# SETS

# 1

# Set Theory

## 1.1 Sets

**Definition 1.1.1** (A set)**.** A set is a collection of well defined objects.

The objects in a set are called the elements of the set. We use uppercase alphabetical letters to denote sets. A set can be described by listing all of its elements. For example,

$$X = \{1, 3, 5, 7, 9\}$$

is read as '$X$ is the set whose elements are $1, 3, 5, 7$ and $9$'. The five elements of the set are separated by commas, and the list is enclosed between curly brackets. If we wish to denote elements of a set using alphabetical letters, then such elements are denoted using lower case letters. A set can also be described by writing a description of its elements between curly brackets. For this purpose we use the set builder notation:

$$X = \{x|P(x)\}$$

where $P(x)$ represents the property that describes the elements of set $X$.The symbol $|$ stands for 'such that'. We can also use full colon for the same purpose. Thus the set $X$ above can also be written as

$$X = \{x|x \text{ is an odd whole number less than } 10\}$$

read as '$X$ is the set of odd whole numbers less than 10'. Note that a set must be well defined. This means that our description of the elements of a set is clear and unambiguous. For example, tall people is not a set, because people tend to disagree about what 'tall' means.

If an element is listed more than once, it is only counted once. For example,

$$\{a, a, b\} = \{a, b\}$$

The set $\{a, a, b\}$ has only the two elements a and b. The second mention of $a$ is an unnecessary repetition and can be ignored. It is normally considered poor notation to list an element more than once.

**Definition 1.1.2** (The symbols $\in$ and $\notin$). The symbol $\in$ stands for 'is an element of or belongs to' while $\notin$ stands for 'is not an element of or does not belong to'. For example, if $A = \{3, 4, 5, 6\}$, then

$$3 \in A (\text{read as '3 is an element of the set A'.})$$

$$8 \notin A (\text{read as '8 is not an element of the set A'.})$$

### 1.1.1 The empty set

**Definition 1.1.3** (Empty set). A set which contains no element is called the empty set. The empty set is aslo called the null or void set. It is generally denoted by the symbol $\emptyset$. It is also represented by $\{\ \}$.

Note that $x \notin \emptyset$, no matter what $x$ may be. There is only one empty set.

**Example 1.1.1.** *Empty sets*

$$\emptyset = \{x | x \neq x\}$$

$$\emptyset = \{x | x \text{ is a a month in a year containing 400 days}\}$$

**Definition 1.1.4** (Set of sets). A set whose elements are sets is called a set of sets.

**Example 1.1.2.** *Set of sets*

$$A = \{\emptyset, \{1, 2\}, \{0\}, \{\text{a}, \text{pen}, \text{book}\}\}$$

**Definition 1.1.5** (The universal set)**.** A universal set is the set of all elements under consideration, denoted by capital $U$ or sometimes $\zeta$.

For example, when we are studying integers then the universal set is the set of all the integers. And when we are working out what sports our friends play then the universal set is the set of all our friends (no matter if they play sport or not).

**Definition 1.1.6** (Subset and Superset)**.** A set $A$ is said to be a subset of set $B$ or $B$ is said to be a superset of set $A$ if every element of set $A$ is also an element of set $B$. We write

$$A \subseteq B$$

In this case we say that set $A$ is contained in set $B$ or set $B$ contains set $A$.

**Remark 1.1.1.** Note that:

- An empty set is always a subset of every set.
- Every set is always a subset of itself.

**Definition 1.1.7** (Proper Subset )**.** A set $A$ is said to be a proper subset of set $B$ if every element of set $A$ is also an element of set $B$, but there is at least one element of set $B$ that is not an member of set $A$. We write

$$A \subset B$$

**Example 1.1.3.**

$$\text{If } A = \{1, 2, 3, 4, 5, 6\} \text{ and } B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$\text{then } A \subseteq B \text{ and in particular } A \subset B$$

**Example 1.1.4.**

$$\text{If } A = \{a, b, c\} \text{ and } B = \{c, a, b\} \text{ then } A \subseteq B \text{ and } B \subseteq A$$

**Definition 1.1.8** (Power set)**.** A Power Set is a set of all the subsets of a set.

**Example 1.1.5.**

$$\text{If } A = \{a, b, c\} \text{ then } \quad P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, A\}$$

It is clear that if a set $A$ contains $n$ elements then the power set of $A$ will contain $2^n$ elements.

