

Mathematical Physical Modelling - Homework 04

Differential Vector Calculus

Antonio Osamu Katagiri Tanaka (A01212611)
Diego Sebastián Cecilio Franco (A01373414)
Katya Michelle Aguilar Pérez (A01750272)

March 8, 2019

Paper trail and process evidence are at the end of this document.

- **VECTORES: PRODUCTO ESCALAR**

1. Encuentra los ángulos del triángulo con vértices $A: (0, 0, 2)$, $B: (3, 0, 2)$ y $C: (1, 1, 1)$. Haz un dibujo del triángulo.
2. Encuentra los ángulos de un paralelogramo si los vértices son $(0, 0)$, $(6, 0)$, $(8, 3)$ y $(2, 3)$.
3. Encuentra la distancia del punto $A: (1, 0, 2)$ al plano $P: 3x + y + z = 9$. Haz un dibujo del problema.
4. ¿Para cuál valor de c serán ortogonales los planos $3x + z = 5$ y $8x - y + cz = 9$?
5. Encuentra la componente del vector \mathbf{a} en la dirección del vector \mathbf{b} en los siguientes casos:
 - (a) $\mathbf{a} = [1, 1, 1]$, $\mathbf{b} = [2, 1, 3]$
 - (b) $\mathbf{a} = [3, 4, 0]$, $\mathbf{b} = [4, -3, 2]$
 - (c) $\mathbf{a} = [8, 2, 0]$, $\mathbf{b} = [-4, -1, 0]$

- **VECTORES Y PRODUCTO ESCALAR TRIPLE**

6. Con respecto a un sistema derecho cartesiano, sean

$$\mathbf{a} = [2, 1, 0], \mathbf{b} = [-3, 2, 0], \mathbf{c} = [1, 4, -2]$$

Mostrando los detalles, encuentra lo siguiente:

- (a) $(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{d}$
 - (b) $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{d})$
7. Encuentra el volumen de un tetraedro con vértices $(1, 1, 1)$, $(5, -7, 3)$, $(7, 4, 8)$ y $(10, 7, 4)$.
 8. Encuentra el volumen de un tetraedro con vértices $(1, 3, 6)$, $(3, 7, 12)$, $(8, 8, 9)$ y $(2, 2, 8)$.

- **FUNCIONES Y CAMPOS ESCALARES Y VECTORIALES**

9. La temperatura T de un tamalito goajaqueño es independiente de z y está dada por una función escalar $T = T(x, y)$. Identifica las isotermas donde $T(x, y) = \text{const.}$ y dibuja algunas de ellas. Puedes usar ayuda computacional para las gráficas.
 - (a) $T = x^2 - y^2$
 - (b) $T = xy$
 - (c) $T = 3x - 4y$
 - (d) $T = \arctan(y/x)$

10. Para cada función, ¿Qué tipo de superficies son las «superficies de nivel» $f(x, y, z) = \text{const.}$?

$$(a) f = 9(x^2 + y^2) + z^2 \quad (b) f = 5x^2 + 2y^2 \quad (c) f = z - \sqrt{x^2 + y^2}$$

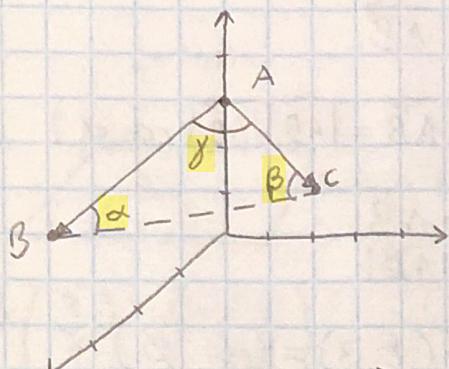
- **CURVAS, TANGENTES, LONGITUD DE CURVA, CURVAS EN MECÁNICA**

11. Dada una curva $C : \mathbf{r}(t)$, encuentra un vector tangente $\mathbf{r}'(t)$, un vector tangente unitario $\mathbf{u}'(t)$ y la tangente de C en P . Esboza la curva y la tangente.
 - (a) $\mathbf{r}(t) = [t, \frac{1}{2}t^2, 1]$, $P : (2, 2, 1)$
 - (b) $\mathbf{r}(t) = [\cos t, \sin t, 9t]$, $P : (1, 0, 18\pi)$
 - (c) $\mathbf{r}(t) = [t \ t^2 \ t^3]$, $P : (1, 1, 1)$

12. Encuentra la longitud total y haz un esbozo de la curva hipocicloide dada por
 $\mathbf{r}(t) = [a \cos^3 t, a \sin^3 t]$.
13. Para las trayectorias en los incisos (a) y (b), encuentra la aceleración tangencial, la aceleración normal, la velocidad y la rapidez.
- (a) Línea recta $\mathbf{r}(t) = [8t, 6t, 0]$
- (b) Elipse $\mathbf{r}(t) = [\cos t, 2 \sin t, 0]$

Vectores: Producto escalar

1- Encuentra los ángulos del triángulo con vértices $A(0, 0, 2)$, $B(3, 0, 2)$ y $C(1, 1, 1)$. Haz un dibujo del triángulo.



γ entre \vec{AB} , \vec{AC}

$$\arccos(\vec{AB} \cdot \vec{AC} / |\vec{AB}| |\vec{AC}| \cos \gamma)$$

$$\vec{AB} = (3, 0, 2) - (0, 0, 2) = (3, 0, 0)$$

$$\vec{AC} = (1, 1, 1) - (0, 0, 2) = (1, 1, -1)$$

$$\frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} = \frac{(3, 0, 0) \cdot (1, 1, -1)}{|(3, 0, 0)| |(1, 1, -1)|} = \frac{(3 \cdot 1) + (0 \cdot 1) + (0 \cdot -1)}{\sqrt{3^2 + 0^2 + 0^2} \sqrt{1^2 + 1^2 + (-1)^2}} = \frac{3}{\sqrt{9}} \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \quad \arccos\left(\frac{1}{\sqrt{3}}\right) = 54.73^\circ$$

α entre \vec{BA} , \vec{BC}

$$\vec{BA} = (0, 0, 2) - (3, 0, 2) = (-3, 0, 0)$$

$$\arccos(\vec{BA} \cdot \vec{BC} / |\vec{BA}| |\vec{BC}| \cos \alpha)$$

$$\vec{BC} = (1, 1, 1) - (3, 0, 2) = (-2, 1, -1)$$

$$\frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} = \frac{(-3, 0, 0) \cdot (-2, 1, -1)}{|(-3, 0, 0)| |(-2, 1, -1)|} = \frac{(-3 \cdot -2) + (0 \cdot 1) + (0 \cdot -1)}{\sqrt{(-3)^2 + 0^2 + 0^2} \sqrt{(-2)^2 + 1^2 + (-1)^2}} = \frac{6}{\sqrt{9}} \frac{1}{\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$\arccos\left(\frac{2}{\sqrt{6}}\right) = 35.26^\circ$$

