PROBLEMS

2.1 An ideal gas at 300 K has a volume of 15 liters at a pressure of 15 atm. Calculate (1) the final volume of the system, (2) the work done by the system, (3) the heat entering or leaving the system, (4) the change in the internal energy, and (5) the change in the enthalpy when the gas undergoes

a. A reversible isothermal expansion to a pressure of 10 atm
b. A reversible adiabatic expansion to a pressure of 10 atm

The constant volume molar heat capacity of the gas, c_v, has the value 1.5 R.

2.2 One mole of a monatomic ideal gas, in the initial state $T = 273$ K, $P = 1$ atm, is subjected to the following three processes, each of which is conducted reversibly:

a. A doubling of its volume at constant pressure,
b. Then a doubling of its pressure at constant volume,
c. Then a return to the initial state along the path $P = 6.643 \times 10^{-4}V^2 + 0.6667$.

Calculate the heat and work effects which occur during each of the three processes.

2.3 The initial state of a quantity of monatomic ideal gas is $P = 1$ atm, $V = 1$ liter, and $T = 373$ K. The gas is isothermally expanded to a volume of 2 liters and is then cooled at constant pressure to the volume V. This volume is such that a reversible adiabatic compression to a pressure of 1 atm returns the system to its initial state. All of the changes of state are conducted reversibly. Calculate the value of V and the total work done on or by the gas.

2.4 Two moles of a monatomic ideal gas are contained at a pressure of 1 atm and a temperature of 300 K. 34,166 joules of heat are transferred to the gas, as a result of which the gas expands and does 1216 joules of work against its surroundings. The process is reversible. Calculate the final temperature of the gas.

2.5 One mole of N_2 gas is contained at 273 K and a pressure of 1 atm. The addition of 3000 joules of heat to the gas at constant pressure causes 832 joules of work to be done during the expansion. Calculate (a) the final state of the gas, (b) the values of ΔU and ΔH for the change of state, and (c) the values of c_v and c_p for N_2. Assume that nitrogen behaves as an ideal gas, and that the above change of state is conducted reversibly.

2.6 Ten moles of ideal gas, in the initial state $P_1 = 10$ atm, $T_1 = 300$ K, are taken round the following cycle:

a. A reversible change of state along a straight line path on the P-V diagram to the state $P = 1$ atm, $T = 300$ K,
b. A reversible isobaric compression to $V = 24.6$ liters, and
c. A reversible constant volume process to $P = 10$ atm.

How much work is done on or by the system during the cycle? Is this work done on the system or by the system?

2.7 One mole of an ideal gas at 25°C and 1 atm undergoes the following reversibly conducted cycle:

a. An isothermal expansion to 0.5 atm, followed by
b. An isobaric expansion to 100°C, followed by
c. An isothermal compression to 1 atm, followed by
d. An isobaric compression to 25°C.

The system then undergoes the following reversible cyclic process.

a. An isobaric expansion to 100°C, followed by
b. A decrease in pressure at constant volume to the pressure P atm, followed by
c. An isobaric compression at P atm to 24.5 liters, followed by
d. An increase in pressure at constant volume to 1 atm.

Calculate the value of P which makes the work done on the gas during the first cycle equal to the work done by the gas during the second cycle.

2.8 Two moles of an ideal gas, in an initial state $P = 10$ atm, $V = 5$ liters, are taken reversibly in a clockwise direction around a circular path give by $(V - 10)^2 + (P - 10)^2 = 25$. Calculate the amount of work done by the gas as a result of the process, and calculate the maximum and minimum temperatures attained by the gas during the cycle.

