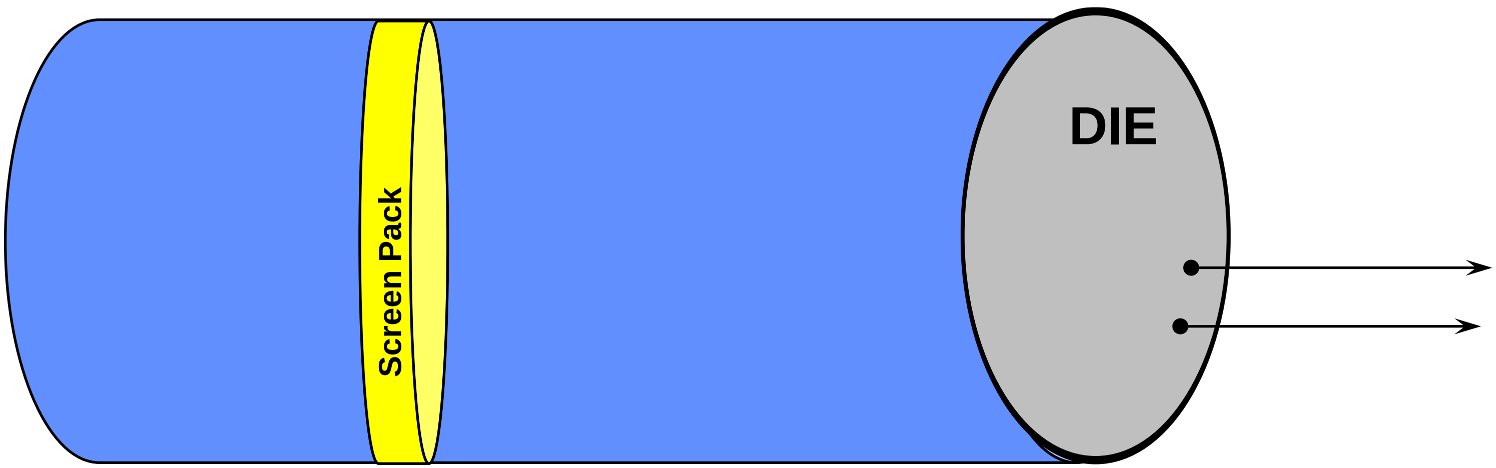
An evaluation of pressure rise across a connical section

