Fourier Analysis. [Chapters 11.7 - 11.9]

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Modelación física matemática (Ene 19 Gpo 1)
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Fourier Integral

Note: a function can be represented as a Fourier Integral if:

the function is piecewise continuous

f(x) is differentiable

and the integral exists

the representation..

$$f(x) = \int_0^\infty [A(w)\cos wx + B(w)\sin wx]\, dx$$

where

$$A(w) = \frac{1}{\pi}\int_{-\pi}^{\infty} f(v)\cos wv\, dv$$

$$B(w) = \frac{1}{\pi}\int_{-\pi}^{\infty} f(v)\sin wv\, dv$$

target

nonperiodic functions

or

interest in the whole x-axis

such as

rectangular waves

if the function is not continuous..

the Fourier Integral is the average of the right & left limits

to save work...

Fourier Cosine Integral

$$f(x) = \int_0^\pi A(w)\cos wx\, dx$$

where

$$A(w) = \frac{2}{\pi}\int_0^\infty f(v)\cos wv\, dv$$

Fourier Sine Integral

$$f(x) = \int_0^\pi B(w)\sin wx\, dx$$

where

$$B(w) = \frac{2}{\pi}\int_0^\infty f(v)\sin wv\, dv$$

Fourier Cosine Transform

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty \hat{f}_c(w)\cos wx\, dw$$

where

$$\hat{f}_c(w) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos wx\, dx$$

Fourier Sine Transform

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty \hat{f}_s(w)\sin wx\, dw$$

where

$$\hat{f}_s(w) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin wx\, dx$$

lets generalize

Fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\underbrace{\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(w)e^{-iwx}\, dw\right]}_{\text{is the Fourier Transform}} e^{iwx}\, dw$$

other transformations..

discrete Fourier Transform $\xrightarrow{\text{for}}$ sampled values

fast Fourier Transform $\xrightarrow{\text{for}}$ high dense sampling

Kreyszig, E. (2011). Advanced Engineering Mathematics. (M. E. Shannon Corliss, Barbara Russiello, Ed.) (10th ed.). New York: JOHN WILEY & SONS, INC. Retrieved from www.ieee.org.