152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x + 10)$ revolutions.
$(x + 10) \times 3\pi = x \times 2\pi \Leftrightarrow 3x + 30 = 2x \Leftrightarrow x = 30.$
Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground = 17.5 m. Radius of inner circle = (17.5 − 1.4) m = 16.1 m.

$$\text{Area of the garden} = \pi \times [(17.5)^2 - (16.1)^2]\ \text{m}^2 = \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right] \text{m}^2$$
$$= \left(\frac{22}{7} \times 33.6 \times 1.4\right) \text{m}^2 = 147.84\ \text{m}^2.$$

154. $2\pi R = 440 \Leftrightarrow 2 \times \frac{22}{7} \times R = 440 \Leftrightarrow R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius = (70 − 7) m = 63 m.

$$\text{Area of the border} = \pi\,[(70)^2 - (63)^2]\ \text{m}^2$$
$$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right] \text{m}^2 = 2926\ \text{m}^2.$$

155. $\pi R_1^2 = 616 \Leftrightarrow R_1^2 = \left(616 \times \frac{7}{22}\right) = 196 \Leftrightarrow R_1 = 14$ cm.

$\pi R_2^2 = 154 \Leftrightarrow R_2^2 = \left(154 \times \frac{7}{22}\right) = 49 \Leftrightarrow R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm = (14 − 7) cm = 7 cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \Leftrightarrow 2\pi\,(R_1 - R_2) = 132 \Leftrightarrow (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

$\therefore$ Required width = 21 m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall = (R + 4) ft.

$$\text{Area of the concrete wall} = \pi\,[(R + 4)^2 - R^2]\ \text{sq. ft}$$
$$= [\pi\,(R + 4 + R)(R + 4 - R)]\ \text{sq. ft} = 8\pi\,(R + 2)\ \text{sq. ft.}$$

$$8\pi\,(R + 2) = \frac{11}{25}\pi R^2 \Leftrightarrow 11R^2 = 200\,(R + 2) \Leftrightarrow 11R^2 - 200R - 400 = 0$$
$$\Leftrightarrow 11R^2 - 220R + 20R - 400 = 0$$
$$\Leftrightarrow 11R\,(R - 20) + 20\,(R - 20) = 0$$
$$\Leftrightarrow (R - 20)(11R + 20) = 0 \Leftrightarrow R = 20.$$

$\therefore$ Radius of the pool = 20 ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \Leftrightarrow \frac{R_1}{R_2} = \frac{23}{22} \Leftrightarrow R_1 = \frac{23}{22} R_2.$

Also, $R_1 - R_2 = 5$ m $\Leftrightarrow \frac{23R_2}{22} - R_2 = 5 \Leftrightarrow R_2 = 110.$

$\therefore$ Diameter of inner circle = (2 × 110) m = 220 m.

159. Area of the semi-circle $= \frac{1}{2}\pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) \text{m}^2 = 77\ \text{m}^2.$

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm = (99 + 63) cm = 162 cm.

161. Given : $\pi R + 2R = 36 \Leftrightarrow (\pi + 2) R = 36 \Leftrightarrow R = \dfrac{36}{\left(\dfrac{22}{7}+2\right)}$ cm $= \left(\dfrac{36 \times 7}{36}\right)$ cm $= 7$ cm.

$\therefore$ Required area $= \pi R^2 = \left(\dfrac{22}{7} \times 7 \times 7\right)$ cm^2 = 154 cm^2.

162. Length of each side of the square $= \sqrt{81}$ cm = 9 cm.

Length of wire $= (9 \times 4)$ cm = 36 cm.

$\pi R + 2R = 36 \Leftrightarrow (\pi + 2) R = 36 \Leftrightarrow R = \dfrac{36}{\left(\dfrac{22}{7}+2\right)} = 7$ cm.

Area of the semi-circle $= \dfrac{1}{2}\pi R^2 = \left(\dfrac{1}{2} \times \dfrac{22}{7} \times 7 \times 7\right)$ cm^2 = 77 cm^2.

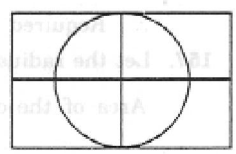
163. Area of the sector $= \left(\dfrac{1}{2} \times \text{arc} \times R\right) = \left(\dfrac{1}{2} \times 3.5 \times 5\right)$ cm^2 = 8.75 cm^2.

164. Area of the sector $= \dfrac{\pi R^2 \theta}{360} = \left(\dfrac{22}{7} \times 7 \times 7 \times \dfrac{108}{360}\right)$ cm^2 = 46.2 cm^2.

165. Side of the square $= \dfrac{120}{4}$ cm = 30 cm.

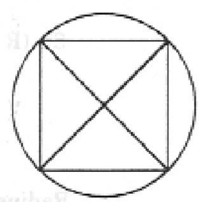
Radius of the required circle $= \left(\dfrac{1}{2} \times 30\right)$ cm = 15 cm.

Area of the required circle $= [\pi \times (15)^2]$ cm^2 $= \left[\dfrac{22}{7} \times (15)^2\right]$ cm^2.



166. Radius of the required circle $= \left(\dfrac{1}{2} \times 14\right)$ cm = 7 cm.

Area of the circle $= \left(\dfrac{22}{7} \times 7 \times 7\right)$ cm^2 = 154 cm^2.



167. $\pi R^2 = 220 \Leftrightarrow R^2 = \left(220 \times \dfrac{7}{22}\right) = 70.$

Now, $R = \dfrac{1}{2} \times (\text{diagonal}) \Leftrightarrow \text{diagonal} = 2R.$

$\therefore$ Area of the square $= \dfrac{1}{2} \times (\text{diagonal})^2$

$= \left(\dfrac{1}{2} \times 4R^2\right) = 2R^2 = (2 \times 70)$ cm^2 = 140 cm^2.

168. Given R = 4 cm. $R = \dfrac{1}{2} \times (\text{diagonal of the square}) \Leftrightarrow \text{diagonal} = 2R = 8$ cm.

Required area $= \pi R^2 - \dfrac{1}{2} \times (8)^2 = (\pi \times 16 - 32) = (16\pi - 32)$ cm^2.

169. $2\pi R = 100 \Leftrightarrow R = \dfrac{100}{2\pi} = \dfrac{50}{\pi}.$

$R = \dfrac{1}{2} \times \text{diagonal} \Leftrightarrow \text{diagonal} = 2R = \dfrac{2 \times 50}{\pi} = \dfrac{100}{\pi}.$

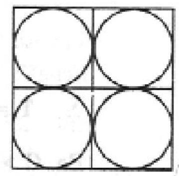
$\therefore$ Area of the square $= \frac{1}{2} \times (\text{diagonal})^2 = \frac{1}{2} \times \left(\frac{100}{\pi}\right)^2$

$\Leftrightarrow \quad a^2 = \frac{1}{2} \times \left(\frac{100}{\pi}\right)^2 \Leftrightarrow a = \frac{1}{\sqrt{2}} \times \frac{100}{\pi} = \frac{50\sqrt{2}}{\pi}$ cm.

170. Side of square paper $= \sqrt{784}$ cm $= 28$ cm.

Radius of each circular plate $= \left(\frac{1}{4} \times 28\right)$ cm $= 7$ cm.

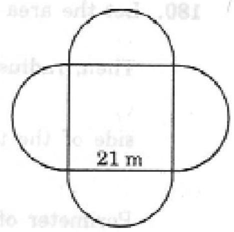
Circumference of each circular plate $= \left(2 \times \frac{22}{7} \times 7\right)$ cm $= 44$ cm.



171. Length of the fence $= 4\pi R$, where $R = \frac{21}{2}$ m

$= \left(4 \times \frac{22}{7} \times \frac{21}{2}\right)$ m $= 132$ m.

Cost of fencing $=$ Rs. $\left(132 \times \frac{25}{2}\right) =$ Rs. 1650.



172. Radius of incircle of an equilateral triangle $= \frac{a}{2\sqrt{3}}$.

Radius of circumcircle of an equilateral triangle $= \frac{a}{\sqrt{3}}$.

$\therefore$ Required ratio $= \frac{\pi a^2}{12} : \frac{\pi a^2}{3} = \frac{1}{12} : \frac{1}{3} = 1 : 4$.

173. Radius of circumcircle $= \left(\frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}}\right)$ cm $= 4\sqrt{3}$ cm.

174. Radius of incircle $= \frac{a}{2\sqrt{3}} = \frac{42}{2\sqrt{3}}$ cm $= 7\sqrt{3}$ cm.

Area of incircle $= \left(\frac{22}{7} \times 49 \times 3\right)$ cm^2 $= 462$ cm^2.

175. Radius of incircle $= \frac{a}{2\sqrt{3}}$. Area of incircle $= \left(\frac{\pi \times a^2}{12}\right)$ cm^2.

$\therefore \quad \frac{\pi a^2}{12} = 154 \Leftrightarrow a^2 = \frac{154 \times 12 \times 7}{22} \Leftrightarrow a = 14\sqrt{3}$.

$\therefore$ Perimeter of the triangle $= (3 \times 14\sqrt{3})$ cm $= (42 \times 1.732)$ cm $= 72.7$ cm (approx.).

176. We have : $a = 6$, $b = 11$, $c = 15$. $s = \frac{1}{2}(6 + 11 + 15) = 16$.

Area of the triangle, $\Delta = \sqrt{16 \times 10 \times 5 \times 1} = 20\sqrt{2}$ cm^2.

Radius of incircle $= \frac{\Delta}{s} = \frac{20\sqrt{2}}{16} = \frac{5\sqrt{2}}{4}$ cm.

177. Let the radius of incircle be r cm. Then, $2\pi r = 88 \Leftrightarrow r = \left(88 \times \frac{7}{22} \times \frac{1}{2}\right) = 14$.

Semi-perimeter, $s = \left(\frac{30}{2}\right)$ cm $= 15$ cm.

$\therefore$ Area of the triangle $= r \times s = (14 \times 15)$ cm^2 $= 210$ cm^2.

178. Radius $= \dfrac{\text{Area}}{\text{Semi-perimeter}} = \left(\text{Area} \times \dfrac{2}{\text{Area}}\right) = 2.$

179. Let the perimeter of each be a.

Then, side of the equilateral triangle $= \dfrac{a}{3}$; side of the square $= \dfrac{a}{4}$;

radius of the circle $= \dfrac{a}{2\pi}$.

$\therefore \quad T = \dfrac{\sqrt{3}}{4} \times \left(\dfrac{a}{3}\right)^2 = \dfrac{\sqrt{3}\,a^2}{36}$; $S = \left(\dfrac{a}{4}\right)^2 = \dfrac{a^2}{16}$; $C = \pi \times \left(\dfrac{a}{2\pi}\right)^2 = \dfrac{a^2}{4\pi} = \dfrac{7a^2}{88}$.

So, $C > S > T$.

180. Let the area of each be a.

Then, radius of the circle $= \dfrac{\sqrt{a}}{\pi}$; side of the square $= \sqrt{a}$;

side of the triangle $= \sqrt{\dfrac{a \times 4}{\sqrt{3}}}$.

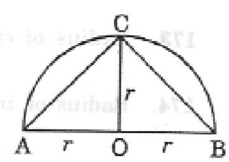
Perimeter of the circle $= 2\pi\sqrt{\dfrac{a}{\pi}} = 2\sqrt{\pi a} = 2\sqrt{3.14 \times a} = 2 \times 1.77\sqrt{a} = 3.54\sqrt{a}$.

Perimeter of the square $= 4\sqrt{a}$;

Perimeter of the triangle $= 3 \times \sqrt{\dfrac{4a}{1.732}} = 3 \times \sqrt{2.31a} = 3 \times 1.52\sqrt{a} = 4.56\sqrt{a}$.

Clearly, perimeter of the triangle is the greatest.

181. Required area $= \dfrac{1}{2} \times \text{base} \times \text{height} = \left(\dfrac{1}{2} \times 2r \times r\right) = r^2$.



182. Required area $= \dfrac{\pi}{2} \times \left(\dfrac{AC}{2}\right)^2 = \dfrac{\pi}{2} \times \dfrac{AC^2}{4} = \dfrac{\pi}{2} \times \dfrac{AB^2 + BC^2}{4}$

$= \dfrac{\pi}{2} \times \left(\dfrac{AB^2}{4} + \dfrac{BC^2}{4}\right) = \dfrac{\pi}{2} \times \left(\dfrac{AB}{2}\right)^2 + \dfrac{\pi}{2} \times \left(\dfrac{BC}{2}\right)^2 = 81 + 36 = 117 \text{ cm}^2$.

183. Let original radius be R cm. Then, original circumference $= (2\pi R)$ cm.

New radius $= (175\% \text{ of R})$ cm $= \left(\dfrac{175}{100} \times R\right)$ cm $= \dfrac{7R}{4}$ cm.

New circumference $= \left(2\pi \times \dfrac{7R}{4}\right)$ cm $= \dfrac{7\pi R}{2}$ cm.

Increase in circumference $= \left(\dfrac{7\pi R}{2} - 2\pi R\right)$ cm $= \dfrac{3\pi R}{2}$ cm.

Increase% $= \left(\dfrac{3\pi R}{2} \times \dfrac{1}{2\pi R} \times 100\right)\% = 75\%$.

184. Let original diameter be d metres. Then, its circumference $= (\pi d)$ metres.

Time taken to cover $(8\pi d)$ m $= 40$ min.

New diameter $= (10d)$ m. Then, its circumference $= (\pi \times 10d)$ m.

$\therefore$ Time taken to go round it once $= \left(\dfrac{40}{8\pi d} \times 10\pi d\right)$ m $= 50$ min.

185. Let the original radius be R cm. New radius $= \left(\frac{106}{100}R\right)$ cm $= \left(\frac{53R}{50}\right)$ cm.

Original area $= \pi R^2$.

$$\text{Increase in area} = \pi\left(\frac{53R}{50}\right)^2 - \pi R^2 = \pi R^2\left[\left(\frac{53}{50}\right)^2 - 1\right] = \frac{\pi R^2\,[(53)^2-(50)^2]}{2500}$$

$$= \frac{\pi R^2\,(103 \times 3)}{2500}\ m^2.$$

$$\text{Increase\%} = \left(\frac{\pi R^2 \times 309}{2500} \times \frac{1}{\pi R^2} \times 100\right)\% = 12.36\%.$$

186. Let the original radius be R cm.

New radius = (90% of R) cm $= \left(\frac{90}{100} \times R\right)$ cm $= \frac{9R}{10}$ cm.

Original area $= \pi R^2$.

$$\text{Diminished area} = \left[\pi R^2 - \pi\left(\frac{9R}{10}\right)^2\right] cm^2 = \left[\left(1 - \frac{81}{100}\right)\pi R^2\right] cm^2 = \left(\frac{19}{100}\pi R^2\right) cm^2.$$

$$\text{Decrease\%} = \left(\frac{19\pi R^2}{100} \times \frac{1}{\pi R^2} \times 100\right)\% = 19\%.$$

187. Let the original radius be R cm. New radius = 2R.

Original area $= \pi R^2$, New area $= \pi\,(2R)^2 = 4\pi R^2$.

Increase in area $= (4\pi R^2 - \pi R^2) = 3\pi R^2$.

$$\text{Increase\%} = \left(\frac{3\pi R^2}{\pi R^2} \times 100\right)\% = 300\%.$$

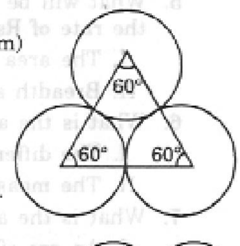
188. $2\pi R_1 = 4\pi$ and $2\pi R_2 = 8\pi \;\Rightarrow\; R_1 = 2$ and $R_2 = 4$

$\Rightarrow$ Original area $= (4\pi \times 2^2) = 16\pi$, Increased area $= (4\pi \times 4^2) = 64\pi$.

Thus, the area quadruples.

189. Required area = (Area of an equilateral Δ of side 7 cm)

− (3 × area of sector with $\theta = 60°$ & $r = 3.5$ cm)

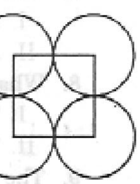


$$= \left[\left(\frac{\sqrt{3}}{4} \times 7 \times 7\right) - \left(3 \times \frac{22}{7} \times 3.5 \times 3.5 \times \frac{60}{360}\right)\right] cm^2$$

$$= \left(\frac{49\sqrt{3}}{4} - 11 \times 0.5 \times 3.5\right) cm^2 = (21.217 - 19.25)\ cm^2 = 1.967\ cm^2.$$

190. Required area $= \left(14 \times 14 - 4 \times \frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) cm^2$

$= (196 - 154)\ cm^2 = 42\ cm^2$.



191. Required area $= \left(63 \times 63 - 4 \times \frac{1}{4} \times \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2}\right) m^2 = 850.5\ m^2$.

EXERCISE 24B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 11*) : *Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and*

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The area of a playground is 1600 m^2. What is its perimeter ? **(Bank P.O. 2003)**
   I. It is a perfect square playground.
   II. It costs Rs. 3200 to put a fence around the playground at the rate of Rs. 20 per metre.
2. What is the area of the rectangle ?
   I. The ratio of the length and the breadth is 3 : 2.
   II. The area of the rectangle is 3.6 times its perimeter.
3. Area of a square is equal to the area of a circle. What is the circumference of the circle ?
   I. The diagonal of the square is x inches.
   II. The side of the square is y inches. **(S.B.I.P.O. 2003)**
4. The area of a rectangle is equal to the area of a right-angled triangle. What is the length of the rectangle ? **(Bank P.O. 2003)**
   I. The base of the triangle is 40 cm.
   II. The height of the triangle is 50 cm.
5. What will be the cost of gardening a strip of land inside around a circular field, at the rate of Rs. 85 per sq. metre ?
   I. The area of the field is 1386 sq. metres.
   II. Breadth and length of the field are in the ratio of 3 : 5 respectively.
6. What is the area of the rectangle ? **(Bank P.O. 2003)**
   I. The difference between the sides is 5 cm.
   II. The measure of its diagonal is 10 cm.
7. What is the area of the circle ?
   I. An arc of length 4 cm subtends an angle of 60° at the centre.
   II. A chord of length 5 cm subtends an angle of 90° at the centre.
8. What is the area of the circle ? **(NABARD, 2002)**
   I. The circumference of the circle is 308 m.
   II. The radius of the circle is 28 m.
9. The area of a rectangle is equal to the area of a circle. What is the length of the rectangle ?
   I. The radius of the circle is equal to the breadth of the rectangle.
   II. The perimeter of the rectangle is 14 cm more than that of the circle.

10. What is the height of the triangle ? (Bank P.O. 2002)
    I. The area of the triangle is 20 times its base.
    II. The perimeter of the triangle is equal to the perimeter of a square of side 10 cm.
11. What will be the cost of painting the inner walls of a room if the rate of painting is Rs. 20 per square foot ? (Bank P.O. 2000)
    I. Circumference of the floor is 44 feet.
    II. The height of the wall of the room is 12 feet.

