Activity on the momentum equation

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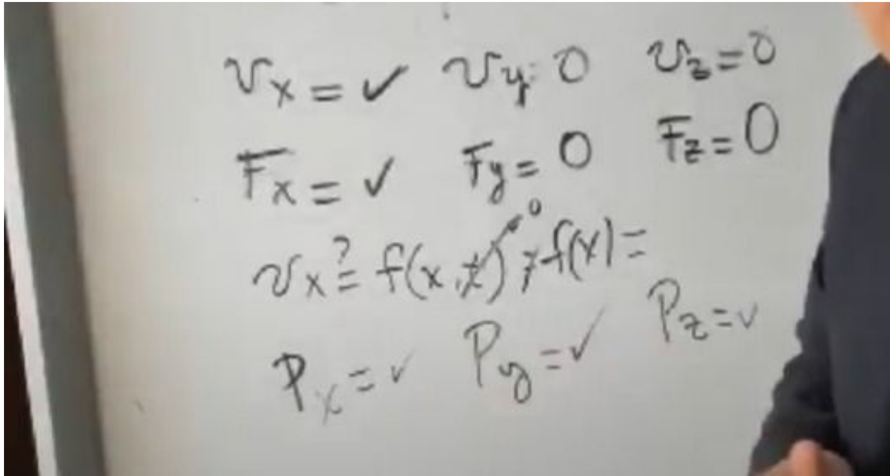
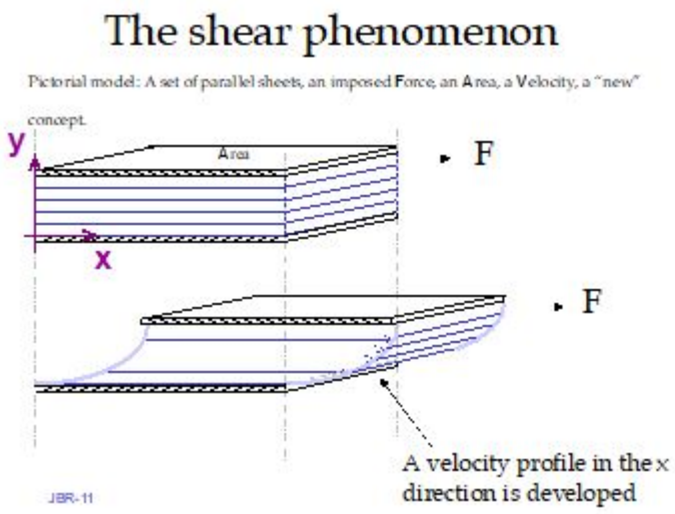
1) Assumptions
   a) Isothermal
   b) Incompressible
   c) Steady state (everything respect to time goes)
2) Choose cartesian system
3) Get the “feeling” of the parameters



The shear phenomenon



Assumptions: Isothermal, Incompressible flow, steady state, *no gravity effects, no pressure in x, pressure in y is due to weight of material, unidirectional flow, negligible body forces.

Coordinate system: cartesian coordinates

Parameters:

Vx=Yes   Vy=Vz=0

Fx=Yes   Fy=No   Fz=No

Vx=f(x,t) is not equal to f(x)

Px=No   Py=Yes   Pz=No

x- component)

$$\rho\left(\cancel{\frac{\partial V_x}{\partial t}} + V_x \frac{\partial V_x}{\partial x} + \cancel{V_y \frac{\partial V_x}{\partial y}} + \cancel{V_z \frac{\partial V_x}{\partial z}}\right) =$$

$$-\cancel{\frac{\partial p}{\partial x}} - \left(\cancel{\frac{\partial \tau_{xx}}{\partial x}} + \frac{\partial \tau_{yx}}{\partial y} + \cancel{\frac{\partial \tau_{zx}}{\partial z}}\right) + \cancel{\rho g_x}$$

y- component)

$$\rho\left(\cancel{\frac{\partial V_y}{\partial t}} + V_x \cancel{\frac{\partial V_y}{\partial x}} + V_y \cancel{\frac{\partial V_y}{\partial y}} + V_z \cancel{\frac{\partial V_y}{\partial z}}\right) =$$

