UNIT-III

Subject-Theory of Computation

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Reduction of Grammar

Example 1: Find a reduced grammar equivalent to the grammar G whose productions are

S->AB | CA

B-> BC | AB

A-> a

G=(VN, sigma, P, S)

C-> aB

C->b

- Useful Symbol={A,C}
- By rule 2
- Useful Symbol={S, B}

S->AB | CA

B-> BC | AB

A-> a

C-> aB

C->b

Example 1 Solution

- Rule 1 the grammar/ symbol that contain only terminal or epsilon is useful grammar or symbol
- By rule 1 we get A & C as useful variable or symbol.

Useful symbol={A,C}

Rule 2: The grammar that contain combination of useful nonterminal and terminal is useful grammar or symbol.

By rule 2 symbol S is also useful

B → Ac

Useful symbol={A,C,S}

Example 1 Solution Cont…

- Useful symbol={A,C,S}
- Useless Symbol={B}
- The reduced grammar is

S→ CA

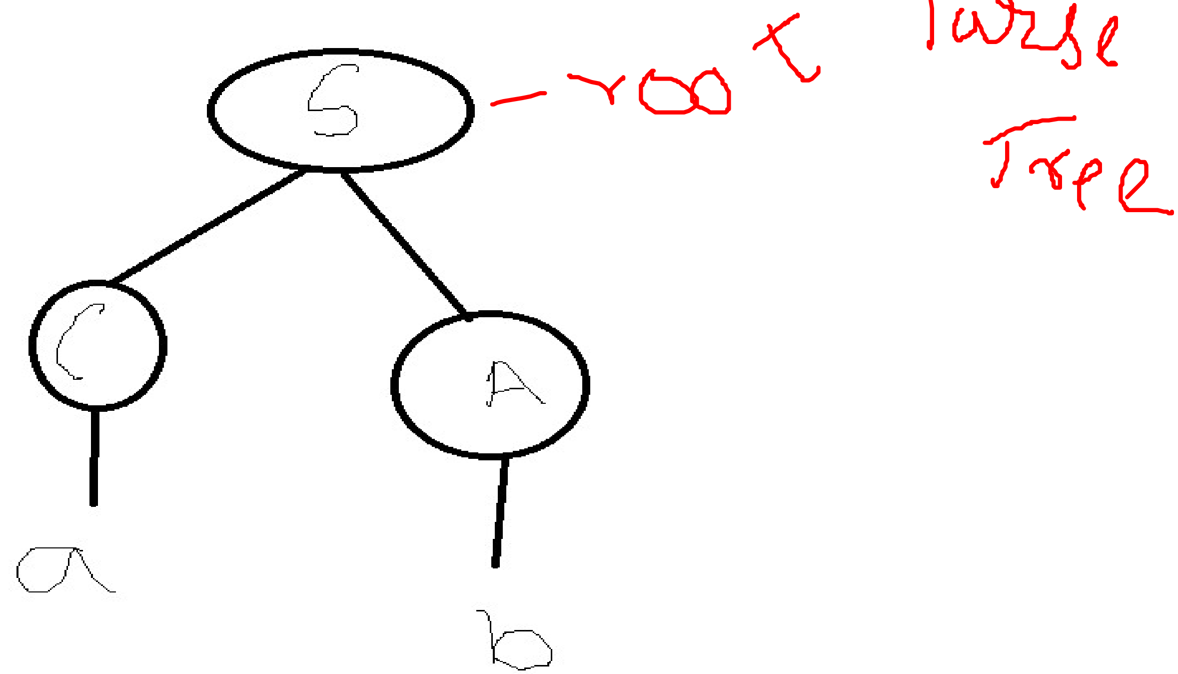
A-> a

C->b

Example 1 Solution…

- By reachable condition
- S,A & C is also useful

S-> CA,A-> a,C->b



Example 1 Solution…

- Final Reduced Grammar will be

S-> CA

A-> a

C->b

- Useful symbol={A,C,S}
- Useless Symbol={B}

Example 2: Find a reduced grammar equivalent to the grammar G whose productions are