Directions (*Questions 12 to 18*) : *Each of the questions below cons'sts of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

12. What is the area of rectangular field ? (Bank P.O. 2004)
    I. The perimeter of the field is 110 metres.
    II. The length is 5 metres more than the width.
    III. The ratio between length and width is 6 : 5 respectively.
    (*a*) I and II only (*b*) Any two of the three (*c*) All I, II and III
    (*d*) I, and either II or III only (*e*) None of these
13. What is the area of the hall ? (Bank P.O. 2003)
    I. Material cost of flooring per square metre is Rs. 2.50.
    II. Labour cost of flooring the hall is Rs. 3500.
    III. Total cost of flooring the hall is Rs. 14,500.
    (*a*) I and II only (*b*) II and III only (*c*) All I, II and III
    (*d*) Any two of the three (*e*) None of these
14. What is the length of the diagonal of the given rectangle ?
    I. The perimeter of the rectangle is 34 cm.
    II. The difference between the length and breadth is 7 cm.
    III. The length is 140% more than the breadth.
    (*a*) Any two of the three (*b*) All I, II and III (*c*) I, and either II or III
    (*d*) I and II only (*e*) II and III only
15. What is the cost of flooring the rectangular hall ? (R.B.I. 2002)
    I. Length and breadth of the hall are in the respective ratio of 3 : 2.
    II. Length of the hall is 48 m and cost of flooring is Rs. 85 per sq. m.
    III. Perimeter of the hall is 160 m and cost of flooring is Rs. 85 per sq. m.
    (*a*) I and II only (*b*) II and III only (*c*) III only
    (*d*) I, and either II or III only (*e*) Any two of the three
16. What is the area of a right-angled triangle ? (S.B.I.P.O. 2000)
    I. The perimeter of the triangle is 30 cm.
    II. The ratio between the base and the height of the triangle is 5 : 12.
    III. The area of the triangle is equal to the area of a rectangle of length 10 cm.
    (*a*) I and II only (*b*) II and III only (*c*) I and III only
    (*d*) III, and either I or II only (*e*) None of these
17. A path runs around a rectangular lawn. What is the width of the path ?
    I. The length and breadth of the lawn are in the ratio of 3 : 1 respectively.
    II. The width of the path is ten times the length of the lawn.
    III. The cost of gravelling the path @ Rs. 50 per m^2 is Rs. 8832.
    (*a*) All I, II and II (*b*) III, and either I or II (*c*) I and III only
    (*d*) II and III only (*e*) None of these

18. What is the area of the isosceles triangle ?

I. Perimeter of the isosceles triangle is 18 metres.

II. Base of the triangle is 8 metres.

III. Height of the triangle is 3 metres.

(a) I and II only (b) II and III only (c) I and III only

(d) II, and either I or III only (e) Any two of the three

Directions (*Questions 19 to 22*) : ***Each of the questions given below is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the given question.***

19. What is the cost of painting the two adjacent walls of a hall at Rs. 5 per m^2 which has no windows or doors ? **(S.B.I.P.O. 2001)**

I. The area of the hall is 24 sq. m.

II. The breadth, length and height of the hall are in the ratio of 4 : 6 : 5 respectively.

III. Area of one wall is 30 sq. m.

(a) I only (b) II only (c) III only

(d) Either I or III (e) All I, II and III are required

20. What is the area of the given rectangle ? **(Bank P.O. 1999)**

I. Perimeter of the rectangle is 60 cm.

II. Breadth of the rectangle is 12 cm.

III. Sum of two adjacent sides is 30 cm.

(a) I only (b) II only (c) III only

(d) I and II only (e) I or III only

21. What is the area of the given right-angled triangle ?

I. Length of the hypotenuse is 5 cm.

II. Perimeter of the triangle is four times its base.

III. One of the angles of the triangle is 60°.

(a) II only (b) III only (c) II or III only (d) II and III both

(e) Information given in all the three statements together is not sufficient to answer the question.

22. What will be the cost of painting the four walls of a room with length, width and height 5 m, 3 m and 8 m respectively ? The room has one door and one window.

I. Cost of painting per sq. m is Rs. 25.

II. Area of window is 2.25 sq. m which is half of the area of the door.

III. Area of the room is 15 sq. m. **(S.B.I.P.O. 1999)**

(a) I only (b) II only (c) III only

(d) II or III only (e) All I, II and III are required

ANSWERS

1. (c)	2. (e)	3. (c)	4. (d)	5. (e)	6. (e)	7. (c)	8. (c)
9. (e)	10. (a)	11. (c)	12. (b)	13. (c)	14. (a)	15. (e)	16. (a)
17. (a)	18. (d)	19. (c)	20. (e)	21. (c)	22. (c)		

SOLUTIONS

1. Area = 1600 m^2.

I. Side = $\sqrt{1600}$ m = 40 m. So, perimeter = (40 × 4) m = 160 m.

∴ I alone gives the answer.

II. Perimeter $= \frac{\text{Total cost}}{\text{Cost per metre}} = \frac{3200}{20}$ m = 160 m.

$\therefore$ II alone gives the answer.

$\therefore$ Correct answer is (c).

2. I. Let $l = 3x$ metres and $b = 2x$ metres. Then, area $= (6x^2)$ m^2.

II. Perimeter $= 2\,(3x + 2x)$ m $= (10x)$ m.

$\therefore\ 6x^2 = 3.6 \times 10x \Leftrightarrow x = \frac{(3.6 \times 10)}{6} = 6.$

$\therefore\ l = 18$ m and $b = 12$ m and so area can be obtained.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (e).

3. I. Area of the circle = Area of the square $= \frac{1}{2}x^2$ sq. inches.

$\Rightarrow\ \pi r^2 = \frac{1}{2}x^2 \Rightarrow r = \sqrt{\frac{x^2}{2\pi}} = \frac{x}{\sqrt{2\pi}}.$

$\therefore$ Circumference of the circle $= 2\pi r$, which can be obtained.

$\therefore$ I alone gives the answer.

II. Area of the circle = Area of the square $= y^2$ sq. inches.

$\Rightarrow\ \pi r^2 = y^2 \Rightarrow r = \frac{y}{\sqrt{\pi}}.$

$\therefore$ Circumference of the circle $= 2\pi r$, which can be obtained.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

4. Given : Area of rectangle = Area of a right-angled triangle

$\Rightarrow\ l \times b = \frac{1}{2} \times B \times H$

I gives, B = 40 cm.

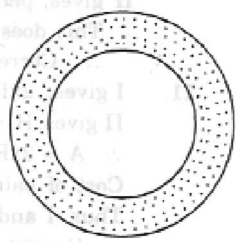
II gives, H = 50 cm.

Thus, to find l, we need b also, which is not given.

$\therefore$ Given data is not sufficient to give the answer.

$\therefore$ Correct answer is (d).

5. I. $\pi R_1^2 = 1386 \Leftrightarrow R_1^2 = \left(1386 \times \frac{7}{22}\right) \Leftrightarrow R_1 = 21$ m.



II. $R_2 = (21 - 1.4)$ m $= 19.6$ m.

$\therefore$ Area $= \pi\,(R_1^2 - R_2^2) = \frac{22}{7} \times [(21)^2 - (19.6)^2]$ m^2.

Thus, the required cost may be obtained.

$\therefore$ I and II together will give the answer.

$\therefore$ Correct answer is (e).

6. I. Let the sides be x cm and $(x + 5)$ cm.

II. $d = \sqrt{(x+5)^2 + x^2} \Leftrightarrow (x + 5)^2 + x^2 = (10)^2 \Leftrightarrow 2x^2 + 10x - 75 = 0$

$\Leftrightarrow\ x = \frac{-10 \pm \sqrt{100 + 600}}{4} = \frac{-10 + \sqrt{700}}{4} = \frac{-10 + 10\sqrt{7}}{4} = \frac{-10 + 10 \times 2.6}{4}$

Thus, sides and therefore area may be known.

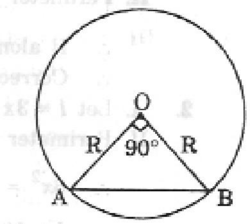
Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

7. I. Length of arc $= \frac{2\pi R\theta}{360} \Leftrightarrow 4 = \left(\frac{2 \times \frac{22}{7} \times R \times 60}{360}\right)$

This gives R and therefore, area of the circle $= \pi R^2$.

Thus, I only gives the answer.



II. $R^2 + R^2 = 5^2 \Leftrightarrow 2R^2 = 25 \Leftrightarrow R^2 = \frac{25}{2}$.

$\therefore$ Area of the circle $= \pi R^2 = \left(\frac{22}{7} \times \frac{25}{2}\right)$ sq. cm.

Thus, II only gives the answer.

$\therefore$ Correct answer is (*c*).

8. I. $2\pi R = 308 \Leftrightarrow 2 \times \frac{22}{7} \times R = 308 \Leftrightarrow R = \left(308 \times \frac{7}{44}\right) = 49$.

Thus, $A = \pi R^2$ can be obtained.

$\therefore$ I alone gives the answer.

II. R = 28 m gives $A = (\pi \times 28 \times 28)\ cm^2$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

9. Given : $l \times b = \pi R^2$. ...(*i*)

I gives, $R = b$. ...(*ii*)

From (*i*) and (*ii*), we get $l = \frac{\pi R^2}{b} = \frac{\pi R^2}{R} = \pi R$. ...(*iii*)

II gives, $2(l + b) = 2\pi R + 14 \Rightarrow l + b = \pi R + 7 \Rightarrow l + R = \pi R + 7$

$\Rightarrow l = \pi R - R + 7$

$\Rightarrow l = l - \frac{l}{\pi} + 7$ [Using (*iii*)]

$\Rightarrow l = 7\pi$.

Thus, I and II together give l.

$\therefore$ Correct answer is (*e*).

10. I. $A = 20 \times B \Rightarrow \frac{1}{2} \times B \times H = 20 \times B \Rightarrow H = 40$.

$\therefore$ I alone gives the answer.

II gives, perimeter of the triangle = 40 cm.

This does not give the height of the triangle.

$\therefore$ Correct answer is (*a*).

11. I gives, $2\pi R = 44$.

II gives, H = 12.

$\therefore A = 2\pi RH = (44 \times 12)$.

Cost of painting = Rs. (44 × 12 × 20).

Thus, I and II together give the answer.

$\therefore$ Correct answer is (*e*).

12. I. $2(l + b) = 110 \Rightarrow l + b = 55$.

II. $l = (b + 5) \Rightarrow l - b = 5$.

III. $\frac{l}{b} = \frac{6}{5} \Rightarrow 5l - 6b = 0$.

These are three equations in l and b. We may solve them pairwise.

$\therefore$ Any two of the three will give the answer.

$\therefore$ Correct answer is (*b*).

13. I. Material cost = Rs. 2.50 per m^2.

II. Labour cost = Rs. 3500.

III. Total cost = Rs. 14,500.

Let the area be A sq. metres.

$\therefore$ Material cost = Rs. (14500 − 3500) = Rs. 11,000.

$\therefore \frac{5A}{2} = 11000 \Leftrightarrow A = \left(\frac{11000 \times 2}{5}\right) = 4400 \text{ m}^2$.

Thus, all I, II and III are needed to get the answer.

$\therefore$ Correct answer is (*c*).

14. I. $2(l + b) = 34 \Rightarrow l + b = 17$ ...(*i*)

II. $(l - b) = 7$ ...(*ii*)

III. $l = (100 + 140)\%$ of $b \Rightarrow l - \frac{240}{100}b = 0$

$\Rightarrow 100l - 240b = 0 \Rightarrow 5l - 12b = 0$ ...(*iii*)

These are 3 equations in l and b. We may solve them pairwise.

$\therefore$ Any two of the three will give the answer.

$\therefore$ Correct answer is (*a*).

15. I. Let $l = 3x$ metres and $b = 2x$ metres.

II. $l = 48$ m, Rate of flooring = Rs. 85 per m^2.

III. $2(l + b) = 160 \Leftrightarrow l + b = 80$, Rate of flooring = Rs. 85 per m^2.

From I and II, we get $3x = 48 \Leftrightarrow x = 16$

$\therefore l = 48$ m, $b = 32$ m $\Rightarrow$ Area of floor = (48×32) m^2.

$\therefore$ Cost of flooring = Rs. $(48 \times 32 \times 85)$.

Thus, I and II give the answer.

From II and III, we get $l = 48$ m, $b = (80 - 48)$ m = 32 m.

$\therefore$ Area of floor and cost of flooring is obtained.

Thus, II and III give the answer.

From III and I, we get $3x + 2x = 80 \Leftrightarrow 5x = 80 \Leftrightarrow x = 16$.

$\therefore l = (3 \times 16)$ m = 48 m and $b = (2 \times 16)$ m = 32 m.

$\therefore$ Area of floor and the cost of flooring is obtained.

Thus, III and I give the answer.

Hence, any two of the three will give the answer.

$\therefore$ Correct answer is (*c*).

16. **From II,** base : height = 5 : 12.

Let base = $5x$ and height = $12x$. Then, hypotenuse = $\sqrt{(5x)^2 + (12x)^2} = 13x$.

From I, perimeter of the triangle = 30 cm.

$\therefore 5x + 12x + 13x = 30 \Leftrightarrow x = 1$.

So, base = $5x$ = 5 cm; height = $12x$ = 12 cm.

$\therefore$ Area = $\left(\frac{1}{2} \times 5 \times 12\right)$ cm^2 = 30 cm^2.

Thus, I and II together give the answer.

Clearly III is redundant, since the breadth of the rectangle is not given.

$\therefore$ Correct answer is (*a*).

17. III gives area of the path = $\frac{8832}{50}$ m^2 = $\frac{4416}{25}$ m^2.

II gives width of path = 10 × (Length of the lawn).

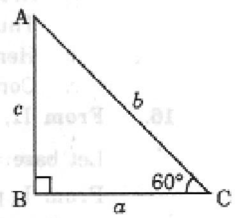
I gives length = $3x$ metres and breadth = x metres
Clearly, all the three will be required to find the width of the path.
$\therefore$ Correct answer is (*a*).

18. II gives base = 8 m.
I gives perimeter = 18 m.
III gives height = 3 m.
From II and I, we get :
$b = 8$ and $a + b + a = 18$ and so $a = 5$.
Thus, the three sides are 5 m, 5 m and 8 m.
From this, the area can be found out.
From II and III, we get : area = $\left(\frac{1}{2} \times 8 \times 3\right) m^2$.
$\therefore$ Correct answer is (*d*).

19. **From II,** let $l = 4x$, $b = 6x$ and $h = 5x$.
Then, area of the hall = $(24x^2)$ m^2.
From I. Area of the hall = 24 m^2.
From II and I, we get $24x^2 = 24 \Leftrightarrow x = 1$.
$\therefore$ $l = 4$ m, $b = 6$ m and $h = 5$ m.
Thus, area of two adjacent walls = $[(l \times h) + (b \times h)]$ m^2 can be found out and so the cost of painting two adjacent walls may be found out.
Thus, III is redundant.
$\therefore$ Correct answer is (*c*).

20. From I and II, we can find the length and breadth of the rectangle and therefore the area can be obtained.
So, III is redundant.
Also, from II and III, we can find the length and breadth and therefore the area can be obtained.
So, I is redundant.
$\therefore$ Correct answer is (*e*).

21. $\frac{BC}{AC} = \cos 60° = \frac{1}{2} \Rightarrow BC = \frac{5}{2}$ cm [$\because$ AC = 5 cm]



From I and III, we get :
$a = \frac{5}{2}$ cm, $b = 5$ cm and $\theta = 60°$.
$\therefore$ $A = \frac{1}{2} ab \sin C$ gives the area.
Thus, I and III give the result.
$\therefore$ II is redundant.
Again, II gives $a + b + c = 4a \Rightarrow b + c = 3a \Rightarrow c = 3a - 5$ [$\because$ $b = 5$ from I]
$a^2 + (3a - 5)^2 = 25$. This gives a and therefore c.
Now, area of Δ ABC = $\frac{1}{2} \times a \times c$, which can be obtained.
Thus I and II give the area.
$\therefore$ III is redundant.
$\therefore$ Correct answer is (*c*).

22. From given length, breadth and height of the room, its area can be obtained.
So, III is redundant.
$\therefore$ Correct answer is (*c*).

25. VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = l, breadth = b and height = h units. Then,

1. ***Volume*** = $(l \times b \times h)$ cubic units.
2. ***Surface area*** = $2\,(lb + bh + lh)$ sq. units.
3. ***Diagonal*** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. CUBE

Let each edge of a cube be of length a. Then,

1. ***Volume*** = a^3 cubic units.
2. ***Surface area*** = $6a^2$ sq. units.
3. ***Diagonal*** = $\sqrt{3}\,a$ units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. ***Volume*** = $(\pi r^2 h)$ cubic units.
2. ***Curved surface area*** = $(2\pi rh)$ sq. units.
3. ***Total surface area*** = $(2\pi rh + 2\pi r^2)$ sq. units $= 2\pi r\,(h + r)$ sq. units.

IV. CONE

Let radius of base = r and Height = h. Then,

1. ***Slant height***, $l = \sqrt{h^2 + r^2}$ units.
2. ***Volume*** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. ***Curved surface area*** = $(\pi r l)$ sq. units.
4. ***Total surface area*** = $(\pi r l + \pi r^2)$ sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. ***Volume*** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. ***Surface area*** = $(4\pi r^2)$ sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. ***Volume*** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. ***Curved surface area*** = $(2\pi r^2)$ sq. units.
3. ***Total surface area*** = $(3\pi r^2)$ sq. units.

Remember : 1 litre = 1000 cm^3.

SOLVED EXAMPLES

Ex. 1. ***Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.***

Sol. Volume = $(16 \times 14 \times 7)$ m^3 = 1568 m^3.

Surface area = $[2(16 \times 14 + 14 \times 7 + 16 \times 7)]$ cm^2 = (2×434) cm^2 = 868 cm^2.

Ex. 2. ***Find the length of the longest pole that can be placed in a room 12 m long, 8 m broad and 9 m high.***

Sol. Length of longest pole = Length of the diagonal of the room

$$= \sqrt{(12)^2 + 8^2 + 9^2} = \sqrt{289} = 17 \text{ m}.$$

Ex. 3. ***The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.***

Sol. Let the breadth of the wall be x metres.

Then, Height = $5x$ metres and Length = $40x$ metres.

$$\therefore \quad x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}.$$

$$\text{So, } x = \frac{4}{10} \text{ m} = \left(\frac{4}{10} \times 100\right) \text{ cm} = 40 \text{ cm}.$$

Ex. 4. ***Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar ?***

Sol. Volume of the wall = $(2400 \times 800 \times 60)$ cu. cm.

