

$$b) A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

> Let's solve the eigen problem...

$$Ax = \lambda x$$

$$Ax - \lambda x = 0$$

$$\begin{cases} ax_1 + bx_2 = \lambda x_1 \\ -bx_1 + ax_2 = \lambda x_2 \end{cases} \Rightarrow \begin{cases} (a-\lambda)x_1 + bx_2 = 0 \\ -bx_1 + (a-\lambda)x_2 = 0 \end{cases}$$

so...

$$\det \begin{pmatrix} a-\lambda & b \\ -b & a-\lambda \end{pmatrix} = 0 \Rightarrow \lambda = \frac{-(-2a) \pm \sqrt{(-2a)^2 - 4(1)(a^2+b^2)}}{2(1)}$$

$$\begin{aligned} (a-\lambda)^2 + b^2 &= 0 \\ \lambda^2 - 2a\lambda + (a^2+b^2) &= 0 \\ \lambda &= \frac{2a \pm 2\sqrt{-b^2}}{2} \\ \lambda &= a \pm ib \end{aligned}$$

the eigen values are:

$$\lambda_1 = a - ib$$

for $\lambda = \lambda_1$, then the eigen vector is

$$\begin{cases} (a - (a - ib))x_1 + bx_2 = 0 \\ -bx_1 + (a - (a - ib))x_2 = 0 \end{cases}$$

$$ix_1 + x_2 = 0$$

$$x_1 = -ix_2$$

$$i(ix_2) + x_2 = 0$$

$$x_2 = x_2$$

$$x = t \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\lambda_2 = a + ib$$

for $\lambda = \lambda_2$, then the eigen vector is

$$\begin{cases} (a - (a + ib))x_1 + bx_2 = 0 \\ -bx_1 + (a - (a + ib))x_2 = 0 \end{cases}$$

$$-ix_1 + x_2 = 0$$

$$x_1 = -ix_2$$

$$-i(-ix_2) + x_2 = 0$$

$$x_2 = x_2$$

$$x = t \begin{pmatrix} -i \\ 1 \end{pmatrix}$$