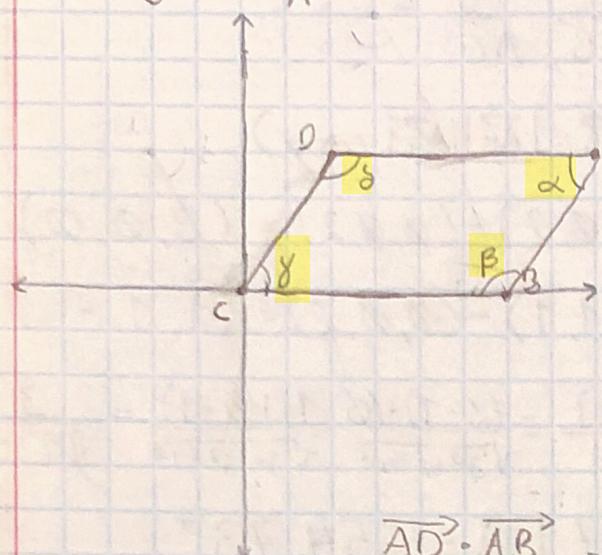
La suma de los ángulos internos de un triángulo es igual a 180°

$$+ 54.73^\circ \\ + 35.26^\circ \\ \hline 89.99^\circ$$

$$- 180^\circ \\ - 89.99^\circ \\ \hline 91.01^\circ$$

$$\gamma = 54.73^\circ \\ \alpha = 35.26^\circ \\ \beta = 91.01^\circ$$

2- Encuentra los ángulos de un paralelogramo si los vértices son (ϕ, ϕ) , $(6, \phi)$, $(8, 3)$, $(2, 3)$



α entre \vec{AD}, \vec{AB}

$$\arccos(\vec{AD} \cdot \vec{AB}) = |\vec{AD}| |\vec{AB}| \cos \alpha$$

$$\alpha = \frac{\vec{AD} \cdot \vec{AB}}{|\vec{AB}| |\vec{AD}|}$$

$$\vec{AD} = (2, 3) - (8, \phi) = (-6, \phi)$$

$$\vec{AB} = (6, \phi) - (8, 3) = (-2, -3)$$

$$\frac{\vec{AD} \cdot \vec{AB}}{|\vec{AD}| |\vec{AB}|} = \frac{(-6, \phi) \cdot (-2, -3)}{|(-6, \phi)| |(-2, -3)|} = \frac{-6 \cdot -2 + \phi \cdot -3}{\sqrt{6^2 + \phi^2} \sqrt{2^2 + 3^2}} = \frac{12 - 3\phi}{\sqrt{13} \sqrt{13}} = \frac{12}{13}$$

$$= \frac{2}{\sqrt{13}} \quad \arccos\left(\frac{2}{\sqrt{13}}\right) = 56.30^\circ$$

β entre \vec{BA}, \vec{BC}

$$\vec{BA} = (8, 3) - (6, \phi) = (2, 3)$$

$$\arccos(\vec{BA} \cdot \vec{BC}) = |\vec{BA}| |\vec{BC}| \cos \beta$$

$$\vec{BC} = (\phi, \phi) - (6, \phi) = (-6, \phi)$$

$$\frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} = \frac{(2, 3) \cdot (-6, \phi)}{|(2, 3)| |(-6, \phi)|} = \frac{2 \cdot -6 + 3 \cdot \phi}{\sqrt{2^2 + 3^2} \sqrt{6^2 + \phi^2}} = \frac{-12 + 3\phi}{\sqrt{13} \sqrt{13}} = \frac{-12}{13} = -\frac{2}{\sqrt{13}} \quad \arccos\left(-\frac{2}{\sqrt{13}}\right) = 123.69^\circ$$

$$\alpha = 56.30^\circ$$

$$\beta = 123.69^\circ$$

$$\gamma = 56.30^\circ$$

$$\delta = 123.69^\circ$$

La suma de dos vértices contiguos de un paralelogramo es igual a 180° y la suma total de los ángulos internos es 360° . Los ángulos opuestos por el vértice son iguales

$\alpha = \gamma$ $\beta = \delta$, para probar esto buscaremos γ y δ con el arcos

$$\gamma \quad \vec{CD} = (2, 3) - (\phi, \phi) = (2, 3)$$

$$\gamma \quad \vec{CB} = (6, \phi) - (\phi, \phi) = (6, \phi)$$

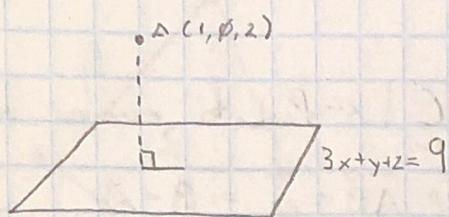
$$\delta \quad \vec{DA} = (8, 3) - (2, 3) = (6, \phi)$$

$$\delta \quad \vec{DC} = (\phi, \phi) - (2, 3) = (-2, -3)$$

$$\frac{\vec{CD} \cdot \vec{CB}}{|\vec{CD}| |\vec{CB}|} = \frac{(2, 3) \cdot (6, \phi)}{|(2, 3)| |(6, \phi)|} = \frac{2 \cdot 6 + 3 \cdot \phi}{\sqrt{2^2 + 3^2} \sqrt{6^2 + \phi^2}} = \frac{12 + 3\phi}{\sqrt{13} \sqrt{13}} = \frac{12}{13} = \frac{2}{\sqrt{13}} \quad \arccos\left(\frac{2}{\sqrt{13}}\right) = 56.30^\circ$$

$$\frac{\vec{DA} \cdot \vec{DC}}{|\vec{DA}| |\vec{DC}|} = \frac{(6, \phi) \cdot (-2, -3)}{|(6, \phi)| |(-2, -3)|} = \frac{6 \cdot -2 + \phi \cdot -3}{\sqrt{6^2 + \phi^2} \sqrt{2^2 + 3^2}} = \frac{-12 - 3\phi}{\sqrt{13} \sqrt{13}} = \frac{-12}{13} = -\frac{2}{\sqrt{13}} \quad \arccos\left(-\frac{2}{\sqrt{13}}\right) = 123.69^\circ$$

