For adiabatic processes,

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{k}{V^{\gamma}}\,dV = k\left[\frac{V^{(1-\gamma)}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$W = P_1 V_1^{\gamma}\left[\frac{V^{(1-\gamma)}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$W = \frac{P_1 V_1^{\gamma}}{1-\gamma}\left[V_2^{(1-\gamma)} - V_1^{(1-\gamma)}\right]$$

$$W = \frac{(1.5\times10^{6})(60\times10^{-3})^{1.125}}{1-1.125}\left[150\times10^{-3\,(1-1.125)} - 60\times10^{-3\,(1-1.125)}\right] \text{Pa m}^3$$

$W = 77\,918,739$ J is the work done by the system

$\left\{ Pa = \frac{Kg}{ms^2} \right.$ & $J = \frac{Kg\,m^2}{s^2}$

1a False. Isobaric process is a thermodynamic process in which the pressure stays constant $\Delta P = 0$. The heat transfered to the system does work, but also changes the internal energy of the system; hence the enthalpy changes.

1b False. Since the volume is constant, the system does no work, however the internal energy can change and that affect the enthalpy

1c False. the change in internal energy is given by: $\Delta U = nC_V\Delta T$

1d False. The energy is transfered only by work; and it's given by $W = \Delta H$

1e For adiabatic processes: True

$Q = 0$; $p\,dV + V\,dp = nR\,dT$; $n\frac{f}{2}R\,dT = -p\,dV$; where f is the number of degrees of freedom.

$$p\,dV + V\,dp = -\frac{2}{f}p\,dV$$

$$V\,dp + \left(1+\frac{2}{f}\right)p\,dV = 0 \quad ; \gamma = 1+\frac{2}{f}$$

$$V\,dp + \gamma p\,dV = 0$$

$$\frac{V\,dp}{pV} + \gamma\frac{p\,dV}{pV} = 0$$

$$\frac{dp}{P} + \gamma \frac{dV}{V} = 0$$

$$\int_{P_i}^{P_f} \frac{1}{P} dp + \gamma \int_{V_i}^{V_f} \frac{dV}{V} = 0$$

$$\ln \frac{P_f}{P_i} + \gamma \ln \frac{V_f}{V_i} = 0$$

$$\ln \frac{P_f V_f^{\gamma}}{P_i V_i^{\gamma}} = 0$$

$$P_f V_f^{\gamma} = P_i V_i^{\gamma} = \text{constant}$$

$P_1V_1 = nRT_1$ & $P_2V_2 = nRT_2$

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{P_1}{P_2} = \frac{V_2^{\gamma}}{V_1^{\gamma}}$$

$$\frac{V_2^{\gamma-1}}{V_1^{\gamma-1}} = \frac{T_1}{T_2}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$\boxed{2}$ for adiabatic processes,

$$W = -\Delta U = -nC_V \Delta T$$
$$= nC_V(T_1 - T_2)$$
$$= \frac{C_V}{R}(P_1V_1 - P_2V_2)$$
$$= \frac{1}{\gamma - 1}(P_1V_1 - P_2V_2)$$

$\boxed{3}$ $n = 1$; $T_1 = 273\ K$; $P_1 = 1\ atm$; $q = 3000\ J$; $w = 832\ J$ / $R = 8.314\ J\ K^{-1} m$
$P_2 = 1\ atm \rightarrow$ constant pressure / $= 0.08206\ L\ atm\ K$

$\boxed{3a}$ $V_1 = \dfrac{nRT_1}{P_1} = \dfrac{(1\ mol)(0.08206\ L\ atm\ K^{-1}\ mol^{-1})(273\ K)}{1\ atm}$

$V_1 = 22.402\ L$

$$\Delta U = q - w = 3000\ J - 832\ J$$
$$= 2168\ J$$

$$w = 832\ J$$
$$= \int P dV = P(V_2 - V_1)$$
$$832\ J = (1\ atm)(V_2 - 22.402\ L)$$
$$V_2 = \frac{832\ J}{1\ atm} + 22.402\ L$$
$$V_2 = 8.211 \times 10^{-3}\ m^3 + 22.402\ L$$
$$V_2 = 30.613\ L$$

$J = \dfrac{Kg\ m^2}{s^2}$; $Pa = \dfrac{Kg}{m\ s^2}$

$1\ atm = 101325\ Pa$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{(1\ atm)(30.613\ L)}{(1\ mol)(0.08206\ L\ atm\ K^{-1}\ mol^{-1})}$$
$$T_2 = 373.056\ K$$

