Behavior of Solids

Summer 2020

Effect of Molecular Weight on Shear Modulus

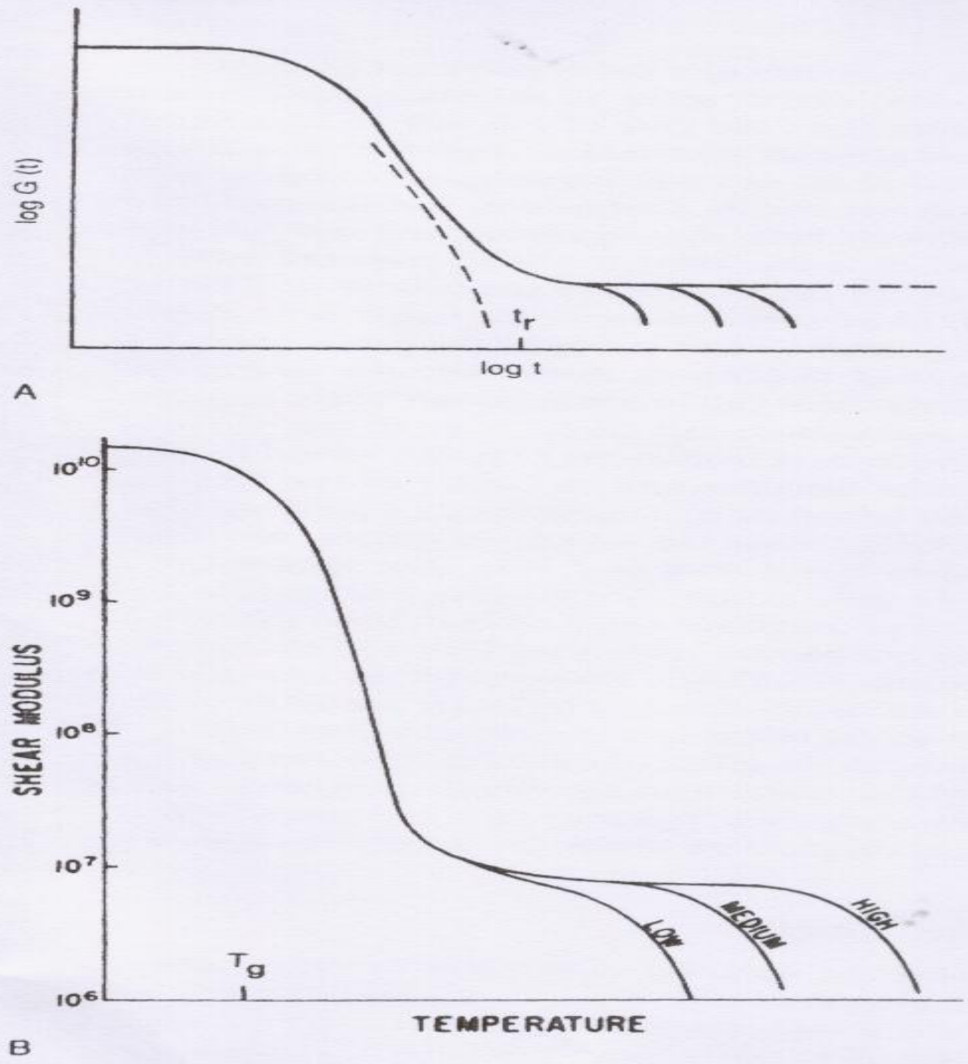


Figure 5 (A) Relaxation modulus at a fixed temperature of a polymer sample: (1) of very low molecular weight (dashed line on left), (2) of moderate to high molecular weight (solid lines), and (3) when cross-linked (dashed line on right). (B) Effect of molecular weight on the modulus–temperature curve of amorphous polymers. Modulus is given in dyn/cm². The characteristic or reference time is t_r; the reference temperature, T_g.

Effect of molecular weight between Crosslinks (Mc) on shear modulus

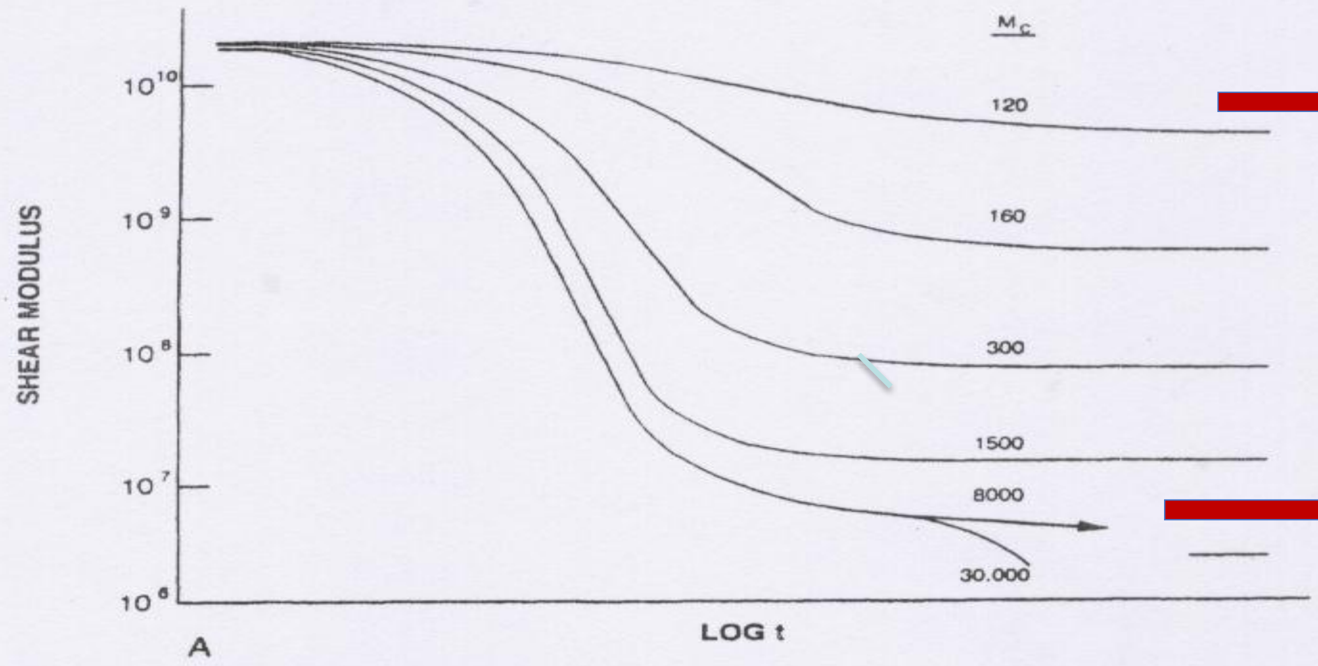
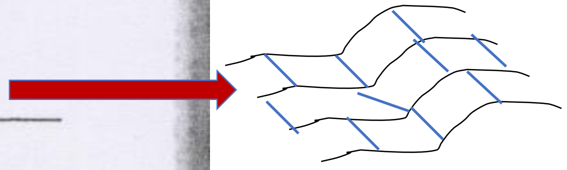
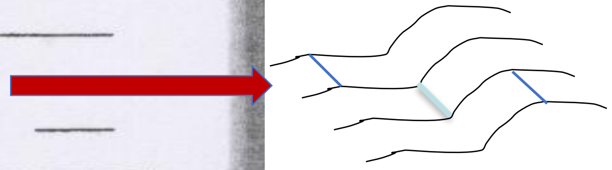


Figure 6 Effect of cross-linking on (A) modulus–time and (B) modulus–temperature curves of amorphous uncross-linked polymers. The numbers on the curves are approximate values of M_c. The value 30,000 refers to an uncross-linked polymer with a molecular weight of roughly 30,000 between entanglement points. Modulus is given in dyn/cm².



The higher the crosslinking the lower the Mc.



