# Modeling and Optimization of the Injection-Molding Process: A Review

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**ABSTRACT:** The purpose of this article is to review the research done in the field of mathematical modeling and optimization of the injection-molding (IM) process. Various papers related to the mathematical description of the filling, postfilling, and plasticating phases of the IM process were assessed, and some recent advances on the IM field are described. In addition, research devoted to the optimization of the IM process based on various techniques is also discussed. These optimization techniques include design of experiments, artificial neural networks, and evolutionary algorithms. The strengths and weaknesses of each approach were discussed. Finally, this paper also discusses the optimization research performed in the IM process regarding some of the specific features associated with the processes such as runner system and cooling channel configurations, process conditions, gate location, and cavity pressure balancing. © 2016 Wiley Periodicals, Inc. Adv Polym Technol 2016, 00, 21683; View this article online at wileyonlinelibrary.com. DOI 10.1002/adv.21683

**KEY WORDS:** Computer modeling, Injection molding, Mathematical modeling, Optimization, Review

## Introduction

In the past three decades, injection molding (IM) had a rapid growth due to the development of new application areas in the fields of automotive/transportation, electronics/appliances, medical, and packaging industries. The complexity of the IM process demands a much better understanding of the material behavior during the basic stages of the process, the physical phenomena taking place, and its relation to the properties and performance of the final molded part.

One of the main goals in IM is the improvement of quality of molded parts at the lowest cost. Meeting required specifications means keeping quality characteristics under control.

The most widely used approach to determine the quality of a molded part consists of optimizing performance indicators as functions of the input variables. This is done by using a model that relates to the behavior of these performance indicators (e.g., warpage, shrinkage, pressure work, cycle time) with the controllable parameters (e.g., part and mold design characteristics and process variables) and then optimizing these models, that is, mathematical-based models or metamodels. Therefore, the development of mathematical models for the individual steps of the injection process is essentially at the industrial level because they are needed for optimizing the process and the product performance.

The aim of this paper is to present the state-of-the-art studies related to both the mathematical modeling and the optimization of the IM process for nonreinforced polymers. First, the state-of-the-art studies concerning the mathematical modeling of the IM process is presented, considering, simultaneously, that the simplifications and advantages of each model are explained, and the most recent developments in the field are introduced. Then, the revision of the literature regarding the optimization will be discussed in detail based on various techniques, including, DOE, ANN and EA, given the relevance of the process optimization for the industry.

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## Theoretical Research

### GENERAL VIEW

IM is one of the most important polymer-processing techniques for producing polymeric parts. The main concern in IM is to produce parts with the desired quality, which is usually related to the mechanical performance, dimensional conformity, and appearance. The major factors affecting part quality are part and mold design and process variables.

The IM process is cyclic and can be characterized by the following main stages: (i) filling, where the molten polymer is injected under pressure into a closed cooled mold; (ii) packing,

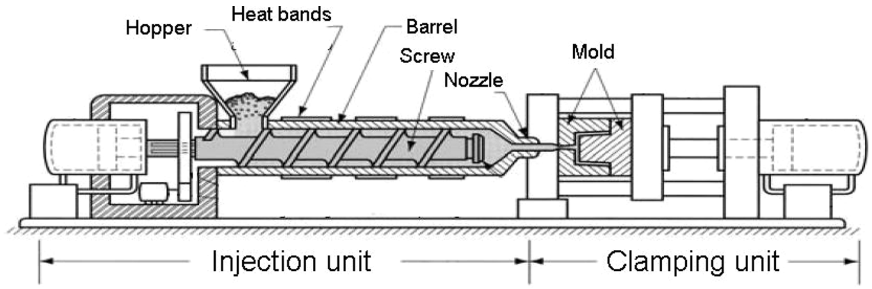


**FIGURE 1.** Units of an injection-molding machine.

where the pressure is maintained high so that the molten polymer stays in the interior of the mold and shrinkage is prevented during solidification; and (iii) cooling/plasticating, involving the part cooling inside the mold until it will be sufficiently rigid to be ejected, and the concomitant melt preparation for the next cycle (plasticating stage). In the plasticating phase, the screw rotates and the solid polymer is conveyed and heated inside the injection barrel until it reaches the molten state to be injected in the next cycle. All these stages occur in the injection and clamping units (see Fig. 1).

## FILLING

The mathematical model for the filling stage considers the effect suffered when the molten polymer fills the cold mold cavity. The main problem for modeling of the filling stage is to select the models to use, when the fluid flow and heat transfer are considered. Different approaches have been used, mainly the Hele–Shaw and Navier–Stokes models. In both cases, the flow through empty and thin cavities is modeled.[1]

The generalized Hele–Shaw model is used for shear flow analysis in a thin cavity (two-dimensional (2D) formulation with midplane thickness). This model considers an incompressible, generalized non-Newtonian fluid under nonisothermal conditions.[2] In this flow, the width of the gap between two plates is assumed to be much smaller than the other dimensions of the flow. Therefore, at a given point, the flow is mostly influenced by the local geometry and hence the lubrication approximation can be applied. In this assumption, the velocity in the gapwise direction is neglected and the pressure is a function of planar coordinates only. As a result, the relevant governing equations describing the Hele–Shaw polymer melt flow are

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{1}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}\left(\eta \frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial z}\left(\eta \frac{\partial v}{\partial z}\right) \tag{3}$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial z^2} + \eta \dot{\gamma}^2 \tag{4}$$

where $x$ and $y$ denote the planar coordinates and $z$ denotes the thickness coordinate, $(u, v, w)$ are the velocity components in the $(x, y, z)$ directions for time $t$ and under pressure $p$, $\rho(T, p)$ is the density of the polymer, $\eta(\dot{\gamma}, T, p)$ is the shear viscosity, where $\dot{\gamma}$ is the shear rate, $T$ is the temperature, $C_p$ is the specific heat, and $k$ is the thermal conductivity. The $z$-coordinate represents the thickness direction, and no flow will take place in that direction. The shear rate is given by

$$\dot{\gamma} = \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2} \tag{5}$$

The boundary and initial conditions for the Hele–Shaw model are given by

$$u = v = 0, \quad T = T_w \quad \text{at } z = h \tag{6}$$

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0 \tag{7}$$

$$p = 0 \quad \text{along the flow front} \tag{8}$$

where $T_w$ is a constant wall temperature and $2h$ is the gap of the cavity (Fig. 2).

Summarizing, the Hele–Shaw model equations are based upon the following simplifications: (i) the flow is inelastic; (ii) body forces, inertial forces, and the fountain flow phenomena are neglected; (iii) normal stresses are neglected; (iv) the conductivity and heat capacity are constant; and (v) thermal convection in the gapwise direction and conduction in the flow direction are neglected.

The Navier–Stokes equations can be expressed in a three-dimensional (3D) formulation by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho\, \mathbf{u} = 0 \tag{9}$$

$$\frac{\partial}{\partial t}(\rho\, \mathbf{u}) + \nabla \cdot (\rho\, \mathbf{u}\mathbf{u} - \boldsymbol{\sigma}) = \rho\, \mathbf{g} \tag{10}$$

$$\boldsymbol{\sigma} = -p\mathbf{I} + \eta\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right) \tag{11}$$

where $\mathbf{u}$ is the velocity vector, $\boldsymbol{\sigma}$ is the total stress tensor, $\mathbf{g}$ is the gravitational force actuating in the body (body forces), $\mathbf{I}$ is the identity matrix, and $\nabla = (\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$. Equation (9) is the unsteady, 3D mass conservation or continuity equation at a point in a compressible fluid. The first term on the left-hand side (l.h.s.) is the rate of change in time of the density (mass per unit volume). The second term on the l.h.s. describes the

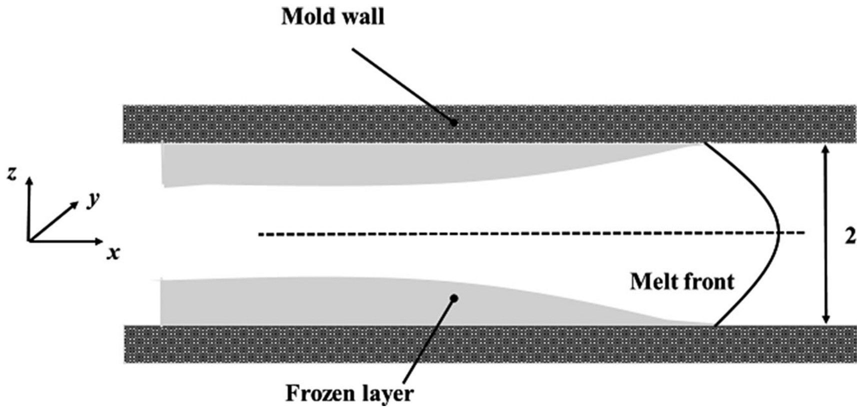


**FIGURE 2.** Cross-sectional view of the flow front in the Hele–Shaw model.

net flow of mass out of the element across its boundaries and is called convective term. Equation (10) is the unsteady, 3D momentum equation at a point in a compressible fluid. The first term on the l.h.s. is the rate of change in time of the momentum of the fluid particle. The second term on the l.h.s. is the total force on the element due to surface stresses (pressure and stress components given by Eq. (11)) and convection of momentum. The term on the right-hand side (r.h.s.) of Eq. (10) represents the rate of increase in momentum due to sources (in this case, only gravitational force). These types of equations are usually used to model incompressible (density is constant) or Newtonian fluids (viscosity is constant).

To complete the system of equations for the filling phase, it is necessary to add the energy equation to take into account the heat transfer. The energy equation is coupled with the equations of flow, and, as a consequence, a system of equations is to be solved. This is due to the dependence of the melt viscosity upon the shear rate (flow), temperature, and pressure. The energy equation can be written in the general form as

$$\rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot k \nabla T + \eta \dot{\gamma}^2 \tag{12}$$

where $C_p$ is the specific heat and $k$ is the thermal conductivity. The first and second terms inside the curved brackets in the l.h.s. of Eq. (12) represent the rate of change of energy of the fluid particle and the net rate of energy transport by convection, respectively. The first term on the r.h.s. of the equation represents the net rate of heat addition to the fluid by conduction, and the second term on the r.h.s. of the equation represents the rate of increase in energy due to viscous dissipation.

