NUMBERS

1

Numbers

1.1 Natural Numbers

Natural numbers or counting numbers are the most basic type of numbers which you learned for the first time as a toddler. They start from 1 and go to infinity, i.e. $1, 2, 3, 4, 5, 6$, and so on. They are also called positive integers. In a set form, they can be written as:

$$\{1, 2, 3, \dots\}$$

The set of natural numbers is denoted by $\mathbb{N}$

Definition 1.1.1 (Prime and Composite Numbers)**.** A prime number is a natural number which has exactly two factors i.e. '1' and number itself. A composite number has more than two factors, which means apart from getting divided by number 1 and itself, it can also be divided by at least one integer.

1.2 Whole Numbers

Whole numbers are the set of natural numbers, including zero. This means they start from 0 and go up to $1, 2, 3$, and so on, i.e.

$$\{0, 1, 2, 3, \dots\}$$

The set of natural numbers is denoted by $\mathbb{W}$

1.3 Integers

Integers are the set of all whole numbers and the negatives of natural numbers. They contain all the numbers which lie between negative infinity and positive infinity. They can be positive, zero, or negative, but cannot be written in decimal or fraction. Integers can be written in set form as

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

We can say that all whole numbers and natural numbers are integers, but not all integers are natural numbers or whole numbers. The set of natural numbers is denoted by $\mathbb{Z}$.

Definition 1.3.1 (Even and Odd Integers)**.** An integer n is even if and only if, n equals twice some integer and odd if n equals twice some integer plus 1(or minus 1). Symbolically,
n is even iff $n = 2k$ for some integer k
n is odd iff $n = 2t + 1$ or equivalently $n = 2t - 1$ for some integer t

1.4 Rational Numbers

Rational numbers are the ones which can be written in fraction form $\frac{p}{q}$, where p and q are integers. Here, the numerator p can be any integer (positive or negative), but the denominator q can never be 0, as the fraction is undefined then. Rational numbers are represented by the symbol $\mathbb{Q}$

1.5 Irrational Numbers

Irrational numbers are those which cannot be written in fraction form, i.e., they cannot be written as the ratio of the two integers. A few examples of irrational numbers are $\sqrt{2}, \sqrt{5}, \pi$. Irrational numbers are represented by the symbol $\overline{\mathbb{Q}}$.

1.6 Real Numbers

Real numbers are the set of all rational and irrational numbers. This includes all the numbers which can be written in the decimal form. All integers are real numbers, but not all real numbers are integers. Real numbers include all the integers, whole numbers, fractions, repeating decimals, terminating decimals, and so on. Real numbers are represented by the symbol $\mathbb{R}$.

1.7 Arithmetic Properties

i. Commutative: $a + b = b + a$, $ab = ba$

ii. Associative: $a + (b + c) = (a + b) + c$, $a(bc) = (ab)c$

iii. Distributive: $a(b + c) = ab + ac$

iv. Identity element: $a + 0 = a$, $a \times 1 = a$

v. Inverse: $a + (-a) = 0, a \times \frac{1}{a} = 1, a \neq 0$

1.8 Intervals

An open interval (a, b) is the set of all real numbers x such that $a < x < b$. A closed interval $[a, b]$ is the set of all real numbers x such that $a \leq x \leq b$. (In other words, an open interval is the set of all real numbers between a and b excluding a and b, and a closed interval is the set of all real numbers between a and b including a and b.) We might also refer to half-open intervals $(a, b]$, where $a < x \leq b$, and $[a, b)$, where $a \leq x < b$. An infinite interval is an interval extending to either $-\infty$ or $+\infty$: note that because ∞ (infinity) is not a real number, however, we always use "open" notation for that portion of the interval. Thus, $(-\infty, a]$ is all x such that $-\infty < x \leq a$, and (b, ∞) is all x such that $b < x < \infty$.