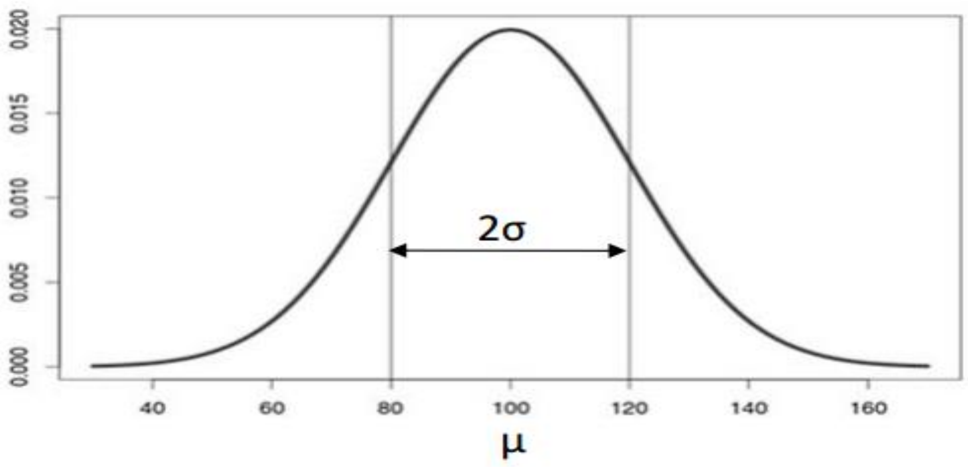
NORMAL DISTRIBUTIONS

UNDERSTANDING AND USING THEM

Properties of Normal Distribution

Basics of Normal Distribution

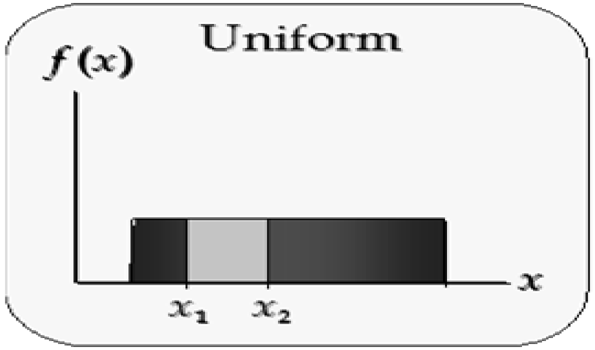


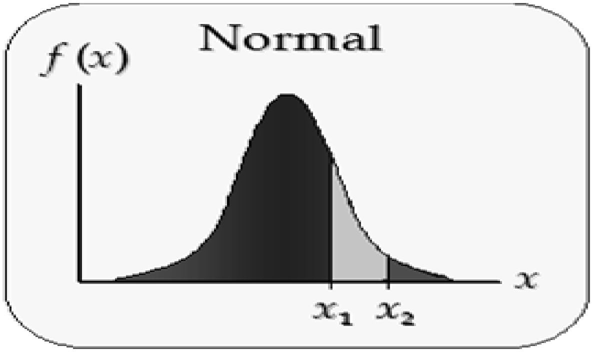
- The graph of the pdf (probability density function) is a bell shaped curve
- The normal random variable takes values from - ∞ to +∞
- It is symmetric and centered around the mean (which is also the median and mode)
- Any normal distribution can be specified with just two parameters – the mean (μ) and the standard deviation (σ)
- We write this as $X \sim N(\mu, \sigma^2)$

Probability Calculation for Continuous Distributions

- The probability associated with any single value of the random variable is always zero

- Probability of values being in a range = Area under the pdf curve in that range