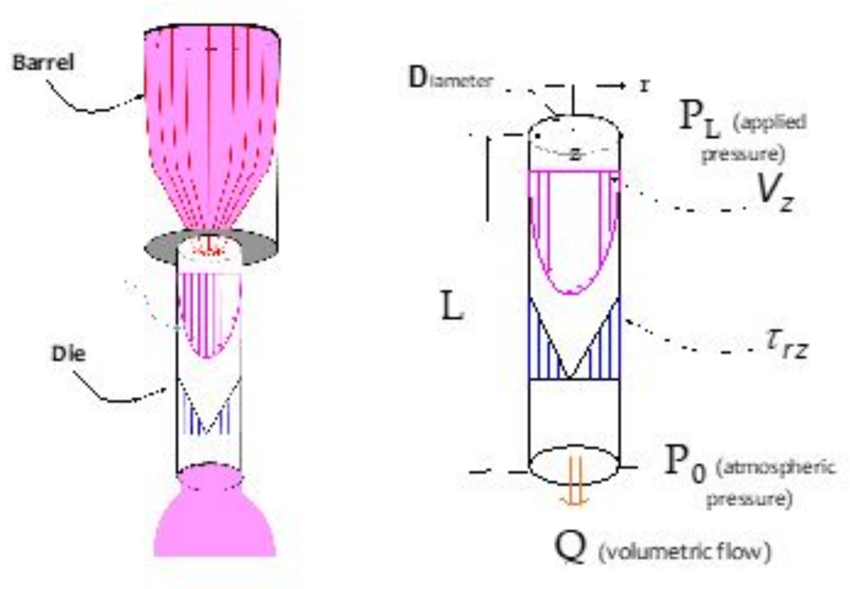
$$-\frac{\partial p}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \cancel{\frac{\partial \tau_{yy}}{\partial y}} + \cancel{\frac{\partial \tau_{zy}}{\partial z}}\right) + \cancel{\rho g_y}$$

z- component)

$$\rho\left(\cancel{\frac{\partial V_z}{\partial t}} + V_x \cancel{\frac{\partial V_z}{\partial x}} + V_y \cancel{\frac{\partial V_z}{\partial y}} + V_z \cancel{\frac{\partial V_z}{\partial z}}\right) =$$

$$-\cancel{\frac{\partial p}{\partial z}} - \left(\cancel{\frac{\partial \tau_{xz}}{\partial x}} + \cancel{\frac{\partial \tau_{yz}}{\partial y}} + \cancel{\frac{\partial \tau_{zz}}{\partial z}}\right) + \cancel{\rho g_z}$$

Capillary rheometry



Assumptions: Isothermal, Incompressible flow, steady state, fully developed flow, negligible body forces.

Coordinate system: cylindrical coordinates

Parameters:

Vr=No Vθ=No Vz=Yes

Fr=No Fθ=No Fz=Yes

Vz=f(z,t) not equal to f(z)

Pr=Yes Pθ=No Pz=Yes

r- component)

$$\rho\left(\cancel{\frac{\partial V_r}{\partial t}} + \cancel{V_r \frac{\partial V_r}{\partial r}} + \cancel{\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}} - \cancel{\frac{V_\theta^2}{r}} + \cancel{V_z \frac{\partial V_r}{\partial z}}\right) =$$
$$-\frac{\partial p}{\partial r} - \left(\cancel{\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})} + \cancel{\frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}} - \cancel{\frac{\tau_{\theta\theta}}{r}} + \frac{\partial \tau_{rz}}{\partial z}\right) + \cancel{\rho g_r}$$

