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Notes Done

Suggested Title: ③ Edit

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a) Each dataset has likelihood function

$$f_{\underline{x}}(\sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left\{-\frac{1}{2} \cdot \frac{x_i}{\sigma^2}\right\}$$

$$= (2\pi\sigma^2)^{-n/2} \cdot \exp\left(-\frac{1}{2\sigma^2} \cdot \sum_{i=1}^{n} x_i^2\right)$$

$$l_{\underline{x}}(\sigma^2) = -\frac{n}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \cdot \sum_{i=1}^{n} x_i^2$$

$$\frac{\partial}{\partial\sigma^2} l_{\underline{x}}(\sigma^2) = -\frac{n}{2} \cdot \frac{\cancel{2\pi}}{\cancel{2\pi}\sigma^2} + \frac{1}{2(\sigma^2)^2} \cdot \sum_{i=1}^{n} x_i^2$$

$$= -\frac{1}{2} \cdot \left[\frac{n}{\sigma^2} - \frac{\sum_{i=1}^{n} x_i^2}{(\sigma^2)^2}\right]$$

...tting this expression to ~~zero~~ zero to find

Setting this expression to ~~zero~~ to find the MLE ...

$$0 = -\frac{1}{2}\left[\frac{n}{\sigma^2} - \frac{\sum_{i=1}^{n} x_i^2}{(\sigma^2)^2}\right]$$

$$\Rightarrow \frac{\sum_{i=1}^{n} x_i^2}{(\sigma^2)^{\cancel{2}}} = \frac{n}{\cancel{\sigma^2}}$$

$$\Rightarrow \hat{\sigma}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

← Use this to get MLE estimates $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$

b) We know

$$\dot{\ell}_{\underline{x}}(\sigma^2) = -\frac{n}{2\sigma^2} + \frac{\sum_{i=1}^{n} x_i^2}{2(\sigma^2)^2}$$

Thus

$$d \sum^{n} x_i^2$$

Thus

$$\ddot{l}_{\underline{x}}(\sigma^2) = \frac{n}{2(\sigma^2)^2} - \frac{\cancel{2}\cdot\sum_{i=1}^{n} x_i^2}{\cancel{2}\,(\sigma^2)^3}$$

$$= \frac{n}{2(\sigma^2)^2} - \frac{\sum_{i=1}^{n} x_i^2}{(\sigma^2)^3}$$

To compute the <u>Observed Fisher Information</u> we use equation:

$$I(\underline{x}) = -\ddot{l}_{\underline{x}}(\sigma^2)$$

$$= \frac{\sum_{i=1}^{n} x_i^2}{(\sigma^2)^3} - \frac{n}{2(\sigma^2)^2}$$

and substitute σ^2 with $\hat{\sigma}^2_{MLE}$.

thus ...

n

and substitute σ^2 with $\hat{\sigma}^2_{MLE}$.

Thus ...

$$I(\underline{X}) = \frac{\cancel{\sum_{i=1}^{n} x_i^2}}{\left(\frac{\sum_{i=1}^{n} x_i^2}{n}\right)^3} - \frac{n}{2\left(\frac{\sum_{i=1}^{n} x_i^2}{n}\right)^2}$$

$$= \frac{n^3}{\left(\sum_{i=1}^{n} x_i^2\right)^2} - \frac{n^3}{2\left(\sum_{i=1}^{n} x_i^2\right)^2}$$

$$= \frac{n^3}{2\left(\sum_{i=1}^{n} x_i^2\right)^2}$$

c) $\hat{\sigma}_i^2 \sim N\left(\frac{n-1}{n} \cdot \sigma^2, \; 1/I(x_i)\right)$

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c) $\hat{\sigma}_1^2 \sim N\left(\underbrace{\frac{n-1}{n} \cdot \sigma^2}, \; {}^1/_{I(x_1)}\right)$

recall $\hat{\sigma}^{2\,MLE}$ is biased.

use sample 1 data and this equation.

$\hat{\sigma}_2^2 \sim N\left(\frac{n-1}{n} \cdot \sigma^2, \; {}^1/_{I(x_2)}\right)$

use sample 2 data to compute.