

Mathematical Physical Modelling - Homework 04

Differential Vector Calculus

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Paper trail and process evidence are at the end of this document.

- **VECTORES: PRODUCTO ESCALAR**

1. Encuentra los ángulos del triángulo con vértices $A: (0, 0, 2)$, $B: (3, 0, 2)$ y $C: (1, 1, 1)$. Haz un dibujo del triángulo.
2. Encuentra los ángulos de un paralelogramo si los vértices son $(0, 0)$, $(6, 0)$, $(8, 3)$ y $(2, 3)$.
3. Encuentra la distancia del punto $A: (1, 0, 2)$ al plano $P: 3x + y + z = 9$. Haz un dibujo del problema.
4. ¿Para cuál valor de c serán ortogonales los planos $3x + z = 5$ y $8x - y + cz = 9$?
5. Encuentra la componente del vector \mathbf{a} en la dirección del vector \mathbf{b} en los siguientes casos:
 - (a) $\mathbf{a} = [1, 1, 1]$, $\mathbf{b} = [2, 1, 3]$
 - (b) $\mathbf{a} = [3, 4, 0]$, $\mathbf{b} = [4, -3, 2]$
 - (c) $\mathbf{a} = [8, 2, 0]$, $\mathbf{b} = [-4, -1, 0]$

- **VECTORES Y PRODUCTO ESCALAR TRIPLE**

6. Con respecto a un sistema derecho cartesiano, sean

$$\mathbf{a} = [2, 1, 0], \mathbf{b} = [-3, 2, 0], \mathbf{c} = [1, 4, -2]$$

Mostrando los detalles, encuentra lo siguiente:

- (a) $(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{d}$
 - (b) $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{d})$
7. Encuentra el volumen de un tetraedro con vértices $(1, 1, 1)$, $(5, -7, 3)$, $(7, 4, 8)$ y $(10, 7, 4)$.
 8. Encuentra el volumen de un tetraedro con vértices $(1, 3, 6)$, $(3, 7, 12)$, $(8, 8, 9)$ y $(2, 2, 8)$.

- **FUNCIONES Y CAMPOS ESCALARES Y VECTORIALES**

9. La temperatura T de un tamalito goajaqueño es independiente de z y está dada por una función escalar $T = T(x, y)$. Identifica las isotermas donde $T(x, y) = \text{const.}$ y dibuja algunas de ellas. Puedes usar ayuda computacional para las gráficas.
 - (a) $T = x^2 - y^2$
 - (b) $T = xy$
 - (c) $T = 3x - 4y$
 - (d) $T = \arctan(y/x)$

10. Para cada función, ¿Qué tipo de superficies son las «superficies de nivel» $f(x, y, z) = \text{const.}$?

$$(a) f = 9(x^2 + y^2) + z^2 \quad (b) f = 5x^2 + 2y^2 \quad (c) f = z - \sqrt{x^2 + y^2}$$

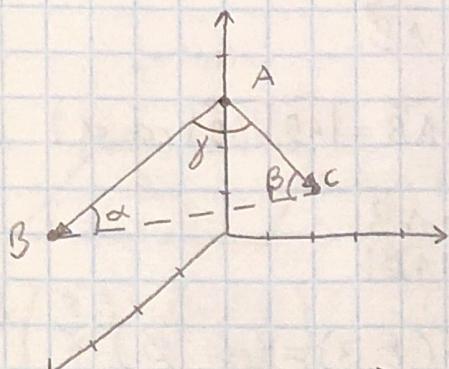
- **CURVAS, TANGENTES, LONGITUD DE CURVA, CURVAS EN MECÁNICA**

11. Dada una curva $C : \mathbf{r}(t)$, encuentra un vector tangente $\mathbf{r}'(t)$, un vector tangente unitario $\mathbf{u}'(t)$ y la tangente de C en P . Esboza la curva y la tangente.
 - (a) $\mathbf{r}(t) = [t, \frac{1}{2}t^2, 1]$, $P : (2, 2, 1)$
 - (b) $\mathbf{r}(t) = [\cos t, \sin t, 9t]$, $P : (1, 0, 18\pi)$
 - (c) $\mathbf{r}(t) = [t \ t^2 \ t^3]$, $P : (1, 1, 1)$

