Problem 2.

You have the following information from a capillary rheometer for a HDPE resin (this is real data, so several runs were made at the same velocity due to possible variabilities in the instrument)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Velocity (in/s) | | Average load (lbf) for the L/D=40 die | | Average load (lbf) for the L/D=20 die | | Average load (lbf) for the L/D=10 die | |
| 0.061 | 170.3 | | 92.6 | | 55.9 | |
| 0.061 | 171.7 | | 92.6 | | 54.1 | |
| 0.307 | 420.5 | | 216.9 | | 121.3 | |
| 0.307 | 417.5 | | 214.4 | | 119.4 | |
| 0.614 | 579.1 | | 296.5 | | 164.6 | |
| 0.614 | 575.8 | | 294.1 | | 162.9 | |
| 3.07 | 848.4 | | 390.7 | | 261.6 | |
| 6.14 | 626.8 | | 360.6 | | 233.8 | |
| 6.14 | 623.6 | | 377.8 | | 264.3 | |
| 13.8 | 939.4 | | 552.4 | | 368.5 | |
| 18.4 | 1098.4 | | 640.3 | | 418.0 | |

Some of the dimensions are:

Barrel diameter 0.68 cm

Capillary diameter: 0.05 inches

Die Lengths: 2 in; 1 in; 0.5 in

The piston moves at constant velocity (inches/second)

The load is given pound force (lbf) and the force sensor is at the top of the piston.

You are asked to:

1. Get the real shear viscosity
2. The pressure at the entrance
3. The elongational viscosity

**A. Rephrase the problem indicating very clearly what you have been asked to do.**

We have to obtain the real shear viscosity , the pressure at the entrance Pe, and the elongational viscosity with the information given from a capillary rheometer for an HDEP resin.

**B. List all the data provided.**

* Velocities (constant)
* Average load
* Barrel diameter
* Die diameter
* Die lengths

**C. Make a list of the assumptions. justifying each of them.**

**D. Write down an algorithm for the solution you are proposing (no calculations are needed at this stage)**

**E. Solve the problem**

# Volumetric flow

The first thing we need to do is calculate the *volumetric flow* (Q) which is given by *the piston velocity* (V) times the *piston cross-section area* (A).

To obtain A we have the *barrel diameter* (0.68 cm) we convert it to inches dividing by 2.54 which gives us a diameter (d) of 0.2575 in.

Since we have different velocities, we will obtain different *volumetric flows*, we will enumerate the different sample trials (by velocity) in order to classify them easily.

Table 1. Volumetric flows from velocities given

|  |  |  |
| --- | --- | --- |
| **No.** | **V (in/s)** | **Q (in3/s)** |
| 1 | 0.061 | 0.003179 |
| 2 | 0.307 | 0.015997 |
| 3 | 0.614 | 0.031994 |
| 4 | 3.07 | 0.15997 |
| 5 | 6.14 | 0.31994 |
| 6 | 13.8 | 0.719083 |
| 7 | 18.4 | 0.958777 |

# Apparent shear rate and shear stress at the wall

Once we have the *volumetric flows* for each *velocity* used, we can obtain the *apparent shear rate* (Γ) given by:

Where the *die radius* R = 0.025 in.

Table 2. Apparent shear rates from volumetric flows.

|  |  |  |
| --- | --- | --- |
| **No.** | **Q (in3/s)** | **Γ (1/s)** |
| 1 | 0.0034 | 279.81 |
| 2 | 0.0173 | 1408.21 |
| 3 | 0.0346 | 2816.43 |
| 4 | 0.1728 | 14082.14 |
| 5 | 0.3456 | 28164.29 |
| 6 | 0.7768 | 63300.84 |
| 7 | 1.0358 | 84401.12 |

To avoid discrepancies, we obtained the following table by averaging the different *load forces* (F) for same *velocities*.

Table 3. Load forces applied for every die configuration

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **F (lbf) L/D=40** | **F (lbf) L/D=20** | **F (lbf) L/D=10** |
| 1 | 171.0 | 92.6 | 55.0 |
| 2 | 419.0 | 215.7 | 120.4 |
| 3 | 577.45 | 295.3 | 163.75 |
| 4 | 848.4 | 390.7 | 261.6 |
| 5 | 625.2 | 369.2 | 249.05 |
| 6 | 939.4 | 552.4 | 368.5 |
| 7 | 1098.4 | 640.3 | 418 |

To obtain the *load pressure* (P) we need the *piston area* (A) and the *load force* (F) applied.

