KNC INNOVATIVE PU COLLEGE, HEBBAL, MYSURU
II PUC - MID-TERM EXAMINATION, SEP - 2022
SUBJECT: MATHEMATICS

MAX MARKS: 100 **MAX TIME: 3 HRS**

PART – A

Answer all the 10 questions **1 x 10 = 10**

1. Relation R on $A = \{1,2,3\}$ defined by $R = \{(1,1)(1,2)(3,3)\}$ is not symmetric, Why?
2. Let $*$ be the binary operation on N given by $a * b = L.C.M\ of\ a\ and\ b$. Find $20 * 60$.
3. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.
4. Write the values of x for which $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$, holds
5. Define a diagonal matrix.
6. If A is a square matrix with $|A| = 8$,then find the value of $|AA^1|$.
7. Differentiate $y = \cos(1-x)$ with respect to x.
8. If $y = \log(\sin x)\ find\ \frac{dy}{dx}$,
9. Evaluate $\int(\sin x + \cos x)dx$
10. Define feasible region.

PART-B

II. Answer any TEN questions. $\mathbf{10 \times 2 = 20}$

11. Show that if $f: A \to B$ and $g: B \to C$ are one-one, then $fog: A \to C$.
12. Write the simplest form of $\tan^{-1}\left(\sqrt{\frac{1-cosx}{1+cosx}}\right), 0 < x < \pi$.
13. Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
14. Solve the equation:$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$.
15. Find value of k ,if area of the triangle is 4 sq. Units and vertices are $(k,0)(4,0)and\ (0,2)$ using determinants.
16. Using determinant show that points $A(a, b+c), B(b, c+a)$ and $C(c, a+b)$ are collinear.
17. Differentiate $\left(x+\frac{1}{x}\right)^x$ with respect to x.
18. Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, if $\mathrm{y} = \log_7(\log \mathrm{x})$
19. If $y + \sin y = \cos x\ \ find \frac{dy}{dx}$.
20. If $x = at^2, y = 2at$ show that
21. Integrate $\sin \mathrm{x}\,.\sin(\cos \mathrm{x})\ \ \mathrm{w.r.t.x}$
22. Integrate $\frac{tan^4\sqrt{x}sec^2\sqrt{x}}{\sqrt{x}}$ with respect to x.
23. Find the approximate change in the volume v of a cube of side x metres caused by increasing the side by 2%.
24. Find two numbers whose sum is 24 and whose product is an large as possible.

PART-C

III. Answer any TEN questions. $\mathbf{10 \times 3 = 30}$

25. Determine whether is the relation in the set $A = \{1,2,3, \dots ,13,14\}$ defined as $R = \{(x, y): 3x - y = 0\}$ is reflexive , symmetric and transitive.
26. Show that the relation R in R defined as $R = \{(a,b) : a \le b\}$, is reflexive and transitive but not symmetric.
27. Write $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ in the simplest form.
28. If A and B are square matrices of the same order ,then show $(AB)^{-1} = B^{-1}A^{-1}$.
29. By using elementary transformation, find the inverse of the matrix A= $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$.
30. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, that is $AB = BA$.
31. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$ with respect to x.
32. Differentiate $(\log x)^{\cos x}$ with r. to x.
33. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.
34. Verify Rolle's Theorem for the function $y = x^2 + 2,\ x \in [-2,2]$.
35. Verify mean value theorem if $f(x) = x^2 - 4\,x - 3$ in the interval$[1,4]$
36. Find two positive numbers *x* and *y* such that $x + y = 60$ and xy^3 is maximum.

PART-D

IV. Answer any EIGHT questions. $\mathbf{8 \times 5 = 40}$

37. Let $f: N \to R$ be defined by $f(x) = 4x^2 + 12x + 15$.Show that $f: N \to S$ where S is the range of the function. Is invertible. Also find f^{-1}.
38. Verify whether the function,$f: N \to Y$ defined by $f(x) = 4x + 3$,where $Y = \{y: y = 4xx + 3, x \in N\}$ is invertible or not.Write the inverse of $f(x)$ if exists.
39. Calculate AC,BC and $(A + B)C$ also verify that $(A + B)C = AC + BC$
$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 4 \\ 5 & -3 & 0 \\ 4 & 5 & -3 \end{bmatrix} and, C = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \\ -1 & 2 & 3 \end{bmatrix}.$$
40. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1 \quad 3 \quad -6]$, verify that $(AB)' = B'A'$.
41. Solve the following system of equation by matrix method $x - y + 2z = 1$,$2y - 3z = 1$ and $3x - 2y + 4z = 2$.
42. Solve the system of equation using matrix method,
$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \quad , \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \qquad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2.$$
43. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.
44. If $y = Ae^{mx} + Be^{nx}$,prove that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$.
45. A sand is pouring from a pipe at the rate of 12cm^3/s. The falling sand form a cone on the ground in such a way that the height of the cone is always one-sixth of the radius

of the base. How fast is the height of the sand cone increasing when the height is $4cm$?

46. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rates of change of (i) the perimeter, and (ii) the area of the rectangle.

PART-E

V. Answer any ONE question. $\mathbf{1 \times 10 = 10}$

47. a) Minimize and maximize $z = 5x + 10y$, subject to the constraints $x + 2y \le 120$, $x + y \ge 60, x - 2y \ge 0$ and $x \ge 0, y \ge 0$ by graphical method.

    b) Find the value of k, if $f(x) = \begin{cases} \frac{1-cos2x}{1-\cos x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$.

48. a) Minimize and Maximize $Z = 600x + 400y$ subject to the constraints $x + 2y \le 12,\ 2x + y \le 12,\ 4x + 5y \le 20$ and $x \ge 0$ *and* $y \ge 0$ by graphical method.

    b) Determine the value of k, if $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & if\ x \neq \frac{\pi}{2} \\ 3, & if\ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$