12. Encuentra la longitud total y haz un esbozo de la curva hipocicloide dada por
 $\mathbf{r}(t) = [a \cos^3 t, a \sin^3 t]$.
13. Para las trayectorias en los incisos (a) y (b), encuentra la aceleración tangencial, la aceleración normal, la velocidad y la rapidez.
- (a) Línea recta $\mathbf{r}(t) = [8t, 6t, 0]$
- (b) Elipse $\mathbf{r}(t) = [\cos t, 2 \sin t, 0]$

Vectores: Producto escalar

- 1- Encuentra los ángulos del triángulo con vértices $A(0, 0, 2)$, $B(3, 0, 2)$ y $C(1, 1, 1)$. Haz un dibujo del triángulo.



γ entre \vec{AB} , \vec{AC}

$$\arccos(\vec{AB} \cdot \vec{AC} / |\vec{AB}| |\vec{AC}|) \cos \gamma$$

$$\vec{AB} = (3, 0, 2) - (0, 0, 2) = (3, 0, 0)$$

$$\vec{AC} = (1, 1, 1) - (0, 0, 2) = (1, 1, -1)$$

$$\frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} = \frac{(3, 0, 0) \cdot (1, 1, -1)}{|(3, 0, 0)| |(1, 1, -1)|} = \frac{(3 \cdot 1) + (0 \cdot 1) + (0 \cdot -1)}{\sqrt{3^2 + 0^2 + 0^2} \sqrt{1^2 + 1^2 + (-1)^2}} = \frac{3}{\sqrt{9}} \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \quad \arccos\left(\frac{1}{\sqrt{3}}\right) = 54.73^\circ$$

α entre \vec{BA} , \vec{BC}

$$\vec{BA} = (0, 0, 2) - (3, 0, 2) = (-3, 0, 0)$$

$$\arccos(\vec{BA} \cdot \vec{BC} / |\vec{BA}| |\vec{BC}|) \cos \alpha$$

$$\vec{BC} = (1, 1, 1) - (3, 0, 2) = (-2, 1, -1)$$

$$\frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} = \frac{(-3, 0, 0) \cdot (-2, 1, -1)}{|(-3, 0, 0)| |(-2, 1, -1)|} = \frac{(-3 \cdot -2) + (0 \cdot 1) + (0 \cdot -1)}{\sqrt{(-3)^2 + 0^2 + 0^2} \sqrt{(-2)^2 + 1^2 + (-1)^2}} = \frac{6}{\sqrt{9}} \frac{1}{\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$\arccos\left(\frac{2}{\sqrt{6}}\right) = 35.26^\circ$$

La suma de los ángulos internos de un triángulo es igual a 180°

$$+ 54.73^\circ$$

$$- 180^\circ$$

$$\gamma = 54.73^\circ$$

$$35.26^\circ$$

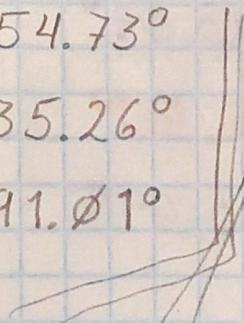
$$- 89.99^\circ$$

$$\alpha = 35.26^\circ$$

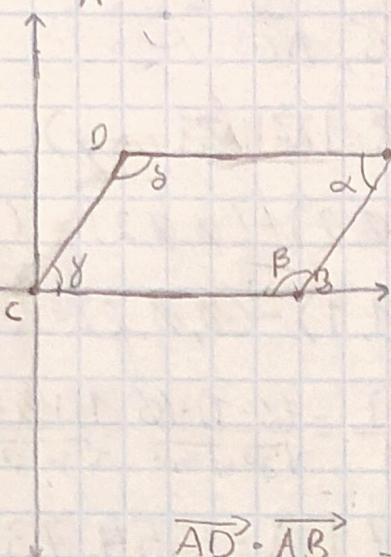
$$89.99^\circ$$

$$- 91.01^\circ$$

$$\beta = 91.01^\circ$$



2- Encuentra los ángulos de un paralelogramo si los vértices son (ϕ, ϕ) , $(6, \phi)$, $(8, 3)$, $(2, 3)$



