

Mathematica Problem Sheet 01

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ITESM Campus Monterrey
Mathematical Physical Modelling F4005
Mathematica problem sheet 1
Due Date: February 5-2019, 23:59.
Professor: Ph.D Daniel López Aguayo

Name and ID: _____

Instructions: this is an individual assignment. The solutions to this assignment must be typed and must include all the Mathematica input or output, you must send them to dlopez.aguayo@tec.mx. You are not allowed to ask questions, but feel free to use the web to read any documentation. The purpose of this activity is to **develop your research skills and motivate you to persevere**. We will discuss the solutions next week (after the deadline). No late homework will be accepted.

[1] Construct a matrix A of size 10×10 whose first row consists of all the integers in $[1, 10]$; the second row consists of all the integers in $[11, 20]$, and so on. For this exercise you will need to read carefully about the commands *Table* and *Range*. Then, use Mathematica to find $a_{4,9}$.

[2] Use Mathematica to compute the inverse of A , where A is the matrix of the previous exercise. In case A is not invertible explain how can you prove it mathematically without many computations.

[3] Use Mathematica to create the following matrix B of size 30×30

$$\begin{bmatrix} \pi & 0 & 0 & \cdots & 0 \\ 0 & \pi & 0 & \cdots & 0 \\ 0 & 0 & \pi & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \pi \end{bmatrix}$$

- (a) Is this an invertible matrix? Verify with Mathematica and also explain your answer mathematically.
- (b) Use Mathematica to compute the inverse of B and also explain how can you compute the inverse of B by hand.
- (c) Compute the rank of B .

[4] Use Mathematica to create the following matrix C of size 16×16

$$\begin{bmatrix} \pi & 0 & 0 & \cdots & 0 & 0 \\ 0 & \pi + 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \pi + 2 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \pi + 3 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \pi + 14 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \pi + 15 \end{bmatrix}$$

- (a) Is this an invertible matrix? Verify with Mathematica and also explain your answer mathematically.
- (b) Use Mathematica to compute the inverse of C and also explain how you can compute the inverse of C by hand.
- (c) Without any computations, how would you find the reduced row echelon form of C ? Once done this, compute the rank of C and verify both answers with Mathematica.

[5] Mathematica is also capable of computing formulas, not only specific calculations. For instance, suppose a user wants to compute the inverse of

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- (a) Save this matrix and compute its inverse.
- (b) What is the error behind the output given by Mathematica? Try to see if an inverse exists when $a = 2, d = 2, b = 1, c = 4$.

The point of this exercise is that you have to be careful with the output given by computers, sometimes they made additional assumptions.

[6] Suppose we want to solve the system

$$\begin{cases} x - y = 3 \\ \pi \cdot x - \pi \cdot y = 3\pi \end{cases}$$

- (a) Use the *Solve* command in Mathematica and analyze the output.
- (b) Read about the *Plot* command and graph both equations.
- (c) Using the above plot, deduce the number of solutions.
- (d) Use matrix inversion and the theorem we saw in class to analyze the number of solutions.

[7] Suppose we want to solve the system

$$\begin{cases} x + y + z = 10 \\ \frac{2}{3}x + \frac{2}{3}y + \frac{2}{3}z = 11 \\ \frac{5}{9}x + \frac{5}{9}y + \frac{5}{9}z = 12 \end{cases}$$

- (a) Read about the *Plot3D* command and plot the above system in Mathematica.
- (b) Read about the term **normal vector** to a plane and include the definition. Using this concept, how can we infer that the system is inconsistent?
- (c) Use Mathematica to find the solution of the system.
- (d) Let A be the coefficient matrix and let $[A|B]$ be the augmented matrix. Using only your logic, how can we know the reduced row echelon form of A without any computations? Find the rank of A .
- (e) Verify with Mathematica the above answer.
- (f) What is the reduced row echelon form of $[A|B]$? Compute the rank of $[A|B]$.
- (g) Apply the theorem given in class to determine whether the system is consistent or inconsistent (i.e compare ranks).

1 Answer to Problem I

```
In[1]:= SQRmatrixSize = 10;  
matrixAentries = Range[1, SQRmatrixSize*SQRmatrixSize];  
matrixAentries =  
    ArrayReshape[matrixAentries, SQRmatrixSize, SQRmatrixSize];  
matrixA =  
    Table[matrixAentries[[i, j]], {i, 1, SQRmatrixSize}, {j, 1, SQRmatrixSize}];  
MatrixForm[matrixA]
```

Out[1]=

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 \\ 31 & 32 & 33 & 34 & 35 & 36 & 37 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 & 49 & 50 \\ 51 & 52 & 53 & 54 & 55 & 56 & 57 & 58 & 59 & 60 \\ 61 & 62 & 63 & 64 & 65 & 66 & 67 & 68 & 69 & 70 \\ 71 & 72 & 73 & 74 & 75 & 76 & 77 & 78 & 79 & 80 \\ 81 & 82 & 83 & 84 & 85 & 86 & 87 & 88 & 89 & 90 \\ 91 & 92 & 93 & 94 & 95 & 96 & 97 & 98 & 99 & 100 \end{pmatrix}$$

2 Answer to Problem II

matrixA is singular, therefore Inverse[matrixA] does not exist.