## POSTFILLING

### Packing

In the packing phase, the mold cavity is filled up by polymer melt. Owing to the thermal shrinkage, more melt is forced to fill the cavity. Therefore, compressible formulation is required to model this behavior. Governing equations for this phase are the same as for the filling phase (Eqs. (9)–(12)), and consideration of the compressibility of the melt is taken into account using a dependency of the specific volume on pressure and temperature. This is called the pvT relationship, which can be modeled using different complexities models, such as Spencer–Gilmore and Tait models.[3] The Spencer–Gilmore model is derived from the ideal gas law by adding a pressure and temperature correction term to the specific volume. If amorphous polymers are to be used in the simulation, a modified Tait model (Eq. (13)) is better to predict the abrupt volumetric change:

$$v(p, T) = v(0, T) \left[ 1 - C \ln \left( 1 + \frac{p}{B(T)} \right) \right] + v_t(p, T) \tag{13}$$

where $v(p, T)$ is the specific volume at temperature and pressure, $v(0, T)$ is the specific volume at zero gauge pressure, $T$ is the temperature, $p$ is the pressure, $C$ is a constant (equal to 0.0894), and $B$ accounts for the pressure sensitivity of the material and is defined below. The upper temperature region ($T > T_t$) can be described by $v(0, T) = b_{1m} + b_{2m}(T - b_5)$, $B(T) = b_{3m}\mathrm{e}^{-b_{4m}(T-b_5)}$, and $v_t(p, T) = 0$ where $b_{1m}$, $b_{2m}$, $b_{3m}$, $b_{4m}$ and $b_{5m}$ are data-fitted coefficients. The lower temperature region ($T < T_t$) can be described by $v(0, T) = b_{1s} + b_{2s}(T - b_5)$, $B(T) = b_{3s}e^{-b_{4s}(T-b_5)}$, and $v_t(p, T) = b_7\mathrm{e}^{b_8(T-b_5)-b_9 p}$, where $b_{1s}$, $b_{2s}$, $b_{3s}$, $b_{4s}$, $b_5$, $b_7$, $b_8$, and $b_9$ are data-fitted coefficients.

### Cooling

A cooling analysis allows the prediction of the coolant flow efficiency and heat transfer capability for each section of the cooling channel, pressure drop in the cooling circuit, and coolant temperature rise. The important phenomenon to take into consideration in the cooling process of IM is the heat transferred from the polymer to the cooling channels by heat conduction, and the heat is then removed with the coolant by heat convection. Moreover, after the part ejection, it continues to cool down to room temperature by dissipating the thermal energy into air.

The governing equation for the cooling stage in mid-plane geometries is the steady-state cycle-averaged temperature field governed by a second-order partial differential equation, Laplace's equation:

$$\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} + \frac{\partial^2 \bar{T}}{\partial z^2} = 0 \tag{14}$$

where $\bar{T}$ is the cycle-average mold temperature.

This equation can be solved with the proper boundary conditions imposed over the different frontiers existent in the mold (cavity surface, cooling channel surface, and external surface).

In the case of a 3D solid model, a full transient method should be used to give the varying mold temperatures during the whole molding cycle. The 3D transient heat conduction gives the temperature field in the mold as follows:

$$k_m\left(\frac{\partial^2 T_m}{\partial x^2}+\frac{\partial^2 T_m}{\partial y^2}+\frac{\partial^2 T_m}{\partial z^2}\right)=\rho_m C_m\frac{\partial T_m}{\partial t} \tag{15}$$

where $T_m$ is the mold temperature, $k_m$ is the mold thermal conductivity, $\rho_m$ is the mold density, and $C_m$ is the specific heat of the mold.

### Stress Development

In IM, there are two main sources for residual stresses: (a) viscoelastic deformations of the thermoplastic material during mold-filling and material-packing stages can produce the so-called "flow-induced" residual stresses and (b) thermally and pressure-induced stresses arise from inhomogeneous cooling of the part in combination with the hydrostatic pressure. Following the general practice, the flow-induced residual stresses are neglected since these are relieved whereas the part resides in the mold at high temperatures prior to ejection.

During material cooling inside the mold, its relaxation time starts to increase and approach the in-mold resident time. Therefore, to accurately predict the thermal stresses, it is mandatory to have the knowledge of the viscoelastic material properties. If infinitesimal strains are considered, the behavior of viscoelastic materials is well described using the anisotropic linear thermo-viscoelasticity[4,5] in the form:

$$\sigma_{ij}=\int_0^t c_{ijkl}\left(\xi\left(t\right)-\xi\left(t'\right)\right)\left(\frac{\partial\varepsilon_{kl}}{\partial t'}-\alpha_{kl}\frac{\partial T}{\partial t'}\right)dt' \tag{16}$$

where $\sigma_{ij}$ and$\varepsilon_{kl}$ are the stress and strain tensors, respectively. The strain tensor is determined from derivatives of the displacements $\mathbf{u}$ as $\varepsilon_{kl}=\frac{1}{2}(u_{k,l}+u_{l,k})$, $c_{ijkl}(t)$ is the fourth-order viscoelastic relaxation tensor, $\alpha_{kl}$ is a tensor describing the linear thermal expansion, and $\xi(t)$ is a "pseudo-timescale" given by

$$\xi\left(t\right)=\int_0^t\frac{1}{a_T}dt' \tag{17}$$

where $a_T$ is the time-dependent shift factor characterized by either the williams-landel-ferry (WLF) equation or the Arrhenius equation depending on the material and the temperature range. The derivative of the strain tensors accounts for density and pressure variations, and temperature derivative accounts for temperature changes.

In summary, the model accounts for the stress developed whereas the material cools under pressure in the mold. In particular, the model accounts for the thermally induced stress which arises from freezing and subsequent shrinkage of the material as well as the pressure-induced stress caused by the action of the melt pressure on the solidified material forming the frozen layer. The prediction of shrinkage and warpage is based on the computed thermally and pressure-induced residual stress distribution.

## PLASTICATING

The dynamic plasticating in IM (assuming an alternative screw IM machine) consists of some basic steps: solids conveying, melting, mixing, homogenization, and melt conveying. The two main steps in the plasticating sequence are melting and melt conveying, consisting of different transport processes. First, there is heat and momentum transport in the thin melt film separating the solid bed from the barrel. These transport operations are diffusional in nature and occur in a direction perpendicular to the down channel direction. Second, there are diffusional momentum and heat transport within the melt channel, also in directions perpendicular to the bulk flow direction. Finally, there is melt and solid convection in the down channel direction.[6] A differential mass balance on the solid bed provides the differential equation for calculating the instantaneous solid bed profile[6]:

$$-\rho_s V_{sz}\frac{\partial\left(HX\right)}{\partial_z}-\phi X^{\frac{1}{2}}=\rho_s\frac{\partial\left(HX\right)}{\partial t} \tag{19}$$

where $\rho_s$ is the density of the solid bed, $X=X(z,t)$ is the width of the solid bed at position $z$ and time $t$, $V_{sz}(t)$ is the down channel velocity of the solid bed, $H=H(z)$ is the channel depth, and the product $\phi(z,t)X^{1/2}$ expresses the rate of melting per unit down channel length.

The melt temperature profile is obtained from a differential energy balance on the melt pool:

$$\begin{aligned}&\rho_m c_m\frac{\partial}{\partial t}\left[H\left(W-X\right)\left(T-T_r\right)\right]+c_m\frac{\partial}{\partial z}\left[G_m\left(T-T_r\right)\right]\\&\quad=c_m\phi X^{\frac{1}{2}}\left(T_f-T_r\right)+q_T+q_v\end{aligned} \tag{20}$$

where $\rho_m$ and $c_m$ are the density and specific heat of the melt, respectively, $W$ is the channel width, $T_r$ and $T_f$ are the reference and film temperatures, respectively, $G_m$ is the mass flow rate of the melt, $q_T$ is the net heat transfer through the walls, and $q_v$ is the heat generated by viscous dissipation.

The next section describes the studies performed concerning the modeling of the IM cycle phases and the recent developments in this field.

## RESEARCH

The main aim of this section is to provide some insights about the merit (advantages/disadvantages) of the modeling approaches used to describe the IM cycle phases. It is intended to focus on the main features of the methods, instead of giving deep and detailed descriptions of the models previously published in the literature.[7]

### Filling, Packing-Holding, and Cooling

Table I summarizes the mathematical modeling research for the filling, packing-holding, and cooling stages of IM process

**TABLE I**
**Mathematical Modeling Research Applied to Filling, Packing, and Cooling Stages of the Injection-Molding Process**

| Authors | Injection Stages | | | Mathematical Models |
|---|---|---|---|---|
| | Filling | Packing | Cooling | |
| Wu et al.[12] | Yes | No | No | 1D |
| Stevenson[13] | Yes | No | No | 1D |
| Stevenson and Chuck[14] | Yes | No | No | 1D |
| Harry and Parrott[15] | Yes | No | No | 1D |
| Lord and Williams[16] | Yes | No | No | 1D |
| Kamal and Kenig[8] | Yes | Yes | Yes | 1D |
| Williams and Lord[17] | Yes | No | No | 1D |
| Nunn and Fenner[18] | Yes | No | No | 1D |
| Tadmor et al.[6] | Yes | No | No | 2D |
| Hieber and Shen[19] | Yes | No | No | 2D |
| Wang et al.[20] | Yes | No | No | 2D |
| Chiang et al.[3] | Yes | Yes | No | 2D |
| Chen and Liu[21] | Yes | Yes | No | 2D |
| Han and Im[22] | Yes | Yes | No | 2D |
| Holm and Langtangen[23] | Yes | Yes | Yes | 2D |
| Himasekhar et al.[24] | No | No | Yes | 2D |
| Chung and Kwon[27] | Yes | No | No | 2D |
| Chiang et al.[9] | Yes | Yes | Yes | 2D |
| Wang et al.[29] | Yes | No | No | 2D |
| Pantani and Titomanlio[30] | No | Yes | Yes | 2D |
| Kabanemi and Crochet[32] | No | No | Yes | 3D |
| Kabanemi et al.[33] | No | No | Yes | 3D |
| Hétu et al.[34] | Yes | No | No | 3D |
| Pichelin and Coupez[35] | Yes | No | No | 3D |
| Pichelin and Coupez[36] | Yes | No | No | 3D |
| Choi and Im[39] | No | Yes | Yes | 3D |
| Xue et al.[37] | Yes | No | No | 3D |
| Zheng et al.[38] | Yes | Yes | Yes | 3D |
| Chang and Yang[40] | Yes | No | No | 3D |
| Rajupalem et al.[43] | Yes | Yes | No | 3D |
| Talwar et al.[44] | Yes | Yes | No | 3D |
| Ilinca and Hétu[10] | Yes | Yes | Yes | 3D |
| Yang et al.[41] | Yes | Yes | Yes | 3D |
| Khayat et al.[45] | Yes | No | No | 3D |
| Cao et al.[46] | Yes | No | No | 3D |
| Hassan et al.[42] | No | No | Yes | 3D |

and includes the main key features of the works developed by the authors. It was necessary circa of three decades for the evolution from one-dimensional (1D) to 3D models. The first authors to present 1D models for filling and postfilling phases of IM were Kamal and Kenig.[8] Only two decades after, Chiang et al.[9] proposed 2D models for the three phases and, finally, Ilinca and Hétu[10] developed the 3D models to describe these injection stages.

A more complete mathematical model for the IM process involves mass, momentum, and energy balance equations, combined with constitutive laws for non-Newtonian fluids and appropriate boundary conditions.[11] The location of the advancing melt front must be determined as a part of the solution. This is hence a free boundary problem. Only a few mathematical models for describing the physical process of IM were developed. In particular, Wu et al.,[12] Stevenson,[13] and Stevenson and Chuck[14] analyzed a 1D flow in a center-gated disk. Harry and Parrott[15] and Lord and Williams[16] studied the 1D filling behavior in rectangular cavity geometries. Kamal and Kenig,[8] Williams and Lord,[17] and Nunn and Fenner[18] modeled 1D tubular flow of polymer melts.