θ- component)

$$\rho\left(\cancel{\frac{\partial V_\theta}{\partial t}} + \cancel{V_r \frac{\partial V_\theta}{\partial r}} + \cancel{\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}} + \cancel{\frac{V_r V_\theta}{r}} + \cancel{V_z \frac{\partial V_\theta}{\partial z}}\right) =$$
$$-\cancel{\frac{1}{r}\frac{\partial p}{\partial \theta}} - \left(\cancel{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta})} + \cancel{\frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta}} + \cancel{\frac{\partial \tau_{\theta z}}{\partial z}}\right) + \cancel{\rho g_\theta}$$

z- component)

$$\rho\left(\cancel{\frac{\partial V_z}{\partial t}} + \cancel{V_r \frac{\partial V_z}{\partial r}} + \cancel{\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}} + V_z \frac{\partial V_z}{\partial z}\right) =$$
$$-\frac{\partial p}{\partial z} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \cancel{\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}} + \cancel{\frac{\partial \tau_{zz}}{\partial z}}\right) + \cancel{\rho g_z}$$

Nabla operator

$$\frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) = \nabla \cdot \vec{\tau}_{ix}$$

$$\frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial y}(\tau_{yy}) + \frac{\partial}{\partial z}(\tau_{zy}) = \nabla \cdot \vec{\tau}_{iy}$$

$$\frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + \frac{\partial}{\partial z}(\tau_{zz}) = \nabla \cdot \vec{\tau}_{iz}$$

$$\left(\frac{d\vec{F}}{dV}\right)_{VISCOUS} = (\nabla \cdot \vec{\tau}_{ix})\hat{i} + (\nabla \cdot \vec{\tau}_{iy})\hat{j} + (\nabla \cdot \vec{\tau}_{iz})\hat{k}$$

$$= \nabla \cdot \overline{\tau_{ij}}, \quad \overline{\tau_{ij}} = \begin{bmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix}$$

Cartesian

x- component)

$$\boxed{\rho(\nabla x \cdot \vec{v}) = \frac{\partial P}{\partial x} - \nabla \tau_{ix} + \rho g_x}$$

$$\rho\left(\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x$$

y- component)

$$\boxed{\rho(\nabla y \cdot \vec{v}) = \frac{\partial P}{\partial y} - \nabla \tau_{iy} + \rho g_y}$$

$$\rho\left(\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho g_y$$

z- component)

$$\boxed{\rho(\nabla z \cdot \vec{v}) = \frac{\partial P}{\partial z} - \nabla \tau_{iz} + \rho g_z}$$

$$\rho\left(\frac{\partial V_z}{\partial t} + V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z$$

$$\boxed{\rho(\nabla \cdot \vec{v}) = \frac{\partial P}{\partial i} - \nabla \tau_{ij} + \rho g_i \text{ for any direction}}$$

J.D. Humphrey, S.L. O'Rourke, An Introduction to Biomechanics, Springer New York, New York, NY, 2015. https://doi.org/10.1007/978-1-4939-2623-7.

Cylindrical

r- component)

$$\boxed{\rho(\nabla r \cdot \vec{v}) = \frac{\partial P}{\partial r} - \nabla \tau_{ir} + \rho g_r}$$

$$\rho\left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z \frac{\partial V_r}{\partial z}\right) =$$

$$-\frac{\partial p}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r$$

θ- component)

$$\boxed{\rho(\nabla \theta \cdot \vec{v}) = \frac{\partial P}{\partial \theta} - \nabla \tau_{i\theta} + \rho g_\theta}$$

$$\rho\left(\frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z \frac{\partial V_\theta}{\partial z}\right) =$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta$$

z- component)

$$\boxed{\rho(\nabla z \cdot \vec{v}) = \frac{\partial P}{\partial z} - \nabla \tau_{iz} + \rho g_z}$$

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z}\right) =$$

$$-\frac{\partial p}{\partial z} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z$$

$$\boxed{\rho(\nabla \cdot \vec{v}) = \frac{\partial P}{\partial i} - \nabla \tau_{ij} + \rho g_i \text{ for any direction}}$$

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