QUANTITATIVE APTITUDE

– R.S. AGARWAL

CONTENTS

SECTION II

DATA INTERPRETATION

1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Numeral :** In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called ***digits*** to represent any number.

A group of digits, denoting a number is called a ***numeral.***

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	8	9	7	4	5	1	3	2

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II. **Place Value or Local Value of a Digit in a Numeral :**

In the above numeral :

Place value of 2 is $(2 \times 1) = 2$; Place value of 3 is $(3 \times 10) = 30$;

Place value of 1 is $(1 \times 100) = 100$ and so on.

Place value of 6 is $6 \times 10^8 = 600000000$.

III. **Face Value :** The ***face value*** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV. **TYPES OF NUMBERS**

1. **Natural Numbers :** Counting numbers 1, 2, 3, 4, 5, ... are called ***natural numbers.***

2. **Whole Numbers :** All counting numbers together with zero form the set of ***whole numbers.*** Thus,

   (*i*) 0 is the only whole number which is not a natural number.

   (*ii*) Every natural number is a whole number.

3. **Integers :** All natural numbers, 0 and negatives of counting numbers *i.e.*, {....., – 3, – 2, – 1, 0, 1, 2, 3,.....} together form the set of integers.

   (*i*) **Positive Integers :** {1, 2, 3, 4, ...} is the set of all positive integers.

   (*ii*) **Negative Integers :** {– 1, – 2, – 3, ...} is the set of all negative integers.

   (*iii*) **Non-Positive and Non-Negative Integers :** 0 is neither positive nor negative. So, {0, 1, 2, 3, ...} represents the set of non-negative integers, while {0, – 1, – 2, – 3, ...} represents the set of non-positive integers.

4. **Even Numbers :** A number divisible by 2 is called an even number. *e.g.*, 2, 4, 6, 8, 10, etc.
5. **Odd Numbers :** A number not divisible by 2 is called an odd number. *e.g.*, 1, 3, 5, 7, 9, 11, etc.
6. **Prime Numbers :** A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

   Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

   Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :

   Find a whole number nearly greater than the square root of p. Let $k > \sqrt{p}$. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.

   e.g., We have to find whether 191 is a prime number or not. Now, $14 > \sqrt{191}$.
   Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.
   191 is not divisible by any of them. So, 191 is a prime number.
7. **Composite Numbers :** Numbers greater than 1 which are not prime, are known as composite numbers. *e.g.*, 4, 6, 8, 9, 10, 12.

   Note : (*i*) 1 is neither prime nor composite.
   (*ii*) 2 is the only even number which is prime.
   (*iii*) There are 25 prime numbers between 1 and 100.
8. **Co-primes :** Two numbers a and b are said to be co-primes, if their H.C.F. is 1. *e.g.*, (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes.

V. TESTS OF DIVISIBILITY

1. **Divisibility By 2 :** A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

   Ex. 84932 is divisible by 2, while 65935 is not.
2. **Divisibility By 3 :** A number is divisible by 3, if the sum of its digits is divisible by 3.

   Ex. 592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.

   But, 864329 is not divisible by 3, since sum of its digits = (8 + 6 + 4 + 3 + 2 + 9) = 32, which is not divisible by 3.
3. **Divisibility By 4 :** A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

   Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.

   But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.
4. **Divisibility By 5 :** A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.
5. **Divisibility By 6 :** A number is divisible by 6, if it is divisible by both 2 and 3.

   Ex. The number 35256 is clearly divisible by 2.
   Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3.
   Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. **Divisibility By 8 :** A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

   Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.

   But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. **Divisibility By 9 :** A number is divisible by 9, if the sum of its digits is divisible by 9.

   Ex. 60732 is divisible by 9, since sum of digits = (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9.

   But, 68956 is not divisible by 9, since sum of digits = (6 + 8 + 9 + 5 + 6) = 34, which is not divisible by 9.

8. **Divisibility By 10 :** A number is divisible by 10, if it ends with 0.

   Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. **Divisibility By 11 :** A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

   Ex. The number 4832718 is divisible by 11, since :

   (sum of digits at odd places) – (sum of digits at even places)

   = (8 + 7 + 3 + 4) – (1 + 2 + 8) = 11, which is divisible by 11.

10. **Divisibility By 12 :** A number is divisible by 12, if it is divisible by both 4 and 3.

    Ex. Consider the number 34632.

    (*i*) The number formed by last two digits is 32, which is divisible by 4.

    (*ii*) Sum of digits = (3 + 4 + 6 + 3 + 2) = 18, which is divisible by 3.

    Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. **Divisibility By 14 :** A number is divisible by 14, if it is divisible by 2 as well as 7.

12. **Divisibility By 15 :** A number is divisible by 15, if it is divisible by both 3 and 5.

13. **Divisibility By 16 :** A number is divisible by 16, if the number formed by the last 4 digits is divisible by 16.

    Ex. 7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. **Divisibility By 24 :** A given number is divisible by 24, if it is divisible by both 3 and 8.

15. **Divisibility By 40 :** A given number is divisible by 40, if it is divisible by both 5 and 8.

16. **Divisibility By 80 :** A given number is divisible by 80, if it is divisible by both 5 and 16.

    Note : If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.

    If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

    Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4 \times 6) = 24$, since 4 and 6 are not co-primes.

VI. MULTIPLICATION BY SHORT CUT METHODS

1. **Multiplication By Distributive Law :**

(i) $a \times (b + c) = a \times b + a \times c$ (ii) $a \times (b - c) = a \times b - a \times c$.

Ex. (i) $567958 \times 99999 = 567958 \times (100000 - 1)$

$= 567958 \times 100000 - 567958 \times 1$

$= (56795800000 - 567958) = 56795232042$.

(ii) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000$.

2. **Multiplication of a Number By 5^n :** Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

Ex. $975436 \times 625 = 975436 \times 5^4 = \dfrac{9754360000}{16} = 609647500$.

VII. BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a^2 - b^2) = (a + b)(a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

Dividend = (Divisor × Quotient) + Remainder

IX.

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a *progression.*

1. **Arithmetic Progression (A.P.) :** If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference of the A.P.* An A.P. with first term a and common difference d is given by a, $(a + d)$, $(a + 2d)$, $(a + 3d)$, ...

The nth term of this A.P. is given by $T_n = a\ (n - 1)\ d$.

The sum of n terms of this A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}\ (\text{first term} + \text{last term}).$$

SOME IMPORTANT RESULTS :

(i) $(1 + 2 + 3 + \ldots + n) = \dfrac{n(n+1)}{2}$

(ii) $(1^2 + 2^2 + 3^2 + \ldots + n^2) = \dfrac{n(n+1)(2n+1)}{6}$.

(iii) $(1^3 + 2^3 + 3^3 + \ldots + n^3) = \dfrac{n^2(n+1)^2}{4}$.

2. **Geometrical Progression (G.P.) :** A progression of numbers in which every term bears a constant ratio with its preceding term, is called a *geometrical progression*. The constant ratio is called the common ratio of the G.P.

A G.P. with first term a and common ratio r is :

$$a, ar, ar^2, ar^3, \ldots$$

In this G.P. $T_n = ar^{n-1}$.

Sum of the n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$.

SOLVED EXAMPLES

Ex. 1. *Simplify :* (*i*) *8888 + 888 + 88 + 8* **(B.S.R.B. 1998)**
(*ii*) *11992 − 7823 − 456* **(Bank Exam, 2003)**

Sol. (*i*)

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \\ \hline \end{array}$$

(*ii*) 11992 − 7823 − 456 = 11992 − (7823 + 456)
= 11992 − 8279 = 3713.

$$\begin{array}{r} 7823 \\ +\ 456 \\ \hline 8279 \\ \hline \end{array} \qquad \begin{array}{r} 11992 \\ -\ 8279 \\ \hline 3713 \\ \hline \end{array}$$

Ex. 2. *What will replace the question mark in each of the following equations ?*
(*i*) *? − 1936248 = 1635773* (*ii*) *8597 − ? = 7429 − 4358* **(Bank P.O. 2000)**

Sol. (*i*) Let x − 1936248 = 1635773. Then, x = 1635773 + 1936248 = 3572021.
(*ii*) Let 8597 − x = 7429 − 4358.
Then, x = (8597 + 4358) − 7429 = 12955 − 7429 = 5526.

