Relations and Functions

(MOST EXPECTED QUESTIONS)

(Only for the Academic year 2020 - 21)

One Mark – 1; Two Marks – 1; Three Marks - 1; Five Marks – 1; Total Marks - 11.

ONE MARK QUESTIONS: (QUESTION NO. 1):

1. Define Reflexive relation on a set.

Ans: A relation R on a set A is called reflexive, if $(a,a)\in R$, for every $a\in A$.

2. Define Symmetric relation on a set.

Ans: A relation R on a set A is called symmetric, if $\forall (a,b)\in R \Rightarrow (b,a)\in R$, $\forall a,b\in A$.

3. Define Transitive relation on a set.

Ans: A relation R on a set A is called transitive if $(a,b)\in R$ & $(b,c)\in R \Rightarrow (a,c)\in R$, $\forall a,b,c\in A$

4. Give an example of a relation which is symmetric only. **(MQP 2)**

Ans: On set $A=\{1,2,3\}$ a relation $R=\{(1,2),(2,1)\}$ is symmetric only.

5. Give an example of a relation which is reflexive and symmetric but not transitive. **(MQP 5)**

Ans: On set $A=\{1,2,3\}$ a relation $R=\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)\}$ is reflexive and symmetric but not transitive.

6. A relation R on $A=\{1,2,3\}$ defined by $R=\{(1,1),(1,2),(3,3)\}$ is not symmetric. Why? **(S 20)(M 14)**

Ans: R is not symmetric because $(1,2)\in R \Rightarrow (2,1)\notin R$.

7. The relation R in the set $\{1,2,3\}$ given by $R=\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ is not transitive. Why? (J 18)

Ans: R is not transitive because $(1,2)\in R$ and $(2,3)\in R$ but $(1,3)\notin R$.

8. Define one-one function.

Ans: A function $f:X\to Y$ is said to be one-one (injective), if the images of distinct elements of X under f are distinct.

9. Define onto function.

Ans: A function $f:X\to Y$ is said to be onto (surjective), if every element of Y is the images of some element of X under f.

10. Define bijective function. (M 18)

Ans: A function $f:X\to Y$ is said to be one-one and onto (bijective), if f is both one-one and onto.

11. Define Binary operation. (M 19)

Ans: A binary operation * on a set A is a function $*:A\times A\to A$. We denote $*(a,\ b)$ by $a*b$.

(or)

An operation * on a set A is said to be a binary operation on A, if $\forall a,b\in A\Rightarrow a*b\in A$.

12. Operation * is defined by $a*b=a$. Is * a binary operation on Z^+? (MQP 1)

Ans: $\forall a,b\in Z^+ \Rightarrow a*b=a\in Z^+$

$\therefore$ * is a binary operation.

13. Let * be a binary operation on N given by $a*b=$ LCM of a and b, find $20*16$.

(MQP 3) (M 17)

Ans: Given, $a*b=$ LCM of a and b, $\forall a,b\in N$.

$20*16=$ LCM of 20 and 16 = 80.

14. Let * be a binary operation defined on set of rational numbers, by $a*b=\dfrac{ab}{4}$. Find the identity element. (M 15)(J 17)

Ans: Given, $a*b=\dfrac{ab}{4}$, $\forall a,b\in Q$.

$\forall a \in Q$, $\exists e \in Q$ such that

$a*e = a$ and $e*a = a$

$\frac{ae}{4} = a$ $\frac{ea}{4} = a$

$e = 4$ $e = 4$

$\therefore$ identity element is 4.

15. Let * be a binary operation on the set of natural numbers given by $a*b =$ LCM of a and b, find $5*7$. **(J 15)(J 19)(M 20)**

Ans: Given, $a*b =$ LCM of a and b, $\forall a,b \in N$.

$5*7$ = LCM of 5 and 7 = 35.