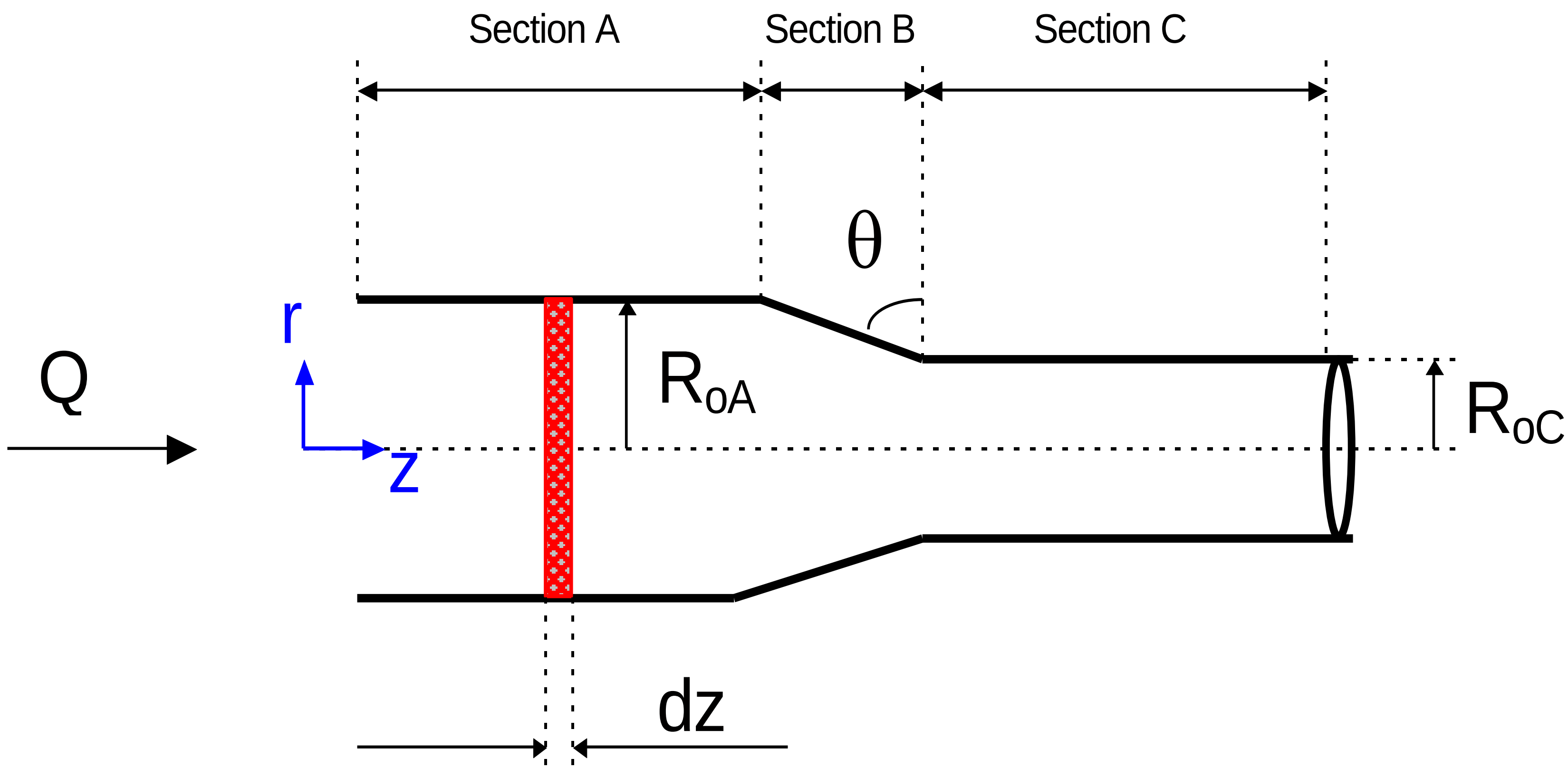
Calculating the pressure drop across a die



Die with 800 holes

General Model

Geometry of each hole



Model assumptions

- Laminar flow,
- No heat due to viscous drag.
- Constant radial temperature.
- A linear temperature profile in the direction of the flow, for non-isothermal cases.

