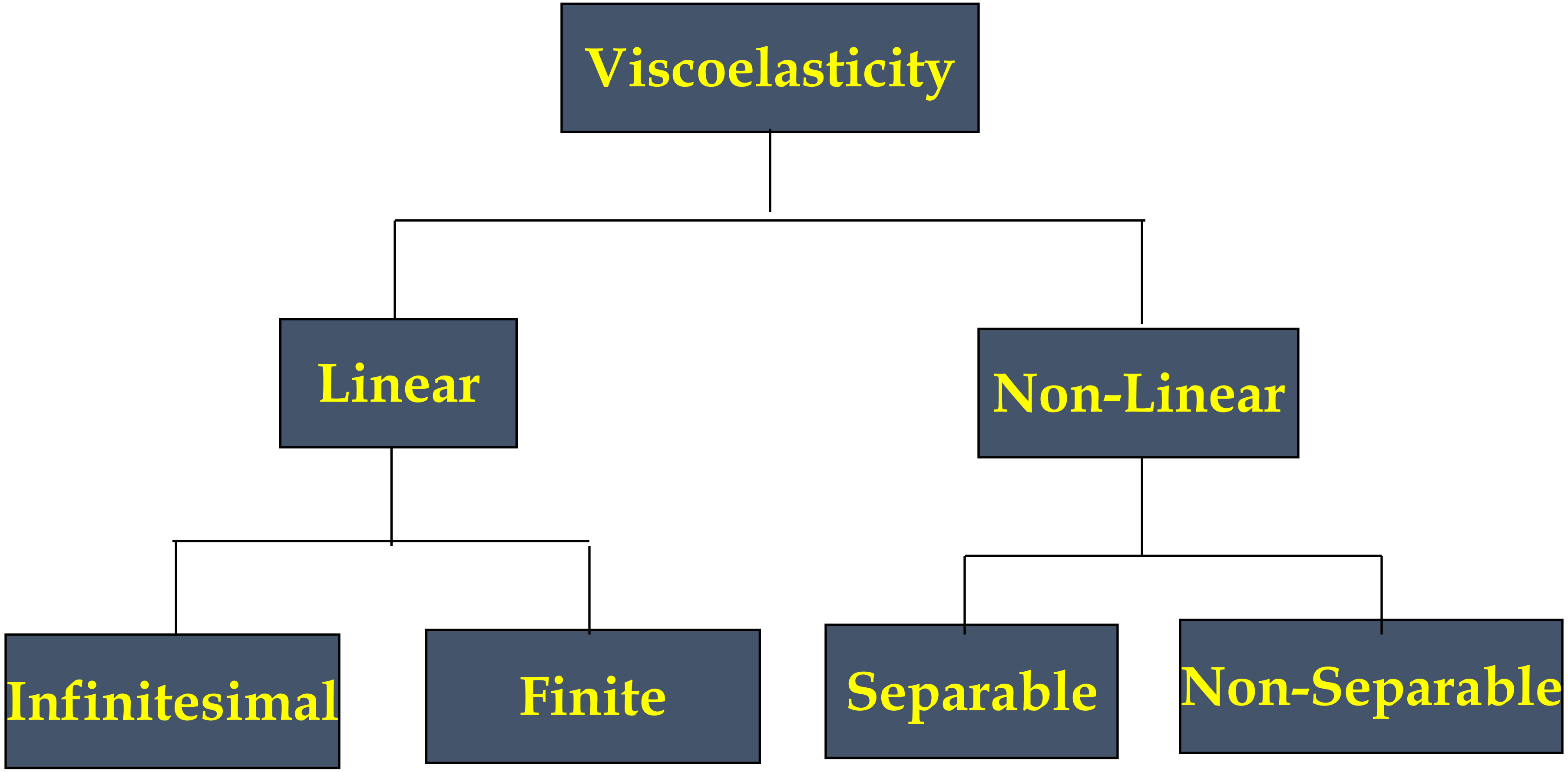
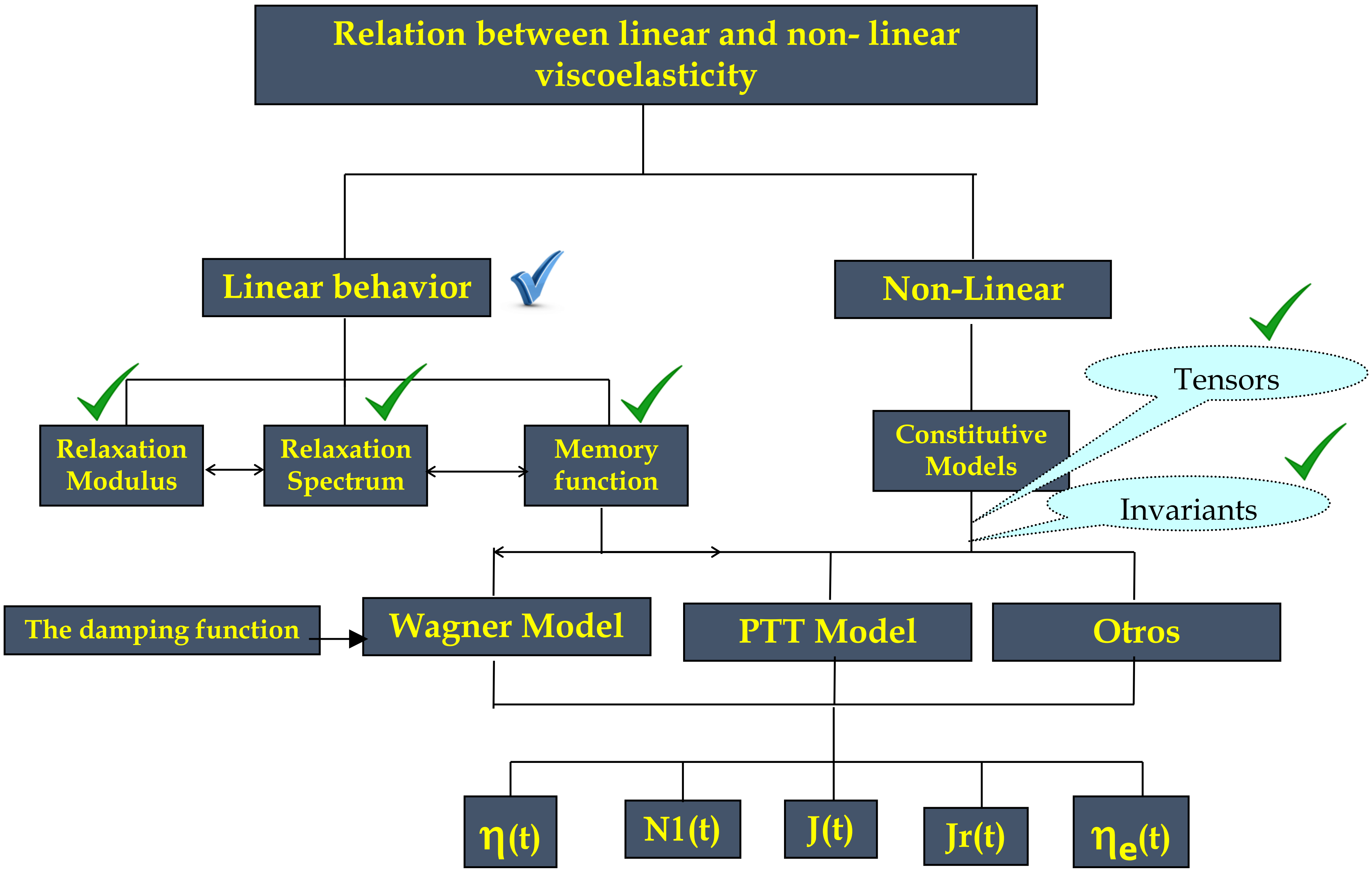
-Team presentations
-Stochastic Model
-Linear Viscoelasticity

