ITESM Campus Monterrey
Mathematical Physical Modelling F4005
HW4: Linear transformations I
Due Date: February 17-2019, 23:59 hrs.
Professor: Ph.D Daniel López Aguayo

Full names of team members: ______________________________

Instructions: Please write neatly on each page of your homework and send it in pdf format to dlopez.aguayo@tec.mx. Typed solutions in LaTeX (only) will be given extra credit; no late homework will be accepted. Each team should consist (of at most) 5 students.

$\boxed{1}$ Consider the map given by $T : \mathbb{R}^3 \to \mathbb{R}$ given by $T(x, y, z) = \frac{1}{(x-2)^2+(z-2)^2+(y-2)^2}$.

(a) Find the domain of T and plot the subset of $\mathbb{R}^3$ that represents the domain.

(b) Is this a linear transformation? justify carefully your answer.

$\boxed{2}$ Consider the map $W : \mathbb{R}^2 \to \mathbb{R}$ given by $W(a, b) = \frac{1}{sin(\frac{a}{2})} + \frac{22222}{\sqrt{b-1}}$.

(a) Find the domain of W.

(b) Is this a linear transformation? justify carefully your answer.

$\boxed{3}$ Consider the map $T : \mathbb{R} \to \mathbb{R}^2$ given by $T(x) = (\frac{sinx}{\pi \cdot e}, \frac{cosx}{\pi \cdot e})$. Prove (mathematically) that the range of T is a circle and find its radius and center.

$\boxed{4}$ Let $N : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $N(a, b) = (a - b, 3b - 3a)$. Compute, mathematically, the range of N and plot it.

$\boxed{5}$ Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x, -y)$.

(a) Find the domain of T.

(b) Prove that T is a linear transformation (**verify both properties**).

(c) Plot some points and deduce the range of T.

(d) What is the geometric interpretation of T? Is it any reflection? What kind?

(e) **Optional**. How can you infer the range of T by using Mathematica? *Hint*: Make use of the *ListPlot* and the *Table* commands, together with a list with two parameters.

$\boxed{6}$ Is the map $P : \mathbb{R} \to \mathbb{R}^2$ given by $P(y) = (y, 0)$ linear? prove in detail your answer.

$\boxed{7}$ Is the map $M : \mathbb{R}^2 \to \mathbb{R}$ given by $M(x, y) = x + y + 2$ linear? prove in detail your answer.

$\boxed{8}$ Is the map $Q : \mathbb{R}^2 \to \mathbb{R}^3$ given by $Q(x, y) = (x, y, \sqrt{2} + \sqrt{31})$ linear? prove in detail your answer.

$\boxed{9}$ Use the following theorem (which I proved and was motivated by a great question by Luis Alejandro Garza Soto!) to answer the questions below it; simply state if the range is a line through the origin; one of the coordinate axes; or the entire plane.

> **Theorem**. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y) = (ax + by, cx + dy)$ where a, b, c, d are arbitrary real numbers.
>
> (a) If $a = b = c = d = 0$, then the range of T is simply the origin in $\mathbb{R}^2$.
>
> (b) If $ad - bc \neq 0$, then the range of T is the whole plane $\mathbb{R}^2$.
>
> (c) If $ad - bc = 0$, and if at least one of the constants a, b, c, d is non-zero, then the range of T is a line through the origin (either a diagonal line, or the y-axis or x-axis).

(i) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (4x - y, 4x + y)$.

(ii) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x, 0)$. What is the geometric interpretation of T?

(iii) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (0, y)$. What is the geometric interpretation of T?

(iv) Use the above theorem to find the range of $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (91x - y, 91x - y)$? What is the graph of T? Hint: it should be familiar to you!

$\boxed{10}$ Suppose x is your grade corresponding to the first partial period; y is the grade corresponding to the second partial period, and z to the final period. Recall that the weighing formula for the final grade of the course is as follows: 30% first partial period, 30% second partial period and 40% final period.

(a) Construct a transformation $T : \mathbb{R}^3 \to \mathbb{R}$ whose image is precisely the final grade of the course.

(b) Is the above function a linear transformation? In case it is, prove it; otherwise explain why not.

$\boxed{11}$ Consider the map $T : \mathbb{R}^3 \to \mathbb{R}$ given by $T(x, y, z) = \frac{y}{x^2+z^2+4000}$. Find the domain of T and make a plot of the subset of $\mathbb{R}^3$ that represents the domain.

$\boxed{12}$ Give a concrete example of a transformation $T : \mathbb{R} \to \mathbb{R}^2$ that satisfies $T(0) = (0, 0)$ but such that T is **not** linear; justify why T is not linear.