S->aAa G={S,A,C,D,E},{a,b}, P,{S})

A->Sb

A-> bCC rule2

A-> DaA

C->abb

C-> DD

E-> aC rule 2

D->aDA

After rule 1

Useful Symbol={C}

By rule 2

Useful Symbol={S,A,E}Useless={D}

Reduced grammar After applying rule1 & rule2

S->aAa

A->Sb

A-> bCC rule2

C->abb

E-> aC rule 2

S->aAa

A->Sb

A-> bCC rule2

C->abb

E-> aC rule 2

Example 2 Solution

- Rule 1: Grammar that contain only epsilon or terminal symbols are useful grammar or symbol.

By rule 1, We get C as useful symbol

Useful Symbol={C}

Rule 2: grammar that contain combination of useful nonterminal & terminal is useful grammar or symbol

By rule 2, We get S,A,E as useful symbols.

Useful Symbol={S,A,E,C}

Useless Symbol={D}

Example 2 Solution Conti…

- Reduced grammar

S->aAa

A->Sb

A-> bCC

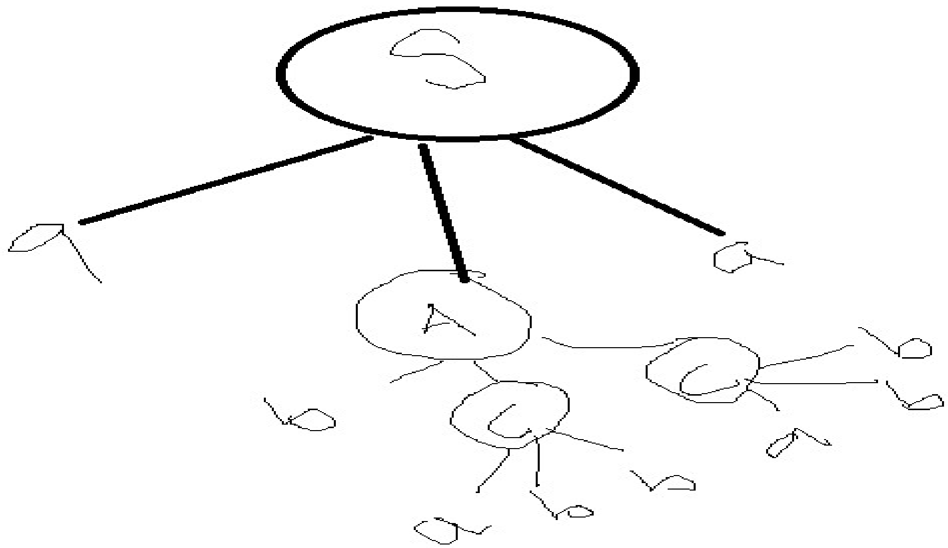
C->abb

E-> aC

Example 2 Solution Conti…

- By Reachable condition

S->aAa,A->Sb,A-> bCC,C->abb, E-> aC



By reachable condition E is also useless symbol.
Useful Symbol={S,A,C}
Useless Symbol={D,E}

Example 2 Solution Conti…

Final Reduced grammar will be:

S->aAa

A->Sb

A-> bCC

C->abb

- Example 3: Find a reduced grammar equivalent to the grammar G whose productions are

S->aAa

A->bBB

B->ab

C->aB

Example 3 Solution Conti…

- Rule 1: Grammar that contain only epsilon or terminal symbols are useful grammar or symbol.