3 Answer to Problem III

```
In[2]:= SQRmatrixSize = 30;  
matrixB =  $\pi$ *IdentityMatrix[SQRmatrixSize];  
MatrixForm[matrixB]
```

Out[2]=

3.a & 3.b

Yes, “A triangular matrix (upper, lower or diagonal) is invertible if and only if no element on its main diagonal is 0.”

“by hand” ...

Since, $\text{Inverse}[A]$ equals $\text{diag}(\frac{1}{a_1}, \frac{1}{a_2}, \dots, \frac{1}{a_n})$, $\text{Inverse}[\text{matrixB}]$ is equal to $\frac{1}{\pi} \text{IdentityMatrix}[30]$

Verification with Mathematica is shown below ...

```
In[3]:= Inverse[matrixB];  
        MatrixForm[%]
```

```
Out[3]=
```

[illegible]

3.c

```
In[4]:= MatrixRank[matrixB]
```

```
Out[4]= 30
```

4 Answer to Problem IV

```

In[5]:= SQRmatrixSize=16;
matrixC=Table[0,{i,1,SQRmatrixSize},{j,1, SQRmatrixSize}];
rows=Dimensions[matrixC][[1]];
cols=Dimensions[matrixC][[2]];
For[i=1,i≤rows,i++,
  For[j=1,j≤cols,j++,
    If[i==j,
      matrixC[[i,j]]=(π+i)-1,
      matrixC[[i,j]]=0];
  ];
];
MatrixForm[matrixC]

```

Out[5]=

$$\begin{pmatrix}
 \pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 2+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 3+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 4+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 5+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 6+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7+\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8+\pi & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9+\pi & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10+\pi & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11+\pi & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12+\pi & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13+\pi & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{pmatrix}$$

4.a & 4.b

Yes, “A triangular matrix (upper, lower or diagonal) is invertible if and only if no element on its main diagonal is 0.”

“by hand” ...

Since, $\text{Inverse}[A]$ equals $\text{diag}(\frac{1}{a_1}, \frac{1}{a_2}, \dots, \frac{1}{a_n})$, $\text{Inverse}[\text{matrixB}]$ is equal to $\frac{1}{(\pi+i)-1} \text{IdentityMatrix}[16]$

Verification with Mathematica is shown below ...

Inverse[matrixC];

MatrixForm[%]

$$\begin{pmatrix} \frac{1}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{5+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{7+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8+\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9+\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{10+\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{11+\pi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{12+\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{13+\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{14+\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{15+\pi} \end{pmatrix}$$

4.c

Multiplying matrixC times the inverse of matrixC. However, since we know that $\text{Inverse}[\text{matrixC}]$ exists, the reduced echelon form is the identity matrix (of size 16x16).

MatrixRank[matrixC];

RowReduce[matrixC];

MatrixForm[%]

[illegible]

5 Answer to Problem V

5.a

$$\mathbf{matrixD} = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$$

`Inverse[matrixD];`

`MatrixForm[%]`

$$\begin{pmatrix} \frac{d}{-bc+ad} & -\frac{b}{-bc+ad} \\ -\frac{c}{-bc+ad} & \frac{a}{-bc+ad} \end{pmatrix}$$

5.b

When $a = 2$; $d = 2$; $b = 1$; $c = 4$, `Inverse[matrixD]` does not exist. Mathematica outputs the following error:

... Inverse: Matrix $\{\{2,1\},\{4,2\}\}$ is singular

6 Answer to Problem VI

6.a

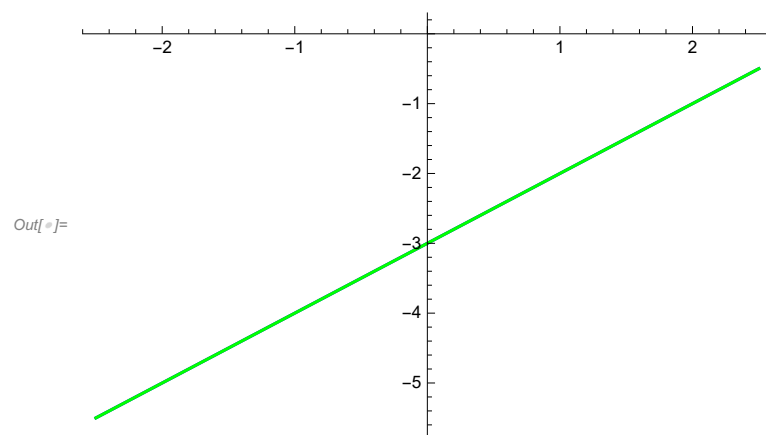
```
Solve[{
  x - y == 3,
  x * π - y * π == 3 * π
}, {x, y}]
```

Solve : Equations may not give solutions for all “solve” variables.

```
{{y → -3 + x}}
```

6.b

```
contourLimits = 2.5;
(*
ContourPlot[{
  x - y == 3,
  x * π - y * π == 3 * π
}, {x, 0.5, 1.0}, {y, -2.0, -2.5}, ContourStyle → {Blue, Orange}]
*)
Show[{
  Plot[x - 3, {x, -contourLimits, contourLimits}, PlotStyle → Blue],
  Plot[x - 3, {x, -contourLimits, contourLimits}, PlotStyle → Green]
}, PlotRange → All, AxesOrigin → {0, 0}]
```



6.c

The system has infinitely many solutions.