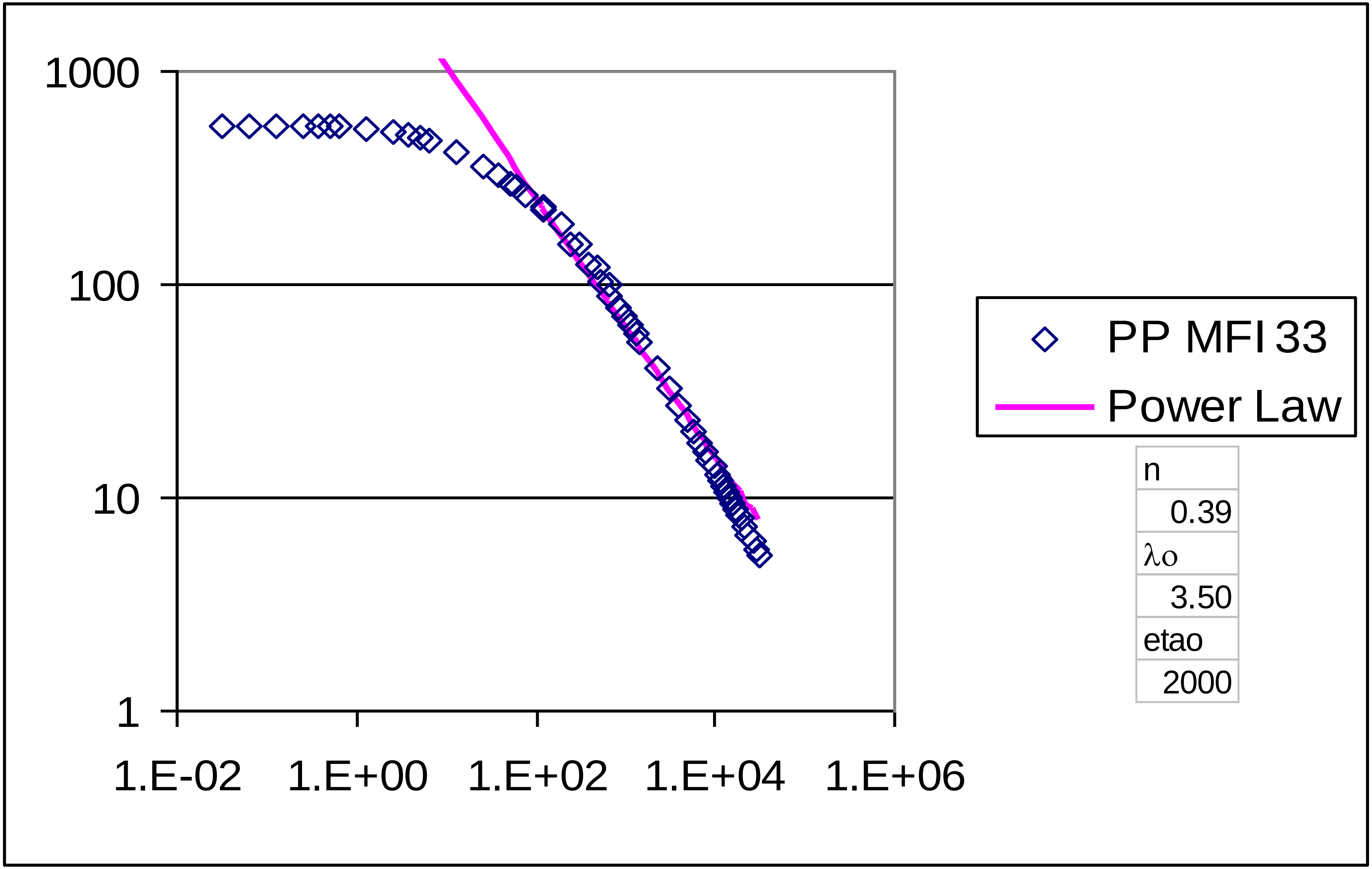
Model requirements

- Actual viscosity data is used in the model

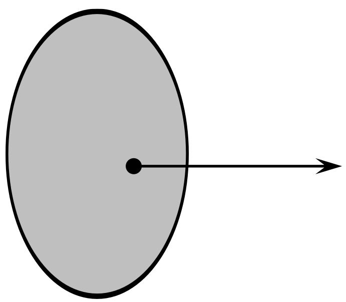
- Viscosity data fit by a power law function:

$$\eta=\eta_0\left(\frac{\dot{\gamma}}{\dot{\gamma}_0}\right)^{n-1}$$

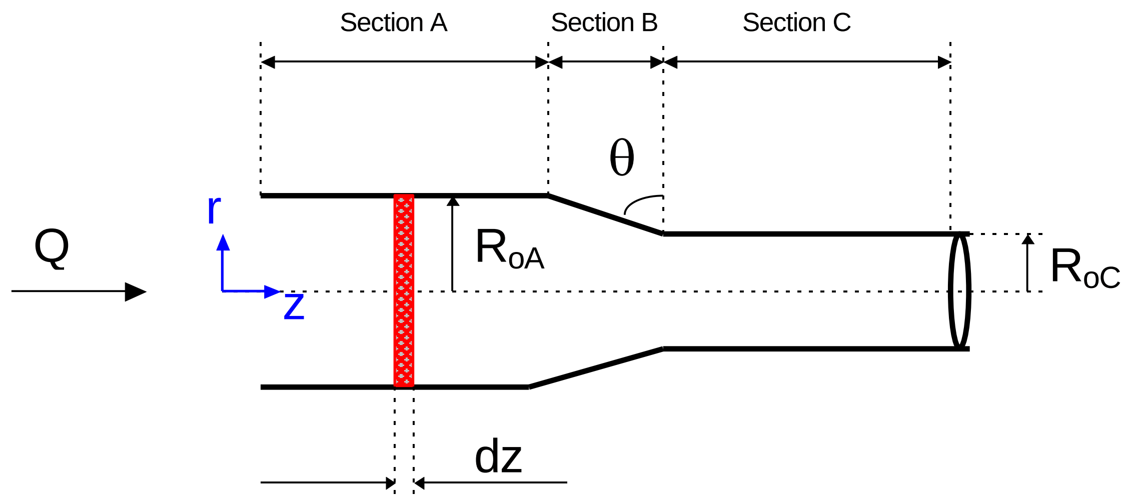
Viscosity vs. Shear rate



Model



Die with 800 holes



$$\dot{\gamma}_0^{n-1}\frac{\rho C_v}{\eta_0}\frac{\partial T}{\partial z}$$

Momentum balance on a thin (dz) slab:

$$\frac{d(r\tau_{rz})}{dr} = \frac{\Delta P}{dz} r$$

$$\tau_{rz} = \frac{\Delta P}{dz}\frac{r}{2}$$

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where $$\eta = \eta_0 \left(\frac{\dot{\gamma}}{\dot{\gamma}_0} \right)^{n-1}$$

$$Q = \langle v_Z \rangle \pi r^2$$

Model (cont.)

$$\tau_{rz} = \frac{\Delta P}{dz}\frac{r}{2} = -\eta_o \left(\frac{\frac{dv_z}{dr}}{\dot{\gamma}_o} \right)^{n-1} \frac{dv_z}{dr}$$

and for each section the pressure drop is calculated as follows

$$\Delta P = \sum_{j=1}^{j=m} \left\{ \left[\frac{Q(3n+1)}{n\pi R_o^3 \dot{\gamma}_o} \right]^n \left[\frac{2\eta_o \dot{\gamma}_o}{R_o K} \right] \right\} \Delta z$$

- *Q is given by the production rate for the process line*
- *n is the power law index (used to fit the non-newtonian region od the viscosity curve*
- *The same is true for eta cero and critical shear rate cero*
- *K is the ratio of the smallest to the biggest radius*

