UNIT-III

Subject-Theory of Computation

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Grammar & Language

3.1.1 DEFINITION OF A GRAMMAR

Definition 3.1 A phrase-structure grammar (or simply a grammar) is (V_N, Σ, P, S), where

(i) V_N is a finite nonempty set whose elements are called variables,
(ii) Σ is a finite nonempty set whose elements are called terminals,
(iii) $V_N \cap \Sigma = \emptyset$,
(iv) S is a special variable (i.e. an element of V_N) called the start symbol, and
(v) P is a finite set whose elements are $\alpha \to \beta$, where α and β are strings on $V_N \cup \Sigma$. α has at least one symbol from V_N. Elements of P are called productions or production rules or rewriting rules.

production, it is not necessary that $AB \to S$ is a

XAMPLE 3.1

$$G = (V_N, \Sigma, P, S) \text{ is a grammar}$$

here

$$V_N = \{\langle\text{sentence}\rangle, \langle\text{noun}\rangle, \langle\text{verb}\rangle, \langle\text{adverb}\rangle\}$$

$$\Sigma = \{\text{Ram, Sam, ate, sang, well}\}$$

$$S = \langle\text{sentence}\rangle$$

onsists of the following productions:

$\langle\text{sentence}\rangle \to \langle\text{noun}\rangle\,\langle\text{verb}\rangle$
$\langle\text{sentence}\rangle \to \langle\text{noun}\rangle\,\langle\text{verb}\rangle\,\langle\text{adverb}\rangle$
$\langle\text{noun}\rangle \to \text{Ram}$
$\langle\text{noun}\rangle \to \text{Sam}$
$\langle\text{verb}\rangle \to \text{ate}$
$\langle\text{verb}\rangle \to \text{sang}$
$\langle\text{adverb}\rangle \to \text{well}$

1. Ram ate.
2. Ram ate well.
3. ate Ram well

(i) Reverse substitution is not permitted. For example, if $S \to AB$ is a production, then we can replace S by AB, but we cannot replace AB by S.

(ii) No inversion operation is permitted. For example, if $S \to AB$ is a production, it is not necessary that $AB \to S$ is a production.

G

Definition 3.4 The language generated by a grammar G (denoted by $L(G)$) is defined as $\{w \in \Sigma^* \mid S \overset{*}{\underset{G}{\Rightarrow}} w\}$. The elements of $L(G)$ are called sentences.

Stated in another way, $L(G)$ is the set of all terminal strings derived from the start symbol S.

G_1 and G_2 are equivalent if $L(G_1) = L(G_2)$.

- If G=({S}, {0,1}, {S->0S1, S->Λ }, S) find L(G).

V_N Σ P S

- If G=({S}, {0,1}, {S->0S1, S->Λ }, S) find L(G).

S->Λ

S->0S1
->0Λ1 =01

S->0S1
->00S11
->00Λ11 =0011

S->0S1
->00S11->000S111->000Λ111 -> 000111

L(G) ={Λ,01,0011,000111,00001111,......}

1. It always starts with 0's
2. it ends with 1. all the 0's are at the begining and 1's are at the end.
3. No of 0's and 1's are equal.

$\boxed{L(G)=\{0^n 1^n \mid n>=0\}}$

- If G=({S}, {a}, {S->SS}, S), find the language generated by G.

S->SS

We can not call SS as a sentence as it contains only varible or NON terminal symbols.

Language generated by this grammar is NULL as no production contains a terminal symbol.

L(G)={ }= ϕ

- Let G=({S,C}, { a, b }, P, S) where P consist of S-> aCa, C->aCa | b. Find L(G).

- Let G=({S,C}, { a, b }, P, S) where P consist of S-> aCa, C->aCa | b. Find L(

S->aCa
->aba

L(G)={aba, aabaa, aaabaaa,aaaabaaaa....}

S->aCa
->aaCaa
->aabaa

1. It always starts and ends with 'a'.
2. All sentence has only one 'b' and it always appea
in the centre.
3. No of leading and trailing 'a' are equal.

->aaCaa
->aaaCaaa
- > aaabaaa

L(G)={ $a^n b a^n$ | n>=1 }

HW

1. S->0S1 | 0A1, A->1A |1
2. S->0S1 | 0A1, A-> 1A0 |10

- Find L(G)
- S->0S1 | 0A1, A->1A |1

$L(G) = \{0^m 1^n \mid n > m, \; n, m \geq 1\}$

S->0S1
->00A11
->001A11
->001111

S->0A1
->011

S->0A1
->01A1
->0111

L(G)={011,0111,001111.....}

1.Lanuage starts with '0' and ends with '1'.
2. No of '0's and 1's are not equal.
3.All 0's appear before 1.
4. No of 1's are greater than no of 1's.

L(G)={ $0^m 1^n \mid m \geq \cancel{0}\, 1, \; n \geq \cancel{0}\, 1, \; n > m$ }

$L(G) = \{0^m 1^n \mid m \geq 1, \; n \geq 1, \; n > m\}$

- Find L(G) $L(G) = \{0^n 1^m 0^m 1^n \mid n \geq 1, m \geq 1\}$
- S->0S1 | 0A1, A-> 1A0 |10

4. No.

S->0S1 | 0A1, A-> 1A0 |10

S->0S1
->00A11
->001011----

001011

S->0A1
->0101----

0101

S->0A1
->01A01
->011001---

011001

S-> 0S1->01A01->011A001->01110001----

1.language starts with '0'.
2.language ends with '1'.
3. No. of 0's and 1's are same.