The "branching flow" approach has been proposed and implemented for the simulation of polymer flow in typically complex mold cavities using these 1D flow representations.[8,12,15] This approach involves lying flat and decomposing the cavity geometry into several conjectured flow paths comprising a series of 1D elements such as strips, disks, fans, and/or tubes. The method requires intelligent judgment from the user because the solution accuracy strongly depends on how the geometry is being branched. Theoretical studies for 2D flow in a thin cavity based on Hele–Shaw flow formulation have been conducted to overcome the deficiency of the branching flow approach.

In particular, for 2D flows, two different approaches have been proposed, namely, the network flow, by Tadmor et al.[6] and the two-step "predictor–corrector" method, by Hieber and Shen.[19] The former discretizes the cavity geometry into a network of rectangular elements. The flow domain defined by the melt-front pattern is calculated based on the velocity at the advancing front. The two-step "predictor–corrector" approach is similar to the network flow approach except that it constantly creates new finite elements at the advancing flow front by taking into account the melt-front velocity and the actual geometry.

Wang et al.[20] simulated the polymer flow in sheet-like cavities with 3D configuration by applying a hybrid finite element method/finite differences method (FEM/FDM)/control-volume approach. This method discretizes the sprue/runner/cavity geometry into a network of 1D tubular elements and 2D triangular thin-shell elements. The advancement of the melt front is automatically tracked by computing the filled volume fraction of each control volume associated with the nodes and thus eliminating the user intervention. This approach became the standard numerical framework for various commercial software packages (e.g., Moldflow, Moldex 3D, and Simpoe) and research codes and has been extended or incorporated by other researchers to simulate the IM packing phase,[3,21–23] the mold cooling,[24] the development of fiber orientation,[25–29] and moldings shrinkage and warpage.[9,30,31] Here, the Hele–Shaw model (Eqs. (1)– (8)) is applied either to Newtonian or non-Newtonian fluids, incompressible or compressible, and isothermal or nonisothermal conditions.

However, the Hele–Shaw flow formulation has its limitations due to the inherent creeping flow and thin-wall assumptions. For example, it cannot accurately predict the 3D flow behaviors within thick and complex components or at the melt flow front (fountain flow), flow junctions, regions where the part thickness changes abruptly or separate melt fronts meet together (weld lines), and regions around special part features such as bosses, corners, and/or ribs. Therefore, to generate complementary and more detailed information related to the flow characteristics and stress distributions in thick-molded parts, a 3D simulation model should be used.[32–42] In these works, both finite-element and finite volume methods were used to predict velocities, pressure, temperature fields, position of the flow front, and residual stresses for the 3D mold filling, packing, and cooling of the IM process. Different 2D and 3D examples were suggested to validate the approaches presented and to illustrate its capabilities.[10,43–46]

Summarizing, it is generally recognized that computer aided engineering (CAE) simulations for IM can be outlined back to the

**TABLE II**
**Mathematical Modeling Research Applied to A Plasticating Stage of the Injection-Molding Process**

| Author | Experimental | Theory |
|---|---|---|
| Donovan et al.[48] | Yes | No |
| Donovan[49] | No | Yes |
| Donovan[50] | Yes | Yes |
| Lipshitz et al.[51] | No | Yes |
| Rauwendaal[52] | No | Yes |
| Potente et al.[53] | Yes | Yes |
| Yung and Xu[54] | No | Yes |
| Yung et al.[55] | No | Yes |
| Steller and Iwko[56] | No | Yes |
| Fernandes et al.[58] | Yes | Yes |

pioneering research done by Kamal and Kenig[8] on the developments of the theoretical principles and fluid mechanics models. To apply 1D flow representations to simulate polymer flow in typically complex mold cavities, a "branching flow" approach was proposed and implemented. However, the solution accuracy strongly depends on how the geometry is being branched, which is unknown a priori and requires intelligent judgment from the user. To overcome the deficiency of the branching flow approach, theoretical studies of 2D flows in a thin cavity based on Hele–Shaw flow formulation were conducted, for example, by Holm and Langtangen[23] and Chiang et al.[9] Based on the Hele–Shaw flow approximations, the continuity and momentum equations for polymer melt flow in IM can be simplified and merged into a single Poisson-like equation in terms of the pressure and fluidity.[19] Despite the successful applications and wide adoption, the Hele–Shaw flow formulation has its limitations owing to the inherent creeping flow and thin-wall assumptions. For example, it cannot accurately model the 3D flow behaviors within thick and complex components or at the melt fronts. As computer speeds increase and the cost of high-performance computers decreases, 3D simulation for polymer IM was anticipated to increase. In particular, Ilinca and Hétu[10] used a pressure-stabilized Petrov–Galerkin method to solve the Navier–Stokes equations in their 3D numerical model for the filling–packing–cooling stages of IM.

## Plasticating

Table II summarizes the mathematical modeling research for the plasticating stage of IM process and includes the main key features of the works developed by the authors.

The plasticating stage in the IM process consists of two main phases. First, a transient phase, in which the screw rotates and retracts to a predetermined distance by the required shot size; and a dwell phase in which the screw is at rest. Polymer melting occurs mainly by two phenomena, heat transfer by conduction from the heated barrel and due to heat generated by viscous dissipation during screw rotation.

A few studies have been conducted about the plasticating phase of the IM process, despite this phase being determinant for the melt quality (e.g., homogenization) and therefore for the quality of the molded part. The existing works are mainly based on models used for the plastication of a single screw extrusion process. The traditional example is the "cooling experiment" made by Maddock[47] to study polymer melting in an extruder.

Donovan et al.[48] studied the nature of the plasticating process in a reciprocating-screw IM machine using the "cooling experiment." Experimental studies were performed with different polymers. The results reveal that the screw recharge process is a transient plasticating extrusion process, which gradually approaches the equilibrium extrusion behavior as the screw rotates. In the same year, Donovan[49] proposed a theoretical model for the transient melting behavior in a reciprocating-screw IM machine. In combination with the steady-state extrusion model, a heuristic approach was used for the transient behavior. Later, Donovan[50] created a plasticating model serving as an analytical design methodology for the plasticating phase of the IM process, which combines a transient melting model with other models to calculate the melt temperature and the pressure profile during the plasticating process. Another approach was proposed by Lipshitz et al.,[51] where a dynamic extrusion melting model for the screw rotation period, a transient heat conduction model with a phase transition for the period when the screw stops, and a model for the drifting occurring at the beginning of melting during the injection cycle were combined. Two decades later, Rauwendaal[52] described quantitatively the solids conveying, melting, and melt conveying in a single screw extruder with both axial and rotational motion of the screw (the model can be considered an extension of the theory for nonreciprocating extruders). Then, Potente et al.[53] proposed a computer program for the simulation of the plastication process in IM. By examining the operating points of large industrial plants, the validity of the computations for different sizes of plasticating units was demonstrated.

Other models were proposed to describe the effects of screw rotation speed, barrel thickness, melt viscosity, barrel heat capacity, and injection velocity on a melting rate. With this aim, Yung and Xu[54] and Yung et al.[55] developed a transient melting model for a reciprocating extruder. More recently, Steller and Iwko[56] presented a model that includes three-zones-screw geometry, to-and-from screw motion characterized by controlled stroke and back pressure, and the existence of a static melting phase with controlled dwell times at back and front screw positions. The dynamic melting phase was characterized by a rotating screw with controlled angular velocity and was computed by applying the theory of dynamic extrusion formulated originally by Tadmor and colleagues,[6] but extended to the three zones screw with axial motion. For the static melting, the equations that result from the solution of the Neumann problem [57] were applied.

Fernandes et al.[58] presented a computational model for the description of polymer flow during the plasticating phase of the IM process. The polymer behavior is determined during the dynamic and static phases of the process. The model takes into account the backward movement of the screw, the presence of a nonreturn valve, and the conduction of heat during the idle times. The model is also used to study the influence of some important operative process parameters, such as screw speed, back-pressure, barrel temperatures, and injection chamber length.

In conclusion, plasticization in a reciprocating-screw IM machine has been generally assumed to be analogous to that of a single-screw extruder. However, there are distinctive differences between melting in reciprocating extruders and nonreciprocating extruders. First, there are three stages in injection cycle of reciprocating screws: feeding (screw rotating and moving backward), stop (no screw movement), and injection (screw moving forward without rotation). Second, there are axial movements of the screw in reciprocating extruders. Third, the time for each stage is limited. Consequently, the resultant balance equations are a lot more complicated in comparison with the similar ones for extrusion and they can be solved only numerically.

## Current Research in Modeling of Injection Molding

Recent research concerning IM modeling is presented, namely in what concerns, crystallization modeling of the solidification process, modeling for prediction of fiber orientation in the cases of short and long fiber composites, modeling for the prediction of the foaming process (such as microcellular foaming), and modeling for the prediction of flow instabilities by the inclusion of viscoelastic constitutive equations in the filling simulation.

Important research effort was devoted to predict the microstructure of injection-molded parts. Under the combined effect of cooling and flow-induced molecular deformation, the resulting material morphology upon crystallization combines both spherulitical- and fibrillar-oriented structures. In both cases, it is necessary to know if a realistic description of the final morphology is required. Pantani et al.[59] gave an extensive review about the models available and the experimental techniques used to predict and characterize the morphology of injection-molded parts. Custódio et al.[60] developed a model for flow-induced crystallization, which revises the underlying ideas of the model proposed by Zuidema et al.[61] and slightly simplifies the model proposed by Steenbakkers et al.,[62] where the morphology of the oriented crystalline phase is predicted. Hierzenberger et al.[63] investigated the application of specific optical tomography to the monitoring of polymer crystallization.[64] Guo et al.[65,66] presented a model to simulate the development of crystallinity and microstructure of IM polymer processing. A good description of the induction times of crystallization under different flow conditions was obtained. In addition, Guo and Narh[67] predicted the crystallinity development in IM using a stress-induced crystallization model that incorporates shear stress relaxation in the postfilling stage. The method used was able to replicate most of the experimental results in the literature using materials with different crystallization properties, such as polyethylene terephthalate (PET) and polypropilene (PP). Kim et al.[68,69] presented a study based on thermodynamic considerations of the entropy changes between unoriented and oriented melts using a nonlinear viscoelastic constitutive equation. Numerical results obtained for the skin-layer thickness matched well with the experiments performed. Kwon et al.[70] developed a model to predict anisotropic shrinkage in the injection-molded parts of semicrystalline polymers. The model based on the frozen-in orientation function and elastic recovery included a crystallinity-dependent viscosity in calculations, which increased the calculated volumetric and thickness shrinkage, leading to a better agreement with experiments. Recently, Zhou et al.[71] simulated the kinetics and morphological evolution of crystallization in the injection molding of isotactic polypropylene. The model considered flow-induced effect in filling, packing, and cooling of the IM process. Energy of dissipation was observed as the major parameter computing the crystallization kinetics.