3- Encuentra la distancia del punto $A(1, \phi, 2)$ al Plano $3x+y+z=9$
haz un dibujo del problema



$$\tilde{P} \cdot \tilde{A} = ax + by + cz = d$$

$$P = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \neq \emptyset \quad A = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$P = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \quad A = \begin{pmatrix} 1 \\ \phi \\ 2 \end{pmatrix}$$

$$D = P \cdot A = \frac{\tilde{P} \cdot \tilde{A}}{|\tilde{P}|}$$

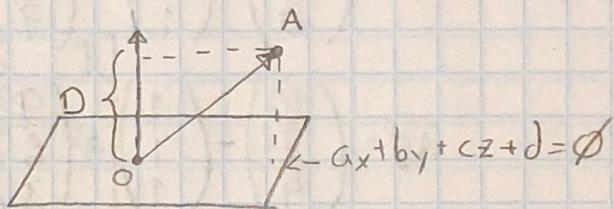
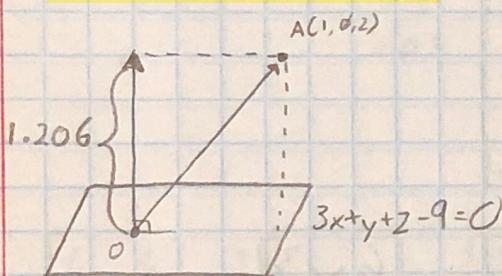
$$\hat{P} \cdot \hat{A} = \frac{d}{|\tilde{P}|}$$

$$P = 3x + y + z - 9 = \emptyset$$

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \text{Proyección } U = \frac{|x \cdot \vec{v}|}{\|\vec{v}\|}$$

$$D = \frac{|(3 \cdot 1) + (1 \cdot \phi) + (1 \cdot 2) - 9|}{\sqrt{3^2 + 1^2 + 1^2}} = \frac{|3 + \phi + 2 - 9|}{\sqrt{11}} = \frac{|4|}{\sqrt{11}}$$

$$D = 1.206 \text{ } \cancel{u}$$



④ C Para cuál valor de C serán ortogonales los planos
 $3x + z = 5$ y $8x - y + Cz = 9$?

▷ Sabiendo que los planos son ortogonales si su producto punto es igual a cero. \therefore Sustituimos:

$$(3, 0, 1) \cdot (8, -1, C) = 24 + 0 + C = 0$$

▷ Obtenemos:

$$24 + C = 0$$

$$\boxed{C = -24}$$

▷ Realizando el producto punto con el valor de " C " tenemos

$$(3, 0, 1) \cdot (8, -1, -24) = \\ (3)(8) + (0)(-1) + (1)(-24) = 24 + 0 - 24 = 0$$

⑤ Encuentra la componente del vector a en la dirección del vector b en los siguientes casos:

(a) $a = [1, 1, 1]$, $b = [2, 1, 3]$ (b) $a = [3, 4, 0]$
 $b = [4, -3, 2]$

(c) $a = [8, 2, 0]$, $b = [-1, -1, 0]$

Para (a) $\Rightarrow a = [1, 1, 1]$, $b = [2, 1, 3]$

Teniendo que $\text{proj}_{\text{ex}} u = \frac{u \cdot v}{|v|}$

$$u = (1, 1, 1) \cdot (2, 1, 3) = \frac{2 + 1 + 3}{\sqrt{14}} = \frac{6}{\sqrt{14}}$$

Para (b) $\Rightarrow a = [3, 4, 0]$, $b = [4, -3, 2]$

$$u = (3, 4, 0) \cdot (4, -3, 2) = \frac{12 - 12 + 0}{\sqrt{16 + 9 + 4}} = \frac{0}{\sqrt{29}} = 0$$

Para (c) $\Rightarrow a = [8, 2, 0]$, $b = [4, -1, 0]$

$$u = (8, 2, 0) \cdot (4, -1, 0) = \frac{32 - 2 + 0}{\sqrt{16 + 1 + 0}} = \frac{30}{\sqrt{17}}$$

⑥ Con respecto a un sistema derecho cartesiano, sean $a = [2, 1, 0]$, $b = [-3, 2, 0]$, $c = [1, 4, -2]$, $d = [5, -1, 3]$

• Mostrando los detalles, encuentra lo siguiente:

$$(a) (b \times c) \cdot d$$

$$(b) b \cdot (c \times d)$$

$$(d)$$

$$(a) b \cdot (c \times d)$$

$$(-3, 2, 0) \cdot [(1, 4, -2) \times (5, -1, 3)] =$$

$$(b) (b \times c) \cdot d = [(b \times c) \cdot d] =$$

$$(b \times c) \cdot d = b \cdot (c \times d)$$

$$\begin{vmatrix} -3 & 2 & 0 \\ 1 & 4 & -2 \\ 5 & -1 & 3 \end{vmatrix} = (-3) \cdot 4 \cdot 3 + 2 \cdot (-2) \cdot 5 + 0 \cdot 1 \cdot (-1) - 0 \cdot 4 \cdot 5 - (-1) \cdot 1 \cdot 3 - (-3) \cdot (-2) \cdot (-1) =$$

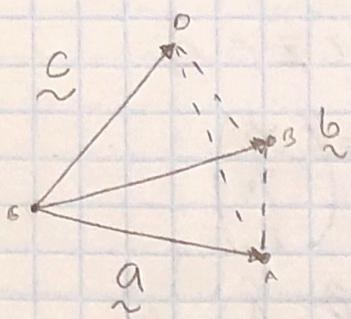
$$(-3, 2, 0) \cdot (-36 + (-20) + 0 - 0 - 6 - (-6)) = -56$$

$$(b \cdot c) \times d = (b \times c) \cdot d$$

$$\begin{vmatrix} 5 & 1 & -3 \\ -3 & 2 & 0 \\ 1 & 4 & -2 \end{vmatrix} = (5 \cdot 2 \cdot (-2)) + (-1) \cdot 0 \cdot 1 + 3 \cdot (-3) \cdot 4 - 3 \cdot 2 \cdot 1 - (-1) \cdot (-3) \cdot (-2) + 5 \cdot 0 \cdot 1 =$$

$$(-20) + 0 + (-36) - 16 - (-6) - 0 = -56$$

7- Encuentra el volumen de un tetraedro con vértices $(1, 1, 1)$, $(5, -7, 3)$, $(7, 4, 8)$ y $(10, 7, 4)$



$$V_T = \frac{1}{6} V_P$$

$$V_P = |(\underline{a} \times \underline{b}) \cdot \underline{c}| \leftarrow \text{Producto escalar triple}$$

$$V_P = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$\vec{AB} = \underline{a} = \underline{B} - \underline{A}$$

$$\vec{AC} = \underline{b} = \underline{C} - \underline{A}$$

$$\vec{AD} = \underline{c} = \underline{D} - \underline{A}$$

$$\underline{A} = (1, 1, 1)$$

$$\underline{B} = (5, -7, 3)$$

$$\vec{AB} = \begin{pmatrix} 5 \\ -7 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 2 \end{pmatrix}$$

