Mathematical Physical Modelling - Homework 04
Differential Vector Calculus

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Paper trail and process evidence are at the end of this document.

SEMANA 4

- **VECTORES: PRODUCTO ESCALAR**

1. Encuentra los ángulos del triángulo con vértices *A:* (0, 0, 2), *B*: (3, 0, 2) y *C*: (1, 1, 1). Haz un dibujo del triángulo.
2. Encuentra los ángulos de un paralelogramo si los vértices son (0, 0), (6, 0), (8, 3) y (2, 3).
3. Encuentra la distancia del punto *A*: (1, 0, 2) al plano $P: 3x + y + z = 9$. Haz un dibujo del problema.
4. ¿Para cuál valor de *c* serán ortogonales los planos $3x + z = 5$ y $8x - y + cz = 9$?
5. Encuentra la componente del vector **a** en la dirección del vector **b** en los siguientes casos:

   (a) $\mathbf{a} = [1,1,1], \mathbf{b} = [2,1,3]$ (b) $\mathbf{a} = [3,4,0], \mathbf{b} = [4,-3,2]$
   (c) $\mathbf{a} = [8,2,0], \mathbf{b} = [-4,-1,0]$

- **VECTORES Y PRODUCTO ESCALAR TRIPLE**

6. Con respecto a un sistema derecho cartesiano, sean

$$\mathbf{a} = [2,1,0], \mathbf{b} = [-3,2,0], \mathbf{c} = [1,4,-2]$$

   Mostrando los detalles, encuentra lo siguiente:
   (a) $(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{d}$ (b) $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{d})$
7. Encuentra el volumen de un tetraedro con vértices (1, 1, 1), (5, –7, 3), (7, 4, 8) y (10, 7, 4).
8. Encuentra el volumen de un tetraedro con vértices (1, 3, 6), (3, 7, 12), (8, 8, 9) y (2, 2, 8).

- **FUNCIONES Y CAMPOS ESCALARES Y VECTORIALES**

9. La temperatura *T* de un tamalito goajaqueño es independiente de *z* y está dada por una función escalar *T* = *T*(*x, y*). Identifica las isotermas donde *T*(*x, y*) = const. y dibuja algunas de ellas. Puedes usar ayuda computacional para las gráficas.
   (a) $T = x^2 - y^2$ (b) $T = xy$
   (c) $T = 3x - 4y$ (d) $T = \arctan(y/x)$

10. Para cada función, ¿Qué tipo de superficies son las «superficies de nivel» $f(x, y, z) = \text{const.}$?

   (a) $f = 9(x^2 + y^2) + z^2$ (b) $f = 5x^2 + 2y^2$ (c) $f = z - \sqrt{x^2 + y^2}$

- **CURVAS, TANGENTES, LONGITUD DE CURVA, CURVAS EN MECÁNICA**

11. Dada una curva $C: \mathbf{r}(t)$, encuentra un vector tangente $\mathbf{r}'(t)$, un vector tangente unitario $\mathbf{u}'(t)$ y la tangente de C en P. Esboza la curva y la tangente.
   (a) $\mathbf{r}(t) = \left[t, \quad \frac{1}{2}t^2, \quad 1\right], \quad P: (2,2,1)$
   (b) $\mathbf{r}(t) = \left[\cos t, \quad \sin t, \quad 9t\right], \quad P: (1,0,18\pi)$
   (c) $\mathbf{r}(t) = \left[t \quad t^2 \quad t^3\right], \quad P: (1,1,1)$

12. Encuentra la longitud total y haz un esbozo de la curva hipocicloide dada por $\mathbf{r}(t) = \left[a\cos^3 t, \quad a\sin^3 t\right]$.

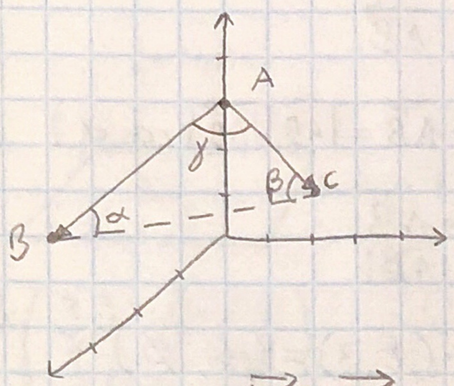
13. Para las trayectorias en los incisos (a) y (b), encuentra la aceleración tangencial, la aceleración normal, la velocidad y la rapidez.