Volume of bricks = 90% of the volume of the wall

$$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right) \text{ cu. cm.}$$

Volume of 1 brick = $(24 \times 12 \times 8)$ cu. cm.

$$\therefore \quad \text{Number of bricks} = \left(\frac{90}{100} \times \frac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000.$$

Ex. 5. ***Water flows into a tank 200 m × 150 m through a rectangular pipe 1.5 m × 1.25 m @ 20 kmph. In what time (in minutes) will the water rise by 2 metres ?***

Sol. Volume required in the tank = $(200 \times 150 \times 2)$ m^3 = 60000 m^3.

$$\text{Length of water column flown in 1 min.} = \left(\frac{20 \times 1000}{60}\right) \text{ m} = \frac{1000}{3} \text{ m}.$$

$$\text{Volume flown per minute} = \left(1.5 \times 1.25 \times \frac{1000}{3}\right) \text{ m}^3 = 625 \text{ m}^3.$$

$$\therefore \quad \text{Required time} = \left(\frac{60000}{625}\right) \text{ min.} = 96 \text{ min.}$$

Ex. 6. ***The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.***

Sol. Volume of the metal used in the box = External Volume − Internal Volume

$= [(50 \times 40 \times 23) - (44 \times 34 \times 20)]$ cm^3

$= 16080$ cm^3.

$$\therefore \quad \text{Weight of the metal} = \left(\frac{16080 \times 0.5}{1000}\right) \text{ kg} = 8.04 \text{ kg}.$$

Ex. 7. ***The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume and surface area.***

Sol. Let the edge of the cube be a.

$\therefore \sqrt{3}\,a = 6\sqrt{3} \Rightarrow a = 6$.

So, Volume $= a^3 = (6 \times 6 \times 6)$ cm^3 $= 216$ cm^3.

Surface area $= 6a^2 = (6 \times 6 \times 6)$ cm^2 $= 216$ cm^2.

Ex. 8. ***The surface area of a cube is 1734 sq. cm. Find its volume.***

Sol. Let the edge of the cube be a. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17$ cm.

$\therefore$ Volume $= a^3 = (17)^3$ cm^3 $= 4913$ cm^3.

Ex. 9. ***A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.***

Sol. Volume of the block $= (6 \times 12 \times 15)$ cm^3 $= 1080$ cm^3.

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

Volume of this cube $= (3 \times 3 \times 3)$ cm^3 $= 27$ cm^3.

$$\text{Number of cubes} = \left(\frac{1080}{27}\right) = 40.$$

Ex. 10. ***A cube of edge 15 cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are 20 cm × 15 cm, find the rise in water level.*** **(R.R.B. 2003)**

Sol. Increase in volume = Volume of the cube $= (15 \times 15 \times 15)$ cm^3.

$$\therefore \text{Rise in water level} = \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{15 \times 15 \times 15}{20 \times 15}\right) \text{cm} = 11.25 \text{ cm}.$$

Ex. 11. ***Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.***

Sol. Volume of new cube $= (1^3 + 6^3 + 8^3)$ cm^3 $= 729$ cm^3.

Edge of new cube $= \sqrt[3]{729}$ cm $= 9$ cm.

$\therefore$ Surface area of the new cube $= (6 \times 9 \times 9)$ cm^2 $= 486$ cm^2.

Ex. 12. ***If each edge of a cube is increased by 50%, find the percentage increase in its surface area.***

Sol. Let original length of each edge $= a$.

Then, original surface area $= 6a^2$.

$$\text{New edge} = (150\% \text{ of } a) = \left(\frac{150}{100}a\right) = \frac{3a}{2}.$$

$$\text{New surface area} = 6 \times \left(\frac{3a}{2}\right)^2 = \frac{27}{2}a^2.$$

$$\text{Increase percent in surface area} = \left(\frac{15}{2}a^2 \times \frac{1}{6a^2} \times 100\right)\% = 125\%.$$

Ex. 13. ***Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.***

Sol. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \text{ or } \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \text{ or } \frac{a}{b} = \frac{1}{3}.$$

$$\therefore \text{Ratio of their surface areas} = \frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 = \frac{1}{9}, \text{ i.e., } 1:9.$$

Ex. 14. ***Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.***

Sol. Volume $= \pi r^2 h = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 40\right)$ cm^3 $= 1540$ cm^3.

Curved surface area $= 2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 40\right)$ cm^2 $= 880$ cm^2.

Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

$= \left[2 \times \frac{22}{7} \times \frac{7}{2} \times (40 + 3.5)\right]$ cm^2 $= 957$ cm^2.

Ex. 15. ***If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.***

Sol. Let the depth of the tank be h metres. Then,

$\pi \times (7)^2 \times h = 1848 \Leftrightarrow h = \left(1848 \times \frac{7}{22} \times \frac{1}{7 \times 7}\right) = 12$ m.

Ex. 16. ***2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm in diameter. Find the length of the wire in metres.***

Sol. Let the length of the wire be h metres. Then,

$\pi \times \left(\frac{0.50}{2 \times 100}\right)^2 \times h = \frac{2.2}{1000} \Leftrightarrow h = \left(\frac{2.2}{1000} \times \frac{100 \times 100}{0.25 \times 0.25} \times \frac{7}{22}\right) = 112$ m.

Ex. 17. ***How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron ?*** **(C.B.I. 1998)**

Sol. Volume of 1 rod $= \left(\frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 7\right)$ cu. m $= \frac{11}{5000}$ cu. m.

Volume of iron $= 0.88$ cu. m.

Number of rods $= \left(0.88 \times \frac{5000}{11}\right) = 400$.

Ex. 18. ***The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio of 2 : 3. Find the ratio of their curved surface areas.***

Sol. Let the radii of the cylinders be $3x$, $5x$ and their heights be $2y$, $3y$ respectively. Then,

Ratio of their curved surface areas $= \frac{2\pi \times 3x \times 2y}{2\pi \times 5x \times 3y} = \frac{2}{5} = 2 : 5$.

Ex. 19. ***If 1 cubic cm of cast iron weighs 21 gms, then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.***

Sol. Inner radius $= \left(\frac{3}{2}\right)$ cm $= 1.5$ cm, Outer radius $= (1.5 + 1) = 2.5$ cm.

$\therefore$ Volume of iron $= [\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100]$ cm^3

$= \frac{22}{7} \times 100 \times [(2.5)^2 - (1.5)^2]$ cm^3 $= \left(\frac{8800}{7}\right)$ cm^3.

$\therefore$ Weight of the pipe $= \left(\frac{8800}{7} \times \frac{21}{1000}\right)$ kg $= 26.4$ kg.

Ex. 20. ***Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.***

Sol. Here, $r = 21$ cm and $h = 28$ cm.

$\therefore$ Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35$ cm.

Volume $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\times\frac{22}{7}\times 21\times 21\times 28\right)\text{ cm}^3 = 12936\text{ cm}^3$.

Curved surface area $= \pi r l = \left(\frac{22}{7}\times 21\times 35\right)\text{ cm}^2 = 2310\text{ cm}^2$.

Total surface area $= (\pi r l + \pi r^2) = \left(2310 + \frac{22}{7}\times 21\times 21\right)\text{ cm}^2 = 3696\text{ cm}^2$.

Ex. 21. ***Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.***

Sol. Here, $r = 7\text{m}$ and $h = 24$ m.

So, $l = \sqrt{r^2+h^2} = \sqrt{7^2+(24)^2} = \sqrt{625} = 25$ m.

Area of canvas $= \pi r l = \left(\frac{22}{7}\times 7\times 25\right)\text{ m}^2 = 550\text{ m}^2$.

$\therefore$ Length of canvas $= \left(\frac{\text{Area}}{\text{Width}}\right) = \left(\frac{550}{1.25}\right)$ m $= 440$ m.

Ex. 22. ***The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.***

Sol. Let the radii of their bases be r and R and their heights be h and $2h$ respectively.

Then, $\frac{2\pi r}{2\pi R} = \frac{3}{4} \Rightarrow \frac{r}{R} = \frac{3}{4} \Rightarrow R = \frac{4}{3}r$.

$\therefore$ Ratio of volumes $= \dfrac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{4}{3}r\right)^2(2h)} = \frac{9}{32} = 9 : 32$.

Ex. 23. ***The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio of their volumes.***

Sol. Let the radii of the cylinder and the cone be $3r$ and $4r$ and their heights be $2h$ and $3h$ respectively.

$\therefore \dfrac{\text{Volume of cylinder}}{\text{Volume of cone}} = \dfrac{\pi\times(3r)^2\times 2h}{\frac{1}{3}\pi\times(4r^2)\times 3h} = \frac{9}{8} = 9 : 8$.

Ex. 24. ***A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.***

Sol. Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

$= \left(\frac{1}{3}\times\frac{22}{7}\times 12\times 12\times 50\right)\text{ cm}^3 = \left(\frac{22\times 4\times 12\times 50}{7}\right)\text{ cm}^3$.

Let the height of the liquid in the vessel be h.

Then, $\frac{22}{7}\times 10\times 10\times h = \frac{22\times 4\times 12\times 50}{7}$ or $h = \left(\frac{4\times 12\times 50}{10\times 10}\right) = 24$ cm.

Ex. 25. ***Find the volume and surface area of a sphere of radius 10.5 cm.***

Sol. Volume $= \frac{4}{3}\pi r^3 = \left(\frac{4}{3}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}\right)\text{ cm}^3 = 4851\text{ cm}^3$.

Surface area $= 4\pi r^2 = \left(4\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\right)\text{ cm}^2 = 1386\text{ cm}^2$.

Ex. 26. ***If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.***

Sol. Let original radius = R. Then, new radius = $\frac{150}{100} R = \frac{3R}{2}$.

Original volume = $\frac{4}{3}\pi R^3$, New volume = $\frac{4}{3}\pi\left(\frac{3R}{2}\right)^3 = \frac{9\pi R^3}{2}$.

Increase % in volume = $\left(\frac{19}{6}\pi R^3 \times \frac{3}{4\pi R^3} \times 100\right)\% = 237.5\%$.

Original surface area = $4\pi R^2$. New surface area = $4\pi\left(\frac{3R}{2}\right)^2 = 9\pi R^2$.

Increase % in surface area = $\left(\frac{5\pi R^2}{4\pi R^2} \times 100\right)\% = 125\%$.

Ex. 27. ***Find the number of lead balls, each 1 cm in diameter that can be made from a sphere of diameter 12 cm.***

Sol. Volume of larger sphere = $\left(\frac{4}{3}\pi \times 6 \times 6 \times 6\right)$ cm^3 = 288π cm^3.

Volume of 1 small lead ball = $\left(\frac{4}{3}\pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$ cm^3 = $\frac{\pi}{6}$ cm^3.

$\therefore$ Number of lead balls = $\left(288\pi \times \frac{6}{\pi}\right) = 1728$.

Ex. 28. ***How many spherical bullets can be made out of a lead cylinder 28 cm high and with base radius 6 cm, each bullet being 1.5 cm in diameter ?*** **(R.R.B. 2003)**

Sol. Volume of cylinder = $(\pi \times 6 \times 6 \times 28)$ cm^3 = $(36 \times 28)\,\pi$ cm^3.

Volume of each bullet = $\left(\frac{4}{3}\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right)$ cm^3 = $\frac{9\pi}{16}$ cm^3.

Number of bullets = $\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}} = \left[(36 \times 28)\,\pi \times \frac{16}{9\pi}\right] = 1792$.

Ex. 29. ***A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.***

Sol. Volume of sphere = $\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3 = 972π cm^3.

Volume of wire = $(\pi \times 0.2 \times 0.2 \times h)$ cm^3.

$\therefore$ $972\pi = \pi \times \frac{2}{10} \times \frac{2}{10} \times h \Rightarrow h = (972 \times 5 \times 5)$ cm $= \left(\frac{972 \times 5 \times 5}{100}\right)$ m = 243 m.

Ex. 30. ***Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a sphere. Find the diameter of the sphere.***

Sol. Volume of sphere = Volume of 2 cones

$$= \left[\frac{1}{3}\pi \times (2.1)^2 \times 4.1 + \frac{1}{3}\pi \times (2.1)^2 \times 4.3\right] \text{cm}^3 = \frac{1}{3}\pi \times (2.1)^2\,(8.4)\ \text{cm}^3.$$

Let the radius of the sphere be R.

$\therefore$ $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi (2.1)^3 \times 4$ or R = 2.1 cm.

Hence, diameter of the sphere = 4.2 cm.

Ex. 31. ***A cone and a sphere have equal radii and equal volumes. Find the ratio of the diameter of the sphere to the height of the cone.***

Sol. Let radius of each be R and height of the cone be H.

Then, $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi R^2 H$ or $\frac{R}{H} = \frac{1}{4}$ or $\frac{2R}{H} = \frac{2}{4} = \frac{1}{2}$.

$\therefore$ Required ratio = 1 : 2.

Ex. 32. ***Find the volume, curved surface area and the total surface area of a hemisphere of radius 10.5 cm.***

Sol. Volume $= \frac{2}{3}\pi r^3 = \left(\frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^3 = 2425.5 \text{ cm}^3$.

Curved surface area $= 2\pi r^2 = \left(2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2 = 693 \text{ cm}^2$.

Total surface area $= 3\pi r^2 = \left(3 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2 = 1039.5 \text{ cm}^2$.

Ex. 33. ***A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?*** (N.I.F.T. 2003)

Sol. Volume of bowl $= \left(\frac{2}{3}\pi \times 9 \times 9 \times 9\right) \text{cm}^3 = 486\pi \text{ cm}^3$.

Volume of 1 bottle $= \left(\pi \times \frac{3}{2} \times \frac{3}{2} \times 4\right) \text{cm}^3 = 9\pi \text{ cm}^3$.

Number of bottles $= \left(\frac{486\pi}{9\pi}\right) = 54$.

Ex. 34. ***A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.***

Sol. Let R be the radius of each.

Height of hemisphere = Its radius = R.

$\therefore$ Height of each = R.

Ratio of volumes $= \frac{1}{3}\pi R^2 \times R : \frac{2}{3}\pi R^3 : \pi R^2 \times R = 1 : 2 : 3$.

EXERCISE 25A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The capacity of a tank of dimensions (8 m × 6 m × 2.5 m) is :
   (*a*) 120 litres (*b*) 1200 litres (*c*) 12000 litres (*d*) 120000 litres
2. Find the surface area of a 10 cm × 4 cm × 3 cm brick. **(R.R.B. 2001)**
   (*a*) 84 sq. cm (*b*) 124 sq. cm (*c*) 164 sq. cm (*d*) 180 sq. cm
3. A cistern 6 m long and 4 m wide contains water up to a depth of 1 m 25 cm. The total area of the wet surface is : **(S.S.C. 2004)**
   (*a*) 49 m^2 (*b*) 50 m^2 (*c*) 53.5 m^2 (*d*) 55 m^2
4. A boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets on it. The mass of man is : **(R.R.B. 2002)**
   (*a*) 12 kg (*b*) 60 kg (*c*) 72 kg (*d*) 96 kg

5. The area of the base of a rectangular tank is 6500 cm^2 and the volume of water contained in it is 2.6 cubic metres. The depth of water in the tank is :
(*a*) 3.5 m (*b*) 4 m (*c*) 5 m (*d*) 6 m

6. Given that 1 cu. cm of marble weighs 25 gms, the weight of a marble block 28 cm in width and 5 cm thick is 112 kg. The length of the block is :
(*a*) 26.5 cm (*b*) 32 cm (*c*) 36 cm (*d*) 37.5 cm

7. Half cubic metre of gold sheet is extended by hammering so as to cover an area of 1 hectare. The thickness of the sheet is :
(*a*) 0.0005 cm (*b*) 0.005 cm (*c*) 0.05 cm (*d*) 0.5 cm

8. In a shower, 5 cm of rain falls. The volume of water that falls on 1.5 hectares of ground is :
(*a*) 75 cu. m (*b*) 750 cu. m (*c*) 7500 cu. m (*d*) 75000 cu. m

9. The height of a wall is six times its width and the length of the wall is seven times its height. If volume of the wall be 16128 cu. m, its width is : **(C.B.I. 1998)**
(*a*) 4 m (*b*) 4.5 m (*c*) 5 m (*d*) 6 m

10. The volume of a rectangular block of stone is 10368 dm^3. Its dimensions are in the ratio of 3 : 2 : 1. If its entire surface is polished at 2 paise per dm^2, then the total cost will be :
(*a*) Rs. 31.50 (*b*) Rs. 31.68 (*c*) Rs. 63 (*d*) Rs. 63.36

11. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. The volume of the cuboid is : **(S.S.C. 1999)**
(*a*) 24 cm^3 (*b*) 48 cm^3 (*c*) 64 cm^3 (*d*) 120 cm^3

12. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm × 6 cm × 2 cm, is :
(*a*) $2\sqrt{13}$ cm (*b*) $2\sqrt{14}$ cm (*c*) $2\sqrt{26}$ cm (*d*) $10\sqrt{2}$ cm

13. Find the length of the longest rod that can be placed in a room 16 m long, 12 m broad and $10\frac{2}{3}$ m high. **(S.S.C. 1999)**
(*a*) $22\frac{1}{3}$ m (*b*) $22\frac{2}{3}$ m (*c*) 23 m (*d*) 68 m

14. How many bricks, each measuring 25 cm × 11.25 cm × 6 cm, will be needed to build a wall 8 m × 6 m × 22.5 cm ? **(B.S.F. 2001)**
(*a*) 5600 (*b*) 6000 (*c*) 6400 (*d*) 7200

15. The number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is : **(M.B.A. 2003)**
(*a*) 3040 (*b*) 5740 (*c*) 6080 (*d*) 8120

16. 50 men took a dip in a water tank 40 m long and 20 m broad on a religious day. If the average displacement of water by a man is 4 m^3, then the rise in the water level in the tank will be : **(N.I.F.T. 2000)**
(*a*) 20 cm (*b*) 25 cm (*c*) 35 cm (*d*) 50 cm

17. A tank 4 m long, 2.5 m wide and 1.5 m deep is dug in a field 31 m long and 10 m wide. If the earth dug out is evenly spread out over the field, the rise in level of the field is :
(*a*) 3.1 cm (*b*) 4.8 cm (*c*) 5 cm (*d*) 6.2 cm

18. A river 1.5 m deep and 36 m wide is flowing at the rate of 3.5 km per hour. The amount of water that runs into the sea per minute (in cubic metres) is :
(*a*) 3150 (*b*) 31500 (*c*) 6300 (*d*) 63000

19. A rectangular water tank is 80 m × 40 m. Water flows into it through a pipe 40 sq. cm at the opening at a speed of 10 km / hr. By how much, the water level will rise in the tank in half an hour ? **(M.B.A. 1997)**
(a) $\frac{3}{2}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{5}{8}$ cm (d) None of these

20. A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of areas of the four walls, the volume of the hall is :
(a) 720 (b) 900 (c) 1200 (d) 1800
(L.I.C. A.A.O. 2003)

21. The sum of the length, breadth and depth of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. It surface area is :
(a) 125 cm^2 (b) 236 cm^2 (c) 361 cm^2 (d) 486 cm^2

