Chapter 3 sols

(1) $f_\mu(x) = \dfrac{e^{-\mu} \cdot \mu^x}{x!}$

Will assume x is one observation, as vector notation not used explicitly in the problem.

$\pi(\mu) = e^{-\mu}$

$$P(\mu|x) \propto \pi(\mu) \cdot f_\mu(x)$$
$$\propto e^{-\mu} \cdot e^{-\mu} \cdot \mu^x$$
$$\propto e^{-2\mu} \cdot \mu^x$$

Compare this with the Gamma distribution

$y \sim Ga(\alpha, \beta)$

then $f(y) = \dfrac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \cdot e^{-\beta y}$

$\propto y^{\alpha-1} \cdot e^{-\beta y}$ ← Dropping the constants & just keeping the kernel

Thus, $y = \mu$, $\beta = 2$ and $\alpha - 1 = x$
$\Rightarrow \alpha = x+1$

and $P(\mu|x) \propto e^{-2\mu} \cdot \mu^x \propto Ga(x+1, 2)$

(3) Going step-by-step, let's first compute the score function.

(2) is ↑

$$\dot{l}_x(\pi) = \frac{\partial}{\partial \pi}\left\{\log\left[\binom{n}{x}\right] + x \cdot \log(\pi) + (n-x) \cdot \log(1-\pi)\right\}$$

below.

$$= \cancel{\frac{\partial}{\partial \pi} \log \binom{n}{x}}^{0} + \frac{x}{\pi} - \frac{(n-x)}{1-\pi}$$

$$= \frac{x \cdot (1-\pi) - (n-x) \cdot \pi}{\pi (1-\pi)} = \frac{x - \cancel{x\pi} - n\pi + \cancel{x\pi}}{\pi (1-\pi)}$$

Now, using the definition of Fisher Information:

$$I_\pi = \sum_{x=0}^{n} \binom{n}{x} \cdot \pi^x \cdot (1-\pi)^{n-x} \cdot \left(\frac{x - n\pi}{\pi(1-\pi)} \right)^2$$

$$= \frac{1}{(\pi(1-\pi))^2} \cdot \sum_{x=0}^{n} \binom{n}{x} \cdot \pi^x \cdot (1-\pi)^{n-x} \cdot (x - n\pi)^2$$

$$= \frac{1}{(\pi(1-\pi))^2} \cdot \underbrace{\sum_{x=0}^{n} \binom{n}{x} \cdot \pi^x \cdot (1-\pi)^{n-x} \cdot (x - E(x))^2}$$

This is the definition of the variance for a Binomial(n, π) distr, which is known & equal to $n \cdot \pi \cdot (1-\pi)$

$$= \frac{n \cancel{\pi} \cancel{(1-\pi)}}{(\pi \cdot (1-\pi))^{\cancel{2}}} = \frac{n}{\pi (1-\pi)}$$

(2) Sonogram

	SS	Diff	
I	1/2	0	1/2

Then $\frac{g(I|s)}{g(F|s)} = \frac{\cancel{1/2}}{\cancel{1/2}} \cdot \frac{1}{1/2} =$

Twins are:

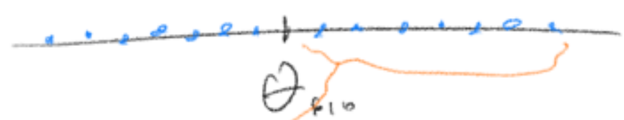
F	1/4	1/4	1/2

$= 2$

Identical twins would be twice as likely as fraternals after sonogram results.

(4) See Jupyter Notebook.

(5) The Effect of gene 610 measured on our sample, X_{610} would have been a different number in other samples from the same population. If we think about θ_{610}, the true effect size vs. $X^{(i)}_{610}$ for different samples, it would have looked like:



The fact that we chose X_{610} because it was large vs. the other gene effects, makes it more likely to be one of the larger measurements across samples, biasing the estimation of its effect.

(6) Posterior of μ given x doesn't change, as our belief of what the parameter μ is only depends on the data (x) through the likelihood.

Now, if we let y: "Information telling us only $x \geq 1$ can be observed", then posterior μ given x & y would change, as the new ... the likelihood func.

information on y would [illegible]