    (a) Línea recta $\mathbf{r}(t) = \left[8t, \quad 6t, \quad 0\right]$

    (b) Elipse $\mathbf{r}(t) = \left[\cos t, \quad 2\sin t, \quad 0\right]$

Vectores: Producto escalar

1- Encuentra los ángulos del triángulo con vértices $A(0,0,2)$ $B(3,0,2)$ y $C(1,1,1)$. Haz un dibujo del triángulo.



γ entre $\overrightarrow{AB}$, $\overrightarrow{AC}$

$\arccos(\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}||\overrightarrow{AC}| \cos\gamma)$

$\overrightarrow{AB} = (3,\varnothing,2)-(\varnothing,\varnothing,2) = (3,\varnothing,\varnothing)$

$\overrightarrow{AC} = (1,1,1)-(\varnothing,\varnothing,2) = (1,1,-1)$

$$\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{(3,\varnothing,\varnothing)\cdot(1,1,-1)}{|3,\varnothing,\varnothing|\ |1,1,1|} = \frac{(3\cdot1)+(\varnothing\cdot1)+(\varnothing\cdot1)}{\sqrt{3^2+\varnothing^2+\varnothing^2}\ \sqrt{1^2+1^2+1^2}} = \frac{\cancel{3}^{\,1}}{\cancel{\sqrt{9}}\ \sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \qquad \arccos\left(\frac{1}{\sqrt{3}}\right) = 54.73^\circ$$

α entre $\overrightarrow{BA}$, $\overrightarrow{BC}$

$\arccos(\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}||\overrightarrow{BC}| \cos\alpha)$

$\overrightarrow{BA} = (\varnothing,\varnothing,2)-(3,\varnothing,2) = (-3,\varnothing,\varnothing)$

$\overrightarrow{BC} = (1,1,1)-(3,\varnothing,2) = (-2,1,-1)$

$$\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \frac{(-3,\varnothing,\varnothing)\cdot(-2,1,-1)}{|3,\varnothing,\varnothing|\ |2,1,1|} = \frac{(-3\cdot-2)+(\varnothing\cdot1)+(0\cdot-1)}{\sqrt{3^2+\varnothing^2+\varnothing^2}\ \sqrt{2^2+1^2+1^2}} = \frac{\cancel{6}^{\,2}}{\cancel{\sqrt{9}}\ \sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$\arccos\left(\frac{2}{\sqrt{6}}\right) = 35.26^\circ$$

La suma de los ángulos internos de un triángulo es igual a 180°

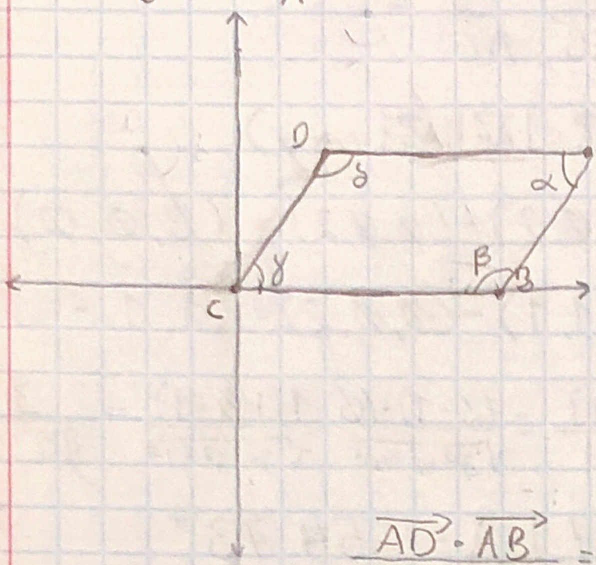
$$\begin{array}{r} 54.73^\circ \\ +\ 35.26^\circ \\ \hline 89.99^\circ \end{array} \qquad \begin{array}{r} 180^\circ \\ -\ 89.99^\circ \\ \hline 91.01^\circ \end{array}$$