22. A swimming pool 9 m wide and 12 m long is 1 m deep on the shallow side and 4 m deep on the deeper side. Its volume is : **(M.A.T. 1998)**
(a) 208 m^3 (b) 270 m^3 (c) 360 m^3 (d) 408 m^3

23. A metallic sheet is of rectangular shape with dimensions 48 m × 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8 m, the volume of the box (in m^3) is : **(M.A.T. 2003)**
(a) 4830 (b) 5120 (c) 6420 (d) 8960

24. An open box is made of wood 3 cm thick. Its external dimensions are 1.46 m, 1.16 m and 8.3 dm. The cost of painting the inner surface of the box at 50 paise per 100 sq. cm is :
(a) Rs. 138.50 (b) Rs. 277 (c) Rs. 415. 50 (d) Rs. 554

25. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is : **(S.S.C. 2003)**
(a) 90 cm (b) 1 dm (c) 1 m (d) 1.1 m

26. If a metallic cuboid weighs 16 kg, how much would a miniature cuboid of metal weigh, if all dimensions are reduced to one-fourth of the original ? **(D.M.R.C. 2003)**
(a) 0.25 kg (b) 0.50 kg (c) 0.75 kg (d) 1 kg

27. The areas of the three adjacent faces of a rectangular box which meet in a point are known. The product of these areas is equal to : **(Section Officers', 2003)**
(a) the volume of the box (b) twice the volume of the box
(c) the square of the volume of the box (d) the cube root of the volume of the box

28. If the areas of the three adjacent faces of a cuboidal box are 120 cm^2, 72 cm^2 and 60 cm^2 respectively, then find the volume of the box. **(S.S.C. 2002)**
(a) 720 cm^3 (b) 864 cm^3 (c) 7200 cm^3 (d) $(72)^2$ cm^3

29. If the areas of three adjacent faces of a rectangular block are in the ratio of 2 : 3 : 4 and its volume is 9000 cu. cm; then the length of the shortest side is :
(a) 10 cm (b) 15 cm (c) 20 cm (d) 30 cm

30. The perimeter of one face a of cube is 20 cm. Its volume must be : **(S.S.C. 1999)**
(a) 125 cm^3 (b) 400 cm^3 (c) 1000 cm^3 (d) 8000 cm^3

31. Total surface area of a cube whose side is 0.5 cm is : **(I.M.T. 2002)**
(a) $\frac{1}{4}$ cm^2 (b) $\frac{1}{8}$ cm^2 (c) $\frac{3}{4}$ cm^2 (d) $\frac{3}{2}$ cm^2

32. The cost of the paint is Rs. 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side ?
(a) Rs. 692 (b) Rs. 768 (c) Rs. 876
(d) Rs. 972 (e) None of these **(Bank P.O. 2002)**

33. The dimensions of a piece of iron in the shape of a cuboid are 270 cm × 100 cm × 64 cm. If it is melted and recast into a cube, then the surface area of the cube will be :
(a) 14400 cm^2 (b) 44200 cm^2 (c) 57600 cm^2 (d) 86400 cm^2

34. The cost of painting the whole surface area of a cube at the rate of 13 paise per sq. cm is Rs. 343.98. Then the volume of the cube is : **(S.S.C. 2003)**
(a) 8500 cm^3 (b) 9000 cm^3 (c) 9250 cm^3 (d) 9261 cm^3

35. If the volume of a cube is 729 cm^3, then the surface area of the cube will be :
(a) 456 cm^2 (b) 466 cm^2 (c) 476 cm^2 (d) 486 cm^2

36. The length of an edge of a hollow cube open at one face is $\sqrt{3}$ metres. What is the length of the largest pole that it can accommodate ? **(M.A.T. 1997)**
(a) $\sqrt{3}$ metres (b) 3 metres (c) $3\sqrt{3}$ metres (d) $\frac{3}{\sqrt{3}}$ metres

37. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$ cm ?
(a) 8 (b) 16 (c) 27 (d) 64
(Hotel Management, 1999)

38. The surface area of a cube is 600 cm^2. The length of its diagonal is :
(a) $\frac{10}{\sqrt{3}}$ cm (b) $\frac{10}{\sqrt{2}}$ cm (c) $10\sqrt{2}$ cm (d) $10\sqrt{3}$ cm

39. If the numbers representing volume and surface area of a cube are equal, then the length of the edge of the cube in terms of the unit of measurement will be :
(a) 3 (b) 4 (c) 5 (d) 6

40. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge ?
(a) 10 (b) 100 (c) 1000 (d) 10000
(R.R.B. 2003)

41. A rectangular box measures internally 1.6 m long, 1 m broad and 60 cm deep. The number of cubical blocks each of edge 20 cm that can be packed inside the box is :
(a) 30 (b) 53 (c) 60 (d) 120

42. How many cubes of 3 cm edge can be cut out of a cube of 18 cm edge ?
(a) 36 (b) 216 (c) 218 (d) 432
(IGNOU, 2003)

43. A cuboidal block of 6 cm × 9 cm × 12 cm is cut up into an exact number of equal cubes. The least possible number of cubes will be : **(Section Officers', 2003)**
(a) 6 (b) 9 (c) 24 (d) 30

44. The size of a wooden block is 5 × 10 × 20 cm. How many such blocks will be required to construct a solid wooden cube of minimum size ?
(a) 6 (b) 8 (c) 12 (d) 16

45. An iron cube of side 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet are in the ratio 1 : 5, the sides are :
(a) 10 cm, 50 cm (b) 20 cm, 100 cm (c) 40 cm, 200 cm (d) None of these
(Hotel Management, 1997)

46. Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube. The edge of the new cube formed is :
(a) 12 cm (b) 14 cm (c) 16 cm (d) 18 cm

47. Five equal cubes, each of side 5 cm, are placed adjacent to each other. The volume of the new solid formed will be :
(a) 125 cm^3 (b) 625 cm^3 (c) 15525 cm^3 (d) None of these

48. A cube of edge 5 cm is cut into cubes each of edge 1 cm. The ratio of the total surface area of one of the small cubes to that of the large cube is equal to : (S.S.C. 2004)
(a) 1 : 5 (b) 1 : 25 (c) 1 : 125 (d) 1 : 625

49. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube ? (M.A.T. 2004)
(a) 2 : 1 (b) 3 : 2 (c) 25 : 18 (d) 27 : 20

50. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The sides of the cubes are : (M.A.T. 2003)
(a) 3 cm, 4 cm, 5 cm (b) 6 cm, 8 cm, 10 cm
(c) 9 cm, 12 cm, 15 cm (d) None of these

51. If the volumes of two cubes are in the ratio 27 : 1, the ratio of their edges is :
(a) 1 : 3 (b) 1 : 27 (c) 3 : 1 (d) 27 : 1
(S.S.C. 1999)

52. The volumes of two cubes are in the ratio 8 : 27. The ratio of their surface areas is :
(a) 2 : 3 (b) 4 : 9 (c) 12 : 9 (d) None of these
(Hotel Management, 2003)

53. Two cubes have volumes in the ratio 1 : 27. Then the ratio of the area of the face of one of the cubes to that of the other is :
(a) 1 : 3 (b) 1 : 6 (c) 1 : 9 (d) 1 : 12

54. If each edge of a cube is doubled, then its volume :
(a) is doubled (b) becomes 4 times
(c) becomes 6 times (d) becomes 8 times

55. If each edge of a cube is increased by 25%, then the percentage increase in its surface area is :
(a) 25% (b) 48.75% (c) 50% (d) 56.25%

56. A circular well with a diameter of 2 metres, is dug to a depth of 14 metres. What is the volume of the earth dug out ? (S.S.C. 1999)
(a) 32 m^3 (b) 36 m^3 (c) 40 m^3 (d) 44 m^3

57. The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base ? (Bank P.O. 2003)
(a) 1.4 m (b) 2.8 m (c) 14 m (d) 28 m (e) None of these

58. The volume of a right circular cylinder whose curved surface area is 2640 cm^2 and circumference of its base is 66 cm, is :
(a) 3465 cm^3 (b) 7720 cm^3 (c) 13860 cm^3 (d) 55440 cm^3

59. If the volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3, then the radius of the cylinder is equal to :
(a) π cm (b) 2 cm (c) 3 cm (d) 4 cm

60. The height of a right circular cylinder is 14 cm and its curved surface is 704 sq. cm. Then its volume is :
(a) 1408 cm^3 (b) 2816 cm^3 (c) 5632 cm^3 (d) 9856 cm^3

61. A closed metallic cylindrical box is 1.25 m high and its base radius is 35 cm. If the sheet metal costs Rs. 80 per m^2, the cost of the material used in the box is :
(a) Rs. 281.60 (b) Rs. 290 (c) Rs. 340.50 (d) Rs. 500

62. The curved surface area of a right circular cylinder of base radius r is obtained by multiplying its volume by :
(a) $2r$ (b) $\frac{2}{r}$ (c) $2r^2$ (d) $\frac{2}{r^2}$

63. The ratio of total surface area to lateral surface area of a cylinder whose radius is 20 cm and height 60 cm, is :
(*a*) 2 : 1 (*b*) 3 : 2 (*c*) 4 : 3 (*d*) 5 : 3

64. A powder tin has a square base with side 8 cm and height 14 cm. Another tin has a circular base with diameter 8 cm and height 14 cm. The difference in their capacities is :
(*a*) 0 (*b*) 132 cm^3 (*c*) 137.1 cm^3 (*d*) 192 cm^3

65. The ratio between the radius of the base and the height of a cylinder is 2 : 3. If its volume is 12936 cu. cm, the total surface area of the cylinder is :
(*a*) 2587.2 cm^2 (*b*) 3080 cm^2 (*c*) 25872 cm^2 (*d*) 38808 cm^2

66. The radius of the cylinder is half its height and area of the inner part is 616 sq. cms. Approximately how many litres of milk can it contain ?
(*a*) 1.4 (*b*) 1.5 (*c*) 1.7 (*d*) 1.9 (*e*) 2.2
(S.B.I.P.O. 2000)

67. The sum of the radius of the base and the height of a solid cylinder is 37 metres. If the total surface area of the cylinder be 1628 sq. metres, its volume is :
(*a*) 3180 m^3 (*b*) 4620 m^3 (*c*) 5240 m^3 (*d*) None of these

68. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. Find the ratio of its diameter to its height. **(S.S.C. 2002)**
(*a*) 3 : 7 (*b*) 7 : 3 (*c*) 6 : 7 (*d*) 7 : 6

69. The height of a closed cylinder of given volume and the minimum surface area is :
(*a*) equal to its diameter (*b*) half of its diameter
(*c*) double of its diameter (*d*) None of these **(R.R.B. 2002)**

70. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original one ?
(*a*) 1 : 2 (*b*) 1 : 4 (*c*) 1 : 8 (*d*) 8 : 1

71. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is :
(*a*) 4 : 9 (*b*) 9 : 4 (*c*) 20 : 27 (*d*) 27 : 20

72. Two right circular cylinders of equal volumes have their heights in the ratio 1 : 2. The ratio of their radii is : **(S.S.C. 1999)**
(*a*) 1 : 2 (*b*) 1 : 4 (*c*) 2 : 1 (*d*) $\sqrt{2} : 1$

73. X and Y are two cylinders of the same height. The base of X has diameter that is half the diameter of the base of Y. If the height of X is doubled, the volume of X becomes :
(*a*) equal to the volume of Y (*b*) double the volume of Y
(*c*) half the volume of Y (*d*) greater than the volume of Y
(C.B.I. 1997)

74. The radius of a wire is decreased to one-third and its volume remains the same. The new length is how many times the original length ?
(*a*) 1 time (*b*) 3 times (*c*) 6 times (*d*) 9 times

75. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by : **(S.S.C. 1999)**
(*a*) $10\frac{1}{2}$ cm (*b*) $11\frac{3}{7}$ cm (*c*) $12\frac{6}{7}$ cm (*d*) 14 cm

76. A well with 14 m inside diameter is dug 10 m deep. Earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. The height of the embankment is :
(*a*) $\frac{1}{2}$ m (*b*) $\frac{2}{3}$ m (*c*) $\frac{3}{4}$ m (*d*) $\frac{3}{5}$ m

77. Water flows through a cylindrical pipe of internal diameter 7 cm at 2 m per second. If the pipe is always half full, then what is the volume of water (in litres) discharged in 10 minutes ? **(S.S.C. 2003)**
(*a*) 2310 (*b*) 3850 (*c*) 4620 (*d*) 9240

78. The number of coins of radius 0.75 cm and thickness 0.2 cm to be melted to make a right circular cylinder of height 8 cm and base radius 3 cm is : **(S.S.C. 2003)**
(*a*) 460 (*b*) 500 (*c*) 600 (*d*) 640

79. Two cylindrical vessels with radii 15 cm and 10 cm and heights 35 cm and 15 cm respectively are filled with water. If this water is poured into a cylindrical vessel 15 cm in height, then the radius of the vessel is :
(*a*) 17.5 cm (*b*) 18 cm (*c*) 20 cm (*d*) 25 cm

80. 66 cubic centimetres of silver is drawn into a wire 1 mm in diameter. The length of the wire in metres will be : **(C.B.I. 1998)**
(*a*) 84 (*b*) 90 (*c*) 168 (*d*) 336

81. A hollow garden roller 63 cm wide with a girth of 440 cm is made of iron 4 cm thick. The volume of the iron used is :
(*a*) 54982 cm^3 (*b*) 56372 cm^3 (*c*) 57636 cm^3 (*d*) 58752 cm^3

82. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.4 cm thick. The volume of the metal is : **(S.S.C. 2003)**
(*a*) 280.52 cm^3 (*b*) 306.24 cm^3 (*c*) 310 cm^3 (*d*) 316 cm^3

83. What length of solid cylinder 2 cm in diameter mush be taken to cast into a hollow cylinder of external diameter 12 cm, 0.25 cm thick and 15 cm long ?
(*a*) 42.3215 cm (*b*) 44.0123 cm (*c*) 44.0625 cm (*d*) 44.6023 cm

84. A hollow iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 g / cm^3, then the weight of the pipe is :
(*a*) 3.6 kg (*b*) 3.696 kg (*c*) 36 kg (*d*) 36.9 kg
(S.S.C. 2004)

85. A circular cylinder can hold 61.6 c.c. of water. If the height of the cylinder is 40 cm and the outer diameter is 16 mm, then the thickness of the material of the cylinder is :
(*a*) 0.2 mm (*b*) 0.3 mm (*c*) 1 mm (*d*) 2 mm

86. The radius of the base and height of a cone are 3 cm and 5 cm respectively whereas the radius of the base and height of a cylinder are 2 cm and 4 cm respectively. The ratio of the volume of cone to that of the cylinder is :
(*a*) 1 : 3 (*b*) 15 : 8 (*c*) 15 : 16 (*d*) 45 : 16

87. The curved surface of a right circular cone of height 15 cm and base diameter 16 cm is : **(S.S.C. 1999)**
(*a*) 60π cm^2 (*b*) 68π cm^2 (*c*) 120π cm^2 (*d*) 136π cm^2

88. What is the total surface area of a right circular cone of height 14 cm and base radius 7 cm ? **(Hotel Management, 2001)**
(*a*) 344.35 cm^2 (*b*) 462 cm^2 (*c*) 498.35 cm^2 (*d*) None of these

89. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is : **(S.S.C. 2000)**
(*a*) 12π cm^3 (*b*) 15π cm^3 (*c*) 16π cm^3 (*d*) 20π cm^3

90. The slant height of a right circular cone is 10 m and its height is 8 m. Find the area of its curved surface. **(R.R.B. 2003)**
(*a*) 30π m^2 (*b*) 40π m^2 (*c*) 60π m^2 (*d*) 80π m^2

91. If a right circular cone of height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is : **(S.S.C. 2003)**
(*a*) 154 cm^2 (*b*) 550 cm^2 (*c*) 704 cm^2 (*d*) 1254 cm^2

4. When B runs 25 m, A runs $\frac{45}{2}$ m.

When B runs 1000 m, A runs $\left(\frac{45}{2} \times \frac{1}{25} \times 1000\right)$ m = 900 m.

$\therefore$ B beats A by 100 m.

5. To reach the winning post A will have to cover a distance of (500 − 140) m, *i.e.*, 360 m.
While A covers 3 m, B covers 4 m.

While A covers 360 m, B covers $\left(\frac{4}{3} \times 360\right)$ m = 480 m.

Thus, when A reaches the winning post, B covers 480 m and therefore remains 20 m behind.

$\therefore$ A wins by 20 m.

6. Ratio of the speeds of A and B = $\frac{5}{3}$: 1 = 5 : 3.

Thus, in a race of 5 m, A gains 2 m over B.
2 m are gained by A in a race of 5 m.

80 m will be gained by A in a race of $\left(\frac{5}{2} \times 80\right)$ m = 200 m.

$\therefore$ Winning post is 200 m away from the starting point.

7. A : B = 100 : 75 and B : C = 100 : 96.

$\therefore$ A : C = $\left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{100}{75} \times \frac{100}{96}\right) = \frac{100}{72}$ = 100 : 72.

$\therefore$ A beats C by (100 − 72) m = 28 m.

8. A : B = 100 : 90 and A : C = 100 : 72.

B : C = $\frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{72} = \frac{90}{72}$.

When B runs 90 m, C runs 72 m.

When B runs 100 m, C runs $\left(\frac{72}{90} \times 100\right)$ m = 80 m.

$\therefore$ B can give C 20 m.

9. A : B = 100 : 90 and A : C = 100 : 87.

$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{87} = \frac{30}{29}$.

When B runs 30 m, C runs 29 m.

When B runs 180 m, C runs $\left(\frac{29}{30} \times 180\right)$ m = 174 m.

$\therefore$ B beats C by (180 − 174) m = 6 m.

10. A : B = 200 : 169 and A : C = 200 : 182.

$\frac{C}{B} = \left(\frac{C}{A} \times \frac{A}{B}\right) = \left(\frac{182}{200} \times \frac{200}{169}\right)$ = 182 : 169.

When C covers 182 m, B covers 169 m.

When C covers 350 m, B covers $\left(\frac{169}{182} \times 350\right)$ m = 325 m.

