INSTITUTO TECNOLÓGICO DE ESTUDIOS SUPERIORES DE MONTERREY
CAMPUS ESTADO DE MÉXICO

Applied Computer Science
Masters in Nanotechnology

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Linear, mixed, and quadratic programming

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Due date: April 11, 2019, 15:59PM

MATLAB Script and Implemented Functions

```matlab
% ************************************************************************
% * AUTHOR(S) :
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% *
% * FILENAME :
% *     HW01.m
% *
% * DESCRIPTION :
% *     Computacióonn Aplicada (Ene 19 Gpo 1)
% *     Homework on Linear, mixed, and quadratic programming
% *
% * NOTES :
% *
% *
% * START DATE :
% *     11 Apr 2019
% ************************************************************************

warning('off')
clc;
clear all;
close all;

%% ************************************************************************
% Problem 1:
% Solve the Following transportation problem:
s = [37.6; 40.4; 44.5]';
d = [20 30 30 40]';
C = [41 27 28 24; 40 29 100 23; 37 30 27 21];

% transportation costs tableau
%             | destination      | supply
%             |  1   2   3   4   |
% ------------+------------------+--------
%    source 1 |  41  27  28  24  | 37.6
%           2 |  40  29 100  23  | 40.4
%           3 |  37  30  27  21  | 44.5
% ------------+------------------+--------
%      demand |  20  30  30  40  |

% Assume that only integer units can be transported.
% Assume that only multiples of 2 units can be transported.
% Upload to Blackboard a pdf file with a MATLAB script that solves both
% cases.
% The pdf should also include the solution matrix A for each case.
disp("Problem 1: Matrix of assignation and cost.");
[assig, total_cost] = EvenTransportation(s,d,C);

% Print the calculations
disp(assig);
disp(total_cost);
disp(" ");

%% ************************************************************************
% Problem 2:
% Using function Investopedia, download the adjusted closing prices for the
% following DJI stocks from January 1, 2018 through January 1, 2019.
%
```

```
% KO Coca-Cola                DIS Disney
% PG Procter & Gamble        MCD McDonald's
% PFE Pfizer                 WMT WalMart
% MRK Merck                  V Visa
% VZ Verizon
%
% Obtain the daily returns for these securities. Plot their expected return
% versus variance.
% Using command quadprog, obtain the optimal portfolios for
% k = 1; 2; 2:5; 4; 7; 9; 11; 20; 50; 1000
% For each value of k, report optimal weights, expected return and
variance.
% Plot this data on the previous graph.
% Upload to Blackboard a pdf file that contains a MATLAB script, any MATLAB
% functions that you implemented, and the required plots and results.

disp("Problem 2: Percentage of investment given a risk-aversion.");

load('Tiingo_data.mat')
PortfolioOptimization(res,data,1);
PortfolioOptimization(res,data,2);
PortfolioOptimization(res,data,2:5);
PortfolioOptimization(res,data,4);
PortfolioOptimization(res,data,7);
PortfolioOptimization(res,data,9);
PortfolioOptimization(res,data,11);
PortfolioOptimization(res,data,20);
PortfolioOptimization(res,data,50);
% WARNING: Your PC may freeze with a big k such as 1000. Uncomment at your
% own risk ...
PortfolioOptimization(res,data,1000);

%% ************************************************************************
% Problem 1 FUNCTION DEFINITION

function [assig, total_cost] = EvenTransportation(s,d,C)

    % Only multiples of 2 units can be transported, so let's assume each
unit
    % contains 2 objects. Let's simulate this by divition by 2.
    s = s/2;
    d = d/2;