Ex. 3. *What could be the maximum value of Q in the following equation ?*
5P9 + 3R7 + 2Q8 = 1114 **(Bank P.O. 1999)**

Sol. We may analyse the given equation as shown :
Clearly, 2 + P + R + Q = 11.
So, the maximum value of Q can be
(11 − 2) *i.e.*, 9 (when P = 0, R = 0).

①	②	
5	P	9
3	R	7
2	Q	8
11	1	4

Ex. 4. *Simplify :* (*i*) *5793405 × 9999* (*ii*) *839478 × 625*

Sol. (*i*) 5793405 × 9999 = 5793405 (10000 − 1) = 57934050000 − 5793405 = 57928256595.

(*ii*) $839478 \times 625 = 839478 \times 5^4 = \dfrac{8394780000}{16} = 524673750.$

Ex. 5. *Evaluate :* (*i*) *986 × 137 + 986 × 863* (*ii*) *983 × 207 − 983 × 107*

Sol. (*i*) 986 × 137 + 986 × 863 = 986 × (137 + 863) = 986 × 1000 = 986000.
(*ii*) 983 × 207 − 983 × 107 = 983 × (207 − 107) = 983 × 100 = 98300.

Ex. 6. *Simplify :* (*i*) *1605 × 1605* (*ii*) *1398 × 1398*

Sol. (*i*) $1605 \times 1605 = (1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5$
$= 2560000 + 25 + 16000 = 2576025.$
(*ii*) $1398 \times 1398 = (1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2$
$= 1960000 + 4 - 5600 = 1954404.$

Ex. 7. *Evaluate :* (*313 × 313 + 287 × 287*).

Sol. $(a^2 + b^2) = \dfrac{1}{2}[(a + b)^2 + (a - b)^2]$

$\therefore \quad (313)^2 + (287)^2 = \dfrac{1}{2}[(313 + 287)^2 + (313 - 287)^2] = \dfrac{1}{2}[(600)^2 + (26)^2]$

$= \dfrac{1}{2}(360000 + 676) = 180338.$

Ex. 8. *Which of the following are prime numbers ?*
(*i*) *241* (*ii*) *337* (*iii*) *391* (*iv*) *571*

Sol. (*i*) Clearly, $16 > \sqrt{241}$. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.
241 is not divisible by any one of them.
∴ 241 is a prime number.

(*ii*) Clearly, $19 > \sqrt{337}$. Prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.
337 is not divisible by any one of them.
$\therefore$ 337 is a prime number.

(*iii*) Clearly, $20 > \sqrt{391}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
We find that 391 is divisible by 17.
$\therefore$ 391 is not prime.

(*iv*) Clearly, $24 > \sqrt{571}$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
571 is not divisible by any one of them.
$\therefore$ 571 is a prime number.

Ex. 9. ***Find the unit's digit in the product $(2467)^{153} \times (341)^{72}$.***

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.
Now, 7^4 gives unit digit 1.
$\therefore$ 7^{152} gives unit digit 1.
$\therefore$ 7^{153} gives unit digit $(1 \times 7) = 7$. Also, 1^{72} gives unit digit 1.
Hence, unit's digit in the product $= (7 \times 1) = 7$.

Ex. 10. ***Find the unit's digit in $(264)^{102} + (264)^{103}$.*** **(S.S.C. 1999)**

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$.
Now, 4^2 gives unit digit 6.
$\therefore$ $(4)^{102}$ gives unit digit 6.
$(4)^{103}$ gives unit digit of the product (6×4) *i.e.*, 4.
Hence, unit's digit in $(264)^{102} + (264)^{103}$ = unit's digit in $(6 + 4) = 0$.

