Quiz 1 - 25 Feb 2019

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$ Find λ's & X

$$\det(\lambda I - A) = 0$$

$$\det\begin{pmatrix} \lambda-3 & -5 & -3 \\ 0 & \lambda-4 & -6 \\ 0 & 0 & \lambda-1 \end{pmatrix} = 0$$

$$(\lambda-3)\begin{pmatrix} \lambda-4 & -6 \\ 0 & \lambda-1 \end{pmatrix} - (-5)\begin{pmatrix} 0 & -6 \\ 0 & \lambda-1 \end{pmatrix} = 0$$

$$(\lambda-3)((\lambda-4)(\lambda-1) - 0(-6)) + 5(0(\lambda-1) - (-6)(0))$$

$$(\lambda-3)((\lambda-4)(\lambda-1)) = 0$$

$\lambda_1 = 3$
$\lambda_2 = 4$
$\lambda_3 = 1$ //

$$Ax = \lambda x$$
$$\rightarrow Ax - \lambda x = 0$$
$$\rightarrow \lambda x - Ax = 0$$

▷ for $\lambda_1 = 3$, $Ax = 3x$

$$(3I - A)x = 0$$

$$\begin{pmatrix} 3-3 & -5 & -3 \\ 0 & 3-4 & -6 \\ 0 & 0 & 3-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & -5 & -3 \\ 0 & -1 & -6 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \qquad \begin{aligned} -5x_2 - 3x_3 &= 0 \\ -x_2 - 6x_3 &= 0 \\ 2x_3 &= 0 \end{aligned}$$

$x_3 = 0 \quad x_2 = 0 \quad x_1 = t$

$$X_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$ //

▷ for $\lambda_2 = 4$, $Ax = 4x$

$$(4I - A)x = 0$$

$$\begin{pmatrix} 4-3 & -5 & -3 \\ 0 & 4-4 & -6 \\ 0 & 0 & 4-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \qquad \begin{aligned} x_1 - 5x_2 - 3x_3 &= 0 \\ -6x_3 &= 0 \\ 3x_3 &= 0 \end{aligned}$$

$x_3 = 0$ $\quad x_1 - 5x_2 = 0$ $\quad x_2 = \frac{1}{5}x_1$

$x_1 = 5x_2$ $\quad x_1 = t$

$x_2 = t$

$$X_2 = \begin{pmatrix} t \\ \frac{1}{5}t \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ \frac{1}{5} \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

or...

$$X_2 = \begin{pmatrix} 5t \\ t \\ 0 \end{pmatrix} = t\begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

for $\lambda_3 = 1$, $Ax = 1x$

$(1I - A)x = 0$

$$\begin{pmatrix} 1-3 & -5 & -3 \\ 0 & 1-4 & -6 \\ 0 & 0 & 1-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \qquad \begin{matrix} -2x_1 - 5x_2 - 3x_3 = 0 \\ -3x_2 - 6x_3 = 0 \end{matrix}$$

.....

needed more time...