**Definition 1.1.9** (Equal sets)**.** Two sets are called equal if they have exactly the same elements. In other words sets $A$ and $B$ are equal if and only if $A \subseteq B$ and $B \subseteq A$

**Example 1.1.6.** *The sets* $\{1, 3, 5\}$ *and* $\{1, 2, 3\}$ *are not equal, because they have different elements. This is written as*

$$\{1, 3, 5\} \neq \{1, 2, 3\}$$

**Example 1.1.7.** *The order in which the elements are written between the curly brackets does not matter at all. For example,*

$$\{1, 3, 5, 7, 9\} = \{3, 9, 7, 5, 1\} = \{5, 9, 1, 3, 7\}$$

### 1.1.2 Finite and infinite sets

All the sets we have seen so far have been finite sets, meaning that we can list all their elements. Here are two more examples:

$$\{\text{whole numbers between } 2000 \text{ and } 2005\} = \{2001, 2002, 2003, 2004\}$$

$$\{\text{whole numbers between } 2000 \text{ and } 3000\} = \{2001, 2002, 2003, \ldots, 2999\}$$

. The three dots '. . . ' in the second example stand for the other 995 numbers in the set. We could have listed them all, but to save space we have used dots instead. This notation can only be used if it is completely clear what it means, as in this situation.

Sets that are not finite are said to be infinite. Here are two examples of infinite sets:

$$\{\text{even whole numbers}\} = \{0, 2, 4, 6, 8, 10, \ldots\}$$

$$\{\text{whole numbers greater than } 2000\} = \{2001, 2002, 2003, 2004, \ldots\}$$

Both of these sets are infinite because no matter how many elements we list, there are always more elements in the set that are not on our list. This time the dots '. . . ' have a slightly different meaning, because they stand for infinitely many elements that we could not possibly list, no matter how long we tried.

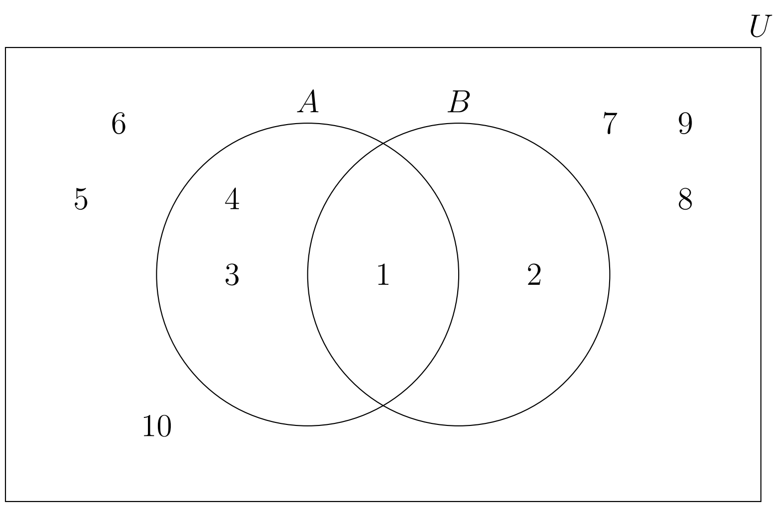
### 1.1.3 The number of elements in a set

If $X$ is a finite set, then $|X|$ denotes the number of elements in set $X$. The number of elements in a set is also called its **cardinality** or **order**. If $S = \{1, 3, 5, 7, 9\}$, then $|S| = 5$.
If $A = \{1001, 1002, 1003, \ldots, 3000\}$, then $|A| = 2000$
If $T = \{\text{letters in the English alphabet}\}$, then $|T| = 26$
Singleton set is a set with just one element. For example If $Y = \{5\}$, then $|Y| = 1$. Note that $5 \in Y$ but $Y \neq 5$

**Definition 1.1.10** (Equivalent sets)**.** Two sets $A$ and $B$ are said to be equivalent if and only if they have the same cardinality.

### 1.1.4 Operations on sets

**Definition 1.1.11** (Venn diagram)**.** A Venn diagram is an illustration of the relationships between and among sets. In a Venn diagram, a rectangular region is used to represent the universal set and all sets are drawn as circles within this region. For example if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 3, 4\}$, and $B = \{1, 2\}$ then we have the following Venn diagram.



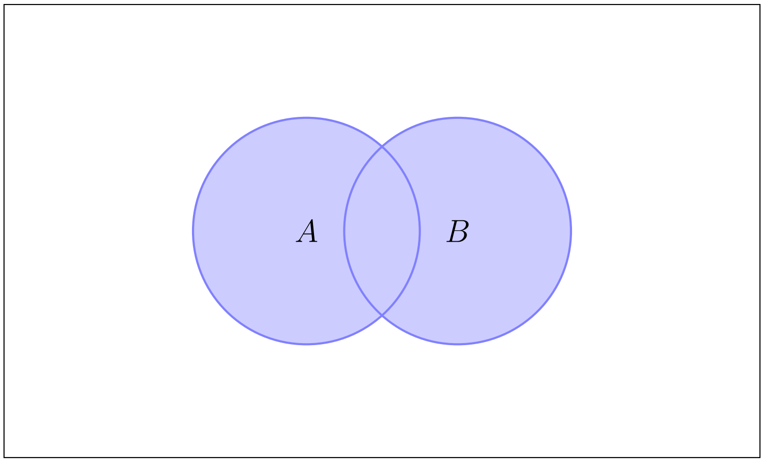

**Union**

**Definition 1.1.12** (Set union)**.** The union of sets $A$ and $B$ denoted $A \cup B$ is the set of all those elements which are either in $A$ or $B$ or both. Symbolically,