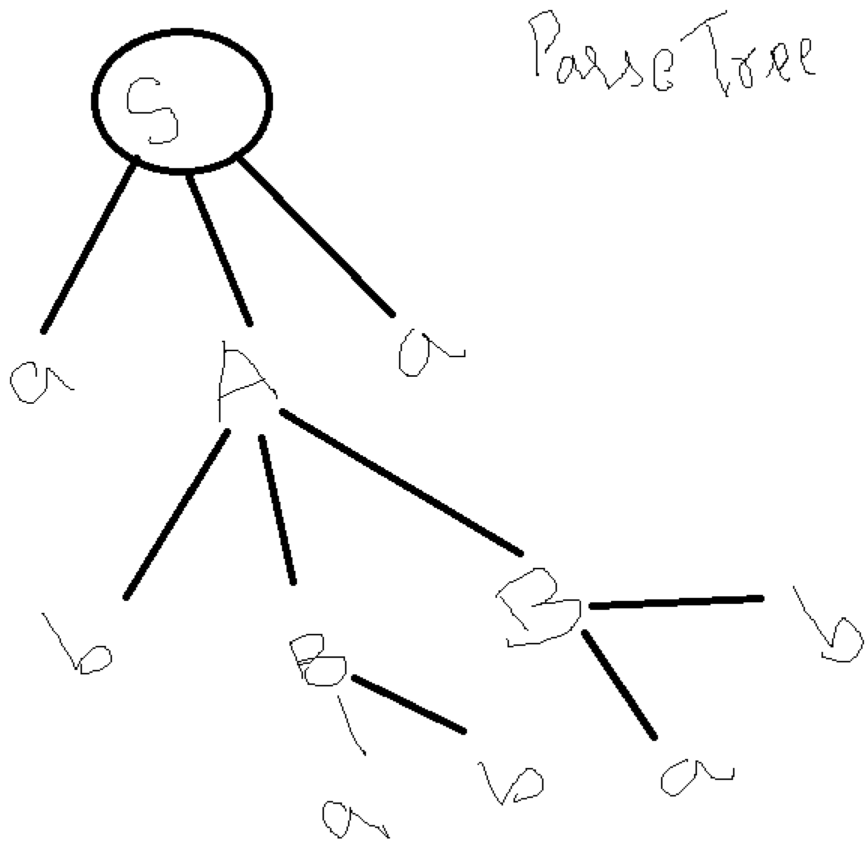
By Rule 1 symbol B is useful

Useful Symbol={B}

- Rule 2: grammar that contain combination of useful nonterminal & terminal is useful grammar or symbol
- By rule 2 Symbol S,A & C is useful
- Useful Symbol={B,S,A,C}
- Useless Symbol={}

Example 3 Solution Conti…

By reachable condition Symbol C is useless.



Useful Symbol={B,S,A}
Useless Symbol={C}

Example 3 Solution Conti…

Reduced grammar is

S->aAa

A->bBB

B->ab

Null productions:

- Null productions are of the form A -> ϵ
- Non-terminal N is called nullable, if there is a production N -> ϵ

- Steps for eliminating Null production from the grammar

1. If A -> ϵ is a production to be eliminated
2. Find all productions, whose right side contains A
3. Replace each occurrence of A in each of these productions to obtain the non ϵ-productions.
4. Add these resultant non ϵ-productions to the grammar to keep the language same.

Ex: 1 Eliminate the null productions from the following grammar

S -> ABAC

A -> aA / ϵ

B -> bB / ϵ

C -> c

To eliminate A->epsilon

S->**A**BAC S-> BAC

S-> AB**A**C S-> ABC

S-> **A**B**A**C S-> BC

A-> aA A->a

To eliminate B->epsilon

S -> A**B**AC S->AAC

B -> b**B** **B->b**

After removing A->epsilon

- S -> ABAC |BAC |ABC | BC

A -> aA / a

B -> bB / ϵ

C -> c

After removing B->epsilpon

- S -> ABAC |BAC |ABC | BC |AAC| AC |C

A -> aA / a

B -> bB / b

C -> c

Ex: 1 Eliminate the null productions from the following grammar

S -> ABAC

A -> aA / ϵ

B -> bB / ϵ

C -> c

To eliminate A->epsilon

S->ABAC S-> ϵBAC , S → BAC ✓

S → ABAC   S → ABϵC   S → ABC ✓

S → ABAC   S → ϵBϵC   S → BC ✓

A → aA   A → aϵ   A → a

Ex: 1 Solution Conti…

- After elimination of A->epsilon, the G becomes
- S -> ABAC | BAC | ABC | BC
  A -> aA | a
  B -> bB / ϵ
  C -> c
- To eliminate B->epsilon, put B-> epsilon on the RHS of production where B is appearing

