Chapter 9 Sols

(1) Formula 9.4 states:

$$S_{ij} = \prod_{k=i}^{j} (1-h_k) \quad . \quad \text{Let } i=1$$

and we obtain $\underbrace{S_j = \prod_{k=1}^{j} (1-h_k)}_{\text{Prob. of living past age } j}$.

The relationship between this expression and 9.1 is given by (9.2). More explicitly, the probability of living past age j can be written as the probability of living exactly j years, plus the probability of living exactly j+1 years and so forth. In math and using slightly different notation from 9.2, we have:

$$S_j = f_j + f_{j+1} + f_{j+2} + \dots$$

$$\Rightarrow S_j = \sum_{k \geq j} f_k \quad \blacksquare$$

(2) We start from 9.4 with with $i=1$

$S_j = \prod^{j} (1-h_k)$, and assume no censoring, thus our data is only a record of n survival times $t_1, t_2, t_3, \ldots, \ldots$

Then, we estimate h_K by $\hat{h}_K = y_K / n_K$ to get

$$\hat{S}_j = \prod_{K=1}^{j} (1 - \hat{h}_K) = \prod_{K=1}^{j} \left(1 - \frac{y_K}{n_K}\right)$$

$$= \prod_{K=1}^{j} \left(\frac{n_K - y_K}{n_K}\right)$$

Then, if we let $t_{(1)} < t_{(2)} < \ldots < t_{(n)}$ be the ordered survival times and use a time unit small enough s.t. there are no ties, then $n_K = n - K + 1$ $\forall K$

The subject whose death will happen in the current time period.

and $y(K) = 1$ (since there are no ties). Thus,

$$\hat{S}_{(j)} = \prod_{K=1}^{j} \left(\frac{n - K + \cancel{1} - \cancel{1}}{n - K + 1}\right) = \prod_{K=1}^{j} \left(\frac{n - K}{n - K + 1}\right)$$

which exactly matches 9.17 if all $d(K)$'s are $= 1$, matching our non-censored data assumption.

(3) See Jupyter Notebook.

(4) See Jupyter Notebook.

(5) Because the numbers on the marginals are not constant from month to month, so we need to control for the changes in n_A and n_B from period to period. We

n_A and

could have used a Binomial model if n_A and n_B were proportional (say with $p = \frac{n_A}{n_A + n_B}$) for all months.

(6) – (8) We skipped the sections of these 3 questions.