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Examen 2

$\boxed{\text{Problema 1}}$ encontrar ∇f y su valor en P

$\boxed{a}$ $f = \dfrac{x}{x^2+y^2}$, $P:(1,1)$

$\dfrac{u'v - uv'}{v^2}$

$\nabla f = \left(\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}\right)$, where:

$$\frac{\partial f}{\partial x} = \frac{-2x^2 + (x^2+y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-2xy}{(x^2+y^2)^2} //$$

$$\nabla f(P) = -\frac{1}{2} //$$

$\boxed{b}$ $f = (x^2+y^2+z^2)^{-1/2}$ $P:(12, 0, 16)$

nx^{n-1}

$$\frac{\partial f}{\partial x} = \left(-\frac{1}{2}(x^2+y^2+z^2)^{-3/2} 2x\right)$$

$$\frac{\partial f}{\partial y} = \left(-\frac{1}{2}(x^2+y^2+z^2)^{-3/2} 2y\right)$$

$$\frac{\partial f}{\partial z} = \left(-\frac{1}{2}(x^2+y^2+z^2)^{-3/2} 2z\right)$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$= \left(-x(x^2+y^2+z^2)^{-3/2}, -y(x^2+y^2+z^2)^{-3/2}, -z(x^2+y^2+z^2)^{-3/2}\right) //$$

$$\nabla f(P) = (-3/2000, 0, -1/500)$$

$$= (-.0015, 0, -.002) //$$

Problema 2 encontrar div v y su valor en P

a $v = [0, \cos(xyz), \sin(xyz)]$, $P: (2, 0.5\cdot\pi, 0)$

$$\operatorname{div} v = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

$$= 0 - \cancel{xz\sin(xyz)} + xy\cos(xyz)$$

$$= xy\cos(xyz) \text{ //}$$

$$\operatorname{div} v(P) = \pi \text{ //}$$

b $v = [v_1(y,z), v_2(z,x), v_3(x,y)]$, $P: (3, 1, -1)$

$$\operatorname{div} v = 0 + 0 + 0$$

$$= 0 \text{ //}$$

& $\operatorname{div} v(P) = 0$ //

c $v = \frac{1}{(x^2+y^2+z^2)^{3/2}}[x, y, z]$, P arbitrario

$u'v + uv'$

$$\frac{\partial v_1}{\partial x} = \frac{1}{(x^2+y^2+z^2)^{3/2}}(1) - \frac{3x}{\cancel{2}}\frac{\cancel{2}x}{(x^2+y^2+z^2)^{5/2}}$$

$$= \frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3x^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial v_2}{\partial y} = \frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3y^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial v_3}{\partial z} = \frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3z^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\operatorname{div} v = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z} \text{ //}$$

$$\operatorname{div} v(P) = \frac{\partial}{\partial x} v_1(x,y,z) + \frac{\partial}{\partial y} v_2(x,y,z) + \frac{\partial}{\partial z}(x,y,z) \text{ //}$$