$$\underline{C} = (7, 4, 8)$$

$$\underline{D} = (10, 7, 4)$$

$$\vec{AC} = \begin{pmatrix} 7 \\ 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix}$$

$$\vec{AD} = \begin{pmatrix} 10 \\ 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix}$$

$$V_P = \begin{bmatrix} 4 & -8 & 2 \\ 6 & 3 & 7 \\ 9 & 6 & 3 \end{bmatrix}$$

Sacamos los determinantes

$$\begin{vmatrix} 4 & -8 & 2 \\ 6 & 3 & 7 \\ 9 & 6 & 3 \end{vmatrix} = \det(4 \begin{bmatrix} 3 & 7 \\ 6 & 3 \end{bmatrix} + (-1 \cdot -8) \begin{bmatrix} 6 & 7 \\ 9 & 3 \end{bmatrix} + 2 \begin{bmatrix} 6 & 3 \\ 9 & 6 \end{bmatrix})$$

$$\det(4((3 \cdot 3) - (6 \cdot 7)) + 8((6 \cdot 3) - (9 \cdot 7)) + 2((6 \cdot 6) - (9 \cdot 3)))$$

$$\det(4(9 - 42) + 8(18 - 63) + 2(36 - 27))$$

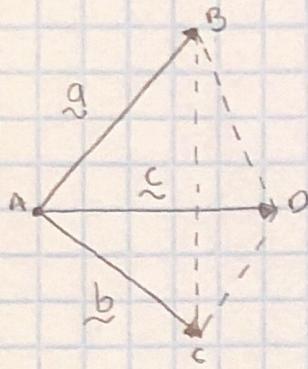
$$\det(4(-33) + 8(-45) + 2(9))$$

$$\det(-132 - 360 + 18) = -474$$

$$V_P = 1474$$

$$V_T = \frac{1}{6} 474 = \underline{\underline{7903}} \quad \checkmark$$

8- Encuentra el volumen de un tetraedro con vértices $(1, 3, 6), (3, 7, 12)$, $(8, 8, 9)$ y $(2, 2, 8)$



$$V_T = \frac{1}{6} V_P$$

$$V_P = |(a \times b) \cdot c|$$

$$\vec{AB} = \underline{a}, \quad \vec{AC} = \underline{b}, \quad \vec{AD} = \underline{c}$$

$$V_P = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$A = (2, 2, 8)$$

$$B = (8, 8, 9)$$

$$\vec{AB} = (8, 8, 9) - (2, 2, 8) = (6, 6, 1)$$

$$C = (3, 7, 12)$$

$$\vec{AC} = (3, 7, 12) - (2, 2, 8) = (1, 5, 4)$$

$$D = (1, 3, 6)$$

$$\vec{AD} = (1, 3, 6) - (2, 2, 8) = (-1, 1, -2)$$

$$V_P = \begin{bmatrix} 6 & 6 & 1 \\ 1 & 5 & 4 \\ -1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 6 & 1 \\ 1 & 5 & 4 \\ -1 & 1 & -2 \end{bmatrix} = \det(6 \begin{bmatrix} 5 & 4 \\ 1 & -2 \end{bmatrix} + (-6) \begin{bmatrix} 1 & 4 \\ -1 & -2 \end{bmatrix} + 1 \begin{bmatrix} 1 & 5 \\ -1 & 1 \end{bmatrix})$$

$$6((5 \cdot -2) - (4 \cdot 1)) - 6((1 \cdot -2) - (-1 \cdot 4)) + 1((1 \cdot 1) - (-1 \cdot 5))$$

$$6(-10 - 4) - 6(-2 + 4) + 1(1 + 5)$$

$$6(-14) - 6(2) + 1(6) = -90$$

$$V_P = 190$$

$$V_T = \frac{1}{6} 90 = \underline{\underline{150^3}}$$

⑨ La temperatura T de un tamalito Catagano es independiente de z y está dada por una función escalar $T = T(x, y)$. Identifica las isotermaas donde $T(x, y) = \text{const.}$ y dibuja algunas de ellas.

$$(a) \Rightarrow T = x^2 - y^2 \quad \text{(Hyperbola)}$$

$$(c) \quad T = 3x - 4y \quad \text{(Parallel straight lines in space)} \quad y = \frac{3}{4}x + C$$

$$(b) \quad T = xy \quad \text{(Hyperbola)}$$

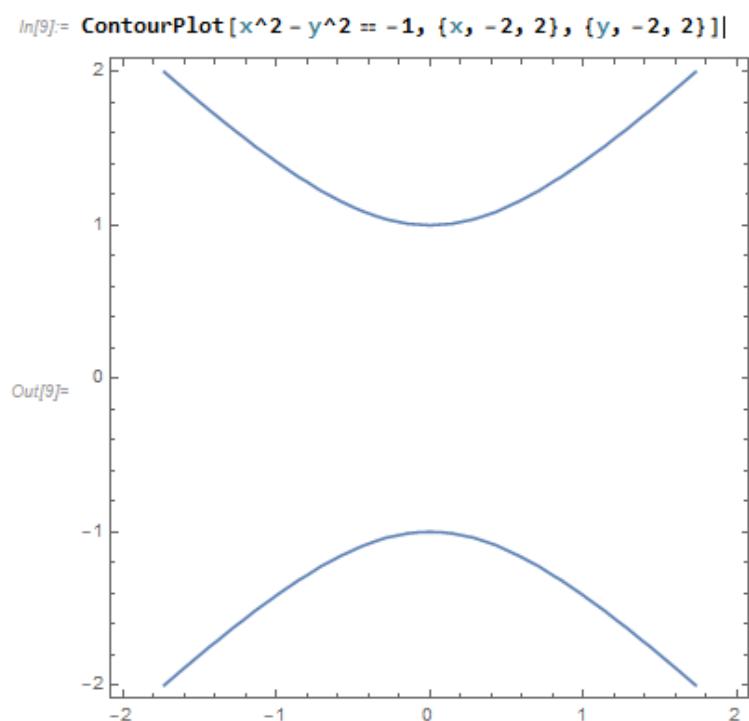
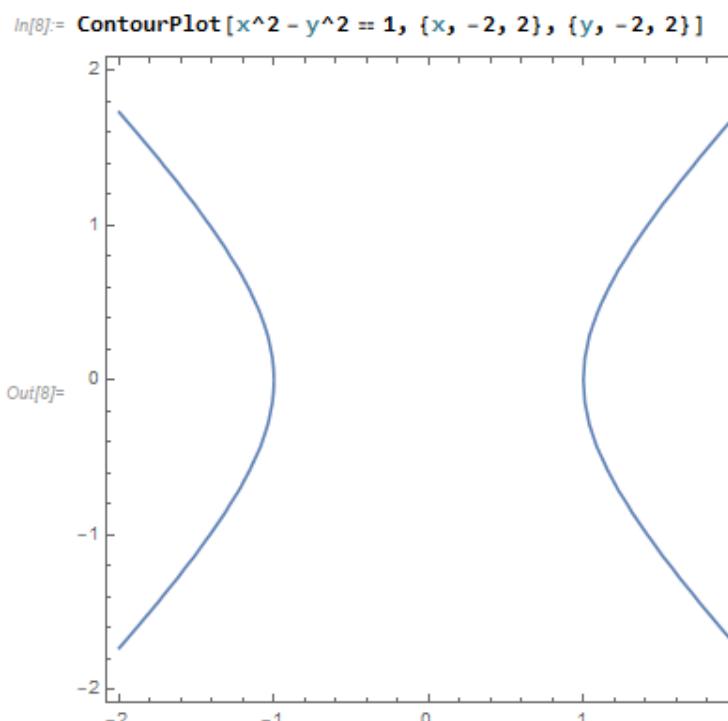
$$(d) \quad T = \arctan(y/x) \quad \text{(Hyperbolas)}$$