6.d

$$\text{matrixA} = \begin{pmatrix} 1 & -1 \\ \pi & -\pi \end{pmatrix};$$

$$\text{matrixB} = \begin{pmatrix} 3 \\ 3 * \pi \end{pmatrix};$$

(*rows = m = i*)(*cols = n = j*)

getNumOfSolutions[varA_, varB_] := Module[{vA = varA, vB = varB, vAB, Arank, ABrank, n},

vAB = ArrayFlatten[{ {vA, vB} }];

Arank = MatrixRank[vA];

ABrank = MatrixRank[vAB];

n = Dimensions[vA][[2]];

If[Arank < ABrank,

Print["The system has no solution"]; ,

If[Arank == ABrank && Arank < n && ABrank < n,

Print["The system has infinitely many solutions"]; ,

If[Arank == ABrank && Arank == n,

Print["The system has a unique solution"]; ,

Print["[ERROR] The system has ? solution(s)\n

Beware, if the system is homogeneous (varB is a zero vector and n>m (more variables than equations), then the system has infinitely many solutions)"];

];

];

];

];

getNumOfSolutions[matrixA, matrixB]

The system has infinitely many solutions

7 Answer to Problem VII

7.a

```
contourLimits = 5;
```

```
(*
```

```
ContourPlot3D[{
```

```
 $x + y + z == 10,$ 
```

```
 $\frac{2}{3} * x + \frac{2}{3} * y + \frac{2}{3} * z == 11,$ 
```

```
 $\frac{5}{9} * x + \frac{5}{9} * y + \frac{5}{9} * z == 12$ 
```

```
},
```

```
{ $x$ , -contourLimits, contourLimits},
```

```
{ $y$ , -contourLimits, contourLimits},
```

```
{ $z$ , -contourLimits, contourLimits}, Axes → True, PlotLegends → “Expressions”]
```

```
*)
```

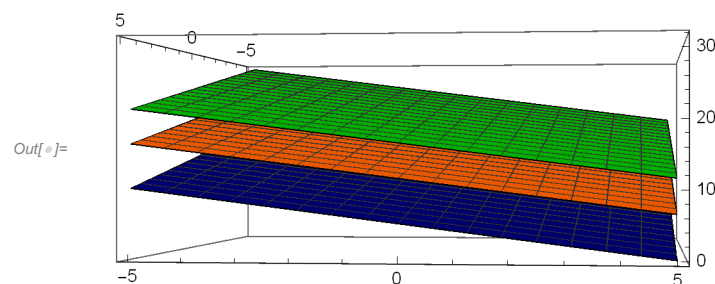
```
Show[{
```

```
Plot3D[ $10 - x - y$ , { $x$ , -contourLimits, contourLimits}, { $y$ , -contourLimits, contourLimits}, PlotStyle → B
```

```
Plot3D [ $\frac{1}{2}(33 - 2x - 2y)$ , { $x$ , -contourLimits, contourLimits}, { $y$ , -contourLimits, contourLimits}, PlotStyl
```

```
Plot3D [ $\frac{1}{5}(108 - 5x - 5y)$ , { $x$ , -contourLimits, contourLimits}, { $y$ , -contourLimits, contourLimits}, PlotSty
```

```
], PlotRange → All, AxesOrigin → {0, 0}]
```



7.b

7.b.1 **normal vector:** A nonzero vector that is perpendicular to the plane is called a normal vector to the plane.

7.b.2 The system is inconsistent because the three normal vectors point to the same direction.

7.c

```
Solve[{
```

```
 $x + y + z == 10,$ 
```


$$\frac{2}{3} * x + \frac{2}{3} * y + \frac{2}{3} * z == 11,$$

$$\frac{5}{9} * x + \frac{5}{9} * y + \frac{5}{9} * z == 12$$

$\}, \{x, y, z\}]$

$\{\}$

7.d & 7.e

The reduced row echelon of A will be $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, since all the entries are the same within each row. `MatrixRank[A]=1`

Verification with Mathematica is shown below ...

$$\text{matrixA} = \begin{pmatrix} 1 & 1 & 1 \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{5}{9} & \frac{5}{9} & \frac{5}{9} \end{pmatrix};$$

$$\text{matrixB} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix};$$

`matrixAB = ArrayFlatten[{{matrixA, matrixB}}];`

`RowReduce[matrixA]`

`MatrixForm[%]`

`MatrixRank[matrixA]`

$\{\{1, 1, 1\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1

7.f

`RowReduce[matrixAB];`

`MatrixForm[%]`

`MatrixRank[matrixAB]`

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2

7.g

getNumOfSolutions[matrixA, matrixB]

The system has no solution

8 Mathematica LaTeX tests