

73. The average age of a husband and his wife was 23 years at the time of their marriage. After five years they have a one-year old child. The average age of the family now is :
(a) 19 years (b) 23 years (c) 28.5 years (d) 29.3 years
(Assistant Grade., 1998)
74. Three years ago, the average age of A and B was 18 years. With C joining them, the average age becomes 22 years. How old is C now ?
(a) 24 years (b) 27 years (c) 28 years (d) 30 years
75. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of the husband is :
(a) 35 years (b) 40 years (c) 50 years (d) None of these
(Hotel Management, 2003)
76. 3 years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is
(a) 1 year (b) $1\frac{1}{2}$ years (c) 2 years (d) 3 years
(S.S.C. 2004)
77. 10 years ago, the average age of a family of 4 members was 24 years. Two children having been born (with age difference of 2 years), the present average age of the family is the same. The present age of the youngest child is :
(a) 1 year (b) 2 years (c) 3 years (d) 5 years
(S.S.C. 2003)
78. After replacing an old member by a new member, it was found that the average age of five members of a club is the same as it was 3 years ago. What is the difference between the ages of the replaced and the new member ?
(a) 2 years (b) 4 years (c) 8 years (d) 15 years
79. The average age of 3 children in a family is 20% of the average age of the father and the eldest child. The total age of the mother and the youngest child is 39 years. If the father's age is 26 years, what is the age of second child ?
(a) 15 years (b) 18 years (c) 20 years (d) Cannot be determined
80. The average age of a group of persons going for picnic is 16 years. Twenty new persons with an average age of 15 years join the group on the spot due to which their average age becomes 15.5 years. The number of persons initially going for picnic is :
(a) 5 (b) 10 (c) 20 (d) 30
(S.S.C. 2003)
81. A certain factory employed 600 men and 400 women and the average wage was Rs. 25.50 per day. If a woman got Rs. 5 less than a man, then what are their daily wages ?
(a) Man : Rs. 25; Woman : Rs. 20 (b) Man : Rs. 27.50, Woman : Rs. 22.50
(c) Man : Rs. 30, Woman : Rs. 25 (d) Man : Rs. 32.50, Woman : Rs. 27.50
82. The arithmetic mean of the scores of a group of students in a test was 52. The brightest 20% of them secured a mean score of 80 and the dullest 25% a mean score of 31. The mean score of remaining 55% is :
(a) 45 (b) 50 (c) 51.4 approx. (d) 54.6 approx.
(S.S.C. 2000)
83. The average salary of all the workers in a workshop is Rs. 8000. The average salary of 7 technicians is Rs. 12000 and the average salary of the rest is Rs. 6000. The total number of workers in the workshop is :
(a) 20 (b) 21 (c) 22 (d) 23
(S.S.C. 2003)
84. In a school with 600 students, the average age of the boys is 12 years and that of the girls is 11 years. If the average age of the school is 11 years 9 months, then the number of girls in the school is :
(a) 150 (b) 250 (c) 350 (d) 450

85. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many papers were there in the examination?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

(SCMHRD, 2001)

86. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is : (S.B.I.P.O. 2002)

- (A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 3 : 5

ANSWERS

1. (e) 2. (e) 3. (d) 4. (b) 5. (b) 6. (c) 7. (b) 8. (c) 9. (b)
10. (b) 11. (c) 12. (d) 13. (b) 14. (c) 15. (d) 16. (a) 17. (c) 18. (c)
19. (b) 20. (d) 21. (d) 22. (b) 23. (d) 24. (b) 25. (c) 26. (a) 27. (b)
28. (c) 29. (d) 30. (d) 31. (d) 32. (b) 33. (c) 34. (b) 35. (d) 36. (c)
37. (d) 38. (a) 39. (a) 40. (a) 41. (a) 42. (d) 43. (c) 44. (c) 45. (e)
46. (c) 47. (b) 48. (d) 49. (d) 50. (c) 51. (c) 52. (a) 53. (d) 54. (d)
55. (a) 56. (d) 57. (b) 58. (c) 59. (c) 60. (a) 61. (b) 62. (d) 63. (a)
64. (c) 65. (c) 66. (d) 67. (b) 68. (a) 69. (d) 70. (c) 71. (c) 72. (c)
73. (a) 74. (a) 75. (b) 76. (c) 77. (c) 78. (d) 79. (d) 80. (c) 81. (b)
82. (c) 83. (b) 84. (a) 85. (d) 86. (b)

SOLUTIONS

1. Average = $\left(\frac{76 + 65 + 82 + 67 + 85}{5} \right) = \left(\frac{375}{5} \right) = 75$
2. Let Arun's weight be X kg. According to question, $65 < X < 72$.
 According to Arun, $65 < X < 72$.
 According to Arun's brother, $60 < X < 70$.
 According to Arun's mother, $X < 68$.
 The values satisfying all the above conditions are 66 and 67.
 Required average = $\left(\frac{66 + 67}{2} \right) = \left(\frac{133}{2} \right) = 66.5$ kg.
3. Average of 20 numbers = 0.
 Sum of 20 numbers = $(0 \times 20) = 0$. It is quite possible that 19 of these numbers may be positive and if their sum is a , then 20th number is $(-a)$.
4. Average = $\left(\frac{10 + 15 + 20 + 25 + 30}{5} \right) = \frac{100}{5} = 20$.
5. Average = $\frac{3(1+2+3+4+5)}{5} = \frac{45}{5} = 9$.
6. Average = $\left(\frac{2+3+5+7+11+13+17+19+23}{9} \right) = \frac{100}{9} = 11\frac{1}{9}$.

7. Clearly, we have $\left(\frac{3+11+7+9+15+13+8+19+17+21+14+x}{12} \right) = 12$
 or $137 + x = 144$ or $x = 144 - 137 = 7$.
8. We have : $\left(\frac{2+7+6+x}{4} \right) = 5$ or $15 + x = 20$ or $x = 5$.
 Also, $\left(\frac{18+1+6+x+y}{5} \right) = 10$ or $25 + 5 + y = 50$ or $y = 20$.
9. We have : $\left[\frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5} \right] = 11$ or $5x + 20 = 55$ or $x = 7$.
 So, the numbers are 7, 9, 11, 13, 15.
 Required mean = $\left(\frac{11+13+15}{3} \right) = \frac{39}{3} = 13$.
10. We have : $\left(\frac{a+b+c}{3} \right) = M$ or $(a+b+c) = 3M$.
 Now, $(a+b+c)^2 = (3M)^2 = 9M^2$.
 $\Leftrightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 9M^2$
 $\Leftrightarrow a^2 + b^2 + c^2 = 9M^2$. [Since $(ab + bc + ca) = 0$]
 Required mean = $\left(\frac{a^2 + b^2 + c^2}{3} \right) = \frac{9M^2}{3} = 3M^2$.
11. Average = $\left(\frac{11+22+33+44+55+66+77+88+99}{9} \right)$
 $= \left[\frac{(11+99)+(22+88)+(33+77)+(44+66)+55}{9} \right]$
 $= \left(\frac{4 \times 110 + 55}{9} \right) = \frac{495}{9} = 55$.
12. Sum of first n natural numbers = $\frac{n(n+1)}{2}$.
 So, average of first n natural numbers = $\frac{n(n+1)}{2n} = \frac{n+1}{2}$.
 Required average = $\left(\frac{50+1}{2} \right) = \frac{51}{2} = 25.5$.
13. $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
 $\therefore 1^2 + 2^2 + 3^2 + \dots + 7^2 = \left(\frac{7 \times 8 \times 15}{6} \right) = 140$.
 So, required average = $\left(\frac{140}{7} \right) = 20$.
14. Sum of odd numbers upto 100 = $1 + 3 + 5 + 7 + \dots + 95 + 97 + 99$.
 $= (1+99) + (3+97) + (5+95) + \dots + \text{upto 25 pairs}$
 $= 100 + 100 + 100 + \dots (25 \text{ times}) = 2500$.
 Average = $\left(\frac{2500}{50} \right) = 50$.

15. Clearly, $b = a + 2$, $c = a + 4$, $d = a + 6$ and $e = a + 8$.

$$\therefore \text{Average} = \frac{a + (a + 2) + (a + 4) + (a + 6) + (a + 8)}{5} = \left(\frac{5a + 20}{5} \right) = (a + 4).$$

16. Let the number be x . Then,

$$\frac{x + x^2}{2} = 5x \Leftrightarrow x^2 - 9x = 0 \Leftrightarrow x(x - 9) = 0 \Leftrightarrow x = 0 \text{ or } x = 9.$$

So, the number is 9.

17. Let the numbers be x , $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + 5$ and $x + 6$.

$$\text{Then, } \frac{x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) + (x + 6)}{7} = 20$$

$$\text{or } 7x + 21 = 140 \text{ or } 7x = 119 \text{ or } x = 17.$$

$$\therefore \text{Largest number} = x + 6 = 23.$$

18. Let the numbers be x , $x + 2$, $x + 4$, $x + 6$ and $x + 8$.

$$\text{Then, } \frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5} = 61 \text{ or } 5x + 20 = 305 \text{ or } x = 57.$$

$$\text{So, required difference} = (57 + 8) - 57 = 8.$$

19. Let the numbers be x , $x + 2$ and $x + 4$.

$$\text{Then, } (x + x + 2 + x + 4) - \frac{(x + x + 2 + x + 4)}{3} = 38$$

$$\text{or } (3x + 6) - \frac{(3x + 6)}{3} = 38 \text{ or } 2(3x + 6) = 114 \text{ or } 6x = 102 \text{ or } x = 17.$$

$$\text{So, first number} = x = 17.$$

20. Clearly, to find the average, we ought to know the number of boys, girls or students in the class, neither of which has been given.

So, the data provided is inadequate.

21. Let the number of other workers be x .

Then, number of agricultural workers = $11x$.

Total number of workers = $12x$.

$$\therefore \text{Average monthly income} = \frac{S \times 11x + T \times x}{12x} = \frac{11S + T}{12}.$$

$$\begin{aligned} 22. \text{Required average} &= \left(\frac{67 \times 2 + 35 \times 2 + 6 \times 3}{2 + 2 + 3} \right) \\ &= \left(\frac{134 + 70 + 18}{7} \right) = \frac{222}{7} = 31\frac{5}{7} \text{ years.} \end{aligned}$$

23. Since the month begins with a Sunday, so there will be five Sundays in the month.

$$\therefore \text{Required average} = \left(\frac{510 \times 5 + 240 \times 25}{30} \right) = \frac{8550}{30} = 285.$$

$$\begin{aligned} 24. \text{Required average} &= \left(\frac{55 \times 50 + 60 \times 55 + 45 \times 60}{55 + 60 + 45} \right) \\ &= \left(\frac{2750 + 3300 + 2700}{160} \right) = \frac{8750}{160} = 54.68. \end{aligned}$$

$$\begin{aligned} 25. \text{Required average} &= \left(\frac{50.25 \times 16 + 45.15 \times 8}{16 + 8} \right) \\ &= \left(\frac{804 + 36120}{24} \right) = \frac{1165.20}{24} = 48.55. \end{aligned}$$

26. Total quantity of petrol consumed in 3 years = $\left(\frac{4000}{7.50} + \frac{4000}{8} + \frac{4000}{8.50} \right)$ litres.
 $\therefore \text{Total quantity} = \frac{4000}{7.50} + \frac{4000}{8} + \frac{4000}{8.50} = 4000 \left(\frac{2}{15} + \frac{1}{8} + \frac{2}{17} \right) = \left(\frac{76700}{51} \right)$ litres.

Total amount spent = Rs. (3×4000) = Rs. 12000.

$\therefore \text{Average cost} = \text{Rs.} \left(\frac{12000 \times 51}{76700} \right) = \text{Rs.} \frac{6120}{767} = \text{Rs.} 7.98.$

27. Clearly, we have : $x = \left(\frac{3y+3z}{6} \right)$ or $2x = y+z$.

28. Let the average expenditure be Rs. x . Then,

$9x = 8 \times 30 + (x + 20)$ or $9x = x + 260$ or $8x = 260$ or $x = 32.50$.

$\therefore \text{Total money spent} = 9x = \text{Rs.} (9 \times 32.50) = \text{Rs.} 292.50$.

29. Sum of 50 numbers = $30 \times 50 = 1500$.

Sum of remaining 48 numbers = $1500 - (35 + 40) = 1425$.

$\therefore \text{Required average} = \left(\frac{1425}{48} \right) = \frac{475}{16} = 29.68$.

30. Excluded number = $(27 \times 5) - (25 \times 4) = 135 - 100 = 35$.

31. Sum of the ages of 14 students = $(16 \times 35) - (14 \times 21) = 560 - 294 = 266$.

$\therefore \text{Required average} = \left(\frac{266}{14} \right) = 19 \text{ years.}$

32. Required average = $\frac{(76 \times 16) - (75 \times 10)}{6} = \frac{(1216 - 750)}{6} = \frac{466}{6} = \frac{233}{3} = 77\frac{2}{3}$.

33. Required average = $\frac{(38.9 \times 10) - (42 \times 6)}{4} = \frac{137}{4} = 34.25$.

34. Sum of the remaining two numbers = $(3.95 \times 6) - [(3.4 \times 2) + (3.85 \times 2)]$

$= (23.70 - 6.8 + 7.7) = 23.70 - 14.5 = 9.20$.

$\therefore \text{Required average} = \left(\frac{9.2}{2} \right) = 4.6$.

35. Let the highest score be x . Then, lowest score = $(x - 172)$.

Then, $(50 \times 40) - [x + (x - 172)] = 38 \times 48 + (100 - x) + 10 + 18$.

$\Leftrightarrow 2x = 2000 + 172 - 1824 \Leftrightarrow 2x = 348 \Leftrightarrow x = 174$.

36. Total price of the two books = Rs. $[(12 \times 10) - (11.75 \times 8)]$
 $= \text{Rs.} (120 - 94) = \text{Rs.} 26$.

Let the price of one book be Rs. x .

Then, the price of other book = Rs. $(x + 60\% \text{ of } x) = \text{Rs.} \left(x + \frac{3}{5}x \right) = \text{Rs.} \left(\frac{8x}{5} \right)$.

So, $x + \frac{8x}{5} = 26 \Leftrightarrow 13x = 130 \Leftrightarrow x = 10$.

$\therefore \text{The prices of the two books are Rs.} 10 \text{ and Rs.} 16$.

37. Average after 11 innings = 36.

$\therefore \text{Required number of runs} = (36 \times 11) - (32 \times 10)$

$= 396 - 320 = 76$.

38. Total sale for 5 months = Rs. $(6435 + 6927 + 6855 + 7230 + 6562) = \text{Rs.} 34009$.

$\therefore \text{Required sale} = \text{Rs.} [(6500 \times 6) - 34009] = \text{Rs.} (39000 - 34009) = \text{Rs.} 4991$.

39. Required average = $\frac{(4375 \times 12) - (4000 \times 3)}{9} = \frac{52500 - 12000}{9} = \frac{40500}{9} = 4500.$

40. Required run rate = $\frac{282 - (3.2 \times 10)}{40} = \frac{250}{40} = 6.25.$

41. Let their prices be $3x$, $5x$ and $7x$.
 Then, $3x + 5x + 7x = (15000 \times 3)$ or $x = 3000.$

∴ Cost of cheapest item = $3x = \text{Rs. } 9000.$

42. Let the fourth number be x .

Then, third number = $5x$, second number = $\frac{5x}{3}$ and first number = $\frac{10x}{3}$.

$$x + 5x + \frac{5x}{3} + \frac{10x}{3} = (24.75 \times 4) \text{ or } 11x = 99 \text{ or } x = 9.$$

So, the numbers are 9, 45, 15 and 30.

∴ Largest number = 45.

43. Let the first number be x .

Then, sum of the four numbers = $x + 4x = 5x$.

$$\text{So, } \frac{5x}{4} = 60 \text{ or } x = \left(\frac{60 \times 4}{5}\right) = 48.$$

44. Let the third number be x . Then, second number = $2x$. First number = $4x$.

$$\therefore \frac{1}{x} + \frac{1}{2x} + \frac{1}{4x} = \left(\frac{7}{72} \times 3\right) \text{ or } \frac{7}{4x} = \frac{7}{24} \text{ or } 4x = 24 \text{ or } x = 6.$$

So, the numbers are 24, 12 and 6.

45. Let the numbers be x , y and z .

$$\text{Then, } \left(\frac{x+y}{2}\right) - \left(\frac{y+z}{2}\right) = 15 \text{ or } (x+y) - (y+z) = 30 \text{ or } x-z = 30.$$

46. Let the eighth number be x . Then, sixth number = $(x-7)$.

Seventh number = $(x-7) + 4 = (x-3)$.

$$\text{So, } \left(2 \times 15\frac{1}{2}\right) + \left(3 \times 21\frac{1}{3}\right) + (x-7) + (x-3) + x = 8 \times 20$$

$$\Leftrightarrow 31 + 64 + (3x-10) = 160 \Leftrightarrow 3x = 75 \Leftrightarrow x = 25.$$

47. A.M. of 75 numbers = 35.

Sum of 75 numbers = $(75 \times 35) = 2625.$

Total increase = $(75 \times 5) = 375.$

Increased sum = $(2625 + 375) = 3000.$

$$\text{Increased average} = \frac{3000}{75} = 40.$$

48. Average of 10 numbers = 7.

Sum of these 10 numbers = $(10 \times 7) = 70.$

∴ $x_1 + x_2 + \dots + x_{10} = 70.$

$$\Rightarrow 12x_1 + 12x_2 + \dots + 12x_{10} = 840$$

$$\Rightarrow \frac{12x_1 + 12x_2 + \dots + 12x_{10}}{10} = 84$$

⇒ Average of new numbers is 84.

