIII

9 Answer to Problem IX

10 Answer to Problem X

11 Answer to Problem XI

12 Answer to Problem XII