Ex. 11. ***Find the total number of prime factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$.***

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{11} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$.
$\therefore$ Total number of prime factors $= (22 + 5 + 2) = 29$.

Ex. 12. ***Simplify :*** ***(i) $896 \times 896 - 204 \times 204$***
(ii) $387 \times 387 + 114 \times 114 + 2 \times 387 \times 114$
(iii) $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$

Sol. (*i*) Given exp. $= (896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(*ii*) Given exp. $= (387)^2 + (114)^2 + 2 \times 387 \times 114$
$= a^2 + b^2 + 2ab$, where $a = 387$, $b = 114$
$= (a + b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(*iii*) Given exp. $= (81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, where $a = 81$, $b = 68$
$= (a - b)^2 = (81 - 68)^2 = (13)^2 = 169$.

Ex. 13. ***Which of the following numbers is divisible by 3 ?***
(i) 541326 ***(ii) 5967013***

Sol. (*i*) Sum of digits in 541326 = (5 + 4 + 1 + 3 + 2 + 6) = 21, which is divisible by 3.
Hence, 541326 is divisible by 3.
(*ii*) Sum of digits in 5967013 = (5 + 9 + 6 + 7 + 0 + 1 + 3) = 31, which is not divisible by 3.
Hence, 5967013 is not divisible by 3.

Ex. 14. ***What least value must be assigned to * so that the number 197*5462 is divisible by 9 ?***

Sol. Let the missing digit be x.
Sum of digits $= (1 + 9 + 7 + x + 5 + 4 + 6 + 2) = (34 + x)$.
For $(34 + x)$ to be divisible by 9, x must be replaced by 2.
Hence, the digit in place of * must be 2.

Ex. 15. ***Which of the following numbers is divisible by 4 ?***

(i) 67920594 ***(ii) 618703572***

Sol. (*i*) The number formed by the last two digits in the given number is 94, which is not divisible by 4.

Hence, 67920594 is not divisible by 4.

(*ii*) The number formed by the last two digits in the given number is 72, which is divisible by 4.

Hence, 618703572 is divisible by 4.

Ex. 16. ***Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5 ?***

Sol. Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

Ex. 17. ***Show that 4832718 is divisible by 11.***

Sol. (Sum of digits at odd places) – (Sum of digits at even places)

$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

Hence, 4832718 is divisible by 11.

Ex. 18. ***Is 52563744 divisible by 24 ?***

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3×8, *i.e.*, 24.

Ex. 19. ***What least number must be added to 3000 to obtain a number exactly divisible by 19 ?***

Sol. On dividing 3000 by 19, we get 17 as remainder.

∴ Number to be added $= (19 - 17) = 2$.

Ex. 20. ***What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?***

Sol. On dividing 2000 by 17, we get 11 as remainder.

∴ Required number to be subtracted = 11.

Ex. 21. ***Find the number which is nearest to 3105 and is exactly divisible by 21.***

Sol. On dividing 3105 by 21, we get 18 as remainder.

∴ Number to be added to $3105 = (21 - 18) = 3$.

Hence, required number $= 3105 + 3 = 3108$.

Ex. 22. ***Find the smallest number of 6 digits which is exactly divisible by 111.***

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

∴ Number to be added $= (111 - 100) = 11$.

Hence, required number = 100011.

Ex. 23. ***On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.***

Sol. $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}} = \dfrac{15968 - 37}{89} = 179.$

Ex. 24. ***A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder ?***

Sol. On dividing the given number by 342, let k be the quotient and 47 as remainder.

Then, number $= 342k + 47 = (19 \times 18k + 19 \times 2 + 9) = 19\,(18k + 2) + 9$.

$\therefore$ The given number when divided by 19, gives $(18k + 2)$ as quotient and 9 as remainder.

Ex. 25. ***A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.***

Sol.

