Homework 2 Solutions:

① a) • Bias calculation:

First, compute $E(\hat{\theta})$

Since $\hat{\theta} = \frac{s+1}{n+2} \Rightarrow E(\hat{\theta}) = \frac{1}{n+2} \cdot E(S+1)$

$= \frac{1}{n+2} \cdot [E(S) + 1]$

Expected # of successes in Binomial setting is np

$= \frac{1}{n+2} \cdot (n\theta + 1)$

$= \frac{n\theta + 1}{n+2}$

Then, compute $\theta - E(\hat{\theta})$

$$\text{Bias} = \theta - \left[\frac{n\theta}{n+2} + \frac{1}{n+2}\right] = \theta\left(\underbrace{\frac{n+2}{n+2}}_{=1} - \frac{n}{n+2}\right) + \frac{1}{n+2}$$

$$= \frac{2\theta + 1}{n+2}$$

• Variance Calculation

$$V(\hat{\theta}) = V\left(\frac{s+1}{n+2}\right) = \frac{1}{(n+2)^2} \cdot V(S+1)$$

$$= \frac{V(S)}{(n+2)^2} = \frac{n \cdot \theta \cdot (1-\theta)}{(n+2)^2}$$

b) $se(\hat{\theta})$ is just $\sqrt{V(\hat{\theta})}$

Thus, $se(\hat{\theta}) = \frac{[n \cdot \theta \cdot (1-\theta)]^{1/2}}{n+2}$

- Since we don't know θ, we use our estimator $\theta = \frac{}{n+2}$ in its place (this is the plug-in principle) and get

$$se(\hat{\theta}) = \frac{\left[n \cdot \left(\frac{s+1}{n+2}\right) \cdot \left(1 - \frac{s+1}{n+2}\right)\right]^{1/2}}{n+2}$$

and then, of course, you can further simplify it.

(2) See Jupyter Notebook w/ sol.

(3) Frequentism assigns the properties of the estimator (like its bias & variance) to the value of it we got from our very specific sample. The reason for doing this (and the link to the repeated sampling assumption), is that your very specific sample is a random one from the infinite possible samples.

(4) The test would asymptotically converge to the t-student's test for larger $n_1 + n_2$, as sample increase would be accompanied by asymptotic convergence of $\hat{\sigma}$ to σ.

(5) See Jupyter Notebook w/ sol.