$\gamma = 54.73^\circ$

$\alpha = 35.26^\circ$

$\beta = 91.01^\circ$

2- Encuentra los ángulos de un paralelogramo si los vértices son $\overset{C}{(\emptyset, \emptyset)}$ $\overset{B}{(6, \emptyset)}$, $\overset{A}{(8, 3)}$, $\overset{D}{(2, 3)}$



α entre $\overrightarrow{AD}, \overrightarrow{AB}$

$\arccos(\overrightarrow{AD} \cdot \overrightarrow{AB} = |\overrightarrow{AD}||\overrightarrow{AB}| \cos\alpha)$

$$\alpha = \frac{\overrightarrow{AD} \cdot AB}{|AD|\,|AB|}$$

$$\overrightarrow{AD} = (2,3) - (8,3) = (-6, \emptyset)$$

$$\overrightarrow{AB} = (6,\emptyset) - (8,3) = (-2,-3)$$

$$\frac{\overrightarrow{AD} \cdot \overrightarrow{AB}}{|\overrightarrow{AD}||\overrightarrow{AB}|} = \frac{(-6,\emptyset)\cdot(-2,-3)}{|6,\emptyset|\,|2,3|} = \frac{-6 \cdot -2 + \emptyset \cdot -3}{\sqrt{6^2}\ \sqrt{2^2+3^2}} = \frac{\overset{2}{\cancel{12}}}{\cancel{6}\ \sqrt{13}}$$

$$= \frac{2}{\sqrt{13}} \qquad \arccos\left(\frac{2}{\sqrt{13}}\right) = 56.30°$$

β entre $\overrightarrow{BA}, \overrightarrow{BC}$ $\qquad \overrightarrow{BA} = (8,3) - (6,\emptyset) = (2,3)$

$\arccos(\overrightarrow{BA} \cdot \overrightarrow{BC} = |BA||BC| \cos\beta)$ $\qquad \overrightarrow{BC} = (\emptyset\emptyset) - (6,\emptyset) = (-6,\emptyset)$

$$\frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \frac{(2\ 3)\cdot(-6,\emptyset)}{|2\ 3|\ |6,\emptyset|} = \frac{2 \cdot -6 + 3 \cdot \emptyset}{\sqrt{2^2+3^2}\ \sqrt{6^2}} = \frac{\overset{-2}{\cancel{-12}}}{\sqrt{13}\ \cancel{6}} = \frac{-2}{\sqrt{13}} \quad \arccos\left(\frac{-2}{\sqrt{13}}\right) = 123.69°$$

$\alpha = 56.30°$

$\beta = 123.69°$

$\gamma = 56.30°$

$\delta = 123.69°$

La suma de dos vértices contiguos de un paralelogramo es igual a 180° y la suma total de los ángulos internos es 360°. Dos ángulos opuestos por el vértice son iguales

$\alpha = \gamma \qquad \beta = \delta$, para probar esto buscaremos γ y δ con el arcos

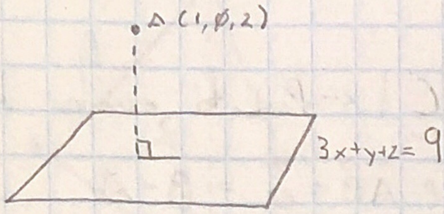
γ $\overrightarrow{CD} = (2,3) - (\emptyset,\emptyset) = (2,3)$ $\qquad$ $\overrightarrow{CB} = (6,\emptyset) - (\emptyset,\emptyset) = (6,\emptyset)$