16. An operation * on Z^+ (the set of all non negative integers) is defined as $a*b = a-b$, $\forall a,b \in Z^+$. Is * a binary operation on Z^+? **(M 16)**

Ans: $\forall a,b \in Z^+ \Rightarrow a*b = a-b \notin Z^+$

$\left[\because 1,2 \in Z^+ \Rightarrow 1*2 = 1-2 = -1 \notin Z^+\right]$

$\therefore$ * is not a binary operation.

17. An operation * on Z^+ (the set of all non negative integers) is defined as $a*b = |a-b|$, $\forall a,b \in Z^+$. Is * a binary operation on Z^+? **(J 16)**

Ans: $\forall a,b \in Z^+ \Rightarrow a*b = |a-b| \in Z^+$

$\therefore$ * is a binary operation.

18. Verify whether * defined on Z^+, by $a*b = ab^2$, $\forall a,b \in Z^+$ is the binary operation or not.

Ans: $\forall a,b \in Z^+ \Rightarrow a*b = ab^2 \in Z^+$

$\therefore$ * is a binary operation.

TWO MARK QUESTIONS: (QUESTION NO. 11):

1. Show that the relation R in the set of integers given by $R = \{(a,b) : 5 \, divides (a-b)\}$ is symmetric and transitive. **(MQP 3)**
2. Check whether the relation ***R*** defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is reflexive or symmetric. **(J-17)**

3. Define binary operation on a set. Verify whether the operation * defined on Z by $a*b=ab+1$ is binary or not. **(MQP 2) (J 18)**

4. Is $*$ defined on the set $\{1,2,3,4,5\}$ by a $*$ b = L.C.M. of a and b a binary operation? Justify your answer.

5. Verify whether the operation * defined on Q, define $a*b=ab+1$ is commutative or not. **(J 14)**

6. Verify whether the operation * defined on Q by $a*b=\frac{ab}{2}$ is commutative or not.

7. Verify whether the operation * defined on Q by $a*b=a+ab$ is commutative or not.

8. Verify whether the operation * defined on Q by $a*b=ab^2$ is commutative or not.

9. A binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a\wedge b = min\{a, b\}$, write the operation table for operation $\wedge$. **(MQP 4)**

10. Let * be a binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by $a*b = HCF\ of\ a\ and\ b$. Write the composition table.

11. Verify whether the operation * defined on Q by $a*b=\frac{ab}{4}$ is associative or not. **(MQP 5)**

12. Verify whether the operation * defined on Q by $a*b=\frac{ab}{2}$ is associative or not.

(M 14) (M 18)(S 20)

13. Let * be a binary operation defined on the set Q of rationals by $a*b=\frac{ab}{4}$. Find identity element if it exist.

THREE MARK QUESTIONS: (QUESTION NO. 25):

1. Show that the relation R in the set of all integers Z defined by $R=\{(a,b):2 \text{ divides } a-b\}$ is an equivalence relation. **(MQP 2) (J 14)**

2. Verify whether the function $f:R-\{3\}\to R-\{1\}$, defined by $f(x)=\frac{x-2}{x-3}$ is one-one and onto or not. Give reason. **(MQP 3)**

3. Show that the relation R in the set $S=\{x: x\in Z\ and\ 0\le x\le 12\}$ given by $R=\{(a, b):|a-b| \text{ is multiple of } 4\}$. Is an equivalence relation? **(MQP 4) (M 16)**

4. Determine whether the relation R in the set $A=\{1, 2, 3, 4, 5, 6\}$ as $R=\{(x, y): y \text{ is divisible by } x\}$ is reflexive, symmetric and transitive. **(MQP 5)**

5. Determine whether the relation R in the set $A=\{1, 2, 3, \ldots\ldots\ldots 13, 14\}$ defined as $R=\{(x, y): 3x-y=0\}$, is reflexive, symmetric and transitive. **(M 15)(J 19)**

6. Prove that the relation R in the set of integer Z defined by $R=\{(x,y): x-y \text{ is an integer }\}$ is an equivalence relation. **(J 15)(S 20)**