2.1

$T_1 = 300K$

$V_1 = 15 l$

$P_1 = 15 atm$

$$n_1 = \frac{P_1 V_1}{RT_1} = \frac{(15)(15)}{(0.08206)(300)} = 9.14 \text{ mol}$$

์ตr $V_2, \omega, q, \Delta U, \Delta H$.

a) Reversible Isothermal $P_2 = 10$ atm.

$$V_2 = \frac{nRT_2}{P_2} = \frac{(9.14)(0.08206)(300)}{10} = 22.5 \ l$$

$$\omega = \int PdV = nRT \ln\left(\frac{V_2}{V_1}\right) = (9.14)(8.314)(300) \ln\left(\frac{22.5}{15}\right) = 9243 \text{ J}$$

$$\Delta U = 0 = q - \omega \rightarrow q = \omega = 9243 \text{ J}.$$

$$\Delta H = n\int C_p dT = 0.$$

b) Reversible Adiabatic $P_2 = 10$ atm $\quad (P_1 V_1^{\gamma} = P_2 V_2^{\gamma})$

$$\Delta U = q - \omega = -\omega = -\int PdV = -\int \frac{nRT}{V} dV$$

$$= n\int C_v dT = n\left(\tfrac{3}{2}\right)(R)(T_2 - T_1) = (9.14)\left(\tfrac{3}{2}\right)(8.314)(T_2 - 300K)$$

$$V_2 = \left(\frac{P_1 V_1^{\gamma}}{P_2}\right)^{\frac{1}{\gamma}} = \left(\frac{(15)(15)^{5/3}}{10}\right)^{3/5} = 19.7 \ l$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{(10)(19.7)}{(9.14)(0.08206)} = 255 \text{ K}$$

$$\Delta U = -\omega = (9.14)\left(\tfrac{3}{2}\right)(8.314)(255 - 300) = -5145 \text{ J}$$

$$\Delta H = n\int C_p dT = (9.14)\left(\tfrac{5}{2}\right)(8.314)(255-300) = -8575 \text{ J}$$

2.2

$T_1 = 273 \text{ K}$

$P_1 = 1 \text{ atm}$

$n = 1 \text{ mol}$

$$V_1 = \frac{nRT_1}{P_1} = \frac{(1)(0.08206)(273)}{1} = 22.4 \ \ell$$

a) Isobaric $V_2 = 2V_1 = 44.8$

$q, w = ?$

$\Delta U_1 = q - w$

$= 5674 - 2270$

$= 3404 \text{ J}$

$$q = n\int C_p dT = (1)\left(\frac{5}{2}\right)(8.314)(T_2 - T_1)$$

$$T_2 = \frac{P_2V_2}{nR} = \frac{(1)(44.8)}{(1)(0.08206)} = 546 \text{ K}$$

$$q = (1)\left(\frac{5}{2}\right)(8.314)(546 - 273) = 5674 \text{ J.}$$

$$w = \int P dV = P(V_2 - V_1) = 1(44.8 - 22.4)(101.325) = 2270 \text{ J.}$$

b) $V_3 = V_2$, $P_3 = 2P_2 = 2(1) = 2\text{atm}$

$\Delta U_2 = 6809$

$$q = n\int C_v dT = (1)\left(\frac{3}{2}\right)(8.314)(T_3 - T_2)$$

$$T_3 = \frac{P_3V_3}{nR} = \frac{(2)(44.8)}{(1)(0.08206)} = 1092 \text{ K.}$$

$$q = (1)\left(\frac{3}{2}\right)(8.314)(1092 - 546) = 6809 \text{ J.}$$

$$w = \int P dV = 0$$

c) $P_4 = P_1 = 1 \text{ atm}$, $V_4 = V_1 = 22.4 \ \ell$, $T_4 = 273 \text{ K}$

$$w = \int P dV = \int \left((6.643\times10^{-4})V^2 + 0.6667\right) dV = \frac{6.643\times10^{-4}}{3}\left[V^3\right]_{44.8}^{22.4} + 0.6667\left[V\right]_{44.8}^{22.4}$$

$$= -32.36 \text{ atm-}\ell = -3278 \text{ J}$$

$$\Delta U = 0 = \Delta U_1 + \Delta U_2 + \Delta U_3 \Rightarrow \Delta U_3 = -(\Delta U_1 + \Delta U_2) = -(3404 + 6809) = -10213 \text{ J}$$

$$q = \Delta U + w = -10213 - 3278 = -13491 \text{ J}$$

2.3

State	P	V	T	
1	1 atm	1 l	373 K	Isothermal
2	P_2	2 l	373 K	Isobaric
3	P_2	V	T_3	Adiabatic
4	1 atm	1 l	373 K.	