You can assume that this is what happens when a rubber is vulcanized, the more crosslinking material you add the shorter the distance between crosslinks

Effect of the degree of crystallinity on the shear modulus of a solid

The higher the crystals content, the more difficult is to shear the material.
The size and shape of the crystals influence the shear modulus

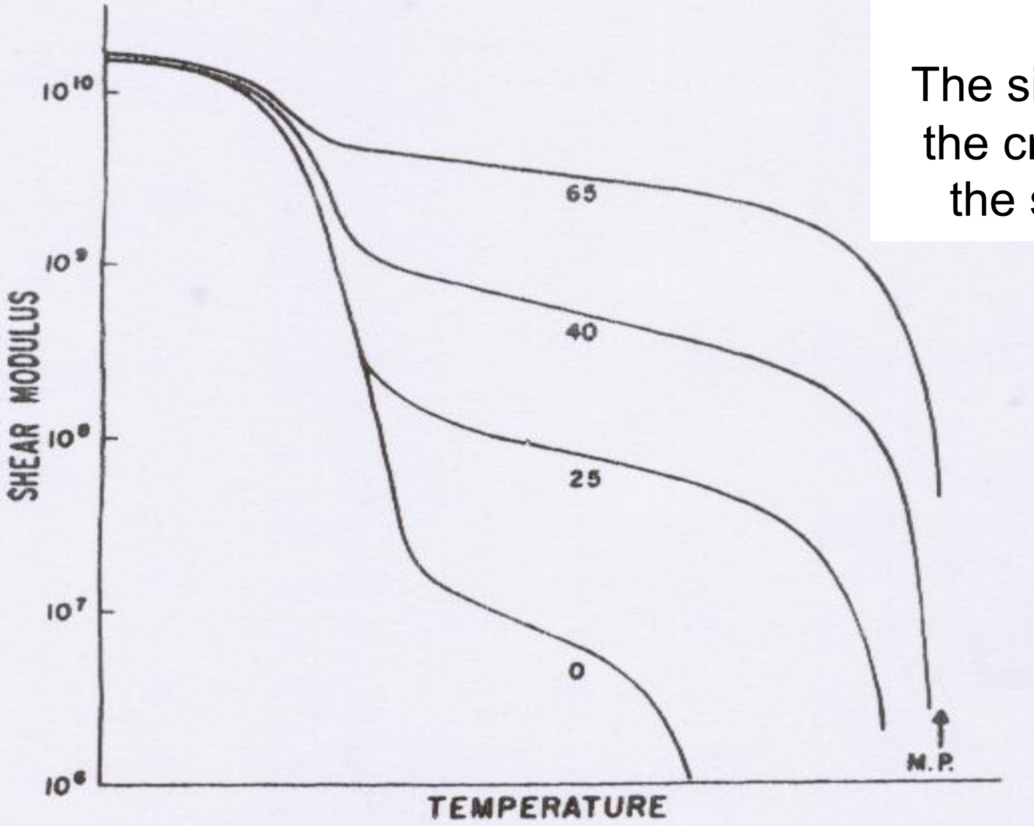


Figure 7 Effect of crystallinity on the modulus–temperature curve. The numbers on the curves are rough approximations of the percent of crystallinity. Modulus is given in dyn/cm².

Effect of temperature on the stress relaxation of a solid and its use to create a master curve

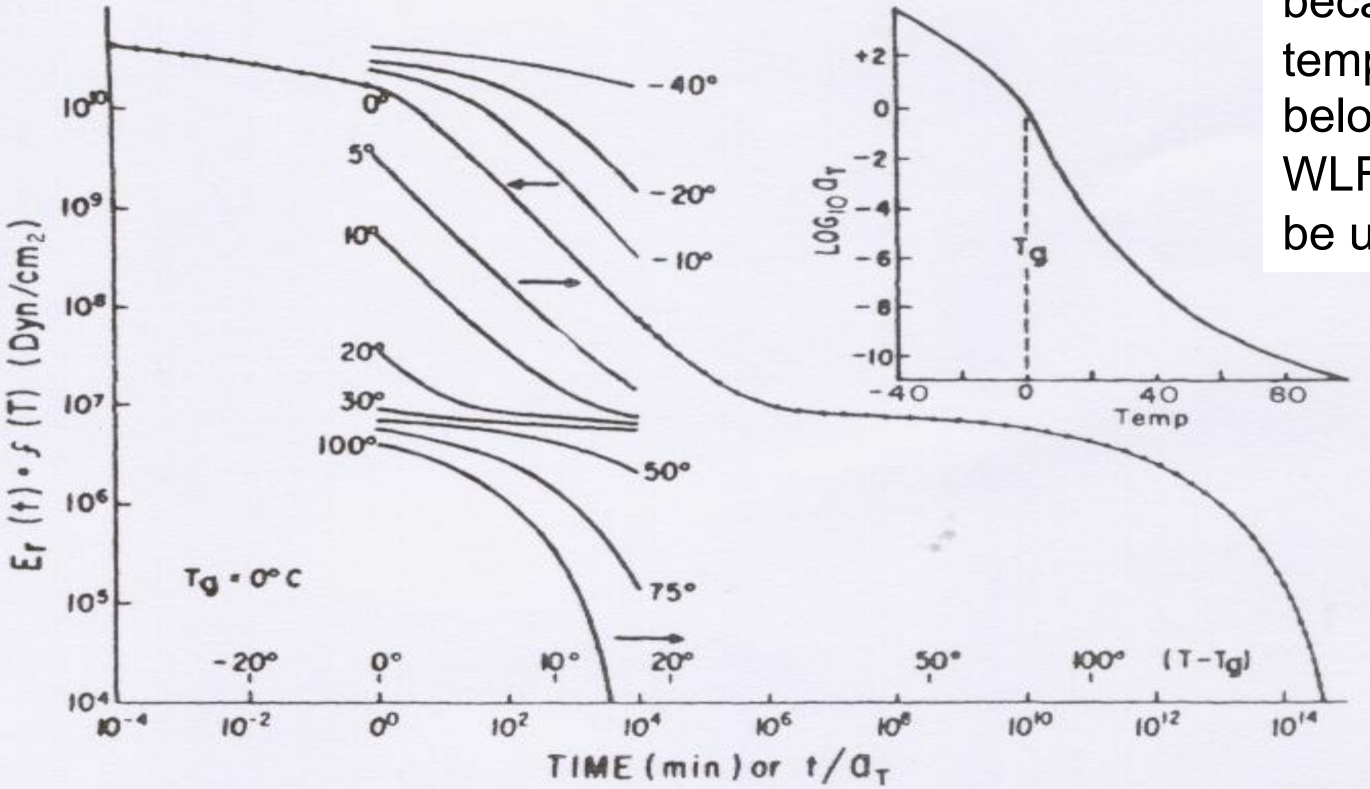


Figure 8 WLF time–temperature superposition applied to stress-relaxation data obtained at several temperatures to obtain a master curve. The master curve, made by shifting the data along the horizontal axis by amounts shown in the insert for a_T, is shown with circles on a line.

Remember that because the temperatures are below Tg+100, then the WLF equation should be used

Behavior when a constant load is applied to a solid (creep)