3b $\Delta U = 2168\,J$

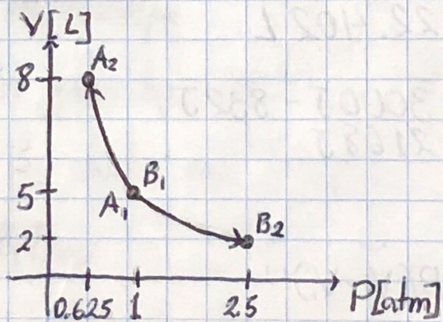
$$H = U + PV$$
$$\Delta H = \Delta(U + PV)$$
$$= \Delta U + \cancel{\Delta PV}\ P\Delta V + V\Delta P$$
$$= 2168\,J + (30.613\,L - 22.402\,L)(1\,atm) + 0 \quad \text{as } \Delta P = 0$$
$$= 2168\,J + (30.613\,L - 22.402\,L)(101325\,Pa)$$
$$\Delta H = 2168\,J + (8.211 \times 10^{-3}\,m^3)(101325\,Pa)$$
$$\Delta H = 2999.980\,J$$

3c
$$\Delta U = n\int C_v\,dT$$
$$2168\,J = (1\,\cancel{mol})\,C_v\,(373.056 - 273\,K)$$
$$C_v = \frac{2168\,J}{100.056\,K}$$
$$C_v = 21.68\,\frac{J}{K}$$

$$\Delta H = n\int C_p\,\Delta T$$
$$2999.980\,J = (1)\,C_p\,(373.056 - 273\,K)$$
$$C_p = 29.983\,\frac{J}{K}$$

4a
$P_{A1} = 1\,atm$ | $P_{B1} = 1\,atm$
$V_{A1} = 5\,L$ | $V_{B1} = 5\,L$
$V_{A2} = 8\,L$ | $V_{B2} = 2\,L$

$$P_{A2} = \frac{P_{A1}V_{A1}}{V_{A2}} \qquad P_{B2} = \frac{P_{B1}V_{B1}}{V_{B2}}$$
$$P_{A2} = 0.625\,atm \qquad P_{B2} = 2.5\,atm$$



4b Assuming isothermal processes,

ΔU for A & B is 0 (no change in internal energy)
ΔH for A & B is 0 (no change in enthalpy)

Final pressure of A $P_{A2} = 0.625\,atm$
Final pressure of B $P_{B2} = 2.5\,atm$

From pdV Work equations for isothermal processes,

$$w = nRT \ln\left(\frac{V_2}{V_1}\right) = P_2 V_2 \ln\left(\frac{V_2}{V_1}\right)$$

$$nRT = P_2 V_2$$

$$T = \frac{P_2 V_2}{nR}$$

$$T_A = \frac{(0.625 \text{ atm})(8L)}{(0.5 \text{ mol})(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})}$$

$$T_A = 121.862 K$$

$$T_B = \frac{(2.5 \text{ atm})(2L)}{(0.5 \text{mol})(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})}$$

$$T_B = 121.862 K$$

5a) $C_V = A\left(\frac{T}{\theta}\right)^3$

$$C_{V_1}(5K) = 3.7\times10^3 \text{ J mol}^{-1} \text{K}^{-1}\left(\frac{5K}{230 K}\right)^3$$

$$C_{V_1}(5K) = 80.435 \text{ J mol}^{-1} \text{K}^{-1}$$

$$C_{V_2}(30K) = 3.7\times10^3 \text{ J mol}^{-1} \text{K}^{-1}\left(\frac{30K}{230 K}\right)$$

$$C_{V_2}(30K) = 482.609 \text{ J mol}^{-1} \text{K}^{-1}$$

5b) $Q_1 = nC_{V_1}(T_2 - T_1)$

$$Q_1 = (2 \text{mol})(80.435 \text{ J mol}^{-1} \text{K}^{-1})(30K - 5K)$$

$$Q_1 = 4021.75 J$$

$$Q_2 = (2 \text{mol})(482.609 \text{ J mol}^{-1} \text{K}^{-1})(30K - 5K)$$

$$Q_2 = 24130.45 J$$

$$\Delta Q = 20108.7 J$$

6a) M & H are intensive properties as they are independent of mass.

6b) The work done on a material by an external magnetic field is given by

$dw = -C\,H\,dM$, where: V = volume of the magnetic field in m^3; μ_0 = permitivity of vacuum in $N\,amp^{-2}$; H and M are in $amp\,m^{-1}$ $\Big\}\ C = V\mu_0$

$\boxed{6c}$ $w = -C\int_{M_1}^{M_2} H\,dM$

$$w = -C\int_0^M H\,dM = \left[HM\right]_0^M(-C)$$

$= -CHM$ Joules

$\boxed{\text{Exercise 4 - 2nd attempt}}$

→ After some discussion, the following assumptions were made:
  i) The thermodynamic process of compartment A is undefined
  ii) The thermodynamic process of compartment B is adiabatic
  iii) The pressure of both compartments are equal to each other but not constant.

→ Let's analyze compartment B.