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

The Venn diagram for set union is given by

$A \cup B$




**Example 1.1.8.** *Determine the union of sets* $A = \{a, b, c, d\}$ *and* $B = \{a, e, f, g\}$

*Solution.*

$$A \cup B = \{a, b, c, d, e, f, g\}$$

□

**Example 1.1.9.** *Determine the union of* $A = \{a, \{a\}, b, c\}$ *and* $B = \{1, 3, 4, 5, c\}$

*Solution.*

$$A \cup B = \{a, \{a\}, b, c, 1, 3, 4, 5\}$$

□

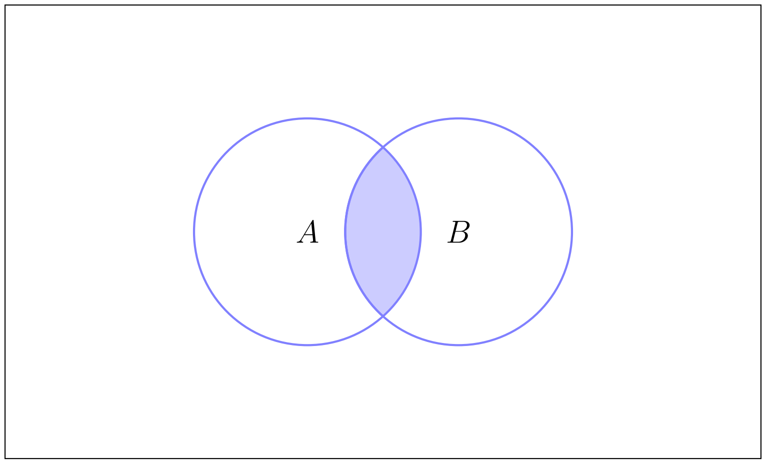
### Intersection

**Definition 1.1.13** (Set intersection)**.** If $A$ and $B$ are two sets, then the intersection $A \cap B$ is the set of all those elements which are common to both sets. Symbolically we write

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

The Venn diagram for $A \cap B$ is given by

$A \cap B$




**Example 1.1.10.** *Determine the intersection of* $A = \{a, b, c, d, e, f\}$ *and* $B = \{0, 3, 4, 5, a, e\}$

*Solution.*

$$A \cap B = \{a, e\}$$

□

**Example 1.1.11.** *Determine the intersection of* $X = \{a, \{a\}, b, c\}$ *and* $Y = \{1, 3, 4, 5\}$

*Solution.*

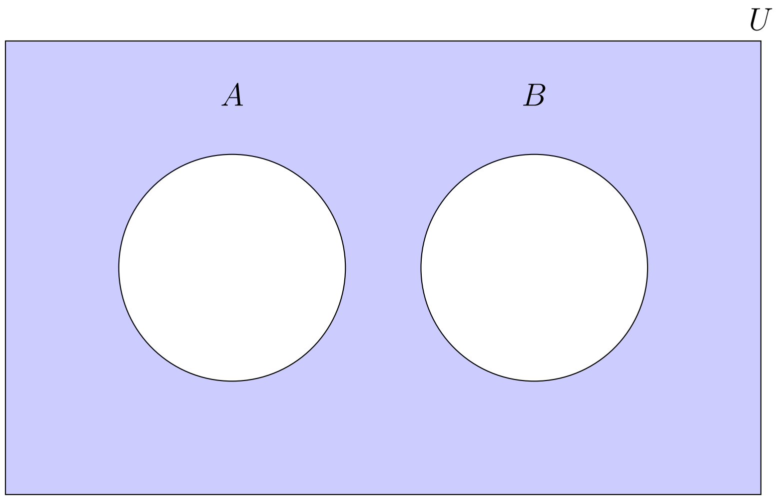
$$X \cap Y = \emptyset$$

□

**Definition 1.1.14** (Disjoint sets)**.** Two sets $A$ and $B$ said to be disjoint or non-overlapping if they have no common element(s). Symbolically we write