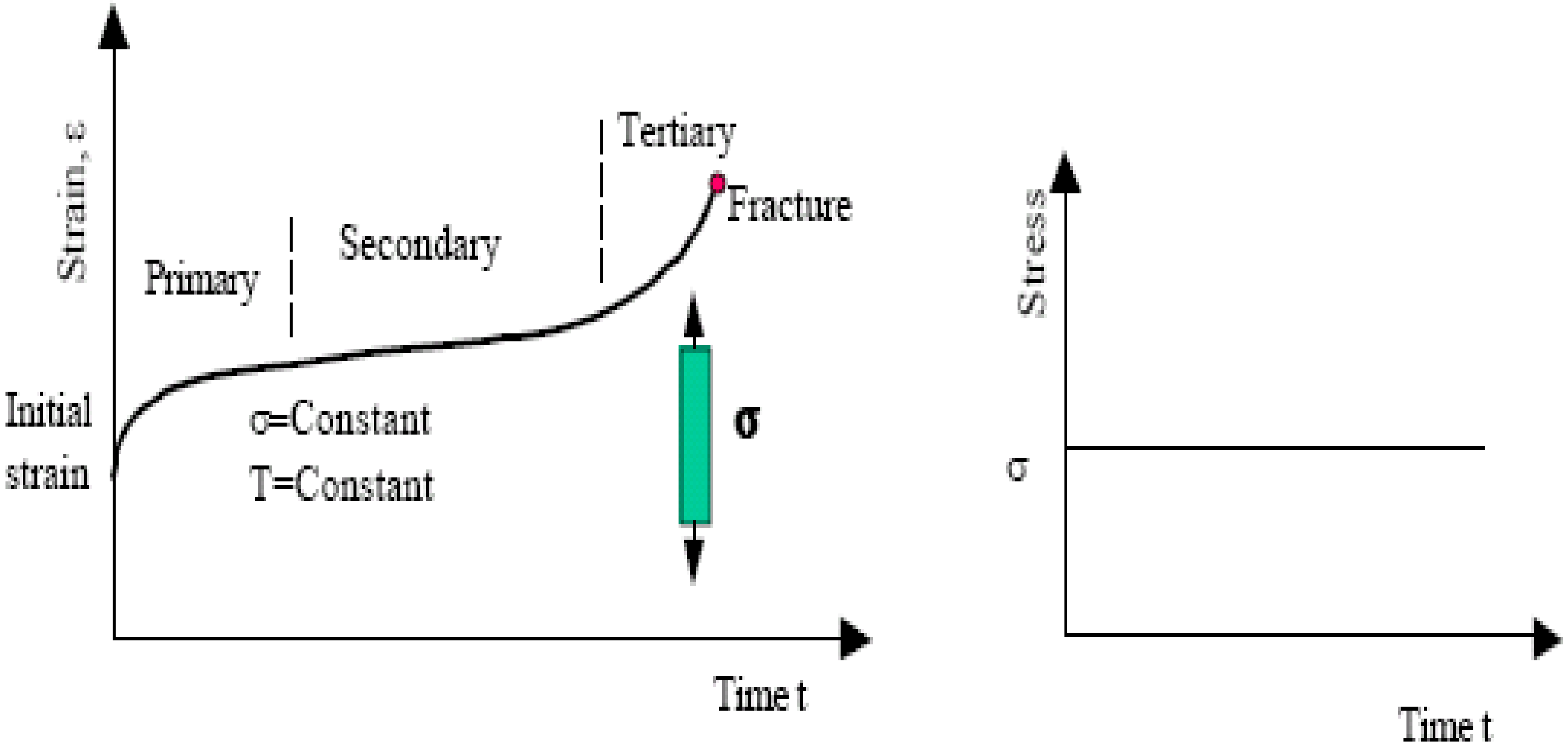


Figure 2. Creep curve for plastics, a constant load is applied [1]

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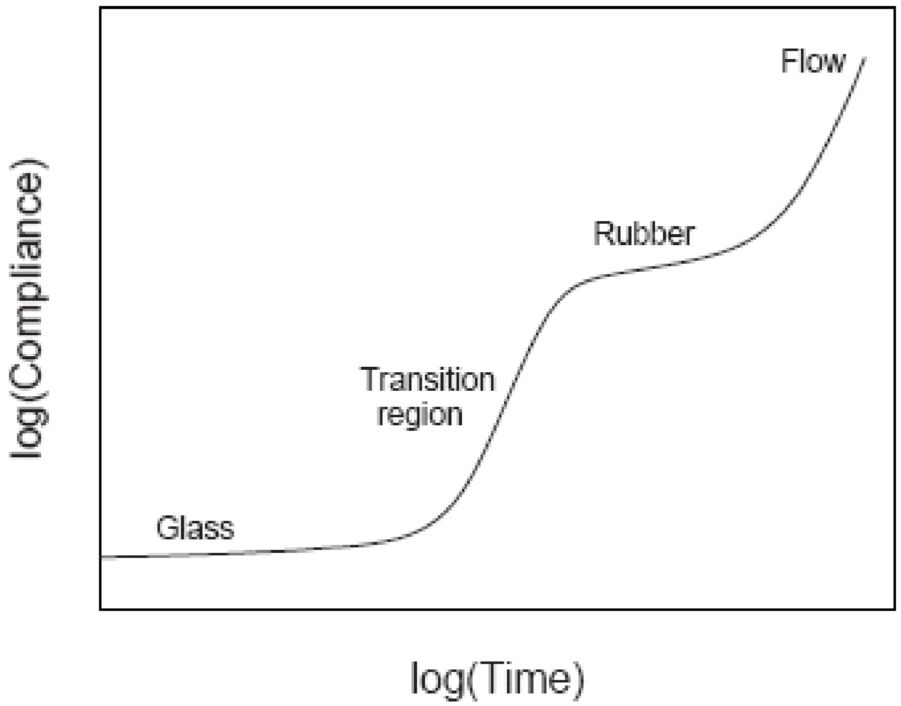


Schematic representation of the linear creep compliance versus time for a polymer glass at a fixed temperature.

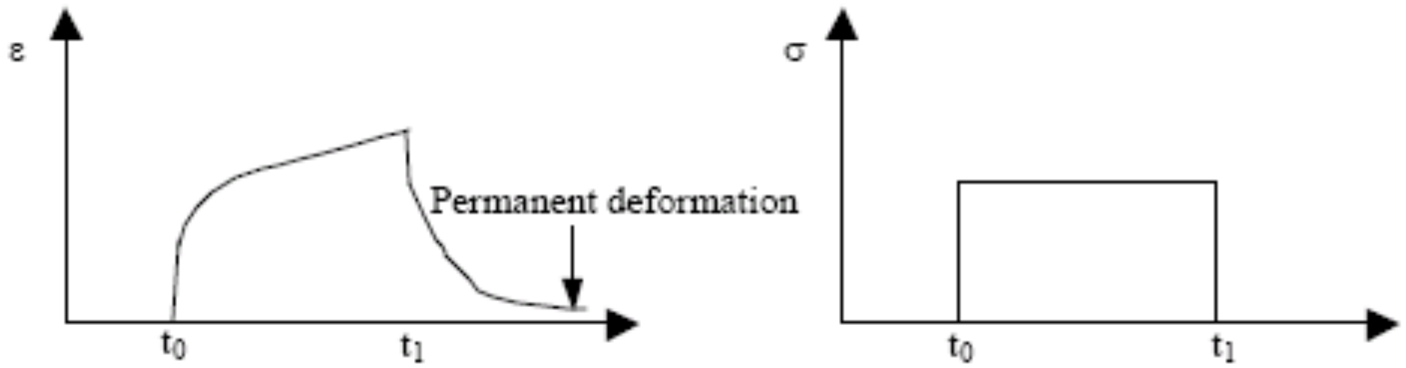
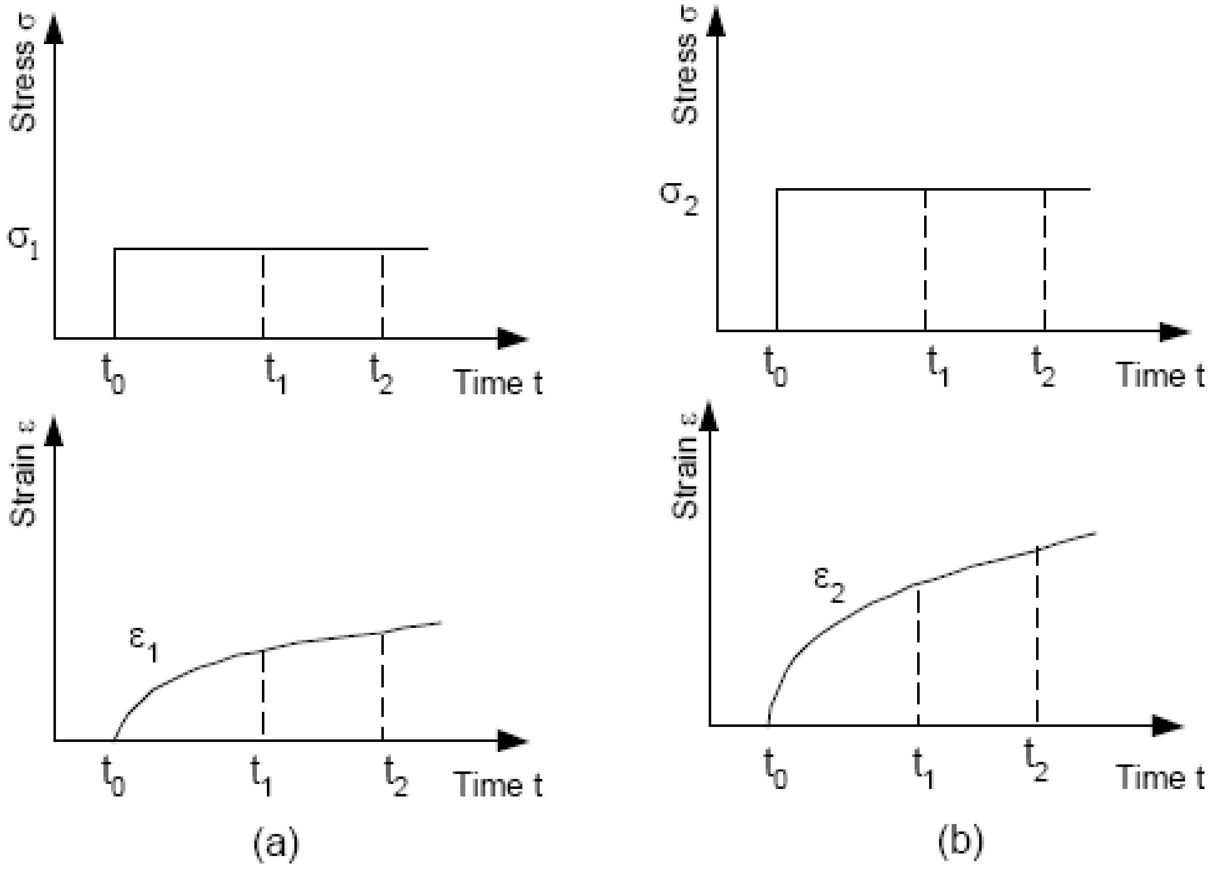