$$n = \frac{PV}{RT} = \frac{(1)(1)}{(0.08206)(373)} = 0.03267$$

$$P_2 = \frac{nRT_2}{V_2} = \frac{(0.03267)(0.08206)(373)}{2} = 0.5 \text{ atm.}$$

State 3 → 4 Adiabatic $\quad P_3V_3^{\gamma} = P_4V_4^{\gamma}$

$$(0.5)(V_3)^{5/3} = (1)(1)^{5/3}$$

$$V_3 = 2^{3/5} = 1.51 \text{ l} \qquad \underline{\text{Ans}}$$

$$T_3 = \frac{P_3V_3}{nR} = \frac{(0.5)(1.51)}{(0.03267)(0.08206)} = 281 \text{ K.}$$

$$W_{1\to 2} = \int PdV = nRT\ln\left(\frac{V_2}{V_1}\right) = (0.03267)(8.314)(373)\ln\left(\frac{2}{1}\right) = 70.2 \text{ J.}$$

$$W_{2\to 3} = \int PdV = P(V_3 - V_2) = 0.5(1.51-2)(101.325) = -24.8 \text{ J.}$$

$$W_{3\to 4} = \int PdV = -\Delta U = -\int nC_v dT = -(0.03267)\left(\frac{3}{2}\right)(8.31)(373-281)$$

$$= -37.5$$

$$W_{total} = 70.2 - 24.8 - 37.5 = 8 \text{ J} \qquad \underline{\text{Ans}}$$

2.4

State 1 $\longrightarrow$ State 2

$n = 2\ \text{mol}$ $\qquad q = 34{,}166\ \text{J}$ $\qquad T_2 = ?$

$P_1 = 1\ \text{atm}$ $\qquad w = 1216\ \text{J}$

$T_1 = 800\ \text{K}$

$$V_1 = \frac{nRT_1}{P_1} = \frac{(2)(0.08206)(300)}{1} = 49.2\ \ell.$$

$$\Delta U = q - w = 34166 - 1216 = 32950\ \text{J}$$

$$\Delta U = \int n C_v dT = 2\left(\frac{3}{2}\right)(8.314)(T_2 - T_1).$$

$$\Delta U = 32950 = 2\left(\frac{3}{2}\right)(8.314)(T_2 - 300)$$

$$T_2 \cong 1620\ \text{K}.$$

Note ไม่ว่า Process จะเปลี่ยนแปลงแบบใด เนื่องจาก U เป็น State Function.

$\Delta U_{1\to2}$ ของทุกแบบ จะเท่ากันเสมอ.

2.5

Gas = N_2

$n = 1$

$T_1 = 273\ K$ $\xrightarrow[q = 3000\ J]{W = 832\ J}$

$P_1 = 1\ atm$ $\qquad\qquad P_2 = 1\ atm$

$$V_1 = \frac{nRT_1}{P_1} = \frac{1(0.08206)(273)}{1} = 22.4\ l$$

$$\Delta U = q - W = 3000 - 832 = 2168\ J$$

$$W = \int PdV = P(V_2 - V_1) = 1(V_2 - 22.4)(101.325) = 832.$$

$$V_2 = \frac{832}{101.325} + 22.4 = 30.6\ l$$

$$T_2 = \frac{P_2V_2}{nR} = \frac{(1)(30.6)}{(1)(0.08206)} = 373\ K$$

a) State 2 : $P_2 = 1\ atm$, $T_2 = 373\ K$, $V_2 = 30.6\ l$.