Table 4. Pressures obtained from the forces and area of the piston

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **P (lbf/in2) L/D=40** | **P (lbf/in2) L/D=20** | **P (lbf/in2) L/D=10** |
| 1 | 3281.7 | 1777.1 | 1055.5 |
| 2 | 8041.1 | 4138.6 | 2309.7 |
| 3 | 11081.9 | 5667.1 | 3142.5 |
| 4 | 16281.7 | 7498.0 | 5020.4 |
| 5 | 11998.3 | 7085.4 | 4779.5 |
| 6 | 18028.1 | 10601.2 | 7071.9 |
| 7 | 21079.5 | 12288.1 | 8021.9 |

Once we have all the *apparent shear rates*, and *pressures* we need to obtain the *apparent shear stress at the wall* (τw)

So we will obtain a τw for every die configuration and for every P.

Table 5. Die configurations

|  |  |  |  |
| --- | --- | --- | --- |
| **L/D** | 40.0 | 20.0 | 10.0 |
| **R (in)** | 0.025 | 0.025 | 0.025 |
| **L (in)** | 2 | 1 | 0.5 |

Table 6. Shear stress at the wall for the different die configurations and pressures

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **τw (lbf/in2) L/D=40** | **τw (lbf/in2) L/D=20** | **τw (lbf/in2) L/D=10** |
| 1 | 18.99 | 20.56 | 24.43 |
| 2 | 46.52 | 47.89 | 53.45 |
| 3 | 64.11 | 65.57 | 72.72 |
| 4 | 94.20 | 86.76 | 116.18 |
| 5 | 69.42 | 81.98 | 110.61 |
| 6 | 104.30 | 122.67 | 163.66 |
| 7 | 121.96 | 142.18 | 185.64 |

# Rabinowitch correction

The treatment above is for a Newtonian fluid, but the shear rates at which these measures were made in the capillary rheometer follow a non-Newtonian behavior, so we have to perform a correction. For this correction we have to plot the log(Γ) vs log(τw) to obtain the *slope* (b) and follow the Rabinowitch correction equation to obtain the *real shear rate at the wall* ().

Table 7. Rabinowitch correction log values for slope calculation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **No.** | **log(Γ)** | **log(τw) L/D=40** | **log(τw) L/D=20** | **log(τw) L/D=10** |
| 1 | 2.447 | 1.278 | 1.313 | 1.388 |
| 2 | 3.149 | 1.668 | 1.680 | 1.728 |
| 3 | 3.450 | 1.807 | 1.817 | 1.862 |
| 4 | 4.149 | 1.974 | 1.938 | 2.065 |
| 5 | 4.450 | 1.841 | 1.914 | 2.044 |
| 6 | 4.801 | 2.018 | 2.089 | 2.214 |
| 7 | 4.926 | 2.086 | 2.153 | 2.269 |

In the next figure, a plot of apparent shear rate vs shear stress at the wall is shown, where we obtain the slopes to apply the Rabinowitch correction.

Once we obtained the *slope* b for each die configuration, we can proceed to substitute values in the Rabinowitch equation stated above and obtain the *real shear rate* () for each die configuration.

Table 8. Slopes obtained for each die configuration

|  |  |
| --- | --- |
| **L/D** | **b** |
| 40 | 0.288 |
| 20 | 0.318 |
| 10 | 0.369 |

Table 9. Real shear rates obtained through the Rabinowitch equation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **No.** | **Γ (1/s)** | **(1/s) L/D=40** | **(1/s) L/D=20** | **(1/s) L/D=10** |
| 1 | 279.81 | 230.00 | 232.10 | 235.67 |
| 2 | 1408.21 | 1157.55 | 1168.11 | 1186.07 |
| 3 | 2816.43 | 2315.10 | 2336.23 | 2372.14 |
| 4 | 14082.14 | 11575.52 | 11681.14 | 11860.68 |
| 5 | 28164.29 | 23151.04 | 23362.27 | 23721.37 |
| 6 | 63300.84 | 52033.29 | 52508.04 | 53315.13 |
| 7 | 84401.12 | 69377.72 | 70010.73 | 71086.84 |

# Bagley correction

Now that we have the shear rate correction, we must do the correction for the pressure. The instrument gives us the *total pressure* P which is given by the *viscous pressure* and the *entrance pressure* which is the one we need to calculate. To obtain this *entrance pressure* we need to extrapolate L/D in to find a theoretical pressure when L/D is 0.