11. A's speed = $\left(5 \times \frac{5}{18}\right)$ m/sec = $\frac{25}{18}$ m/sec.

92. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km^2. The height of the mountain is : (S.S.C. 2002)
(a) 2.2 km (b) 2.4 km (c) 3 km (d) 3.11 km

93. If the area of the base of a right circular cone is 3850 cm^2 and its height is 84 cm, then the curved surface area of the cone is :
(a) 10001 cm^2 (b) 10010 cm^2 (c) 10100 cm^2 (d) 11000 cm^2

94. Volume of a right circular cone having base radius 70 cm and curved surface area 40040 cm^2 is :
(a) 823400 cm^3 (b) 824000 cm^3 (c) 840000 cm^3 (d) 862400 cm^3

95. The radius and height of a right circular cone are in the ratio 3 : 4. If its volume is 96π cm^3, what is its slant height ? (C.B.I. 1997)
(a) 8 cm (b) 9 cm (c) 10 cm (d) 12 cm

96. The length of canvas 1.1 m wide required to build a conical tent of height 14 m and the floor area 346.5 sq. m is :
(a) 490 m (b) 525 m (c) 665 m (d) 860 m

97. If the radius of the base and the height of a right circular cone are doubled, then its volume becomes : (Asstt. Grade, 2003)
(a) 2 times (b) 3 times (c) 4 times (d) 8 times

98. If both the radius and height of a right circular cone are increased by 20%, its volume will be increased by : (S.S.C. 2004)
(a) 20% (b) 40% (c) 60% (d) 72.8%

99. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, then the volume of the cone : (S.S.C. 2000)
(a) remains unaltered (b) decreases by 25%
(c) increases by 25% (d) increases by 50%

100. If the height of a cone be doubled and radius of base remains the same, then the ratio of the volume of the given cone to that of the second cone will be : (S.S.C. 2003)
(a) 1 : 2 (b) 2 : 1 (c) 1 : 8 (d) 8 : 1

101. Two cones have their heights in the ratio of 1 : 3 and radii 3 : 1. The ratio of their volumes is :
(a) 1 : 1 (b) 1 : 3 (c) 3 : 1 (d) 2 : 3

102. The radii of two cones are in the ratio 2 : 1, their volumes are equal. Find the ratio of their heights. (C.B.I. 1998)
(a) 1 : 8 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

103. If the volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5, then the ratio of their heights is :
(a) 1 : 5 (b) 5 : 4 (c) 5 : 16 (d) 25 : 64

104. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm is : (M.A.T. 2002)
(a) 13.6 cm^3 (b) 89.8 cm^3 (c) 121 cm^3 (d) 147.68 cm^3

105. A cone of height 7 cm and base radius 3 cm is carved from a rectangular block of wood 10 cm × 5 cm × 2 cm. The percentage of wood wasted is :
(a) 34% (b) 46% (c) 54% (d) 66%

106. A right circular cone and a right circular cylinder have equal base and equal height. If the radius of the base and the height are in the ratio 5 : 12, then the ratio of the total surface area of the cylinder to that of the cone is :
(a) 3 : 1 (b) 13 : 9 (c) 17 : 9 (d) 34 : 9

107. A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. The radius of the cone will be : (R.R.B. 2003)
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

108. A right cylindrical vessel is full of water. How many right cones having the same radius and height as those of the right cylinder will be needed to store that water ?
(a) 2 (b) 3 (c) 4 (d) 8

109. A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to form cones, each of height 1 cm and base radius 1 mm. The number of cones is :
(a) 450 (b) 1350 (c) 4500 (d) 13500

110. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm ?
(a) 48 min. 15 sec. (b) 51 min. 12 sec. (c) 52 min. 1 sec. (d) 55 min.

111. A solid cylindrical block of radius 12 cm and height 18 cm is mounted with a conical block of radius 12 cm and height 5 cm. The total lateral surface of the solid thus formed is : (Hotel Management, 1998)
(a) 528 cm^2 (b) $1357\frac{5}{7}$ cm^2 (c) 1848 cm^2 (d) None of these

112. Consider the volumes of the following : (Civil Services, 2002)
1. A parallelopiped of length 5 cm, breadth 3 cm and height 4 cm
2. A cube of each side 4 cm
3. A cylinder of radius 3 cm and length 3 cm
4. A sphere of radius 3 cm

The volumes of these in the decreasing order is :
(a) 1, 2, 3, 4 (b) 1, 3, 2, 4 (c) 4, 2, 3, 1 (d) 4, 3, 2, 1

113. The volume of a sphere is 4851 cu. cm. Its curved surface area is :
(a) 1386 cm^2 (b) 1625 cm^2 (c) 1716 cm^2 (d) 3087 cm^2

114. The curved surface area of a sphere is 5544 sq. cm. Its volume is :
(a) 22176 cm^3 (b) 33951 cm^3 (c) 38808 cm^3 (d) 42304 cm^3

115. The volume of a sphere of radius r is obtained by multiplying its surface area by :
(a) $\frac{4}{3}$ (b) $\frac{r}{3}$ (c) $\frac{4r}{3}$ (d) $3r$

116. If the volume of a sphere is divided by its surface area, the result is 27 cm. The radius of the sphere is : (R.R.B. 2003)
(a) 9 cm (b) 36 cm (c) 54 cm (d) 81 cm

117. Spheres A and B have their radii 40 cm and 10 cm respectively. The ratio of the surface area of A to the surface area of B is : (S.S.C. 2003)
(a) 1 : 4 (b) 1 : 16 (c) 4 : 1 (d) 16 : 1

118. Surface area of a sphere is 2464 cm^2. If its radius be doubled, then the surface area of the new sphere will be :
(a) 4928 cm^2 (b) 9856 cm^2 (c) 19712 cm^2 (d) Data insufficient

119. If the radius of a sphere is doubled, how many times does its volume become ?
(a) 2 times (b) 4 times (c) 6 times (d) 8 times

120. If the radius of a sphere is increased by 2 cm, then its surface area increases by 352 cm^2. The radius of the sphere before the increase was : (C.B.I. 2003)
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

121. If the measured value of the radius is 1.5% larger, the percentage error (correct to one decimal place) made in calculating the volume of a sphere is : (C.B.I. 1997)
(a) 2.1 (b) 3.2 (c) 4.6 (d) 5.4

122. The volumes of two spheres are in the ratio of 64 : 27. The ratio of their surface areas is : **(R.R.B. 2002)**
(*a*) 1 : 2 (*b*) 2 : 3 (*c*) 9 : 16 (*d*) 16 : 9

123. If the surface areas of two spheres are in the ratio of 4 : 25, then the ratio of their volumes is :
(*a*) 4 : 25 (*b*) 25 : 4 (*c*) 125 : 8 (*d*) 8 : 125

124. If three metallic spheres of radii 6 cms, 8 cms and 10 cms are melted to form a single sphere, the diameter of the new sphere will be : **(D.M.R.C. 2003)**
(*a*) 12 cms (*b*) 24 cms (*c*) 30 cms (*d*) 36 cms

125. A solid metallic sphere of radius 8 cm is melted and recast into spherical balls each of radius 2 cm. The number of spherical balls, thus obtained, is :
(*a*) 16 (*b*) 48 (*c*) 64 (*d*) 82

126. A spherical ball of lead, 3 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm respectively. The diameter of the third ball is :
(*a*) 2.5 cm (*b*) 2.66 cm (*c*) 3 cm (*d*) 3.5 cm

127. If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the radius of each such ball is :
(*a*) 1.25 cm (*b*) 2.5 cm (*c*) 3.75 cm (*d*) 5 cm

128. A hollow spherical metallic ball has an external diameter 6 cm and is $\frac{1}{2}$ cm thick. The volume of metal used in the ball is : **(S.S.C. 2004)**
(*a*) $37\frac{2}{3}$ cm^3 (*b*) $40\frac{2}{3}$ cm^3 (*c*) $41\frac{2}{3}$ cm^3 (*d*) $47\frac{2}{3}$ cm^3

129. A solid piece of iron of dimensions $49 \times 33 \times 24$ cm is moulded into a sphere. The radius of the sphere is : **(Hotel Management, 1999)**
(*a*) 21 cm (*b*) 28 cm (*c*) 35 cm (*d*) None of these

130. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in diameter ?
(*a*) 1347 (*b*) 2541 (*c*) 2662 (*d*) 5324

131. How many lead shots each 3 mm in diameter can be made from a cuboid of dimensions 9 cm $\times$ 11 cm $\times$ 12 cm ?
(*a*) 7200 (*b*) 8400 (*c*) 72000 (*d*) 84000

132. A sphere and a cube have equal surface areas. The ratio of the volume of the sphere to that of the cube is :
(*a*) $\sqrt{\pi} : \sqrt{6}$ (*b*) $\sqrt{2} : \sqrt{\pi}$ (*c*) $\sqrt{\pi} : \sqrt{3}$ (*d*) $\sqrt{6} : \sqrt{\pi}$

133. The ratio of the volume of a cube to that of a sphere which will fit inside the cube is :
(*a*) $4 : \pi$ (*b*) $4 : 3\pi$ (*c*) $6 : \pi$ (*d*) $2 : \pi$

134. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is :
(*a*) 3 cm (*b*) 4 cm (*c*) 6 cm (*d*) 12 cm
(S.S.C. 2002)

135. The diameter of the iron ball used for the shot-put game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder ? **(S.S.C. 2004)**
(*a*) 14 cm (*b*) $\frac{14}{3}$ cm (*c*) 28 cm (*d*) $\frac{28}{3}$ cm

136. The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is : **(C.B.I. 1997)**

(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

137. How many spherical bullets can be made out of a lead cylinder 15 cm high and with base radius 3 cm, each bullet being 5 mm in diameter ?

(a) 6000 (b) 6480 (c) 7260 (d) 7800

138. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of spherical balls is :

(a) 12 (b) 16 (c) 24 (d) 48

139. The diameter of a sphere is 8 cm. It is melted and drawn into a wire of diameter 3 mm. The length of the wire is :

(a) 36.9 m (b) 37.9 m (c) 38.9 m (d) 39.9 m

140. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by : **(M.B.A. 2000)**

(a) $\frac{2}{9}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{9}{4}$ cm (d) $\frac{9}{2}$ cm

141. 12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is : **(S.S.C. 2000)**

(a) $\sqrt{3}$ cm (b) 2 cm (c) 3 cm (d) 4 cm

142. A cylindrical tub of radius 12 cm contains water upto a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. The radius of the ball is :

(a) 4.5 cm (b) 6 cm (c) 7.25 cm (d) 9 cm

143. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is : **(C.B.I. 2003)**

(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm

144. A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden solid sphere of radius 9 cm. The percentage of the wood wasted is : **(S.S.C. 2000)**

(a) 25% (b) 25π% (c) 50% (d) 75%

145. A metallic cone of radius 12 cm and height 24 cm is melted and made into spheres of radius 2 cm each. How many spheres are there ?

(a) 108 (b) 120 (c) 144 (d) 180

146. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. The height of the cone is : **(R.R.B. 2002)**

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

147. In what ratio are the volumes of a cylinder, a cone and a sphere, if each has the same diameter and the same height ?

(a) 1 : 3 : 2 (b) 2 : 3 : 1 (c) 3 : 1 : 2 (d) 3 : 2 : 1

148. The total surface area of a solid hemisphere of diameter 14 cm, is :

(a) 308 cm^2 (b) 462 cm^2 (c) 1232 cm^2 (d) 1848 cm^2

149. Volume of a hemisphere is 19404 cu. cm. Its radius is :

(a) 10.5 cm (b) 17.5 cm (c) 21 cm (d) 42 cm

150. The capacities of two hemispherical vessels are 6.4 litres and 21.6 litres. The areas of inner curved surfaces of the vessels will be in the ratio of :

(a) $\sqrt{2} : \sqrt{3}$ (b) 2 : 3 (c) 4 : 9 (d) 16 : 81

151. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, the volume of the beverage in the cylindrical vessel is : (I.A.S. 1999)

(a) $66\frac{2}{3}\%$ (b) $78\frac{1}{2}\%$ (c) 100%

(d) More than 100% (i.e., some liquid will be left in the bowl).

152. A metallic hemisphere is melted and recast in the shape of a cone with the same base radius (R) as that of the hemisphere. If H is the height of the cone, then :

(a) H = 2R (b) H = 3R (c) $H = \sqrt{3}R$ (d) $H = \frac{2}{3}R$

(S.S.C. 1999)

153. A hemisphere of lead of radius 6 cm is cast into a right circular cone of height 75 cm. The radius of the base of the cone is :

(a) 1.4 cm (b) 2 cm (c) 2.4 cm (d) 4.2 cm

154. A hemisphere and a cone have equal bases. If their heights are also equal, then the ratio of their curved surfaces will be : (S.S.C. 2002)

(a) 1 : 2 (b) 2 : 1 (c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$

155. A sphere of maximum volume is cut out from a solid hemisphere of radius r. The ratio of the volume of the hemisphere to that of the cut out sphere is :

(a) 3 : 2 (b) 4 : 1 (c) 4 : 3 (d) 7 : 4

ANSWERS

1. (d)	2. (c)	3. (a)	4. (b)	5. (b)	6. (b)	7. (b)	8. (b)
9. (a)	10. (d)	11. (b)	12. (c)	13. (b)	14. (c)	15. (c)	16. (b)
17. (c)	18. (a)	19. (c)	20. (c)	21. (b)	22. (b)	23. (b)	24. (b)
25. (a)	26. (a)	27. (c)	28. (c)	29. (b)	30. (a)	31. (d)	32. (c)
33. (d)	34. (d)	35. (d)	36. (b)	37. (d)	38. (d)	39. (d)	40. (c)
41. (d)	42. (b)	43. (c)	44. (b)	45. (b)	46. (a)	47. (b)	48. (b)
49. (c)	50. (b)	51. (c)	52. (b)	53. (c)	54. (d)	55. (d)	56. (d)
57. (e)	58. (c)	59. (b)	60. (b)	61. (a)	62. (b)	63. (c)	64. (d)
65. (b)	66. (b)	67. (b)	68. (b)	69. (a)	70. (b)	71. (c)	72. (d)
73. (c)	74. (d)	75. (b)	76. (b)	77. (c)	78. (d)	79. (d)	80. (a)
81. (d)	82. (b)	83. (c)	84. (b)	85. (c)	86. (c)	87. (d)	88. (c)
89. (a)	90. (c)	91. (b)	92. (b)	93. (b)	94. (d)	95. (c)	96. (b)
97. (d)	98. (d)	99. (b)	100. (a)	101. (c)	102. (b)	103. (d)	104. (b)
105. (a)	106. (c)	107. (d)	108. (b)	109. (d)	110. (b)	111. (d)	112. (d)
113. (a)	114. (c)	115. (b)	116. (d)	117. (d)	118. (b)	119. (d)	120. (d)
121. (c)	122. (d)	123. (d)	124. (b)	125. (c)	126. (a)	127. (d)	128. (d)
129. (a)	130. (b)	131. (d)	132. (d)	133. (c)	134. (c)	135. (c)	136. (a)
137. (b)	138. (d)	139. (b)	140. (c)	141. (d)	142. (d)	143. (b)	144. (d)
145. (a)	146. (b)	147. (c)	148. (b)	149. (c)	150. (c)	151. (c)	152. (a)
153. (c)	154. (d)	155. (b)					

SOLUTIONS

1. Capacity of the bank = Volume of the tank

$$= \left(\frac{8 \times 100 \times 6 \times 100 \times 2.5 \times 100}{1000}\right) \text{ litres} = 120000 \text{ litres.}$$

2. Surface area = $[2\ (10 \times 4 + 4 \times 3 + 10 \times 3)]\ cm^2 = (2 \times 82)\ cm^2 = 164\ cm^2$.

3. Area of the wet surface = $[2\ (lb + bh + lh) - lb] = 2\ (bh + lh) + lb$

$= [2\ (4 \times 1.25 + 6 \times 1.25) + 6 \times 4]\ m^2 = 49\ m^2$.

4. Volume of water displaced = $(3 \times 2 \times 0.01)\ m^3 = 0.06\ m^3$.

$\therefore$ Mass of man = Volume of water displaced $\times$ Density of water

$= (0.06 \times 1000)$ kg = 60 kg.

5. Volume = $(2.6 \times 100 \times 100 \times 100)$ cu. cm.

$$\text{Depth} = \frac{\text{Volume}}{\text{Area of the base}} = \left(\frac{2.6 \times 100 \times 100 \times 100}{6500}\right) \text{ cm} = 400 \text{ cm} = 4 \text{ m.}$$

6. Let length = x cm. Then, $x \times 28 \times 5 \times \frac{25}{1000} = 112$

$$\therefore \quad x = \left(112 \times \frac{1000}{25} \times \frac{1}{28} \times \frac{1}{5}\right) \text{ cm} = 32 \text{ cm.}$$

7. Volume of gold = $\left(\frac{1}{2} \times 100 \times 100 \times 100\right) cm^3$.

Area of sheet = $10000\ m^2 = (10000 \times 100 \times 100)\ cm^2$

$$\therefore \quad \text{Thickness of the sheet} = \left(\frac{1 \times 100 \times 100 \times 100}{2 \times 10000 \times 100 \times 100}\right) \text{ cm} = 0.005 \text{ cm.}$$

8. Area = $(1.5 \times 10000)\ m^2 = 15000\ m^2$.

$$\text{Depth} = \frac{5}{100} \text{ m} = \frac{1}{20} \text{ m.}$$

$$\therefore \quad \text{Volume} = (\text{Area} \times \text{Depth}) = \left(15000 \times \frac{1}{20}\right) m^3 = 750\ m^3.$$

9. Let the width of the wall be x metres.

Then, Height = $(6x)$ metres and Length = $(42x)$ metres.

$$\therefore \quad 42x \times x \times 6x = 16128 \iff x^3 = \left(\frac{16128}{42 \times 6}\right) = 64 \iff x = 4.$$

10. Let the dimensions be $3x$, $2x$ and x respectively. Then,

$$3x \times 2x \times x = 10368 \iff x^3 = \left(\frac{10368}{6}\right) = 1728 \iff x = 12.$$

So, the dimensions of the block are 36 dm, 24 dm, and 12 dm.

Surface area = $[2\ (36 \times 24 + 24 \times 12 + 36 \times 12)]\ dm^2$

$= [2 \times 144\ (6 + 2 + 3)]\ dm^2 = 3168\ dm^2$.

$$\therefore \quad \text{Cost of polishing} = \text{Rs.} \left(\frac{2 \times 3168}{100}\right) = \text{Rs. } 63.36.$$

11. Let the dimensions of the cuboid be x, $2x$ and $3x$.

Then, $2\ (x \times 2x + 2x \times 3x + x \times 3x) = 88$

$\iff 2x^2 + 6x^2 + 3x^2 = 44 \iff 11x^2 = 44 \iff x^2 = 4 \iff x = 2$.

$\therefore$ Volume of the cuboid = $(2 \times 4 \times 6)\ cm^3 = 48\ cm^3$.

12. Required length $= \sqrt{8^2 + 6^2 + 2^2}$ cm $= \sqrt{104}$ cm $= 2\sqrt{26}$ cm.

13. Required length $= \sqrt{(16)^2 + (12)^2 + \left(\frac{32}{3}\right)^2}$ m $= \sqrt{256 + 144 + \frac{1024}{9}}$ m

$= \sqrt{\frac{4624}{9}}$ m $= \frac{68}{3}$ m $= 22\frac{2}{3}$ m.