Another field in development is the simulation of fiber suspensions in the injecting-molding process, which can feasibly improve the product quality. Generally, most simulations reported in the literature, for this case, use the continuous mechanics approach to solve the governing Navier–Stokes equations for the flow, in which the contribution of the fiber motion is included in the various stress terms. Currently, most commercial software use the Folgar–Tucker model[26] to predict the fiber orientation. Dou et al.[72] proposed a methodology for simulating the front evolution of fiber suspension flow using both the projection and level set methods. The dilute suspensions of short fibers in Newtonian fluids were considered. The objective of this study was to investigate the effect of fiber behavior on the front shape and velocity field as well as on fiber orientation. Wang et al.[28] modified the Folgar–Tucker model and proposed a reduced strain closure model, which is characterized by isotropic rotary diffusion. However, it cannot distinguish if the fiber alignment in the flow direction is smaller for the case of long-fiber materials or for the case of short-fiber materials. Thus, the isotropic diffusion was replaced by the anisotropic rotary diffusion, which is defined on the surface of the unit sphere traced by all orientations of the unit vector.[73] The first continuum model that accounts for the orientation evolution of semiflexible fibers was proposed by Strautins and Latz.[74] which is referred as the Bead–Rod model. This model, originally formulated for dilute suspensions, was extended to nondilute suspensions by Ortman.[75] Moreover, Ortman et al.[76] assessed the performance of both the Bead–Rod and Folgar–Tucker orientation models to predict fiber orientation within an injection-molded center-gated disk.

Another recent field of research in IM is the microcellular IM. In this case, the blend of atmospheric gas (usually nitrogen or carbon dioxide) in a supercritical state with a polymer melt inside the injection machine barrel is made to produce a single-phase polymer–gas solution. During the molding process, the gas emerges from the melt generating numerous microcells. The internal pressure that arises from the cell growth eliminates the need of packing pressure, while the parts are produced with excellent dimensional stability, shorter cycle time, and less material. Zheng et al.[77] applied the unit cell model[78] to this process. Han et al.[79] proposed a different simulation in which the model used simulates the development of gas cells in the melt during IM. Gramann et al.[80] described a numerical simulation to predict nucleation and cell growth through the IM process. This model was based on coupling the solidification–nucleation process, considering mold and melt temperature, injection pressure, and material properties. Osorio and Turng[81] developed a mathematical formulation solved by numerical simulations for nonisothermal cell growth during the postfilling stage of microcellular IM. Wang and Hu[82] introduced in the model the microcellular foam IM principle and some surface defects. Summaries of the technologies to improve surface quality, such as gas counter pressure, rapid heat cycle molding (RHCM), and film insulation, were also described. In summary, microcellular foam IM parts have many advantages, namely, saving material and energy, reducing cycle time, and parts with excellent dimensional stability. However, the low part's surface quality (swirl marks, silver streak, surface blistering, postblow, and large surface roughness) limits its application scope seriously. To overcome this, some technologies need to be applied as mentioned above.

Among other developments in IM, the last one that this paper describes is the flow instabilities which can cause nonuniform surface reflectivity. The cause of the instabilities is of an elastic nature. There can be significant difficulties with incorporating elasticity into simulations of the free surface flow because of the geometric singularity which exists at the contact point where the free surface intersects the mold. Bogaerds et al.[83] developed a time marching scheme that is able to handle the complex stability problem of viscoelastic flows with nonsteady computational domains, which result from perturbed free surfaces or fluid interfaces. It was shown that this method is able to accurately predict the stability characteristics of a number of single and multilayer shear flows of upper convected Maxwell (UCM) fluids. However, the UCM model cannot take into account material functions that are present in realistic flows of polymer melts. The choice of a constitutive relation is not a trivial task. Grillet et al.[84] and Bogaerds et al.[85] have shown that models such as the Phan–Thien–Tanner, the Giesekus, and the Pom–Pom model have completely different linear stability characteristics in simple shear flows. Bogaerds et al.[86] developed a model to examine the stability problem of the IM process using the extended Pom–Pom constitutive equations that allow for controlling the degree of strain hardening of the fluids without affecting the shear behavior considerably. They use a transient finite element algorithm that is able to efficiently handle time-dependent viscoelastic flow problems and includes a free surface description to take perturbations of the computational domain into account.

Other recent developments in IM could be described in this paper; however, the main scope of the paper is the studies addressed to the optimization of the process (section Injection-Molding Optimization) due to the importance of this issue to the industry.

In summary, the attempts to solve the nonlinear equations that govern the flow and heat transfer in IM can be described taking into account four main lines of research: (i) analytical solutions of an integrated form of the equation for the heat transport; (ii) analytical approximations for the unidirectional flow, combined with heat transport, based on local balance equations, numerical simulations of the flow, and the heat transport through simple cavities; (iii) models based on the local balance equations, and, at last, (iv) numerical analysis of the multidirectional flow and the heat transport through complex-shaped cavities. These lines are characterized by the level of approximation used to reach to a set of equations that can be solved either by applied mathematical rules (analytical solutions) or numerical techniques (FEM and/or FDM). The application of the numerical simulation approach will be discussed later on in this work.

# Injection-Molding Optimization

In this section, the goal is to present some of the optimization studies performed on the IM process using different approaches. The methodology followed on the description of the works that will be presented is based on their main features with the objective of helping the readers in finding a similar issue to their own problem. More detailed discussions can be found elsewhere.[7,87]

## TRIAL-AND-ERROR APPROACHES

Setting the process parameters of IM is a highly skilled job based on the operator's "know-how" and intuitive sense acquired through long-term experience rather than through a theoretical and analytical approach. The methods based on this approach to set up the process parameters for IM are called trial-and-error techniques. Attempts were also made in utilizing the process simulation results to determine a set of optimized molding parameters. Some tend to define a defect-free molding region or working window for IM (process window approach, PWA). Others applied DOE techniques to define an acceptable parameter setting. However, these approaches require a certain number of experimental trial runs over predefined molding programs. Some works that used the PWA or DOE techniques in IM are presented in the next two subsections.

### Process Window Approach

Table III summarizes the research on process window optimization approaches that have been done in the IM process, identifying the main aspect studied by the authors, namely, runner system, process conditions, gate location, cavity balancing, and cooling channel layout.

In the PWA, the aim is to determine a feasible process conditions zone for which the quality of molded parts is achieved. Several attempts that aim at determining a feasible process zone for IM were developed since the quality of molded parts is greatly affected by the conditions under which they are processed. This process zone is always referred as a process window. In the development of a process window for the IM process, various techniques were introduced. Some researchers developed a process window through a number of simulation trials with different combinations of molding parameters.

Pandelidis and Zou[88] studied the optimal injection gate locations with a quality function consisting of temperature differences, overpack, and frictional heating terms. A combination of simulated annealing and the hill-climbing method was then used to find the optimal node. The process variables were also optimized in a later publication by Pandelidis and Zou.[89] The methodology presented is independent of the simulation package, thermoplastic material, and mold geometry. Also, Seaman et al.[90] implemented an algorithm that runs in real time and provides immediate feedback to a process operator about the current operating point in a manufacturing process. In this model, the output measurements are product properties and inputs are process command signals and parameters of process controllers. This model was used within a multiobjective optimization algorithm: the quality controller. The optimization took place with respect to three quality characteristics: cycle time, flashing, and mold underfill (short shot). Later, Seaman et al.[91] explored the use of multiobjective optimization for the purpose of extending proportional integral derivative controllers to applications that require the optimization of multiple objectives. The model was applied to the plastication phase of IM with similar results in the cases of experimental and simulation data. Tan and Yuen[92] developed a computer system for computing an initial variables setting for an IM machine. The computations were made using fuzzy logic, but the results shown were considered acceptable only for simple geometries. Li et al.[93] used the feature warpage

**TABLE III**

**Process Window, Design of Experiments, and Numerical Simulation Research Applied to the Injection-Molding Process**

| Author | Runner System | Process Conditions | GateLocation | Cavity Balancing | Cooling Channel |
|---|---|---|---|---|---|
| | | Process window | | | |
| Pandelidis and Zou[88] | No | No | Yes | No | No |
| Pandelidis and Zou[89] | No | Yes | No | No | No |
| Seaman et al.[90,91] | No | No | No | No | No |
| Tan and Yuen[92] | No | No | No | No | No |
| Li et al.[93] | No | No | Yes | No | No |
| | | Design of experiments | | | |
| Chang and Faison[97] | No | Yes | No | No | No |
| Huang and Tai[98] | No | Yes | No | No | No |
| Viana and Cunha[99] | No | Yes | No | No | No |
| Park and Ahn[100] | Yes | Yes | Yes | No | Yes |
| Wu and Liang[101] | No | Yes | No | No | No |
| Feng et al.[102] | No | Yes | No | No | No |
| Tang et al.[103] | No | Yes | No | No | No |
| Dong et al.[104] | No | Yes | No | No | No |
| AlKaabneh et al.[105] | No | Yes | No | No | No |
| Chen and Kurniawan[106] | No | Yes | No | No | No |
| | | Numerical simulation | | | |
| Jong and Wang[107] | Yes | No | No | No | No |
| Lee and Kim[108] | No | Yes | No | No | No |
| Lee and Kim[109] | No | No | Yes | No | No |
| Seow and Lam[110] | No | No | No | Yes | No |
| Douglas et al.[111] | No | Yes | Yes | No | No |
| Tang et al.[112,113] | No | No | No | No | Yes |
| Park and Kwon[114–116] | No | No | No | No | Yes |
| Lam and Seow[117] | No | No | No | Yes | No |
| Lam and Jin[118] | No | No | Yes | No | No |
| Huang and Fadel[119] | No | No | No | No | Yes |
| Kumar et al.[120] | No | Yes | No | No | No |
| Courbebaisse and Garcia[121,122] | No | No | Yes | No | No |
| Shen et al.[123] | No | No | Yes | No | No |
| Mathey et al.[124] | No | No | No | No | Yes |
| Zhai et al.[125,127] | No | No | Yes | No | No |
| Zhai et al.[126] | Yes | No | No | No | No |
| Kim and Turng[128] | Yes | Yes | Yes | No | No |
| Park et al.[129] | No | Yes | No | No | No |
| Pirc et al.[130] | No | No | No | No | Yes |
| Pirc et al.[131] | No | No | No | No | Yes |
| Ozcelik and Sonat[132] | No | Yes | No | No | No |
| Agazzi et al.[133] | No | No | No | No | Yes |

to describe the warpage of injection-molded parts that are evaluated based on numerical simulation software. The method results in an optimal gate location, by which the part is satisfactory for the manufacturer.

## Design of Experiments

For an ad hoc experimental design approach, an experiment is traditionally conducted in which only one process parameter is varied at a time until the quality of the molded part is found satisfactory. Experiments are performed to investigate characteristics of a system. Responses or characteristics are the results evaluated from experiments. The responses should be analyzed appropriately according to the objectives of the experiments. The factors are the sources that affect the experiments. DOE determines the allocation and method of experiments to satisfy the objectives. Generally, DOE proceeds as illustrated in Fig. 3. Design using DOE can be regarded as a method using metamodels. The metamodel is the model of a model. A function can be approximated by some specific function values. The approximated function is a metamodel. In a design using DOE, well-developed DOE methods are employed to define the metamodel, and the design variable vector is obtained.