b). $\Delta U = 2168\ J$

$$\Delta H = \Delta(U + PV) = \Delta U + P\Delta V + V\Delta P = 2168 + (1)(30.6 - 22.4)(101.325)$$

$$= 2999\ J$$

c)

$$\Delta U = n\int C_V dT$$

$$2168 = (1)C_V(373 - 273)$$

$$C_V = 21.7\ \frac{J}{mol}$$

$$\Delta H = n\int C_p dT$$

$$2999 = (1)\ C_p\ (373 - 273)$$

$$C_p = 30.0\ \frac{J}{mol}$$

2.6

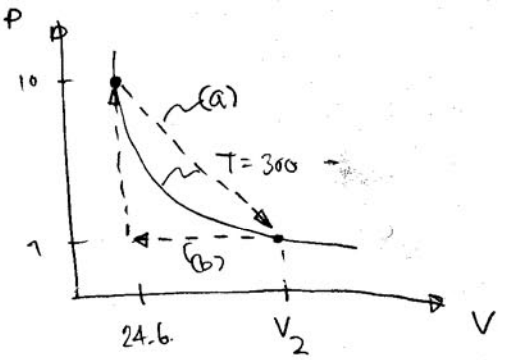
$n = 10$ mol Gas = ideal.

$P_1 = 10$ atm

$T_1 = 300$ K

$$V_1 = \frac{nRT_1}{P_1} = \frac{(10)(0.08206)(300)}{10} = 24.6\ l$$

a)



$$V_2 = \frac{nRT_2}{P_2} = \frac{(10)(0.08206)(300)}{1}$$

$$= 246\ l$$

$$W = \int P dV = \text{area under P-V} = \frac{1}{2}(10+1)(246-24.6)(101.325)$$

$$= 123383 \text{ J}$$

b). $P_3 = P_2 = 1$ atm

$V_3 = 24.6\ l$

$W =$ พ.ท. ใต้กราฟ $= -(1)(246-24.6)(101.325) = -22,433$ J

c). $W = \int P dV = 0$

$W_{total} = 123383 - 22433 = 100,950$ J Ans

(ระบบทำงาน. 100,950 จ)

2.7 $n = 1$ mol. Gas = ideal.

$T_1 = 298$ K $\qquad V_1 = \frac{nRT_1}{P_1} = \frac{(1)(0.08206)(298)}{1} = 24.45\ l$

$P_1 = 1$ atm

a). $T_2 = 298$ K, $P_2 = 0.5$ atm, $V_2 = \frac{nRT_2}{P_2} = \frac{(1)(0.08206)(298)}{0.5} = 48.9\ l$

$\Delta U = 0 = q - w \rightarrow w = q = \int P dV = nRT \ln\left(\frac{V_2}{V_1}\right) = (1)(8.314)(298) \ln\left(\frac{48.9}{24.45}\right)$

$w_a = 1717$ J

b). $P_3 = P_2 = 0.5$ atm, $T_3 = 373$ K, $V_3 = \frac{nRT_3}{P_3} = \frac{(1)(0.08206)(373)}{0.5} = 61.2\ l$

$w_b = \int P dV = P(V_3 - V_2) = (0.5)(61.2 - 48.9)(101.325) = 624$ J.

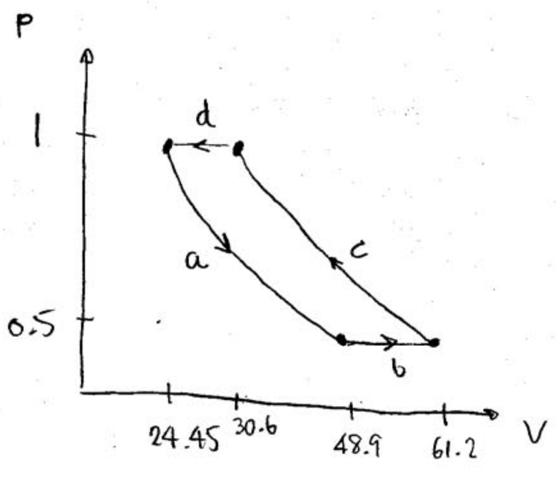
c) $T_4 = T_3 = 373$ K, $P_4 = 1$ atm, $V_4 = \frac{nRT_4}{P_4} = \frac{(1)(0.08206)(373)}{1} = 30.6\ l$