14. Number of bricks $= \frac{\text{Volume of the wall}}{\text{Volume of 1 brick}} = \left(\frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6}\right) = 6400.$

15. Volume of the bricks = 95% of volume of wall $= \left(\frac{95}{100} \times 600 \times 500 \times 50\right)$ cm^3.

Volume of 1 brick = $(25 \times 12.5 \times 7.5)$ cm^3.

$\therefore$ Number of bricks $= \left(\frac{95}{100} \times \frac{600 \times 500 \times 50}{25 \times 12.5 \times 7.5}\right) = 6080.$

16. Total volume of water displaced $= (4 \times 50)$ m^3 = 200 m^3.

$\therefore$ Rise in water level $= \left(\frac{200}{40 \times 20}\right)$ m = 0.25 m = 25 cm.

17. Volume of earth dug out $= \left(4 \times \frac{5}{2} \times \frac{3}{2}\right)$ m^3 = 15 m^3.

Area over which earth is spread $= \left(31 \times 10 - 4 \times \frac{5}{2}\right)$ m^2 = 300 m^2.

$\therefore$ Rise in level $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{15}{300} \times 100\right)$ cm = 5 cm.

18. Length of water column flown in 1 min. $= \left(\frac{3.5 \times 1000}{60}\right)$ m $= \frac{175}{3}$ m.

$\therefore$ Volume flown per minute $= \left(\frac{175}{3} \times 36 \times \frac{3}{2}\right)$ m^3 = 3150 m^3.

19. Length of water column flown in 1 min. $= \left(\frac{10 \times 1000}{60}\right)$ m $= \frac{500}{3}$ m.

Volume flown per minute $= \left(\frac{500}{3} \times \frac{40}{100 \times 100}\right)$ m^3 $= \frac{2}{3}$ m^3.

Volume flown in half an hour $= \left(\frac{2}{3} \times 30\right)$ m^3 = 20 m^3.

$\therefore$ Rise in water level $= \left(\frac{20}{40 \times 80}\right)$ m $= \left(\frac{1}{160} \times 100\right)$ cm $= \frac{5}{8}$ cm.

20. $2(15 + 12) \times h = 2(15 \times 12)$ or $h = \frac{180}{27}$ m $= \frac{20}{3}$ m.

$\therefore$ Volume $= \left(15 \times 12 \times \frac{20}{3}\right)$ m^3 = 1200 m^3.

21. $(l + b + h) = 19$ and $\sqrt{l^2 + b^2 + h^2} = 5\sqrt{5}$ and so $(l^2 + b^2 + h^2) = 125$.

Now, $(l + b + h)^2 = 19^2 \Rightarrow (l^2 + b^2 + h^2) + 2(lb + bh + lh) = 361$

$\Rightarrow 2(lb + bh + lh) = (361 - 125) = 236.$

$\therefore$ Surface area = 236 cm^2.

22. Volume $= \left[12 \times 9 \times \left(\frac{1+4}{2}\right)\right] m^3 = (12 \times 9 \times 2.5)\ m^3 = 270\ m^3$.

23. Clearly, $l = (48 - 16)$ m $= 32$ m, $b = (36 - 16)$ m $= 20$ m, $h = 8$ m.

$\therefore$ Volume of the box $= (32 \times 20 \times 8)\ m^3 = 5120\ m^3$.

24. Internal length $= (146 - 6)$ cm $= 140$ cm.

Internal breadth $= (116 - 6)$ cm $= 110$ cm.

Internal depth $= (83 - 3)$ cm $= 80$ cm.

Area of inner surface $= [2\ (l + b) \times h] + lb$

$= [2\ (140 + 110) \times 80 + 140 \times 110]\ cm^2 = 55400\ cm^2$.

$\therefore$ Cost of painting $=$ Rs. $\left(\frac{1}{2} \times \frac{1}{100} \times 55400\right) =$ Rs. 277.

25. Let the thickness of the bottom be x cm.

Then, $[(330 - 10) \times (260 - 10) \times (110 - x)] = 8000 \times 1000$

$\Leftrightarrow \quad 320 \times 250 \times (110 - x) = 8000 \times 1000 \Leftrightarrow (110 - x) = \frac{8000 \times 1000}{320 \times 250} = 100$

$\Leftrightarrow \quad x = 10$ cm $= 1$ dm.

26. Let the dimensions of the bigger cuboid be x, y and z.

Then, Volume of the bigger cuboid $= xyz$.

Volume of the miniature cuboid $= \left(\frac{1}{4}x\right)\left(\frac{1}{4}y\right)\left(\frac{1}{4}z\right) = \frac{1}{64}xyz$.

$\therefore$ Weight of the miniature cuboid $= \left(\frac{1}{64} \times 16\right)$ kg $= 0.25$ kg.

27. Let length $= l$, breadth $= b$ and height $= h$. Then,

Product of areas of 3 adjacent faces $= (lb \times bh \times lh) = (lbh)^2 = (\text{Volume})^2$.

28. Let the length, breadth and height of the box be l, b and h respectively. Then,

Volume $= lbh = \sqrt{(lbh)^2} = \sqrt{lb \times bh \times lh} = \sqrt{120 \times 72 \times 60} = 720\ cm^3$.

29. Let $lb = 2x$, $bh = 3x$ and $lh = 4x$.

Then, $24x^3 = (lbh)^2 = 9000 \times 9000 \Rightarrow x^3 = 375 \times 9000 \Rightarrow x = 150$.

So, $lb = 300$, $bh = 450$, $lh = 600$ and $lbh = 9000$.

$\therefore \quad h = \frac{9000}{300} = 30$, $l = \frac{9000}{450} = 20$ and $b = \frac{9000}{600} = 15$.

Hence, shortest side $= 15$ cm.

30. Edge of the cube $= \left(\frac{20}{4}\right)$ cm $= 5$ cm.

$\therefore$ Volume $= (5 \times 5 \times 5)\ cm^3 = 125\ cm^3$.

31. Surface area $= \left[6 \times \left(\frac{1}{2}\right)^2\right] cm^2 = \frac{3}{2}\ cm^2$.

32. Surface area of the cube $= (6 \times 8^2)$ sq. ft. $= 384$ sq. ft.

Quantity of paint required $= \left(\frac{384}{16}\right)$ kg $= 24$ kg.

$\therefore$ Cost of painting $=$ Rs. $(36.50 \times 24) =$ Rs. 876.

33. Volume of the cube $= (270 \times 100 \times 64)\ cm^3$.

Edge of the cube $= \sqrt[3]{270 \times 100 \times 64}$ cm $= (3 \times 10 \times 4)$ cm $= 120$ cm.

$\therefore$ Surface area $= (6 \times 120 \times 120)\ cm^2 = 86400\ cm^2$.

34. Surface area $= \left(\frac{34398}{13}\right) = 2646\ cm^2$.

$\therefore\ 6a^2 = 2646 \Rightarrow a^2 = 441 \Rightarrow a = 21.$

So, Volume $= (21 \times 21 \times 21)\ cm^3 = 9261\ cm^3$.

35. $a^3 = 729 \Rightarrow a = 9.$

$\therefore$ Surface area $= (6 \times 9 \times 9)\ cm^2 = 486\ cm^2$.

36. Required length = Diagonal $= \sqrt{3}\,a = (\sqrt{3} \times \sqrt{3})\ m = 3\ m.$

37. $\sqrt{3}\,a = 4\sqrt{3} \Rightarrow a = 4.$

$\therefore$ Volume $= (4 \times 4 \times 4)\ cm^3 = 64\ cm^3$.

38. $6a^2 = 600 \Rightarrow a^2 = 100 \Rightarrow a = 10.$

$\therefore$ Diagonal $= \sqrt{3}\,a = 10\sqrt{3}$ cm.

39. $a^3 = 6a^2 \Rightarrow a = 6.$

40. Number of cubes $= \left(\frac{100 \times 100 \times 100}{10 \times 10 \times 10}\right) = 1000.$

41. Number of blocks $= \left(\frac{160 \times 100 \times 60}{20 \times 20 \times 20}\right) = 120.$

42. Number of cubes $= \left(\frac{18 \times 18 \times 18}{3 \times 3 \times 3}\right) = 216.$

43. Volume of block $= (6 \times 9 \times 12)\ cm^3 = 648\ cm^3$.

Side of largest cube = H.C.F. of 6 cm, 9 cm, 12 cm = 3 cm.

Volume of this cube $= (3 \times 3 \times 3) = 27\ cm^3$.

$\therefore$ Number of cubes $= \left(\frac{648}{27}\right) = 24.$

44. Side of smallest cube = L.C.M. of 5 cm, 10 cm, 20 cm = 20 cm.

Volume of the cube $= (20 \times 20 \times 20)\ cm^3 = 8000\ cm^3$.

Volume of the block $= (5 \times 10 \times 20)\ cm^3 = 1000\ cm^3$.

$\therefore$ Number of blocks $= \left(\frac{8000}{1000}\right) = 8.$

45. Let the sides of the sheet be x and $5x$. Then,

Volume of the sheet = Volume of the cube

$\Rightarrow\ x \times 5x \times \frac{1}{2} = 10 \times 10 \times 10 \Rightarrow 5x^2 = 2000 \Rightarrow x^2 = 400 \Rightarrow x = 20.$

$\therefore$ The sides are 20 cm and 100 cm.

46. Volume of the new cube $= (6^3 + 8^3 + 10^3)\ cm^3 = 1728\ cm^3$.

Let the edge of the new cube be a cm.

$\therefore\ a^3 = 1728 \Rightarrow a = 12.$

47. The new solid formed is a cuboid of length 25 cm, breadth 5 cm and height 5 cm.

$\therefore$ Volume $= (25 \times 5 \times 5)\ cm^3 = 625\ cm^3$.

48. Required ratio $= \frac{6 \times 1 \times 1}{6 \times 5 \times 5} = \frac{1}{25} = 1:25.$

49. Volume of the large cube $= (3^3 + 4^3 + 5^3)\ cm^3 = 216\ cm^3$.

Let the edge of the large cube be a.

So, $a^3 = 216 \Rightarrow a = 6$ cm.

$\therefore$ Required ratio $= \frac{6 \times (3^2 + 4^2 + 5^2)}{6 \times 6^2} = \frac{50}{36} = 25:18.$

50. Let the sides of the three cubes be $3x$, $4x$ and $5x$.
Then, Volume of the new cube $= [(3x)^3 + (4x)^3 + (5x)^3] = 216x^3$.
Edge of the new cube $= (216x^3)^{1/3} = 6x$.
Diagonal of the new cube $= 6\sqrt{3}\,x$.

$\therefore\quad 6\sqrt{3}\,x = 12\sqrt{3} \;\Rightarrow\; x = 2$.

So, the sides of the cubes are 6 cm, 8 cm and 10 cm.

51. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{27}{1} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{3}{1}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{3}{1} \Leftrightarrow a : b = 3 : 1.$$

52. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{8}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{2}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{4}{9} \Leftrightarrow \frac{6a^2}{6b^2} = \frac{4}{9}.$$

53. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{1}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{1}{9}.$$

54. Let original edge $= a$. Then, volume $= a^3$.
New edge $= 2a$. So, new volume $= (2a)^3 = 8a^3$.
$\therefore$ Volume becomes 8 times.

55. Let original edge $= a$. Then, surface area $= 6a^2$.

New edge $= \frac{125}{100}a = \frac{5a}{4}$.

New surface area $= 6 \times \left(\frac{5a}{4}\right)^2 = \frac{75a^2}{8}$.

Increase in surface area $= \left(\frac{75a^2}{8} - 6a^2\right) = \frac{27a^2}{8}$.

$\therefore$ Increase % $= \left(\frac{27a^2}{8} \times \frac{1}{6a^2} \times 100\right)\% = 56.25\%$.

56. Volume $= \pi r^2 h = \left(\frac{22}{7} \times 1 \times 1 \times 14\right) \text{m}^3 = 44 \text{ m}^3$.

57. Volume of the tank $= 246.4$ litres $= 246400 \text{ cm}^3$.
Let the radius of the base be r cm. Then,

$$\left(\frac{22}{7} \times r^2 \times 400\right) = 246400 \Leftrightarrow r^2 = \left(\frac{246400 \times 7}{22 \times 400}\right) = 196 \Leftrightarrow r = 14.$$

$\therefore$ Diameter of the base $= 2r = 28$ cm.

58. $2\pi r = 66 \Rightarrow r = \left(66 \times \frac{1}{2} \times \frac{7}{22}\right) = \frac{21}{2}$ cm.

$$\frac{2\pi rh}{2\pi r} = \left(\frac{2640}{66}\right) \Rightarrow h = 40 \text{ cm}.$$

$\therefore$ Volume $= \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40\right) \text{cm}^3 = 13860 \text{ cm}^3$.

59. Let the radius and height be r cm each.

Then, $\frac{22}{7} \times r^2 \times r = \frac{176}{7} \Rightarrow r^3 = \left(\frac{176}{7} \times \frac{7}{22}\right) = 8 \Rightarrow r = 2.$

60. $\frac{2\pi rh}{h} = \frac{704}{14} \Rightarrow 2\pi r = \frac{704}{14}.$

$\therefore \quad r = \left(\frac{704}{14} \times \frac{1}{2} \times \frac{7}{22}\right) = 8$ cm.

$\therefore \quad$ Volume $= \left(\frac{22}{7} \times 8 \times 8 \times 14\right) \text{cm}^3 = 2816 \text{ cm}^3.$

61. Total surface area $= 2\pi r(h + r) = \left[2 \times \frac{22}{7} \times \frac{35}{100} \times (1.25 + 0.35)\right] \text{m}^2$

$= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times \frac{16}{10}\right) \text{m}^2 = 3.52 \text{ m}^2.$

$\therefore$ Cost of the material = Rs. (3.52 × 80) = Rs. 281.60.

62. Curved surface area $= 2\pi rh = (\pi r^2 h) \cdot \frac{2}{r} = \left(\text{Volume} \times \frac{2}{r}\right).$

63. $\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi rh + 2\pi r^2}{2\pi rh} = \frac{(h + r)}{h} = \frac{80}{60} = \frac{4}{3}.$

64. Difference in capacities $= \left(8 \times 8 \times 14 - \frac{22}{7} \times 4 \times 4 \times 14\right) \text{cm}^3 = 192 \text{ cm}^3.$

65. Let radius $= 2x$ and height $= 3x$. Then,

$\frac{22}{7} \times (2x)^2 \times 3x = 12936 \Leftrightarrow x^3 = \left(12936 \times \frac{7}{22} \times \frac{1}{12}\right) = 343 = 7^3$

$\therefore \quad x = 7$. So, radius = 14 cm and height = 21 cm.

$\therefore \quad$ Total surface area $= 2 \times \frac{22}{7} \times 14 \times (21 + 14) = \left(2 \times \frac{22}{7} \times 14 \times 35\right) \text{cm}^2 = 3080 \text{ cm}^2.$

66. It is given that $r = \frac{1}{2}h$ and $2\pi rh + \pi r^2 = 616 \text{ m}^2$

$\therefore \quad 2\pi \times \frac{1}{2}h \times h + \pi \times \frac{1}{4}h^2 = 616$

$\Rightarrow \quad \frac{5}{4} \times \frac{22}{7} \times h^2 = 616 \Rightarrow h^2 = \left(616 \times \frac{28}{110}\right) = \frac{28 \times 28}{5}.$

$\therefore \quad$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{1}{4} h^2 \times h = \frac{22}{7} \times \frac{1}{4} \times \frac{28 \times 28}{5} \times \frac{28}{\sqrt{5}} \text{ cm}^3$

$= \left(\frac{22 \times 28 \times 28}{25} \times \sqrt{5}\right) \text{cm}^3 = \left(\frac{22 \times 28 \times 28 \times 2.23}{25 \times 1000}\right)$ litres $= 1.53$ litre.

67. $(h + r) = 37$ and $2\pi r(h + r) = 1628.$

$\therefore \quad 2\pi r \times 37 = 1628$ or $r = \left(\frac{1628}{2 \times 37} \times \frac{7}{22}\right) = 7.$

So, $r = 7$ m and $h = 30$ m.

$\therefore \quad$ Volume $= \left(\frac{22}{7} \times 7 \times 7 \times 30\right) \text{m}^3 = 4620 \text{ m}^3.$

68. $\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow r = \left(\frac{924}{264} \times 2\right) = 7$ m.

And, $2\pi rh = 264 \Rightarrow h = \left(264 \times \frac{7}{22} \times \frac{1}{2} \times \frac{1}{7}\right) = 6$ m.

$\therefore$ Required ratio $= \frac{2r}{h} = \frac{14}{6} = 7:3.$

69. $V = \pi r^2 h$ and $S = 2\pi rh + 2\pi r^2$

$\Rightarrow S = 2\pi r(h + r)$, where $h = \frac{V}{\pi r^2}$

$\Rightarrow S = 2\pi r\left(\frac{V}{\pi r^2} + r\right) = \frac{2V}{r} + 2\pi r^2 \Rightarrow \frac{dS}{dr} = \frac{-2V}{r^2} + 4\pi r$ and $\frac{d^2S}{dr^2} = \left(\frac{4V}{r^3} + 4\pi\right) > 0$

$\therefore$ S is minimum when $\frac{dS}{dr} = 0$

$\Leftrightarrow \frac{-2V}{r^2} + 4\pi r = 0 \Leftrightarrow V = 2\pi r^3 \Leftrightarrow \pi r^2 h = 2\pi r^3 \Leftrightarrow h = 2r.$

70. Let original radius = R. Then, new radius $= \frac{R}{2}$.

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{R}{2}\right)^2 \times h}{\pi \times R^2 \times h} = \frac{1}{4}.$$

71. Let their radii be $2x$, $3x$ and heights be $5y$, $3y$.

Ratio of their volumes $= \frac{\pi \times (2x)^2 \times 5y}{\pi \times (3x)^2 \times 3y} = \frac{20}{27}.$

72. Let their heights be h and $2h$ and radii be r and R respectively. Then,

$\pi r^2 h = \pi R^2 (2h) \Rightarrow \frac{r^2}{R^2} = \frac{2h}{h} = \frac{2}{1} \Rightarrow \frac{r}{R} = \frac{\sqrt{2}}{1}$ *i.e.* $\sqrt{2}:1.$

73. Let the height of X and Y be h, and their radii be r and $2r$ respectively. Then,
Volume of X $= \pi r^2 h$ and Volume of Y $= \pi (2r)^2 h = 4\pi r^2 h.$

New height of X $= 2h.$

So, new volume of X $= \pi r^2 (2h) = 2\pi r^2 h = \frac{1}{2}(4\pi r^2 h) = \frac{1}{2} \times$ (Volume of Y).

74. Let original radius $= r$ and original length $= h$.

New radius $= \frac{r}{3}$ and let new length = H.

Then, $\pi r^2 h = \pi \left(\frac{r}{3}\right)^2 \times H$ or $H = 9h.$

75. Let the drop in the water level be h cm. Then,

$\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times h = 11000 \Leftrightarrow h = \left(\frac{11000 \times 7 \times 4}{22 \times 35 \times 35}\right)$ cm $= \frac{80}{7}$ cm $= 11\frac{3}{7}$ cm.