49. $\frac{x_1 + x_2 + \dots + x_{10}}{10} = \text{new average}$ $\Rightarrow x_1 + x_2 + \dots + x_{10} = 10\bar{x}$
 $\text{old sum} = 110 \times 100 = 1100$ $\Rightarrow x_1 + x_2 + \dots + x_{10} = 1100$ \Rightarrow $x_1 + x_2 + \dots + x_{10} = 10\bar{x}$
 $\text{new } \bar{x} = \frac{110}{100} \bar{x}_1 + \frac{110}{100} \bar{x}_2 + \dots + \frac{110}{100} \bar{x}_{10} = \frac{110}{100} \times 10\bar{x}$
 $\text{new } \bar{x} = \frac{110}{100} \bar{x}_1 + \frac{110}{100} \bar{x}_2 + \dots + \frac{110}{100} \bar{x}_{10} = \frac{11}{10} \bar{x}$
 $\bar{x} = \bar{x} \Rightarrow \bar{x} = \bar{x} \Rightarrow \bar{x} = \bar{x} \Rightarrow \bar{x} = \bar{x}$
 $\Rightarrow \text{Average is increased by } 10\%.$

50. Correct sum = $(36 \times 50 + 48 - 23) = 1825$.

$$\therefore \text{Correct mean} = \frac{1825}{50} = 36.5.$$

51. Let there be x pupils in the class.

$$\text{Total increase in marks} = \left(x \times \frac{1}{2} \right) = \frac{x}{2}.$$

$$\therefore \frac{x}{2} = (83 - 63) \Rightarrow \frac{x}{2} = 20 \Rightarrow x = 40.$$

52. Age of the 15th student = $[15 \times 15 - (14 \times 5 + 16 \times 9)] = (225 - 214) = 11$ years.

53. Middle number = $[(10.5 \times 6 + 11.4 \times 6) - 10.9 \times 11] = (131.4 - 119.9) = 11.5$.

54. Total weight of $(A + B + C) = \left(54 \frac{1}{3} \times 3 \right)$ kg = 163 kg.

Total weight of $(B + D + E) = (53 \times 3)$ kg = 159 kg.

Adding both, we get : $A + 2B + C + D + E = (163 + 159)$ kg = 322 kg.

So, to find the average weight of A, B, C, D and E, we ought to know B's weight, which is not given. So, the data is inadequate.

55. Sum of temperatures on 1st, 2nd, 3rd and 4th days = $(58 \times 4) = 232$ degrees ... (i)

Sum of temperatures on 2nd, 3rd, 4th and 5th days = $(60 \times 4) = 240$ degrees ... (ii)

Subtracting (i) from (ii), we get :

Temp. on 5th day - Temp. on 1st day = 8 degrees.

Let the temperatures on 1st and 5th days be $7x$ and $8x$ degrees respectively.

Then, $8x - 7x = 8$ or $x = 8$.

\therefore Temperature on the 5th day = $8x = 64$ degrees.

56. Let A, B, C represent their respective weights. Then, we have :

$$A + B + C = (45 \times 3) = 135 \quad \dots(i)$$

$$A + B = (40 \times 2) = 80 \quad \dots(ii)$$

$$B + C = (43 \times 2) = 86 \quad \dots(iii)$$

$$\text{Adding (ii) and (iii), we get : } A + 2B + C = 166 \quad \dots(iv)$$

$$\text{Subtracting (i) from (iv), we get : } B = 31. \quad \dots(v)$$

$$\therefore B's \text{ weight} = 31 \text{ kg.}$$

57. Let P, Q and R represent their respective monthly incomes. Then, we have :

$$P + Q = (5050 \times 2) = 10100 \quad \dots(i)$$

$$Q + R = (6250 \times 2) = 12500 \quad \dots(ii)$$

$$P + R = (5200 \times 2) = 10400 \quad \dots(iii)$$

$$\text{Adding (i), (ii) and (iii), we get : } 2(P + Q + R) = 33000 \text{ or } P + Q + R = 16500 \quad \dots(iv)$$

$$\text{Subtracting (ii) from (iv), we get } P = 4000.$$

$$\therefore P's \text{ monthly income} = \text{Rs. } 4000.$$

58. Age of the teacher = $(37 \times 15 - 36 \times 14)$ years = 51 years.
59. Manager's monthly salary = Rs. $(1600 \times 21 - 1500 \times 20)$ = Rs. 3600.
60. Weight of the teacher = $(35.4 \times 25 - 35 \times 24)$ kg = 45 kg.
61. Age of the mother = $(12 \times 7 - 7 \times 6)$ years = 42 years.
62. Let the average age of the whole team be x years.
 $\therefore 11x - (26 + 29) = 9(x - 1) \Rightarrow 11x - 9x = 46 \Rightarrow 2x = 46 \Rightarrow x = 23$.
 So, average age of the team is 23 years.
63. Sum of heights of the 5 boys = $(25 \times 1.4 - 20 \times 1.55)$ m = 4 m.
 \therefore Required average = $\left(\frac{4}{5}\right)$ m = 0.8 m.
64. Total weight increased = (8×2.5) kg = 20 kg.
 Weight of new person = $(65 + 20)$ kg = 85 kg.
65. Sum of the weights of the students after replacement
 $= [(52 \times 45) - (48 \times 5) + (54 \times 5)]$ kg = 2370 kg.
 \therefore New average = $\left(\frac{2370}{45}\right)$ kg = $53\frac{2}{3}$ kg.
66. Total age increased = (8×2) years = 16 years.
 Sum of ages of two new men = $(21 + 23 + 16)$ years = 60 years.
 \therefore Average age of two new men = $\left(\frac{60}{2}\right)$ years = 30 years.
67. Let five consecutive numbers be $x, x + 1, x + 2, x + 3$ and $x + 4$.
 Their average = $\frac{5x + 10}{5} = (x + 2)$.
 Average of 7 numbers = $\frac{(5x + 10) + (x + 5) + (x + 6)}{7} = \frac{7x + 21}{7} = (x + 3)$.
 So, the average increased by 1.
68. Let average for 10 innings be x . Then,
 $\frac{10x + 108}{11} = x + 6 \Rightarrow 11x + 66 = 10x + 108 \Rightarrow x = 42$.
 \therefore New average = $(x + 6) = 48$ runs.
69. Let the number of wickets taken till the last match be x . Then,
 $\frac{12.4x + 26}{x + 5} = 12 \Rightarrow 12.4x + 26 = 12x + 60 \Rightarrow 0.4x = 34 \Rightarrow x = \frac{34}{0.4} = \frac{340}{4} = 85$.
70. Let the total score be x .
 $\therefore \frac{x + 92 - 85}{8} = 84 \Rightarrow x + 7 = 672 \Rightarrow x = 665$.
71. Average speed = $\frac{2xy}{x+y}$ km/hr = $\left(\frac{2 \times 50 \times 30}{50+30}\right)$ km/hr = 37.5 km/hr.
72. Let A, B, C, D and E represent their respective weights. Then,
 $A + B + C = (84 \times 3) = 252$ kg, $A + B + C + D = (80 \times 4) = 320$ kg.
 $\therefore D = (320 - 252)$ kg = 68 kg, $E = (68 + 3)$ kg = 71 kg.
 $B + C + D + E = (79 \times 4) = 316$ kg.
 Now, $(A + B + C + D) - (B + C + D + E) = (320 - 316)$ kg = 4 kg.
 $\therefore A - E = 4 \Rightarrow A = (4 + E) = 75$ kg.

73. Sum of the present ages of husband, wife and child = $(23 \times 2 + 5 \times 2) + 1 = 57$ years.
 \therefore Required average = $\left(\frac{57}{3}\right) = 19$ years.
74. Present age of $(A + B) = (18 \times 2 + 3 \times 2)$ years = 42 years.
 Present age of $(A + B + C) = (22 \times 3)$ years = 66 years.
 \therefore C's age = $(66 - 42)$ years = 24 years.
75. Sum of the present ages of husband, wife and child = $(27 \times 3 + 3 \times 3)$ years = 90 years.
 Sum of the present ages of wife and child = $(20 \times 2 + 5 \times 2)$ years = 50 years.
 \therefore Husband's present age = $(90 - 50)$ years = 40 years.
76. Total age of 5 members, 3 years ago = (17×5) years = 85 years.
 Total age of 5 members now = $(85 + 3 \times 5)$ years = 100 years.
 Total age of 6 members now = (17×6) years = 102 years.
 \therefore Age of the baby = $(102 - 100)$ years = 2 years.
77. Total age of 4 members, 10 years ago = (24×4) years = 96 years.
 Total age of 4 members now = $(96 + 10 \times 4)$ years = 136 years.
 Total age of 6 members now = (24×6) years = 144 years.
 Sum of the ages of 2 children = $(144 - 136)$ years = 8 years.
 Let the age of the younger child be x years.
 Then, age of the elder child = $(x + 2)$ years.
 $\therefore x + x + 2 = 8 \Leftrightarrow 2x = 6 \Leftrightarrow x = 3$.
 \therefore Age of younger child = 3 years.
78. Age decreased = (5×3) years = 15 years.
 So, the required difference = 15 years.
79. Since the total or average age of all the family members is not given, the given data is inadequate. So, the age of second child cannot be determined.
80. Let the initial number of persons be x . Then,
 $16x + 20 \times 15 = 15.5(x + 20) \Leftrightarrow 0.5x = 10 \Leftrightarrow x = 20$.
81. Let the daily wage of a man be Rs. x .
 Then, daily wage of a woman = Rs. $(x - 5)$.
 Now, $600x + 400(x - 5) = 25.50 \times (600 + 400) \Leftrightarrow 1000x = 27500 \Leftrightarrow x = 27.50$.
 \therefore Man's daily wages = Rs. 27.50; Woman's daily wages = $(x - 5)$ = Rs. 22.50.
82. Let the required mean score be x . Then,
 $20 \times 80 + 25 \times 31 + 55 \times x = 52 \times 100$
 $\Leftrightarrow 1600 + 775 + 55x = 5200 \Leftrightarrow 55x = 2825 \Leftrightarrow x = \frac{565}{11} = 51.4$.
83. Let the total number of workers be x . Then,
 $8000x = (12000 \times 7) + 6000(x - 7) \Leftrightarrow 2000x = 42000 \Leftrightarrow x = 21$.
84. Let the number of girls be x . Then, number of boys = $(600 - x)$.
 Then, $\left(11\frac{3}{4} \times 600\right) = 11x + 12(600 - x) \Leftrightarrow x = 7200 - 7050 \Leftrightarrow x = 150$.
85. Let the number of papers be x . Then, $63x + 20 + 2 = 65x$ or $2x = 22$ or $x = 11$.
86. Let the ratio be $k : 1$. Then,
 $k \times 16.4 + 1 \times 15.4 = (k + 1) \times 15.8$
 $\Leftrightarrow (16.4 - 15.8)k = (15.8 - 15.4) \Leftrightarrow k = \frac{0.4}{0.6} = \frac{2}{3}$.
 \therefore Required ratio = $\frac{2}{3} : 1 = 2 : 3$.

Rising TD = 1 + (M × 3 + S × 22) × 6

EXERCISE 6B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (Questions 1 to 10) : Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The total of the present ages of A, B, C and D is 96 years. What is B's present age ?
 - I. The average age of A, B and D is 20 years.
 - II. The average age of C and D is 25 years.
2. What is the average age of children in the class ? (Bank P.O. 2003)
 - I. Age of the teacher is as many years as the number of children.
 - II. Average age increased by 1 year if the teacher's age is also included.
3. What is the average weight of the three new team members who are recently included in the team ?
 - I. The average weight of the team increases by 20 kg.
 - II. The three new men substitute earlier members whose weights are 64 kg, 75 kg and 66 kg.
4. The average age of P, Q, R and S is 30 years. How old is R ? (R.B.I. 2003)
 - I. The sum of ages of P and R is 60 years.
 - II. S is 10 years younger than R.
5. How old will C be after 10 years ?
 - I. Five years ago, the average age of A and B was 15 years.
 - II. Average age of A, B and C today is 20 years.
6. How many children are there in the group ? (Bank P.O. 2000)
 - I. Average age of the children in this group is 15 years. The total age of all the children in this group is 240 years.
 - II. The total age of all the children in the group and the teacher is 264 years. The age of the teacher is 9 years more than the average age of the children.
7. Deepak's marks in Hindi are 15 more than the average marks obtained by him in Hindi, Economics, Sociology and Philosophy. What are his marks in Philosophy ?
 - I. The total marks obtained by him in Hindi and Philosophy together is 120.
 - II. The difference between the marks obtained by him in Sociology and Economics is 120.
8. How many candidates were interviewed everyday by the panel A out of the three panels A, B and C ? (Bank P.O. 1999)
 - I. The three panels on an average interview 15 candidates everyday.
 - II. Out of a total of 45 candidates interviewed everyday by the three panels, the number of candidates interviewed by panel A is more by 2 than the candidates interviewed by panel C and is more by 1 than the candidates interviewed by panel B.

9. The average age of teacher and students in a class is 3 years more than the average age of students. What is the age of the class teacher ? (Bank P.O. 2000)

I. There are 11 students in the class.

(a) 8 (b) 12 (c) 15 (d) 18 (e) 20

II. The average age of teacher and students is 14 years.

(a) 10 (b) 12 (c) 14 (d) 16 (e) 18

10. What will be the average weight of the remaining class ? (Bank P.O. 1999)

I. Average weight of 30 children out of total 46 in the class is 22.5 kg and that of the remaining children is 29.125 kg. A child having weight more than 40 kg is excluded.

II. Average weight of a class of 46 children is 23.5 kg. A child weighing 46 kg is dropped out.

Directions (Questions 11 to 13) : Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.

11. How many marks did Tarun secure in English ? (S.B.I.P.O. 2000)

I. The average marks obtained by Tarun in four subjects including English is 60.

II. The total marks obtained by him in English and Mathematics together is 170.

III. The total marks obtained by him in Mathematics and Science together is 180.

(a) I and II only (b) II and III only (c) I and III only

(d) All I, II and III (e) None of these

12. The mean temperature of Monday to Wednesday was 37°C and of Tuesday to Thursday was 34°C. What was the temperature on Thursday ? (S.B.I.P.O. 2000)

I. The temperature on Thursday was $\frac{4}{5}$ th that of Monday.

II. The mean temperature of Monday and Thursday was 40.5°C.

III. The difference between the temperature on Monday and that on Thursday was 9°C.

(a) I and II only (b) II and III only (c) Either I or II

(d) Either I, II or III (e) Any two of the three

13. In a cricket eleven, the average age of eleven players is 28 years. What is the age of the captain ?

I. The captain is eleven years older than the youngest player.

II. The average age of 10 players, other than the captain is 27.3 years.

III. Leaving aside the captain and the youngest player, the average ages of three groups of three players each are 25 years, 28 years and 30 years respectively.

(a) Any two of the three (b) All I, II and III

(c) II only or I and III only (d) II and III only

(e) None of these

Directions (Question 14) : The given question is followed by three statements labelled I, II and III. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the given question.

14. What is the average salary of 15 employees ? (S.B.I.P.O. 2001)

I. Average salary of 7 clerical cadre (out of the 15 employees) is Rs. 8500.

II. Average salary of 5 officer cadre (out of the 15 employees) is Rs. 10000.

III. Average salary of the 3 sub-staff employees (out of the 15 employees) is Rs. 2500.

(a) None (b) Only I

(c) Only II (d) Only III

(e) Question cannot be answered even with information in all the three statements

ANSWERS

1. (d) 2. (d) 3. (d) 4. (d) 5. (e) 6. (a) 7. (d) 8. (b) 9. (e)
 10. (b) 11. (e) 12. (c) 13. (c) 14. (a)

SOLUTIONS

1. $A + B + C + D = 96$... (i)
 I gives, $A + B + D = (3 \times 20) \Rightarrow A + B + D = 60$... (ii)
 II gives, $C + D = (2 \times 50) \Rightarrow C + D = 100$... (iii)
 From (i), (ii) and (iii) also, we cannot find B.
 ∴ Correct answer is (d).
2. Let there be x children.
 I gives, age of teacher = x years.
 II gives, average age of $(x + 1)$ persons = $(x + 1)$ years.
 ∴ Teacher's age = $(x + 1)(x + 1) - x^2 = (x^2 + 1 + 2x) - x^2 = (1 + 2x)$.
 Thus, teacher's age cannot be obtained.
 ∴ Correct answer is (d).
3. Let the number of team members be n .
 I. Total increase in weight on replacement = $(20n)$ kg.
 II. Total weight of new members = $[(64 + 75 + 66) + 20n]$ kg
 $= (205 + 20n)$ kg.
 ∴ Required average = $\frac{(205 + 20n)}{3}$ kg and we need n to get the answer.
 ∴ Correct answer is (d).
4. $P + Q + R + S = (30 \times 4) \Rightarrow P + Q + R + S = 120$... (i)
 I. $P + R = 60$... (ii)
 II. $S = (R - 10)$... (iii)
 From (i), (ii) and (iii), we cannot find R.
 ∴ Correct answer is (d).
5. I. $A + B = (15 \times 2) + (5 \times 2) \Rightarrow A + B = 40$... (i)
 II. $A + B + C = (20 \times 3) \Rightarrow A + B + C = 60$... (ii)
 From (i) and (ii), we get $C = 20$.
 C's age after 10 years = $(20 + 10)$ years = 30 years.
 ∴ Correct answer is (e).
6. Let there be x children in the group.
 I. Average age = 15 years.
 II. Total age of x children = $15x$ years.
 ∴ $15x = 240 \Leftrightarrow x = \frac{240}{15} \Leftrightarrow x = 16$.
 So, there are 16 children in the group.
 II. Total age of x children and 1 teacher is 264 years.
 Age of teacher = $(15 + 9)$ years = 24 years.
 Total age of x children = $(264 - 24)$ years = 240 years.
 This does not give the answer.
 ∴ Correct answer is (a).