- Area under the entire curve always equals 1

Z-scores, Standard Normal Distribution

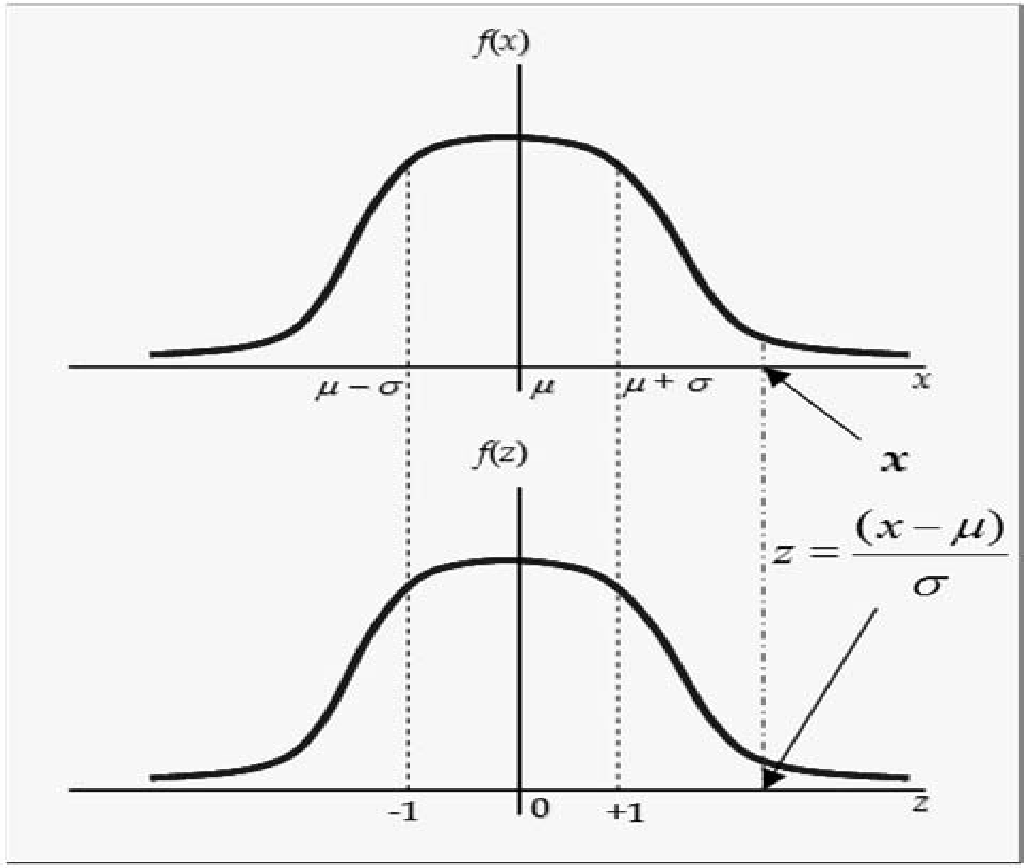
- For every value (x) of the random variable X, we can calculate its Z-score:

  $$z_i = \frac{x_i - \mu}{\sigma}$$

- Interpretation – How many standard deviations away is the value from the mean?

- If X~N(μ, σ^2), then
  - Z-scores have a normal distribution with μ=0 and σ=1

  i.e. Z ~ N(0,1)
  - Standard Normal Distribution

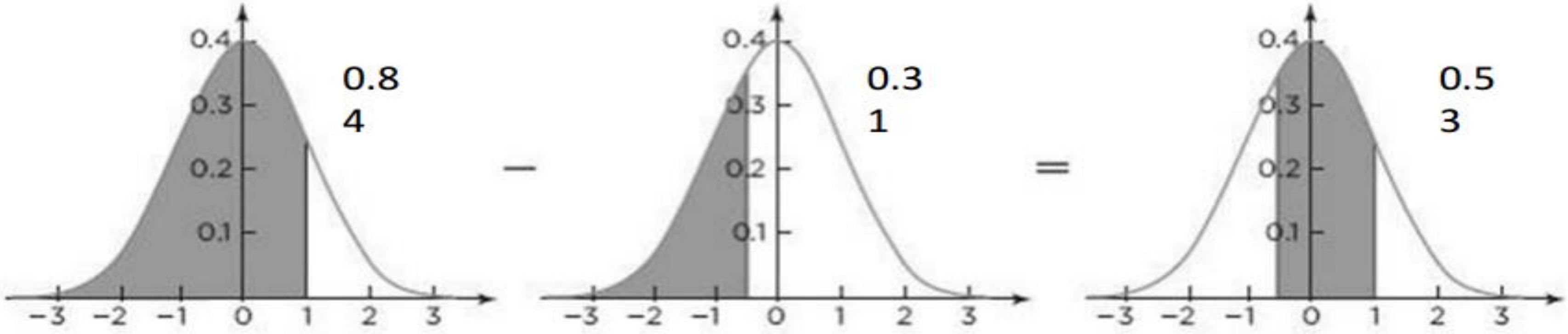


Probability Calculation for Normal Distribution (GMAT Scores)