δ $\overrightarrow{DA} = (8,3) - (2,3) = (6,\emptyset)$ $\qquad$ $\overrightarrow{DC} = (\emptyset,\emptyset) - (2,3) = (-2,-3)$

$$\frac{\overrightarrow{CD} \cdot \overrightarrow{CB}}{|\overrightarrow{CD}||\overrightarrow{CB}|} = \frac{(2,3)\cdot(6,\emptyset)}{|2,3|\ |6,\emptyset|} = \frac{2 \cdot 6 + 3 \cdot \emptyset}{\sqrt{2^2+3^2}\ \sqrt{6^2}} = \frac{\overset{2}{\cancel{12}}}{\sqrt{13}\ \cancel{6}} = \frac{2}{\sqrt{13}} \quad \arccos\left(\frac{2}{\sqrt{13}}\right) = 56.30°$$

$$\frac{\overrightarrow{DA} \cdot \overrightarrow{DC}}{|\overrightarrow{DA}||\overrightarrow{DC}|} = \frac{(6,\emptyset)\cdot(-2,-3)}{|6,\emptyset|\ |2,3|} = \frac{6 \cdot -2 + \emptyset \cdot -3}{\sqrt{6^2}\ \sqrt{2^2+3^2}} = \frac{\overset{-2}{\cancel{-12}}}{\cancel{6}\ \sqrt{13}} = \frac{-2}{\sqrt{13}} \quad \arccos\left(\frac{-2}{\sqrt{13}}\right) = 123.69°$$

3- Encuentra la distancia del punto $A(1, \emptyset, 2)$ al Plano $3x+y+z=9$ haz un dibujo del problema



$$\underset{\sim}{P}\cdot\underset{\sim}{A} = ax+by+cz=d$$

$$P=\begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq \emptyset \qquad A=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

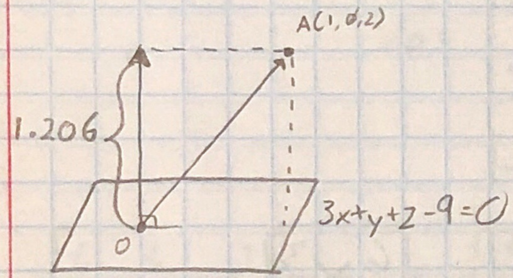
$$P=\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \quad A=\begin{pmatrix} 1 \\ \emptyset \\ 2 \end{pmatrix} \qquad D = P\cdot A = \frac{\underset{\sim}{P}\cdot\underset{\sim}{A}}{|\underset{\sim}{P}|} \qquad \hat{P}\cdot\hat{A} = \frac{d}{|P|}$$

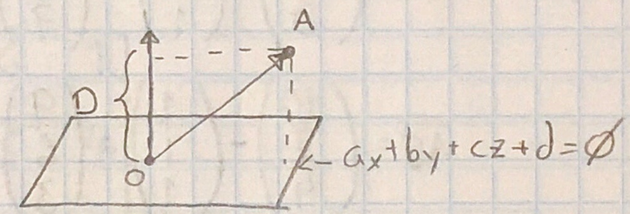
$$D = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}} = \text{Proyección } U = \frac{|\underset{\sim}{u}\cdot\underset{\sim}{v}|}{|\underset{\sim}{v}|}$$

$$P = 3x+y+z-9=\emptyset$$

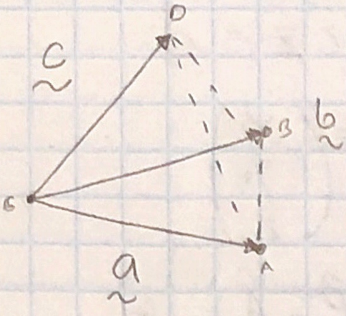
$$D = \frac{|(3\cdot 1)+(1\cdot\emptyset)+(9\cdot 2)-9)|}{\sqrt{3^2+1^2+1^2}} = \frac{|3+\emptyset+2-9|}{\sqrt{11}} = \frac{|4|}{\sqrt{11}}$$

$$\underline{D = 1.206\ u}\ //$$





7- Encuentra el volumen de un tetaedro con vértices $(1,1,1)$ $(5,-7,3)$ $(7,4,8)$ y $(10,7,4)$



$$V_T = \frac{1}{6} V_p$$

$$V_p = |(\underset{\sim}{a} \times \underset{\sim}{b}) \cdot \underset{\sim}{c}|$$ ← Producto escalar triple

$$V_p = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$\overrightarrow{AB} = \underset{\sim}{a} = B - A$

