Obtaining the actual shear viscosity and the elongational viscosity.

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Problem-solving steps

1. Paraphrase the problem and clearly indicate what is being asked.
2. List data.
3. List of assumptions (indicating the reason for them).
4. Solution algorithm.
5. Problem solution.
6. Question whether the result is reasonable.
7. Reference list.

Problem description

- To obtain the actual shear viscosity and the elongation viscosity of a polymer using the information obtained in a capillary rheometer.
- There are available measurements of the different loads (forces) required for a polymer to flow to the steady state at different speeds and with different die L/D.
- The loads have been measured with a sensor located at the tip of the piston, which has a diameter similar to that of the barrel.

Data

- D_{barrel} = 0.68 cm = 0.2677 in
- D_{die} = 0.05 in

Table 1. Corresponding die lengths and die L/D

Die length [in]	L/D [-]
2.0	40
1.0	20
0.5	10

Table 2. Loads (forces [lbf]) against speeds [in / s] at steady state, for different die L/D

Velocidad (in/s)	Carga (lbf) L/D=40	Carga (lbf) L/D=20	Carga (lbf) L/D=10
0.06142	170.3	92.6	55.9
0.06142	171.7	92.6	54.1
0.30694	420.5	216.9	121.3
0.30694	417.5	214.4	119.4
0.61394	579.1	296.5	164.6
0.61394	575.8	294.1	162.9
3.06938	848.4	390.7	261.6
6.13938	626.8	360.6	233.8
6.13938	623.6	377.8	264.3
13.81250	939.4	552.4	368.5
18.41875	1098.4	640.3	418.0

Assumptions

- The power law corresponds to a good fit for the viscosity of this polymer.
- To obtain the Rabinowitch correction, the data of the die with the longest L/D is used, since it is the one that causes the greatest shear stress.
- When obtaining the slope for the Rabinowitch correction, any polynomial can be used. In this case, a squared polynomial was adjusted, expecting a good fit for the range being managed ($R2 = 0.965$). Thus, the slope (b) for the Rabinowitch correction is the derivative of the obtained equation and varies with respect to log (τw).

Solution algorithm

A) ACTUAL SHEAR VISCOSITY CURVE

- Bagley correction - correction factor for L / D:

  The purpose is to obtain the die inlet pressure for each speed to correct the effect of the pressure drop.

Obtain the average force in case two values are at the same speed

Plot the force at each L/D

Obtain the force when L / D = 0 (intersection with y), which is the input force.

Subtract the input force from the force used in the calculations, since the balance of forces was made on the die.

Solution algorithm

A) ACTUAL SHEAR VISCOSITY CURVE

- Rabinowitch correction

The data of the longest die (L / D = 40) is used.

1. Calculate the flow through the barrel (Q)
2. Calculate the apparent shear rate (Γ)
3. Calculate the pressure drop (ΔP)
4. Calculate the shear stress on the wall (τw)
5. Obtain apparent viscosity (ηA)
6. Plotting log (Γ) vs log (τw)
7. Obtain the slope (b) of the graph
8. Obtain the real shear rate (γw)
9. Obtain actual shear viscosity (η)
10. Plot the viscosity against the shear rate.

$$Q = P_v A_{transversal,barrel}$$

$$\Gamma = \dot{\gamma}_a = \frac{4Q}{\pi R^3{}_{die}}$$

$$\Delta P_g = \frac{F_z}{A_{transversal,barrel}}$$

$$\tau_w = \frac{\Delta P_g R_{die}}{2L_{die}}$$

$$\eta_A = \frac{\tau_w}{\Gamma}$$

$$\dot{\gamma}_w = \left(\frac{3+b}{4}\right)\Gamma$$

$$\eta = \frac{\tau_w}{\dot{\gamma}_w}$$

Solution algorithm

B) ELONGATIONAL VISCOSITY

- Adjust the viscosity vs. shear rate plot to obtain the parameter *n* of the power law.
- Obtain the pressure drop at the inlet ΔP_{in}
- Obtain the elongation viscosity for each apparent shear rate and apparent viscosity (without Bagley and Rabinowitch corrections):

$$\Delta P_{in} = \frac{F_{L/D=0}}{A_{barrel}} \qquad \dot{\varepsilon}_A = \frac{4\dot{\gamma}_A^2 \eta_A}{3(n+1)\Delta P_{in}} \qquad \sigma_e = \frac{3(n+1)\Delta P_{in}}{8} \qquad \eta_e = \frac{\sigma_e}{\dot{\varepsilon}_A}$$