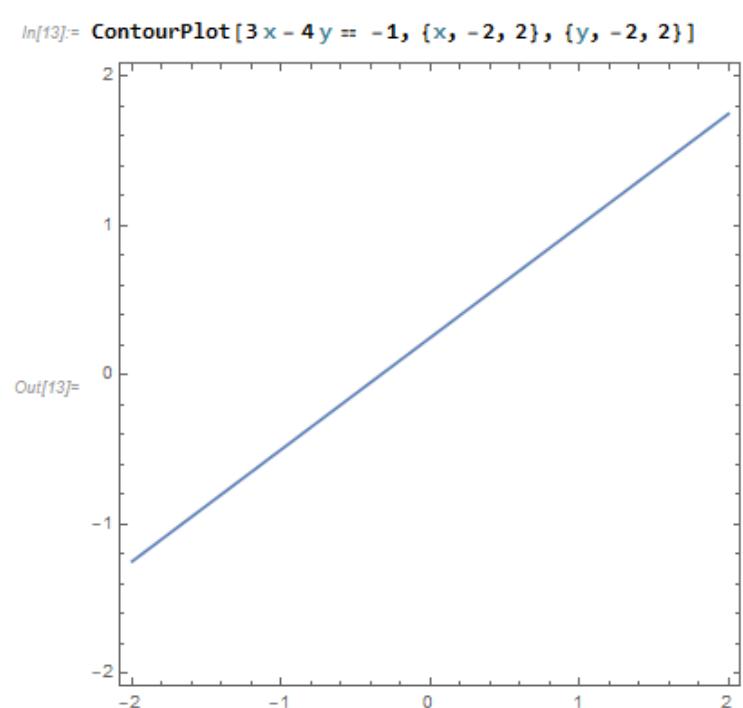
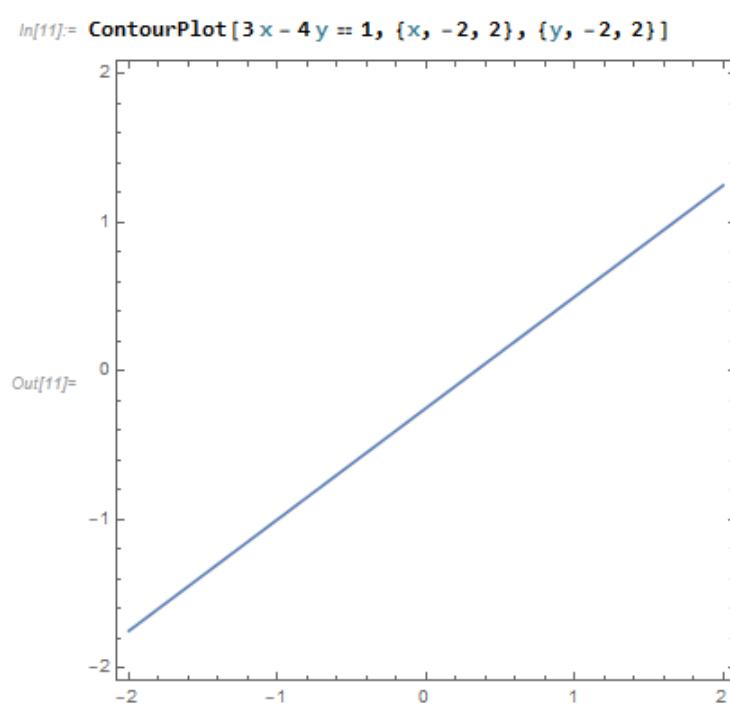
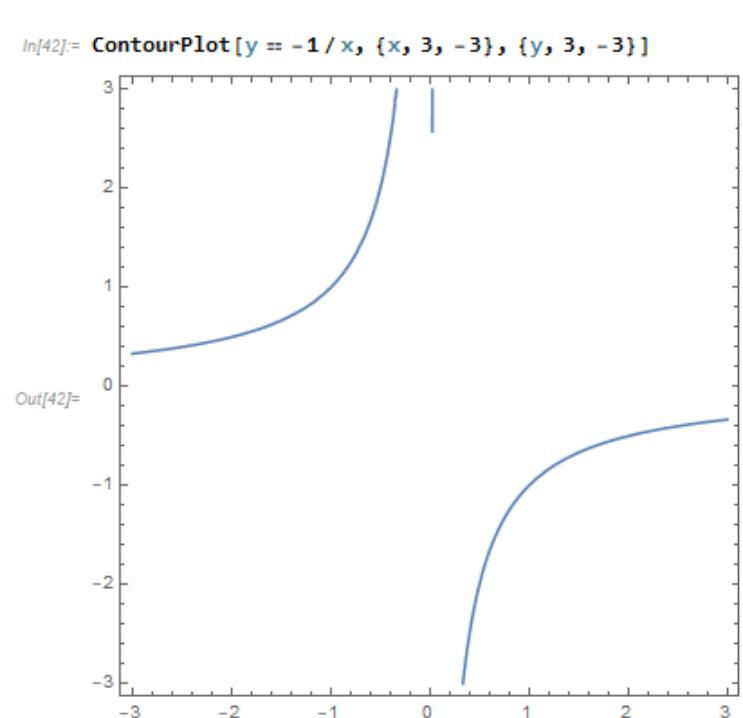
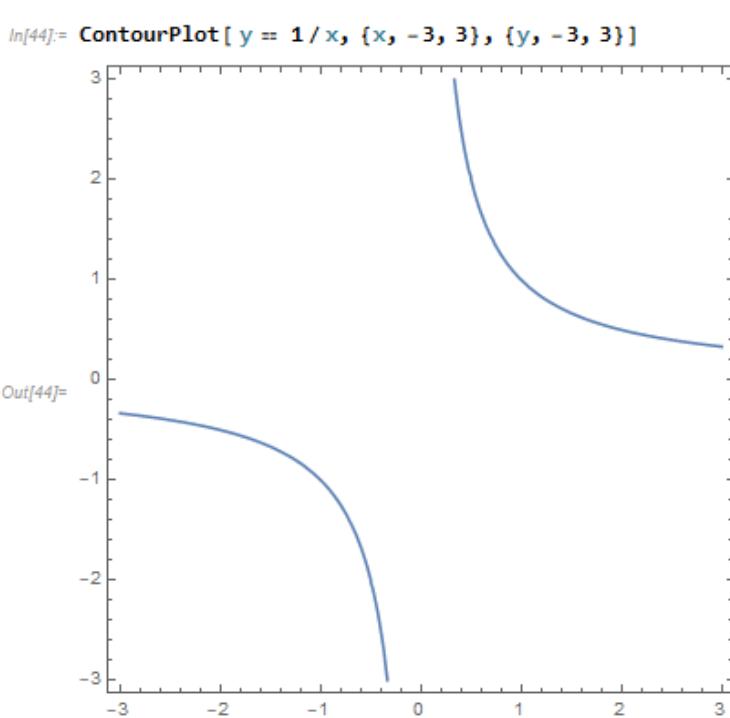
a = hyperbola