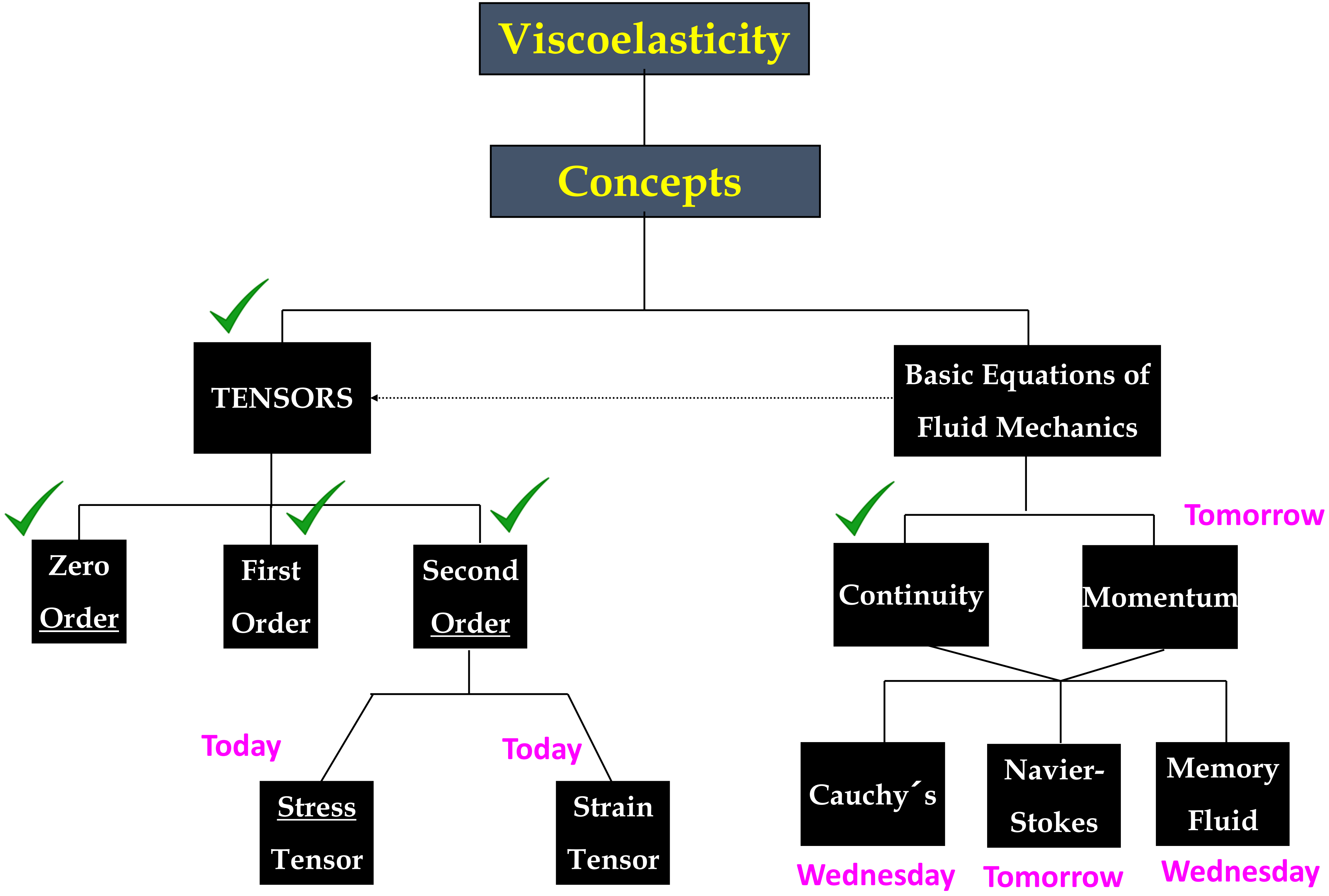
Agenda

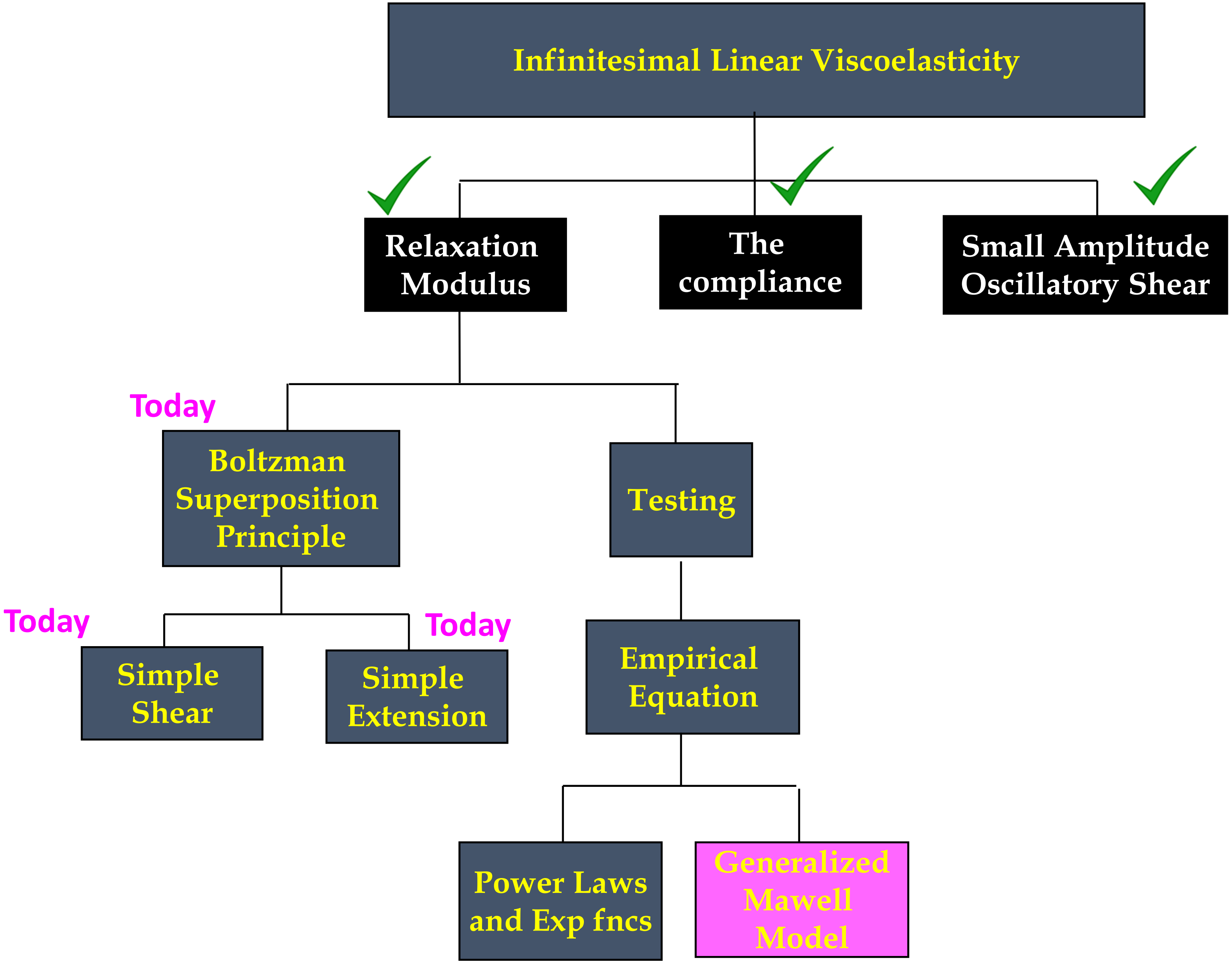
Conceptual Map



Conceptual Map




Viscoelasticity
Concepts
TENSORS
Basic Equations of Fluid Mechanics
Zero Order
First Order
Second Order
Stress Tensor
Strain Tensor
Today
Today
Continuity
Momentum
Tomorrow
Cauchy´s
Navier-Stokes
Memory Fluid
Wednesday
Tomorrow
Wednesday


Infinitesimal Linear Viscoelasticity
Relaxation Modulus
The compliance
Small Amplitude Oscillatory Shear
Today
Boltzman Superposition Principle
Testing
Today
Simple Shear
Today
Simple Extension
Empirical Equation
Power Laws and Exp fncs
Generalized Mawell Model

Infinitesimal Linear

VISCOELASTICITY

Linear Infinitesimal Viscoelasticity

- The simplest type of viscoelastic behavior
- "This is observed when the deformation is sufficiently mild that the molecules of a polymeric material are disturbed from their equilibrium configuration and entanglement state to a negligible extent" (Dealy, 1990)

Linear Infinitesimal Viscoelasticity

- The relaxation processes due to the Brownian motion are always acting to return the molecules to their equilibrium state (such as in the **Newtonian region** of the viscosity curve)

- Therefore the deformations are very small or very slow when dealing with linear infinitesimal viscoelasticity

Linear Infinitesimal Viscoelasticity

- "Since the deformations that occur in plastics processing are neither very small nor very slow, one might wonder how the theory of linear viscoelasticity (LVE) could be put to practical use." (Dealy, 1990)

- "**It is clearly of little use in processing modeling**. In fact, its principal utility is as a method for characterizing the molecules in their equilibrium state." (Dealy, 1990).