For this correction we ignore the Rabinowitch correction in order to maintain the same shear rate, using Γ, for every die configuration L/D.

Table 10. Pressures obtained for different shear rates and configurations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **No.** | **Γ (1/s)** | **P (lbf/in2) L/D=40** | **P (lbf/in2) L/D=40** | **P (lbf/in2) L/D=40** |
| 1 | 279.81 | 3037.8 | 1645.0 | 977.1 |
| 2 | 1408.21 | 7443.4 | 3831.0 | 2138.0 |
| 3 | 2816.43 | 10258.3 | 5245.9 | 2909.0 |
| 4 | 14082.14 | 15071.6 | 6940.7 | 4647.3 |
| 5 | 28164.29 | 11106.5 | 6558.8 | 4424.3 |
| 6 | 63300.84 | 16688.2 | 9813.3 | 6546.3 |
| 7 | 84401.12 | 19512.8 | 11374.8 | 7425.7 |

Then we plot the data so that we can do a linear regression and find the *pressure* at L/D=0, which will be given at the y intercept. In the figure we can observe the linear fit for different velocities to determine the *entrance pressure* Pe.

From the figure we can obtain the y intercepts which are considered the *entrance pressure* Pe at every *shear rate* evaluated.

Table 11. Entrance pressures by shear rates

|  |  |  |
| --- | --- | --- |
| **No.** | **Γ (1/s)** | **Pe (lbf/in2)** |
| 1 | 279.808 | 280.68 |
| 2 | 1408.214 | 331.76 |
| 3 | 2816.429 | 402.82 |
| 4 | 14082.14 | 581.8 |
| 5 | 28164.29 | 2150.4 |
| 6 | 63300.84 | 3108.8 |
| 7 | 84401.12 | 3356.7 |

And by subtracting the entrance pressure to the total pressure we obtain the viscous pressure

Table 12. Viscous pressures obtained by subtracting P-Pe

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **Pv (lbf/in2) L/D=40** | **Pv (lbf/in2) L/D=20** | **Pv (lbf/in2) L/D=10** |
| 1 | 2757.1 | 1364.3 | 696.4 |
| 2 | 7111.7 | 3499.2 | 1806.2 |
| 3 | 9855.4 | 4843.1 | 2506.2 |
| 4 | 14489.8 | 6358.9 | 4065.5 |
| 5 | 8956.1 | 4408.4 | 2273.9 |
| 6 | 13579.4 | 6704.5 | 3437.5 |
| 7 | 16156.1 | 8018.1 | 4069.0 |

And with the *viscous pressure* we can obtain the *real sheer stress* τ.

Table 13. Real sheer stress from Pv

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **τ (lbf/in2) L/D=40** | **τ (lbf/in2) L/D=20** | **τ(lbf/in2) L/D=10** |
| 1 | 17.23 | 17.05 | 17.41 |
| 2 | 44.45 | 43.74 | 45.16 |
| 3 | 61.60 | 60.54 | 62.65 |
| 4 | 90.56 | 79.49 | 101.64 |
| 5 | 55.98 | 55.10 | 56.85 |
| 6 | 84.87 | 83.81 | 85.94 |
| 7 | 100.98 | 100.23 | 101.72 |

In the following figure we plotted the apparent shear stress (left) and the corrected shear stress vs apparent shear rate to show the changes.

# Shear viscosity

For obtaining the *real* *shear viscosity* ηs we need to divide the *real* *shear stress* over the *real shear rate.*

The real shear viscosity can be found in Table 12.

Table 12. Real shear viscosities

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **(lbf s/in2) L/D=40** | **(lbf s/in2) L/D=20** | **(lbf s/in2) L/D=10** |
| 1 | 0.07492 | 0.073478 | 0.073873 |
| 2 | 0.038398 | 0.037445 | 0.038072 |
| 3 | 0.026606 | 0.025913 | 0.026412 |
| 4 | 0.007824 | 0.006805 | 0.008569 |
| 5 | 0.002418 | 0.002359 | 0.002396 |
| 6 | 0.001631 | 0.001596 | 0.001612 |
| 7 | 0.001455 | 0.001432 | 0.001431 |