Table III summarizes the research on DOEs optimization that has been done in the IM process, identifying the main aspect studied by the authors.

DOE techniques are attempted to obtain the understanding of the IM process. Among the various DOE techniques, the Taguchi method was widely used to determine optimal process parameters for IM. Taguchi methods use a special design of orthogonal arrays to study the entire factor space with only a small number of experiments. The Taguchi method uses the signal-to-noise (S/N) ratio to convert the trial result data into a value for the

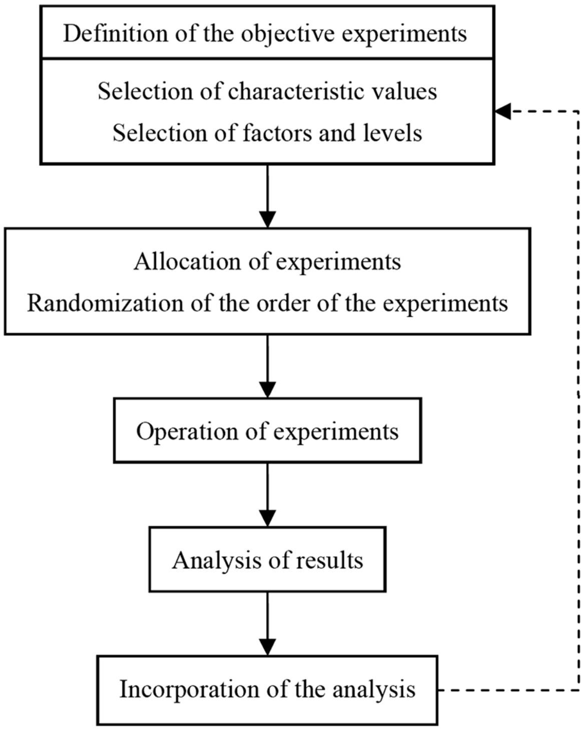


**FIGURE 3.** General DOE process (adapted from Park[94]).

characteristic in the optimum setting analysis. The S/N ratio reflects both the average and the variation of the quality characteristic. The standard S/N ratios generally used are as follows: nominal is best, lower the better, and higher the better. The optimal setting is the parameter combination that has the highest S/N ratio.

Another type of orthogonal array design that allows experimenters to study main effects and desired interaction effects in a minimum number of trials or experimental runs is called a fractional factorial design. These designs are generally represented in the form $2^{(k-p)}$, where $k$ is the number of factors and $(1/2)^p$ represents the fraction of the full factorial $2^k$. For example, $2^{(5-2)}$ is a 1/4th fraction of a $2^5$ full factorial experiment. This means that one may be able to study five factors at two levels in just eight experimental trials instead of 32 trials.

Analysis-of-variance (ANOVA) procedures separate the variation observable in a response variable into two basic components: variation due to assignable causes and to uncontrolled or random variation. Assignable causes refer to known or suspected sources of variation from variables that are controlled (experimental factors) or measured (covariates) during the conduct of an experiment. Random variation includes the effects of all other sources not controlled or measured during the experiment. Assignable causes corresponding to controllable factors can be either fixed or random effects. Details for this method can be found elsewhere.[95,96]

Chang and Faison[97] applied the Taguchi method to systematically investigate the effects of process conditions on the shrinkage of three plastics: high-density polyethylene, polystyrene (PS), and acrylonitrile butadiene styrene. The model predicted well the shrinkage behavior for the PS. In the same way, Huang and Tai[98] analyzed the effective factors of warpage in injection-molded items applying the Taguchi method. Filling time, mold temperature, gate dimensions, melt temperature, packing pressure, and packing time were identified as the effective factors that affect the warpage of injection-molded products. It is mentioned that regression analysis could also be employed to achieve an ideal set of processing conditions for minimizing warpage. Viana and Cunha[99] presented a study of the influence of the processing conditions on the impact behavior of the molded plates in two locations: in and away from a weld line. The simulation studies were based on a DOE approach where the operative variables are changed in three levels. The experimental results were analyzed by the ANOVA statistical tool. The simulation and experimental results were compared, and they concluded that all the considered processing variables are significant in determining the impact response of the moldings. Park and Ahn[100] developed an optimal design for IM using DOE procedure and FEM analysis. The methodology used considered two-way interactions between the factors and main effects of individual factors. The approach was applied successively for both mold design and process design of a high geometric complexity part. The methodology proposed by these authors can be used to successfully improve the productivity of products manufactured by IM processes. Wu and Liang[101] discussed the effect of various process parameters on the characteristics of the weld line of an injection-molded part. The Taguchi method was applied to study the individual contribution of process parameters on the tensile strength of the injected moldings. One of the main conclusions is that the mechanical strength obtained from a standard tensile test is not suitable for microinjection-molding processes. Feng et al.[102] examined multiple quality optimization of the IM for polyether-ether-ketone. This study applies the Taguchi method to cut down on the number of experiments (resources) and combines gray relational analysis to determine the optimal processing parameters for multiple quality characteristics. The efficiency of the system was assessed experimentally, thus the optimization model can be further applied to the planning of other related processes. Tang et al.[103] studied the effective processing variables to minimize the warpage defect by applying the experimental design of the Taguchi method. The processing variables used were melt temperature, filling time, packing pressure, and packing time. It was concluded that melt temperature was the processing variable that affects mostly the warpage, and filling time has influenced slightly the warpage. Dong et al.[104] presented a parametric sensitivity study of the replication of a cross-micro-channel by IM with a nickel mold insert. The effects of processing variables were quantitatively characterized by means of DOE based on the Taguchi method by varying mold and melt temperatures, injection velocity, and packing pressure. The sensitivity analysis showed that mold temperature is the most sensitive processing variable. AlKaabneh et al.[105] suggested a new scheme of analytic hierarchy process—Taguchi experimental design—FEM—ANOVA for evaluating optimal processing settings. The technique evidenced an agreement between process design and manufacturer preferences. Finally, Chen et al.[106] developed a systematic technique using the Taguchi method, ANOVA, BPNN, GA, and hybrid particle swarm optimization-genetic algorithms (PSO-GA) to optimize length and warpage of plastic parts. The proposed system was found to be very effective to increase quality of plastic parts and to stabilize variability in IM manufacturing industry.

These studies show that the DOE approach can reveal subtle interactions between IM process variables with a minimum number of test runs. In addition, to improve the success of DOE in optimizing the IM process, other methods can be incorporated in conjunction with DOE.

## APPLICATION OF NUMERICAL SIMULATION APPROACH

In the range of all the approaches used in the optimization of the IM process, the combination of mathematical models, numerical methods, and user interface programming was the basis of some typical numerical simulation models. In fact, through the simulation packages, a number of process parameters for IM can be obtained. Table III summarizes the research that has been carried out about the application of the numerical simulation approach in the optimization of the IM process, identifying the main aspect studied by the authors.

Numerical models were developed to simulate the IM process over its all phases: filling, postfilling, and cooling. Some of the research work that provides numerical simulation results for different aspects of the IM process is presented here.

Jong and Wang[107] described an automatic and optimal design of runner systems in IM based on flow simulations. Lee and Kim[108] investigated optimal gate locations using different evaluation criteria: warpage, weld and meld lines formation, and Izod impact toughness. Also, Lee and Kim[109] used a modified complex method to reduce warpage by optimizing the thicknesses of different surfaces, the warpage being further reduced by obtaining the optimum processing conditions. Seow and Lam[110] used a method that focuses on cavity balancing to reduce part distortion. By balancing the flow, overpacking and residual stresses are reduced. The method was implemented and adapted to a commercial software. Douglas et al.[111] are the first to design a mold by combining polymer process modeling, design sensitivity analysis, and numerical optimization. Tang et al.[112,113] used 2D transient FEM simulations coupled with a Powell's optimization scheme to optimize the cooling channel geometry to get uniform temperature distribution in the polymeric part. Park and Kwon[114] developed 2D and 3D stationary boundary element method (BEM) simulations in the injection molds coupled with 1D transient analytical computation in the polymer part (through the thickness). The heat transfer integral equation is differentiated to get the sensitivities of a cost function to the considered parameters.[115] The calculated sensitivities are then used to optimize the position of linear cooling channels for simple mold cavity shapes.[116] Later, Lam and Seow[117] presented a new method for flow path generation using the hill-climbing algorithm. It overcomes the difficulties of the straight flow path assumption by deriving the flow paths from the fill pattern of the cavity. Lam and Jin[118] put forward gate location optimization schemes based on the flow path concept. Standard deviation of flow path length, standard deviation of fill time, or range of fill times were employed as the objective function. Huang and Fadel[119] used 2D transient FEM simulations to optimize the use of mold materials according to part temperature uniformity or cycle time. Kumar et al.[120] presented the optimization of the molding conditions based on the simulation results for the nonisothermal mold-filling and an empirical relation predicting cooling time. The results obtained are in agreement with the analytical solutions, and the optimization study was able to provide a way to select which type of material was recommended to produce the part. Courbebaisse and Garcia[121] presented a new approach toward optimization of polymer IM by doing a shape analysis. Subsequently, they developed this methodology further and applied it to a single-gate location optimization of an L-shaped part.[122] Shen et al.[123] developed a general methodology that automatically predicts the optimal gate location based on IM simulations. The main success of the methodology lies in the fact that the algorithm can be effectively used for complicated parts without human interaction during the optimization. Mathey et al.[124] developed an optimization procedure to improve the cooling of injection molds. The BEM was used to solve the heat transfer equation, and the results obtained from the optimization showed that elliptical cooling channels produce better results; however, more complex shapes should be investigated. Zhai et al.[125] presented the two gate location optimization of one molding cavity by an efficient search method based on pressure gradient, and subsequently positioned weld lines to the desired locations by varying runner sizes for multigated parts.[126] Moreover, Zhai et al.[127] investigated optimal gate location with evaluation criteria of injection pressure at the end of filling. This work showed that the method is effective in repositioning the weld lines to the desired region of the part. Kim and Turng[128] developed 3D numerical simulations for the filling stage of the IM process using different algorithms and finite element types. In addition, the numerical algorithm was further applied to a molding analysis of a regular sized part with microsurface features. It was concluded that the numerical implementation worked well in predicting the melt front advancements. Park et al.[129] presented a flow simulation of thin wall IM using the rapid thermal response molding process. The proposed simulation provides reliable flow estimation by accounting for the effects of the thermal boundary condition. Pirc et al.[130] used the BEM and dual reciprocity method applied to unsteady heat transfer of injection molds. They presented a practical methodology to optimize both the position and the shape of the cooling channels in the IM process. Later, Pirc et al.[131] extended this methodology to optimize both the position and the shape of the cooling channels in 3D IM simulations. The results for the simple geometries were encouraging, but the efficiency of the optimization could be improved to handle more complex shape variations. Ozcelik and Sonat[132] studied the warpage values resulted by Moldflow simulations. The effect of each injection parameter on each material studied was examined by ANOVA analysis. Finally, Agazzi et al.[133] proposed a methodology for the optimal design of the cooling system. The expected result was an improvement of the part quality in terms of shrinkage and warpage. Particularly, the methodology does not require the a priori specification of the number of cooling channels.