    % Only integer units can be transported, so round toward negative
infinity.
    s = floor(s);

    % The following is based on:
    %
http://web.tecnico.ulisboa.pt/mcasquilho/compute/_linpro/TaylorB_module_b.p
df
    % f = [C(1,:) C(2,:) C(3,:)]';
    n = length(s);
    m = length(d);
    f = reshape(C',n*m,1);
    A = zeros(n,n*m);

    for i=1:n
        A(i,1+(i-1)*4:i*4) = 1;
    end
    %
```

```matlab
    b = s;
    Aeq = zeros(n,n*m);
    %
    for j=1:m
        Aeq(j,j:m:n*m) = 1;
    end
    %
    beq = d;
    LB = zeros(n*m,1);
    UB = Inf(n*m,1);
    %
    x = linprog(f,A,b,Aeq,beq,LB,UB);
    %
    assig = reshape(x,m,n)';
    total_cost = sum(sum(assig.*C));

    % Up to this point calculations have been made by units of 2 objects,
let's
    % multiply the result by 2 to get the transportation assignments and
const
    % by product
    assig = assig*2;
    total_cost = total_cost*2;
end

%% ************************************************************************
% Problem 2 FUNCTION DEFINITION

function [out] = PortfolioOptimization(res,data,k)
    % Let's convert the closing prices into returns
    % CLOSING PRICES
    CP_KO  = res.KO;
    CP_PG  = res.PG;
    CP_PFE = res.PFE;
    CP_MRK = res.MRK;
    CP_VZ  = res.VZ;
    CP_DIS = res.DIS;
    CP_MCD = res.MCD;
    CP_WMT = res.WMT;
    CP_V   = res.V;

    % RETURNS
    R_KO  = (CP_KO(2:end)-CP_KO(1:end-1))./CP_KO(1:end-1);
    R_PG  = (CP_PG(2:end)-CP_PG(1:end-1))./CP_PG(1:end-1);
    R_PFE = (CP_PFE(2:end)-CP_PFE(1:end-1))./CP_PFE(1:end-1);
    R_MRK = (CP_MRK(2:end)-CP_MRK(1:end-1))./CP_MRK(1:end-1);
    R_VZ  = (CP_VZ(2:end)-CP_VZ(1:end-1))./CP_VZ(1:end-1);
    R_DIS = (CP_DIS(2:end)-CP_DIS(1:end-1))./CP_DIS(1:end-1);
    R_MCD = (CP_MCD(2:end)-CP_MCD(1:end-1))./CP_MCD(1:end-1);
    R_WMT = (CP_WMT(2:end)-CP_WMT(1:end-1))./CP_WMT(1:end-1);
    R_V   = (CP_V(2:end)-CP_V(1:end-1))./CP_V(1:end-1);

    Returns = [R_KO R_PG R_PFE R_MRK R_VZ R_DIS R_MCD R_WMT R_V];

    fields = transpose(fieldnames(res)); % asset names
    nAssets = length(fields); % number of assets

    % Normalized Asset Prices
    assetP = data./data(1, :); %%NormalizedPrice
    figure(1);
    plot(assetP);
```

```
    xlabel('Day count');
    ylabel('Normalized Price');
    title('Normalized Asset Prices');
    for i = 1:nAssets
        text(length(assetP(:,i)),assetP(end,i),strcat( "  ", fields(i)) );
    end

    % Expected returns / mean of the returns per security
    muR  = mean(Returns);

    % % Risk-Adjusted Returns
    % assetRisk =  std(Returns);

    % Returns variance per security / risk
    C = cov(Returns); % std(R)^2 variances are in the main diagonal ...
    sigmaR = diag(C); % extract the main diagonal of C

    % Plot their expected return versus variance.
    figure(2);
    scatter(sigmaR,muR,2);
    title("Pareto Front");
    xlabel("Risk (Std Dev of Return)");
    ylabel("Expected Return");
    for i = 1:nAssets
        text(sigmaR(i),muR(i),strcat( "  ", fields(i)) );
    end

    % Quadratic Programming Portfolio Optimization
    % based on: openExample('optim/PortfolioMIQPExample')

    % Load the data for the problem
    r = transpose(muR); % returns
    Q = C;%sigmaR;         % risk
    % Set the number of assets
    N = length(r);
    % Create continuous variables xvars representing the asset allocation
    % fraction
    xvars = optimvar('xvars',N,1,'LowerBound',0,'UpperBound',1);
    % binary variables vvars representing whether or not the associated
    % % xvars is zero or strictly positive
    % vvars = 
optimvar('vvars',N,1,'Type','integer','LowerBound',0,'UpperBound',1);