$L(G)=\{0^m 1^n 0^n 1^m \mid m, n \geq 1$

$L(G) = \{0^m 1^n 0^n 1^m \mid m, n \geq 1\}$

Ans

- Let L be the set of all palindromes over $\{a, b\}^*$. Construct G generating L.

$G=\{V_N, \Sigma, P, S\}$

$\Sigma = \{a, b\}$

palindrome: it starts with and ends with same symbol. if we read it it from the front or from back it is a same string
e.g. MADAM

a, b, aa, bb, aaa, bbb

L(G)={Λ,aba,bab,abba,baab,aabaa,bbabb......}

G=({S},{a,b}, P, {S})
where prductions P are given->
below
S->Λ S->a, S->b
S->aSa, S->bSb

- Find a grammar generating $L=\{a^n b^n c^i \mid n>=1, i>=0\}$

- Find a grammar generating $L=\{a^n b^n c^i \mid n>=1, i>=0\}$

$L(G)=\{ab, abc, abcc, aabb, abccc, aabbc, ...\}$

Language consist of 2 types of sentences,1. which of the type anbn another which is ending with c. Symbol 'C' can be present or absent in the sentence.

$G=(\{S,A\},\{a,b,c\},P,\{S\})$

where P is given below

$S \rightarrow A \mid Sc$

$A \rightarrow ab \mid aAb$

- Find a grammar generating $\{a^j b^n c^n \mid n >= 1, j >= 0\}$

G=({S,A},{a,b,c},P,{S})
where P is given below

S-> A | aS
A-> bc | bAc

$\wedge c \wedge$ $\quad c$ $\quad w = \wedge$ $\quad w^T = \wedge$

$\wedge c a$ $\quad = w = a$ $\quad w^T = a$

- Construct a grammar generating

L= {wcw^T | w { a, b}*}

$L(G) =$

$L(G) = \{c, aca, bcb, abcba, bacab, aacaa \ldots\}$

$w = b \quad w^T = b$ - bcb

$w = ab \quad w^T = ba$

$w = ba \quad w^T = ab$

$G = \{V_N, \Sigma, P, S\}$

$S \to c$ $\quad P_1$

$S \to aSa$ $\quad P_2$

$S \to bSb$ $\quad P_3$

abcba

$G = (\{S\}, \{a, b\}, P, \{S\})$

abcba

- Construct a grammar generating

$L = \{wcw^T \mid w \in \{a, b\}^*\}$

$w = \lambda \quad w^T = \lambda$
$= a \quad = a$
$= b \quad = b$
$= ab \quad = ba$
$= ba \quad = ab$

$L(G) = \{c, aca, bcb, abcba, bacab, \ldots\}$

$G = (\{S\}, \{a, c, b\}, P, S)$

where P is

$\boxed{S \to c,\ S \to aSa,\ S \to bSb}$ Ans

- Find a grammar generating $\{0^n 1^{2n} \mid n >= 1\}$

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$L(G)=\{011,001111,000111111,......\}$

$G=(\{S\},\{0,1\},\{S\text{->}011,S\text{->}0S11,\{S\})$

$S\text{->}011 \mid 0S11$

$L(G) = \{011, 001111 ......\}$

$S \Rightarrow 011 \mid 0S11$

Language constraints

1.starts with 0 and ends with1.

2. all strings contain 0in the begining and all 1's at the end.

3. no. of 1's are twice to the no. of 0's.

- Find a grammar generating
- $L(G)=\{0^n1^n \mid n >=1\} U \{1^m0^m \mid m >=1\}$

- Find a grammar generating
- $L(G)=\{0^n1^n \mid n >=1\} \cup \{1^m0^m \mid m >=1\}$

$L(G) = L(G_1) \cup L(G_2)$

$S \to A \mid B$

$A \to 01 \mid 0A1$

$B \to 10 \mid 1B0$

$G = (\{S, A, B\}, \{0, 1\}, P, \{S\})$

- HW

Find a grammar generating $\{0^n 1^m 0^m 1^n \mid n >= 1, m >= 1\}$

Find a grammar generating $\{0^n 1^m 0^m 1^n \mid n >= 1, m >= 1\}$

Lang
1. it starts with 0 and end with1.
2.the no. of leading 0's and 1's are equal.
3. in the middle the sequencw 1 and 0's and here yhe no. of 1's and 0's are equal.

L(G)={0101,011001,001011,......}

S->0S1 | 0A1

A-> 1A0 | 10

G=({S,A},{0,1},{S->0S1,S->0A1, A->1A0, A->10}, {S})

S->0A1->01A01->01A01->011001

- $\{0^n 1^m 0^n \mid m,n>=1\} \cup \{0^n 1^m 2^m \mid m,n>=1\}$
-
- S->0A0 |B
- A->0A0 | 1 |1A
- B->0C | 0B
- C-> 12 |1C2

Ans