The studies showed that CAE simulation tools can replace trial-and-error approaches and contribute to select material, design the part and mold, and define the processing conditions in a more effective way. Additionally, specific CAE software uses built in criteria and rules to suggest optimal processing conditions, which help in producing better parts and reducing the probability of defective parts. However, the application of the numerical simulation approach in the optimization of IM has also limitations. For instance, the assumptions and

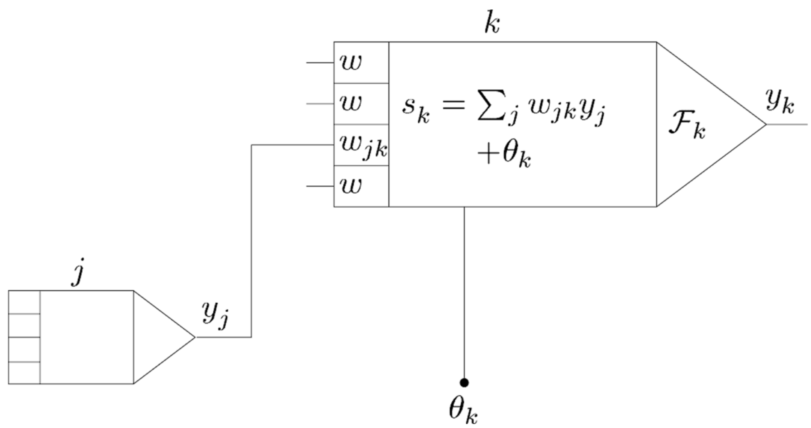


**FIGURE 4.** The basic components of an artificial neural network. The propagation rule used here is the standard weighted summation.

simplifications in the models used can generate differences between the real optimal scheme and the one obtained by the model. In addition, the users should not only be trained using these software but also in traditional molding and design.

## ARTIFICIAL INTELLIGENCE APPROACHES

Some artificial intelligence (AI) techniques were also used in the determination of the parameter setting for IM. ANN and EA are emerging as the new approaches in the determination of the process parameters for IM. A trained neural network system can quickly provide a set of molding parameters according to the results of the predicted quality of molded parts. However, the time required in the training and retraining a neural network could be very high. By using an EA approach, the system can locally optimize the molding parameters even without the knowledge about the process. Actually some studies combine different techniques to improve the optimization of the IM process. The following two subsections describe some works involving the AI techniques applied to IM.

### Artificial Neural Network

ANN is a technique that emulates the neural reasoning behavior of biological neural systems. An input layer accepts scaled input values and passes these values to a series of hidden layers and finally to an output layer. A neural network must be trained by methodically examining sets of input values and their associated outputs (supervised learning method). A trained neural network system has the ability to transform nonlinear mathematical modeling into a simplified black-box structure that is able of generalizing the set of previously learned instances.[134,135] An ANN consists of a pool of simple processing units that communicate by sending signals to each other over a large number of weighted connections. Figure 4 illustrates the basic components of an ANN. A set of major aspects of a parallel distributed model can be distinguished[136]: a set of processing units (neurons and cells); a state of activation $y_k$ for every unit, which is equivalent to the output of the unit; connections between the units (generally, each connection is defined by a weight $w_{jk}$ that determines the effect which the signal of unit $j$ has on unit $k$); a propagation rule, which determines the effective input $s_k$ of a unit from its external inputs; an activation function $F_k$, which determines the new level of activation based on the effective input $s_k(t)$ and the current activation $y_k(t)$ (i.e., the update); an external input (bias and offset) $\theta_k$ for each unit; a method for information gathering (the learning rule); and an environment within which the system must operate, providing input signals and, if necessary, error signals.

The neural network approach was also applied in building a metamodel for the control of the quality in IM. The inputs to the networks are the processing conditions, and the outputs are the quality characteristics. Several systems are developed based on the rule-based approach to provide solutions to molding problems. Some recently developed systems are based on the case-based approach to assist in setting the molding conditions.

Table IV summarizes the ANN optimization research that has been done in the IM process, identifying the main aspect studied by the authors.

Zhao and Gao[137] measured and analyzed the melt temperature at an injection nozzle exit. They proposed a systematic method for predicting the melt temperature profile by modeling the three consecutive stages: plastication, dwell, and injection. The modeling of the melt temperature during plastication was carried out by a set of ANN. The mathematical model for the dwell period was developed and solved by the FDM. The melt flow during the injection phase was modeled as a free boundary problem and solved by the FEM. Combining the modeling of these three stages, they predicted efficiently the melt temperature at the nozzle exit. Sadeghi[138] used an ANN method for offline planning and control of quality, using a CAE software. This work can be improved by integrating more process parameters as inputs to increase the capability of the model in the outputs side. Yarlagadda and Khong[139] developed a neural network to set up plastic IM process parameters. Additionally, Yarlagadda[140] presented an integrated neural network system for establishing process parameters such as injection pressure and injection time in the metal IM process. The implemented approach was validated with published and simulation results obtained using Moldflow. Huang and Lee[141] employed two intelligent neural network control strategies to adjust the injection velocity of the filling stage and control the nozzle pressure of the postfilling phase. It can be concluded that neural

**TABLE IV**

**Artificial Neural Networks and Evolutionary Algorithms Research Applied to the Injection-Molding Process**

| Authors | Runner System | Process Conditions | Gate Location | Cavity Balancing | Cooling Channel |
|---|---|---|---|---|---|
| | | Artificial neural networks | | | |
| Zhao and Gao[137] | No | Yes | No | No | No |
| Sadeghi[138] | No | Yes | No | No | No |
| Yarlagadda and Khong[139] | No | Yes | No | No | No |
| Yarlagadda[140] | No | Yes | No | No | No |
| Huang and Lee[141] | No | No | No | No | No |
| Chen et al.[142] | No | No | No | No | No |
| Liao et al.[143] | No | Yes | No | No | No |
| Castro et al.[144] | No | No | Yes | No | No |
| Chen et al.[145] | No | Yes | No | No | No |
| Shie[146] | No | Yes | No | No | No |
| Yin et al.[147] | No | Yes | No | No | No |
| Tsai and Luo[148] | No | Yes | No | No | No |
| | | Evolutionary algorithms | | | |
| Young[154] | No | No | Yes | No | No |
| Kim et al.[155] | No | Yes | No | No | No |
| Ye and Wang[156] | No | No | No | No | No |
| Kim et al.[157] | No | No | Yes | No | No |
| Lee et al.[158] | No | No | No | No | No |
| Mok et al.[159] | No | Yes | No | No | No |
| Turng and Peic[160] | No | Yes | No | No | No |
| Shi et al.[161] | No | No | No | No | No |
| Alam and Kamal[162–164] | Yes | No | No | No | No |
| Lam et al.[165] | No | Yes | No | No | Yes |
| Kurtaran et al.[166] | No | Yes | No | No | No |
| Ozcelik and Erzurumlu[168] | No | Yes | No | No | No |
| Ozcelik and Erzurumlu[169] | No | Yes | No | No | No |
| Lam et al.[170] | No | Yes | No | No | No |
| Qiao[171] | No | No | No | No | Yes |
| Shen et al.[172] | No | Yes | No | No | No |
| Chen et al.[173] | No | Yes | No | No | No |
| Fernandes et al.[174] | No | Yes | No | No | No |
| Ding et al.[176] | No | Yes | No | No | No |
| Yin et al.[177] | No | Yes | No | No | No |
| Wu et al.[175] | Yes | Yes | No | No | No |
| Lu et al.[179] | No | Yes | No | No | No |
| Fernandes et al.[180] | No | Yes | No | No | Yes |
| Natalini et al.[181] | No | Yes | No | No | No |
| Cheng et al.[182] | Yes | Yes | No | No | No |
| Kitayama and Natsume[184] | No | Yes | No | No | No |
| Tsai and Luo[185] | No | Yes | No | No | No |

controllers are better than an open-loop controller and that approach improved the performance of an industrial IM machine. Chen et al.[142] proposed an online soft-sensor scheme to predict online the melt-flow length via a recurrent neural network model using online measurable process variables as inputs. The methodology was capable to work using different mold shapes. However, some improvements are needed in the training mold set to apply the methodology in an industrial context. Liao et al.[143] used an approach so that the fluctuation of injected plastic part mass can be predicted using ANN quite accurately when key molding process variables are changed. The methodology used can be extended to develop a fully integrated adaptive control mechanism for the IM industry. Castro et al.[144] proposed a method utilizing CAE, statistical testing, ANNs, and data envelopment analysis to find the best compromises between multiple performance measures to prescribe the values for the controllable process variables in IM, while considering their variability explicitly. An assumption of the model consists of assuming a uniform distribution for the variability of the performace measures (PMs). The work can be extended by using different statistical distributions. Chen et al.[145] presented a self-organizing map plus a back propagation neural network (BPNN) model for the dynamic quality predictor. In addition, another BPNN model was employed for comparison. The numerical results revealed that the proposed dynamic model not only effectively increased the prediction performance of product quality but also it obtained more-reliable product quality in advance. Shie[146] proposed a hybrid method integrating a

Input: $N$ *(population size)*
$T$ *(maximum number of generations)*
$p_c$ *(crossover probability)*
$p_m$ *(mutation rate)*
Output: $\boldsymbol{A}$ *(nondominated set)*

Step 1: ***Initialization***: *Set* $\boldsymbol{P}_0 = \emptyset$ *and* $t = 0$. *For* $i = 1, \ldots, N$ *do*

*a)* *Choose* $\boldsymbol{i} \in \boldsymbol{I}$ *according to some probability distribution.*

*b)* *Set* $\boldsymbol{P}_0 = \boldsymbol{P}_0 + \{\boldsymbol{i}\}$.

Step 2: ***Fitness assignment***: *For each individual* $\boldsymbol{i} \in \boldsymbol{P}_t$ *determine the encoded decision vector* $\boldsymbol{x} = \boldsymbol{m}(\boldsymbol{i})$ *as well as the objective vector* $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ *and calculate the scalar fitness value* $F(\boldsymbol{i})$.

Step 3: ***Selection***: *Set* $\boldsymbol{P}' = \emptyset$. *For* $i = 1, \ldots, N$ *do*

*a)* *Select one individual* $\boldsymbol{i} \in \boldsymbol{P}_t$ *according to a given scheme and based on its fitness value* $F(\boldsymbol{i})$.

*b)* *Set* $\boldsymbol{P}' = \boldsymbol{P}' + \{\boldsymbol{i}\}$.

*The temporary population* $\boldsymbol{P}'$ *is called the* mating pool.

Step 4: ***Recombination***: *Set* $\boldsymbol{P}'' = \emptyset$. *For* $i = 1, \ldots, \frac{N}{2}$ *do*

*a)* *Choose two individuals* $\boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{P}'$ *and remove them from* $\boldsymbol{P}'$.