    % and zvar representing the  variable, a positive scalar.
    zvar = optimvar('zvar',1,'LowerBound',0);
    % Set the Optimization Problem
    qpprob = optimproblem('ObjectiveSense','maximize');

    % Set the risk-aversion: lambda = k
    % and iterate if k is a vector
    for lambda = k(1):k(end)
        % Define the objective function
        qpprob.Objective = r'*xvars - lambda*zvar;
        % solving the problem with the current constraints
        options = optimoptions(@intlinprog,'Display','off'); % Suppress 
iterative display
        [xLinInt,~,~,~] = solve(qpprob,'options',options);
        % stop iterating when the slack variable is within 0.01% of the 
true quadratic value
        thediff = 1e-4;
        iter = 1; % iteration counter
        assets = xLinInt.xvars;
```

```
        truequadratic = assets'*Q*assets;
        zslack = zeros(length(truequadratic),1);
        % keep a history of the computed true quadratic and slack variables
for plotting.
        history = [truequadratic,zslack];
        options = optimoptions(options,'LPOptimalityTolerance',1e-
10,'RelativeGapTolerance',1e-8,...
                              'ConstraintTolerance',1e-
9,'IntegerTolerance',1e-6);
        % Compute the quadratic and slack values.
        while abs((zslack - truequadratic)/truequadratic) > thediff %
relative error
            % If the quadratic and slack values differ,
            % then add another linear constraint and solve again.
            constr = 2*assets'*Q*xvars - zvar <= assets'*Q*assets;
            newname = ['iteration',num2str(iter)];
            qpprob.Constraints.(newname) = constr;
            % Solve the problem with the new constraints
            [xLinInt,~,~,~] = solve(qpprob,'options',options);
            assets = (assets+xLinInt.xvars)/2; % Midway from the previous
to the current
            %assets = xLinInt(xvars); % Use the previous line or this one
            truequadratic = xLinInt.xvars'*Q*xLinInt.xvars;
            zslack = xLinInt.zvar;
            history = [history;truequadratic,zslack];
            iter = iter + 1;
        end

        % Convert the porfolio weights into percentages of investment per
        % asset
        Percentage_of_Investment = xLinInt.xvars/sum(xLinInt.xvars);

        % Prepare variables to print
        for i = 1:nAssets
            print(i) = ...
                strcat( ...
                    fields(i), ...
                    " : ", ...
                    num2str(round(Percentage_of_Investment(i),4)), ...
                    "%" ...
                );
        end
        % Retun the final calculation
        out = transpose(print);
        % and print
        disp(strcat( ...
            "Percentage of Investment with ", ...
            "risk-aversion k = ", num2str(lambda)));
        disp(out);

        % PLOT Efficient Frontier
        % Let's do sigmaR.^2 as it (re)calculates the standard deviation
        % which is the square-root of variance)
        p = Portfolio('AssetMean',muR,
'AssetCovar',sigmaR.^2,'AssetList',fields);
        p = setDefaultConstraints(p);
        p = setSolver(p, 'quadprog');
        hold on
        plotFrontier(p)
        hold off

%         % PLOT Percentage_of_Investment
```

```
%           figure;
%           bar(Percentage_of_Investment, 0.125);
%           grid on;
%           xlabel('Asset index');
%           ylabel('Proportion of investment');
%           title(strcat("Optimal asset allocation with k =
",num2str(lambda)));
%           for i = 1:nAssets
%               text(i,Percentage_of_Investment(i),fields(i));
%           end
    end
end
```

