42 Find $D_w f$ at $(1, 2, 0)$

- The directional derivative $D_w f$ of $df/ds$ of a function $f(x,y,z) = zy + yx$ at a point $P:(1,2,0)$ in the direction of a vector $w = [y^2, z^2, x^2]$ is defined by:

$$D_w f = \frac{df}{ds} = \frac{\partial f}{\partial x}x' + \frac{\partial f}{\partial y}y' + \frac{\partial f}{\partial z}z'$$ where the primes denote derivatives with respect s

$$= w \cdot \text{grad } f$$

- If the direction is given by a vector a of any length $(\neq 0)$, then:

$$D_a f = \frac{1}{|a|} a \cdot \text{grad } f$$

- Let's compute $D_w f$ =

$$D_w f = \frac{1}{|w|} w \cdot \text{grad } f$$

$$= \frac{1}{\sqrt{y^4 + z^4 + x^4}} [y^2, z^2, x^2] \cdot \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right]$$

$$= \frac{1}{\sqrt{x^4 + y^4 + z^4}} [y^2, z^2, x^2] \cdot [y, x+z, y]$$

$$= \frac{1}{\sqrt{x^4 + y^4 + z^4}} [y^3 + xz^2 + z^3 + yx^2]$$

$D_w f$ at $P:(1, 2, 0)$ gives:

$$\frac{1}{\sqrt{1^4 + 2^4 + 0^4}} [2^3 + 1(0)^2 + 0^3 + 2(1)^2] = \frac{10\sqrt{17}}{17}$$

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