$\overrightarrow{AC} = \underset{\sim}{b} = C - A$

$AD = \underset{\sim}{c} = D - A$

$A = (1,1,1)$

$B = (5,-7,3)$

$C = (7,4,8)$

$D = (10,7,4)$

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ -7 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 2 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 7 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix}$$

$$\overrightarrow{AD} = \begin{pmatrix} 10 \\ 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix}$$

$$V_p = \begin{bmatrix} 4 & -8 & 2 \\ 6 & 3 & 7 \\ 9 & 6 & 3 \end{bmatrix}$$

Sacamos los determinantes

$$\begin{bmatrix} 4 & -8 & 2 \\ 6 & 3 & 7 \\ 9 & 6 & 3 \end{bmatrix} = \det\left(4\begin{bmatrix} 3 & 7 \\ 6 & 3 \end{bmatrix} + (-1 \cdot -8)\begin{bmatrix} 6 & 7 \\ 9 & 3 \end{bmatrix} + 2\begin{bmatrix} 6 & 3 \\ 9 & 6 \end{bmatrix}\right)$$

$\det(4((3\cdot3)-(6\cdot7)) + 8((6\cdot3)-(9\cdot7)) + 2((6\cdot6)-(9\cdot3)))$

$\det(4(9-42) + 8(18-63) + 2(36-27))$

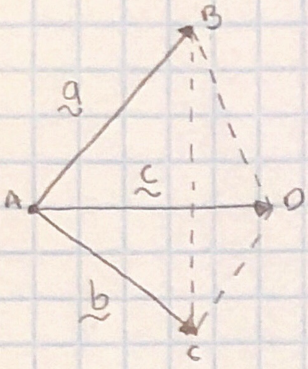
$\det(4(-33) + 8(-45) + 2(9))$

$\det(-132 - 360 + 18) = -474$

$V_p = |474|$

$V_T = 1/6 \; 474 = \underline{79 \, u^3}$

8- Encuentra el volumen de un tetaedro con vértices $(1,3,6)$, $(3,7,12)$ $(8,8,9)$ y $(2,2,8)$



$$V_T = \frac{1}{6} V_p$$

$$V_p = |(a \times b) \cdot c|$$

$$\overrightarrow{AB} = \underset{\sim}{a} \qquad \overrightarrow{AC} = \underset{\sim}{b} \qquad \overrightarrow{AD} = \underset{\sim}{c}$$

$$V_p = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$A = (2, 2, 8)$

$B = (8, 8, 9)$ $\qquad \overrightarrow{AB} = (8, 8, 9) - (2, 2, 8) = (6, 6, 1)$

$C = (3, 7, 12)$ $\qquad \overrightarrow{AC} = (3, 7, 12) - (2, 2, 8) = (1, 5, 4)$

$D = (1, 3, 6)$ $\qquad \overrightarrow{AD} = (1, 3, 6) - (2, 2, 8) = (-1, 1, -2)$

$$V_p = \begin{bmatrix} 6 & 6 & 1 \\ 1 & 5 & 4 \\ -1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 6 & 1 \\ 1 & 5 & 4 \\ -1 & 1 & -2 \end{bmatrix} = det\left(6\begin{bmatrix} 5 & 4 \\ 1 & -2 \end{bmatrix} + (-6)\begin{bmatrix} 1 & 4 \\ -1 & -2 \end{bmatrix} + 1\begin{bmatrix} 1 & 5 \\ -1 & 1 \end{bmatrix}\right)$$

$$6((5 \cdot -2) - (4 \cdot 1)) - 6((1 \cdot -2) - (-1 \cdot 4)) + 1((1 \cdot 1) - (-1 \cdot 5))$$

$$6(-10 - 4) - 6(-2 + 4) + 1(1 + 5)$$

$$6(-14) - 6(2) + 1(6) = -90$$

$$V_p = |90|$$

$$V_T = \frac{1}{6}\, 90 = \underline{15\, u^3}$$

Homework 04 - Differential Vector Calculus

Funciones y Campos Escalares y Vectoriales

$\boxed{10}$ Para cada función, ¿Qué tipo de "superficies de nivel" $f(x, y, z) = \text{const.}$?