$$A \cap B = \emptyset$$

Below is the Venn diagram for disjoint sets $A$ and $B$

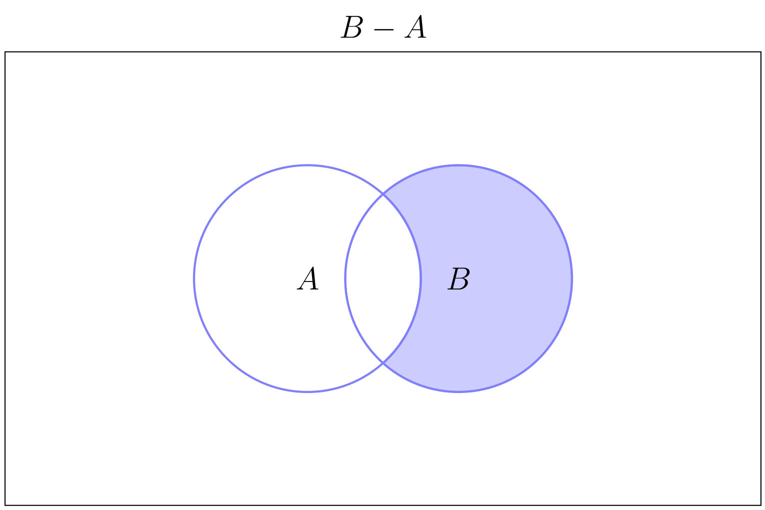



**Difference**

**Definition 1.1.15** (Set difference)**.** Given two sets $A$ and $B$, the difference $B - A$ or $B \backslash A$ is the set of all those elements which are in set $B$ but not in set $A$. Symbolically we write

$$B - A = \{x | x \in B \text{ and } x \notin A\}$$

The Venn diagram for $B - A$ is given by



**Example 1.1.12.** *Given* $A = \{a, b, c, d, e, f\}$ *and* $B = \{0, 3, 4, 5, a, e\}$, *find* $B - A$.

*Solution.*

$$B - A = \{0, 3, 4, 5\}$$

□

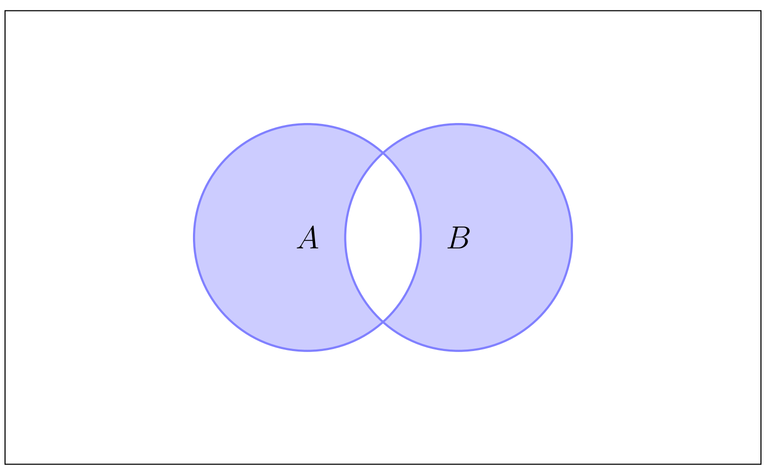
### Symmetric Difference

**Definition 1.1.16** (Symmetric difference)**.** Given two sets $A$ and $B$, the symmetric difference $A \triangle B$ or $A \oplus B$ is the set of all those elements which are either in $A$ or in $B$ but not in both. Symbolically we write

$$A \triangle B = (A - B) \cup (B - A)$$

The Venn diagram for $A \triangle B$ is given by

$A \triangle B$




**Example 1.1.13.** *Given* $A = \{0, 1, a, b, c, d, e, f\}$ *and* $B = \{0, 3, 4, 5, a, e\}$, *find* $A \triangle B$.

*Solution.*

$$A \triangle B = \{1, 3, 4, 5, b, c, d, f\}$$

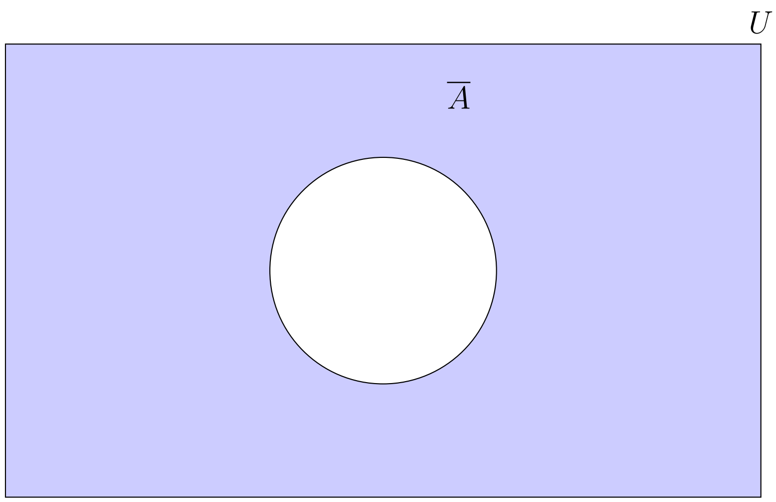
□

### Complement

**Definition 1.1.17** (Set complement)**.** Given set $A$ then the complement of $A$ denoted $A^c$ or $A'$ or $\overline{A}$ is the set of all those elements which are in the universal set $U$ but not in set $A$. Symbolically we write