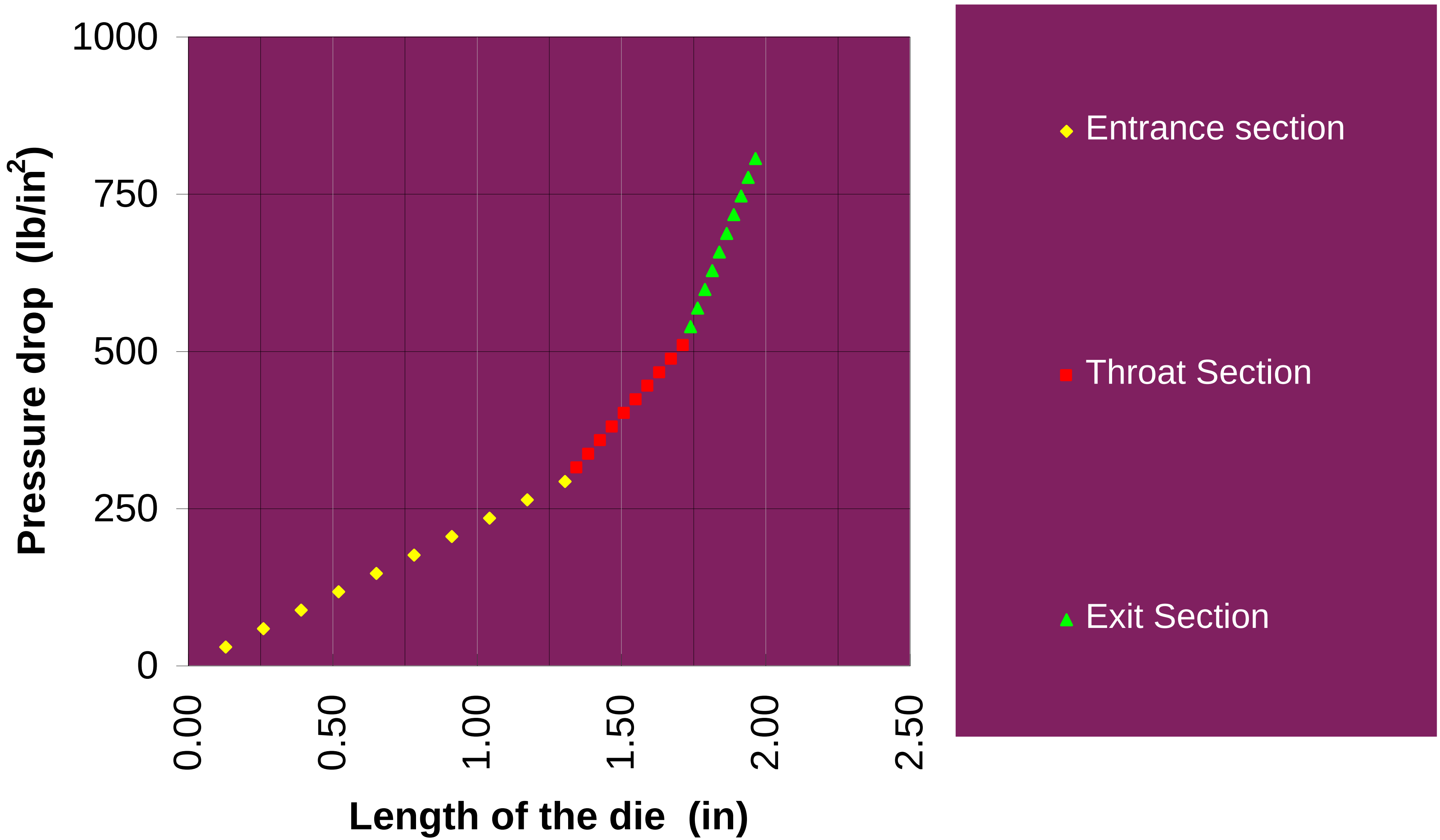
where $K = \frac{\mathrm{Rsmall}}{\mathrm{Rbig}}$

Model capabilities

The user can:

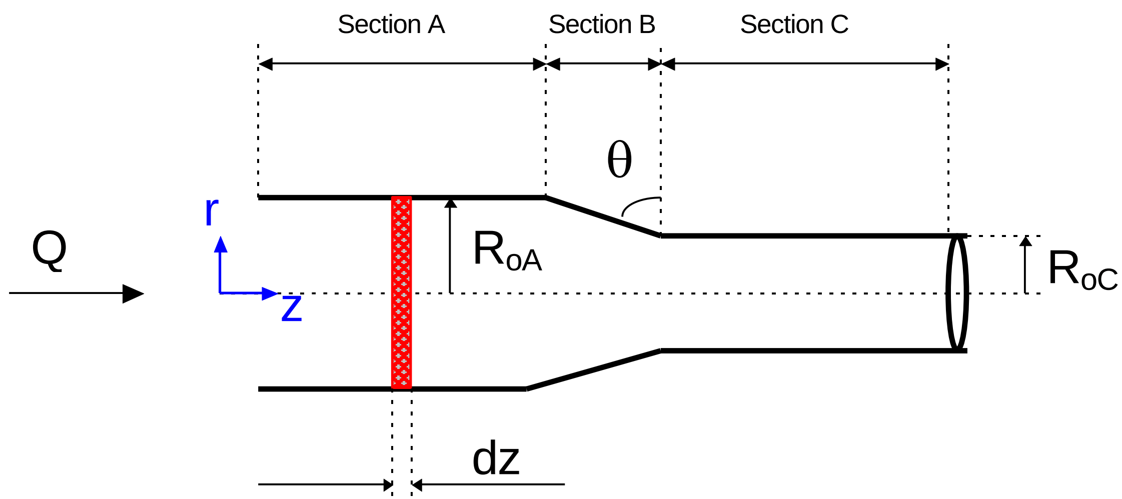
- define the die dimensions or choose a given die from a list of dies.
- set the temperature, in degrees Fahrenheit, for the melt at the entrance and at the exit of the die.
- define the throughput in lb./hr.
- select the resins from a list of resins for which viscosity data has been measured.