$w_c = \int P dV = nRT \ln\left(\frac{V_4}{V_3}\right) = (1)(8.314)(373) \ln\left(\frac{30.6}{61.2}\right) = -2149$

d) $P_5 = P_4 = 1$ atm, $T_5 = 298$ K, $V_5 = \frac{nRT_5}{P_5} = \frac{(1)(0.08206)(298)}{1} = 24.45\ l$

$w_d = \int P dV = P(V_2 - V_1) = (1)(24.45 - 30.6)(101.325) = -624$ J.

* $w_{total} = w_a + w_b + w_c + w_d = -432$ J



a). $P_6 = P_5 = 1 \text{ atm}$, $T_6 = 373\,K$, $V_6 = \dfrac{nRT_6}{P_6} = \dfrac{(1)(0.08206)(373)}{1} = 30.6\ \ell$

$W_a = \int P dV = P(V_6 - V_5) = 1(30.6 - 24.45)(101.325) = 624\ J.$

b) $P_7 = P$, $V_7 = V_6 = 30.6$, $T_7 = \dfrac{P(30.6)}{(1)(0.08206)} = 373\,P$

$W_b = \int P dV = 0$

c) $P_8 = P_7 = P$, $V_8 = 24.5\ \ell$, $T_8 = \dfrac{P(24.5)}{(1)(0.08206)} = 298\,P$

$W_c = \int P dV = P(24.5 - 30.6)(101.325) = -618\,P$

d) $P_9 = 1 \text{ atm}$, $V_9 = V_8 = 24.5\ \ell$, $T_9 = \dfrac{(1)(24.5)}{(1)(0.08206)} = 298$

$W_d = 0$

$W_{total} = 624 + 0 - 618P + 0 = 432$

$P = \dfrac{(432 - 624)}{-618} = 0.3 \text{ atm}$ Ans

28

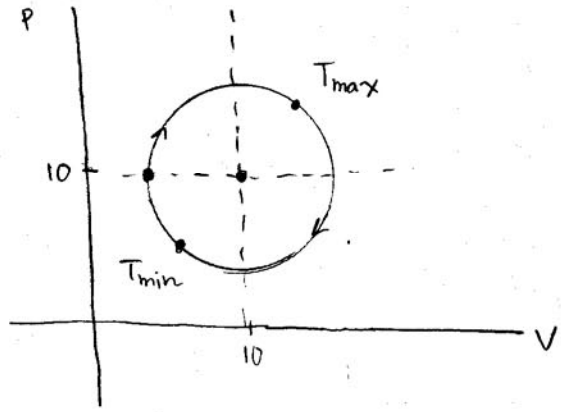
$n = 2$

$P_1 = 10$ atm

$V_1 = 5\ l$

$T_1 = \dfrac{P_1 V_1}{nR} = \dfrac{(10)(5)}{(2)(0.08206)}$

$= 305$ K



At T_{max} :

$$\frac{(P-10)}{(V-10)} = \tan 45^\circ \rightarrow (P-10) = (V-10) \quad — ①$$

$$(P-10)^2 + (V-10)^2 = 25 \quad — ②$$

$$2(P-10)^2 = 25$$

$$P = \sqrt{12.5} + 10 = 13.5$$

$$V = 13.5$$

$$T_{max} = \frac{PV}{nR} = \frac{(13.5)^2}{(2)(0.08206)} = 1110 \text{ K. } \underline{\underline{\text{Ans}}}$$

At T_{min} :

$$(10-P) = (10-V) \quad — ①$$

$$(P-10)^2 + (V-10)^2 = 25$$

$$2(10-P)^2 = 25$$

$$P = -\sqrt{12.5} + 10 = 6.5$$

$$V = 6.5$$

$$T_{min} = \frac{(6.5)^2}{(2)(0.08206)} = 257 \text{ K. } \underline{\underline{\text{Ans}}}$$