Linear Infinitesimal Viscoelasticity (LVE)

- Therefore, LVE is useful, mostly, for comparing resins.
- Sometimes it is possible to correlate LVE properties with the average molecular weight or MWD.

The relaxation modulus under LVE

- Remember that when a sudden deformation γ_o is applied at time t = 0, the stress changes and is measured as function of time

$$G(t, g_o) \text{ o } \frac{s(t)}{g_o}$$

- For very low strains such stress is strain independent

$$G(t) = \frac{\sigma(t)}{\gamma_o}$$

$$\sigma(t) = G(t)\gamma_o$$

The Boltzman Superposition Principle

- The shear stress at time t_1:

$$\sigma(t) = G(t - t_1)\delta\gamma(t_1)$$

If $t_1=0$, then …what ?

- For a sequence of small shear strains, the stress resulting from the strain introduced at time t_i is independent from any other strains introduced earlier.

$$\sigma(t) = G(t - t_1)\delta\gamma(t_1) + ... + G(t - t_i)\delta\gamma(t_i)$$

- Therefore, we can add or superpose the stress resulting from N imposed strains as follows:

$$\sigma(t) = \sum_{i=1}^{N} G(t - t_i)\delta\gamma(t_i) \quad (t > t_N)$$

The Boltzman Superposition Principle

- Therefore by making the increment very small:

$$\sigma(t) = \int_{-\infty}^{t} G(t - t')d\gamma(t')$$

And by dividing and multiplying by dt

$$\sigma(t) = \int_{-\infty}^{t} G(t - t')\,\dot{\gamma}(t')\,dt'$$

- But since, the experiment starts at t = 0 :

$$\sigma(t) = \int_{0}^{t} G(t - t')d\gamma(t')$$

- This equation applies only to shearing deformations, but it can be generalized by applying the fact that in LVE the relaxation process is not only independent of the strain but also of the type of kinematics.

Tensorial forms of the "Boltzman Superposition Principle"

- We can replace the shear strain by the strain tensor for infinitesimal strain, and replace the shear stress by the stress tensor to obtain:

$$\tau_{ij}(t) = \int_{-\infty}^{t} G(t - t')\, d\gamma_{ij}(t')$$

$$\tau_{ij}(t) = \int_{-\infty}^{t} G(t - t')\, \dot{\gamma}_{ij}(t')\, dt'$$

$$\tau_{ij}(t) = G(t)\gamma_{ij} \qquad \text{for } (t \geq 0)$$

Using the tensorial form of the BSP for **steady simple shear**

Using this shear rate tensor:

$$\dot{\gamma}_{ij}(t \geq 0) = \begin{bmatrix} 0 & \dot{\gamma} & 0 \\ \dot{\gamma} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the tensorial form of the BSP:

$$\tau_{ij}(t) = \int_{-\infty}^{t} G(t - t')\,\dot{\gamma}_{ij}(t')dt'$$

then: $\tau_{12}(t) = \tau_{21}(t) = \int_{-\infty}^{t} G(t - t')\,\dot{\gamma}_{ij}(t')dt'$

Since steady shear

$$\tau_{12} = \tau_{21} = \dot{\gamma} \int_{-\infty}^{t} G(t - t')\; dt'$$

since $\tau_{12}(t) = \sigma$

and since $\dot{\gamma}$ is very small

$$\frac{\sigma}{\dot{\gamma}} = \int_{-\infty}^{t} G(t - t')\; dt' = \eta_0$$

Using the tensorial form of the BSP for **steady simple extension**

- We can replace the shear strain by the strain tensor for infinitesimal strain, and replace the shear stress by the stress tensor to obtain:

$$\tau_{ij}(t) = \int_{-\infty}^{t} G(t-t')\,d\gamma_{ij}(t') \qquad \tau_{ij}(t) = \int_{-\infty}^{t} G(t-t')\,\dot{\gamma}_{ij}(t')\,dt'$$