Typical Plot



An evaluation of temperature rise of a polymer flowing through a pipe

Model



Momentum balance on a thin (dz) slab:

$$\frac{d(r\tau_{rz})}{dr} = \frac{\Delta P}{dz} r \qquad (1)$$

$$\tau_{rz} = \frac{\Delta P}{dz}\frac{r}{2} \qquad (2)$$

Model assumptions

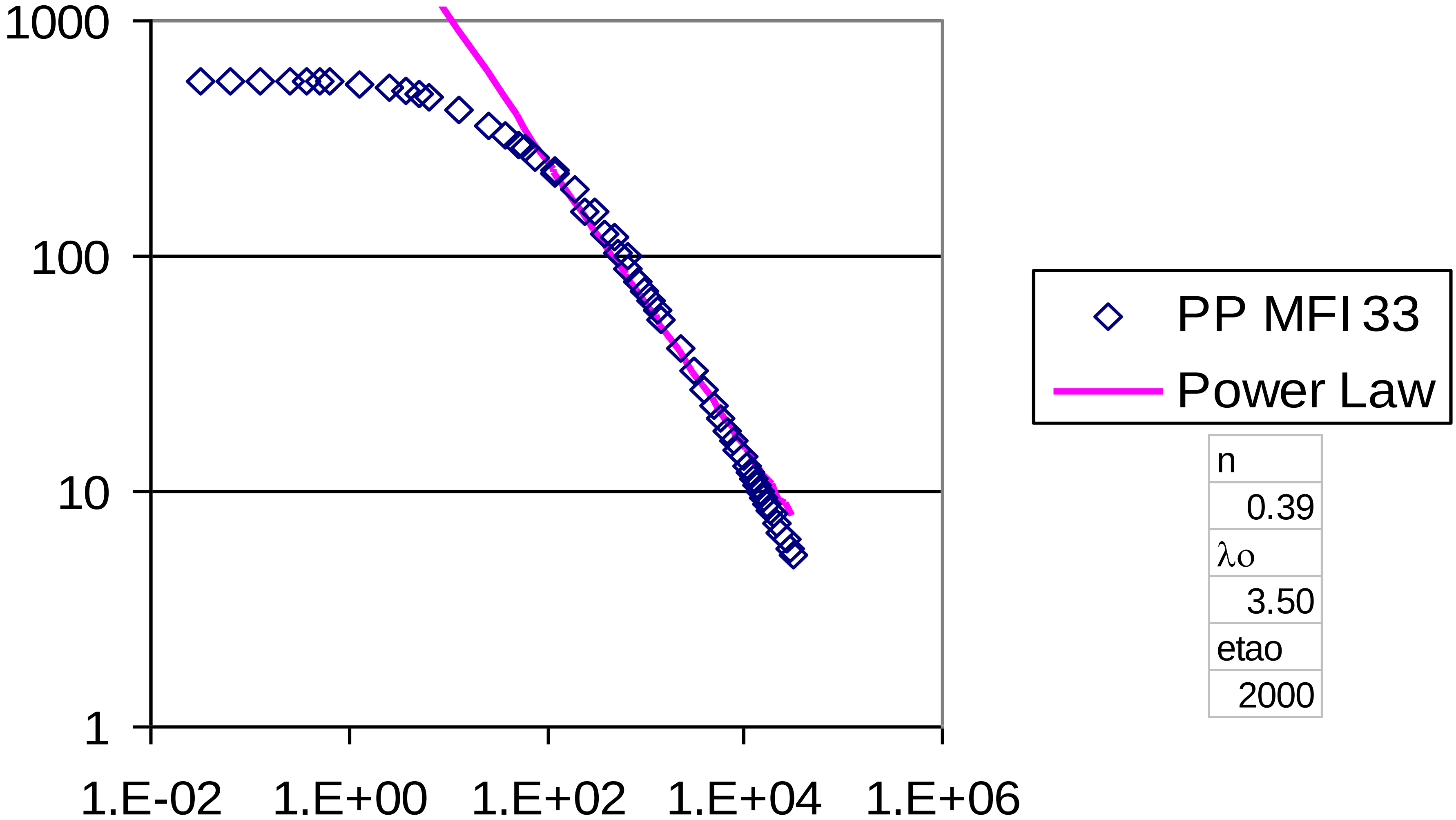
- Laminar flow,
- Heat due to viscous drag,
- Constant radial temperature and,
- No heat is conducted in the axial direction.