Remark: The inlet pressure drop (ΔP_{in}) corresponds to the elongational pressure drop and this is the pressure drop that is subtracted in the Bagley correction

Solution

A) ACTUAL SHEAR VISCOSITY CURVE

- Bagley correction - correction factor for L/D

Table 3. Obtaining the force when L/D=0 and correction for L/D=40

Data	Velocity (in/s)	Average load (lbf) L/D = 40	Average load (lbf) L/D = 20	Average load (lbf) L/D = 10	Load (force) L/D = 0 (intercepto)	Corrected load lbf L/D = 40
1,2	0.06142	171.0	92.6	55.0	15.8	155.2
3,4	0.30694	419.0	215.7	120.4	18.7	400.3
5,6	0.61394	577.5	295.3	163.8	22.7	554.8
8,9	6.13938	625.2	369.2	249.1	121.1	504.2
10	13.81250	939.4	552.4	368.5	175.0	764.4
11	18.41875	1098.4	640.3	418.0	189.0	909.5

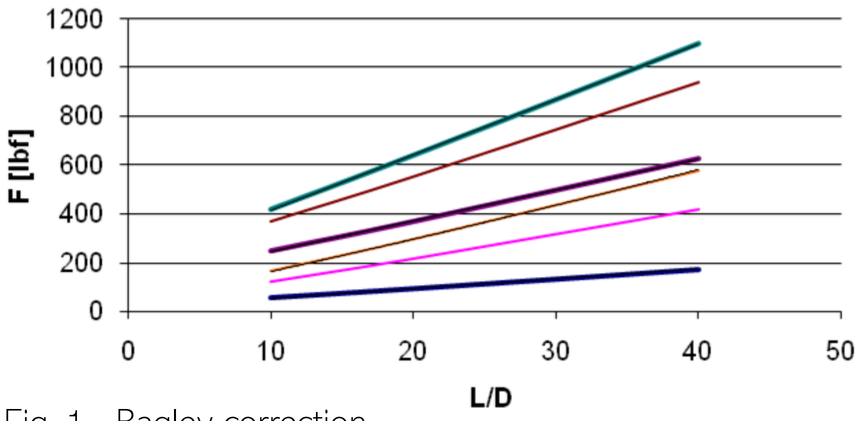

Fig. 1 . Bagley correction

Remark: The data corresponding to data 7 were not used; (speed of 3.069375 in / s), since there was only one measurement that was an *outlier* compared to the others.

Solution

A) ACTUAL SHEAR VISCOSITY CURVE

- Corrección de Rabinowitch

Table 4. Obtaining the corrected cutting viscosity

	Load lbf	ΔPg (lbf/in^2)	τ_w (lbf/in^2)	τ_w ($Pa=N/m^2$)	Q (in^3/s)	$\Gamma = \gamma_{app}$ (1/s)	η_A (Pa*s)	b	γ_w (1/s)	η (Pa s)
1,2	155.2	2757.09	17.23	118809.30	0.0035	281.7	421.71	-0.764	157.51	754.305
3,4	400.3	7111.68	44.45	306458.32	0.0173	1407.9	217.67	3.691	2355.21	130.119
5,6	554.8	9855.45	61.60	424693.48	0.0346	2816.1	150.81	5.225	5790.90	73.338
8,9	504.2	8956.11	55.98	385938.83	0.3456	28161.4	13.70	4.775	54741.68	7.050
10	764.4	13579.39	84.87	585166.41	0.7775	63358.2	9.24	6.732	154154.55	3.796
11	909.5	16156.18	100.98	696205.64	1.0368	84487.1	8.24	7.549	222816.61	3.125