Plots and Results

Problem 1: Matrix of assignation and cost.

Optimal solution found.

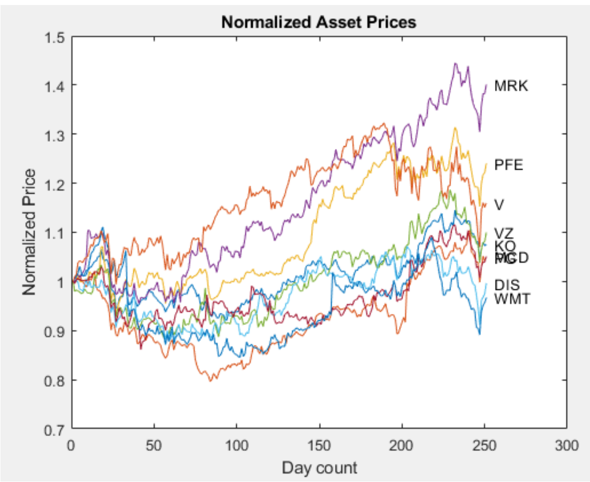
Matrix A:

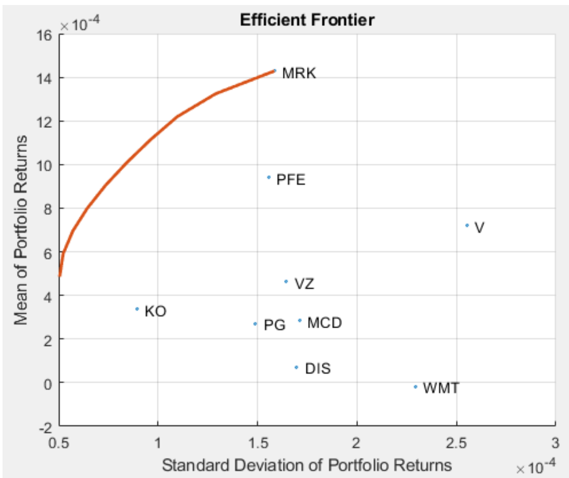
```
     0    30     6     0
     0     0     0    40
    20     0    24     0
```

Optimal Cost: **3286**

Problem 2: Percentage of investment given a risk-aversion.
Percentage of Investment with risk-aversion **k = 1**

```
    "KO : 0%"
    "PG : 0%"
    "PFE : 0.3258%"
    "MRK : 0.3258%"
    "VZ : 0.1509%"
    "DIS : 0%"
    "MCD : 0%"
    "WMT : 0%"
    "V : 0.1974%"
```

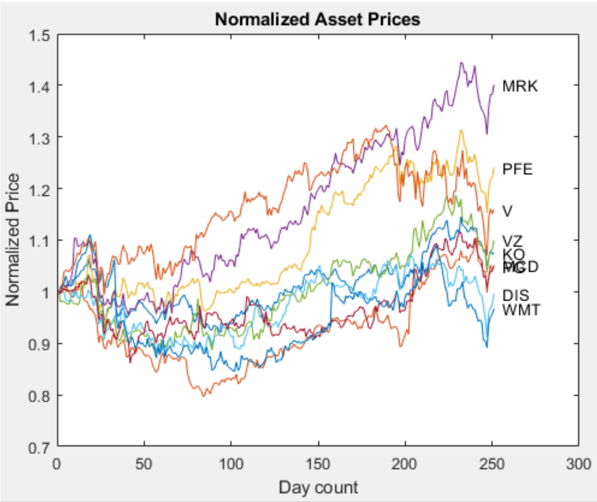


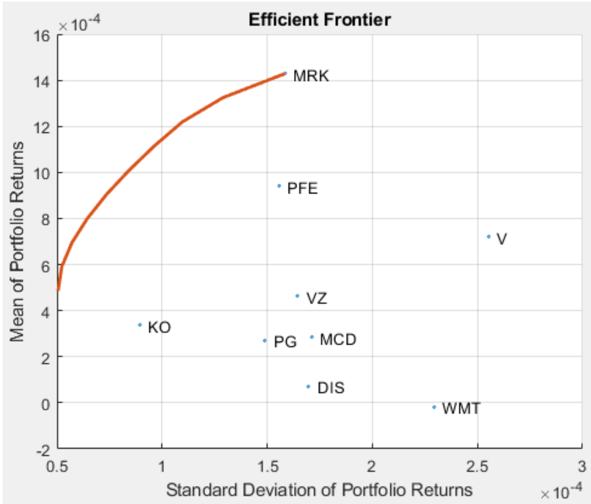


Percentage of Investment with risk-aversion **k = 2**

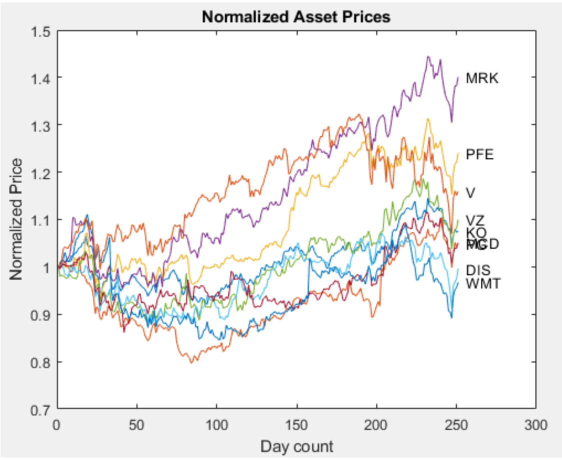
```
    "KO : 0%"
    "PG : 0%"
    "PFE : 0.4578%"
    "MRK : 0.5325%"
```