7. $H = \frac{(H + E + S + P)}{4} + 15$...
 $\Rightarrow 4(H - 15) = H + E + S + P \Rightarrow 3H - 60 = E + S + P$...
 I. $H + P = 120$...
 II. $S - E = 120$...
 From (i), (ii) and (iii), we cannot find P.
 \therefore Correct answer is (d).
8. I. Total candidates interviewed by 3 panels = $(15 \times 3) = 45$.
 II. Let x candidates be interviewed by C.
 Number of candidates interviewed by A = $(x + 2)$.
 Number of candidates interviewed by B = $(x + 1)$.
 $\therefore x + (x + 2) + (x + 1) = 45 \Leftrightarrow 3x = 42 \Leftrightarrow x = 14$.
 So, the number of candidates interviewed by A is 14.
 Hence, the correct answer is (b).
9. Average age of 11 students and 1 teacher = 14 years
 \Rightarrow Total age of (11 students and 1 teacher) = (14×12) years = 168 years.
 Average age of (11 students and 1 teacher) = (Average age of 11 students) + 3
 \Rightarrow Average age of 11 students = $(14 - 3)$ years = 11 years
 \Rightarrow Total age of 11 students = (11×11) years = 121 years.
 \therefore Age of the teacher = $(168 - 121)$ years = 47 years.
 Thus, both I and II are needed to get the answer.
 \therefore Correct answer is (e).
10. I. Total weight of 46 children = $[(22.5 \times 30) + (29.125 \times 16)]$ kg = 1141 kg.
 Weight excluded is not exact. So, average of remaining class cannot be obtained.
 II. Total weight of 45 children = $[(23.5 \times 46) - 46]$ kg = 1035 kg.
 Average weight of 45 children = $\frac{1035}{45}$ kg = 23 kg.
 \therefore Data in II is sufficient to answer the question, while the data in I is not sufficient.
 \therefore Correct answer is (b).
11. I gives, total marks in 4 subjects = $(60 \times 4) = 240$.
 II gives, $E + M = 170$
 III gives, $M + S = 180$.
 Thus, none of (a), (b), (c), (d) is true.
 \therefore Correct answer is (e).
12. I. $M + T + W = (37 \times 3) \Rightarrow M + T + W = 111$...
 $T + W + Th = (34 \times 3) \Rightarrow T + W + Th = 102$...
 II gives, $Th = \frac{4}{5}M \Rightarrow M = \frac{5}{4}Th$
 Using it in (i), we get :
 $\frac{5}{4}Th + T + W = 111$...
 On subtracting (ii) from (iii), we get : $\frac{1}{4}Th = 9 \Rightarrow Th = 36$.
 Thus, I alone gives the answer.
 III gives, $M + Th = (40.5 \times 2) \Rightarrow M + Th = 81$...
 On subtracting (ii) from (i), we get $M - Th = 9$...
 From (iv) and (v), we get $Th = 36$.

Thus, II alone gives the answer.

III gives, $M - Th = 9$.

Clearly, III with given results, does not give the answer.

∴ Correct answer is (c).

13. Total age of 11 players = (28×11) years = 308 years.

$$I. C = Y + 11 \Rightarrow C - Y = 11 \quad \dots(i)$$

- II. Total age of 10 players (excluding captain) = (27.3×10) years = 273 years.

$$\therefore \text{Age of captain} = (308 - 273) \text{ years} = 35 \text{ years.}$$

Thus, $C = 35$.

From (i) and (ii), we get $Y = 24$.

- III. Total age of 9 players = $[(25 \times 3) + (28 \times 3) + (30 \times 3)]$ years = 249 years.

$$\therefore C + Y = (308 - 249) = 59 \quad \dots(ii)$$

From (i) and (iii), we get $C = 35$.

Thus, II alone gives the answer.

Also, I and III together give the answer.

∴ Correct answer is (c).

14. I. gives, total salary of 7 clerks = Rs. (8500×7) = Rs. 59500.

- II. gives, total salary of 5 officers = Rs. (10000×5) = Rs. 50000.

- III. gives, total salary of 3 sub-staff members = Rs. (2500×3) = Rs. 7500.

Total salary of 15 employees = Rs. $(59500 + 50000 + 7500)$ = Rs. 117000.

$$\therefore \text{Average salary} = \text{Rs. } \left(\frac{117000}{15} \right) = \text{Rs. } 7800.$$

All given statements are needed. Hence, none is redundant.

∴ Correct answer is (a).

7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyse the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex. 1. A number is as much greater than 36 as is less than 86. Find the number.

Sol. Let the number be x . Then, $x - 36 = 86 - x \Leftrightarrow 2x = 86 + 36 = 122 \Leftrightarrow x = 61$.
 Hence, the required number is 61.

Ex. 2. Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twice the number. (Hotel Management, 2002)

Sol. Let the number be x . Then, $7x - 15 = 2x + 10 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.
 Hence, the required number is 5.

Ex. 3. The sum of a rational number and its reciprocal is $\frac{13}{6}$. Find the number. (S.S.C. 2000)

Sol. Let the number be x .

$$\begin{aligned} \text{Then, } x + \frac{1}{x} &= \frac{13}{6} \Leftrightarrow \frac{x^2 + 1}{x} = \frac{13}{6} \Leftrightarrow 6x^2 - 13x + 6 = 0 \\ &\Leftrightarrow 6x^2 - 9x - 4x + 6 = 0 \Leftrightarrow (3x - 2)(2x - 3) = 0 \\ &\Leftrightarrow x = \frac{2}{3} \text{ or } x = \frac{3}{2}. \end{aligned}$$

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

Ex. 4. The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Sol. Let the numbers be x and $(184 - x)$. Then,

$$\frac{x}{3} - \frac{(184 - x)}{7} = 8 \Leftrightarrow 7x - 3(184 - x) = 168 \Leftrightarrow 10x = 720 \Leftrightarrow x = 72.$$

So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Sol. Let the numbers be x and y . Then,

$$x - y = 11 \quad \dots(i) \quad \text{and} \quad \frac{1}{5}(x + y) = 9 \Rightarrow x + y = 45 \quad \dots(ii)$$

Adding (i) and (ii), we get : $2x = 56$ or $x = 28$. Putting $x = 28$ in (i), we get : $y = 17$.
 Hence, the numbers are 28 and 17.

Ex. 6. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers. (S.S.C. 2003)

Sol. Let the numbers be x and y . Then, $x + y = 42$ and $xy = 437$.

$$x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{(42)^2 - 4 \times 437} = \sqrt{1764 - 1748} = \sqrt{16} = 4.$$

Required difference = 4.

Ex. 7. The sum of two numbers is 15 and the sum of their squares is 113. Find the numbers.

Sol. Let the numbers be x and $(15 - x)$.

$$\text{Then, } x^2 + (15 - x)^2 = 113 \Leftrightarrow x^2 + 225 + x^2 - 30x = 113$$

$$\Leftrightarrow 2x^2 - 30x + 112 = 0 \Leftrightarrow x^2 - 15x + 56 = 0$$

$$\Leftrightarrow (x - 7)(x - 8) = 0 \Leftrightarrow x = 7 \text{ or } x = 8.$$

So, the numbers are 7 and 8.

Ex. 8. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the four consecutive even numbers be $x, x + 2, x + 4$ and $x + 6$.

$$\text{Then, sum of these numbers} = (27 \times 4) = 108.$$

$$\text{So, } x + (x + 2) + (x + 4) + (x + 6) = 108 \text{ or } 4x = 96 \text{ or } x = 24.$$

$$\therefore \text{Largest number} = (x + 6) = 30.$$

Ex. 9. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

Sol. Let the numbers be $x, x + 2$ and $x + 4$.

$$\text{Then, } x^2 + (x + 2)^2 + (x + 4)^2 = 2531 \Leftrightarrow 3x^2 + 12x - 2511 = 0$$

$$\Leftrightarrow x^2 + 4x - 837 = 0 \Leftrightarrow (x - 27)(x + 31) = 0 \Leftrightarrow x = 27.$$

Hence, the required numbers are 27, 29 and 31.

Ex. 10. Of two numbers, 4 times the smaller one is less than 3 times the larger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.

Sol. Let the numbers be x and y , such that $x > y$.

$$\text{Then, } 3x - 4y = 5 \quad \dots(i) \text{ and } (x + y) - 6(x - y) = 6 \Rightarrow -5x + 7y = 6 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 59$ and $y = 43$.

Hence, the required numbers are 59 and 43.

Ex. 11. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

Sol. Let the ten's digit be x . Then, unit's digit = $(x + 3)$.

$$\text{Sum of the digits} = x + (x + 3) = 2x + 3. \text{ Number} = 10x + (x + 3) = 11x + 3.$$

$$\therefore \frac{11x + 3}{2x + 3} = \frac{4}{1} \Leftrightarrow 11x + 3 = 4(2x + 3) \Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$$

$$\text{Hence, required number} = 11x + 3 = 36.$$

Ex. 12. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Sol. Let the ten's digit be x . Then, unit's digit = $(9 - x)$.

$$\text{Number} = 10x + (9 - x) = 9x + 9.$$

$$\text{Number obtained by reversing the digits} = 10(9 - x) + x = 90 - 9x.$$

$$\therefore (9x + 9) - 63 = 90 - 9x \Leftrightarrow 18x = 144 \Leftrightarrow x = 8.$$

So, ten's digit = 8 and unit's digit = 1.

Hence, the required number is 81.

Ex. 13. A fraction becomes $\frac{2}{3}$ when 1 is added to both, its numerator and denominator. And, it becomes $\frac{1}{2}$ when 1 is subtracted from both the numerator and denominator. Find the fraction.

(S.S.C. 2000) A sum of money is divided between two persons A and B such that A gets $\frac{1}{3}$ rd of what B gets. Then,

$$\frac{x+1}{y+1} = \frac{2}{3} \Rightarrow 3x - 2y = -1 \quad \dots(i) \text{ and } \frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 3$, $y = 5$.

$$\therefore \text{Required fraction} = \frac{3}{5}$$

Ex. 14. 50 is divided into two parts such that the sum of their reciprocals is $\frac{1}{12}$.

Find the two parts.

Sol. Let the two parts be x and $(50 - x)$.

$$\text{Then, } \frac{1}{x} + \frac{1}{50-x} = \frac{1}{12} \Leftrightarrow \frac{50-x+x}{x(50-x)} = \frac{1}{12} \Rightarrow x^2 - 50x + 600 = 0 \\ \Rightarrow (x-30)(x-20) = 0 \Rightarrow x = 30 \text{ or } x = 20.$$

So, the parts are 30 and 20.

Ex. 15. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers. (S.S.C. 2000)

Sol. Let the numbers be x , y and z . Then,

$$x+y=10 \quad \dots(i) \quad y+z=19 \quad \dots(ii) \quad x+z=21 \quad \dots(iii)$$

Adding (i), (ii) and (iii), we get : $2(x+y+z) = 50$ or $(x+y+z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.

EXERCISE 7A

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

- The difference between a number and its three-fifth is 50. What is the number ?
 (a) 75 (b) 100 (c) 125 (d) None of these
(Bank P.O. 2003)
- If a number is decreased by 4 and divided by 6, the result is 8. What would be the result if 2 is subtracted from the number and then it is divided by 5 ?
 (a) $9\frac{2}{3}$ (b) 10 (c) $10\frac{1}{5}$ (d) $11\frac{1}{5}$ (e) None of these
(Bank P.O. 2000)
- If one-third of one-fourth of a number is 15, then three-tenth of that number is :
 (a) 35 (b) 36 (c) 45 (d) 54
(N.I.F.T. 2003)
- A number is doubled and 9 is added. If the resultant is trebled, it becomes 75. What is that number ?
 (a) 3.5 (b) 6 (c) 8 (d) None of these
(S.S.C. 1999)
- Three-fourth of a number is 60 more than its one-third. The number is :
 (a) 84 (b) 108 (c) 144 (d) None of these
(R.D. 2000)
- When 24 is subtracted from a number, it reduces to its four-seventh. What is the sum of the digits of that number ?
 (a) 1 (b) 9 (c) 11 (d) None of these
 (e) Data inadequate
(R.D. 2000)

22. The sum of a positive number and its reciprocal is thrice the difference of the number and its reciprocal. The number is : (S.S.C. 1999)
- (a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
23. The product of two natural numbers is 17. Then, the sum of the reciprocals of their squares is : (S.S.C. 1999)
- (a) $\frac{1}{289}$ (b) $\frac{289}{290}$ (c) $\frac{290}{289}$ (d) 289
24. If $2\frac{1}{2}$ is added to a number and the sum multiplied by $4\frac{1}{2}$ and 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient becomes 25. What is the number ? (R.R.B. 2002)
- (a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$ (c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$
25. Three numbers are in the ratio $4 : 5 : 6$ and their average is 25. The largest number is : (S.S.C. 1999)
- (a) 30 (b) 32 (c) 36 (d) 42
26. Three numbers are in the ratio of $3 : 4 : 6$ and their product is 1944. The largest of these numbers is : (S.S.C. 1999)
- (a) 6 (b) 12 (c) 18 (d) None of these
27. Two numbers are such that the square of one is 224 less than 8 times the square of the other. If the numbers be in the ratio of $3 : 4$, the numbers are : (S.S.C. 1999)
- (a) 6, 8 (b) 9, 12 (c) 12, 16 (d) None of these
28. Two numbers are such that the ratio between them is $4 : 7$. If each is increased by 4, the ratio becomes $3 : 5$. The larger number is : (S.S.C. 1999)
- (a) 36 (b) 48 (c) 56 (d) 64
29. The sum of three numbers is 264. If the first number be twice the second and third number be one-third of the first, then the second number is : (R.R.B. 2004)
- (a) 48 (b) 54 (c) 72 (d) 84
30. The sum of two numbers is 22. Five times one number is equal to 6 times the other. The bigger of the two numbers is : (C.B.I. 1998)
- (a) 10 (b) 12 (c) 15 (d) 16
31. One-fifth of a number is equal to $\frac{5}{8}$ of another number. If 35 is added to the first number, it becomes four times of the second number. The second number is : (Bank P.O. 1999)
- (a) 25 (b) 40 (c) 70 (d) 125
32. The sum of two numbers is 25 and their difference is 13. Find their product. (L.I.C. 2003)
- (a) 104 (b) 114 (c) 315 (d) 325
33. If the sum of two numbers is 33 and their difference is 15, the smaller number is : (C.B.I. 1997)
- (a) 9 (b) 12 (c) 15 (d) 18
34. The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is : (S.S.C. 2000)
- (a) 11 : 9 (b) 11 : 18 (c) 21 : 19 (d) 22 : 9

35. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers ? (Bank P.O. 1999)
(a) 12 (b) 14 (c) 16 (d) 18 (e) None of these
36. The difference between two integers is 5. Their product is 500. Find the numbers.
(a) 15, 20 (b) 20, 25 (c) 30, 25 (d) 21, 26
(Hotel Management, 2003)
37. Two numbers differ by 5. If their product is 336, then the sum of the two numbers is :
(a) 21 (b) 28 (c) 37 (d) 51
(S.S.C. 1999)
38. Two different natural numbers are such that their product is less than their sum. One of the numbers must be :
(a) 1 (b) 2 (c) 3 (d) None of these
39. The product of two numbers is 9375 and the quotient, when the larger one is divided by the smaller, is 15. The sum of the numbers is : (S.S.C. 2004)
(a) 380 (b) 395 (c) 400 (d) 425
40. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is :
(a) 240 (b) 270 (c) 295 (d) 360
41. The sum of two numbers is 40 and their product is 375. What will be the sum of their reciprocals ? (S.S.C. 1999)
(a) $\frac{1}{40}$ (b) $\frac{8}{75}$ (c) $\frac{75}{4}$ (d) $\frac{75}{8}$
42. The sum of two positive integers multiplied by the bigger number is 204, and their difference multiplied by the smaller number is 35. The numbers are :
(a) 12, 5 (b) 13, 4 (c) 14, 3 (d) 24, 10
43. If the sum and difference of two numbers are 20 and 8 respectively, then the difference of their squares is : (S.S.C. 2000)
(a) 12 (b) 28 (c) 160 (d) 180
44. The product of two numbers is 120 and the sum of their squares is 289. The sum of the numbers is : (R.R.B. 2004)
(a) 20 (b) 23 (c) 169 (d) None of these
45. The product of two numbers is 45 and the sum of their squares is 106. The numbers are : (R.R.B. 2002)
(a) 3 and 5 (b) 5 and 9 (c) 5 and 19 (d) 45 and 1
46. The sum of the squares of two numbers is 3341 and the difference of their squares is 891. The numbers are :
(a) 25, 36 (b) 25, 46 (c) 35, 46 (d) None of these
47. The difference between two positive integers is 3. If the sum of their squares is 369, then the sum of the numbers is : (S.S.C. 2003)
(a) 25 (b) 27 (c) 33 (d) 81
48. If the sum of two numbers is 22 and the sum of their squares is 404, then the product of the numbers is : (S.S.C. 2000)
(a) 40 (b) 44 (c) 80 (d) 88
49. The difference between the squares of two numbers is 256000 and the sum of the numbers is 1000. The numbers are :
(a) 600, 400 (b) 628, 372 (c) 640, 360 (d) None of these
50. If the difference of two numbers is 3 and the difference of their squares is 39, then the larger number is :
(a) 8 (b) 9 (c) 12 (d) 13

51. The sum of three consecutive numbers is 87. The greatest among these three numbers is : (Hotel Management, 2003)
- (a) 26 (b) 28 (c) 29 (d) 30
52. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is : (M.B.A. 1998)
- (a) 9 (b) 11 (c) 13 (d) 15
53. The sum of four consecutive even integers is 1284. The greatest of them is : (S.S.C. 2002)
- (a) 320 (b) 322 (c) 324 (d) 326
54. The sum of three consecutive odd numbers is 20 more than the first of these numbers. What is the middle number ? (S.B.I.P.O. 1997)
- (a) 7 (b) 9 (c) 11
- (d) Data inadequate (e) None of these
55. The product of three consecutive even numbers when divided by 8 is 720. The product of their square roots is : (Hotel Management, 2001)
- (a) $12\sqrt{10}$ (b) $24\sqrt{10}$ (c) 120 (d) None of these
56. The sum of three consecutive multiples of 3 is 72. What is the largest number ? (S.S.C. 1999)
- (a) 21 (b) 24 (c) 27 (d) 36
57. What is the sum of two consecutive even numbers, the difference of whose squares is 84 ? (S.S.C. 2003)
- (a) 34 (b) 38 (c) 42 (d) 46
58. The sum of the squares of three consecutive natural numbers is 2030. What is the middle number ? (S.S.C. 2000)
- (a) 25 (b) 26 (c) 27 (d) 28
59. There are two numbers such that the sum of twice the first and thrice the second is 39, while the sum of thrice the first and twice the second is 36. The larger of the two is : (L.I.C. 2003)
- (a) 6 (b) 8 (c) 9 (d) 12
60. In a two-digit number, the digit in the unit's place is four times the digit in ten's place and sum of the digits is equal to 10. What is the number ? (Bank P.O. 1999)
- (a) 14 (b) 41 (c) 82
- (d) Data inadequate (e) None of these
61. A number of two digits has 3 for its unit's digit, and the sum of digits is $\frac{1}{7}$ of the number itself. The number is : (L.I.C. 2003)
- (a) 43 (b) 53 (c) 63 (d) 73
62. A two-digit number exceeds the sum of the digits of that number by 18. If the digit at the unit's place is double the digit in the ten's place, what is the number ? (Data inadequate)
- (a) 24 (b) 42 (c) 48 (d) Data inadequate
63. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number ? (B.S.R.B. 2003)
- (a) 69 (b) 78 (c) 96
- (d) Cannot be determined (e) None of these
64. In a two-digit number, if it is known that its unit's digit exceeds its ten's digit by 2 and that the product of the given number and the sum of its digits is equal to 144, then the number is : (C.B.I. 2003)
- (a) 24 (b) 26 (c) 42 (d) 46

65. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by :
(a) 3 (b) 5 (c) 9 (d) 11 (S.S.C. 2003)
66. The sum of the digits of a two-digit number is 9 less than the number. Which of the following digits is at unit's place of the number ?
(a) 1 (b) 2 (c) 4 (d) Data inadequate
67. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number ?
(a) 3 (b) 4 (c) 9 (Bank P.O. 2003)
(d) Cannot be determined (e) None of these
68. The difference between a two-digit number and the number obtained by interchanging the two digits is 63. Which is the smaller of the two numbers ? (Bank P.O. 2003)
(a) 29 (b) 70 (c) 92
(d) Cannot be determined (e) None of these
69. The sum of the digits of a two-digit number is $\frac{1}{5}$ of the difference between the number and the number obtained by interchanging the positions of the digits. What is definitely the difference between the digits of that number ?
(a) 5 (b) 7 (c) 9 (Bank P.O. 2000)
(d) Data inadequate (e) None of these
70. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true ? (Bank P.O. 2003)
(a) Sum of the digits is a two-digit number.
(b) Digit in the unit's place is twice the digit in the ten's place.
(c) Digits in the unit's place and the ten's place are equal.
(d) Digit in the unit's place is half of the digit in the ten's place.
(e) None of these
71. If the number obtained on interchanging the digits of a two-digit number is 18 more than the original number and the sum of the digits is 8, then what is the original number ?
(a) 26 (b) 35 (c) 53 (S.B.I.P.O. 2002)
(d) Cannot be determined (e) None of these
72. The difference between a two-digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1 : 2 ? (M.A.T. 1999)
(a) 4 (b) 8 (c) 16 (d) None of these
73. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two and the number will be increased by 99 if its digits are reversed. The number is :
(a) 145 (b) 253 (c) 370 (d) 352 (Hotel Management, 2003)
74. A two-digit number becomes five-sixth of itself when its digits are reversed. The two digits differ by one. The number is :
(a) 45 (b) 54 (c) 56 (d) 65

75. A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit's place. Three times the number added to $\frac{6}{7}$ times the number obtained by reversing the digits equals 108. The sum of the digits in the number is :
(a) 6 (b) 7 (c) 8 (d) 9 (e) 10 (S.S.C. 2003)
76. The digit in the unit's place of a number is equal to the digit in the ten's place of half of that number and the digit in the ten's place of that number is less than the digit in unit's place of half of the number by 1. If the sum of the digits of the number is 7, then what is the number ?
(a) 34 (b) 52 (c) 84 (d) 162 (e) None of these (S.B.I.P.O. 2001)
77. In a two-digit number, the digit in the unit's place is more than twice the digit in ten's place by 1. If the digits in the unit's place and the ten's place are interchanged, difference between the newly formed number and the original number is less than the original number by 1. What is the original number ? (Bank P.O. 1999)
(a) 25 (b) 37 (c) 49 (d) 52 (e) 73
78. A certain number of two digits is three times the sum of its digits and if 45 be added to it, the digits are reversed. The number is : (L.I.C.A.A.O. 2003)
(a) 23 (b) 27 (c) 32 (d) 72
79. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is : (M.B.A. 2003)
(a) 18 (b) 24 (c) 42 (d) 81
80. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater fraction is :
(a) $\frac{4}{5}$ (b) $\frac{7}{6}$ (c) $\frac{7}{4}$ (d) $\frac{7}{3}$ (S.S.C. 2002)
81. In a pair of fractions, fraction A is twice the fraction B and the product of two fractions is $\frac{2}{25}$. What is the value of fraction A ? (Bank P.O. 1999)
(a) $\frac{1}{5}$ (b) $\frac{1}{25}$ (c) $\frac{2}{5}$ (d) Data inadequate
82. The sum of the numerator and denominator of a fraction is 11. If 1 is added to the numerator and 2 is subtracted from the denominator, it becomes $\frac{2}{3}$. The fraction is :
(a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) $\frac{3}{8}$ (d) $\frac{8}{3}$
83. The denominator of a fraction is 3 more than the numerator. If the numerator as well as the denominator is increased by 4, the fraction becomes $\frac{4}{5}$. What was the original fraction ? (S.B.I.P.O. 1999)
(a) $\frac{8}{11}$ (b) $\frac{5}{8}$ (c) $\frac{10}{13}$ (d) $\frac{7}{10}$
84. The difference between the numerator and the denominator of a fraction is 5. If 5 is added to its denominator, the fraction is decreased by $1\frac{1}{4}$. Find the value of the fraction.
(a) $\frac{1}{6}$ (b) $2\frac{1}{4}$ (c) $3\frac{1}{4}$ (d) 6 (M.B.A. 1997)

85. The numerator and denominator of a fraction are in the ratio of 2 : 3. If 6 is subtracted from the numerator, the result is a fraction that has a value $\frac{2}{3}$ of the original fraction. The numerator of the original fraction is : (S.S.C. 1999)
(a) 6 (b) 18 (c) 27 (d) 36
86. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator of the fraction, the fraction becomes $\frac{1}{3}$. The fraction is : (C.B.I. 1997)
(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$
87. If the numerator of a fraction is increased by 2 and the denominator is increased by 3, the fraction becomes $\frac{7}{9}$, and if both the numerator as well as the denominator are decreased by 1, the fraction becomes $\frac{4}{5}$. What is the original fraction ? (S.B.I.P.O. 1999)
(a) $\frac{5}{6}$ (b) $\frac{9}{11}$ (c) $\frac{13}{16}$ (d) $\frac{17}{21}$
88. When the numerator of a fraction increases by 4, the fraction increases by $\frac{2}{3}$. The denominator of the fraction is :
(a) 2 (b) 3 (c) 4 (d) 6
89. 54 is to be divided into two parts such that the sum of 10 times the first and 22 times the second is 780. The bigger part is :
(a) 24 (b) 34 (c) 30 (d) 32
90. 243 has been divided into three parts such that half of the first part, one-third of the second part and one-fourth of the third part are equal. The largest part is :
(a) 74 (b) 86 (c) 92 (d) 108
91. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers ? (S.B.I.P.O. 2000)
(a) 21 (b) 27 (c) 32 (d) None of these
92. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is : (Hotel Management, 1999)
(a) 20 (b) 30 (c) 40 (d) None of these
93. The sum of three numbers is 136. If the ratio between first and second be 2 : 3 and that between second and third is 5 : 3, then the second number is :
(a) 40 (b) 48 (c) 60 (d) 72
94. Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55 and the sum of the third and thrice the first is 90. The third number is :
(a) 20 (b) 25 (c) 30 (d) 3

ANSWERS

1. (c) 2. (b) 3. (d) 4. (c) 5. (c) 6. (c) 7. (a) 8. (c) 9. (d)
10. (a) 11. (d) 12. (c) 13. (c) 14. (b) 15. (e) 16. (b) 17. (b) 18. (a)
19. (a) 20. (a) 21. (c) 22. (a) 23. (c) 24. (b) 25. (a) 26. (c) 27. (a)
28. (c) 29. (c) 30. (b) 31. (b) 32. (b) 33. (a) 34. (a) 35. (a) 36. (b)

37. (c) 38. (a) 39. (c) 40. (b) 41. (b) 42. (a) 43. (c) 44. (b) 45. (b)
 46. (c) 47. (b) 48. (a) 49. (b) 50. (a) 51. (d) 52. (d) 53. (c) 54. (b)
 55. (b) 56. (c) 57. (c) 58. (b) 59. (c) 60. (e) 61. (c) 62. (a) 63. (d)
 64. (a) 65. (d) 66. (d) 67. (b) 68. (d) 69. (d) 70. (b) 71. (b) 72. (b)
 73. (b) 74. (b) 75. (a) 76. (b) 77. (b) 78. (b) 79. (b) 80. (b) 81. (c)
 82. (c) 83. (a) 84. (b) 85. (b) 86. (b) 87. (a) 88. (d) 89. (b) 90. (d)
 91. (c) 92. (a) 93. (c) 94. (c)

SOLUTIONS

1. Let the number be x . Then, $x - \frac{3}{5}x = 50 \Leftrightarrow \frac{2}{5}x = 50 \Leftrightarrow x = \left(\frac{50 \times 5}{2}\right) = 125$.

2. Let the number be x . Then, $\frac{x-4}{6} = 8 \Leftrightarrow x-4 = 48 \Leftrightarrow x = 52$.

$$\therefore \frac{x-2}{5} = \frac{52-2}{5} = \frac{50}{5} = 10.$$

3. Let the number be x . Then, $\frac{1}{3}$ of $\frac{1}{4}$ of $x = 15 \Leftrightarrow x = 15 \times 12 = 180$.

$$\text{So, required number} = \left(\frac{3}{10} \times 180\right) = 54.$$

4. Let the number be x . Then, $3(2x+9) = 75 \Leftrightarrow 2x+9=25 \Leftrightarrow 2x=16 \Leftrightarrow x=8$.

5. Let the number be x . Then, $\frac{3}{4}x - \frac{1}{3}x = 60 \Leftrightarrow \frac{5x}{12} = 60 \Leftrightarrow x = \left(\frac{60 \times 12}{5}\right) = 144$.

6. Let the number be x . Then,

$$x-24 = \frac{4}{7}x \Leftrightarrow x - \frac{4}{7}x = 24 \Leftrightarrow \frac{3}{7}x = 24 \Leftrightarrow x = \left(\frac{24 \times 7}{3}\right) = 56.$$

$$\therefore \text{Sum of the digits} = (5+6) = 11.$$

7. Let the number be x . Then, $15x - x = 196 \Leftrightarrow 14x = 196 \Leftrightarrow x = 14$.

8. Let the number be x . Then, $\frac{x}{4} = x-21 \Leftrightarrow x = 4x-84 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28$.

9. Let the number be x . Then, $\left(\frac{1}{5}x + 4\right) = \left(\frac{1}{4}x - 10\right) \Leftrightarrow \frac{x}{20} = 14 \Leftrightarrow x = 14 \times 20 = 280$.

10. Let the larger number be x .

$$\text{Then, } x-12 = 20\% \text{ of } x \Leftrightarrow x - \frac{x}{5} = 12 \Leftrightarrow \frac{4x}{5} = 12 \Leftrightarrow x = \left(\frac{12 \times 5}{4}\right) = 15.$$

11. Let the number be x . Then, $\frac{1}{7}x - \frac{1}{11}x = 100 \Leftrightarrow \frac{4x}{77} = 100 \Leftrightarrow x = \frac{7700}{4} = 1925$.

12. Let the number be x .

$$\text{Then, } \left(\frac{1}{2}x + \frac{1}{5}x\right) - \frac{1}{3}x = \frac{22}{3} \Leftrightarrow \frac{11x}{30} = \frac{22}{3} \Leftrightarrow x = \left(\frac{22 \times 30}{3 \times 11}\right) = 20.$$

13. Let the number be x . Then, $2x+20 = 8x-4 \Leftrightarrow 6x = 24 \Leftrightarrow x = 4$.

14. Let the number be x .

$$\text{Then, } \frac{2}{3}x - 50 = \frac{1}{4}x + 40 \Leftrightarrow \frac{2}{3}x - \frac{1}{4}x = 90 \Leftrightarrow \frac{5x}{12} = 90 \Leftrightarrow x = \left(\frac{90 \times 12}{5}\right) = 216.$$

15. Let the number be x . (a) 38 (b) 14 (c) 04 (d) 98 (e) 88 (f) 76
 (g) Then, $x + x^2 = 182 \Leftrightarrow x^2 + x - 182 = 0 \Leftrightarrow (x + 14)(x - 13) = 0 \Leftrightarrow x = 13$.
16. Let the integer be x . (a) 09 (b) 06 (c) 80 (d) 75 (e) 88 (f) 80
 (g) Then, $x^2 - 20x = 96 \Leftrightarrow x^2 - 20x - 96 = 0 \Leftrightarrow (x + 4)(x - 24) = 0 \Leftrightarrow x = 24$.
17. Let the number be x . (d) 87 (d) 77 (d) 87 (d) 87 (d) 87 (d) 87
 (b) Then, $3x^2 - 4x = x + 50 \Leftrightarrow 3x^2 - 5x - 50 = 0 \Leftrightarrow (3x + 10)(x - 5) = 0 \Leftrightarrow x = 5$.
18. Let the number be x . Then, $x + \frac{1}{x} = \frac{34}{8} \Leftrightarrow \frac{x^2 + 1}{x} = \frac{34}{8} \Leftrightarrow 8x^2 - 34x + 8 = 0$
 $\Leftrightarrow 4x^2 - 17x + 4 = 0 \Leftrightarrow (4x - 1)(x - 4) = 0 \Leftrightarrow x = 4$.
 [neglecting $x = \frac{1}{4}$, as x is a natural no.]
 ∴ Required number = $4 \times \sqrt{4} = 4 \times 2 = 8$.
19. Let the number be x .
 Then, $\frac{2}{3}x = \frac{25}{216} \times \frac{1}{x} \Leftrightarrow x^2 = \frac{25}{216} \times \frac{3}{2} = \frac{25}{144} \Leftrightarrow x = \sqrt{\frac{25}{144}} = \frac{5}{12}$.
20. Let the number be x .
 Then, $x + 17 = \frac{60}{x} \Leftrightarrow x^2 + 17x - 60 = 0 \Leftrightarrow (x + 20)(x - 3) = 0 \Leftrightarrow x = 3$.
21. Let the number be x .
 Then, $x - 4 = \frac{21}{x} \Leftrightarrow x^2 - 4x - 21 = 0 \Leftrightarrow (x - 7)(x + 3) = 0 \Leftrightarrow x = 7$.
22. Let the number be x . Then, $x + \frac{1}{x} = 3\left(x - \frac{1}{x}\right) \Leftrightarrow \frac{x^2 + 1}{x} = 3\left(\frac{x^2 - 1}{x}\right)$
 $\Leftrightarrow x^2 + 1 = 3x^2 - 3 \Leftrightarrow 2x^2 = 4 \Leftrightarrow x^2 = 2 \Leftrightarrow x = \sqrt{2}$.
23. Let the numbers be a and b . Then, $ab = 17 \Rightarrow a = 1$ and $b = 17$.
 So, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2 + b^2}{a^2 b^2} = \frac{1^2 + (17)^2}{(1 \times 17)^2} = \frac{290}{289}$.
24. Let the number be x . Then,
 $\frac{4}{2}\left(x + \frac{1}{2}\right) + 3 = 25 \Leftrightarrow \frac{9}{2}\left(x + \frac{5}{2}\right) + 3 = 25$
 $\Leftrightarrow \frac{9x}{2} + \frac{45}{4} + 3 = 25 \times \frac{6}{5} = 30 \Leftrightarrow \frac{9x}{2} = 30 - \frac{57}{4} \Leftrightarrow \frac{9x}{2} = \frac{63}{4}$
 $\Leftrightarrow x = \left(\frac{63}{4} \times \frac{2}{9}\right) = \frac{7}{2} = 3\frac{1}{2}$.
25. Let the numbers be $4x$, $5x$ and $6x$. Then, $\frac{4x + 5x + 6x}{3} = 25 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.
 ∴ Largest number = $6x = 30$.
26. Let the numbers be $3x$, $4x$ and $6x$.
 Then, $3x \times 4x \times 6x = 1944 \Leftrightarrow 72x^3 = 1944 \Leftrightarrow x^3 = 27 \Leftrightarrow x = 3$.
 ∴ Largest number = $6x = 18$.
27. Let the numbers be $3x$ and $4x$. Then,
 $(4x)^2 = 8 \times (3x)^2 - 224 \Leftrightarrow 16x^2 = 72x^2 - 224 \Leftrightarrow 56x^2 = 224 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
 So, the numbers are 6 and 8.

28. Let the numbers be $4x$ and $7x$. Then, $\frac{4x+4}{7x+4} = \frac{3}{5} \Leftrightarrow 5(4x+4) = 3(7x+4) \Leftrightarrow x = 8$.
 \therefore Larger number = $7x = 56$.
29. Let the second number be x . Then, first number = $2x$ and third number = $\frac{2x}{3}$.
 $\therefore 2x + x + \frac{2x}{3} = 264 \Leftrightarrow \frac{11x}{3} = 264 \Leftrightarrow x = \left(\frac{264 \times 3}{11}\right) = 72$.
30. Let the numbers be x and $(22 - x)$. Then, $5x = 6(22 - x) \Leftrightarrow 11x = 132 \Leftrightarrow x = 12$.
 \therefore So, the numbers are 12 and 10.
31. Let the numbers be x and y . Then, $\frac{1}{5}x = \frac{5}{8}y \Leftrightarrow y = \frac{8}{25}x$.
 \therefore Now, $x + 35 = 4y \Leftrightarrow x + 35 = \frac{32}{25}x \Leftrightarrow \frac{7}{25}x = 35 \Leftrightarrow x = \left(\frac{35 \times 25}{7}\right) = 125$.
 \therefore Second number = $y = \frac{8}{25}x = \left(\frac{8}{25} \times 125\right) = 40$.
32. Let the numbers be x and y . Then, $x + y = 25$ and $x - y = 13$.
 $4xy = (x + y)^2 - (x - y)^2 = (25)^2 - (13)^2 = 625 - 169 = 456 \Rightarrow xy = 114$.
33. Let the numbers be x and y .
 \therefore Then, $x + y = 33$... (i) and $x - y = 15$... (ii)
 \therefore Solving (i) and (ii), we get : $x = 24$, $y = 9$.
 \therefore Smaller number = 9.
34. Let the numbers be x and y . Then, $\frac{x+y}{x-y} = \frac{40}{4} = 10 \Leftrightarrow (x+y) = 10(x-y) \Leftrightarrow 9x = 11y \Leftrightarrow \frac{x}{y} = \frac{11}{9}$.
35. Let the numbers be x and $(28 - x)$. Then,
 $x(28 - x) = 192 \Leftrightarrow x^2 - 28x + 192 = 0 \Leftrightarrow (x - 16)(x - 12) = 0 \Leftrightarrow x = 16$ or $x = 12$.
 \therefore So, the numbers are 16 and 12.
36. Let the integers be x and $(x + 5)$. Then,
 $x(x + 5) = 500 \Leftrightarrow x^2 + 5x - 500 = 0 \Leftrightarrow (x + 25)(x - 20) = 0 \Leftrightarrow x = 20$.
 \therefore So, the numbers are 20 and 25.
37. Let the numbers be x and y . Then, $x - y = 5$ and $xy = 336$.
 $(x + y)^2 = (x - y)^2 + 4xy = 25 + 4 \times 336 = 1369 \Rightarrow x + y = \sqrt{1369} = 37$.
38. Since $1 < x < 1 + x$, so one of the numbers is 1.
39. Let the numbers be x and y . Then, $xy = 9375$ and $\frac{x}{y} = 15$.
 $\therefore \frac{xy}{(x/y)} = \frac{9375}{15} \Leftrightarrow y^2 = 625 \Leftrightarrow y = 25 \Rightarrow x = 15y = (15 \times 25) = 375$.
 \therefore Sum of the numbers = $375 + 25 = 400$.
40. Let the numbers be x and $(x + 1365)$.
 \therefore Then, $x + 1365 = 6x + 15 \Leftrightarrow 5x = 1350 \Leftrightarrow x = 270$.
41. Let the numbers be x and y . Then, $x + y = 40$ and $xy = 375$.
 $\therefore \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{40}{375} = \frac{8}{75}$.