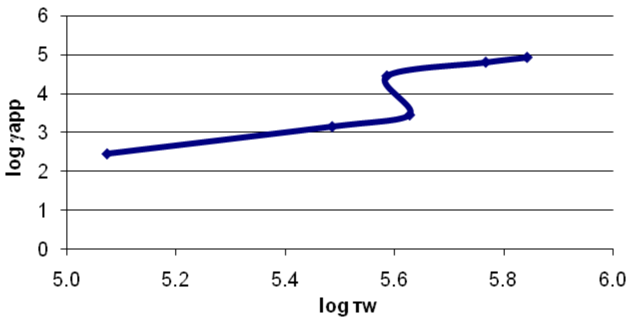

Fig. 2. Curve containing all of the values

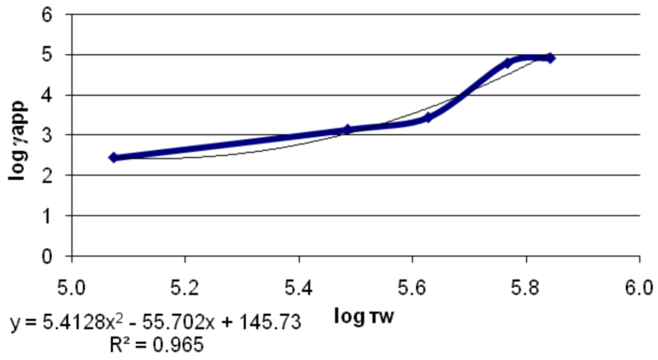

Fig. 3. Curve without value "7" to obtain b

Solution

A) CORRECTED SHEAR VISCOSITY CURVE

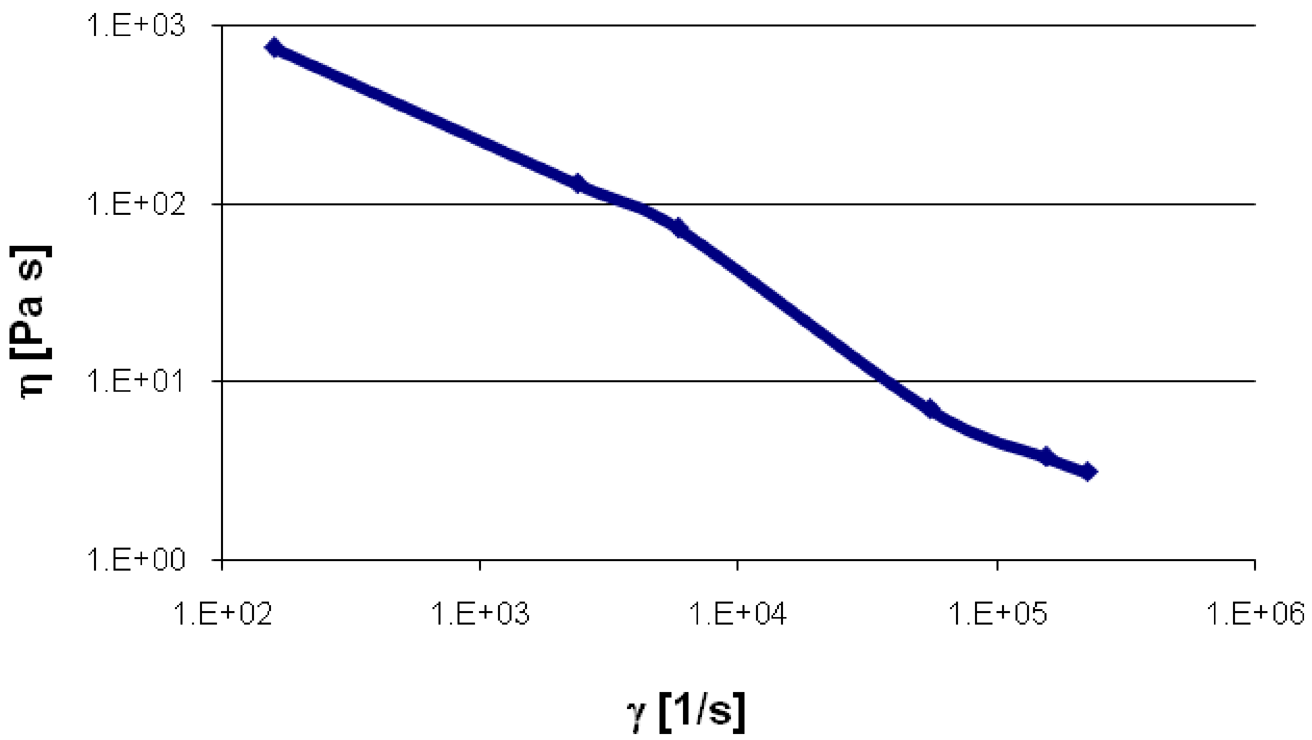

Fig 4. Corrected shear viscosity curve

Solution

B) CORRECTED ELONGATIONAL VISCOSITY CURVE

Table 4. Obtaining the elongation viscosity

ΔP_{in} (N/m^2)	γ_{app} $(1/s)$	η_A $(Pa*s)$	σ_e (Pa)	ε_A $(1/s)$	η_e $(Pa*s)$
1.94E+06	2.82E+02	4.65E+02	1.08E+06	1.71E+01	6.32E+04
2.29E+06	1.41E+03	2.28E+02	1.28E+06	1.77E+02	7.21E+03
2.78E+06	2.82E+03	1.57E+02	1.55E+06	4.02E+02	3.86E+03
1.48E+07	2.82E+04	1.70E+01	8.27E+06	8.15E+02	1.01E+04
2.14E+07	6.34E+04	1.14E+01	1.20E+07	1.91E+03	6.27E+03
2.31E+07	8.45E+04	9.95E+00	1.29E+07	2.75E+03	4.69E+03
1.94E+06	2.82E+02	4.65E+02	1.08E+06	1.71E+01	6.32E+04