$$\overline{A} = \{x|\ x \in U \text{ and } x \notin A\}$$

The Venn diagram for $\overline{A}$ is given by




**Example 1.1.14.** *Given* $U = \{0, 1, 2, \ldots, 10\}$ *and* $A = \{0, 3, 4, 5\}$, *find* $\overline{A}$.

*Solution.*

$$\overline{A} = \{1, 2, 6, 7, 8, 9, 10\}$$

□

**Example 1.1.15.** *Show that* $A \cap B \subseteq A$.

*Solution.* We must show that any element in $A \cap B$ is also in $A$. Let $x$ be an element in $A \cap B$. Since $x$ is in the intersection of $A$ and $B$, then $x$ must be an element of $A$ and $B$. Therefore $x$ is in $A$, which in turn implies $A \cap B \subseteq A$. □

**Example 1.1.16.** *Prove that* $(A - B) = A \cap \overline{B}$

*Solution.* Let $x \in A - B$, so $x \in A$ and $x \notin B$ implying $x \in \overline{B}$. Therefore, $x \in A \cap \overline{B}$. Since $x$ is an arbitrary element of $A - B$ we have

$$(i)\ \ (A - B) \subseteq A \cap \overline{B}$$

.

Conversely, let $y \in A \cap \overline{B}$. Therefore, $y \in A$ and $y \in \overline{B}$ i.e $y \in A$ and $y \notin B$. It therefore follows that $y \in A - B$. Again since $y$ was arbitrary we conclude that

$$(ii)\ \ A \cap \overline{B} \subseteq (A - B)$$

From $(i)$ and $(ii)$ it follows that $(A - B) = A \cap \overline{B}$ □

**Example 1.1.17.** *Prove* $(A \cap B) \cup (\overline{A} \cap B) = B$

*Solution.* Let $x \in (A \cap B) \cup (\overline{A} \cap B)$. Then either $x \in (A \cap B)$ or $x \in (\overline{A} \cap B)$ or both. If $x \in (A \cap B)$ then $x \in A$ and $x \in B$. If $x \in (\overline{A} \cap B)$ then $x \in \overline{A}$ and $x \in B$. In either case $x \in B$. Since $x$ is an arbitrary element of $(A \cap B) \cup (\overline{A} \cap B)$ we have,

$$(i)\ \ (A \cap B) \cup (\overline{A} \cap B) \subseteq B$$

Conversely let $y \in B$. Then $y \in A \cap B$ or $y \in \overline{A} \cap B$. Since $y$ is in either we have $y \in (A \cap B) \cup (\overline{A} \cap B)$. Similarly since $y$ is an arbitrary element of $B$ we have,

$$(ii)\ \ B \subseteq (A \cap B) \cup (\overline{A} \cap B)$$

From $(i)$ and $(ii)$ it follows that $(A \cap B) \cup (\overline{A} \cap B) = B$ □

**Example 1.1.18.** *Prove that* $\overline{A \cap B} = \overline{A} \cup \overline{B}$

*Solution.* Let $x \in \overline{A \cap B}$. This implies that $x \notin A \cap B$. Therefore $x \in \overline{A}$ or $x \in \overline{B}$ or both. It therefore follows that $x \in (\overline{A} \cup \overline{B})$. Now since $x$ is an arbitrary element of $\overline{A \cap B}$ we have,

$$(i)\ \overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$$

Conversely let $y \in (\overline{A} \cup \overline{B})$. Then $y \in \overline{A}$ or $y \in \overline{B}$ or both . Therefore by definition $y \notin A$ or $y \notin B$ or both which implies that $y \in \overline{A \cap B}$. Now since $y$ is an arbitrary element of $\overline{A} \cup \overline{B}$ we have,

$$(ii)\ \overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$$

From $(i)$ and $(ii)$ it follows that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. □

**Example 1.1.19.** *Show that $A \cup (B - A) = A \cup B$.*

*Solution.* Let $x \in A \cup (B - A)$. It follows that $x \in A$ or $x \in (B - A)$. Note that $x$ cannot be an element of both by the definition of $(B - A)$. If $x \in A$, then $x \in A \cup B$. If $x \in (B - A)$, then $x \in B$ and $x \notin A$ by the definition of $(B - A)$. In this case, too, $x \in A \cup B$. Thus,

$$(i)\ A \cup (B - A) \subseteq A \cup B$$

.

Conversely let $y \in A \cup B$. Therefore, $y \in A$ or $y \in B$ or both. So, if $y \in A$ or both then $y \in A \cup (B - A)$. The other case is $y \in B$, $y \notin A$. In this case $y \in (B - A)$ by definition of $(B - A)$. Therefore,

$$(ii)\ A \cup B \subseteq A \cup (B - A)$$

. From $(i)$ and $(ii)$ it follows that $A \cup (B - A) = A \cup B$ □

**Definition 1.1.18** (Membership table)**.** Membership tables show all the combinations of sets an element can belong to. In the membership table 1 means the element belongs to a set while 0 means it does not belong to.