3	x
5	$y - 1$
8	$z - 4$
	$1 - 7$

$\therefore$ $z = (8 \times 1 + 7) = 15;\; y = (5z + 4) = (5 \times 15 + 4) = 79;\; x = (3y + 1) = (3 \times 79 + 1) = 238.$

Now,

8	238
5	$29 - 6$
3	$5 - 4$
	$1 - 2$

$\therefore$ Respective remainders are 6, 4, 2.

Ex. 26. ***Find the remainder when 2^{31} is divided by 5.***

Sol. $2^{10} = 1024$. Unit digit of $2^{10} \times 2^{10} \times 2^{10}$ is 4 [as $4 \times 4 \times 4$ gives unit digit 4].

$\therefore$ Unit digit of 2^{31} is 8.

Now, 8 when divided by 5, gives 3 as remainder.

Hence, 2^{31} when divided by 5, gives 3 as remainder.

Ex. 27. ***How many numbers between 11 and 90 are divisible by 7 ?***

Sol. The required numbers are 14, 21, 28, 35, ..., 77, 84.

This is an A.P. with $a = 14$ and $d = (21 - 14) = 7$.

Let it contain n terms.

Then, $T_n = 84 \Rightarrow a + (n - 1)\,d = 84$

$\Rightarrow 14 + (n - 1) \times 7 = 84$ or $n = 11$.

$\therefore$ Required number of terms = 11.

Ex. 28. ***Find the sum of all odd numbers upto 100.***

Sol. The given numbers are 1, 3, 5, 7, ..., 99.

This is an A.P. with $a = 1$ and $d = 2$.

Let it contain n terms. Then,

$1 + (n - 1) \times 2 = 99$ or $n = 50$.

$$\therefore \text{ Required sum } = \frac{n}{2}\,(\text{first term} + \text{last term})$$

$$= \frac{50}{2} \times (1 + 99) = 2500.$$

Ex. 29. ***Find the sum of all 2 digit numbers divisible by 3.***

Sol. All 2 digit numbers divisible by 3 are :

12, 51, 18, 21, ..., 99.

This is an A.P. with $a = 12$ and $d = 3$.

Let it contain n terms. Then,

$12 + (n - 1) \times 3 = 99$ or $n = 30$.

$$\therefore \text{ Required sum } = \frac{30}{2} \times (12 + 99) = 1665.$$

Ex. 30. ***How many terms are there in 2, 4, 8, 16, ..., 1024 ?***

Sol. Clearly 2, 4, 8, 16, ..., 1024 form a G.P. with $a = 2$ and $r = \frac{4}{2} = 2$.

Let the number of terms be n. Then,

$2 \times 2^{n-1} = 1024$ or $2^{n-1} = 512 = 2^9$.

$\therefore$ $n - 1 = 9$ or $n = 10$.

Ex. 31. $2 + 2^2 + 2^3 + ... + 2^8 = ?$

Sol. Given series is a G.P. with $a = 2$, $r = 2$ and $n = 8$.

$$\therefore \quad \text{Sum} = \frac{a(r^n - 1)}{(r-1)} = \frac{2 \times (2^8 - 1)}{(2-1)} = (2 \times 255) = 510.$$