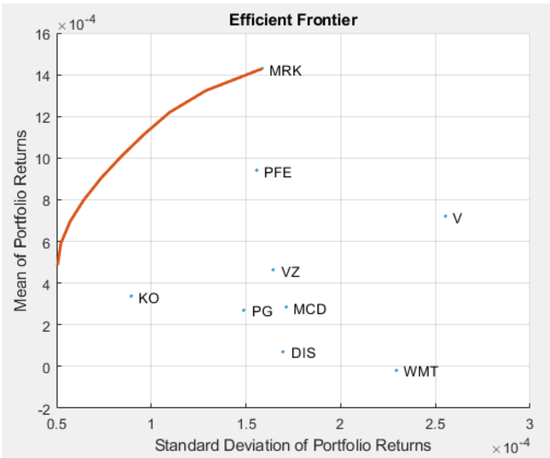
"VZ : 0%"
"DIS : 0%"
"MCD : 0%"
"WMT : 0%"
"V : 0.0097%"



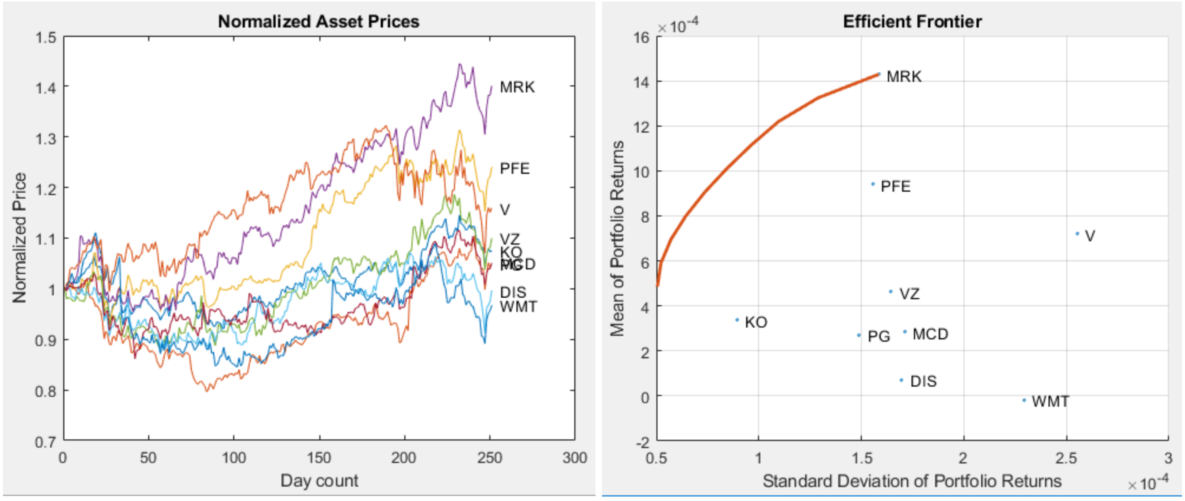


Percentage of Investment with risk-aversion **k = 2.5**
"KO : 0%"
"PG : 0%"
"PFE : 0.358%"
"MRK : 0.642%"
"VZ : 0%"
"DIS : 0%"
"MCD : 0%"
"WMT : 0%"
"V : 0%"