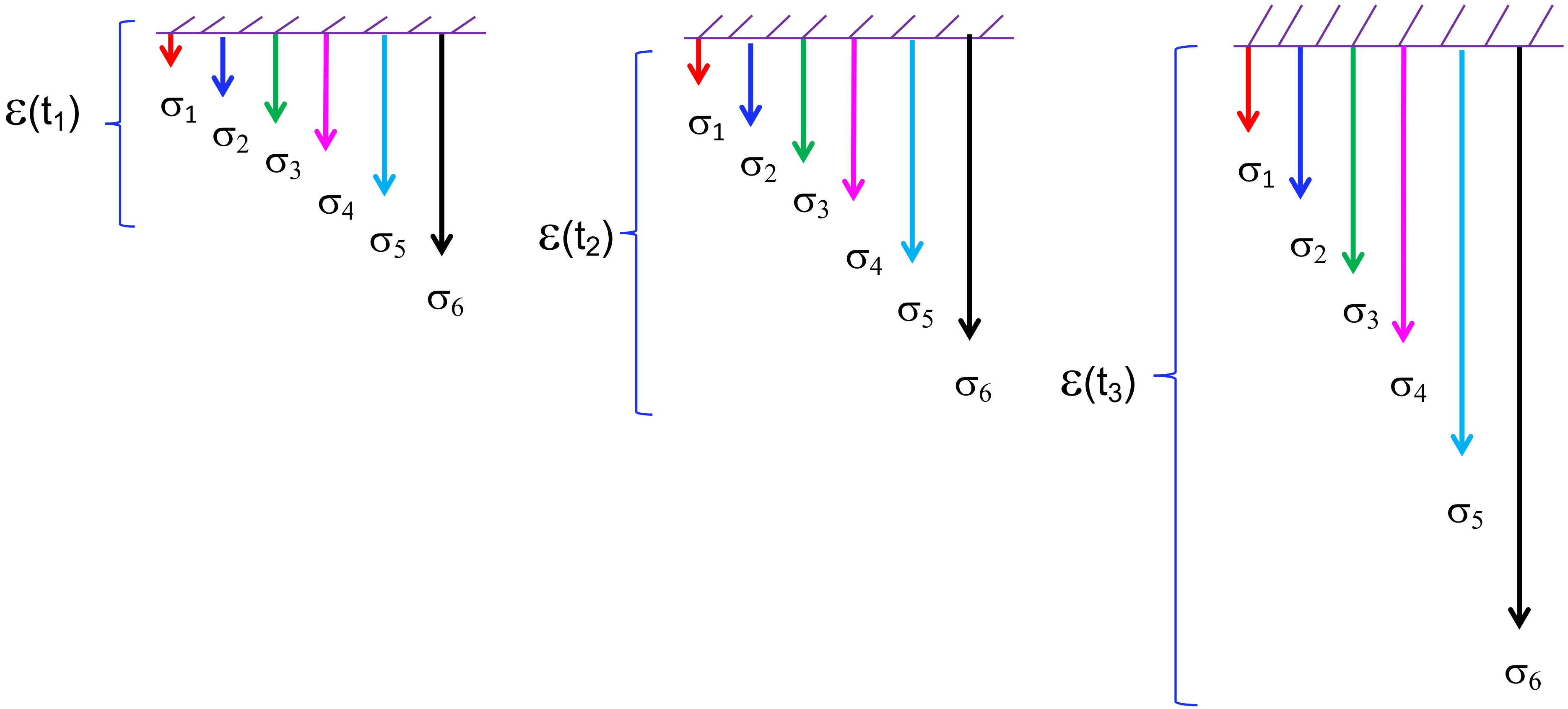
Figure 3. Creep curve with recovery. A constant load is applied at t_0 and removed at t_1

Applied stress and resulting strain

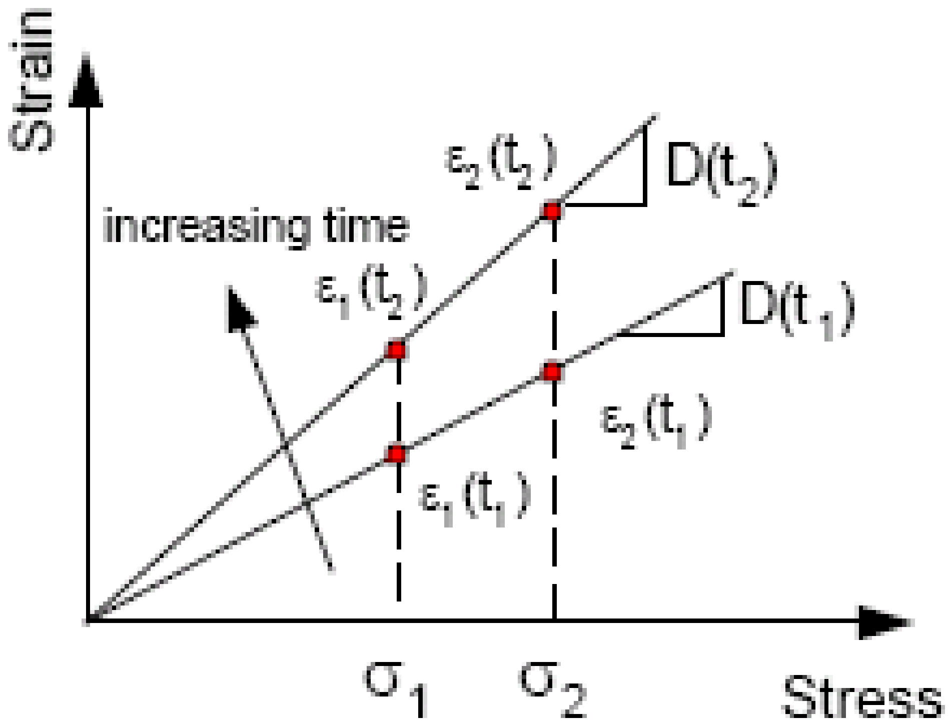


$$\underbrace{\frac{\text{Strain}}{\text{Stress}}}_{\textit{Compliance}} = \mathbf{D}$$

Creating Isochrones



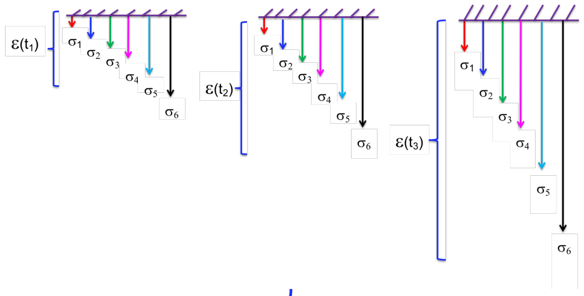
Building isochrones using information obtained from constant stress-strain curves *(see previous slide)*