α entre \vec{AD}, \vec{AB}

$$\cos(\vec{AD} \cdot \vec{AB}) = |\vec{AD}| |\vec{AB}| \cos \alpha$$

$$\alpha = \frac{\vec{AD} \cdot \vec{AB}}{|\vec{AD}| |\vec{AB}|}$$

$$\vec{AD} = (2, 3) - (8, \phi) = (-6, \phi)$$

$$\vec{AB} = (6, \phi) - (8, 3) = (-2, -3)$$

$$\frac{\vec{AD} \cdot \vec{AB}}{|\vec{AD}| |\vec{AB}|} = \frac{(-6, \phi) \cdot (-2, -3)}{|(-6, \phi)| |(-2, -3)|} = \frac{-6 \cdot -2 + \phi \cdot -3}{\sqrt{6^2 + \phi^2} \sqrt{2^2 + 3^2}} = \frac{12 - 3\phi}{\sqrt{13} \sqrt{13}} = \frac{12}{13}$$

$$= \frac{2}{\sqrt{13}} \quad \cos\left(\frac{2}{\sqrt{13}}\right) = 56.30^\circ$$

β entre \vec{BA}, \vec{BC}

$$\vec{BA} = (8, 3) - (6, \phi) = (2, 3)$$

$$\cos(\vec{BA} \cdot \vec{BC}) = |\vec{BA}| |\vec{BC}| \cos \beta \quad \vec{BC} = (\phi, \phi) - (6, \phi) = (-6, \phi)$$

$$\frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} = \frac{(2, 3) \cdot (-6, \phi)}{|(2, 3)| |(-6, \phi)|} = \frac{2 \cdot -6 + 3 \cdot \phi}{\sqrt{2^2 + 3^2} \sqrt{6^2 + \phi^2}} = \frac{-12 + 3\phi}{\sqrt{13} \sqrt{13}} = \frac{-12}{13} \quad \cos\left(\frac{-12}{13}\right) = 123.69^\circ$$

$$\alpha = 56.30^\circ$$

La suma de dos vértices contiguos de un paralelogramo es igual a 180° y la suma total de los ángulos internos es 360° . Los ángulos opuestos por el vértice son iguales

$\alpha = \gamma \quad \beta = \delta$, para probar esto buscaremos γ y δ con el arcos

$$\delta = 123.69^\circ$$

$$\gamma \quad \vec{CD} = (2, 3) - (\phi, \phi) = (2, 3)$$

$$\vec{CB} = (6, \phi) - (\phi, \phi) = (6, \phi)$$

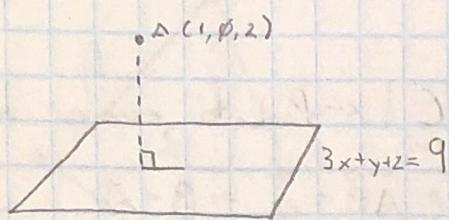
$$\vec{DA} = (8, 3) - (2, 3) = (6, \phi)$$

$$\delta \quad \vec{DC} = (\phi, \phi) - (2, 3) = (-2, -3)$$

$$\frac{\vec{CD} \cdot \vec{CB}}{|\vec{CD}| |\vec{CB}|} = \frac{(2, 3) \cdot (6, \phi)}{|(2, 3)| |(6, \phi)|} = \frac{2 \cdot 6 + 3 \cdot \phi}{\sqrt{2^2 + 3^2} \sqrt{6^2 + \phi^2}} = \frac{12}{\sqrt{13} \sqrt{13}} = \frac{12}{13} \quad \cos\left(\frac{12}{13}\right) = 56.30^\circ$$

$$\frac{\vec{DA} \cdot \vec{DC}}{|\vec{DA}| |\vec{DC}|} = \frac{(6, \phi) \cdot (-2, -3)}{|(6, \phi)| |(-2, -3)|} = \frac{6 \cdot -2 + \phi \cdot -3}{\sqrt{6^2 + \phi^2} \sqrt{2^2 + 3^2}} = \frac{-12}{\sqrt{13} \sqrt{13}} = \frac{-12}{13} \quad \cos\left(\frac{-12}{13}\right) = 123.69^\circ$$