$\boxed{a}$ $9(x^2+y^2) + z^2 = 1 \Rightarrow$ spheroid

$\boxed{b}$ $5x^2 + 2y^2 = 1 \Rightarrow$ ellipse

$\boxed{c}$ $z - \sqrt{x^2+y^2} = 0 \Rightarrow$ cone

Homework 04 - Differential Vector Calculus

Curvas, Tangentes, Longitud de Curva, Curvas en Mecánica.

[11] Dada una curva $C: r(t)$, encuentra un vector tangente $r'(t)$, un vector tangente unitario $u'(t)$ y la tangente de C en P. Esboza la curva y la tangente. (pp 410)

Let's solve the "Tangent to a Curve" problems...

Note: If $r'(t) \neq 0$, we call $r'(t)$ a tangent vector of C at P, and u the unit tangent vector.

[a] $r(t) = \left[t, \frac{1}{2}t^2, 1\right]$, $P: (2, 2, 1)$

$$r'(t) = [1, t, 0] \;//$$

$$u = \frac{1}{|r'(t)|} r'(t) = \frac{1}{\sqrt{1^2 + t^2 + 0^2}} (1, t, 0)$$

$$= \frac{1}{\sqrt{1+t^2}} (1, t, 0) \;//$$

at $t = 2$, $r(t) = P$...

$$r'(2) = (1, 2, 0)$$

$$u = \frac{1}{\sqrt{5}} (1, 2, 0) \;//$$

[b] $r(t) = [\cos(t), \sin(t), 9t]$, $P: (1, 0, 18\pi)$

$$r'(t) = [-\sin(t), \cos(t), 9] \;//$$

$$u = \frac{1}{\sqrt{(-\sin(t))^2 + (\cos(t))^2 + 9^2}} (-\sin(t), \cos(t), 9) \quad \Big\{ \sin^2(t) + \cos^2(t) = 1$$

$$= \frac{1}{\sqrt{82}} (-\sin(t), \cos(t), 9) \;//$$

at $t = 2\pi$, $r(t) = P$

$$r'(2\pi) = (0, 1, 9)$$

$$u = \frac{1}{\sqrt{82}}(0,1,9)$$

$\boxed{c}$ $r(t) = [t, t^2, t^3]$, $P: (1,1,1)$

$$r'(t) = [1, 2t, 3t^2]$$

$$u = \frac{1}{\sqrt{1^2 + (2t)^2 + (3t^2)^2}}(1, 2t, 3t^2)$$

$$= \frac{1}{\sqrt{1 + 4t^2 + 9t^4}}(1, 2t, 3t^2)$$

at $t = 1$, $r(t) = P$

$$r'(1) = (1, 2, 3)$$

$$u = \frac{1}{\sqrt{1+4+9}}(1,2,3)$$

$$= \frac{1}{\sqrt{14}}(1,2,3)$$

12 Encuentra la longitud total y haz un esbozo de la curva hipocicloide dada por $r(t) = [a\cos^3 t, a\sin^3 t]$ (pp 411)

Note: The length ℓ is given by:

$\ell = \int_a^b \sqrt{r'(t) \cdot r'(t)}\, dt$; where ℓ represents the limit of the lengths of broken lines of n chords. for each n, the interval $a \leq t < b$ is subdivided by points.

Note: ℓ is called the length of C, and C is called rectifiable.

$$r'(t) = \frac{d}{dt} r(t)$$
$$= \frac{d}{dt}\left[a\cos^3(t), a\sin^3(t)\right]$$
$$= \left[-3a\cos^2(t)\sin(t), 3a\cos(t)\sin^2(t)\right]$$

$$r'(t) \cdot r'(t) = \left(9a^2\cos^4(t)\sin^2(t)\right) + \left(9a^2\cos^2(t)\sin^4(t)\right)$$
$$= 9a^2\cos^2(t)\sin^2(t)$$
$$= \frac{9a^2}{4}\sin^2(2t)$$