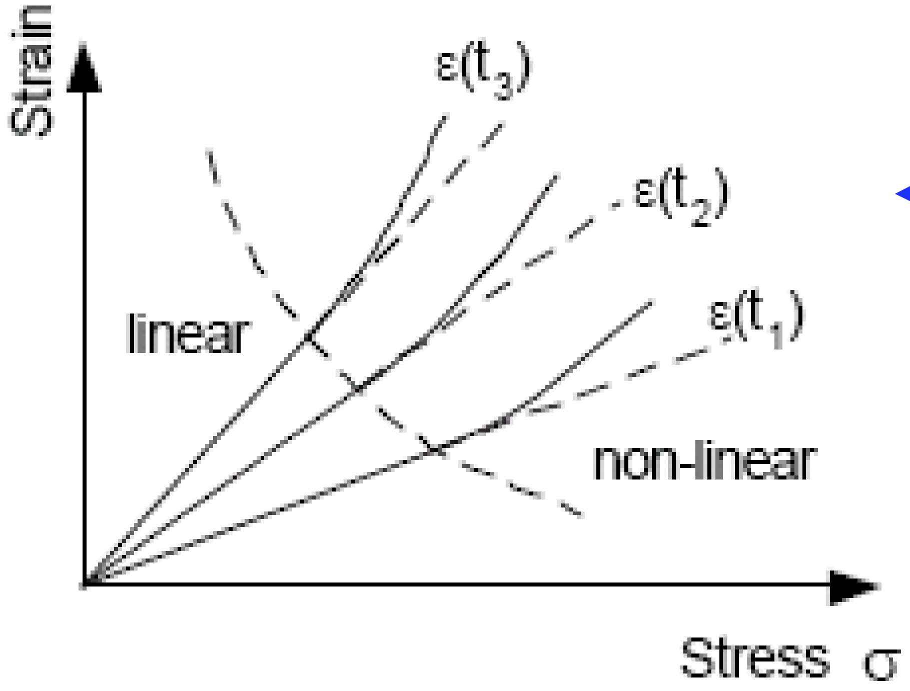
$$\frac{\varepsilon_1(t)}{\sigma_1(t)} = \frac{\varepsilon_2(t)}{\sigma_2(t)}$$

$$D(t) = \varepsilon(t) / \sigma$$

Isochrones and onset of the non-linear behavior



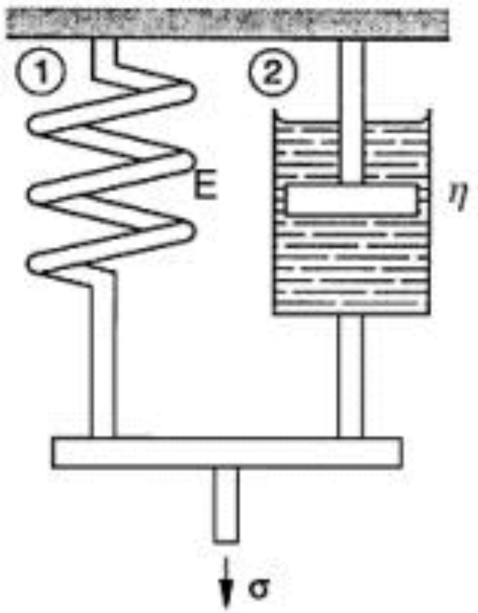
$t_3 > t_2 > t_1$



Linear-nonlinear transition of stress strain relationship with respect to different time levels [7]

With lower stresses the non- linear behavior starts at higher times

Modeling the compliance with the Kelvin Model



Schematic diagram of Kelvin model [16]

$$\sigma = E\varepsilon + \eta\dot{\varepsilon},$$

$$\varepsilon(t) = \frac{\sigma_0}{E}(1 - e^{-t/\tau})$$

$$\tau = \eta / E$$

Tau has units of time and is called the retardation time.

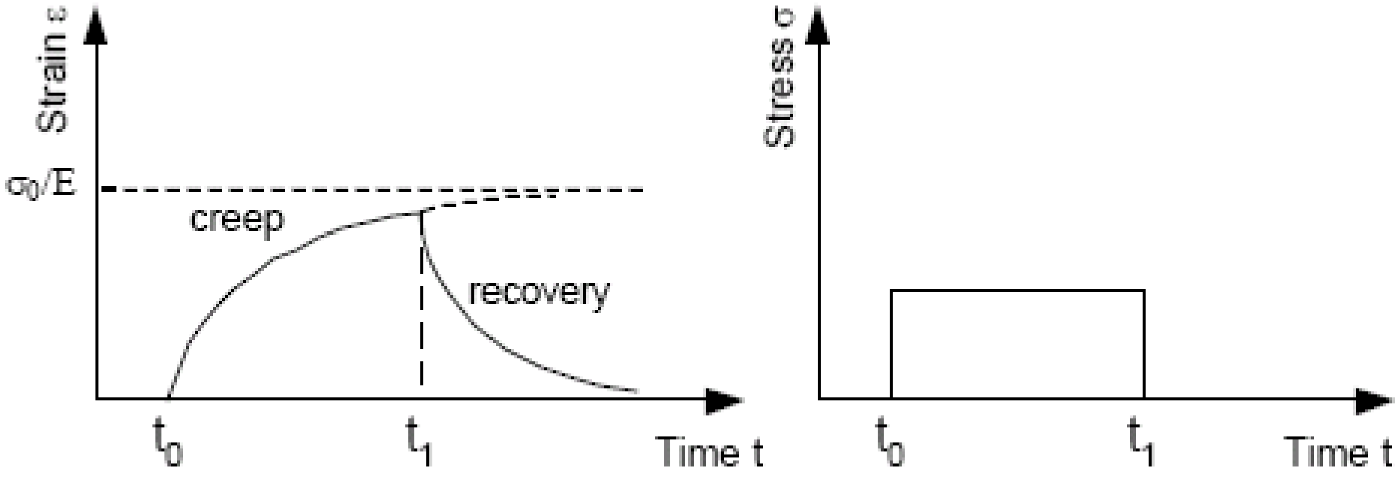


Figure 7. Creep and creep recovery response of Kelvin model

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Stress superposition principle

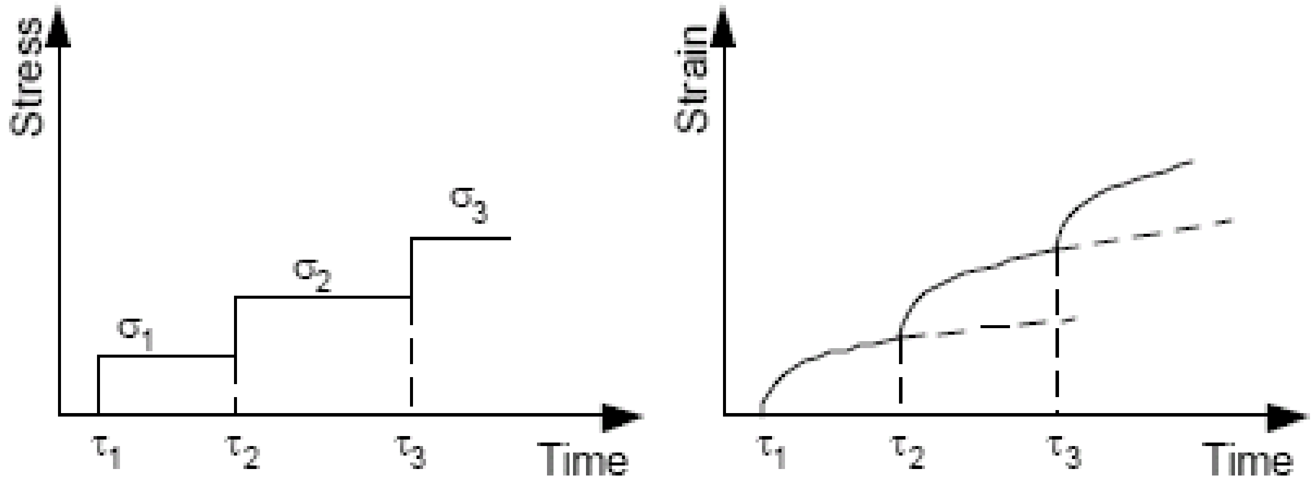


Figure 9. Boltzmann superposition principle

$$\varepsilon(t) = D(t-\tau_1)\sigma_1 + D(t-\tau_2)(\sigma_2-\sigma_1) + \cdots + D(t-\tau_i)(\sigma_i-\sigma_{i-1})$$