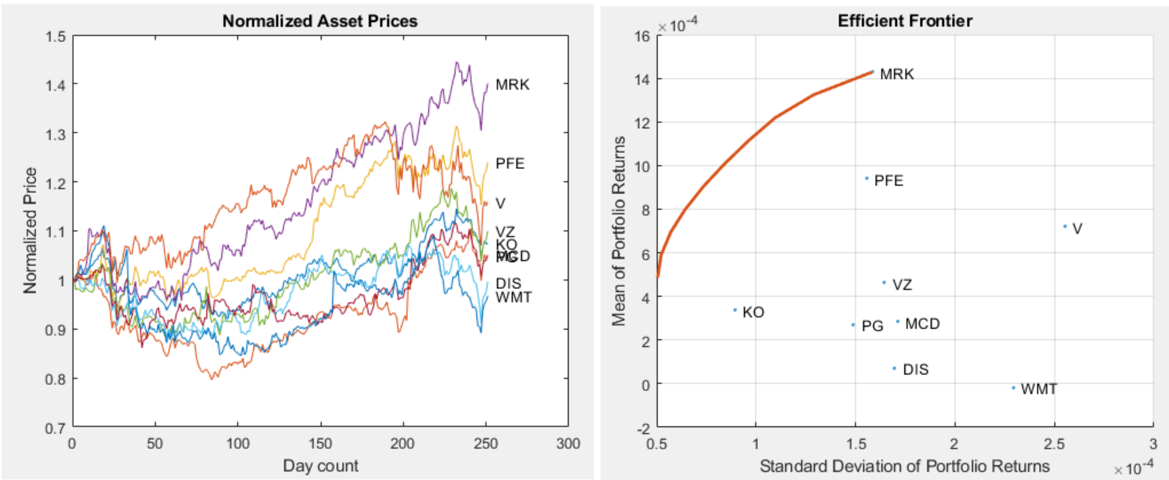


Percentage of Investment with risk-aversion **k = 4**
"KO : 0%"
"PG : 0%"
"PFE : 0.0974%"
"MRK : 0.9026%"
"VZ : 0%"
"DIS : 0%"
"MCD : 0%"
"WMT : 0%"
"V : 0%"



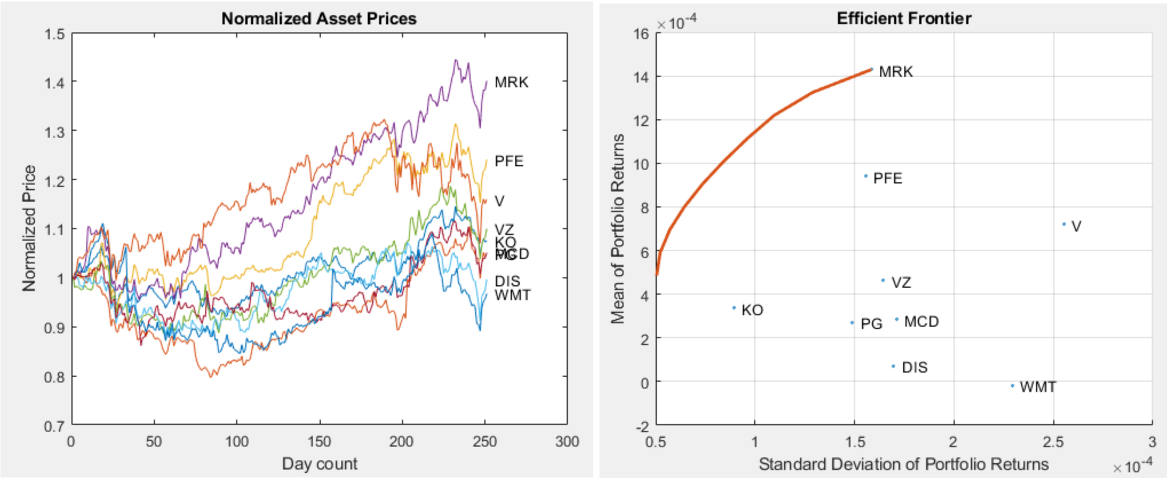
Percentage of Investment with risk-aversion **k = 7**

```
    "KO : 0%"
    "PG : 0%"
    "PFE : 0.04%"
    "MRK : 0.96%"
    "VZ : 0%"
    "DIS : 0%"
    "MCD : 0%"
    "WMT : 0%"
    "V : 0%"
```