*b)* *Recombine* $\boldsymbol{i}$ *and* $\boldsymbol{j}$. *The resulting children are* $\boldsymbol{k}, \boldsymbol{l} \in \boldsymbol{I}$.

*c)* *Add* $\boldsymbol{k}, \boldsymbol{l}$ *to* $\boldsymbol{P}''$ *with probability* $p_c$. *Otherwise add* $\boldsymbol{i}, \boldsymbol{j}$ *to* $\boldsymbol{P}''$.

Step 5: ***Mutation***: *Set* $\boldsymbol{P}''' = \emptyset$. *For each individual* $\boldsymbol{i} \in \boldsymbol{P}''$ *do*

*a)* *Mutate* $\boldsymbol{i}$ *with mutation rate* $p_m$. *The resulting individual is* $\boldsymbol{j} \in \boldsymbol{I}$.

*b)* *Set* $\boldsymbol{P}''' = \boldsymbol{P}''' + \{\boldsymbol{j}\}$.

Step 6: ***Termination***: *Set* $\boldsymbol{P}_{t+1} = \boldsymbol{P}'''$ *and* $t = t + 1$. *If* $t \geq T$ *or another stopping criterion is satisfied then set* $\boldsymbol{A} = p(\boldsymbol{m}(\boldsymbol{P}_t))$ *else go to Step 2.*

**FIGURE 5.** General evolutionary algorithm (adapted from Zitzler[153]).

trained generalized regression neural network and a sequential quadratic programming method to determine an optimal variable setting of the IM process. The methodology proposed gives better prediction in the optimal run, than regression models based on the DOE and the ANOVA, moreover, eliminates the trial-and-error process increasing the efficiency to obtain an optimal solution. Yin et al.[147] carried out FEM simulations to study the effect of critical process parameters on the warpage of the plastics during PIM. Afterward, a mathematical function mapping the relationship between warpage value and process parameters was computed using a BPNN. The results confirmed that a better solution can be obtained to satisfy the manufacture demands. Finally, Tsai and Luo[148] used ANN and response surface method (RSM) to obtain the lens form accuracy prediction model. It was concluded that the ANN model is better than the RSM approach for optimizing the geometry of the molded parts.

Commonly, the ANN approach is applied to build a process model for quality control in IM. Also, it can be stated that networks created from processing variables can predict the part quality in a most efficient way than the ones based on process conditions. In addition, the ANN model should be trained with a set of data capable of describing the process in an adequately way; otherwise, the results will not have a physical meaning.

## Evolutionary Algorithms

EAs draw inspiration from the natural search and selection processes leading to the survival of the fittest individuals. The evolutionary computation techniques can be classified into four main categories: genetic algorithms (GAs),[149] evolution strategies,[150] evolutionary programming,[151] and genetic programming.[152] This classification is based on some details and historical development facts rather than on major functional differences. In fact, their biological basis is essentially the same. With the aid of the evolutionary strategies and GAs, the optimum parameter setting of IM can be obtained even without prior knowledge of the IM process. In general, an EA is characterized by three facts: (i) a set of solution candidates is maintained, which (ii) undergoes a selection process, and (iii) is manipulated by genetic operators, usually recombination and mutation. The basic structure of an EA is formalized in Fig. 5. By analogy to natural evolution, the solution candidates are called individuals and the set of solution candidates is called the population. Each individual represents a possible solution, that is, a decision vector, to the problem. The set of all possible vectors constitutes the individual space $I$. Therefore, the population is a set of vectors $i \in I$. In the selection process, which can be either stochastic or completely deterministic, low-quality individuals are removed from the population, while high-quality individuals are reproduced. The quality of an individual with respect to the optimization task is represented by a scalar value, the so-called fitness. Given an individual $i \in I$, a mapping function $m$ encapsulates the decoding algorithm to derive the decision vector $x = m(i)$ from $i$. Applying $f$ to $x$ yields the corresponding objective vector on the basis of which a fitness value is assigned to $i$. Recombination and mutation aim at generating new solutions within the search space by the variation of the existing ones. Based on the above concepts, natural evolution is simulated by an iterative computation process.

EA-based methods are also proposed to address the problem of processing conditions optimization. Also, recent works tend to develop a system for parameter setting of the IM process that combines and mutually strengthens the different AI technologies.

Table IV also summarizes the EAs optimization research that has been done in the IM process, identifying the main aspect studied by the authors, whereas Table V presents specific features concerning these works.

Young[154] developed a gate location optimization method for liquid composite molding based on the minimization of the mold-filling pressure, uneven-filling pattern, and temperature difference during mold filling. In this case, a GA was used, which in general is a better method to find the global optimum, that is, when compared with gradient search-based methods, especially when multiple variables and a large search space were present. Kim et al.[155] applied GA in the development of systems for the optimization of the processing parameters. The performance of the process was quantified using a weighted sum of the temperature difference during filling, “overpacking,” and frictionally overheating criteria. Ye[156] employed a GA in the optimization of the IM process. The application of this algorithm to determine the gate location and the optimal processing conditions in IM was demonstrated with examples. Kim et al.[157] developed a numerical simulation for isothermal mold filling in the resin transfer molding process by using control volume finite element method (CVFEM). This CVFEM, coupled interactively with GA, was used as an optimum gate locator to minimize the filling time in complex geometries. Lee et al.[158] developed a GA-based process planning system to optimize concurrently the selection of operations and sequences for the manufacture of mold bases. This methodology was able to assist users in the optimization of process plans for the whole mold base. Mok et al.[159] described a hybrid neural network and GA approach to determine a set of initial process parameters for IM. In this case, the time to obtain the initial process parameters can be greatly reduced and the solutions generated can produce good quality molded parts. However, experimental studies are needed to validate the effectiveness of the system. Turng and Peic[160] presented a system implementation and experimental verifications of an integrated CAE optimization tool that couples a process simulation program with optimization algorithms to determine the optimal design and process variables for IM. Shi et al.[161] presented a strategy that combines a neural network and a GA to solve the optimization model for the IM process. The results obtained shown that the optimization strategy is effective, but it could be improved with the incorporation of a rule-based knowledge system. Alam and Kamal[162] proposed a new approach for runner balancing, which accounts for the product costs by including runner system volume and cycle time. The optimization problem was solved with a multiobjective GA, nondominated sorted genetic algorithm. Later, Alam and Kamal[163] reformulated the runner-balancing problem. The objectives of this problem include the minimization of the runner-system volume, of the cycle time, and of the maximum difference in shrinkage among the parts. Finally, Alam and Kamal[164] presented a multiobjective GA coupled with a small sensitivity matrix to simulate the effects of process variation. The optimal solutions of balanced runner systems exhibited trade-offs between material costs and the percentage of rejected parts. Lam et al.[165] explored an approach to optimize both cooling channel design and processing condition setup simultaneously through an EA. The objective was to achieve the most uniform cavity surface temperature to assure product quality. The algorithm used shown its efficacy in the case of the optimization of cooling systems, where other traditional methods, as gradient-based optimization, can be trapped in a local optimum. Kurtaran et al.[166] determined the best IM processing conditions for a bus ceiling lamp to enable minimum warpage. In finding optimum values, the power of finite element software Moldflow, ANN, statistical DOEs, and GA were exploited. They concluded that, for the problem studied, the warpage is inversely proportional to mold temperature, melt temperature, packing pressure, and packing time but is directly proportional to cooling time. Additionally, the methodology can be applied to improve part design and find material properties for different injection-molding parts. Gaspar-Cunha and Viana[167] proposed the use of an automatic optimization methodology based on multi-objective evolutionary algorithms (MOEA) for the maximization of the mechanical properties of injected moldings. The methodology proposed is general enough to be used with any IM simulator code and is able of optimizing multiobjectives, producing results with physical meaning when applied to polymer-processing techniques. Ozcelik and Erzurumlu[168] studied the minimization of warpage on thin shell-like plastic parts by integrating finite element analysis (FEA), response surface methodology, and GA. Later, Ozcelik and Erzurumlu[169] introduced an efficient optimization methodology using ANN and a GA in minimizing the warpage of thin plastic parts manufactured by IM. Lam et al.[170] proposed a strategy for the use of a hybrid approach in IM conditions optimization. This hybrid approach employs the GA method, first to identify candidate local minimum regions and, then they are used as a starting point to initiate the gradient method while searching for the local minimum. Qiao[171] implemented a systematic computer-aided methodology for the optimization of cooling system design. He proposed a novel hybrid optimizer by combining the Davidon–Fletcher–Powell method with the simulated annealing. In this way, a global optimization method was developed with high efficiency. The application of the optimizer to cooling system design showed that great improvement on the temperature uniformity along the cavity surface is obtained. Shen et al.[172] proposed a combining ANN and GA method to optimize the IM process, improving the quality index

**TABLE V**
**Genetic Algorithms and Evolutionary Strategies Characterization in Injection-Molding Process**

| | Genetic Algorithms and Evolutionary Strategies Approach | | |
|---|---|---|---|
| Author | Fitness Evaluation | Optimization or Decision Variable(s) | Aim |
| Ye and Wang[156] | Single objective | Gate locations and process conditions | Maximum pressure at the end of filling and maximum curvature among all elements of the part |
| Kim et al.[157] | Single objective | Gate locations | Fill time |
| Lee et al.[158] | Single objective | Operation selections and sequences for the manufacture of mold bases | Overall machining Time |
| Shi et al.[161] | Single objective | Mold temperature, melt temperature, injection time, injection pressure | Maximum shear stress |
| Lam et al.[165] | Single objective | Co-ordinates of centers of cooling channels, sizes of cooling channels, flow rate, inlet temperature of the coolant in each cooling channel, packing time, cooling time, clamp open time | Standard deviation of cavity surface temperature |
| Kurtaran et al.[166] | Single objective | Mold temperature, melt temperature, packing pressure, packing pressure time, cooling time | Warpage |
| Ozcelik and Erzurumlu[168] | Single objective | Dimensional parameters | Warpage |
| Ozcelik and Erzurumlu[169] | Single objective | Mold temperature, melt temperature, packing pressure, packing pressure time, cooling time | Warpage |
| Qiao[171] | Single objective | Locations of cooling channels | Standard deviation of cavity surface temperature |
| Shen et al.[172] | Single objective | Melt temperature, mold temperature, injection time, packing time, holding pressure | Volumetric shrinkage variation |
| Wu et al.[175] | Single objective | Runner size, molding conditions and part geometry | Maximum part warpage |
| Tsai and Luo[185] | Single objective | Mold temperature, packing pressure, packing time, cooling time | Form accuracy |
| Turng and Peic[160] | Single objective/ Aggregation function | Melt temperature, injection speed, packing pressure, packing time, mold temperature, cooling time | Shrinkage in length, cycle time, volumetric shrinkage |
| Young[154] | Aggregation function | Gate locations | Mold-filling pressure, uneven-filling pattern and temperature difference during mold filling |
| Kim et al.[155] | Aggregation function | Mold temperature, melt temperature, filling time | Temperature difference, overpack, frictionally overheating |
| Mok et al.[159] | Aggregation function | Part design parameters, mold design parameters, process parameters of injection molding | Quality measures (maximum wall shear stress, maximum representative shear rate, maximum temperature difference, cycle time) |
| Lam et al.[170] | Aggregation function | Melt temperature, mold temperature, injection time | Maximum shear stress, maximum cooling time |
| Chen et al.[173] | Aggregation function | Melt temperature, injection velocity, injection pressure, VP switch, packing pressure, packing time | Product's length and weight |
| Yin et al.[177] | Aggregation function | Mold temperature, melt temperature, packing pressure, packing time, cooling time | Warpage defects and maximum clamp force |
| Natalini et al.[181] | Aggregation function | Packing pressure, packing time, melt temperature and cooling time | Warpage of a specified surface and two diameter sizes |
| Alam and Kamal[162–164] | Multiobjective | Runner diameters and lengths, processing conditions | Runner-system volume, cycle time, maximum difference in shrinkage |