c = Parallel - straight lines
in space $y = \frac{3}{4}x + C$

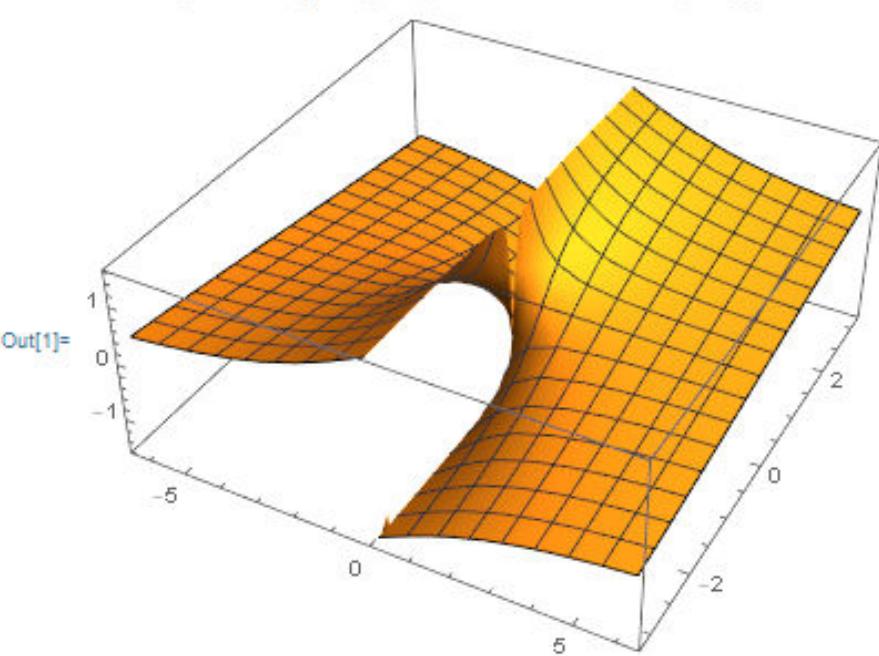
$$b = y = \frac{1}{x}$$

$$d = \arctan(y/x) \Rightarrow 3D$$





In[1]:= `Plot3D[ArcTan[y/x], {x, -2*Pi, 2*Pi}, {y, -Pi, Pi}]`

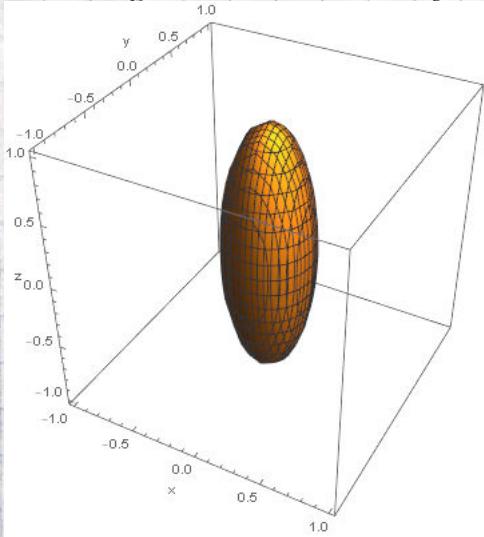


Homework 04 - Differential Vector Calculus

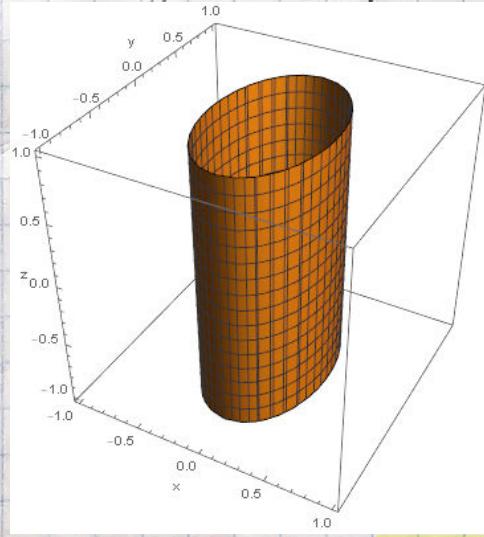
Funciones y Campos Escalares y Vectoriales

10) Para cada función, ¿Qué tipo de "superficies de nivel" $f(x, y, z) = \text{const.}$?