Consider the following membership table: The third row is all elements that belong to set $B$ but not set $A$. Thus, these elements are in the union, but not the intersection or difference.

| $A$ | $B$ | $A \cup B$ | $A \cap B$ | $A - B$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Table 1.1: Example of a membership table

**Example 1.1.20.** *Use membership table to prove* $A \cap B = B - (B - A)$

| $A$ | $B$ | $A \cap B$ | $B - A$ | $B - (B - A)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |

*Solution.* Because the columns for $A \cap B$ and $B - (B - A)$ have the same values, the two expressions are identical. □

### 1.1.5 Laws of algebra of sets

1. **Idempotent laws**
   - $A \cup A = A$
   - $A \cap A = A$
2. **Associative laws**
   - $A \cup (B \cup C) = (A \cup B) \cup C$
   - $A \cap (B \cap C) = (A \cap B) \cap C$
3. **Commutative laws**
   - $A \cup B = B \cup A$

- $A \cap B = B \cap A$

4. **Distributive laws**
   - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. **Identity laws**
   - $A \cup U = U$
   - $A \cap \emptyset = \emptyset$
   - $A \cup \emptyset = A$
   - $A \cap U = A$

6. **Complement laws**
   - $A \cup \overline{A} = U$
   - $A \cap \overline{A} = \emptyset$
   - $\overline{U} = \emptyset$
   - $\overline{\emptyset} = U$

7. **De Morgan's laws**
   - $\overline{A \cup B} = \overline{A} \cap \overline{B}$
   - $\overline{A \cap B} = \overline{A} \cup \overline{B}$

8. **Involution law**: $\overline{(\overline{A})} = A$

9. **Absorption laws**
   - $A \cup (A \cap B) = A$
   - $A \cap (A \cup B) = A$

**Example 1.1.21.** *Use set identities to prove that* $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$.

*Solution.*

$$
\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{B \cap C} \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A}
\end{aligned}
$$

$\square$

**Example 1.1.22.** *Use set identities to simplify* $(A - B) \cup (A \cap B)$

*Solution.*

$$
\begin{aligned}
(A - B) \cup (A \cap B) &= (A \cap \overline{B}) \cup (A \cap B) \\
&= ((A \cap \overline{B}) \cup A) \cap ((A \cap \overline{B}) \cup B) \\
&= A \cap ((A \cup B) \cap (\overline{B} \cup B)) \\
&= A \cap (A \cup B) \\
&= A
\end{aligned}
$$

$\square$

**Example 1.1.23.** *Simplify* $A - (A - B)$

*Solution.*

$$
\begin{aligned}
A - (A - B) &= A - (A \cap \overline{B}) \\
&= A \cap \overline{(A \cap \overline{B})} \\
&= A \cap (\overline{A} \cup \bar{(}\overline{B})) \\
&= A \cap (\overline{A} \cup B) \\
&= (A \cap \overline{A}) \cup (A \cap B) \\
&= \emptyset \cup (A \cap B) \\
&= A \cap B
\end{aligned}
$$

$\square$

### 1.1.6 Inclusion-Exclusion Principle

For finite sets $A_1, A_2, \ldots, A_n$, if the sets are pairwise disjoint ($A_i \cap A_j = \emptyset, i \neq j$) then

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$$

What if the sets are not pairwise disjoint? Given sets $A_1, A_2, \ldots, A_n$ the cardinality of the union is: The sum of the individual cardinalities, minus all the cardinalities of intersections of two sets, plus the cardinalities of intersections of three sets, minus the cardinalities of intersections of four sets, etc. This alternating sum ends with plus or minus the cardinality of the intersection of all $n$ sets.

**Two sets:**

$$|A \cup B| = |A| + |B| - |A \cap B|$$

.

**Three sets:**

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

**Four sets:**

$$\begin{aligned}|A \cup B \cup C \cup D| = {} & |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| \\ & - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| \\ & + |A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|\end{aligned}$$

**Example 1.1.24.** *A travel agent surveyed* 100 *people to find out how many of them had visited the cities of Melbourne and Brisbane. In the survey 31 people had visited Melbourne,* 26 *people had been to Brisbane, and* 12 *people had visited both cities. Find the number of people who had visited:*