(*Continued*)

**TABLE V**
**Continued**

| | Genetic Algorithms and Evolutionary Strategies Approach | | |
|---|---|---|---|
| Author | Fitness Evaluation | Optimization or Decision Variable(s) | Aim |
| Fernandes et al.[174] | Multiobjective | Melt and mold temperatures, injection time and holding pressure | Temperature difference on the molding at the end of filling, the maximum cavity pressure, the pressure work, the linear shrinkage and the cycle time |
| Ding et al.[176] | Multiobjective | Screw design parameters, material properties parameters and process parameters | Plasticizing capacity, injecting pressure and injecting power |
| Lu et al.[179] | Multiobjective | Mold temperature, injection velocity, packing time, packing pressure | Energy conception per cycle and part weight |
| Fernandes et al.[180] | Multiobjective | Cooling channels design, injection time, melt and mold temperatures and holding pressure | Warpage of the part measured by the deformation angle |
| Cheng et al.[182] | Multiobjective | Mold and process parameters | Volumetric shrinkage, total volume of the runner system and sum of injection time, packing time and cooling time |
| Kitayama and Natsume[184] | Multiobjective | Injection pressure, packing pressure, injection time, packing time, melt temperature, mold temperature | Volume shrinkage and clamping force |

of the volumetric shrinkage variation in the injected part. Chen et al.[173] presented a research that integrates Taguchi's DOE methods with BPNNs, GAs, and engineering optimization concepts, to optimize the initial process settings of the IM equipment. This approach demonstrated to be feasible and effective for process parameter optimization and can help the industry in achieving competitive advantages on quality and costs. Fernandes et al.[174] presented a study based on MOEA capable of defining the values of important process operating conditions yielding the best performance in terms of prescribed criteria (cycle time, powerwork, volumetric shrinkage). Wu et al.[175] presented a study to consider simultaneously the combination of different classes of design factors, as well as the weld line as a design constraint in which, both, their length and position are taken into account in the IM optimization. The study adopts an enhanced GA, called distributed multi-population genetic algorithm, which combines a preexisting optimization algorithm with commercial Moldflow software and a master slave distributed architecture. Ding et al.[176] proposed a new multiobjective optimization algorithm K means of joint support vector clustering-strength Pareto evolutionary algorithm (KSVC)-SPEA. Making use of the KSVC method and the SPEA, they effectively achieved the multiobjective optimization design of the overall performance of the IM machine. Yin et al.[177] established a multiobjective optimization model (DOE, BPNN, and GA) to optimize the process parameters during plastic IM on the basis of the finite element simulation software Moldflow. The proposed optimization method can adjust the process parameters accurately and effectively to satisfy the demand of real manufactured parts. Liao et al.[178] discussed a method of combining Gaussian process (GP) surrogate model and Gaussian genetic algorithm (GGA) to optimize the IM process. The GP surrogate model is constructed to map the complex nonlinear relationship between process conditions and quality indexes of IM parts. GGA combined with Gaussian mutation and hybrid GA are employed to evaluate the model to search the global optimal solutions. Lu et al.[179] developed a multiobjective parameter optimization framework for energy saving in the IM process. It combines an experimental design by Taguchi's method, a process analysis by ANOVA, a process modeling algorithm by ANN, and a multiobjective parameter optimization algorithm by the GA-based lexicographic method. The implementation of the proposed framework can reduce the energy consumption significantly in laboratory-scale tests, and the product quality can meet the predetermined requirements. Fernandes et al.[180] presented a study based on MOEA capable of defining the best position and layout of the cooling channels and/or setting the processing conditions for IM of polymeric materials. The algorithm proposed is independent of the simulation code used and produced results with physical meaning by taking some advantages of the use of Pareto curves. Natalini et al.[181] studied the effects of process parameters that heavily influence the warpage of a specified surface and two diameter sizes of a thermoplastic injected part. Pareto front was extracted using a GA, and the best set of parameters was determined after application of specific selection criteria and a weighted objective function. Numerical and experimental results were compared, and it was concluded that the methodology developed was capable to provide good results for warpage. Cheng et al.[182] proposed a novel methodology integrating variable complexity methods, constrained non-dominated sorted genetic algorithm, BPNNs, and Moldflow analyses to locate the Pareto optimal solutions to a constrained multiobjective optimization problem. The methodology was applied for the optimization of mold and process parameters for manufacturing mice with a compound-cavity mold. The proposed optimization model enabled the achievement of high product quality, low cost, and efficient production at the same time. Chen et al.[183] developed a two-phase optimization system to find out the optimal process parameters of polymer IM. In the first phase of the system the Taguchi method and ANOVA are employed to determine the initial process parameters, and in the second phase a BPNN and a GA search are employed for the optimal parameter combination. Experimental results were also obtained and satisfy the quality specification and also improve stability of the IM process. Kitayama and Natsume[184] described a multiobjective design optimization where volume shrinkage and clamping force were minimized simultaneously. A radial basis function network is adopted for the sequential approximate optimization, and the Pareto frontier is identified with a small number of simulation runs. Finally, Tsai and Luo[185] investigated the form accuracy of a plastic optical lens and proposed a hybrid DOE, ANN, and GA modeling approach for optimization of process parameter in IM.

From the above applications, it can be seen that several AI methods are frequently combined together in IM optimization to exploit their respective advantages. The success of the GA is much dependent on the design of a fitness function and on the setup of the operating range of process parameters. This way, to develop this kind of systems, experienced molding people are needed. In addition, the definition of the GA parameters (population size, crossover rate, and mutation rate) must be done accurately. Otherwise, the exploitation and exploration of the decision space will not be balanced. Finally, these algorithms have the disadvantage that they need several evaluations to achieve convergence. This means that if each function evaluation is time consuming, the method will be unreasonable. This way, as it could be noticed by the described literature, the GAs are hardly used alone, instead they are integrated with other methods.

## Concluding Remarks

This work presents a review of research performed in the mathematical description of the various stages of the IM process and in the techniques used to optimize this process. Concerning the former, a number of research works for filling, postfilling, and plasticating stages of IM have been described. For the last, several approaches related to trial-and-error, application of numerical simulation, and AI were presented.

The complete mathematical model of the IM process involves coupling mass, momentum, and energy balance equations, combined with constitutive laws for non-Newtonian fluids and state equations and appropriate boundary conditions. The dynamic plasticating model in IM consists of the following basic mechanisms: solids conveying, melting, and melt conveying. The existing studies are mainly based on the models used in the plastication of the single screw extrusion process. The model of the filling stage of the IM process implies the effect suffered when the melt polymer fills the mold cavity. Usually, the simulations of the filling process are based on two types of equations for the fluid flow and heat transfer, the Hele–Shaw and the Navier–Stokes equations. The postfilling stages of the IM include the packing phase, the time of solidification (cooling analysis), and the changes that the polymer suffers in the solidification, such as shrinkage and warpage. These phenomena are due to the factors that take part in the process of phase changing.

To effectively optimize the IM process, a compromise must be found between all the performance measures of interest.

One advantage of the numerical simulation is that it could provide useful information in part design, mold design, and process design of IM. A number of processing parameters can be obtained through the simulation packages. However, this approach involves cresting a finite element model and running a number of simulations, to obtain the acceptable molding parameters.

The methods based on the operator’s “know-how” and intuitive sense acquired through long-term experience to determine the process parameters for IM are called techniques from the trial-and-error approach (e.g., process window and DOEs). Normally, an acceptable result can be yielded based on the PWA. However, the development of a full set of process windows would be very difficult because of the large number of process parameters involved and the number of possible interactions among the parameters. Interactions among independent process variables, with a minimum number of test runs, could be found with the Taguchi parameter design. Furthermore, the analysis of the experimental results could help on the development of a model for the IM process, which allows the prediction of part characteristics as a function of process conditions. Such a metamodel can then be used to find the optimal parameters setting. Thus, the DOE approach requires a certain measure of expert knowledge of both statistics and process engineering in the experiment planning.

Some AI techniques were used in the optimization of IM. ANNs and EAs are emerging as the new approaches in the optimization of the IM process.

Neural networks generate their own rules by learning from certain training examples. In the ANN approach, a trained neural network system can quickly provide a set of molding parameters according to the results of the predicted quality of molded parts. However, the time required for training and retraining a neural network could be very high and, if the initial condition changes, a new model must be build up. In addition, neural networks cannot communicate how they work for the users so that it may be difficult to see what they are doing wrong. As a consequence, users cannot gain the understanding of the IM process through the neural networks.

By using an EA approach, the system can locally optimize the molding parameters even without the knowledge about the process. However, the effectiveness of the GA and evolutionary strategies is much dependent on the choice of the operating range of processing parameters and design of a fitness function that quantifies the quality of molded parts. Besides, the control parameters of GA and evolutionary strategies, such as population size, crossover, and mutation rates, must be properly defined. Actually, some studies combine different techniques to improve the optimization of the IM process.

Summarizing, the main limitations of the above-described methodologies are the following: First, most of the approaches proposed either consider one objective or use an aggregation function of the various objectives. This is not the better way to deal with the multiobjective nature of these optimization problems, since this implies the aggregation in a single function of very different measures with different meanings. Therefore, the optimization effect is restricted by the determination of some weighting factors. This type of methodology is not able to capture the trade-off between the objectives, which can bias the solution found. Second, the methodologies proposed are not totally integrated, that is, they do not consider the optimization and modeling processes as a whole process. Third, in some cases, the extension of the proposed methodology to more complicated geometries is not obvious. Also, most of the optimizations are either unconstrained or subject to the constraints on the design parameters and manufacturing operations, ignoring important features of IM like weld lines on the part, cooling channels diameter and locations, runner system geometry, mold materials, etc.

Future research activities would tend to develop an integrated system for production of high-quality and enhanced properties injection-molded products that can combine wisely experimental and simulated data and the different AI technologies, which should be able to exploit completely the multiphase, repetitive, and periodic nature of the process, to deal with complex and intrinsic nonlinear and time-changing features and with the inherent process variability. One open question is, therefore, to develop an effective and robust design procedure, potentially integrated in a holistic product life management environment, forming the basis for intelligent process setting, optimization, and control, to allow obtaining optimized processes and products, higher quality, higher productivity, and cost reduction, as currently main industrial endeavors.

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