Model requirements

- **Actual viscosity data can be used in the model**
- **Viscosity data fit by a power law function:**

$$\eta=\eta_0\left(\frac{\dot{\gamma}}{\dot{\gamma}_0}\right)^{n-1}$$

Viscosity vs. Shear rate



n	
0.39	
λo	
3.50	
etao	
2000	

Model (contd)

$$\tau_{rz} = \frac{\Delta P}{dz}\frac{r}{2} = -\eta\frac{dv_z}{dr} \qquad \textbf{(3)}$$

where $$\eta = \eta_0\left(\frac{\dot{\gamma}}{\dot{\gamma}_0}\right)^{n-1} \qquad \textbf{(4)}$$

For Newtonian liquids $$Q = \langle v_z \rangle \pi r^2 \qquad \textbf{(5)}$$

$$\frac{\Delta P}{dz}\frac{r}{2} = -\eta_o\left(\frac{1}{\dot{\gamma}_o}\right)\dot{\gamma}^{n-1}\dot{\gamma} \qquad \textbf{(6)}$$

$$\frac{\Delta P}{dz}\frac{r}{2} = -\eta_o\left(\frac{1}{\dot{\gamma}_o}\right)^{n-1}\left(\frac{dv_z}{dr}\right)^{n} \qquad \textbf{(7)}$$

Model (contd)

$$\frac{\Delta P}{dz}\frac{r}{2} = -\eta_o\left(\frac{1}{\dot{\gamma}_o}\right)^{n-1}\left(\frac{dv_z}{dr}\right)^{n} \qquad \textbf{(8)}$$

$$\eta_o = \mu_o \exp\left(-\frac{Ea}{R(T-T_o)}\right) \qquad \textbf{(9)}$$

Sustituyendo 9 en 8

$$dv_z = \left[\frac{1}{2}\frac{\Delta P}{dz}\dot{\gamma}_o^{n-1} / [\mu_o \exp(-\frac{Ea}{R(T-To)})]\right]^{1/n} r^{1/n} dr$$

(10)

Integrando

$$v_z = \left[\frac{1}{2}\frac{\Delta P}{dz}\dot{\gamma}_o^{n-1} / [\eta_o \exp(-\frac{Ea}{R(T-To)})]\right]^{s} R^{s+1}\left[(r^{*\,s+1} - 1\right] \qquad \textbf{(11)}$$

Model (contd)

Energy Balance :

$$\rho C_p v_z \frac{dT}{dz} = k\left\{\frac{1}{r}\frac{\partial(r\frac{\partial T}{\partial r})}{\partial r} - \frac{\partial^2 T}{\partial z^2}\right\} + \tau_{rz}\frac{\partial v_z}{\partial r} \qquad (12)$$

$$\rho C_p v_z \frac{dT}{dz} = \tau_{rz}\frac{\partial v_z}{\partial r} \qquad (13)$$

$$\rho C_p v_z \frac{dT}{dz} = -m_o\left\{\exp\left(-\frac{Ea}{R(T-T_o)}\right)\right\}\left(\frac{1}{\dot{\gamma}_o^n}\right)\left(\frac{\partial v_z}{\partial r}\right)^{(n+1)} \qquad (14)$$

Model (contd)

Energy Balance :