76. Volume of earth dug out $= \left(\frac{22}{7} \times 7 \times 7 \times 10\right) m^3 = 1540 \ m^3$.

Area of embankment $= \frac{22}{7} \times \left[(28)^2 - (7)^2\right] = \left(\frac{22}{7} \times 35 \times 21\right) m^2 = 2310 \ m^2$.

Height of embankment $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{1540}{2310}\right) m = \frac{2}{3} m$.

77. Volume of water flown in 1 sec. $= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 200\right) cm^3 = 7700 \ cm^3$.

Volume of water flown in 10 min. $= (7700 \times 60 \times 10) \ cm^3$

$= \left(\frac{7700 \times 60 \times 10}{1000}\right)$ litres $= 4620$ litres.

78. Volume of one coin $= \left(\frac{22}{7} \times \frac{75}{100} \times \frac{75}{100} \times \frac{2}{10}\right) cm^3 = \frac{99}{280} \ cm^3$.

Volume of larger cylinder $= \left(\frac{22}{7} \times 3 \times 3 \times 8\right) cm^3$.

$\therefore$ Number of coins $= \left(\frac{22 \times 9 \times 8}{7} \times \frac{280}{99}\right) = 640$.

79. Let the radius of the vessel be R. Then,

$\pi R^2 \times 15 = \pi \times (15)^2 \times 35 + \pi \times (10)^2 \times 15$

$\Leftrightarrow \pi R^2 \times 15 = 9375\pi \Leftrightarrow R^2 = 625 \Leftrightarrow R = 25$ cm.

80. Let the length of the wire be h.

Radius $= \frac{1}{2}$ mm $= \frac{1}{20}$ cm. Then,

$\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times h = 66 \Leftrightarrow h = \left(\frac{66 \times 20 \times 20 \times 7}{22}\right) = 8400 \text{ cm} = 84$ m.

81. Circumference of the girth = 440 cm.

$\therefore \quad 2\pi R = 440 \Rightarrow R = \left(440 \times \frac{1}{2} \times \frac{7}{22}\right) = 70$ cm.

So, Outer radius = 70 cm. Inner radius = (70 − 4) cm = 66 cm.

Volume of iron $= \pi \left[(70)^2 - (66)^2\right] \times 63 = \left(\frac{22}{7} \times 136 \times 4 \times 63\right) cm^3 = 58752 \ cm^3$.

82. Internal radius $= \left(\frac{11.2}{2}\right)$ cm = 5.6 cm, External radius = (5.6 + 0.4) cm = 6 cm.

Volume of metal $= \left\{\frac{22}{7} \times [(6)^2 - (5.6)^2] \times 21\right\} cm^3 = (66 \times 11.6 \times 0.4) \ cm^3 = 306.24 \ cm^3$.

83. External radius = 6 cm, Internal radius = (6 − 0.25) cm = 5.75 cm.

Volume of material in hollow cylinder

$= \left\{\frac{22}{7} \times [(6)^2 - (5.75)^2] \times 15\right\} cm^3 = \left(\frac{22}{7} \times 11.75 \times 0.25 \times 15\right) cm^3$

$= \left(\frac{22}{7} \times \frac{1175}{100} \times \frac{25}{100} \times 15\right) cm^3 = \left(\frac{11 \times 705}{56}\right) cm^3$.

Let the length of solid cylinder be h. Then,

$\frac{22}{7} \times 1 \times 1 \times h = \left(\frac{11 \times 705}{56}\right) \Leftrightarrow h = \left(\frac{11 \times 705}{56} \times \frac{7}{22}\right) \text{ cm} = 44.0625$ cm.

84. External radius = 4 cm, Internal radius = 3 cm.

Volume of iron = $\left\{\frac{22}{7}\times[(4)^2-(3)^2]\times 21\right\}\text{ cm}^3 = \left(\frac{22}{7}\times 7\times 1\times 21\right)\text{ cm}^3 = 462\text{ cm}^3$.

$\therefore$ Weight of iron = (462 × 8) gm = 3696 gm = 3.696 kg.

85. Let the internal radius of the cylinder be x. Then,

$\frac{22}{7}\times r^2\times 40 = \frac{616}{10} \Leftrightarrow r^2 = \left(\frac{616\times 7}{10\times 22\times 40}\right) = 0.49 \Leftrightarrow r = 0.7.$

So, internal radius = 0.7 cm = 7 mm.

$\therefore$ Thickness = (8 − 7) mm = 1 mm.

86. $\frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{\frac{1}{3}\times\pi\times(3)^2\times 5}{\pi\times(2)^2\times 4} = \frac{45}{48} = \frac{15}{16}.$

87. $h = 15$ cm, $r = 8$ cm. So, $l = \sqrt{r^2+h^2} = \sqrt{8^2+(15)^2} = 17$ cm.

$\therefore$ Curved surface area = $\pi rl = (\pi\times 8\times 17)\text{ cm}^2 = 136\pi\text{ cm}^2$.

88. $h = 14$ cm, $r = 7$ cm. So, $l = \sqrt{(7)^2+(14)^2} = \sqrt{245} = 7\sqrt{5}$ cm.

$\therefore$ Total surface area = $\pi rl + \pi r^2 = \left(\frac{22}{7}\times 7\times 7\sqrt{5} + \frac{22}{7}\times 7\times 7\right)\text{ cm}^2$

$= [154(\sqrt{5}+1)]\text{ cm}^2 = (154\times 3.236)\text{ cm}^2 = 498.35\text{ cm}^2$.

89. Clearly, we have $r = 3$ cm and $h = 4$ cm.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\times\pi\times 3^2\times 4\right)\text{ cm}^3 = 12\pi\text{ cm}^3$.

90. $l = 10$ m, $h = 8$ m. So, $r = \sqrt{l^2-h^2} = \sqrt{(10)^2-8^2} = 6$ m.

$\therefore$ Curved surface area = $\pi rl = (\pi\times 6\times 10)\text{ m}^2 = 60\pi\text{ m}^2$.

91. $\frac{1}{3}\times\frac{22}{7}\times r^2\times 24 = 1232 \Leftrightarrow r^2 = \left(\frac{1232\times 7\times 3}{22\times 24}\right) = 49 \Leftrightarrow r = 7.$

Now, $r = 7$ cm, $h = 24$ cm. So, $l = \sqrt{(7)^2+(24)^2} = 25$ cm.

$\therefore$ Curved surface area = $\left(\frac{22}{7}\times 7\times 25\right)\text{ cm}^2 = 550\text{ cm}^2$.

92. Let the radius of the base be r km. Then,

$\pi r^2 = 1.54 \Rightarrow r^2 = \left(\frac{1.54\times 7}{22}\right) = 0.49 \Rightarrow r = 0.7$ km.

Now, $l = 2.5$ km, $r = 0.7$ km.

$\therefore$ $h = \sqrt{(2.5)^2-(0.7)^2}$ km $= \sqrt{6.25-0.49}$ km $= \sqrt{5.76}$ km = 2.4 km.

So, height of the mountain = 2.4 km.

93. $\pi r^2 = 3850 \Rightarrow r^2 = \left(\frac{3850\times 7}{22}\right) = 1225 \Rightarrow r = 35.$

Now, $r = 35$ cm, $h = 84$ cm.

So, $l = \sqrt{(35)^2+(84)^2} = \sqrt{1225+7056} = \sqrt{8281} = 91$ cm.

$\therefore$ Curved surface area = $\left(\frac{22}{7}\times 35\times 91\right)\text{ cm}^2 = 10010\text{ cm}^2$.

94. $\frac{22}{7} \times 70 \times l = 40040 \Rightarrow l = \left(\frac{40040 \times 7}{22 \times 70}\right) = 182.$

Now, $l = 182$ cm, $r = 70$ cm.

So, $h = \sqrt{(182)^2 - (70)^2} = \sqrt{252 \times 112} = 168$ cm.

$\therefore$ Volume $= \left(\frac{1}{3} \times \frac{22}{7} \times 70 \times 70 \times 168\right) \text{cm}^3 = 862400 \text{ cm}^3.$

95. Let the radius and the height of the cone be $3x$ and $4x$ respectively. Then,

$\frac{1}{3} \times \pi \times (3x)^2 \times 4x = 96\pi \Leftrightarrow 36x^3 = (96 \times 3) \Rightarrow x^3 = \left(\frac{96 \times 3}{36}\right) = 8 \Leftrightarrow x = 2.$

$\therefore$ Radius = 6 cm, Height = 8 cm.

Slant height $= \sqrt{6^2 + 8^2}$ cm $= \sqrt{100}$ cm $= 10$ cm.

96. $\pi r^2 = 346.5 \Rightarrow r^2 = \left(346.5 \times \frac{7}{22}\right) = \frac{441}{4} \Rightarrow r = \frac{21}{2}.$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{\frac{441}{4} + (14)^2} = \sqrt{\frac{1225}{4}} = \frac{35}{2}.$

So, area of canvas needed $= \pi r l = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2}\right) \text{m}^2 = \left(\frac{33 \times 35}{2}\right) \text{m}^2.$

$\therefore$ Length of canvas $= \left(\frac{33 \times 35}{2 \times 1.1}\right)$ m $= 525$ m.

97. Let the original radius and height of the cone be r and h respectively.
Then, new radius $= 2r$. New height $= 2h$.

$$\therefore \frac{\text{New Volume}}{\text{Original Volume}} = \frac{\frac{1}{3} \times \pi \times (2r)^2 \times 2h}{\frac{1}{3} \times \pi \times r^2 \times h} = \frac{8}{1}.$$

98. Let the original radius and height of the cone be r and h respectively.

Then, Original volume $= \frac{1}{3} \pi r^2 h.$

New radius $= \frac{120}{100} r = \frac{6}{5} r$, New height $= \frac{6}{5} h.$

New volume $= \frac{1}{3} \pi \times \left(\frac{6}{5} r\right)^2 \times \left(\frac{6}{5} h\right) = \frac{216}{125} \times \frac{1}{3} \pi r^2 h.$

Increase in volume $= \frac{91}{125} \times \frac{1}{3} \pi r^2 h.$

$$\therefore \text{Increase } \% = \left(\frac{\frac{91}{125} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100\right) \% = 72.8\%.$$

99. Let the original radius and height of the cone be r and h respectively.

Then, original volume $= \frac{1}{3} \pi r^2 h.$

New radius $= \frac{r}{2}$ and new height $= 3h.$

New volume $= \frac{1}{3} \times \pi \times \left(\frac{r}{2}\right)^2 \times 3h = \frac{3}{4} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Decrease % $= \left(\frac{\frac{1}{4} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100 \right) \% = 25\%.$

100. Required ratio $= \frac{\frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 \times (2h)} = \frac{1}{2}$

101. Let their heights be x, $3x$ and their radii be $3y$, y.

Then, Ratio of volumes $= \frac{\frac{1}{3} \times \pi \times (3y)^2 \times x}{\frac{1}{3} \times \pi \times y^2 \times (3x)} = \frac{9}{3} = 3 : 1.$

102. Let their radii be $2x$, x and their heights be h and H respectively. Then,

$\frac{1}{3} \times \pi \times (2x)^2 \times h = \frac{1}{3} \times \pi \times x^2 \times \text{H}$ or $\frac{h}{\text{H}} = \frac{1}{4}.$

103. Let their radii be $4x$ and $5x$, and their heights be h and H respectively. Then,

$\frac{\frac{1}{3} \times \pi \times (4x)^2 \times h}{\frac{1}{3} \times \pi \times (5x)^2 \times \text{H}} = \frac{1}{4}$ or $\frac{h}{\text{H}} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}.$

104. Volume of the largest cone

= Volume of the cone with diameter of base 7 cm and height 7 cm

$= \left(\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7\right) \text{cm}^3 = \left(\frac{269.5}{3}\right) \text{cm}^3 = 89.8 \text{ cm}^3.$

105. Volume of the block = $(10 \times 5 \times 2)$ cm^3 = 100 cm^3.

Volume of the cone carved out $= \left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7\right) \text{cm}^3 = 66 \text{ cm}^3.$

$\therefore$ Wood wasted = $(100 - 66)\%$ = 34%.

106. Let their radius and height be $5x$ and $12x$ respectively.

Slant height of the cone, $l = \sqrt{(5x)^2 + (12x)^2} = 13x.$

$\frac{\text{Total surface area of cylinder}}{\text{Total surface area of cone}} = \frac{2\pi r (h + r)}{\pi r (l + r)} = \frac{2 (h + r)}{(l + r)} = \frac{2 \times (12x + 5x)}{(13x + 5x)} = \frac{34x}{18x} = \frac{17}{9}.$

107. Let the radius of the cone be r cm.

Then, $\frac{1}{3} \pi \times r^2 \times 6 = \pi \times 8 \times 8 \times 2 \iff r^2 = \left(\frac{8 \times 8 \times 2 \times 3}{6}\right) = 64 \iff r = 8$ cm.

108. Let radius of each be r and height of each be h.

Then, number of cones needed $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = 3.$

109. Volume of cylinder = $(\pi \times 3 \times 3 \times 5)$ cm^3 = 45π cm^3.

Volume of 1 cone $= \left(\frac{1}{3} \pi \times \frac{1}{10} \times \frac{1}{10} \times 1\right) \text{cm}^3 = \frac{\pi}{300} \text{cm}^3.$

$\therefore$ Number of cones $= \left(45\pi \times \frac{300}{\pi}\right) = 13500.$

110. Volume flown in conical vessel $= \frac{1}{3}\pi \times (20)^2 \times 24 = 3200\pi$.

Volume flown in 1 min. $= \left(\pi \times \frac{2.5}{10} \times \frac{2.5}{10} \times 1000\right) = 62.5\pi$.

$\therefore$ Time taken $= \left(\frac{3200\pi}{62.5\pi}\right) = 51$ min. 12 sec.

111. Slant height of the cone, $l = \sqrt{(12)^2 + (5)^2} = 13$ cm.

Lateral surface of the solid = Curved surface of cone + Curved surface of cylinder + Surface area of bottom

$= \pi rl + 2\pi rh + \pi r^2$, where h is the height of the cylinder

$= \pi r\,(l + h + r) = \left[\frac{22}{7} \times 12 \times (13 + 18 + 12)\right] \text{cm}^2$

$= \left(\frac{22}{7} \times 12 \times 43\right) \text{cm}^2 = \left(\frac{11352}{7}\right) \text{cm}^2 = 1621\frac{5}{7} \text{cm}^2$.

112. Volume of parallelopiped $= (5 \times 3 \times 4)$ cm^3 = 60 cm^3.

Volume of cube $= (4)^3$ cm^3 = 64 cm^3.

Volume of cylinder $= \left(\frac{22}{7} \times 3 \times 3 \times 3\right) \text{cm}^3 = 84.86 \text{ cm}^3$.

Volume of sphere $= \left(\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3\right) = 113.14 \text{ cm}^3$.

113. $\frac{4}{3} \times \frac{22}{7} \times R^3 = 4851 \Rightarrow R^3 = \left(4851 \times \frac{3}{4} \times \frac{7}{22}\right) = \left(\frac{21}{2}\right)^3 \Rightarrow R = \frac{21}{2}$.

$\therefore$ Curved surface area $= \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2 = 1386 \text{ cm}^2$.

114. $4\pi R^2 = 5544 \Rightarrow R^2 = \left(5544 \times \frac{1}{4} \times \frac{7}{22}\right) = 441 \Rightarrow R = 21$.

$\therefore$ Volume $= \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right) \text{cm}^3 = 38808 \text{ cm}^3$.

115. Volume $= \frac{4}{3}\pi r^3 = \frac{r}{3}(4\pi r^2) = \frac{r}{3} \times$ Surface area.

116. $\frac{\frac{4}{3}\pi R^3}{4\pi R^2} = 27 \Rightarrow R = 81$ cm.

117. Let the radii of A and B be r and R respectively.

$\therefore$ Required ratio $= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{40}{10}\right)^2 = 16:1$.

118. Let the original radius be r.

Then, original surface area $= 4\pi r^2 = 2464$ cm^2 (given).

New radius $= 2r$.

$\therefore$ New surface area $= 4\pi\,(2r)^2 = 4 \times 4\pi r^2 = (4 \times 2464)$ cm^2 = 9856 cm^2.

119. Let the original radius be r. Then, original volume $= \frac{4}{3}\pi r^3$.

New radius $= 2r$.

$\therefore$ New volume $= \frac{4}{3}\pi\,(2r)^3 = 8 \times \frac{4}{3}\pi r^3 = 8 \times$ original volume.

120. $4\pi (r+2)^2 - 4\pi r^2 = 352 \Leftrightarrow (r+2)^2 - r^2 = \left(352 \times \frac{7}{22} \times \frac{1}{4}\right) = 28.$

$\Leftrightarrow (r+2+r)(r+2-r) = 28 \Leftrightarrow 2r + 2 = 14 \Rightarrow r = \left(\frac{14}{2} - 1\right) = 6$ cm.

121. Let the correct radius be 100 cm. Then, measured radius = 101.5 cm.

$\therefore$ Error in volume $= \frac{4}{3}\pi\,[(101.5)^3 - (100)^3]\ \text{cm}^3$

$= \frac{4}{3}\pi\,(1045678.375 - 1000000)\ \text{cm}^3 = \left(\frac{4}{3} \times \pi \times 45678.375\right) \text{cm}^3.$

$\therefore$ Error % $= \left\{\dfrac{\frac{4}{3}\pi\,(45678.375)}{\frac{4}{3}\pi\,(100 \times 100 \times 100)} \times 100\right\}\% = 4.56\% = 4.6\%$ (app.).

122. Let their radii be R and r. Then,

$$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \frac{64}{27} = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{4}{3}.$$

Ratio of surface areas $= \dfrac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}.$

123. Let their radii be R and r. Then,

$$\frac{4\pi R^2}{4\pi r^2} = \frac{4}{25} \Rightarrow \left(\frac{R}{r}\right)^2 = \left(\frac{2}{5}\right)^2 \Rightarrow \frac{R}{r} = \frac{2}{5}.$$

$\therefore$ Ratio of volumes $= \dfrac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}.$

124. Volume of new sphere $= \left[\frac{4}{3}\pi \times (6)^3 + \frac{4}{3}\pi \times (8)^3 + \frac{4}{3}\pi \times (10)^3\right] \text{cm}^3$

$= \left\{\frac{4}{3}\pi\,[(6)^3 + (8)^3 + (10)^3]\right\} \text{cm}^3$

$= \left(\frac{4}{3}\pi \times 1728\right) \text{cm}^3 = \left[\frac{4}{3}\pi \times (12)^3\right] \text{cm}^3.$

Let the radius of the new sphere be R. Then,

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (12)^3 \Rightarrow R = 12$ cm.

$\therefore$ Diameter = 2R = 24 cm.