3- Encuentra la distancia del punto $A(1, \phi, 2)$ al Plano $3x+y+z=9$
haz un dibujo del problema



$$\tilde{P} \cdot \tilde{A} = ax + by + cz = d$$

$$P = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq \emptyset \quad A = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$P = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \quad A = \begin{pmatrix} 1 \\ \phi \\ 2 \end{pmatrix}$$

$$D = P \cdot A = \frac{\tilde{P} \cdot \tilde{A}}{|\tilde{P}|}$$

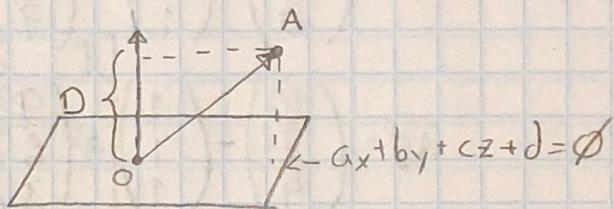
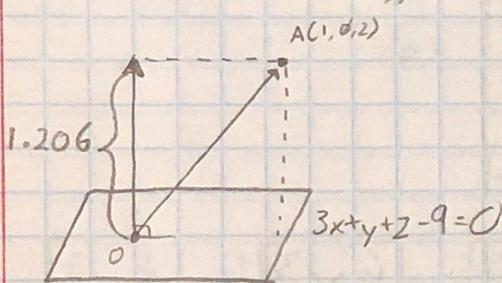
$$\hat{P} \cdot \hat{A} = \frac{d}{|\tilde{P}|}$$

$$P = 3x + y + z - 9 = \emptyset$$

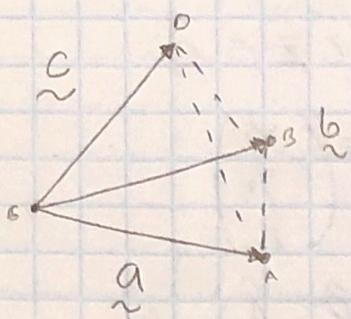
$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \text{Proyección } U = \frac{|x \cdot \vec{v}|}{\|\vec{v}\|}$$

$$D = \frac{|(3 \cdot 1) + (1 \cdot \phi) + (1 \cdot 2) - 9|}{\sqrt{3^2 + 1^2 + 1^2}} = \frac{|3 + \phi + 2 - 9|}{\sqrt{11}} = \frac{|4|}{\sqrt{11}}$$

$$D = \underline{1.206} \text{ } \cancel{U}$$



7- Encuentra el volumen de un tetraedro con vértices $(1, 1, 1)$, $(5, -7, 3)$, $(7, 4, 8)$ y $(10, 7, 4)$



$$V_t = \frac{1}{6} V_p$$

$$V_p = |(\underline{a} \times \underline{b}) \cdot \underline{c}| \leftarrow \text{Producto escalar triple}$$

$$V_p = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$\vec{AB} = \underline{a} = \underline{B} - \underline{A}$$

$$\vec{AC} = \underline{b} = \underline{C} - \underline{A}$$

$$\vec{AD} = \underline{c} = \underline{D} - \underline{A}$$

$$\underline{A} = (1, 1, 1)$$

$$\underline{B} = (5, -7, 3)$$

$$\vec{AB} = \begin{pmatrix} 5 \\ -7 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 2 \end{pmatrix}$$

$$\underline{C} = (7, 4, 8)$$

$$\underline{D} = (10, 7, 4)$$

$$\vec{AC} = \begin{pmatrix} 7 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix}$$

$$\vec{AD} = \begin{pmatrix} 10 \\ 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix}$$