$$\varepsilon(t) = \int_{-\infty}^{t} D(t-\tau)d\sigma(t)$$

Design criteria

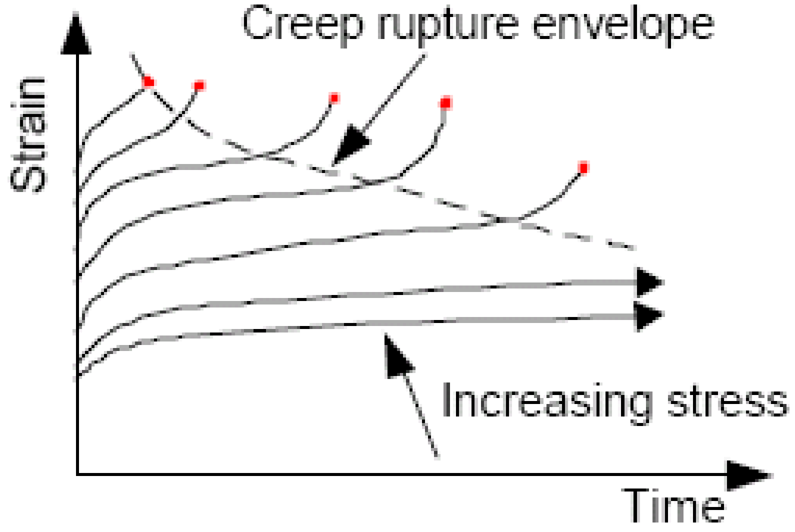


Figure 4. Creep rupture envelope [1]

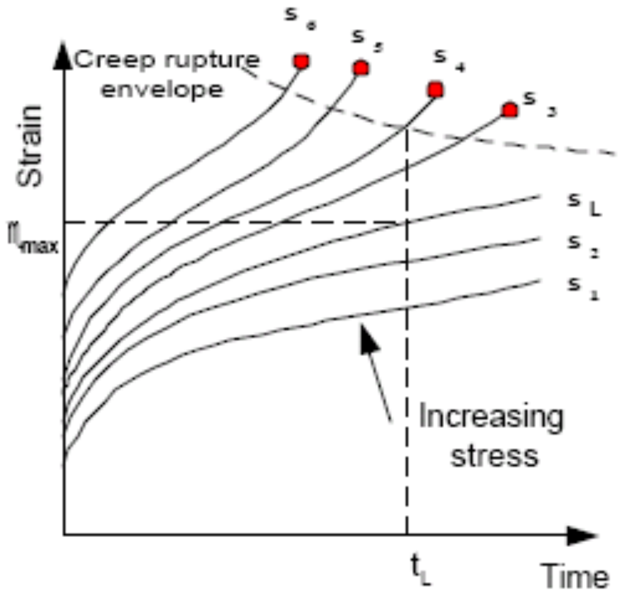


Figure 5. Design criteria by creep curves

Stress Relaxation Modulus for Solid PIB

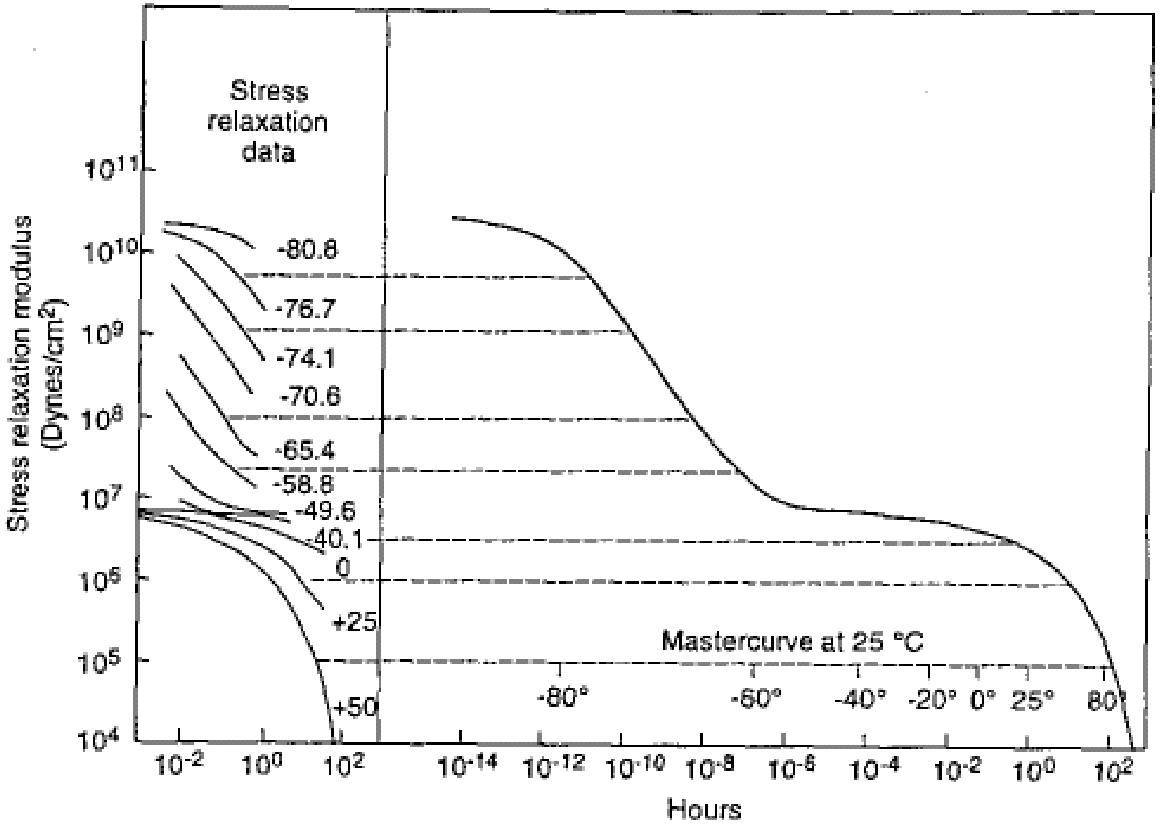
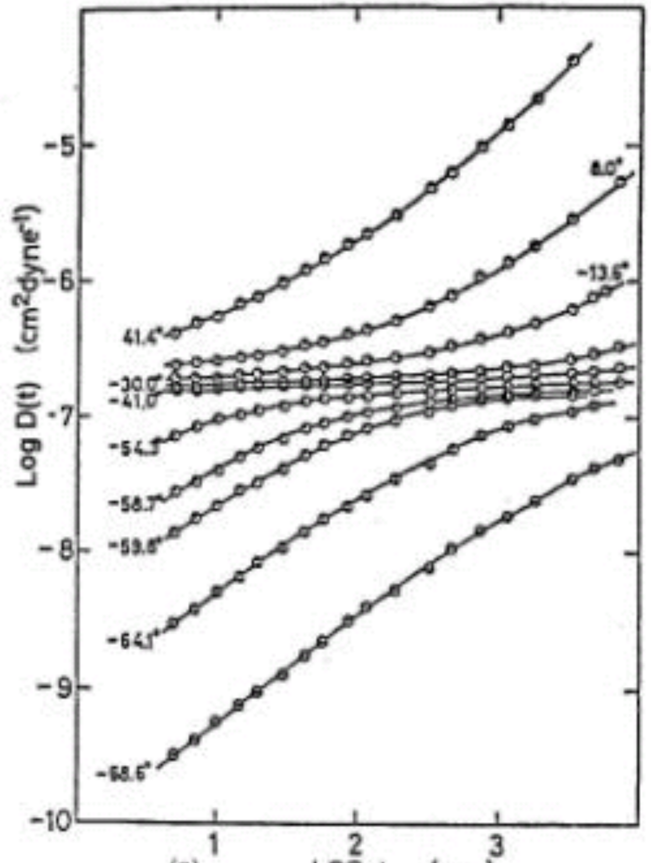


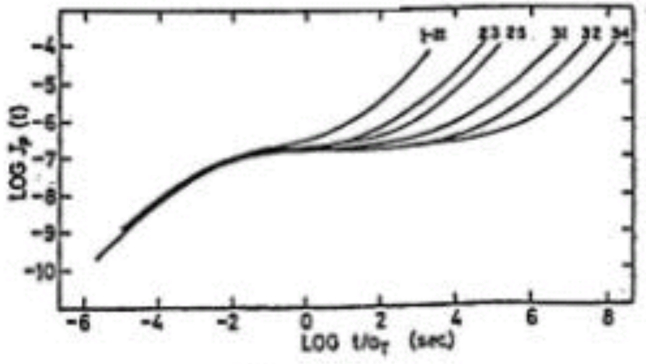
Figure 10. Relaxation modulus curves for polyisobutylene and corresponding master curve at 25 ^{0}C [24]