*i Melbourne or Brisbane.*

*ii Brisbane but not Melbourne.*

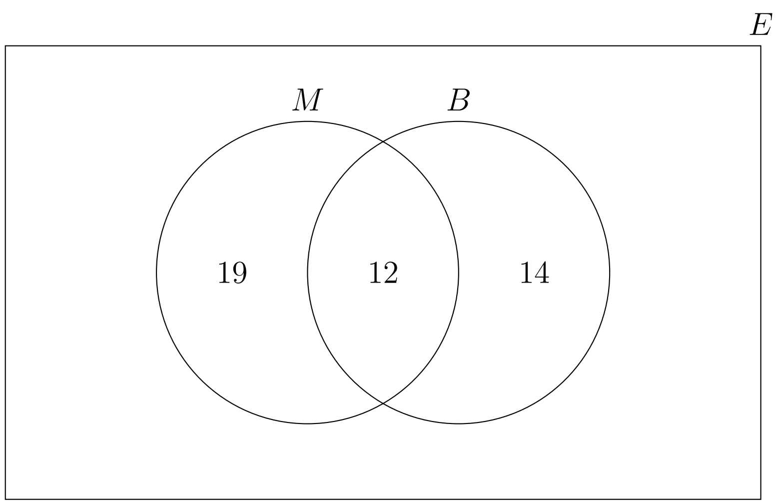
*iii Only one of the two cities.*

*iv Neither city.*

*Solution.* Let $M$ be the set of people who had visited Melbourne, and let $B$ be the set of people who had visited Brisbane. Let the universal set $E$ be the set of people surveyed. The information given in the question can now be rewritten as

$$|M| = 31, |B| = 26, |M \cap B| = 12, |E| = 100$$

Hence number of people who had visited $M$ only $= 31–12 = 19$ and those who had visited $B$ only$= 26–12 = 14$. This information can be represented on the Venn diagram as shown below.




i Melbourne or Brisbane $= 19 + 14 + 12 = 45$.

ii Brisbane but not Melbourne$= 14$.

iii Only one of the two cities $= 19 + 14 = 33$.

iv Neither city $= 100–45 = 55$.

□

**Example 1.1.25.** *Use the following information to answer the questions below:*

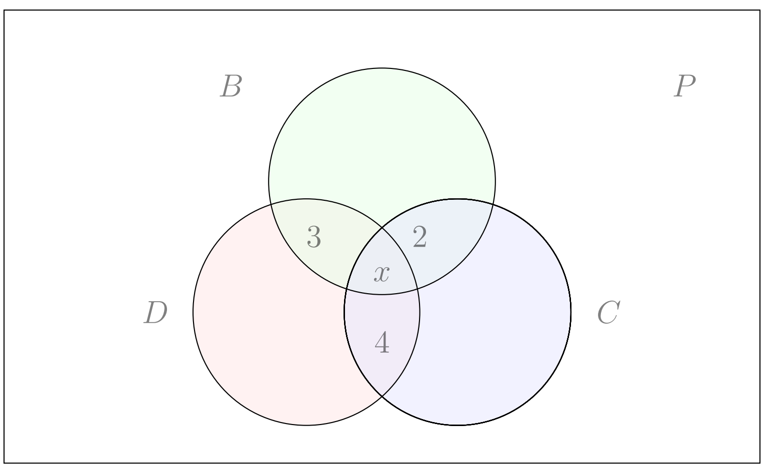
- *28 children have a dog, a cat, or a bird.*
- *13 children have a dog.*

- *13 children have a cat.*
- *13 children have a bird.*
- *4 children have only a dog and a cat.*
- *3 children have only a dog and a bird.*
- *2 children have only a cat and a bird.*
- *No child has two of each type of pet.*

*i How many children have the three different types of pets?*

*ii How many children have only one type of pet?*

*Solution.* P: be the set of children with pets, C: be the set of children with cats, B: be the set of children with birds, D: be the set of children with dogs. If $x$ represent the number of children with the three different types of pets, then above information can be represented on a Venn diagram as shown below




$$|B \cup C \cup D| = |B| + |C| + |D| - |B \cap C| - |B \cap D| - |C \cap D| + |B \cap C \cap D|$$

$$28 = 13 + 13 + 13 - (x + 2) - (x + 3) - (x + 4) + x$$
$$28 = 30 - 2x$$
$$x = 1$$

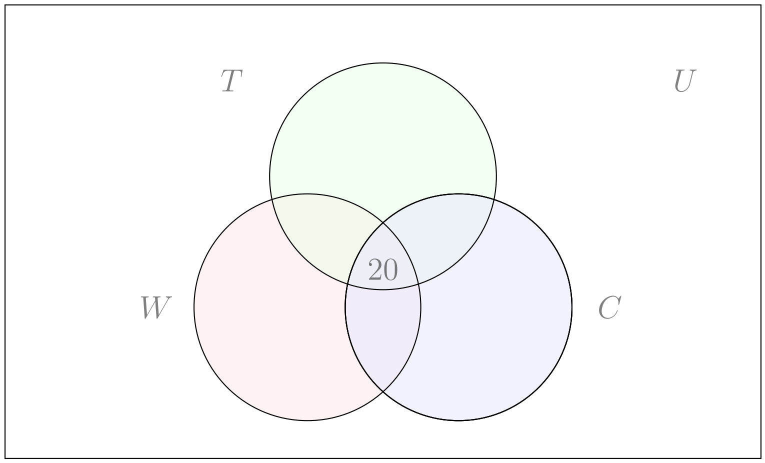
*i*. Therefore one child has three different types of pets.
*ii*. Number of children with one type of pet=Total number of children-Number of children with more than one type of pet.
$= 28 - (1 + 2 + 3 + 4) = 18$
Therefore 18 children have one type of pet. □

