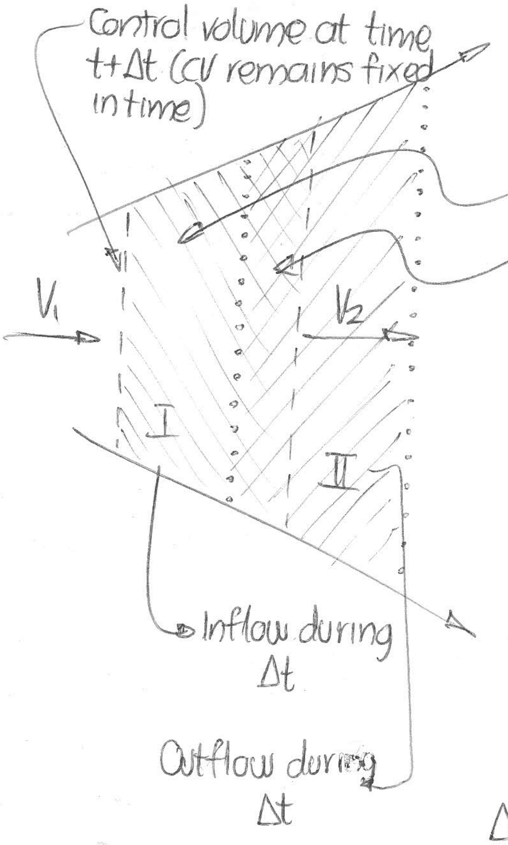
THE REYNOLDS TRANSPORT THEOREM



$B \rightarrow$ any extensive property

$b = B/m \rightarrow$ corresponding intensive property

$\dot{B} \rightarrow$ flow rate

$$B_{sys,t} = B_{cv,t}$$

$$B_{sys,t+\Delta t} = B_{cv,t+\Delta t} - B_{I,t+\Delta t} + B_{II,t+\Delta t}$$

$$\frac{\Delta B_{sys}}{\Delta t} = \frac{B_{sys,t+\Delta t} - B_{sys,t}}{\Delta t}$$

$$= \frac{B_{cv,t+\Delta t} - B_{I,t+\Delta t} + B_{II,t+\Delta t} - B_{cv,t}}{\Delta t}$$

$$= \frac{B_{cv,t+\Delta t} - B_{cv,t}}{\Delta t} - \frac{B_{I,t+\Delta t}}{\Delta t} + \frac{B_{II,t+\Delta t}}{\Delta t}$$

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta B_{sys}}{\Delta t} = \frac{dB_{cv}}{dt} - \dot{B}_{in} + \dot{B}_{out}$$

$$B_{I,t+\Delta t} = b_1 m_{I,t+\Delta t} = b_1 \rho_1 V_{I,t+\Delta t} = b_1 \rho_1 V_1 \Delta t A_1$$

$$B_{II,t+\Delta t} = b_2 m_{II,t+\Delta t} = b_2 \rho_2 V_{II,t+\Delta t} = b_2 \rho_2 V_2 \Delta t A_2$$

$$\dot{B}_{in} = \dot{B}_I = \lim_{\Delta t \to 0} \frac{B_{I,t+\Delta t}}{\Delta t} = \lim_{\Delta t \to 0} \frac{b_1 \rho_1 V_1 \cancel{\Delta t} A_1}{\cancel{\Delta t}} = b_1 \rho_1 V_1 A_1$$

$$\dot{B}_{out} = \dot{B}_{II} = \lim_{\Delta t \to 0} \frac{B_{II,t+\Delta t}}{\Delta t} = \lim_{\Delta t \to 0} \frac{b_2 \rho_2 V_2 \cancel{\Delta t} A_2}{\cancel{\Delta t}} = b_2 \rho_2 V_2 A_2$$

$$\therefore \frac{dB_{sys}}{dt} = \frac{dB_{cv}}{dt} - b_1 \rho_1 V_1 A_1 + b_2 \rho_2 V_2 A_2$$

if velocities are normal to the areas

$$\dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in} = \int_{CS} \rho b \vec{V} \cdot \vec{n} \, dA \quad \text{(inflow if negative)}$$

$$B_{CV} = \int_{CV} \rho b \, dV$$

$$\therefore \frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho b \, dV + \int_{CS} \rho b \vec{V} \cdot \vec{n} \, dA$$

if CV is not moving or deforming with time

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t} (\rho b) \, dV + \int_{CS} \rho b \vec{V} \cdot \vec{n} \, dA$$