$$V_p = \begin{bmatrix} 4 & -8 & 2 \\ 6 & 3 & 7 \\ 9 & 6 & 3 \end{bmatrix}$$

Sacamos los determinantes

$$\begin{vmatrix} 4 & -8 & 2 \\ 6 & 3 & 7 \\ 9 & 6 & 3 \end{vmatrix} = \det(4 \begin{bmatrix} 3 & 7 \\ 6 & 3 \end{bmatrix} + (-1 \cdot -8) \begin{bmatrix} 6 & 7 \\ 9 & 3 \end{bmatrix} + 2 \begin{bmatrix} 6 & 3 \\ 9 & 6 \end{bmatrix})$$

$$\det(4((3 \cdot 3) - (6 \cdot 7)) + 8((6 \cdot 3) - (9 \cdot 7)) + 2((6 \cdot 6) - (9 \cdot 3)))$$

$$\det(4(9 - 42) + 8(18 - 63) + 2(36 - 27))$$

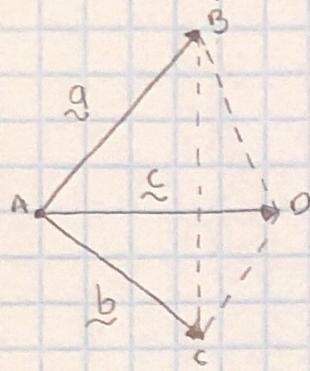
$$\det(4(-33) + 8(-45) + 2(9))$$

$$\det(-132 - 360 + 18) = -474$$

$$V_p = 1474$$

$$V_t = \frac{1}{6} 474 = \underline{\underline{79}} \quad \checkmark$$

8- Encuentra el volumen de un tetraedro con vértices $(1, 3, 6), (3, 7, 12)$, $(8, 8, 9)$ y $(2, 2, 8)$



$$V_T = \frac{1}{6} V_P$$

$$V_P = |(a \times b) \cdot c|$$

$$\vec{AB} = \underline{a}, \quad \vec{AC} = \underline{b}, \quad \vec{AD} = \underline{c}$$

$$V_P = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$A = (2, 2, 8)$$

$$B = (8, 8, 9)$$

$$\vec{AB} = (8, 8, 9) - (2, 2, 8) = (6, 6, 1)$$

$$C = (3, 7, 12)$$

$$\vec{AC} = (3, 7, 12) - (2, 2, 8) = (1, 5, 4)$$

$$D = (1, 3, 6)$$

$$\vec{AD} = (1, 3, 6) - (2, 2, 8) = (-1, 1, -2)$$

$$V_P = \begin{bmatrix} 6 & 6 & 1 \\ 1 & 5 & 4 \\ -1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 6 & 1 \\ 1 & 5 & 4 \\ -1 & 1 & -2 \end{bmatrix} = \det(6 \begin{bmatrix} 5 & 4 \\ 1 & -2 \end{bmatrix} + (-6) \begin{bmatrix} 1 & 4 \\ -1 & -2 \end{bmatrix} + 1 \begin{bmatrix} 1 & 5 \\ -1 & 1 \end{bmatrix})$$

$$6((5 \cdot -2) - (4 \cdot 1)) - 6((1 \cdot -2) - (-1 \cdot 4)) + 1((1 \cdot 1) - (-1 \cdot 5))$$

$$6(-10 - 4) - 6(-2 + 4) + 1(1 + 5)$$

$$6(-14) - 6(2) + 1(6) = -90$$

$$V_P = 190$$

$$V_T = \frac{1}{6} 90 = \underline{15 \text{ u}^3} \quad \cancel{\text{X}}$$

Homework 04 - Differential Vector Calculus

Curvas, Tangentes, Longitud de Curva, Curvas en Mecánica

11 Dada una curva $C: r(t)$, encuentra un vector tangente $r'(t)$, un vector tangente unitario $u'(t)$ y la tangente de C en P . Esboza la curva y la tangente. (pp 410)

Let's solve the "Tangent to a Curve" problems...