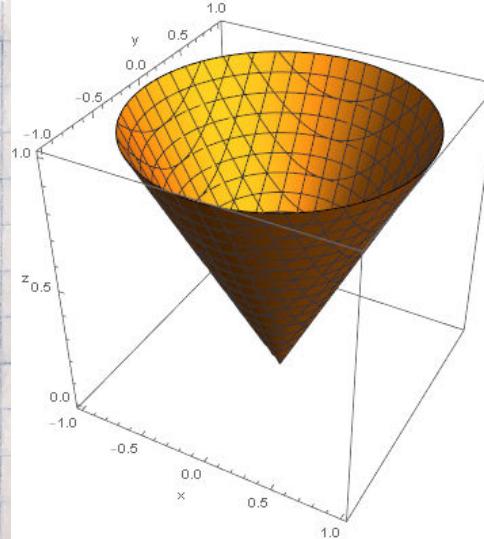
a) $9(x^2 + y^2) + z^2 = 1 \Rightarrow \text{spheroid}$



b) $5x^2 + 2y^2 = 1 \Rightarrow \text{ellipse}$



c) $z - \sqrt{x^2 + y^2} = 0 \Rightarrow \text{cone}$



Homework 04 - Differential Vector Calculus

Curvas, Tangentes, Longitud de Curva, Curvas en Mecánica

[11] Dada una curva $C: r(t)$, encuentra un vector tangente $r'(t)$, un vector tangente unitario $u'(t)$ y la tangente de C en P . Esboza la curva y la tangente. (pp 410)

Let's solve the "Tangent to a Curve" problems...

Note: If $r'(t) \neq 0$, we call $r'(t)$ a tangent vector of C at P , and u the unit tangent vector.

a $r(t) = \left[t, \frac{1}{2}t^2, 1 \right]$, $P: (2, 2, 1)$

$$r'(t) = \left[1, t, 0 \right] \quad / \parallel$$

$$\begin{aligned} u &= \frac{1}{\|r'(t)\|} r'(t) = \frac{1}{\sqrt{1+t^2}} (1, t, 0) \\ &= \frac{1}{\sqrt{1+t^2}} (1, t, 0) \quad / \parallel \end{aligned}$$

at $t=2$, $r(t) = P$...

$$r'(2) = (1, 2, 0)$$

$$u = \frac{1}{\sqrt{5}} (1, 2, 0) \quad / \parallel$$

b $r(t) = [\cos(t), \sin(t), 9t]$, $P: (1, 0, 18\pi)$

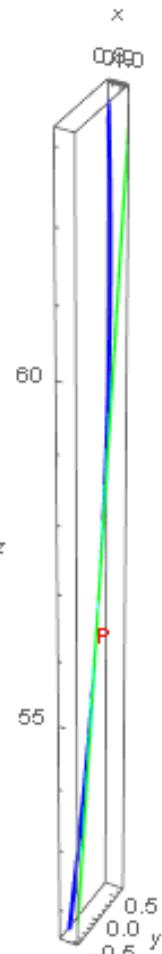
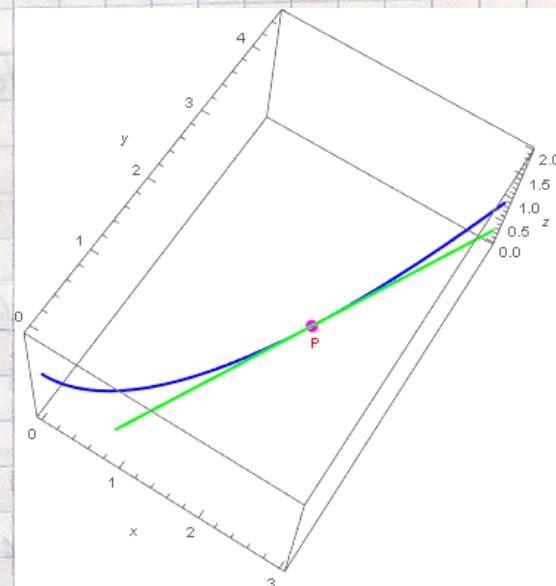
$$r'(t) = [-\sin(t), \cos(t), 9] \quad / \parallel$$

$$u = \frac{1}{\sqrt{(-\sin(t))^2 + (\cos(t))^2 + 9^2}} (-\sin(t), \cos(t), 9) \quad \left. \begin{array}{l} \sin^2(t) + \cos^2(t) = 1 \\ \end{array} \right\}$$

$$= \frac{1}{\sqrt{82}} (-\sin(t), \cos(t), 9) \quad / \parallel$$

at $t=2\pi$, $r(t) = P$

$$r'(2\pi) = (0, 1, 9)$$



$$U = \frac{1}{\sqrt{82}} (0, 1, 9) \cancel{\mu}$$

C) $r(t) = [t, t^2, t^3]$, $P: (1, 1, 1)$

$$r'(t) = [1, 2t, 3t^2] \cancel{\mu}$$

$$U = \frac{1}{\sqrt{1^2 + (2t)^2 + (3t^2)^2}} (1, 2t, 3t^2)$$

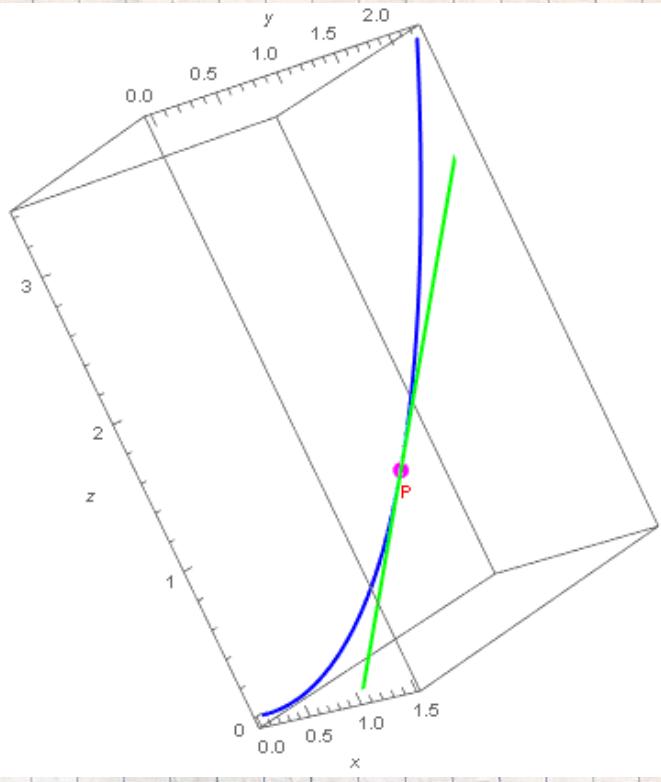
$$= \frac{1}{\sqrt{1 + 4t^2 + 9t^4}} (1, 2t, 3t^2) \cancel{\mu}$$

at $t=1$, $r(t) = P$

$$r'(1) = (1, 2, 3)$$

$$U = \frac{1}{\sqrt{1+4+9}} (1, 2, 3)$$

$$= \frac{1}{\sqrt{14}} (1, 2, 3) \cancel{\mu}$$



12 Encuentra la longitud total y haz un esbozo de la curva hipocicloide dada por $r(t) = [a\cos^3 t, a\sin^3 t]$ (pp 411)

Note: The length ℓ is given by:

$\ell = \int_a^b \sqrt{r'(t) \cdot r'(t)} dt$; where ℓ represents the limit of the lengths of broken lines of n chords. For each n , the interval $a \leq t \leq b$ is subdivided by points.