Ex: 1 Solution Conti…

- S -> ABAC | BAC | ABC | BC
  A -> aA | a
  B -> bB | b
  C -> c
- S-> ABAC S->A AC

$S \to AAC$ ✓

$S \to BAC$ $S \to \epsilon AC$ $S \to AC$ ✓

$S \to ABC$ $S \to A\epsilon C$ $S \to AC$ ✓

$S \to BC$ $S \to \epsilon C$ $S \to C$ ✓

Ex: 1 Solution Conti…

- After elimination of B->epsilon, the G becomes
- S -> ABAC | BAC | ABC | BC |AAC |AC | C
  A -> aA | a
  B -> bB | b
  C -> c

Example 2: Eliminate the null productions from the following grammar
S -> aSb/aAb/ab/a

- A -> ε

Example 2: Eliminate the null productions from the following grammar

S -> aSb/aAb/ab/a

- A -> ε
- To eliminate A-> epsilon, put A->epsilon wherever A is appearing on the R.H.S of productions

S -> a ∈ b   S -> ab

- S->aAb

The new G will be

S -> aSb/ab/a/ab

Rewrite the G as

S -> aSb/a/ab

Ex: 2 Solution Conti…

S -> aSb/aAb/ab/a

- A -> ε
- To eliminate A-> epsilon, put A->epsilon wherever A is appearing on the R.H.S of productions

- Put A→ ε in the given production
- S->aAb

S → aεb S → ab

After elimination of A→ epsilon

1. we get the S→ ab as the new production.
2. This new production will be added in the grammar & A->epsilon will be deleted from the grammar

Ex: 2 Solution Conti…

- New Grammar After elimination of A→epsilon is

S -> aSb/ab/ab/a

We can Rewrite the grammar as

S -> aSb/ab/a

Example 3: Eliminate the null productions from the following grammar

S -> AB

A -> aAA/ε

B -> bBB/ε

To eliminate A-> ε

A->a A A   A -> aεA -> aA ✓

A -> aAε   A -> aA ✓

A -> aεε   A -> a ✓

S -> AB   S -> εB   S -> B ✓

Solution of Ex 3 Conti….

Elimination of Null Productions

- Grammar After elimination of A-> ϵ
- Is S -> AB | B

A -> aAA/aA | aA | a

B -> bBB/ε

To eliminate B-> ϵ

$S \to AB \quad S \to A\epsilon \quad S \to A$ ✓

$S \to B \quad S \to \epsilon$ ✓ $\quad B \to bB$ ✓

$B \to bBB \quad B \to b\epsilon B \quad B \to bB$ ✓

$B \to bB\epsilon \to bB \quad B \to b\epsilon\epsilon \quad B \to b$ ✓

S -> ABCd

A -> BC

B -> bB | λ

C -> cC | λ

To eliminate B->∧

Put B->∧ in the RHS of production wherever its appering.

S->A∧Cd->ACd

Put B->∧ in A -> BC

A -> ^C->C

Put B->∧ in B->bB

B-> b ^ → b

After eliminating B->∧, the grammar becomes

S->ABCd |ACd

A-> BC | C

B-> bB | b

C-> cC |^

- To eliminate C->∧
- Put C->∧ in the RHS of production wherever its appearing
- Put C->∧ in S -> ABCd
- S->AB^d->Abd
- Put C->∧ in S->**ACd**
- **S-> A^d→ Ad**

Put C->∧ in **A-> BC | C**

A-> B,

Put C->∧ in **C-> cC it will generate C→c**

- **S->ABCd |Acd |Abd |Ad**
- **A-> BC | C | B**
- **B-> bB | b**
- **C-> cC |c**
-

Unit Production:

Unit productions are the productions in which one non-terminal gives another non-terminal. Use the following steps to remove unit production:

$A \to B$

Steps for Elimination Unit Productions

- **Step 1:** To remove $X \rightarrow Y$, add production $X \rightarrow a$ to the grammar rule whenever $Y \rightarrow a$ occurs in the grammar.
- **Step 2:** Now delete $X \rightarrow Y$ from the grammar.
- **Step 3:** Repeat step 1 and step 2 until all unit productions are remove

Example 1: Eliminate Unit productions from the following grammar.

