Homework 2 Solutions:

(1)

a) · Bias calculation:

First, compute $E(\hat{\theta})$

Since $\hat{\theta} = \dfrac{s+1}{n+2}$ $\Rightarrow$ $E(\hat{\theta}) = \dfrac{1}{n+2} \cdot E(s+1)$

Expected # of successes in Binomial setting is np

$$= \dfrac{1}{n+2} \cdot [E(s) + 1]$$

$$= \dfrac{1}{n+2} \cdot (n\theta + 1)$$

$$= \dfrac{n\theta + 1}{n+2}$$

Then, compute $\theta - E(\hat{\theta})$

$$\text{Bias} = \theta - \left[\dfrac{n\theta}{n+2} + \dfrac{1}{n+2}\right] = \theta\left(\underbrace{\dfrac{n+2}{n+2}}_{=1} - \dfrac{n}{n+2}\right) - \dfrac{1}{n+2}$$

$$= \dfrac{2\theta - 1}{n+2}$$

· Variance Calculation

$$V(\hat{\theta}) = V\left(\dfrac{s+1}{n+2}\right) = \dfrac{1}{(n+2)^2} \cdot V(s+1)$$

$$= \dfrac{V(s)}{(n+2)^2} = \dfrac{n \cdot \theta \cdot (1-\theta)}{(n+2)^2}$$

b) $se(\hat{\theta})$ is just $\sqrt{V(\hat{\theta})}$

Thus, $se(\hat{\theta}) = \dfrac{[n \cdot \theta \cdot (1-\theta)]^{1/2}}{n+2}$

$\hat{\theta}$ $\;\;$ $\frac{s+1}{}$

- Since we don't know θ, we use our estimator $\hat{\theta} = \frac{\overline{}}{n+2}$ in its place (this is the <u>plug-in principle</u>) and get

$$se(\hat{\theta}) = \frac{\left[n \cdot \left(\frac{s+1}{n+2}\right) \cdot \left(1 - \frac{s+1}{n+2}\right) \right]^{1/2}}{n+2}$$

and then, of course, you can further simplify it.

② See Jupyter Notebook w/ sol.

③ Frequentism assigns the properties of the estimator (like its <u>bias & variance</u>) to the value of it we got from our very specific sample. The reason for doing this (and the link to the <u>repeated sampling assumption</u>), is that your very specific sample is a <u>random one</u> from the ~~the~~ infinite possible samples.

④ The test would asymptotically converge to the t-student's test for larger $\underline{n_1 + n_2}$, as sample increase would be accompanied by asymptotic convergence of $\underline{\hat{\sigma} \text{ to } \sigma}$.

⑤ See Jupyter Notebook w/ sol.