Note: If $r'(t) \neq 0$, we call $r'(t)$ a tangent vector of C at P , and u the unit tangent vector.

a $r(t) = \left[t, \frac{1}{2}t^2, 1 \right]$, $P: (2, 2, 1)$

$$r'(t) = \left[1, t, 0 \right] \quad \cancel{\text{/}}\cancel{\text{/}}$$

$$\begin{aligned} u &= \frac{1}{|r'(t)|} r'(t) = \frac{1}{\sqrt{1^2 + t^2 + 0^2}} (1, t, 0) \\ &= \frac{1}{\sqrt{1+t^2}} (1, t, 0) \quad \cancel{\text{/}}\cancel{\text{/}} \end{aligned}$$

at $t = 2$, $r(t) = P$...

$$r'(2) = (1, 2, 0)$$

$$u = \frac{1}{\sqrt{5}} (1, 2, 0) \quad \cancel{\text{/}}\cancel{\text{/}}$$

b $r(t) = [\cos(t), \sin(t), 9t]$, $P: (1, 0, 18\pi)$

$$r'(t) = [-\sin(t), \cos(t), 9] \quad \cancel{\text{/}}\cancel{\text{/}}$$

$$\begin{aligned} u &= \frac{1}{\sqrt{(-\sin(t))^2 + (\cos(t))^2 + 9^2}} (-\sin(t), \cos(t), 9) \quad \left. \begin{array}{l} \sin^2(t) + \cos^2(t) = 1 \end{array} \right. \\ &= \frac{1}{\sqrt{82}} (-\sin(t), \cos(t), 9) \quad \cancel{\text{/}}\cancel{\text{/}} \end{aligned}$$

at $t = 2\pi$, $r(t) = P$

$$r'(2\pi) = (0, 1, 9)$$

$$U = \frac{1}{\sqrt{82}} (0, 1, 9) \cancel{\mu}$$

C $r(t) = [t, t^2, t^3]$, $P: (1, 1, 1)$

$$r'(t) = [1, 2t, 3t^2] \cancel{\mu}$$

$$U = \frac{1}{\sqrt{1^2 + (2t)^2 + (3t^2)^2}} (1, 2t, 3t^2)$$

$$= \frac{1}{\sqrt{1 + 4t^2 + 9t^4}} (1, 2t, 3t^2) \cancel{\mu}$$

at $t=1$, $r(t) = P$

$$r'(1) = (1, 2, 3)$$

$$U = \frac{1}{\sqrt{1+4+9}} (1, 2, 3)$$

$$= \frac{1}{\sqrt{14}} (1, 2, 3) \cancel{\mu}$$

12 Encuentra la longitud total y haz un esbozo de la curva hipocicloide dada por
 $r(t) = [a\cos^3 t, a\sin^3 t]$ (pp 411)

Note: The length ℓ is given by:

$\ell = \int_a^b \sqrt{r'(t) \cdot r'(t)} dt$; where ℓ represents the limit of the lengths of broken lines of n chords. For each n , the interval $a \leq t \leq b$ is subdivided by points.

Note: ℓ is called the length of C , and C is called rectifiable.

$$\begin{aligned} r'(t) &= \frac{d}{dt} r(t) \\ &= \frac{d}{dt} [\cos^3(t), \sin^3(t)] \\ &= [-3a\cos^2(t)\sin(t), 3a\cos(t)\sin^2(t)] \end{aligned}$$

$$\begin{aligned} r'(t) \cdot r'(t) &= (9a^2 \cos^4(t) \sin^2(t)) + (9a^2 \cos^2(t) \sin^4(t)) \\ &= 9a^2 \cos^2(t) \sin^2(t) \\ &= \frac{9a^2}{4} \sin^2(2t) \end{aligned}$$

$$\ell = \int_0^{2\pi} \sqrt{\frac{9a^2}{4} \sin^2(2t)} dt = \frac{3a}{2} \int_0^{2\pi} \sin(2t) dt$$

$$\begin{aligned} &= \frac{3a}{2} \left[-\frac{1}{2} \cos(2t) \Big|_0^{\pi} \right] \\ &= \frac{3a}{2} \left[\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) \right] \\ &= \frac{3a}{2} u \end{aligned}$$