Parameters of the power law

k = 8520

n = 0.4874

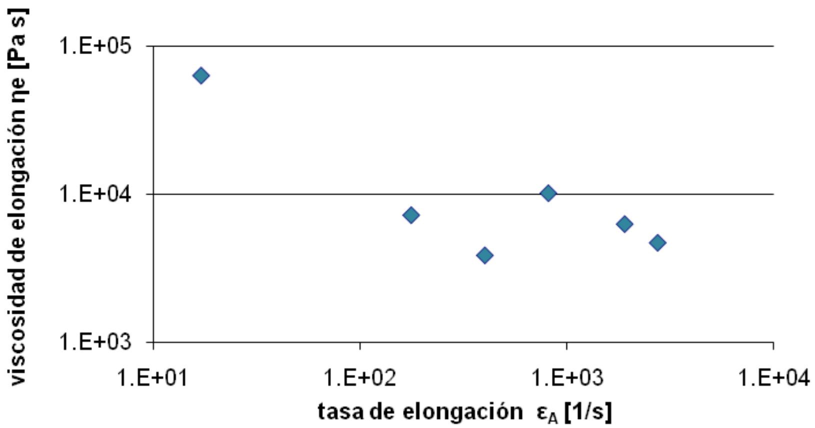

Fig. 5. Elongational viscosity curve

Solution Analysis

- The obtained viscosity values are high due to the high load (force) values and the thickness of the capillary.
- The elongation viscosity values are two or three orders of magnitude greater than the apparent viscosity. This is:
  - Due to the drastic change in diameters between the barrel and the die, which has a diameter more than 5 times less than the barrel.
  - Effect of the rearrangement of chains that is required for the polymer to enter the die.

References

- Bonilla, Jaime. Apuntes de la clase de *Ingeniería de Plásticos*. Tecnológico de Monterrey. Temas: *Capillary Rheometry.*
- Progelhof, Richard C. y Throne, James L. *Polymer Engineering Principles.* Hanser, 1993.