42. Let the numbers be x and y such that $x > y$. Then,

$$x(x+y) = 204 \Rightarrow x^2 + xy = 204 \quad \dots(i) \text{ and } y(x-y) = 35 \Rightarrow xy - y^2 = 35 \quad \dots(ii)$$

Subtracting (ii) from (i), we get : $x^2 + y^2 = 169$.

The only triplet satisfying this condition is (12, 5, 13). Thus, $x = 12$, $y = 5$.

43. Let the numbers be x and y . Then, $x+y = 20$ and $x-y = 8$.

$$\therefore x^2 - y^2 = (x+y)(x-y) = 20 \times 8 = 160.$$

44. Let the numbers be x and y . Then, $xy = 120$ and $x^2 + y^2 = 289$.

$$\therefore (x+y)^2 = x^2 + y^2 + 2xy = 289 + 240 = 529.$$

$$\therefore x+y = \sqrt{529} = 23.$$

45. Let the numbers be x and y . Then, $xy = 45$ and $x^2 + y^2 = 106$.

$$(x+y) = \sqrt{(x^2 + y^2) + 2xy} = \sqrt{106 + 90} = \sqrt{196} \Rightarrow x+y = 14 \quad \dots(i)$$

$$(x-y) = \sqrt{(x^2 + y^2) - 2xy} = \sqrt{106 - 90} = \sqrt{16} \Rightarrow x-y = 4 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 9$ and $y = 5$.

46. Let the numbers be x and y . Then,

$$x^2 + y^2 = 3341 \quad \dots(i) \text{ and } x^2 - y^2 = 891 \quad \dots(ii)$$

Adding (i) and (ii), we get : $2x^2 = 4232$ or $x^2 = 2116$ or $x = 46$.

Subtracting (ii) from (i), we get : $2y^2 = 2450$ or $y^2 = 1225$ or $y = 35$.

So, the numbers are 35 and 46.

47. Let the numbers be x and $(x+3)$. Then,

$$x^2 + (x+3)^2 = 369 \Leftrightarrow x^2 + x^2 + 9 + 6x = 369 \quad \dots(i)$$

$$\Leftrightarrow 2x^2 + 6x - 360 = 0 \Leftrightarrow x^2 + 3x - 180 = 0 \Leftrightarrow (x+15)(x-12) = 0 \Leftrightarrow x = 12.$$

So, the numbers are 12 and 15.

$$\therefore \text{Required sum} = (12 + 15) = 27.$$

48. Let the numbers be x and y . Then, $(x+y) = 22$ and $x^2 + y^2 = 404$.

$$\text{Now, } 2xy = (x+y)^2 - (x^2 + y^2) = (22)^2 - 404 = 484 - 404 = 80 \Rightarrow xy = 40.$$

49. Let the numbers be x and y . Then, $x^2 - y^2 = 256000$ and $x+y = 1000$.

On dividing, we get : $x-y = 256$.

Solving $x+y = 1000$ and $x-y = 256$, we get : $x = 628$ and $y = 372$.

50. Let the numbers be x and y . Then, $x^2 - y^2 = 39$ and $x-y = 3$.

On dividing, we get : $x+y = 13$.

Solving $x+y = 13$ and $x-y = 3$, we get : $x = 8$ and $y = 5$.

$$\therefore \text{Larger number} = 8. \quad \dots(i)$$

51. Let the numbers be x , $x+1$ and $x+2$.

$$\text{Then, } x + (x+1) + (x+2) = 87 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28.$$

$$\therefore \text{Greatest number} = (x+2) = 30.$$

52. Let the three integers be x , $x+2$ and $x+4$. Then, $3x = 2(x+4) + 3 \Leftrightarrow x = 11$.

$$\therefore \text{Third integer} = x+4 = 15.$$

53. Let the four integers be x , $x+2$, $x+4$ and $x+6$.

$$\text{Then, } x + (x+2) + (x+4) + (x+6) = 1284 \Leftrightarrow 4x = 1272 \Leftrightarrow x = 318.$$

$$\therefore \text{Greatest integer} = x+6 = 324.$$

54. Let the numbers be x , $x+2$ and $x+4$.

$$\text{Then, } x + (x+2) + (x+4) = x+20 \Leftrightarrow 2x = 14 \Leftrightarrow x = 7.$$

$$\therefore \text{Middle number} = x+2 = 9.$$

55. Let the numbers be x , $x + 2$ and $x + 4$.

$$\text{Then, } \frac{x(x+2)(x+4)}{8} = 720 \Leftrightarrow x(x+2)(x+4) = 5760.$$

$$\therefore \sqrt{x} \times \sqrt{(x+2)} \times \sqrt{(x+4)} = \sqrt{x(x+2)(x+4)} = \sqrt{5760} = 24\sqrt{10}.$$

56. Let the numbers be $3x$, $3x + 3$ and $3x + 6$.

$$\text{Then, } 3x + (3x + 3) + (3x + 6) = 72 \Leftrightarrow 9x = 63 \Leftrightarrow x = 7.$$

$$\therefore \text{Largest number} = 3x + 6 = 27.$$

57. Let the numbers be x and $x + 2$.

$$\text{Then, } (x+2)^2 - x^2 = 84 \Leftrightarrow 4x + 4 = 84 \Leftrightarrow 4x = 80 \Leftrightarrow x = 20.$$

$$\therefore \text{Required sum} = x + (x+2) = 2x + 2 = 42.$$

58. Let the numbers be x , $x + 1$ and $x + 2$.

$$\text{Then, } x^2 + (x+1)^2 + (x+2)^2 = 2030 \Leftrightarrow 3x^2 + 6x - 2025 = 0$$

$$\Leftrightarrow x^2 + 2x - 675 = 0 \Leftrightarrow (x+27)(x-25) = 0 \Leftrightarrow x = 25.$$

$$\therefore \text{Middle number} = (x+1) = 26.$$

59. Let the numbers be x and y . Then, $2x + 3y = 39$... (i) and $3x + 2y = 36$... (ii)

On solving (i) and (ii), we get : $x = 6$ and $y = 9$.

$$\therefore \text{Larger number} = 9.$$

60. Let the ten's digits be x . Then, unit's digit = $4x$.

$$\therefore x + 4x = 10 \Leftrightarrow 5x = 10 \Leftrightarrow x = 2.$$

So, ten's digit = 2, unit's digit = 8.

Hence, the required number is 28.

61. Let the ten's digit be x . Then, number = $10x + 3$ and sum of digits = $(x+3)$.

$$\text{So, } (x+3) = \frac{1}{7}(10x+3) \Leftrightarrow 7x+21 = 10x+3 \Leftrightarrow 3x = 18 \Leftrightarrow x = 6.$$

Hence, the number is 63.

62. Let the ten's digit be x . Then, unit's digit = $2x$.

Number = $10x + 2x = 12x$; Sum of digits = $x + 2x = 3x$.

$$\therefore 12x - 3x = 18 \Leftrightarrow 9x = 18 \Leftrightarrow x = 2.$$

Hence, required number = $12x = 24$.

63. Let the ten's digit be x and unit's digit be y .

$$\text{Then, } x + y = 15 \text{ and } x - y = 3 \text{ or } y - x = 3.$$

Solving $x + y = 15$ and $x - y = 3$, we get : $x = 9$, $y = 6$.

Solving $x + y = 15$ and $y - x = 3$, we get : $x = 6$, $y = 9$.

So, the number is either 96 or 69. Hence, the number cannot be determined.

64. Let the ten's digit be x . Then, unit's digit = $x + 2$.

Number = $10x + (x+2) = 11x + 2$; Sum of digits = $x + (x+2) = 2x + 2$.

$$\therefore (11x + 2)(2x + 2) = 144 \Leftrightarrow 22x^2 + 26x - 140 = 0 \Leftrightarrow 11x^2 + 13x - 70 = 0$$

$$\Leftrightarrow (x-2)(11x+35) = 0 \Leftrightarrow x = 2 \Leftrightarrow$$

Hence, required number = $11x + 2 = 24$.

65. Let the ten's digit be x and unit's digit be y . Then, number = $10x + y$.

Number obtained by interchanging the digits = $10y + x$.

$$\therefore (10x + y) + (10y + x) = 11(x + y), \text{ which is divisible by 11.}$$

66. Let the ten's digit be x and unit's digit be y . Then, $(10x + y) - (x + y) = 9$ or $x = 1$.

From this data, we cannot find y , the unit's digit. So, the data is inadequate.

67. Let the ten's digit be x and unit's digit be y . Then, $(10x + y) - (10y + x) = 36 \Leftrightarrow 9(x - y) = 36 \Leftrightarrow x - y = 4$.
68. Let the ten's digit be x and unit's digit be y . Then, $(10x + y) - (10y + x) = 63 \Leftrightarrow 9(x - y) = 63 \Leftrightarrow x - y = 7$. Thus, none of the numbers can be determined.
69. Let the ten's digit be x and unit's digit be y . Then, $x + y = \frac{1}{5}[(10x + y) - (10y + x)] \Leftrightarrow 5x + 5y = 9x - 9y \Leftrightarrow 4x = 14y$. Thus, the value of $(x - y)$ cannot be determined from the given data.
70. Let the ten's digit be x and unit's digit be y . Then, $10 \times 2x + \frac{1}{2}y = 10y + x \Leftrightarrow 20x - x = 10y - \frac{y}{2} \Leftrightarrow 19x = \frac{19}{2}y \Leftrightarrow y = 2x$. Thus, the unit's digit is twice the ten's digit.
71. Let ten's digit = x . Then, unit's digit = $(8 - x)$. $\therefore [10(8 - x) + x] - [10x + (8 - x)] = 18 \Leftrightarrow 18x = 54 \Leftrightarrow x = 3$. So, ten's digit = 3 and unit's digit = 5. Hence, original number = 35.
72. Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit. Let the ten's and unit's digits be $2x$ and x respectively. Then, $(10 \times 2x + x) - (10x + 2x) = 36 \Leftrightarrow 9x = 36 \Leftrightarrow x = 4$. \therefore Required difference = $(2x + x) - (2x - x) = 2x = 8$.
73. Let the middle digit be x . Then, $2x = 10$ or $x = 5$. So, the number is either 253 or 352. Since the number increases on reversing the digits, so the hundred's digit is smaller than the unit's digit. Hence, required number = 253.
74. Since the number reduces on reversing the digits, so ten's digit is greater than the unit's digit. Let the unit's digit be x . Then, ten's digit = $(x + 1)$. $\therefore 10x + (x + 1) = \frac{5}{6}[10(x + 1) + x] \Leftrightarrow 66x + 6 = 55x + 50 \Leftrightarrow 11x = 44 \Leftrightarrow x = 4$. Hence, required number = 54.
75. Let the unit's digit be x . Then, ten's digit = $(x - 2)$. $\therefore 3[10(x - 2) + x] + \frac{6}{7}[10x + (x - 2)] = 108 \Leftrightarrow 231x - 420 + 66x - 12 = 756 \Leftrightarrow 297x = 1188 \Leftrightarrow x = 4$. Hence, sum of the digits = $x + (x - 2) = 2x - 2 = 6$.
76. Let the ten's digit be x and unit's digit be y . Then, $\frac{10x + y}{2} = 10y + (x + 1)$
 $\Leftrightarrow 10x + y = 20y + 2x + 2 \Leftrightarrow 8x - 19y = 2 \quad \dots(i)$ and $x + y = 7 \quad \dots(ii)$
 Solving, (i) and (ii), we get : $x = 5$, $y = 2$. Hence, required number = 52.
77. Let the ten's digit be x . Then, unit's digit = $2x + 1$. $\therefore [10x + (2x + 1)] - [(10(2x + 1) + x) - (10x + (2x + 1))] = 1 \Leftrightarrow (12x + 1) - (9x + 9) = 1 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3$. So, ten's digit = 3 and unit's digit = 7. Hence, original number = 37.
78. Let the ten's digit be x and unit's digit be y . Then, $10x + y = 3(x + y) \Rightarrow 7x - 2y = 0 \quad \dots(i)$
 $10x + y + 45 = 10y + x \Rightarrow y - x = 5 \quad \dots(ii)$
 Solving (i) and (ii), we get : $x = 2$ and $y = 7$. \therefore Required number = 27.

79. Let the ten's and unit's digit be x and $\frac{8}{x}$ respectively.

$$\text{Then, } \left(10x + \frac{8}{x}\right) + 18 = 10 \times \frac{8}{x} + x \Leftrightarrow 10x^2 + 8 + 18x = 80 + x^2 \text{ and add and } \\ \Leftrightarrow 9x^2 + 18x - 72 = 0 \Leftrightarrow x^2 + 2x - 8 = 0 \Leftrightarrow (x+4)(x-2) = 0 \Leftrightarrow x = 2.$$

So, ten's digit = 2 and unit's digit = 4. Hence, required number = 24.

80. Let the two fractions be a and b . Then, $ab = \frac{14}{15}$ and $\frac{a}{b} = \frac{35}{24}$.

$$\frac{ab}{(a/b)} = \frac{\left(\frac{14}{15} \times \frac{24}{35}\right)}{\left(\frac{14}{15} \times \frac{35}{24}\right)} \Leftrightarrow b^2 = \frac{16}{25} \Leftrightarrow b = \frac{4}{5}, ab = \frac{14}{15} \Rightarrow a = \left(\frac{14}{15} \times \frac{5}{4}\right) = \frac{7}{6}.$$

Since $a > b$, so greater fraction is $\frac{7}{6}$.

$$81. A = 2B \Rightarrow B = \frac{1}{2}A. \text{ So, } AB = \frac{2}{25} \Rightarrow \frac{1}{2}A^2 = \frac{2}{25} \Rightarrow A^2 = \frac{4}{25} \Rightarrow A = \frac{2}{5}.$$

82. Let the fraction be $\frac{x}{y}$. Then, $x+y=11$ (i)

$$\frac{x+1}{y-2} = \frac{2}{3} \Rightarrow 3(x+1) = 2(y-2) \Rightarrow 3x+3 = 2y-4 \Rightarrow 3x-2y = -7 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 3$ and $y = 8$. So, the fraction is $\frac{3}{8}$.

83. Let the numerator be x . Then, denominator = $x+3$.

$$\text{Now, } \frac{x+4}{(x+3)+4} = \frac{4}{5} \Leftrightarrow 5(x+4) = 4(x+7) \Leftrightarrow x = 8.$$

∴ The fraction is $\frac{8}{11}$.

84. Let the denominator be x . Then, numerator = $x+5$.

$$\text{Now, } \frac{x+5}{x} - \frac{x+5}{x+5} = \frac{5}{4} \Leftrightarrow \frac{x+5}{x} = \frac{5}{4} + 1 = \frac{9}{4} = 2\frac{1}{4}.$$

So, the fraction is $2\frac{1}{4}$.

$$85. \text{Let the fraction be } \frac{2x}{3x}. \text{ Then, } \frac{2x-6}{3x} = \frac{2}{3} \times \frac{2x}{3x} \Leftrightarrow \frac{2x-6}{3x} = \frac{4x}{9x} \Leftrightarrow 18x^2 - 54x = 12x^2 \\ \Leftrightarrow 6x^2 - 54x = 0 \Leftrightarrow 6x(x-9) = 0 \Leftrightarrow x = 9.$$

Hence, numerator of the original fraction = $2x = 18$.

86. Let the fraction be $\frac{x}{y}$. Then,

$$\frac{x}{y+1} = \frac{1}{2} \Leftrightarrow 2x-y=1 \quad \dots(i) \text{ and } \frac{x+1}{y} = 1 \Leftrightarrow x-y=-1 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 2$, $y = 3$. Hence, the required fraction is $\frac{2}{3}$.

87. Let the fraction be $\frac{x}{y}$. Then, $x+y=1$ and $\frac{x-1}{y-1} = \frac{4}{5} \Leftrightarrow 5x-4y=1$.

$$\frac{x+2}{y+3} = \frac{7}{9} \Leftrightarrow 9x-7y=3 \quad \dots(i) \text{ and } \frac{x-1}{y-1} = \frac{4}{5} \Leftrightarrow 5x-4y=1 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 5$, $y = 6$. Hence, the original fraction is $\frac{5}{6}$.

88. Let the fraction be $\frac{x}{y}$. Then, $\frac{x+4}{y} - \frac{x}{y} = \frac{2}{3} \Leftrightarrow \frac{4}{y} = \frac{2}{3} \Leftrightarrow y = \left(\frac{4 \times 3}{2}\right) = 6$.

\therefore Denominator = 6. $\Rightarrow 0 = 8 - x + \frac{5x}{6} \Rightarrow 0 = 8t - x + \frac{5x}{6} \Rightarrow$

89. Let the two parts be $(54 - x)$ and x . Then, $10(54 - x) + 22x = 780 \Leftrightarrow 12x = 240 \Leftrightarrow x = 20$.

\therefore Bigger part = $(54 - x) = 34$.

90. Let the three parts be A, B and C.

Let $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x$. Then, $A = 2x$, $B = 3x$ and $C = 4x$. So, $A : B : C = 2 : 3 : 4$.

\therefore Largest part = $\left(243 \times \frac{4}{9}\right) = 108$.

91. Let the four numbers be A, B, C and D. Let $A + 3 = B + 3 = 3C = \frac{D}{3} = x$.

(i). Then, $A = x - 3$, $B = x + 3$, $C = \frac{x}{3}$ and $D = 3x$.