The effect of the average molecular weight on the master curve

Mw Increases



(a)

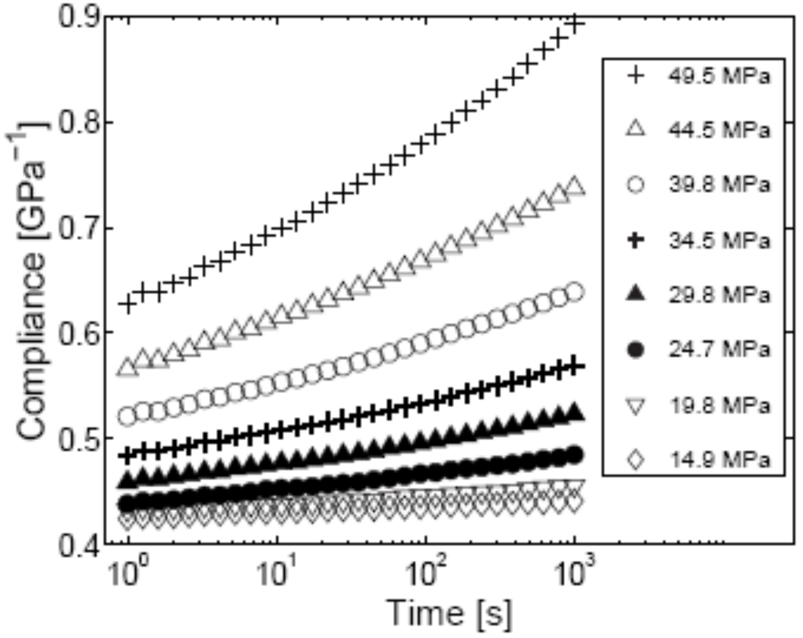


(b)

Curve Number	Mw
I-21	5.76×10^4
23	1.03×10^5
25	1.59×10^5
31	3.95×10^5
32	6.20×10^5
34	1.12×10^6

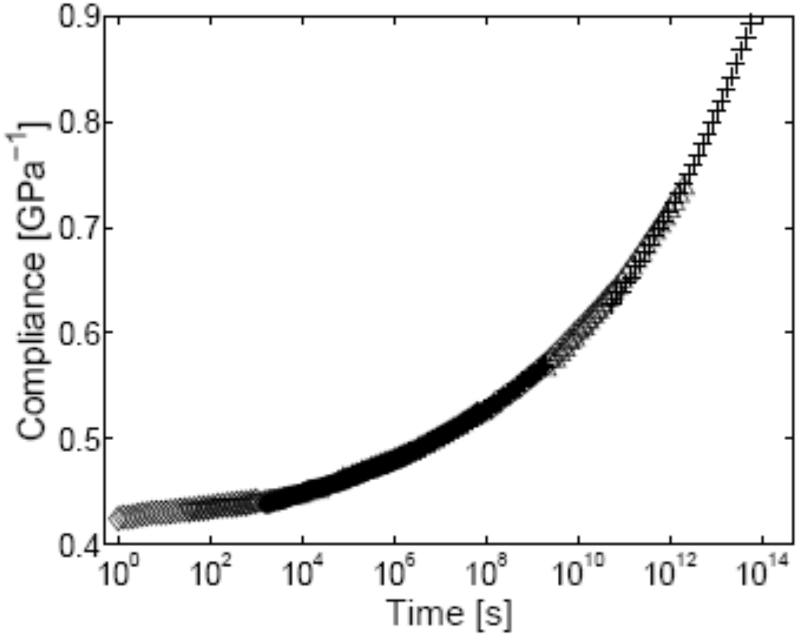
Figure 11. (a) Creep compliance of polyisoprene at different temperatures. Data for molecular wieght $1.12*10^6$; (b) Master curves for creep compliance of Polyisoprene with different molecular weight at reference temperature of -30^0C [26]

σ [MPa]	$\log(a_{15})$
14.9	0.00
19.8	-1.55
24.7	-3.25
29.8	-4.80
34.5	-6.25
39.8	-7.80
44.5	-9.30
49.5	-10.75



(a)

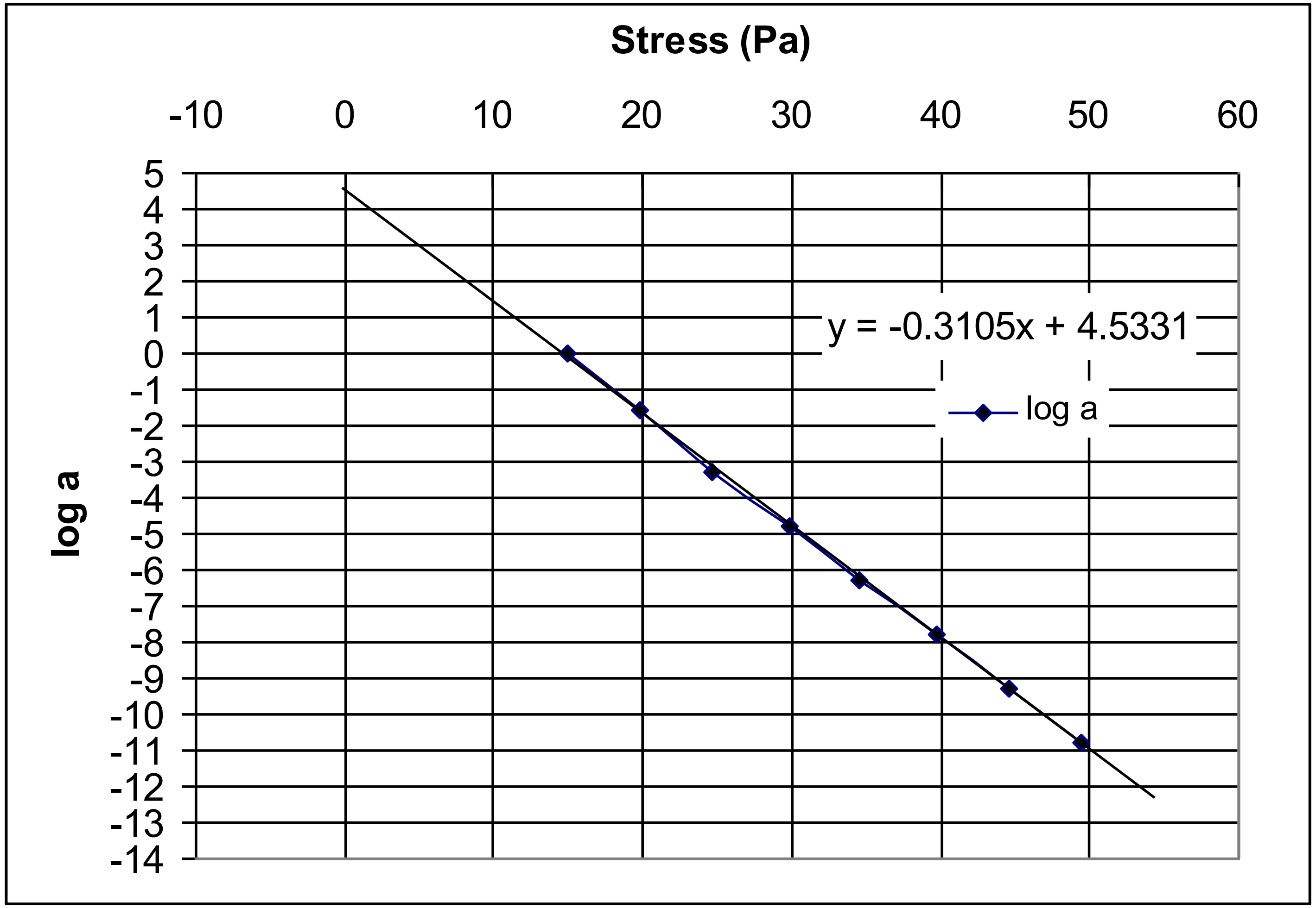
Creep compliance of policarbonate at 22^{o}C and at several loads



(b)