- Suppose GMAT scores are distributed normally with $\mu = 711$ and $\sigma = 29$.
- What is $P(X \leq 680)$?
- Step 1: Calculate Z-score corresponding to 680
  - $Z = (680-711)/29 = -1.06$
- Step 2: Calculate the probabilities using Z-tables
  - $P(Z \leq -1) = 0.14$

More Normal Distribution Calculations

- What is $P(697 \leq X \leq 740)$?
- Step 1: Use $P(x_1 \leq X \leq x_2)$ = Use $P(X \leq x_2) - P(X \leq x_1)$
- Step 2: Calculate $P(P(X \leq x_2)$ and $P(X \leq x_1)$ as before
  - $P(X \leq 740) = P(Z \leq 1) = 0.84$
  - $P(X \leq 697) = P(Z \leq -0.5) = 0.31$
- Step 3: Calculate $P(697 \leq X \leq 740) = 0.84 - 0.31 = 0.53$



Another Example

- Suppose a packaging system fills boxes such that the weights are normally distributed with a μ = 16.3 oz and σ = 0.2 oz. The package label states the weight as 16 oz.

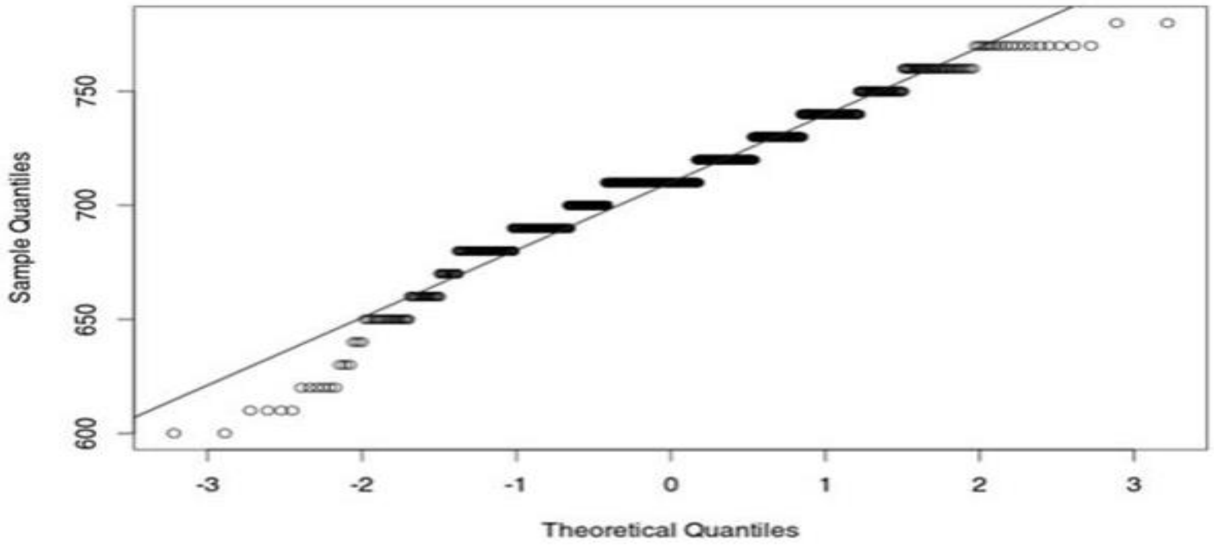
  - What is the probability that a randomly picked box is underweight?

  - To what weight should the mean of the process be adjusted so that the chance of an underweight box is only 0.005?
  - Step 1: Find z such that $P(Z \leq z) = 0.005$
  - Using the second page of Z-tables, z = -2.5758
  - Step 2: Find new mean weight (μ) for process

$$\frac{16-\mu}{0.2} = -2.5758$$

$$\Rightarrow \mu = 16 + 0.2(2.5758) \approx 16.52$$

Normal Quantile (Q-Q) Plot



- Nearly normal if the data track the diagonal reference line on the plot
- Deviations often likely at extremes, and the bands help judge the severity of the deviation

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Summary of Session

- Normal distribution is a symmetric, continuous probability distribution that is uniquely specified by a mean and standard deviation
- Every normal distribution can be converted into a standard normal distribution (Z-score)
- Sum of normally distributed random variables is a normally distributed random variable