Converting to Pa·s we obtain the following viscosities:

Table 12. Real shear viscosities in Pa s

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **(Pa·s) L/D=40** | **(Pa·s) L/D=20** | **(Pa·s) L/D=10** |
| 1 | 52.67415 | 51.65983 | 51.93788 |
| 2 | 26.99661 | 26.32649 | 26.76708 |
| 3 | 18.70611 | 18.2187 | 18.56979 |
| 4 | 5.500478 | 4.784144 | 6.024736 |
| 5 | 1.699917 | 1.658322 | 1.684893 |
| 6 | 1.146774 | 1.122136 | 1.133267 |
| 7 | 1.023281 | 1.0065 | 1.006082 |

Figure x. Shear viscosity plot

We compared our results with the Carreau-Yasuda model to be certain of our calculations, in the following figure (la tuya) we plotted the fitted model alongside our findings, and as can be seen they are very similar, thus our calculations are a close representation of the model.

# Elongational viscosity

The elongational viscosity is defined as the ratio of elongational stress to elongational strain rate. It measures the resistance of a fluid against elongational deformation [1].

To measure the elongational viscosity , first we need to calculate the Elongational rate which is given by,

Where is the *apparent* *shear viscosity*, the *apparent shear rate*, Pe the entrance pressure, and *n* the *power law index* [1].

We know that using Carreau-Yasuda model for this resin we obtain *n*=0.48 so we obtain the following values

Table 13. Elongational rates

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **(in/s) L/D=40** | **(in/s) L/D=20** | **(in/s) L/D=10** |
| 1 | 18.82721 | 18.46466 | 18.56405 |
| 2 | 206.777 | 201.6443 | 205.0189 |
| 3 | 472.008 | 459.7094 | 468.5683 |
| 4 | 2402.391 | 2089.525 | 2631.366 |
| 5 | 803.4992 | 783.8387 | 796.398 |
| 6 | 1894.015 | 1853.324 | 1871.707 |
| 7 | 2782.648 | 2737.014 | 2735.879 |

And then we proceed to calculate with the following equation,

Which results can be found in the following Table 14.

Table 14. Elongational viscosities

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **(lbf s/in2)**  **L/D=40** | **(lbf s/in2) L/D=40** | **(lbf s/in2) L/D=40** |
| 1 | 7.882716 | 8.03749 | 7.99446 |
| 2 | 0.848344 | 0.869939 | 0.855619 |
| 3 | 0.451245 | 0.463317 | 0.454557 |
| 4 | 0.12805 | 0.147223 | 0.116908 |
| 5 | 1.41509 | 1.450584 | 1.427708 |
| 6 | 0.86788 | 0.886935 | 0.878224 |
| 7 | 0.63783 | 0.648464 | 0.648733 |

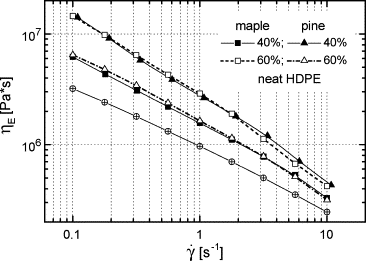
Converting to Pa·s we obtain the following viscosities:

Table 14. Elongational viscosities in Pa s

|  |  |  |  |
| --- | --- | --- | --- |
| **No.** | **(Pa·s) L/D=40** | **(Pa·s) L/D=20** | **(Pa·s) L/D=10** |
| 1 | 5542.088 | 5650.904 | 5620.652 |
| 2 | 596.4441 | 611.6262 | 601.5586 |
| 3 | 317.2558 | 325.7433 | 319.5848 |
| 4 | 90.02806 | 103.508 | 82.19404 |
| 5 | 994.9052 | 1019.86 | 1003.776 |
| 6 | 610.179 | 623.5761 | 617.4514 |
| 7 | 448.4378 | 455.9145 | 456.1037 |

In the following plot, we can find the behavior of the elongational viscosity.

What an elongation viscosity graph should look like



**F. Ask yourself if the result is reasonable and, if needed check in the web for technical papers to support your answer.**

We believe our results of the shear viscosity are reasonable because we compared with the careau-yasuda model. But our results form the elongation viscosity show some inconsistencies with other elongation viscosity graphs.

**References**

[1] D. Sarkar and M. Gupta, “Estimation of Elongational Viscosity,” *Asme*, vol. 90, pp. 309–318, 2000.