CONSERVATION OF MASS

$B = m$

$b = m/m = 1$

$$\left(\frac{dm}{dt}\right)_{sys} = 0$$

Using RTT

$$0 = \frac{d}{dt}\left(\int_{cv} \rho \, dV\right) + \int_{cs} \rho (\vec{V} \cdot \vec{n}) \, dA$$

$$\int_{cv} \frac{\partial \rho}{\partial t} dV + \int_{cs} \rho (\vec{V} \cdot \vec{n}) \, dA = 0$$

$$\int_{cv} \frac{\partial \rho}{\partial t} dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

THE LINEAR MOMENTUM EQUATION

$\vec{B} = m\vec{V}$

$\vec{b} = m\vec{V}/m = \vec{V}$

Using RTT

$$\frac{d}{dt}(m\vec{V})_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{cv} \vec{V} \rho \, dV\right) + \int_{cs} \vec{V} \rho (\vec{V} \cdot \vec{n}) \, dA$$

$$\sum \vec{F} = \frac{d}{dt}\left(\int_{cv} \vec{V} \rho \, dV\right) + \sum (\dot{m}_i \vec{V}_i)_{out} - \sum (\dot{m}_i \vec{V}_i)_{in}$$

THE ENERGY EQUATION

$$\delta Q - \delta W = dE \qquad Q \rightarrow \text{heat}$$
$$\dot{Q} - \dot{W} = \frac{dE}{dt} \qquad W \rightarrow \text{work}$$

$B = E$

$b = \frac{E}{m} = e$

Using RTT

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{cv} e\rho \, dV\right) + \int_{cs} e\rho(\vec{V}\cdot\vec{n})\,dA$$

$$e = e_{internal} + e_{kinetic} + e_{potential} + \cancel{e_{other}}^{\,0}$$

$$e = \hat{U} + \frac{1}{2}V^2 + gz$$

$$\dot{W} = \dot{W}_{shaft} + \dot{W}_{press} + \dot{W}_{\substack{viscous\\stresses}} = \dot{W}_s + \dot{W}_p + \dot{W}_v$$

$$d\dot{W}_p = -(p\,dA)V_{n,in} = -p(-\vec{V}\cdot\vec{n})\,dA$$

because it goes in

$$\dot{W}_p = \int_{cs} p(\vec{V}\cdot\vec{n})\,dA$$

$$d\dot{W}_v = -\vec{\tau}\cdot\vec{V}\,dA \qquad \vec{\tau} \rightarrow \text{stress vector}$$

$$\dot{W}_v = -\int_{cs} \vec{\tau}\cdot\vec{V}\,dA$$

$$\dot{W} = \dot{W}_s + \int_{cs} p(\vec{V}\cdot\vec{n})\,dA - \int_{cs} (\vec{\tau}\cdot\vec{V})_{ss}\,dA$$

$$\dot{Q} - \dot{W}_s - \int_{cs} p(\vec{V}\cdot\vec{n})\,dA - \dot{W}_v = \frac{d}{dt}\left(\int_{cv} e\rho\,dV\right) + \int_{cs} e\rho(\vec{V}\cdot\vec{n})\,dA$$

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{d}{dt}\left(\int_{cv} e\rho\,dV\right) + \int_{cs} e\rho(\vec{V}\cdot\vec{n})\,dA + \int_{cs} p(\vec{V}\cdot\vec{n})\,dA$$

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \int_{cv} \frac{\partial}{\partial t}(e\rho)\,dV + \int_{cs}\left(e + \frac{p}{\rho}\right)\rho(\vec{V}\cdot\vec{n})\,dA$$