(ii). $A + B + C + D = 64 \Rightarrow (x - 3) + (x + 3) + \frac{x}{3} + 3x = 64 \Rightarrow \frac{11x}{3} = 64 \Rightarrow$

$$\Rightarrow 5x + \frac{x}{3} = 64 \Rightarrow 16x = 192 \Rightarrow x = 12.$$

Thus, the numbers are 9, 15, 4 and 36.

\therefore Required difference = $(36 - 4) = 32$.

92. Let the numbers be a , b and c . Then, $a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \times 131 = 400$$

$$\Rightarrow (a + b + c) = \sqrt{400} = 20.$$

93. $A : B = 2 : 3$ and $B : C = 5 : 3 = \frac{3}{5} \times 5 : \frac{3}{5} \times 3 = 3 : \frac{9}{5}$.

So, $A : B : C = 2 : 3 : \frac{9}{5} = 10 : 15 : 9$.

\therefore Second number = $\left(136 \times \frac{15}{34}\right) = 60$.

94. Let the numbers be x , y and z . Then, $x + y = 45$, $y + z = 55$ and $3x + z = 90$.

$$\Rightarrow y = 45 - x, z = 55 - y = 55 - (45 - x) = 10 + x$$

$$\therefore 3x + 10 + x = 90 \text{ or } x = 20.$$

$$y = (45 - 20) = 25 \text{ and } z = (10 + 20) = 30.$$

\therefore Third number = 30.

EXERCISE 7B

$$(i) \quad I = y - x \Leftrightarrow \frac{I}{y} = \frac{x}{y}$$

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (Questions 1 to 6) : Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

(000) Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the two-digit number ? (S.B.I.P.O. 2003)

I. The difference between the two digits is 9.

II. The sum of the digits is equal to the difference between the two digits.

2. What is the difference between the digits of a two-digit number ? (Bank P.O. 1999)

I. The sum of the digits of that number is 8.

II. One-fifth of that number is 15 less than half of 44.

3. What is the ratio between the two numbers ? (H.S.C. 2001)

I. The sum of two numbers is twice their difference.

II. The smaller number is 6.

4. What is the two-digit number whose first digit is a and the second digit is b ? The number is greater than 9. (M.A.T. 2000)

I. The number is a multiple of 51.

II. The sum of the digits a and b is 6.

5. The difference between the digits of a two-digit number is 4. What is the digit in the unit's place ?

I. The difference between the number and the number obtained by interchanging the positions of the digits is 36.

II. The sum of the digits of that number is 12.

6. What is the number ? (Bank P.O. 2000)

I. The sum of the two digits is 8. The ratio of the two digits is 1 : 3.

II. The product of two digits of a number is 12. The quotient of two digits is 3.

Directions (Questions 7 to 10) : Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the given question.

7. What is the two-digit number ? (M.B.A. 2002)

I. Sum of the digits is 7.

II. Difference between the number and the number obtained by interchanging the digits is 9.

III. Digit in the ten's place is bigger than the digit in the unit's place by 1.

(a) I and II only (b) II and III only (c) I and III only

(d) All I, II and III (e) None of these

8. What is the sum of the digits of the two-digit number ?

I. The ratio between the ten's digit and unit's digit of the number is 3 : 2.

II. The number obtained on reversing the order of its digits is 18 less than the original number.

III. The product of the digits is 24.

(a) Any two of the three (b) I only or II and III only (c) All I, II and III

(d) I and II only (e) None of these

9. What will be the sum of two numbers ? (S.B.I.P.O. 2000)
 I. Among the two numbers, the bigger number is greater than the smaller number by 6.
 II. 40% of the smaller number is equal to 30% of the bigger number.

III. The ratio between half of the bigger number and one-third of the smaller number is 2 : 1.

- (a) I and II only (b) II and III only (c) All I, II and III
 (d) Any two of the three (e) None of these

10. What is the two-digit number ? (R.B.I. 2002)

I. The difference between the two-digit number and the number formed by interchanging the digits is 27.

II. The difference between the two digits is 3.

III. The digit at unit's place is less than that at ten's place by 3.

- (a) I and II only (b) I and III only (c) All I, II and III

(d) I, and either II or III (e) Even with all I, II and III, answer cannot be given.

ANSWERS

1. (e) 2. (e) 3. (a) 4. (e) 5. (b) 6. (b) 7. (e) 8. (a)
 9. (a) 10. (c)

SOLUTIONS

1. Let the tens and unit digits be x and y respectively. Then,

$$\text{I. } x - y = 9.$$

$$\text{II. } x + y = x - y.$$

From I and II, we get $x - y = 9$ and $x + y = 9$.

On solving, we get $x = 9$ and $y = 0$.

Required number is 90. Thus, both I and II are needed to get the answer.

Thus, both I and II are needed to get the answer.

Correct answer is (e).

2. Let the tens and unit digits be x and y respectively. Then,

$$\text{I. } x + y = 8$$

$$\text{II. } \left(\frac{1}{2} \times 44\right) - \frac{1}{5}(10x + y) = 15 \Rightarrow 10x + y = 35 \quad \dots(ii)$$

On solving (i) and (ii), we get $x = 3$ and $y = 5$.

Thus, I and II together give the answer.

Correct answer is (e).

3. Let the two numbers be x and y .

I gives, $x + y = 2(x - y) \Leftrightarrow x = 3y \Leftrightarrow \frac{x}{y} = \frac{3}{1} \Leftrightarrow x : y = 3 : 1$.

Thus, I only gives the answer.

II does not give the answer.

∴ Correct answer is (a).

4. Number = $10b + a$.

$$\text{I. } 10b + a = 51 \times c, \text{ where } c = 1, 2, 3 \text{ etc.} \quad \dots(i)$$

$$\text{II. } a + b = 6 \quad \dots(ii)$$

Taking $c = 1$, we get $10b + (6 - b) = 51 \Leftrightarrow 9b = 45 \Leftrightarrow b = 5$.

$\therefore a = 1, b = 5$. So, number = 51.

Thus, I and II together give the answer.

\therefore Correct answer is (e).

5. Let the ten's digit be x and unit's digit be y .

Then, $x - y = \pm 4$... (i)

$$\text{I. } (10x + y) - (10y + x) = 36 \Leftrightarrow x - y = 4 \quad \text{... (ii)}$$

$$\text{II. } x + y = 12 \quad \text{... (iii)}$$

Thus, (i) and (iii) together give the answer.

\therefore II alone gives the answer and I alone does not give the answer.

\therefore Correct answer is (b).

6. Let the tens and units digit be x and y respectively. Then, $x + y = 8$.

$$\text{I. } x + y = 8 \text{ and } \frac{x}{y} = \frac{1}{3} \quad \text{... (i)}$$

$$\text{II. } xy = 12 \text{ and } \frac{x}{y} = \frac{3}{1} \quad \text{... (ii)}$$

\therefore II gives, $x^2 = 36 \Leftrightarrow x = 6$. So, $3y = 6 \Leftrightarrow y = 2$.

Thus, II alone gives the number. Clearly, I alone does not give the answer.

\therefore Correct answer is (b).

7. Let the tens and units digit be x and y respectively.

$$\text{I. } x + y = 7 \quad \text{... (i)}$$

$$\text{II. } (10x + y) - (10y + x) = 9 \Rightarrow x - y = 1. \quad \text{... (ii)}$$

$$\text{III. } x - y = 1. \quad \text{... (iii)}$$

Thus, I and II as well as I and III give the answer.

\therefore Correct answer is (e).

8. I. Let the tens and units digit be $3x$ and $2x$ respectively.

$$\text{II. } (30x + 2x) - (20x + 3x) = 18 \Leftrightarrow x = 2.$$

$$\text{III. } 3x \times 2x = 24 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2.$$

Thus, any two of the three will give the answer.

\therefore Correct answer is (a).

9. Let the required numbers be x and y , where $x > y$.

$$\text{I. } x - y = 6 \quad \text{... (i)}$$

$$\text{II. } \frac{30}{100}x = \frac{40}{100}y \Leftrightarrow 3x - 4y = 0 \quad \text{... (ii)}$$

$$\text{III. } \frac{\frac{1}{2}x}{\frac{1}{3}y} = \frac{2}{1} \Leftrightarrow \frac{3x}{2y} = \frac{2}{1} \Leftrightarrow \frac{x}{y} = \frac{4}{3}.$$

Thus, I and II only give the answer.

\therefore Correct answer is (a).

10. Let the tens and units digit be x and y respectively.

$$\text{I. } (10x + y) - (10y + x) = 27 \Leftrightarrow x - y = 3.$$

$$\text{II. } x - y = 3.$$

$$\text{III. } x - y = 3.$$

Thus, even all the given three statements together do not give the answer.

\therefore Correct answer is (e).

8. PROBLEMS ON AGES

SOLVED EXAMPLES

Ex. 1. Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ? (Hotel Management, 2002)

Sol. Let Rajeev's present age be x years. Then,

$$\text{Rajeev's age after 15 years} = (x + 15) \text{ years.}$$

$$\text{Rajeev's age 5 years back} = (x - 5) \text{ years.}$$

$$\therefore x + 15 = 5(x - 5) \Leftrightarrow x + 15 = 5x - 25 \Leftrightarrow 4x = 40 \Leftrightarrow x = 10.$$

Hence, Rajeev's present age = 10 years.

Ex. 2. The ages of two persons differ by 16 years. If 6 years ago, the elder one be 3 times as old as the younger one, find their present ages. (A.A.O. Exam, 2003)

Sol. Let the age of the younger person be x years.

$$\text{Then, age of the elder person} = (x + 16) \text{ years.}$$

$$\therefore (x - 6) = (x + 16 - 6) \Leftrightarrow 3x - 18 = x + 10 \Leftrightarrow 2x = 28 \Leftrightarrow x = 14.$$

Hence, their present ages are 14 years and 30 years.

Ex. 3. The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, what is Nikita's age ? (S.B.I.P.O. 1999)

Sol. Let Ankit's age be x years. Then, Nikita's age = $\frac{240}{x}$ years.

$$\therefore 2 \times \frac{240}{x} - x = 4 \Leftrightarrow 480 - x^2 = 4x \Leftrightarrow x^2 + 4x - 480 = 0 \\ \Leftrightarrow (x + 24)(x - 20) = 0 \Leftrightarrow x = 20.$$

Hence, Nikita's age = $\left(\frac{240}{20}\right)$ years = 12 years.

Ex. 4. The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. Find the present age of the father. (S.S.C. 2003)

Sol. Let the son's present age be x years. Then, father's present age = $(3x + 3)$ years.

$$\therefore (3x + 3 + 3) = 2(x + 3) + 10 \Leftrightarrow 3x + 6 = 2x + 16 \Leftrightarrow x = 10.$$

Hence, father's present age = $(3x + 3) = (3 \times 10 + 3)$ years = 33 years.

Ex. 5. Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages ?

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = $4x$ years.

$$\text{Son's age after 8 years} = (x + 8) + 8 = (x + 16) \text{ years.}$$

$$\text{Rohit's age after 8 years} = (4x + 8) + 8 = (4x + 16) \text{ years.}$$

$$\therefore 2(x + 16) = 4x + 16 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8.$$

Hence, son's present age = $(x + 8) = 16$ years.

$$\text{Rohit's present age} = (4x + 8) = 40 \text{ years.}$$

Ex. 6. One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively. Four years hence, this ratio would become 7 : 8. How old is Sachin ? (NABARD, 2002)

Sol. Let Gaurav's and Sachin's ages one year ago be $6x$ and $7x$ years respectively. Then,
 Gaurav's age 4 years hence = $(6x + 1) + 4 = (6x + 5)$ years.

Sachin's age 4 years hence = $(7x + 1) + 4 = (7x + 5)$ years.

$$\frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x+5) = 7(7x+5) \Leftrightarrow 48x+40 = 49x+35 \Leftrightarrow x = 5.$$

Hence, Sachin's present age = $(7x + 1) = 36$ years.

Ex. 7. Abhay's age after six years will be three-seventh of his father's age. Ten years ago, the ratio of their ages was $1 : 5$. What is Abhay's father's age at present?

Sol. Let the ages of Abhay and his father 10 years ago be x and $5x$ years respectively. Then,

Abhay's age after 6 years = $(x + 10) + 6 = (x + 16)$ years.

Father's age after 6 years = $(5x + 10) + 6 = (5x + 16)$ years.

$$\therefore \frac{(x+16)}{(5x+16)} = \frac{3}{7} \Leftrightarrow 7(x+16) = 3(5x+16) \Leftrightarrow 7x+112 = 15x+48 \Leftrightarrow 8x = 64 \Leftrightarrow x = 8.$$

Hence, Abhay's father's present age = $(5x + 10) = 50$ years.

EXERCISE 8A

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer:

1. Sachin is younger than Rahul by 4 years. If their ages are in the respective ratio of $7 : 9$, how old is Sachin? (Bank P.O. 2003)

(a) 16 years (b) 18 years (c) 28 years
 (d) Cannot be determined (e) None of these
2. The ratio between the present ages of P and Q is $6 : 7$. If Q is 4 years old than P, what will be the ratio of the ages of P and Q after 4 years? (S.B.I.P.O. 1998)

(a) $3 : 4$ (b) $3 : 5$ (c) $4 : 3$
 (d) Data inadequate (e) None of these
3. The ratio between the present ages of P and Q is $5 : 7$ respectively. If the difference between Q's present age and P's age after 6 years is 2, what is the total of P's and Q's present ages? (Bank P.O. 1999)

(a) 48 years (b) 52 years (c) 56 years
 (d) Cannot be determined (e) None of these
4. At present, the ratio between the ages of Arun and Deepak is $4 : 3$. After 6 years, Arun's age will be 26 years. What is the age of Deepak at present? (R.R.B. 2003)

(a) 12 years (b) 15 years (c) $19\frac{1}{2}$ years (d) 21 years
 (e) None of these
5. Present ages of X and Y are in the ratio $5 : 6$ respectively. Seven years hence this ratio will become $6 : 7$ respectively. What is X's present age in years? (Bank P.O. 2003)

(a) 35 (b) 42 (c) 49
 (d) Cannot be determined (e) None of these
6. Present ages of Sameer and Anand are in the ratio of $5 : 4$ respectively. Three years hence, the ratio of their ages will become $11 : 9$ respectively. What is Anand's present age in years? (R.B.I. 2003)

(a) 24 (b) 27 (c) 40
 (d) Cannot be determined (e) None of these

7. Six years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present ? (Bank P.O. 2004)
- (a) 16 years (b) 18 years (c) 20 years
(d) Cannot be determined (e) None of these
8. The total of the ages of Jayant, Prem and Saransh is 93 years. Ten years ago, the ratio of their ages was 2 : 3 : 4. What is the present age of Saransh ?
- (a) 24 years (b) 32 years (c) 34 years (d) 38 years
9. The ratio of the present ages of two brothers is 1 : 2 and 5 years back, the ratio was 1 : 3. What will be the ratio of their ages after 5 years ? (S.S.C. 2002)
- (a) 1 : 4 (b) 2 : 3 (c) 3 : 5 (d) 5 : 6
10. Hitesh is 40 years old and Ronnie is 60 years old. How many years ago was the ratio of their ages 3 : 5 ?
- (a) 5 years (b) 10 years (c) 20 years (d) 37 years
11. The ratio of the father's age to his son's age is 7 : 3. The product of their ages is 756. The ratio of their ages after 6 years will be :
- (a) 5 : 2 (b) 2 : 1 (c) 11 : 7 (d) 13 : 9
12. The present ages of three persons are in proportions 4 : 7 : 9. Eight years ago, the sum of their ages was 56. Find their present ages (in years). (I.M.T. 2002)
- (a) 8, 20, 28 (b) 16, 28, 36 (c) 20, 35, 45 (d) None of these
13. The ratio of the ages of a man and his wife is 4 : 3. After 4 years, this ratio will be 9 : 7. If at the time of marriage, the ratio was 5 : 3, then how many years ago were they married ?
- (a) 8 years (b) 10 years (c) 12 years (d) 15 years
14. The ratio between the school ages of Neelam and Shaan is 5 : 6 respectively. If the ratio between the one-third age of Neelam and half of Shaan's age is 5 : 9, then what is the school age of Shaan ? (Bank P.O. 2002)
- (a) 25 years (b) 30 years (c) 36 years
(d) Cannot be determined (e) None of these
15. The ratio between the present ages of A and B is 5 : 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence is 1 : 1. What is the ratio between A's age 4 years hence and B's age 4 years ago ? (SIDBI, 2000)
- (a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 4 : 1 (e) None of these
16. Ten years ago, A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages ?
- (a) 20 years (b) 30 years (c) 45 years (d) None of these
17. A is two years older than B who is twice as old as C. If the total of the ages of A, B and C be 27, then how old is B ? (Hotel Management, 2003)
- (a) 7 (b) 8 (c) 9 (d) 10 (e) 11
18. A man is 24 years older than his son. In two years, his age will be twice the age of his son. The present age of the son is : (R.R.B. 2003)
- (a) 14 years (b) 18 years (c) 20 years (d) 22 years
19. Eighteen years ago, a father was three times as old as his son. Now the father is only twice as old as his son. Then the sum of the present ages of the son and the father is : (S.S.C. 2003)
- (a) 54 (b) 72 (c) 105 (d) 108
20. A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present ? (IGNOU. 2003)
- (a) 32 years (b) 36 years (c) 40 years (d) 48 years