125. Volume of bigger sphere $= \left[\frac{4}{3}\pi \times (8)^3\right] \text{cm}^3 = \left(\frac{4}{3}\pi \times 512\right) \text{cm}^3.$

Volume of 1 ball $= \left[\frac{4}{3}\pi \times (2)^3\right] \text{cm}^3 = \left(\frac{4}{3}\pi \times 8\right) \text{cm}^3.$

$\therefore$ Number of balls $= \left(\dfrac{\frac{4}{3}\pi \times 512}{\frac{4}{3}\pi \times 8}\right) = \frac{512}{8} = 64.$

126. Let the radius of the third ball be R cm. Then,

$$\frac{4}{3}\pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi \times (1)^3 + \frac{4}{3}\pi \times R^3 = \frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \frac{27}{64} + 1 + R^3 = \frac{27}{8} \Rightarrow R^3 = \frac{125}{64} = \left(\frac{5}{4}\right)^3 \Rightarrow R = \frac{5}{4}.$$

$\therefore$ Diameter of the third ball $= 2R = \frac{5}{2}$ cm $= 2.5$ cm.

127. Volume of each ball $= \frac{1}{8} \times \left(\frac{4}{3}\pi \times 10 \times 10 \times 10\right) \text{cm}^3$.

$$\therefore \frac{4}{3}\pi R^3 = \frac{1}{8} \times \frac{4}{3}\pi \times 10 \times 10 \times 10 \Rightarrow R^3 = \left(\frac{10}{2}\right)^3 = 5^3 \Rightarrow R = 5.$$

128. External radius = 3 cm, Internal radius = (3 − 0.5) cm = 2.5 cm.

$$\text{Volume of the metal} = \left[\frac{4}{3} \times \frac{22}{7} \times \{(3)^3 - (2.5)^3\}\right] \text{cm}^3$$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times \frac{91}{8}\right) \text{cm}^3 = \left(\frac{143}{3}\right) \text{cm}^3 = 47\frac{2}{3} \text{ cm}^3.$$

129. Volume of the solid = $(49 \times 33 \times 24)$ cm^3.

Let the radius of the sphere be r.

$$\text{Then, } \frac{4}{3}\pi r^3 = (49 \times 33 \times 24) \Leftrightarrow r^3 = \left(\frac{49 \times 33 \times 24 \times 3 \times 7}{4 \times 22}\right) = (21)^3 \Leftrightarrow r = 21.$$

130. $$\text{Number of bullets} = \frac{\text{Volume of the cube}}{\text{Volume of 1 bullet}} = \left(\frac{22 \times 22 \times 22}{\frac{4}{3} \times \frac{22}{7} \times 1 \times 1 \times 1}\right) = 2541.$$

131. $$\text{Volume of each lead shot} = \left[\frac{4}{3}\pi \times \left(\frac{0.3}{2}\right)^3\right] \text{cm}^3 = \left(\frac{4}{3} \times \frac{22}{7} \times \frac{27}{8000}\right) \text{cm}^3 = \frac{99}{7000} \text{ cm}^3.$$

$$\therefore \text{ Number of lead shots} = \left(9 \times 11 \times 12 \times \frac{7000}{99}\right) = 84000.$$

132. $$4\pi R^2 = 6a^2 \Rightarrow \frac{R^2}{a^2} = \frac{3}{2\pi} \Rightarrow \frac{R}{a} = \frac{\sqrt{3}}{\sqrt{2\pi}}.$$

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi R^3}{a^3} = \frac{4}{3}\pi \cdot \left(\frac{R}{a}\right)^3 = \frac{4}{3}\pi \cdot \frac{3\sqrt{3}}{2\pi\sqrt{2\pi}} = \frac{2\sqrt{3}}{\sqrt{2\pi}} = \frac{\sqrt{12}}{\sqrt{2\pi}} = \frac{\sqrt{6}}{\sqrt{\pi}}.$$

133. Let the edge of the cube be a. Then, volume of the cube = a^3.

Radius of the sphere = $(a/2)$.

$$\text{Volume of the sphere} = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{\pi a^3}{6}.$$

$$\therefore \text{ Required ratio} = a^3 : \frac{\pi a^3}{6} = 6 : \pi.$$

134. $$4\pi R^2 = 2\pi \times 6 \times 12 \Rightarrow R^2 = \left(\frac{6 \times 12}{2}\right) = 36 \Rightarrow R = 6 \text{ cm}.$$

135. Let the radius of the cylinder be R.

Then, $\pi \times R^2 \times \frac{7}{3} = \frac{4}{3}\pi \times 7 \times 7 \times 7$

$\Rightarrow \quad R^2 = \left(\frac{4 \times 7 \times 7 \times 7}{3} \times \frac{3}{7}\right) = 196 = (14)^2 \Rightarrow R = 14$ cm.

$\therefore$ Diameter = 2R = 28 cm.

136. Required volume = Volume of a sphere of radius 1 cm

$= \left(\frac{4}{3}\pi \times 1 \times 1 \times 1\right) \text{cm}^3 = \frac{4}{3}\pi \text{ cm}^3.$

137. Volume of cylinder $= \pi \times (3)^2 \times 15 = 135\pi$ cm^3.

Radius of 1 bullet $= \frac{5}{2}$ mm $= \frac{5}{20}$ cm $= \frac{1}{4}$ cm.

Volume of 1 bullet $= \left(\frac{4}{3}\pi \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) \text{cm}^3 = \frac{\pi}{48} \text{ cm}^3.$

$\therefore$ Number of bullets $= \left(135\pi \times \frac{48}{\pi}\right) = 6480.$

138. Let the radius of the cylindrical rod be r.

Then, height of the rod $= 8r$ and radius of one ball $= \frac{r}{2}$.

$\therefore$ Number of balls $= \frac{\pi \times r^2 \times 8r}{\frac{4}{3}\pi \times \left(\frac{r}{2}\right)^3} = \left(\frac{8 \times 8 \times 3}{4}\right) = 48.$

139. Let the length of the wire be h.

Then, $\pi \times \frac{3}{20} \times \frac{3}{20} \times h = \frac{4}{3}\pi \times 4 \times 4 \times 4$

$\Leftrightarrow \quad h = \left(\frac{4 \times 4 \times 4 \times 4 \times 20 \times 20}{3 \times 3 \times 3}\right) \text{cm} = \left(\frac{102400}{27}\right) \text{cm} = 3792.5 \text{ cm} = 37.9 \text{ m}.$

140. Let the rise in the water level be h cm.

Then, $\pi \times 4 \times 4 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{3 \times 3}{4}\right) = \frac{9}{4}$ cm.

141. Let the radius of each sphere be r cm.

Then, Volume of 12 spheres = Volume of cylinder

$\Rightarrow \quad 12 \times \frac{4}{3}\pi \times r^3 = \pi \times 8 \times 8 \times 2 \Rightarrow r^3 = \left(\frac{8 \times 8 \times 2 \times 3}{12 \times 4}\right) = 8 \Rightarrow r = 2$ cm.

$\therefore$ Diameter of each sphere $= 2r = 4$ cm.

142. Let the radius of the ball be r cm.

Volume of ball = Volume of water displaced by it

$\therefore \quad \frac{4}{3}\pi r^3 = \pi \times 12 \times 12 \times 6.75 \Rightarrow r^3 = 9 \times 9 \times 9 \Rightarrow r = 9$ cm.

143. Let the height of the cone be h cm. Then,

$\frac{1}{3}\pi \times 6 \times 6 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{36 \times 3}{36}\right) = 3$ cm.

144. Volume of sphere $= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3.

Volume of cone $= \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3.

Volume of wood wasted $= \left[\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) - \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)\right]$ cm^3
$= (\pi \times 9 \times 9 \times 9)$ cm^3.

$\therefore$ Required percentage $= \left(\dfrac{\pi \times 9 \times 9 \times 9}{\frac{4}{3}\pi \times 9 \times 9 \times 9} \times 100\right)\% = \left(\frac{3}{4} \times 100\right)\% = 75\%$.

145. Number of spheres $= \dfrac{\text{Volume of cone}}{\text{Volume of 1 sphere}} = \dfrac{\frac{1}{3}\pi \times 12 \times 12 \times 24}{\frac{4}{3}\pi \times 2 \times 2 \times 2} = 108.$

146. Volume of material in the sphere $= \left[\frac{4}{3}\pi \times \{(4)^3 - (2)^3\}\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 56\right)$ cm^3.

Let the height of the cone be h cm.

Then, $\frac{1}{3}\pi \times 4 \times 4 \times h = \left(\frac{4}{3}\pi \times 56\right) \Leftrightarrow h = \left(\frac{4 \times 56}{4 \times 4}\right) = 14$ cm.

147. Let radius = R and height = H. Then,

Ratio of their volumes $= \pi R^2 H : \frac{1}{3}\pi R^2 H : \frac{4}{3}\pi R^3 = H : \frac{1}{3}H : \frac{4}{3}R$

$= H : \frac{1}{3}H : \frac{4}{3} \times \frac{H}{2}$ $\left[\text{In sphere, } H = 2R \text{ or } R = \frac{H}{2}\right]$

$= 3 : 1 : 2.$

148. Total surface area $= 3\pi R^2 = \left(3 \times \frac{22}{7} \times 7 \times 7\right)$ cm^2 $= 462$ cm^2.

149. Let the radius be R cm. Then,

$\frac{2}{3} \times \frac{22}{7} \times R^3 = 19404 \Leftrightarrow R^3 = \left(19404 \times \frac{21}{44}\right) = (21)^3 \Leftrightarrow R = 21$ cm.

150. Let their radii be R and r. Then,

$\dfrac{\frac{2}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \dfrac{6.4}{21.6} \Leftrightarrow \left(\frac{R}{r}\right)^3 = \frac{8}{27} = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{R}{r} = \frac{2}{3}.$

$\therefore$ Ratio of curved surface areas $= \dfrac{2\pi R^2}{2\pi r^2} = \left(\frac{R}{r}\right)^2 = \frac{4}{9}.$

151. Let the height of the vessel be x. Then, radius of the bowl = radius of the vessel $= \frac{x}{2}$.

Volume of the bowl, $V_1 = \frac{2}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{12}\pi x^3.$

Volume of the vessel, $V_2 = \pi\left(\frac{x}{2}\right)^2 x = \frac{1}{4}\pi x^3.$

Since $V_2 > V_1$, so the vessel can contain 100% of the beverage filled in the bowl.

152. $\frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 H \Rightarrow H = 2R.$

153. Let the radius of the cone be R cm. Then,

$$\frac{1}{3}\pi \times R^2 \times 75 = \frac{2}{3}\pi \times 6 \times 6 \times 6$$

$$\Leftrightarrow R^2 = \left(\frac{2\times 6\times 6\times 6}{75}\right) = \left(\frac{144}{25}\right) = \left(\frac{12}{5}\right)^2 \Leftrightarrow R = \frac{12}{5}\text{ cm} = 2.4\text{ cm}.$$

154. Let the radius of each be R. Height of hemisphere, H = R.

So, height of cone = height of hemisphere = R.

Slant height of cone $= \sqrt{R^2 + R^2} = \sqrt{2}\,R.$

$$\frac{\text{Curved surface area of hemisphere}}{\text{Curved surface area of cone}} = \frac{2\pi R^2}{\pi R \times \sqrt{2}\,R} = \sqrt{2} : 1.$$

155. Volume of hemisphere $= \frac{2}{3}\pi r^3.$

Volume of biggest sphere = Volume of sphere with diameter $r = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{1}{6}\pi r^3$

$$\therefore \text{ Required ratio} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{6}\pi r^3} = \frac{4}{1} \text{ i.e. } 4:1.$$

EXERCISE 25B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 10*) : ***Each of the questions given below consists of a statement and / or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is / are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the weight of the iron beam ?
   I. The beam is 9 m long, 40 cm wide and 20 cm high.
   II. Iron weighs 50 kg per cubic metre.
2. What is the volume of 32 metre high cylindrical tank ? **(Bank P.O. 2003)**
   I. The area of its base is 154 m^2.
   II. The diameter of the base is 14 m.
3. What is the volume of a cube ? **(Bank P.O. 2003)**
   I. The area of each face of the cube is 64 square metres.
   II. The length of one side of the cube is 8 metres.

4. What is the total cost of painting the inner surface of an open box at the rate of 50 paise per 100 sq. cm ?
   I. The box is made of wood 3 cm thick.
   II. The external dimensions of the box are 50 cm, 40 cm and 23 cm.
5. What is the capacity of a cylindrical tank ? **(I.B.P.S. 2002)**
   I. Radius of the base is half of its height which is 28 metres.
   II. Area of the base is 616 sq. metres and its height is 28 metres.
6. What is the volume of the cylinder ? **(Bank P.O. 2003)**
   I. Height is equal to the diameter.
   II. Perimeter of the base is 352 cm.
7. What will be the total cost of whitewashing the conical tomb at the rate of 80 paise per square metre ?
   I. The diameter and the slant height of the tomb are 28 m and 50 m.
   II. The height of the tomb is 48 m and the area of its base is 616 sq. m.
8. What is the height of a circular cone ? **(Bank P.O. 1999)**
   I. The area of that cone is equal to the area of a rectangle whose length is 33 cm.
   II. The area of the base of that cone is 154 sq. cm.
9. Is a given rectangular block, a cube ? **(M.A.T. 1999)**
   I. At least 2 faces of the rectangular block are squares.
   II. The volume of the block is 64.
10. A spherical ball of given radius x cm is melted and made into a right circular cylinder. What is the height of the cylinder ? **(S.B.I.P.O. 2003)**
    I. The volume of the cylinder is equal to the volume of the ball.
    II. The area of the base of the cylinder is given.

Directions (*Questions 11-13*) : ***Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.***

11. What is the capacity of the cylindrical tank ? **(R.B.I. 2003)**
    I. The area of the base is 61,600 sq. cm.
    II. The height of the tank is 1.5 times the radius.
    III. The circumference of base is 880 cm.
    (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
    (*d*) Any two of the three (*e*) Only II and either I or III
12. A solid metallic cone is melted and recast into a sphere. What is the radius of the sphere ?
    I. The radius of the base of the cone is 2.1 cm.
    II. The height of the cone is four times the radius of its base.
    III. The height of the cone is 8.4 cm.
    (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
    (*d*) Any two of the three (*e*) All I, II and III
13. What is the total surface area of the cone ?
    I. The area of the base of the cone is 154 cm^2.
    II. The curved surface area of the cone is 550 cm^2.
    III. The volume of the cone is 1232 cm^3.
    (*a*) I, and either II or III (*b*) II, and either I or III
    (*c*) III, and either I or II (*d*) Any two of the three
    (*e*) None of these

ANSWERS

1. (e) 2. (c) 3. (c) 4. (e) 5. (c) 6. (e) 7. (c) 8. (d)
9. (d) 10. (b) 11. (e) 12. (d) 13. (a)

SOLUTIONS

1. I gives, $l = 9$ m, $b = \frac{40}{100}$ m $= \frac{2}{5}$ m and $h = \frac{20}{100}$ m $= \frac{1}{5}$ m.

This gives, volume $= (l \times b \times h) = \left(9 \times \frac{2}{5} \times \frac{1}{5}\right) m^3 = \frac{18}{25} m^3$.

II gives, weight of iron is 50 kg / m^3.

$\therefore$ Weight $= \left(\frac{18}{25} \times 50\right)$ kg $= 36$ kg.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

2. Given, height = 32 m.

I gives, area of the base = 154 m^2.

$\therefore$ Volume = (area of the base × height) = $(154 \times 32)\ m^3 = 4928\ m^3$.

Thus, I alone gives the answer.

II gives, radius of the base = 7 m.

$\therefore$ Volume $= \pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 32\right) m^3 = 4928\ m^3$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

3. Let each edge be a metres. Then,

I. $a^2 = 64 \Rightarrow a = 8$ m $\Rightarrow$ Volume $= (8 \times 8 \times 8)\ m^3 = 512\ m^3$.

Thus, I alone gives the answer.

II. $a = 8$ m $\Rightarrow$ Volume $= (8 \times 8 \times 8)\ m^3 = 512\ m^3$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

4. I gives, thickness of the wall of the box = 3 cm.

II gives, Internal length = (50 − 6) cm = 44 cm, Internal breadth = (40 − 6) = 34 cm,

Internal height = (23 − 3) cm = 20 cm.

Area to be painted = (area of 4 walls + area of floor) $= [2(l + b) \times h + (l \times b)]$

$= [2(44 + 34) \times 20 + (44 \times 34)]\ cm^2 = 4616\ cm^2$.

Cost of painting = Rs. $\left(\frac{1}{2 \times 100} \times 4616\right)$ = Rs. 23.08.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

5. I gives, $h = 28$ m and $r = 14$ cm.

$\therefore$ Capacity $= \pi r^2 h$, which can be obtained.

Thus, I alone gives the answer.

II gives, $\pi r^2 = 616\ m^2$ and $h = 28$ m.

$\therefore$ Capacity $= (\pi r^2 \times h) = (616 \times 28)\ m^3$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

6. I gives, $h = 2r$.

II gives, $2\pi r = 352 \Rightarrow r = \left(\frac{352}{2} \times \frac{7}{22}\right)$ cm $= 56$ cm.

From I and II, we have $r = 56$ cm, $h = (2 \times 56)$ cm $= 112$ cm.

Thus, we can find the volume.

$\therefore$ Correct answer is (e).

7. I gives, $r = 14$ m, $l = 50$ m.

$\therefore$ Curved surface $= \pi rl = \left(\frac{22}{7} \times 14 \times 50\right) m^2 = 2200\ m^2$.

Cost of whitewashing = Rs. $\left(2200 \times \frac{80}{100}\right)$ = Rs. 1760.

Thus, I alone gives the answer.

II gives, $h = 48$ m, $\pi r^2 = 616\ m^2$.

These results give r and h and so l can be found out.

$\therefore$ Curved surface $= \pi rl$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

8. II gives the value of r.

But, in I, the breadth of rectangle is not given.

So, we cannot find the surface area of the cone.

Hence, the height of the cone cannot be determined.

$\therefore$ Correct answer is (d).

9. I gives, any two of l, b, h are equal.

II gives, $lbh = 64$.

From I and II, the values of l, b, h may be (1, 1, 64), (2, 2, 16), (4, 4, 4).

Thus, the block may be a cube or cuboid.

$\therefore$ Correct answer is (d).

10. Clearly, I is not needed, since it is evident from the given question.

From II, we get radius of the base of the cylinder.

Now, $\frac{4}{3}\pi x^3 = \pi r^2 h$ in which x and r are known.

$\therefore$ h can be determined.

$\therefore$ Correct answer is (b).

11. Capacity $= \pi r^2 h$.

I gives, $\pi r^2 = 61600$. This gives r.

II gives, $h = 1.5\ r$.

Thus, I and II give the answer.

Again, III gives $2\pi r = 880$. This gives r.

So, II and III also give the answer.

$\therefore$ Correct answer is (e).