Creep compliance master curve at a reference stress of14.9 Mpa (does not consider aging)

Shift factor for the stress-time superposition



Stres-strain-time

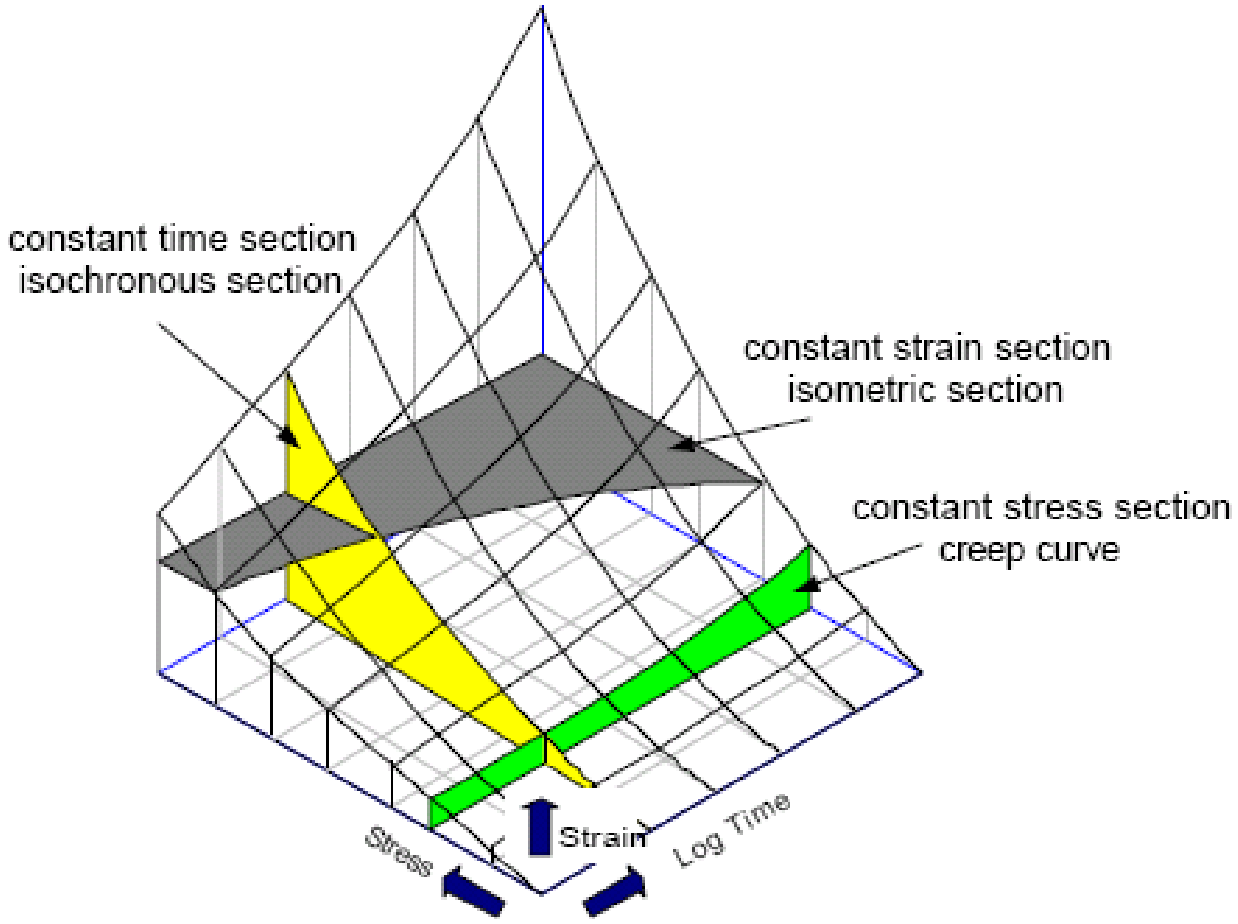
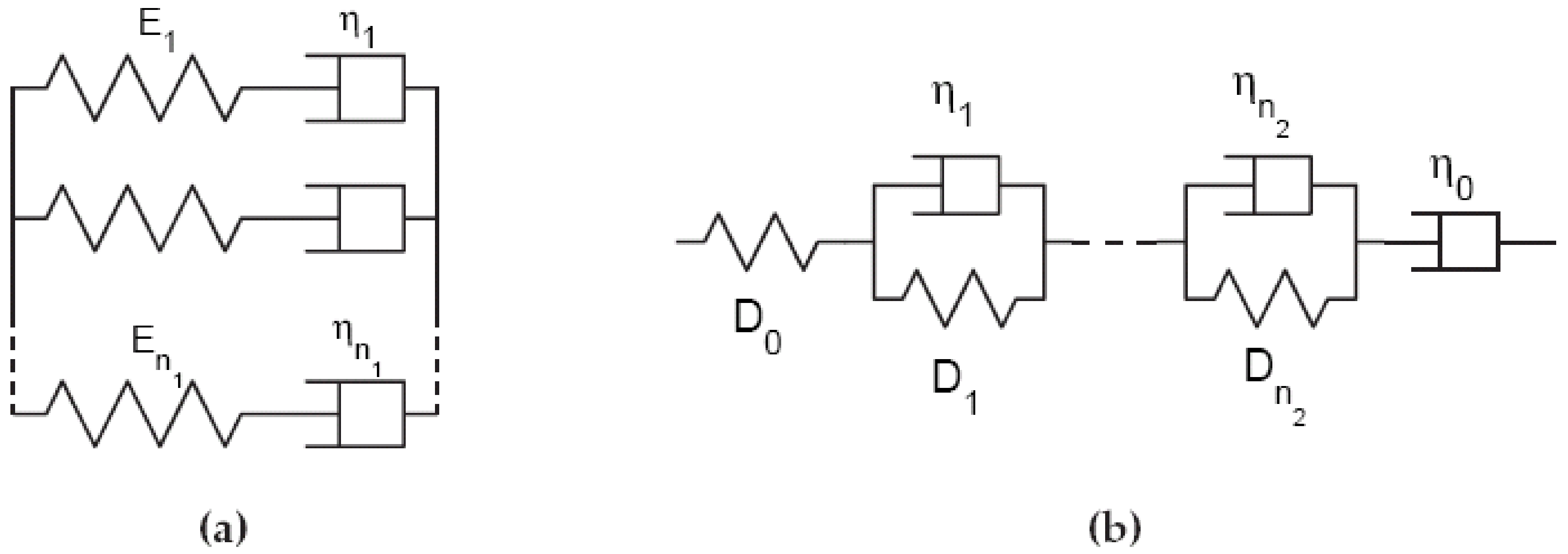


Figure 1. Constant stress-strain-time coordinates [1]

Modeling the creep compliance with more than one Kelvin-Voight Model

$$E(t) = \sum_{i=1}^{n_1} E_i \exp\left(-\frac{t}{\tau_i}\right)$$

$$D(t) = D_0 + \sum_{i=1}^{n_2} D_i \left[1 - \exp\left(-\frac{t}{\tau_i}\right)\right] + \frac{t}{\eta_0}$$



Schematic representation of the generalized linear Maxwell model (a) and the generalized Kelvin-Voigt model (b).

Comparing relaxation modulus vs. Compliance modulus models

Relaxation modulus

$$E(t) = E_0 \exp\left(-\frac{t}{\tau(\sigma)}\right)$$

$$\tau(\sigma) = \tau_0 a_\sigma(\sigma)$$

$$\tau_0 = \eta_0 / E_0$$

For "n_1" elements

$$E(t) = \sum_{i=1}^{n_1} E_i \exp\left(-\frac{t}{\tau_i(\sigma)}\right)$$

Compliance modulus

$$D(t) = D_0 + \frac{t}{\eta(\sigma)}$$

$$D_0 = 1/E_0$$

For "n_2" elements

$$D(t) = D_0 + \sum_{i=1}^{n_2} D_i \left[1 - \exp\left(-\frac{t}{\tau_i(\sigma)}\right)\right] + \frac{t}{\eta_0(\sigma)}$$

Pseudo-Elastic Design Method

Supuestos:

i) La deformaciones son pequeñas

ii) El modulo es constante

iii) La deformación es independiente de la rapidez con que se aplique el stress o la historia de los esfuerzos aplicados y la recuperación es inmediata

iv) El material es isotrópico

v) El material se comporta de la misma manera en tensión que en compresión.