- The tensorial form can be used to determine how the extensional flow properties are related to the shear relaxation properties:

$$\tau_{ij}(t) = G(t)\gamma_{ij} \qquad \text{for } (t \geq 0)$$

- Where

$$\gamma_{ij}(t \geq 0) = \begin{bmatrix} 2\varepsilon_0 & 0 & 0 \\ 0 & -\varepsilon_0 & 0 \\ 0 & 0 & -\varepsilon_0 \end{bmatrix}$$

Using the tensorial form of the BSP for steady simple extension

$$\tau_{ij}(t) = G(t)\begin{bmatrix} 2\varepsilon_0 & 0 & 0 \\ 0 & -\varepsilon_0 & 0 \\ 0 & 0 & -\varepsilon_0 \end{bmatrix} = \quad \tau_{ij}(t) = \begin{bmatrix} 2G(t)\varepsilon_0 & 0 & 0 \\ 0 & -G(t)\varepsilon_0 & 0 \\ 0 & 0 & -G(t)\varepsilon_0 \end{bmatrix} =$$

Since the stretching stress is

$$\sigma_E = \tau_{11} - \tau_{22} = 2G(t)\varepsilon_0 - (-G(t)\varepsilon_0)$$

$$\sigma_E = \tau_{11} - \tau_{22} = 3G(t)\varepsilon_0$$

- Then $$\frac{\sigma_E}{\varepsilon_0} = \boxed{} = 3G(t)$$

Then, the Infinitesimal Rheological Behavior considers

An infinitesimal strain tensor $\dot{\gamma}_{ij}$

A relaxation modulus, G(s), independent of strain (in other words, independent of the previous strain history).

where s=t-t´

The Boltzman Superposition Principle

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since $\tau_{12}(t) = \sigma$

and since $\dot{\gamma}$ is very small

$$\rightarrow \quad \frac{\sigma}{\dot{\gamma}} = \int_{-\infty}^{t} G(t - t')\; dt' = \eta_0$$

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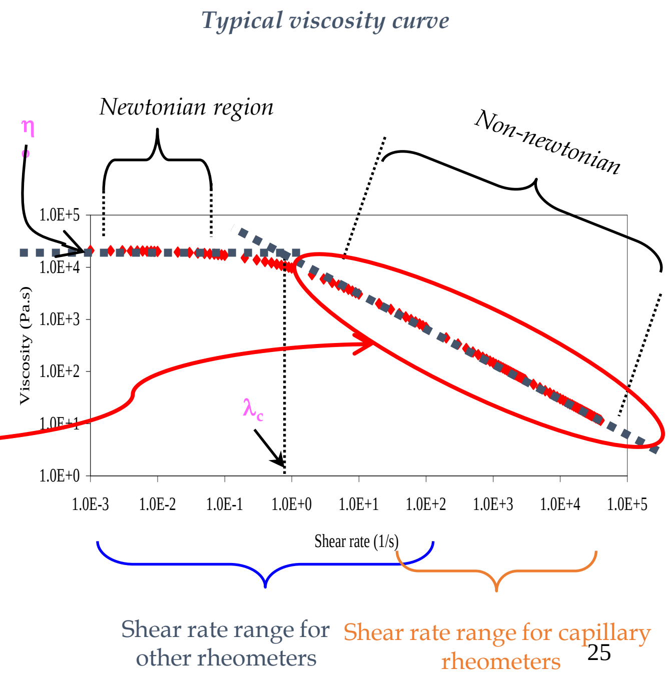
But...

It is well known that:

- there is a dependence of the viscosity on the shear rate and that



- there is a non-zero normal stress difference as the shear rate increases (Weissenberg effect)

$$\dot{\gamma}_{ij}(t \geq 0) = \begin{bmatrix} 0 & \dot{\gamma} & 0 \\ \dot{\gamma} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then…

a different theory or a modification of the infinitesimal theory is required:

The Finite Linear Viscoelasticity <u>or</u>

The Rubberlike liquid <u>or</u>

Lodge Network Theory