21. Tanya's grandfather was 8 times older to her 16 years ago. He would be 3 times of her age 8 years from now. Eight years ago, what was the ratio of Tanya's age to that of her grandfather ?
(a) 1 : 2 (b) 1 : 5 (c) 3 : 8 (d) None of these
(S.S.C. 2003)
22. The age of father 10 years ago was thrice the age of his son. Ten years hence, father's age will be twice that of his son. The ratio of their present ages is :
(a) 5 : 2 (b) 7 : 3 (c) 9 : 2 (d) 13 : 4
(L.I.C.A.A.O. 2003)
23. Four years ago, the father's age was three times the age of his son. The total of the ages of the father and the son after four years, will be 64 years. What is the father's age at present ?
(a) 32 years (b) 36 years (c) 44 years
(d) Data inadequate (e) None of these
(S.S.C. 2003)
24. One year ago, Promila was four times as old as her daughter Sakshi. Six years hence, Promila's age will exceed her daughter's age by 9 years. The ratio of the present ages of Promila and her daughter is :
(a) 9 : 2 (b) 11 : 3 (c) 12 : 5 (d) 13 : 4
(R.R.B. 2000)
25. The sum of the present ages of a father and his son is 60 years. Six years ago, father's age was five times the age of the son. After 6 years, son's age will be :
(a) 12 years (b) 14 years (c) 18 years (d) 20 years
(R.R.B. 2000)
26. The total age of A and B is 12 years more than the total age of B and C. C is how many years younger than A ?
(a) 12 (b) 24 (c) C is elder than A
(d) Data inadequate (e) None of these
(S.I.D.B.I., 2000)
27. Q is as much younger than R as he is older than T. If the sum of the ages of R and T is 50 years, what is definitely the difference between R and Q's age ?
(a) 1 year (b) 2 years (c) 25 years
(d) Data inadequate (e) None of these
(Bank P.O. 1999)
28. The age of a man is three times the sum of the ages of his two sons. Five years hence, his age will be double of the sum of the ages of his sons. The father's present age is :
(a) 40 years (b) 45 years (c) 50 years (d) 55 years
(R.R.B. 2000)
29. The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages was 34. The ages of the son and the father are respectively :
(a) 6 and 39 (b) 7 and 38 (c) 9 and 36 (d) 11 and 34
(S.S.C. 2003)
30. Rajan got married 8 years ago. His present age is $\frac{6}{5}$ times his age at the time of his marriage. Rajan's sister was 10 years younger to him at the time of his marriage. The age of Rajan's sister is :
(a) 32 years (b) 36 years (c) 38 years (d) 40 years
(U.P.S.C. 2003)
31. The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child ?
(a) 4 years (b) 8 years (c) 10 years (d) None of these
(S.S.C. 2000)
32. Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age ?
(a) 2 times (b) $2\frac{1}{2}$ times (c) $2\frac{3}{4}$ times (d) 3 times
(C.B.I. 1998)

33. The difference between the ages of two persons is 10 years. Fifteen years ago, the elder one was twice as old as the younger one. The present age of the elder person is :
(a) 25 years (b) 35 years (c) 45 years (d) 55 years
34. A father said to his son, "I was as old as you are at present at the time of your birth." If the father's age is 38 years now, the son's age five years back was :
(a) 14 years (b) 19 years (c) 33 years (d) 38 years
(Assistant Grade, 1998)
35. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is :
(a) 19 years (b) 29 years (c) 39 years (d) 49 years
36. Sneh's age is $\frac{1}{6}$ th of her father's age. Sneh's father's age will be twice of Vimal's age after 10 years. If Vimal's eighth birthday was celebrated two years before, then what is Sneh's present age ?
(a) $6\frac{2}{3}$ years (b) 24 years (c) 30 years (d) None of these
37. If 6 years are subtracted from the present age of Gagan and the remainder is divided by 18, then the present age of his grandson Anup is obtained. If Anup is 2 years younger to Madan whose age is 5 years, then what is Gagan's present age ?
(a) 48 years (b) 60 years (c) 84 years (d) 96 years
38. Ayesha's father was 38 years of age when she was born while her mother was 36 years old when her brother four years younger to her was born. What is the difference between the ages of her parents ?
(Hotel Management, 2002)
(a) 2 years (b) 4 years (c) 6 years (d) 8 years
39. My brother is 3 years elder to me. My father was 28 years of age when my sister was born while my mother was 26 years of age when I was born. If my sister was 4 years of age when my brother was born, then, what was the age of my father and mother respectively when my brother was born ?
(a) 32 yrs, 23 yrs (b) 32 yrs, 29 yrs (c) 35 yrs, 29 yrs (d) 35 yrs, 33 yrs
40. A person was asked to state his age in years. His reply was, "Take my age three years hence, multiply it by 3 and then subtract three times my age three years ago and you will know how old I am." What was the age of the person ?
(S.S.C. 2004)
(a) 18 years (b) 20 years (c) 24 years (d) 32 years

ANSWERS

1. (e) 2. (e) 3. (a) 4. (b) 5. (a) 6. (a) 7. (a) 8. (d)
9. (c) 10. (b) 11. (b) 12. (b) 13. (c) 14. (d) 15. (c) 16. (d) 17. (d) 18. (d) 19. (d) 20. (c) 21. (d) 22. (b) 23. (e) 24. (d)
25. (d) 26. (a) 27. (d) 28. (b) 29. (a) 30. (c) 31. (a) 32. (a)
33. (b) 34. (b) 35. (c) 36. (d) 37. (b) 38. (c) 39. (a) 40. (a)

SOLUTIONS

1. Let Rahul's age be x years. Then, Sachin's age = $(x - 7)$ years.
$$\therefore \frac{x - 7}{x} = \frac{7}{9} \Leftrightarrow 9x - 63 = 7x \Leftrightarrow 2x = 63 \Leftrightarrow x = 31.5.$$

Hence, Sachin's age = $(x - 7) = 24.5$ years.

2. Let P's age and Q's age be $6x$ years and $7x$ years respectively.
 Then, $7x - 6x = 4 \Leftrightarrow x = 4$.
 ∴ Required ratio = $(6x + 4) : (7x + 4) = 28 : 32 = 7 : 8$.
3. Let the present ages of P and Q be $5x$ years and $7x$ years respectively.
 Then, $7x - (5x + 6) = 2 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4$.
 ∴ Required sum = $5x + 7x = 12x = 48$ years.
4. Let the present ages of Arun and Deepak be $4x$ years and $3x$ years respectively. Then,
 $4x + 6 = 26 \Leftrightarrow 4x = 20 \Leftrightarrow x = 5$.
 ∴ Deepak's age = $3x = 15$ years.
5. Let the present ages of X and Y be $5x$ years and $6x$ years respectively.
 Then, $\frac{5x + 7}{6x + 7} = \frac{6}{7} \Leftrightarrow 7(5x + 7) = 6(6x + 7) \Leftrightarrow x = 7$.
 ∴ X's present age = $5x = 35$ years.
6. Let the present ages of Sameer and Anand be $5x$ years and $4x$ years respectively.
 Then, $\frac{5x + 3}{4x + 3} = \frac{11}{9} \Leftrightarrow 9(5x + 3) = 11(4x + 3) \Leftrightarrow x = 6$.
 ∴ Anand's present age = $4x = 24$ years.
7. Let the ages of Kunal and Sagar 6 years ago be $6x$ and $5x$ years respectively.
 Then, $\frac{(6x + 6) + 4}{(5x + 6) + 4} = \frac{11}{10} \Leftrightarrow 10(6x + 10) = 11(5x + 10) \Leftrightarrow 5x = 10 \Leftrightarrow x = 2$.
 ∴ Sagar's present age = $(5x + 6) = 16$ years.
8. Let the ages of Jayant, Prem and Saransh 10 years ago be $2x$, $3x$ and $4x$ years respectively.
 Then, $(2x + 10) + (3x + 10) + (4x + 10) = 93 \Leftrightarrow 9x = 63 \Leftrightarrow x = 7$.
 ∴ Saransh's present age = $(4x + 10) = 38$ years.
9. Let the present ages of the two brothers be x years and $2x$ years respectively.
 Then, $\frac{x - 5 + 1}{2x - 5} = \frac{1}{3} \Leftrightarrow 3(x - 5) = (2x - 5) \Leftrightarrow x = 10$.
 ∴ Required ratio = $(x + 5) : (2x + 5) = 15 : 25 = 3 : 5$.
10. Suppose, the ratio was $3 : 5$, x years ago.
 Then, $\frac{40 - x}{60 - x} = \frac{3}{5} \Leftrightarrow 5(40 - x) = 3(60 - x) \Leftrightarrow 2x = 20 \Leftrightarrow x = 10$.
 ∴ Their present ages are 40 and 60 years respectively.
11. Let the present ages of the father and son be $7x$ and $3x$ years respectively.
 Then, $7x \times 3x = 756 \Leftrightarrow 21x^2 = 756 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6$.
 ∴ Required ratio = $(7x + 6) : (3x + 6) = 48 : 24 = 2 : 1$.
12. Let their present ages be $4x$, $7x$ and $9x$ years respectively.
 Then, $(4x - 8) + (7x - 8) + (9x - 8) = 56 \Leftrightarrow 20x = 80 \Leftrightarrow x = 4$.
 ∴ Their present ages are 16 , 28 and 36 years respectively.
13. Let the present ages of the man and his wife be $4x$ and $3x$ years respectively.
 Then, $\frac{4x + 4}{3x + 4} = \frac{9}{7} \Leftrightarrow 7(4x + 4) = 9(3x + 4) \Leftrightarrow x = 8$.
 So, their present ages are 32 and 24 years respectively.
 Suppose they were married z years ago.
 Then, $\frac{32 - z}{24 - z} = \frac{5}{3} \Leftrightarrow 3(32 - z) = 5(24 - z) \Leftrightarrow 12z = 24 \Leftrightarrow z = 12$.

14. Let the school ages of Neelam and Shaan be $5x$ and $6x$ years respectively. Then,

$$\frac{\frac{1}{3} \times 5x}{\frac{1}{2} \times 6x} = \frac{5}{9} \Leftrightarrow \left(\frac{1}{3} \times 9 \times 5x \right) = \left(\frac{5}{2} \times 6x \right) \Leftrightarrow 15 = 15. \text{ Hence, } x \text{ cannot be determined.}$$

Thus, Shaan's age cannot be determined.

15. Let the present ages of A and B be $5x$ and $3x$ years respectively.

$$\text{Then, } \frac{5x - 4}{3x + 4} = \frac{1}{1} \Leftrightarrow 5x - 4 = 3x + 4 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4.$$

∴ Required ratio = $(5x + 4) : (3x - 4) = 24 : 8 = 3 : 1$.

16. Let the ages of A and B 10 years ago be x and $2x$ years respectively.

$$\text{Then, } \frac{x + 10}{2x + 10} = \frac{3}{4} \Leftrightarrow 4(x + 10) = 3(2x + 10) \Leftrightarrow 2x = 10 \Leftrightarrow x = 5.$$

∴ Sum of their present ages = $(x + 10) + (2x + 10) = (3x + 20) = 35$ years.

17. Let C's age be x years. Then, B's age = $2x$ years. A's age = $(2x + 2)$ years.

$$\therefore (2x + 2) + 2x + x = 27 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5.$$

Hence, B's age = $2x = 10$ years.

18. Let the son's present age be x years. Then, man's present age = $(x + 24)$ years.

$$\therefore (x + 24) + 2 = 2(x + 2) \Leftrightarrow x + 26 = 2x + 4 \Leftrightarrow x = 22.$$

19. Let the present ages of the father and son be $2x$ and x years respectively.

$$\text{Then, } (2x - 18) = 3(x - 18) \Leftrightarrow x = 36.$$

∴ Required sum = $(2x + x) = 3x = 108$ years.

20. Let the mother's present age be x years. Then, the person's present age = $\left(\frac{2}{5}x\right)$ years.

$$\therefore \left(\frac{2}{5}x + 8\right) = \frac{1}{2}(x + 8) \Leftrightarrow 2(2x + 40) = 5(x + 8) \Leftrightarrow x = 40.$$

21. 16 years ago, let T = x years and G = $8x$ years.

After 8 years from now, T = $(x + 16 + 8)$ years and G = $(8x + 16 + 8)$ years.

$$\therefore 8x + 24 = 3(x + 24) \Leftrightarrow 5x = 48.$$

$$\text{8 years ago, } \frac{T}{G} = \frac{x+8}{8x+8} = \frac{\frac{48}{5}+8}{8 \times \frac{48}{5}+8} = \frac{88}{424} = \frac{11}{53}.$$

22. Let the ages of father and son 10 years ago be $3x$ and x years respectively.

$$\text{Then, } (3x + 10) + 10 = 2[(x + 10) + 10] \Leftrightarrow 3x + 20 = 2x + 40 \Leftrightarrow x = 20.$$

∴ Required ratio = $(3x + 10) : (x + 10) = 70 : 30 = 7 : 3$.

23. Let the ages of father and son 4 years ago be $3x$ and x years respectively.

$$\text{Then, } [(3x + 4) + 4] + [(x + 4) + 4] = 64 \Leftrightarrow 4x = 48 \Leftrightarrow x = 12.$$

∴ Father's present age = $3x = 36$ years.

24. Let the ages of Promila and Sakshi 1 year ago be $4x$ and x years respectively.

$$\text{Then, } [(4x + 1) + 6] - [(x + 1) + 6] = 9 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$$

∴ Required ratio = $(4x + 1) : (x + 1) = 13 : 4$.

25. Let the present ages of son and father be x and $(60 - x)$ years respectively.

$$\text{Then, } (60 - x) - 6 = 5(x - 6) \Leftrightarrow 54 - x = 5x - 30 \Leftrightarrow 6x = 84 \Leftrightarrow x = 14.$$

∴ Son's age after 6 years = $(x + 6) = 20$ years.

26. $(A + B) - (B + C) = 12 \Leftrightarrow A - C = 12$.

27. $R - Q = R - T \Rightarrow Q = T$. Also, $R + T = 50 \Rightarrow R + Q = 50$.

So, $(R - Q)$ cannot be determined.

28. Let the sum of present ages of the two sons be x years.

Then, father's present age = $3x$ years.

$$\therefore (3x + 5) = 2(x + 10) \Leftrightarrow 3x + 5 = 2x + 20 \Leftrightarrow x = 15.$$

Hence, father's present age = 45 years.

29. Let the ages of father and son be x and $(45 - x)$ years respectively.

$$\begin{aligned} \text{Then, } (x - 5)(45 - x - 5) &= 34 \Leftrightarrow (x - 5)(40 - x) = 34 \Leftrightarrow x^2 - 45x + 234 = 0 \\ &\Leftrightarrow (x - 39)(x - 6) = 0 \Leftrightarrow x = 39 \text{ or } x = 6. \end{aligned}$$

\therefore Father's age = 39 years and son's age = 6 years.

30. Let Rajan's present age be x years. Then, his age at the time of marriage = $(x - 8)$ years.

$$\therefore x = \frac{6}{5}(x - 8) \Leftrightarrow 5x = 6x - 48 \Leftrightarrow x = 48.$$

Rajan's sister's age at the time of his marriage = $(x - 8) - 10 = (x - 18) = 30$ years.

\therefore Rajan's sister's present age = $(30 + 8)$ years = 38 years.

31. Let the ages of the children be x , $(x + 3)$, $(x + 6)$, $(x + 9)$ and $(x + 12)$ years.

$$\text{Then, } x + (x + 3) + (x + 6) + (x + 9) + (x + 12) = 50 \Leftrightarrow 5x = 20 \Leftrightarrow x = 4.$$

\therefore Age of the youngest child = $x = 4$ years.

32. Let Ronit's present age be x years. Then, father's present age = $(x + 3x)$ years = $4x$ years.

$$\therefore (4x + 8) = \frac{5}{2}(x + 8) \Leftrightarrow 8x + 16 = 5x + 40 \Leftrightarrow 3x = 24 \Leftrightarrow x = 8.$$

$$\text{Hence, required ratio} = \frac{(4x + 16)}{(x + 16)} = \frac{48}{24} = 2.$$

33. Let their ages be x years and $(x + 10)$ years respectively.

$$\text{Then, } (x + 10) - 15 = 2(x - 15) \Leftrightarrow x - 5 = 2x - 30 \Leftrightarrow x = 25.$$

\therefore Present age of the elder person = $(x + 10) = 35$ years.

34. Let the son's present age be x years. Then, $(38 - x) = x \Leftrightarrow 2x = 38 \Leftrightarrow x = 19$.

\therefore Son's age 5 years back = $(19 - 5)$ years = 14 years.

35. Let B's present age = x years. Then, A's present age = $(x + 9)$ years.

$$\therefore (x + 9) + 10 = 2(x - 10) \Leftrightarrow x + 19 = 2x - 20 \Leftrightarrow x = 39.$$

36. Vimal's age after 10 years = $(8 + 2 + 10)$ years = 20 years.

Sneh's father's age after 10 years = 40 years. Sneh's father's present age = 30 years.

$$\therefore \text{Sneh's age} = \left(\frac{1}{6} \times 30\right) \text{ years} = 5 \text{ years.}$$

37. Anup's age = $(5 - 2)$ years = 3 years. Let Gagan's age be x years.

$$\text{Then, } \frac{x-6}{18} = 3 \Leftrightarrow x - 6 = 54 \Leftrightarrow x = 60.$$

38. Mother's age when Ayesha's brother was born = 36 years.

Father's age when Ayesha's brother was born = $(38 + 4)$ years = 42 years.

\therefore Required difference = $(42 - 36)$ years = 6 years.

39. Clearly, my brother was born 3 years before I was born and 4 years after my sister was born.

So, father's age when brother was born = $(28 + 4)$ years = 32 years;

mother's age when brother was born = $(26 - 3)$ years = 23 years.

40. Let the present age of the person be x years.

$$\text{Then, } 3(x + 3) - 3(x - 3) = x \Leftrightarrow (3x + 9) - (3x - 9) = x \Leftrightarrow x = 18.$$

EXERCISE 8B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (Questions 1 to 8) : Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The sum of the ages of P, Q and R is 96 years. What is the age of Q ?
 - I. P is 6 years older than R.
 - II. The total of the ages of Q and R is 56 years.

(Bank P.O. 2003)
2. What is Sonia's present age ?
 - I. Sonia's present age is five times Deepak's present age.
 - II. Five years ago her age was twenty-five times Deepak's age at that time.
3. How old is C now ?
 - I. Three years ago, the average of A and B was 18 years.
 - II. With C joining them now, the average becomes 22 years.
4. What is Reena's present age ?
Reena's present age is five times her son's present age.
Reena's age two years hence will be three times her daughter's age at that time.

(Bank P.O. 2003)
5. What is the average age of A and B ?
 - I. The ratio between one-fifth of A's age and one-fourth of B's age is 1 : 2.
 - II. The product of their ages is 20 times B's age.
6. Average age of employees working in a department is 30 years. In the next year, ten workers will retire. What will be the average age in the next year ? (I.M.T. 2002)
 - I. Retirement age is 60 years.
 - II. There are 50 employees in the department.
7. What is the ratio between the ages of the father and the son ?
 - I. The sum of their ages is 50 years.
 - II. 3 times the sum of their ages is equal to 5 times the father's age.
8. Divya is twice as old as Shruti. What is the difference in their ages ?
Divya's age is twice that of Shruti's age.