$\cos^2(t)\sin^2(t) = \frac{\sin^2(2t)}{4}$

$$\ell = \int_0^{2\pi} \sqrt{\frac{9a^2}{4}\sin^2(2t)}\, dt = \frac{3a}{2}\int_0^{2\pi} \sin(2t)\, dt$$
$$= \frac{3a}{2}\left[-\frac{1}{2}\cos(2t)\Big|_0^{2\pi}\right]$$
$$= \frac{3a}{2}\left[\cancel{\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)}^{\,0}\right]$$
$$= \frac{3a}{2}\,u$$

///

13 Para las trayectorias en los incisos (a) y (b), encuentra la aceleración tangencial, la aceleración normal, la velocidad y la rapidez. (pp. 412)

Note: The tangent vector of C is called the velocity vector v. (as the tangent points at the instantaneous directions of motion and its length gives the speed $|v|$. The second derivative of $r(t)$ is called the acceleration vector a. $|a|$ (a's length) is called the acceleration of the motion.

$$v(t) = r'(t)$$
$$|v| = |r'| = \sqrt{r' \cdot r'} = ds/dt \qquad \& \qquad a(t) = v'(t) = r''(t)$$

Note: The acceleration vector can be splited into directional components: a_{tan} & a_{norm}
The tangential acceleration vector a_{tan} is tangent to the path and the normal acceleration vector a_{norm} is perpendicular (normal) to the path.

That is...

$$u(s) = \frac{dr}{ds} \; ; \; v(t) = u(s)\frac{ds}{dt} \; ; \; a(t) = \frac{du}{ds}\left(\frac{ds}{dt}\right)^2 + u(s)\frac{d^2s}{dt^2}$$

$$a_{tan} = \frac{a \cdot v}{v \cdot v} v \; ; \; a_{norm} = a - a_{tan}$$

a Línea recta $r(t) = [8t, 6t, 0]$

$$v(t) = r'(t) = [8, 6, 0] \, //$$

$$|v| = \sqrt{r' \cdot r'}$$
$$= \sqrt{8^2 + 6^2 + 0^2}$$
$$= 10 \, //$$

$$a(t) = v'(t) = [0, 0, 0] \, //$$

$$a_{tan} = \frac{a \cdot v}{v \cdot v} = \frac{0 + 0 + 0}{8^2 + 6^2 + 0^2}[8, 6, 0]$$
$$= [0, 0, 0] \, //$$

$$a_{norm} = a - a_{tan}$$
$$= [0, 0, 0] \, //$$

b Elipse $r(t) = [\cos t, 2\sin t, 0]$

$$v(t) = r'(t) = [-\sin t, 2\cos t, 0]$$

$$|v| = \sqrt{(-\sin t)^2 + (2\cos t)^2 + 0}$$
$$= \sqrt{4\cos^2(t) + \sin^2(t)}$$

$$a(t) = v'(t) = [-\cos t, -2\sin t, 0]$$

$$a_{tan} = \frac{a \cdot v}{v \cdot v} = \frac{(-\cos t)(-\sin t) + (-2\sin t)(2\cos t) + 0}{\sin^2 t + 4\cos^2 t + 0}$$
$$= \frac{-3\cos(t)\sin(t)}{\sin^2(t) + 4\cos^2(t)}$$
$$= \frac{3\sin(2t)}{5 + 3\cos(2t)}[-\sin(t), 2\cos(t), 0]$$

$$a_{norm} = a - a_{tan}$$
$$= [-\cos(t), -2\sin(t), 0] - \underbrace{\frac{3\sin(2t)}{5+3\cos(2t)}}_{\alpha}[-\sin(t), 2\cos(t), 0]$$
$$= [(-\cos(t) - \alpha(-\sin(t))), (-2\sin(t) - \alpha(2\cos(t), 0]$$
$$= \left[-\cos(t) - \frac{6\sin(t)\sin(2t)}{5+3\cos(2t)}, -2\sin(t) + \frac{6\cos(t)\sin(2t)}{5+3\cos(2t)}, 0\right]$$
$$= \left[-\cos(t) - \frac{6\sin(t)\sin(2t)}{5+3\cos(2t)}, -\frac{4\sin(t)}{5+3\cos(2t)}, 0\right]$$