Simplificando: para un determinado shear rate $\dot{\gamma} = \frac{dv_z}{dr}$

y evaluando a una velocidad vz promedio $\langle v_z \rangle$

donde

$$\langle v_z \rangle = \frac{\int_0^{2\pi} \int_0^{R} v_z r dr d\theta}{\int_0^{2\pi} \int_0^{R} r dr d\theta} \qquad \textbf{(15)}$$

Sustituir ecuación 11 en 15 y luego 15 en 16

Model (contd)

Energy Balance :

$$\frac{dT}{dz} = -m_o\left\{\exp\left(-\frac{Ea}{R(T-T_o)}\right)\right\}\left(\frac{1}{\dot{\gamma}_o^n}\right)\frac{1}{\rho C_p\langle v_z\rangle}(\dot{\gamma})^{(n+1)}$$

(16)

$$\Delta T = \left[-m_o\left\{\exp\left(-\frac{Ea}{R(T-T_o)}\right)\right\}\left(\frac{1}{\dot{\gamma}_o^n}\right)\frac{1}{\rho C_p\langle v_z\rangle}(\dot{\gamma})^{(n+1)}\right]\Delta z$$

(17)

Donde la ecuación 17 debe evaluarse a varias temperaturas de salida T hasta que haya convergencia. Esto para un dado de longitud L y haciendo las evaluaciones en pequeños intervalos de delta zeta.

Comments

In general:

- **Pressure drop calculations :**
  - **require knowing the Cp, k and ρ as a function of temperature**
  - **the greater the shear rate the higher the viscous dissipation**
  - **is affected by the flow activation energy of the polymer**

Viscous Dissipation

From Table 10.2-2 del Bird

Assumption: Adiabatic

$$\rho C_v v_z \frac{\partial T}{\partial z} = \tau_{rz} \frac{\partial v_z}{\partial r}$$

$$\tau_{rz} = -\eta_o \left(\frac{\frac{dv_z}{dr}}{\dot{\gamma}_o} \right)^{n-1} \frac{dv_z}{dr}$$

$$\rho C_v v_z \frac{\partial T}{\partial z} = -\eta_0 \left(\frac{1}{\dot{\gamma}_0} \right)^{n-1} \frac{\partial v_z}{\partial r}^{n+1}$$

$$v_z \frac{\partial T}{\partial z} = -\frac{\eta_0}{\rho C_v v_z} \left(\frac{1}{\dot{\gamma}_0} \right)^{n-1} \left(\frac{\partial v_z}{\partial r} \right)^{n+1}$$

$$v_z \frac{\partial T}{\partial z} = -\frac{\eta_0}{\rho C_v v_z}\left(\frac{1}{\dot{\gamma}_0}\right)^{n-1}\left(\frac{\partial v_z}{\partial r}\right)^{n+1}$$

$$v_z \frac{\partial T}{\partial z} = -\frac{\eta_0}{\rho C_v v_z}\left(\frac{1}{\dot{\gamma}_0}\right)^{n-1}\left(\frac{\partial v_z}{\partial r}\right)^{n+1}$$

$$\left(\frac{\rho C_v v_z v_z \frac{\partial T}{\partial z}}{\frac{\eta_0}{}\left(\frac{1}{\dot{\gamma}_0}\right)^{n-1}}\right)^{\frac{1}{n+1}} = \frac{\partial v_z}{\partial r}$$

$$\int_0^R \left(\frac{\rho C_v \dfrac{\partial T}{\partial z}}{\dfrac{\eta_0}{\left(\dfrac{1}{\dot{\gamma}_0}\right)^{n-1}}} \right)^{\frac{1}{n+1}} dr = \int \frac{\partial v_z}{\left(v_z\right)^{\frac{2}{n+1}}}$$

$$\int_0^R \left(\dot{\gamma}_0^{n-1} \frac{\rho C_v}{\eta_0} \frac{\partial T}{\partial z} \right)^{\frac{1}{n+1}} dr = \int \frac{\partial v_z}{\left(v_z\right)^{\frac{2}{n+1}}}$$