Note: ℓ is called the length of C , and C is called rectifiable.

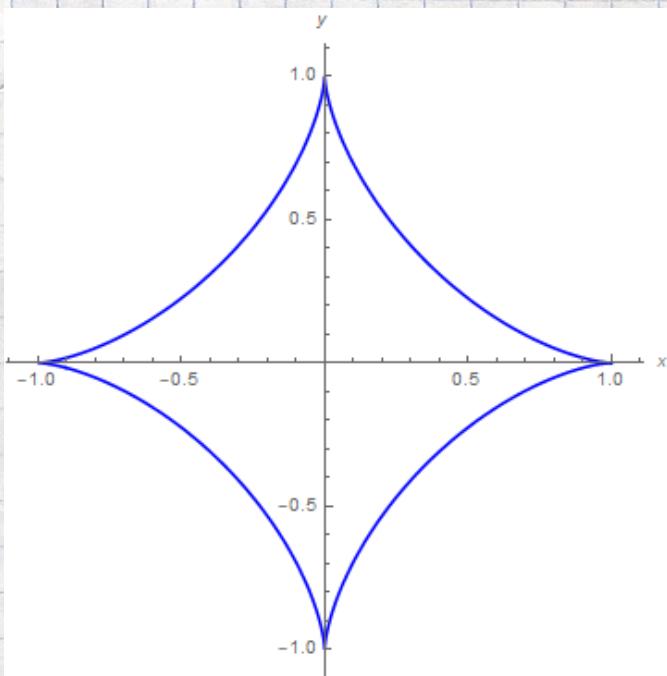
$$\begin{aligned} r'(t) &= \frac{d}{dt} r(t) \\ &= \frac{d}{dt} [\cos^3(t), \sin^3(t)] \\ &= [-3a\cos^2(t)\sin(t), 3a\cos(t)\sin^2(t)] \end{aligned}$$

$$\begin{aligned} r'(t) \cdot r'(t) &= (9a^2 \cos^4(t) \sin^2(t)) + (9a^2 \cos^2(t) \sin^4(t)) \\ &= 9a^2 \cos^2(t) \sin^2(t) \\ &= \frac{9a^2}{4} \sin^2(2t) \end{aligned}$$

$$\ell = \int_0^{2\pi} \sqrt{\frac{9a^2}{4} \sin^2(2t)} dt = \frac{3a}{2} \int_0^{2\pi} \sin(2t) dt$$

$$\begin{aligned} &= \frac{3a}{2} \left[-\frac{1}{2} \cos(2t) \Big|_0^{2\pi} \right] \\ &= \frac{3a}{2} \left[\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) \right] \end{aligned}$$

$$= \frac{3a}{2} \cancel{u}$$



[13] Para las trayectorias en los incisos (a) y (b), encuentra la aceleración tangencial, la aceleración normal, la velocidad y la rapidez. (pp. 412)

Note: The tangent vector of C is called the velocity vector v . (as the tangent points at the instantaneous directions of motion and its length gives the speed $|v|$). The second derivative of $r(t)$ is called the acceleration vector a . $|a|$ (a 's length) is called the acceleration of the motion.

$$v(t) = r'(t) \quad |v| = |r'| = \sqrt{r' \cdot r'} = ds/dt \quad \& \quad a(t) = v'(t) = r''(t)$$

Note: The acceleration vector can be split into directional components: a_{tan} & a_{norm} . The tangential acceleration vector a_{tan} is tangent to the path and the normal acceleration vector a_{norm} is perpendicular (normal) to the path.

That is...

$$u(s) = \frac{dr}{ds} ; \quad v(t) = u(s) \frac{ds}{dt} ; \quad a(t) = \frac{du}{ds} \left(\frac{ds}{dt} \right)^2 + u(s) \frac{d^2s}{dt^2}$$

$$a_{tan} = \frac{\mathbf{a} \cdot \mathbf{v}}{v \cdot v} \mathbf{v} ; \quad a_{norm} = \mathbf{a} - a_{tan}$$

[a] Línea recta $r(t) = [8t, 6t, 0]$

$$v(t) = r'(t) = [8, 6, 0] \quad /4$$

$$\begin{aligned} |v| &= \sqrt{r' \cdot r'} \\ &= \sqrt{8^2 + 6^2 + 0^2} \\ &= 10 \end{aligned}$$

$$a(t) = v'(t) = [0, 0, 0] \quad /4$$

$$\begin{aligned} a_{tan} &= \frac{\mathbf{a} \cdot \mathbf{v}}{v \cdot v} = \frac{0+0+0}{8^2+6^2+0^2} [8, 6, 0] \\ &= [0, 0, 0] \quad /4 \end{aligned}$$

$$\begin{aligned} a_{norm} &= \mathbf{a} - a_{tan} \\ &= [0, 0, 0] \quad /4 \end{aligned}$$

b) Ellipse $r(t) = [\cos t, 2\sin t, 0]$

$$v(t) = r'(t) = [-\sin t, 2\cos t, 0] \quad //$$

$$\|v\| = \sqrt{(-\sin t)^2 + (2\cos t)^2 + 0} \\ = \sqrt{4\cos^2 t + \sin^2 t} \quad //$$

$$a(t) = v'(t) = [-\cos t, -2\sin t, 0] \quad //$$

$$a_{tan} = \frac{a \cdot v}{v \cdot v} = \frac{(-\cos t)(-\sin t) + (-2\sin t)(2\cos t) + 0}{\sin^2 t + 4\cos^2 t + 0} \\ = \frac{-3\cos(t)\sin(t)}{\sin^2(t) + 4\cos^2(t)} \\ = \frac{3\sin(2t)}{5+3\cos(2t)} [-\sin(t), 2\cos(t), 0] \quad //$$

$$a_{norm} = a - a_{tan} \\ = [-\cos(t), -2\sin(t), 0] - \underbrace{\frac{3\sin(2t)}{5+3\cos(2t)} [-\sin(t), 2\cos(t), 0]}_{\alpha} \\ = \left[(-\cos(t) - \alpha(-\sin(t)), -2\sin(t) - \alpha(2\cos(t)), 0) \right] \\ = \left[-\cos(t) - \frac{6\sin(t)\sin(2t)}{5+3\cos(2t)}, -2\sin(t) + \frac{6\cos(t)\sin(2t)}{5+3\cos(2t)}, 0 \right] \\ = \left[-\cos(t) - \frac{6\sin(t)\sin(2t)}{5+3\cos(2t)}, -\frac{4\sin(t)}{5+3\cos(2t)}, 0 \right] \quad //$$