(Bank P.O. 2003)

 - I. Five years hence, the ratio of their ages would be 9 : 5.
 - II. Ten years back, the ratio of their ages was 3 : 1.

Directions (Questions 9 to 13) : Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statements is/are necessary to answer the question.

9. What is the present age of A ?
 - I. The sum of the ages of A and B is 21 years.
 - II. The difference of the ages of A and B is 5 years.

- III. The product of the ages of A and B is 104 years. (give III to II to I (a))
(a) I and II only (b) II and III only (c) I and III only
(d) Any two of the three (e) None of these
10. What is the present age of Tanya ? (Bank P.O. 2004)
I. The ratio between the present ages of Tanya and her brother Rahul is 3 : 4 respectively.
II. After 5 years the ratio between the ages of Tanya and Rahul will be 4 : 5.
III. Rahul is 5 years older than Tanya.
(a) I and II only (b) II and III only (c) I and III only
(d) All I, II and III (e) Any two of the three
11. What is the difference between the ages of Y and X ?
I. The ratio between the ages of X and Y is 2 : 3.
II. Y's age is 50% more than X's age.
III. One-fourth of X's age is equal to one-sixth of Y's age.
(a) All I, II and III (b) Any two of the three
(c) III, and either I or II (d) Only I and II
(e) Question cannot be answered even with information in all three statements
12. What is Arun's present age ? (M.B.A. 2002)
I. Five years ago, Arun's age was double that of his son's age at that time.
II. Present ages of Arun and his son are in the ratio of 11 : 6 respectively.
III. Five years hence, the respective ratio of Arun's age and his son's age will become 12 : 7.
(a) Only I and II (b) Only II and III (c) Only I and III
(d) Any two of the three (e) None of these
13. What is Ravi's present age ? (R.B.I. 2002)
I. The present age of Ravi is half of that of his father.
II. After 5 years, the ratio of Ravi's age to that of his father's age will be 6 : 11.
III. Ravi is 5 years younger than his brother.
(a) I and II only (b) II and III only
(c) I and III only (d) All I, II and III
(e) Even with all the three statements answer cannot be given.
- Directions (Questions 14 to 16) : Each of these questions is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is redundant and can be dispensed with while answering the given question.
14. What is the ratio of the present ages of Anna and her mother ?
I. The sum of the ages of Anna, her mother and her father is 62.
II. Five years ago, Anna's age was one-fifth of her father's age.
III. Two years ago, the sum of the ages of Anna and her father was 36.
(a) I or II only (b) II or III only (c) III only
(d) I or III only (e) All I, II and III are required.
15. What will be the ratio between ages of Sam and Albert after 5 years ? (Bank P.O. 1999)
I. Sam's present age is more than Albert's present age by 4 years.
II. Albert's present age is 20 years.
III. The ratio of Albert's present age to Sam's present age is 5 : 6.

- (a) I or II or III only (b) II only (c) III only
 (d) I or III only (e) II or III only

16. What is the difference between the present ages of Ayush and Deepak ?
 (S.B.I.P.O. 1998)

- I. The ratio between Ayush's present age and his age after 8 years is 4 : 5.
 II. The ratio between the present ages of Ayush and Deepak is 4 : 3.
 III. The ratio between Deepak's present age and his age four years ago is 6 : 5.
 (a) Any two of I, II and III (b) I or III only
 (c) Any one of the three (d) All I, II and III are required
 (e) Even with all I, II and III, the answer cannot be obtained.

ANSWERS

1. (e) 2. (e) 3. (e) 4. (d) 5. (e) 6. (e) 7. (b) 8. (c)
 9. (d) 10. (e) 11. (e) 12. (d) 13. (a) 14. (e) 15. (a) 16. (c)

SOLUTIONS

1. Given : $P + Q + R = 96$... (i)
 I. $P = R + 6$... (ii)
 II. $Q + R = 56$... (iii)
 On subtracting (iii) from (i), we get $P = 40$.
 Putting $P = 40$ in (ii), we get $R = 34$. Putting $R = 34$ in (iii), we get $Q = 22$.
 Thus, I and II both together give the answer. So, correct answer is (e).

2. I. $S = 5D \Rightarrow D = \frac{S}{5}$... (i)
 II. $S - 5 = 25(D - 5) \Leftrightarrow S = 25D - 120$... (ii)
 Using (i) in (ii), we get $S = \left(25 \times \frac{S}{5}\right) - 120 \Leftrightarrow 4S = 120 \Leftrightarrow S = 30$.

Thus, I and II both together give the answer. So, correct answer is (e).

3. I. 3 years ago, $\frac{1}{2}(A + B) = 18 \Rightarrow 3$ years ago, $(A + B) = 36$
 Now, $(A + B) = (36 + 3 + 3) = 42 \Rightarrow A + B = 42$... (i)
 II. Now, $\frac{1}{3}(A + B + C) = 22 \Rightarrow A + B + C = 66$... (ii)
 From I and II, we get $C = (66 - 42) = 24$.

Thus, I and II both together give the answer. So, correct answer is (e).

4. I. Reena's Present age = $5 \times$ (Her son's present age).
 II. Reena's age 2 years hence = 3 times her daughter's age at that time.
 Clearly, data even in I and II is not sufficient to get Reena's present age.
 ∴ Correct answer is (d).

5. I. $\frac{A}{5} : \frac{B}{4} = 1 : 2 \Leftrightarrow \frac{A}{5} \times \frac{4}{B} = \frac{1}{2} \Leftrightarrow \frac{A}{B} = \left(\frac{1}{2} \times \frac{5}{4}\right) = \frac{5}{8} \Leftrightarrow A : B = 5 : 8$.
 II. $20B = AB$.

Let A's age be $5x$ years. Then, B's age is $8x$ years.

$$\therefore 20 \times 8x = 5x \times 8x \Leftrightarrow 40x = 160 \Leftrightarrow x = 4.$$

$$\therefore A = 20 \text{ and } B = 32.$$

Thus, I and II together give the answer. So, correct answer is (e).

6. I. Retirement age is 60 years.
 II. There are 50 employees in the department.

Average age of 50 employees = 30 years.

Total age of 50 employees = (50×30) years = 1500 years.

Number of employees next year = 40.

Total age of 40 employees next year = $(1500 + 40 - 60 \times 10) = 940$.

$$\text{Average age next year} = \frac{940}{40} \text{ years} = 23\frac{1}{2} \text{ years.}$$

Thus, I and II together give the answer. So, correct answer is (e).

7. I. $F + S = 50$...(i) II. $3(F + S) = 5F$...
 From II, we get $2F = 3S \Leftrightarrow \frac{F}{S} = \frac{3}{2}$

Thus, II alone gives the answer, but I alone does not give the answer.

∴ Correct answer is (5).

8. Let Divya's present age be D years and Shruti's present age be S years.

Then, $D = 2 \times S \Leftrightarrow D - 2S = 0$

$$\text{I. } \frac{D+5}{S+5} = \frac{9}{5} \quad \dots(\text{ii})$$

$$\text{II. } \frac{D-10}{S-10} = \frac{3}{1} \quad \dots(\text{iii})$$

From (ii), we get $5D + 25 = 9S + 45 \Leftrightarrow 5D - 9S = 20$...
 From (iii), we get $D - 10 = 3S - 30 \Leftrightarrow D - 3S = -20$...
 Thus from (i) and (ii), we get the answer.

Also, from (i) and (iii), we get the answer.

∴ I alone as well as II alone gives the answer. Hence, the correct answer is (c).

9. I. $A + B = 21$. II. $A - B = 5$. III. $AB = 104$.

Clearly, any two of three will give the answer. So, correct answer is (d).

10. I. Let the present ages of Tanya and Rahul be $3x$ years and $4x$ years.

II. After 5 years, (Tanya's age) : (Rahul's age) = 4 : 5.

III. (Rahul's age) = (Tanya's age) + 5.

$$\text{From I and II, we get } \frac{3x+5}{4x+5} = \frac{4}{5}. \text{ This gives } x.$$

∴ Tanya's age = $3x$ can be found. Thus, I and II give the answer.

From I and III, we get $4x = 3x + 5$. This gives x .

∴ Tanya's age = $3x$ can be found. Thus, I and III give the answer.

From III : Let Tanya's present age be t years.

Then, Rahul's present age = $(t + 5)$ years.

$$\text{Thus, from II and III, we get : } \frac{t}{t+5} = \frac{4}{5}. \text{ This gives } t.$$

Thus, II and III give the answer.

∴ Correct answer is (e).

$$\text{11. I. } X : Y = 2 : 3 \Rightarrow \frac{X}{Y} = \frac{2}{3} \Rightarrow 3X = 2Y.$$

$$\text{II. } Y = \frac{150}{100} X \Rightarrow Y = \frac{3X}{2} \Rightarrow 3X = 2Y.$$

$$\text{III. } \frac{1}{4} X = \frac{1}{6} Y \Rightarrow 6X = 4Y \Rightarrow 3X = 2Y.$$

Thus, even I, II and III together do not give the answer.

∴ Correct answer is (e).

12. II. Let the present ages of Arun and his son be $11x$ and $6x$ years respectively.

I. 5 years ago, Arun's age = $2 \times$ His son's age.

III. 5 years hence, $\frac{\text{Arun's age}}{\text{Son's age}} = \frac{12}{7}$.

Clearly, any two of the above will give Arun's present age.

∴ Correct answer is (d).

13. I. Let Ravi's present age be x years. Then, his father's present age = $2x$ years.

II. After 5 years, $\frac{\text{Ravi's age}}{\text{Father's age}} = \frac{6}{11}$.

III. Ravi is younger than his brother.

From I and II, we get $\frac{x+5}{2x+5} = \frac{6}{11}$. This gives x , the answer.

Thus, I and II together give the answer. Clearly, III is redundant.

∴ Correct answer is (a).

14. I. $A + M + F = 62$.

II. $(A - 5) = \frac{1}{5}(F - 5)$.

III. $(A - 2) + (F - 2) = 36$.

From II and III, we may get A and F.

Putting these values in I, we get M.

Thus, all I, II and III are required to get the answer.

∴ Correct answer is (e).

15. Clearly, any two of the given statements will give the answer and in each case, the third is redundant.

∴ Correct answer is (a).

16. Clearly, any two of the given statements will give the answer and in each case, the third is redundant.

∴ Correct answer is (c).

9. SURDS AND INDICES

IMPORTANT FACTS AND FORMULAE

1. LAWS OF INDICES :

$$(i) a^m \times a^n = a^{m+n} \quad (ii) \frac{a^m}{a^n} = a^{m-n} \quad (iii) (a^m)^n = a^{mn}$$

$$(iv) (ab)^n = a^n b^n \quad (v) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (vi) a^0 = 1$$

2. SURDS : Let a be a rational number and n be a positive integer such that $a^{\frac{1}{n}} = \sqrt[n]{a}$ is irrational. Then, $\sqrt[n]{a}$ is called a surd of order n .

3. LAWS OF SURDS :

$$(i) \sqrt[n]{a} = a^{\frac{1}{n}} \quad (ii) \sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} \quad (iii) \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$(iv) (\sqrt[n]{a})^n = a \quad (v) \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \quad (vi) (\sqrt[n]{a})^m = \sqrt[n]{a^m}.$$

SOLVED EXAMPLES

Ex. 1. Simplify : (i) $(27)^{\frac{2}{3}}$ (ii) $(1024)^{-\frac{4}{5}}$ (iii) $\left(\frac{8}{125}\right)^{\frac{4}{3}}$

$$\text{Sol. } (i) (27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\left(3 \times \frac{2}{3}\right)} = 3^2 = 9.$$

$$(ii) (1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left[5 \times \left(-\frac{4}{5}\right)\right]} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}.$$

$$(iii) \left(\frac{8}{125}\right)^{\frac{4}{3}} = \left\{\left(\frac{2}{5}\right)^3\right\}^{\frac{4}{3}} = \left(\frac{2}{5}\right)^{\left[3 \times \frac{4}{3}\right]} = \left(\frac{2}{5}\right)^4 = \left(\frac{5}{2}\right)^{-4} = \frac{5^4}{2^4} = \frac{625}{16}.$$

Ex. 2. Evaluate : (i) $(.00032)^{\frac{3}{5}}$ (ii) $(256)^{0.16} \times (16)^{0.18}$

$$\text{Sol. } (i) (.00032)^{\frac{3}{5}} = \left(\frac{32}{100000}\right)^{\frac{3}{5}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left(\frac{2}{10}\right)^5 = \left(\frac{1}{5}\right)^3 = \frac{1}{125}.$$

$$(ii) (256)^{0.16} \times (16)^{0.18} = [(16)^2]^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18}$$

$$= (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32 + 0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4.$$

Ex. 3. What is the quotient when $(x^{-1} - 1)$ is divided by $(x - 1)$?

$$\text{Sol. } \frac{x^{-1} - 1}{x - 1} = \frac{\frac{1}{x} - 1}{x - 1} = \frac{(1 - x)}{x} \times \frac{1}{(x - 1)} = -\frac{1}{x}.$$

Hence, the required quotient is $-\frac{1}{x}$.

Ex. 4. If $2^{x-1} + 2^{x+1} = 1280$, then find the value of x .

$$\begin{aligned} \text{Sol. } 2^{x-1} + 2^{x+1} &= 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280 \\ &\Leftrightarrow 2^{x-1} = \frac{1280}{5} = 256 = 2^8 \Leftrightarrow x - 1 = 8 \Leftrightarrow x = 9. \end{aligned}$$

Hence, $x = 9$.

Ex. 5. Find the value of $\left[5 \left(8^{\frac{1}{3}} + 27^{\frac{1}{3}} \right)^3 \right]^{\frac{1}{4}}$.

$$\begin{aligned} \text{Sol. } \left[5 \left(8^{\frac{1}{3}} + 27^{\frac{1}{3}} \right)^3 \right]^{\frac{1}{4}} &= \left[5 \left((2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}} \right)^3 \right]^{\frac{1}{4}} = \left[5 \left(2^{\left(3 \times \frac{1}{3} \right)} + 3^{\left(3 \times \frac{1}{3} \right)} \right)^3 \right]^{\frac{1}{4}} \\ &= (5(2 + 3)^3)^{\frac{1}{4}} = (5 \times 5^3)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\left(4 \times \frac{1}{4} \right)} = 5^1 = 5. \end{aligned}$$

Ex. 6. Find the value of $\left\{ (16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}} \right\}$.

$$\begin{aligned} \text{Sol. } \left[(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}} \right] &= \left[(4^2)^{\frac{3}{2}} + (4^2)^{-\frac{3}{2}} \right] = 4^{\left(2 \times \frac{3}{2} \right)} + 4^{\left\{ 2 \times \left(-\frac{3}{2} \right) \right\}} \\ &= 4^3 + 4^{-3} = 4^3 + \frac{1}{4^3} = \left(64 + \frac{1}{64} \right) = \frac{4097}{64}. \end{aligned}$$

Ex. 7. If $\left(\frac{1}{5}\right)^{3y} = 0.008$, then find the value of $(0.25)^y$.

$$\begin{aligned} \text{Sol. } \left(\frac{1}{5}\right)^{3y} &= 0.008 = \frac{8}{1000} = \frac{1}{125} = \left(\frac{1}{5}\right)^3 \Leftrightarrow 3y = 3 \Leftrightarrow y = 1. \\ \therefore (0.25)^y &= (0.25)^1 = 0.25. \end{aligned}$$

Ex. 8. Find the value of $\frac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}}$.

$$\begin{aligned} \text{Sol. } \frac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}} &= \frac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \frac{3^{\left(5 \times \frac{n}{5} \right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} \\ &= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9. \end{aligned}$$

Ex. 9. Find the value of $\left(2^{\frac{1}{4}} - 1 \right) \left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1 \right)$. (N.I.E.T. 2003)

Sol. Putting $2^{\frac{1}{4}} = x$, we get :

$$\begin{aligned} \left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right) &= (x-1)(x^3 + x^2 + x + 1), \text{ where } x = 2^{\frac{1}{4}} \\ &= (x-1)[x^2(x+1) + (x+1)] \\ &= (x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1) \\ &= (x^4-1) = \left[\left(\frac{1}{2^4}\right)^4 - 1\right] = \left[2^{\left(\frac{1}{4} \times 4\right)} - 1\right] = (2-1) = 1. \end{aligned}$$

Ex. 10. Find the value of $\frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

Sol.
$$\frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \frac{6^{\frac{2}{3}} \times (6^7)^{\frac{1}{3}}}{(6^6)^{\frac{1}{3}}} = \frac{6^{\frac{2}{3}} \times 6^{\left(\frac{7 \times \frac{1}{3}}{3}\right)}}{6^{\left(\frac{6 \times \frac{1}{3}}{3}\right)}} = \frac{6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}\right)}}{6^2}$$

$$= 6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}-2\right)} = 6^{\frac{2}{3}} \times 6^{\frac{1}{3}} = 6^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 6^1 = 6.$$

Ex. 11. If $x = y^a$, $y = z^b$ and $z = x^c$, then find the value of abc.

Sol.
$$\begin{aligned} z^1 &= x^c = (y^a)^c \quad [\because x = y^a] \\ &= y^{(ac)} = (z^b)^{ac} \quad [\because y = z^b] \\ &= z^{b(ac)} = z^{abc}. \end{aligned}$$

$\therefore abc = 1.$

Ex. 12. Simplify : $\left(\frac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$

Sol. Given Expression = $(x^{(a-b)})^{(a^2+b^2+ab)} \cdot (x^{(b-c)})^{(b^2+c^2+bc)} \cdot (x^{(c-a)})^{(c^2+a^2+ca)}$

$$= x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}$$

$$= x^{(a^3-b^3)} \cdot x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$$

Ex. 13. Which is larger $\sqrt{2}$ or $\sqrt[3]{3}$?

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6.

Changing each to a surd of order 6, we get :

$$\begin{aligned} \sqrt{2} &= 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2} \times \frac{3}{3}\right)} = 2^{\frac{3}{6}} = (2^3)^{\frac{1}{6}} = (8)^{\frac{1}{6}} = \sqrt[6]{8} \\ \sqrt[3]{3} &= 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3} \times \frac{2}{2}\right)} = 3^{\frac{2}{6}} = (3^2)^{\frac{1}{6}} = (9)^{\frac{1}{6}} = \sqrt[6]{9}. \end{aligned}$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Ex. 14. Find the largest from among $\sqrt[4]{6}$, $\sqrt{2}$ and $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C.M. is 12.

Changing each to a surd of order 12, we get :