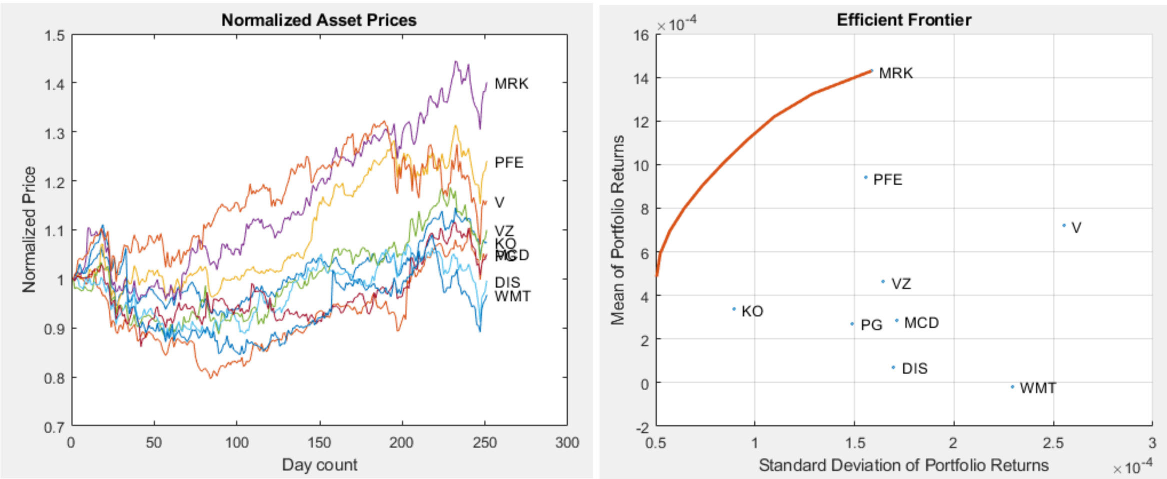
Percentage of Investment with risk-aversion **k = 9**

```
    "KO : 0%"
    "PG : 0%"
    "PFE : 0.0447%"
    "MRK : 0.9553%"
    "VZ : 0%"
    "DIS : 0%"
    "MCD : 0%"
    "WMT : 0%"
    "V : 0%"
```



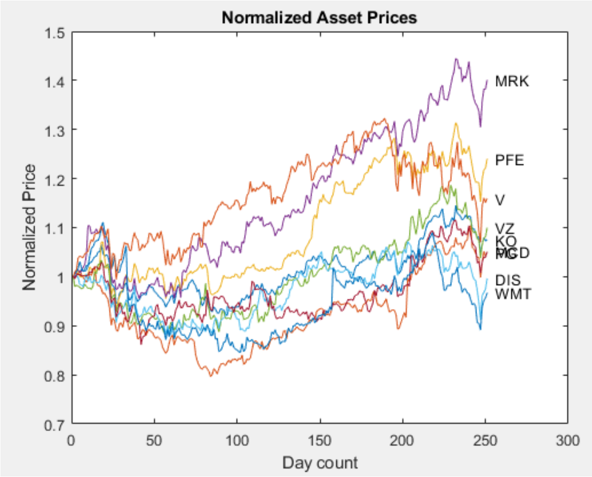
Percentage of Investment with risk-aversion **k = 11**

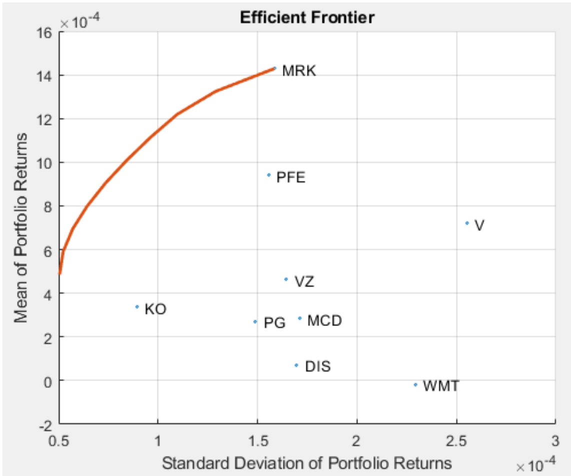
"KO : 0%"
"PG : 0%"
"PFE : 0.0387%"
"MRK : 0.9613%"
"VZ : 0%"
"DIS : 0%"
"MCD : 0%"
"WMT : 0%"
"V : 0%"



Percentage of Investment with risk-aversion **k = 20**

"KO : 0%"
"PG : 0%"
"PFE : 0.0406%"
"MRK : 0.9594%"
"VZ : 0%"
"DIS : 0%"
"MCD : 0%"
"WMT : 0%"
"V : 0%"






```
Percentage of Investment with risk-aversion k = 50
    "KO : 0%"
    "PG : 0%"
    "PFE : 0.0463%"
    "MRK : 0.9537%"
    "VZ : 0%"
    "DIS : 0%"
    "MCD : 0%"
    "WMT : 0%"
    "V : 0%"
```