- For an adiabatic process in an ideal monatomic gas, the polytropic index n is 3/2.
  if $n = \frac{1}{\gamma - 1} \Rightarrow \gamma = \frac{5}{3}$; where γ is the ratio of specific heats.

- Therefore PV^n is constant
  $P_1 V_1^n = P_2 V_2^n$
  $(1\,atm)(5L)^n = P_2(2L)^n$
  $(101325\,Pa)(5\times10^{-3}\,m^3)^n = P_2(2\times10^{-3}\,m^3)^n$
  $P_2 = 400522.2299\,Pa$ or $3.953\,atm$

- As the heat transfer is 0, then
  $\Delta U = W$
  $= PdV$
  $= PV^\gamma \int_{V_1}^{V_2} \frac{1}{V^\gamma} dV$
  $= \frac{P_1 V_1^\gamma (V_2^{1-\gamma} - V_1^{1-\gamma})}{1-\gamma}$
  $\Delta U = -639.879\,J$

- Let's compute the temperatures of compartment B
  $PV = nRT$
  $(101325\,Pa)(5\times10^{-3}\,m^3) = (0.5\,mol)(8.314\,JK^{-1}mol^{-1})\,T_1$
  $T_1 = 121.873\,K$

  $(400522.2299\,Pa)(2\times10^{3}\,m^3) = (0.5\,mol)(8.314\,JK^{-1}mol^{-1})\,T_2$
  $T_2 = 192.698\,K$

→ Let's analyze compartment A

- Due to assumption iii), $P_2 = 400522.2299\ Pa$

$P_1 V_1^n = P_2 V_2^n$

$(1\,atm)(5L)^n = (3.953\,atm)(8L)^n$

$n = 2.92439$

$\gamma = 1 + \frac{1}{n} = 1.34195$

$PV = nRT$

$(101325\,Pa)(5\times10^{-3}\,m^3) = (0.5\,mol)(8.314\,J\,K^{-1}\,mol^{-1})T_1$

$T_1 = 121.873\,K$

$(400\,522.2299\,Pa)(8\times10^{-3}\,m^3)$

$T_2 = 194.996\,K$

- For an ideal monoatomic gas:

$Q = C_v n \Delta T$; $Q = \Delta U + P\Delta V = \Delta U$

$C_v = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{3}{2}R = 12.471 \frac{J}{K\,mol}$; $C_p = C_v + R$, $C_p = 20.785\,J\,K^{-1}\,mol^{-1}$

$\Delta U = (12.471\,J\,K^{-1}\,mol^{-1})(0.5\,mol)(T_2 - T_1)$

$= 455.958\,J$

$\Delta U = \Delta Q - PdV$; where $PdV = \frac{(101325\,Pa)(5\times10^{-3}\,m^3)^{\gamma}[(8\times10^{-3}\,m^3)^{1-\gamma} - (5\times10^{-3}\,m^3)^{1-\gamma}]}{1-\gamma}$

$\Delta Q = 675.924\,J$

$PdV = 219.966\,J$

→ Let's calculate ΔH for compartments A & B

for A, $\Delta H = \Delta Q - VdP$

$= \Delta Q - m C_p (T_2 - T_1)$

$= 675.924 - (0.5\,mol)(20.785\,J\,K^{-1}\,mol^{-1})(194.996\,K - 121.873\,K)$

$\Delta H = -84.007\,J$

for B, $\Delta H = \Delta Q - VdP$

$= 0 - m C_p (T_2 - T_1)$

$= (0.5\,mol)(20.785\,J\,K^{-1}\,mol^{-1})(192.698\,K - 121.873\,K)$

$= -60.125\,J$

→ Let's analize the surroundings.

$$\Delta U_{surr} = \Delta U_A + \Delta U_B$$
$$= 455.958\,J + (-639.879\,J)$$
$$= -183.921\,J$$

$$\Delta H_{surr} = \Delta H_A - \Delta H_B$$
$$= -84.007\,J + (-60.125\,J)$$
$$= -144.132\,J$$

→ For the universe,

$\Delta U = 0$ & $\Delta H = 0$

→ So:

-Initial State

- Compartment A
  $P = 1\,atm$
  $V = 5\,L$
  $T = 121.873\,K$

- Compartment B
  $P = 1\,atm$
  $V = 5\,L$
  $T = 121.873\,K$

-Final State

- Compartment A
  $P = 3.953\,atm$
  $V = 8\,L$
  $T = 194.996\,K$
  $\Delta U = 455.958\,J$
  $\Delta H = -84.007\,J$

- Compartment B
  $P = 3.953\,atm$
  $V = 2\,L$
  $T = 192.698\,K$
  $\Delta U = -639.879\,J$
  $\Delta H = -60.125\,J$

- Surroundings
  $\Delta U = -183.921\,J$
  $\Delta H = -144.132\,J$

- Universe
  $\Delta U = 0$
  $\Delta H = 0$