7. Show that the relation R in the set $A=\{1, 2, 3, 4, 5\}$ given by $R=\{(a,b): |a-b| \text{ is even }\}$ is an equivalence relation. **(J 16) (M 18)**

8. Show that the relation R in ***R*** (set of real numbers) is defined as $R=\{(a,b): a\le b\}$ is reflexive and transitive but not symmetric. **(M 17)**

9. Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R=\{(a,b): b=a+1\}$ is reflexive or symmetric. **(J 17)**

10. Show that the relation ***R*** defined in the set A of all triangles as $R=\{(T_1,T_2): T_1 \text{ is similar to } T_2\}$ is an equivalence relation. **(M 20)**

11. Let T be the set of all triangles in a plane with R a relation in T given by $R=\{(T_1, T_2): T_1 \text{ is congruent to } T_2\}$. Show that R is an equivalence relation.

12. Let *'L'* be the set of all lines in XY plane and R be the relation in L defines as $R=\{(L_1, L_2): L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation.

13. Show that the relation R in the set of real numbers R defined as $R=\{(a, b):\ a\le b^2\}$ is neither reflexive nor symmetric nor transitive. **(MQP 1)**

14. Check whether the relation R in ***R*** defined by $R=\{(a,b): a\le b^3\}$ is reflexive, symmetric or transitive. **(M 19)**

15. Verify whether the operation * defined on Q by $a*b=a^2+b^2$ is commutative or associative or not.

16. Verify whether the operation * defined on Q by $a*b=a+ab$ is commutative or associative or not.

FIVE MARK QUESTIONS: (QUESTION NO. 39):

1. Check the injectivity and surjectivity of the function $f: R\to R$ defined by $f(x)=3-4x$. Is it a bijective function?

2. Verify whether the function $f: N\to N$ defined by $f(x)=x^2$ is one-one, onto and bijective.

3. Show that the function $f: R_* \to R_*$ defined $f(x)=\frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero real numbers.
4. Show that the Modulus function $f : R \to R$ given by $f(x)=|x|$, is neither one-one nor onto.
5. Prove that the Greatest integer function $f : R \to R$ given by $f(x)=[x]$ is neither one-one nor onto.
6. Show that the signum function $f : R \to R$, given by $f(x)=\begin{cases} -1 & if \quad x<0 \\ 0 & if \quad x=0 \\ 1 & if \quad x>0 \end{cases}$ is neither one-one nor onto.
7. Check the injectivity and surjectivity of the function $f: R \to R$ defined by $f(x)=1+x^2$. Is it a bijective function?
8. Verify whether the function $f: R-\{3\} \to R-\{1\}$, defined by $f(x)=\frac{x-2}{x-3}$ is one-one and onto or not. Justify your answer.
9. Show that $f: N \to N$,given by $f(1)=f(2)=1$ and $f(x)=x-1$, for every $x>2$, is onto but not one-one.
10. Verify whether the function $f: Z \to Z$ defined by $f(x)=x^2$ is one-one, onto and bijective.
11. Verify whether the function $f: R \to R$ defined by $f(x)=x^2$ is one-one, onto and bijective.
12. Verify whether the function $f: N \to N$ defined by $f(x)=x^3$ is one-one, onto and bijective.
13. Verify whether the function $f: Z \to Z$ defined by $f(x)=x^3$ is one-one, onto and bijective.
14. Verify whether the function $f: R \to R$ defined by $f(x)=x^3$ is one-one, onto and bijective.
15. Verify whether the function $f: R \to R$ given by $f(x)=2x$, is one-one, onto and bijective.
16. Verify whether the function $f: N \to N$ given by $f(x)=2x$, is one-one, onto and bijective.

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NOTE :
Those who need assistance in I PUC & II PUC mathematics for board examination and KCET/JEE join my ***Telegram channel***: ***LG_Maths***.