$S \to 0A \mid 1B \mid C$

$A \to 0S \mid 00$

$B \to 1 \mid A$

$C \to 01$

$S \rightarrow 0A \mid 1B \mid$ **01**

$A \rightarrow 0S \mid 00$

$B \rightarrow 1 \mid A$

$C \rightarrow 01$

Put C->01 in S->C

S->01

To remove B→ A

Put A→ 0S ,A→ 00 in B→ A

B→ 0S ,B→ 00 , B→ 1

- After removal of unit production the grammar becomes
- $S \rightarrow 0A \mid 1B \mid$ **01**
- $A \rightarrow 0S \mid 00$
- $B \rightarrow 1 \mid 0S \mid 00$
- $C \rightarrow 01$

$S \rightarrow 0A \mid 1B \mid C$

Example 1 Solution Conti…

$A \rightarrow 0S \mid 00$

$B \rightarrow 1 \mid A$

$C \rightarrow 01$

To remove S->C

We will put C->01 in S->C

We will get new production S->01

So G after removal of unit production S→ C will be:

$S \rightarrow 0A \mid 1B \mid 01$

$A \rightarrow 0S \mid 00$

$B \rightarrow 1 \mid A$

$C \rightarrow 01$

Example 1 Solution Conti…

- To remove the unit production B→A, we will put all the production of variable A i.e.

$A \rightarrow 0S \mid 00$ in the above production.

So the new production will be

B→0S & B→00

This new production will be added to the grammar.

After removal of Unit production B→ A

The G will be:

$S \rightarrow 0A \mid 1B \mid 01$

$A \rightarrow 0S \mid 00$

$B \rightarrow 1 \mid 0S \mid 00$

$C \rightarrow 01$

Example 1 Solution Conti…

- Grammar after elimination of B->A

$S \rightarrow 0A \mid 1B \mid 01$

$A \rightarrow 0S \mid 00$

$B \rightarrow 1 \mid 0S \mid 00$

$C \rightarrow 01$

Example 2: T-> T+R| R

R-> R*V| V

V→(T)| u

To remove Unit production R→ V, Put

V→(T)| u in R→ V

New productions of R →(T)| u

After removal of unit production R→ V the grammar becomes

T-> T+R| R

R-> R*V| (T)| u

V→(T)| u

T-> T+R| R

R-> R*V| (T)| u

V→(T)| u

To remove Unit production T→ R, Put

R-> R*V| (T)| u in T→ R

We will get new productions of T as

T-> R*V| (T)| u

After removal of unit production T→R the grammar becomes

T-> T+R| R*V| (T)| u

R-> R*V| (T)| u

V→(T)| u

Eliminate Unit productions from the following grammar

Example 2: T-> T+R| R

R-> R*V| V

V->(T)| u

To eliminate T→ R

T-→ T+ R | R* V | V

Grammar will be:

T-> T+R| R*V | V

R-> R*V| V

V->(T)| u

Example 2 Solution Conti…

- To eliminate T-> V put RHS of V in this production.
- T→>(T)| u this new production will be generated

G will be after removal of T→ V

T-> T+R| R*V | (T)| u

R-> R*V| V

V->(T)| u

To eliminate R→ V put RHS of V in this production

New production we will get as below

R-→(T)| u

And this will be added in the grammar & R—V will be deleted

Example 2 Solution Conti…

Grammar after removal of R→ V will be

T-> T+R| R*V | (T)| u

R-> R*V| (T)| u

V->(T)| u

G=({T,R,V},{u,+,*,(,)},P,{T})