III

[13] Para las trayectorias en los incisos (a) y (b), encuentra la aceleración tangencial, la aceleración normal, la velocidad y la rapidez. (pp. 412)

Note: The tangent vector of C is called the velocity vector v . (as the tangent points at the instantaneous directions of motion and its length gives the speed $|v|$). The second derivative of $r(t)$ is called the acceleration vector a . $|a|$ (a 's length) is called the acceleration of the motion.

$$v(t) = r'(t) \quad |v| = |r'| = \sqrt{r' \cdot r'} = ds/dt \quad \& \quad a(t) = v'(t) = r''(t)$$

Note: The acceleration vector can be split into directional components: a_{tan} & a_{norm} . The tangential acceleration vector a_{tan} is tangent to the path and the normal acceleration vector a_{norm} is perpendicular (normal) to the path.

That is...

$$u(s) = \frac{dr}{ds} ; \quad v(t) = u(s) \frac{ds}{dt} ; \quad a(t) = \frac{du}{ds} \left(\frac{ds}{dt} \right)^2 + u(s) \frac{d^2s}{dt^2}$$

$$a_{tan} = \frac{\mathbf{a} \cdot \mathbf{v}}{v \cdot v} \mathbf{v} ; \quad a_{norm} = \mathbf{a} - a_{tan}$$

[a] Línea recta $r(t) = [8t, 6t, 0]$

$$v(t) = r'(t) = [8, 6, 0] \quad /4$$

$$\begin{aligned} |v| &= \sqrt{r' \cdot r'} \\ &= \sqrt{8^2 + 6^2 + 0^2} \\ &= 10 \end{aligned}$$

$$a(t) = v'(t) = [0, 0, 0] \quad /4$$

$$\begin{aligned} a_{tan} &= \frac{\mathbf{a} \cdot \mathbf{v}}{v \cdot v} = \frac{0+0+0}{8^2+6^2+0^2} [8, 6, 0] \\ &= [0, 0, 0] \end{aligned}$$

$$\begin{aligned} a_{norm} &= \mathbf{a} - a_{tan} \\ &= [0, 0, 0] \end{aligned}$$

b) Ellipse $r(t) = [\cos t, 2\sin t, 0]$

$$v(t) = r'(t) = [-\sin t, 2\cos t, 0] \quad //$$

$$\|v\| = \sqrt{(-\sin t)^2 + (2\cos t)^2 + 0} \\ = \sqrt{4\cos^2 t + \sin^2 t} \quad //$$

$$a(t) = v'(t) = [-\cos t, -2\sin t, 0] \quad //$$

$$a_{tan} = \frac{a \cdot v}{v \cdot v} = \frac{(-\cos t)(-\sin t) + (-2\sin t)(2\cos t) + 0}{\sin^2 t + 4\cos^2 t + 0} \\ = \frac{-3\cos(t)\sin(t)}{\sin^2(t) + 4\cos^2(t)} \\ = \frac{3\sin(2t)}{5+3\cos(2t)} [-\sin(t), 2\cos(t), 0] \quad //$$

$$a_{norm} = a - a_{tan}$$

$$= [-\cos(t), -2\sin(t), 0] - \underbrace{\frac{3\sin(2t)}{5+3\cos(2t)} [-\sin(t), 2\cos(t), 0]}_{\alpha} \\ = \left[(-\cos(t) - \alpha(-\sin(t)), -2\sin(t) - \alpha(2\cos(t)), 0) \right] \\ = \left[-\cos(t) - \frac{6\sin(t)\sin(2t)}{5+3\cos(2t)}, -2\sin(t) + \frac{6\cos(t)\sin(2t)}{5+3\cos(2t)}, 0 \right] \\ = \left[-\cos(t) - \frac{6\sin(t)\sin(2t)}{5+3\cos(2t)}, -\frac{4\sin(t)}{5+3\cos(2t)}, 0 \right] \quad //$$