**Example 1.1.26.** *Shannon's high school starts a campaign to encourage students to use green transport (walking or cycling) for travelling to and from school. At the end of the semester, Shannon's class surveys the 750 students at the school to see if the campaign is working. They obtain these results:*

- *370 students use public transport.*
- *100 students cycle and use public transport.*
- *80 students walk and use public transport.*
- *35 students walk and cycle.*
- *20 students walk, cycle and use public transport.*
- *445 students cycle or use public transport.*
- *265 students walk or cycle.*

*How many students use green as the only mean of transport for travelling to and from school?*

*Solution.* Let:
$U$ : be the universal set (Students who attend Shannon's school)
$T$ : be set of students who use public transport
$W$ : be set of students who walk
$C$ : be set of students who cycle



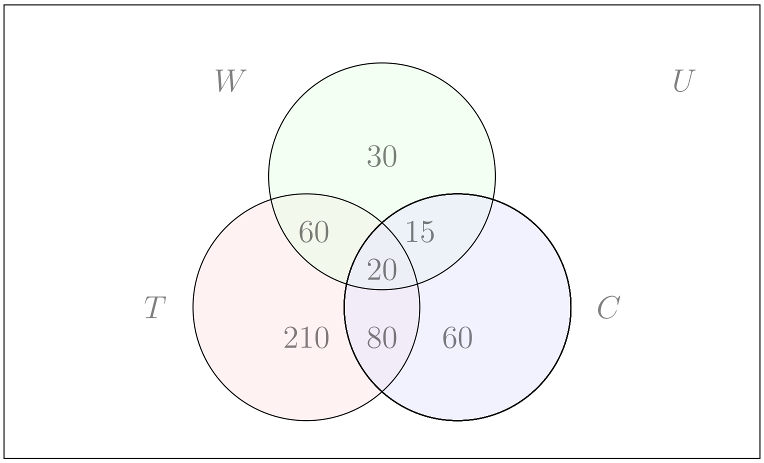

The number of those who walk and use public transport is 80. Of those students, 20 have already been counted( those who use the three means of transport). That leaves 80-20=60 to go into the other region of the intersection between $T$ and $W$.

The number of those who cycle and walk is 35. Of those students, 20 have already been counted i.e those who use the three means of transport. Again 35-20=15 goes into the other region of the intersection between $C$ and $W$.

Those who cycle and use public transport is 100. Therefore, 100-20=80 goes into the other region of the intersection between $C$ and $T$.

The number of students who use public transport only is $370 - (20 + 60 + 80) = 210$.
The number of students who cycle but do not use public transport is $445 - 370 = 75$. Therefore, the number of those who cycle only is $75 - 15 = 60$.

The number of students who only walk is $265 - (60+80+15+20+60) = 30$.




The total number of students who use green transport is $30 + 15 + 60 = 105$. □

### 1.1.7 Cartesian product

**Definition 1.1.19** (Cartesian product)**.** Suppose that A and B are non-empty sets. The cartesian product of $A$ and $B$ denoted $A \times B$ is the set of all ordered pairs $(a, b)$ such that $a \in A$ and $b \in B$. We write:

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$$

Given sets $A_1, A_2, A_3, \ldots, A_n$ we have

$$A_1 \times A_2 \times A_3 \times \cdots \times A_n = \{a_1, a_2, a_3, \ldots, a_n | a_1 \in A_1, a_2 \in A_2, a_3 \in A_3, \ldots, a_n \in A_n\}$$

and

$$|A_1 \times A_2 \times A_3 \times \cdots \times A_n| = |A_1| \times |A_2| \times |A_3| \times \cdots \times |A_n|$$

Note that in general $A \times B \neq B \times A$.

**Example 1.1.27.** *Given that* $A = \{1, 2\}$ *and* $B = \{a, b, c\}$*. Find* $A \times B$

*Solution.*

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

□

**Some properties of the cartesian product**

i $A \times B \neq B \times A$ in general. $A \times B = B \times A$ if and only if $A = B$.

ii $A \times (B \times C) \neq (A \times B) \times C$.

iii $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

iv $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

v $A \times (B \backslash C) = (A \times B) \backslash (A \times C)$.

vi If $A \subseteq B$, then $A \times B \subseteq B \times C$ for any set $C$.

vii $A \times B = \emptyset$ is either $A = \emptyset$ or $B = \emptyset$