EXERCISE 1

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The difference between the local value and face value of 7 in the numeral 657903 is :
(a) 0 (b) 7896 (c) 6993 (d) 903
2. The difference between the place values of 7 and 3 in the number 527435 is :
(a) 4 (b) 5 (c) 45 (d) 6970
(R.R.B. 2001)
3. The sum of the smallest six-digit number and the greatest five-digit number is :
(a) 199999 (b) 201110 (c) 211110 (d) 1099999
4. If the largest three-digit number is subtracted from the smallest five-digit number, then the remainder is : **(S.S.C. 1998)**
(a) 1 (b) 9000 (c) 9001 (d) 90001
5. 5978 + 6134 + 7014 = ? **(Bank P.O. 1999)**
(a) 16226 (b) 19126 (c) 19216 (d) 19226
6. 18265 + 2736 + 41328 = ? **(Bank P.O. 2000)**
(a) 61329 (b) 62239 (c) 62319 (d) 62329
7. 39798 + 3798 + 378 = ? **(Bank P.O. 2002)**
(a) 43576 (b) 43974 (c) 43984 (d) 49532
8. 9358 − 6014 + 3127 = ? **(SIDBI, 2000)**
(a) 6381 (b) 6471 (c) 6561 (d) 6741
9. 9572 − 4018 − 2164 = ?
(a) 3300 (b) 3390 (c) 3570 (d) 7718
10. 7589 − ? = 3434 **(Bank P.O. 2003)**
(a) 721 (b) 3246 (c) 4155 (d) 11023
11. 9548 + 7314 = 8362 + ? **(S.B.I.P.O. 2000)**
(a) 8230 (b) 8410 (c) 8500 (d) 8600
12. 7845 − ? = 8461 − 3569
(a) 2593 (b) 2773 (c) 3569 (d) None of these
13. 3578 + 5729 − ?486 = 5821
(a) 1 (b) 2 (c) 3 (d) None of these
14. If 6x43 − 46y9 = 1904, which of the following should come in place of x ?
(a) 4 (b) 6 (c) 9
(d) Cannot be determined (e) None of these

15. What should be the maximum value of B in the following equation ? **(Bank P.O. 2000)**
5A9 − 7B2 + 9C6 = 823
(*a*) 5 (*b*) 6 (*c*) 7 (*d*) 9

16. In the following sum, '?' stands for which digit ? **(M.B.A. 1998)**
? + 1? + 2? + ?3 + ?1 = 21?
(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 9

17. 5358 × 51 = ?
(*a*) 273258 (*b*) 273268 (*c*) 273348 (*d*) 273358

18. 360 × 17 = ? **(R.B.I. 2003)**
(*a*) 5120 (*b*) 5320 (*c*) 6120 (*d*) 6130

19. 587 × 999 = ? **(M.B.A. 1998)**
(*a*) 586413 (*b*) 587523 (*c*) 614823 (*d*) 615173

20. 469157 × 9999 = ?
(*a*) 4586970843 (*b*) 4686970743 (*c*) 4691100843 (*d*) 584649125

21. 8756 × 99999 = ?
(*a*) 796491244 (*b*) 815491244 (*c*) 875591244 (*d*) None of these

22. The value of 112 × 5^4 is : **(M.B.A. 2002)**
(*a*) 6700 (*b*) 70000 (*c*) 76500 (*d*) 77200

23. 935421 × 625 = ?
(*a*) 575648125 (*b*) 584638125 (*c*) 584649125 (*d*) 585628125

24. 12846 × 593 + 12846 × 407 = ?
(*a*) 12846000 (*b*) 14203706 (*c*) 24038606 (*d*) 24064000

25. 1014 × 986 = ?
(*a*) 998804 (*b*) 998814 (*c*) 998904 (*d*) 999804

26. 1307 × 1307 = ?
(*a*) 1601249 (*b*) 1607249 (*c*) 1701249 (*d*) 1708249

27. 1399 × 1399 = ?
(*a*) 1687401 (*b*) 1901541 (*c*) 1943211 (*d*) 1957201

28. 106 × 106 + 94 × 94 = ?
(*a*) 20032 (*b*) 20072 (*c*) 21032 (*d*) 23032

29. 217 × 217 + 183 × 183 = ? **(Hotel Management, 2002)**
(*a*) 79698 (*b*) 80578 (*c*) 80698 (*d*) 81268

30. 12345679 × 72 is equal to : **(S.S.C. 2000)**
(*a*) 88888888 (*b*) 888888888 (*c*) 898989898 (*d*) 999999998

31. What number should replace *x* in this multiplication problem ?

$$\begin{array}{r} 3x4 \\ 4 \\ \hline 1216 \end{array}$$

(Hotel Management, 2000)
(*a*) 0 (*b*) 2 (*c*) 4 (*d*) 5

32. A positive integer, which when added to 1000, gives a sum which is greater than when it is multiplied by 1000. This positive integer is : **(M.A.T. 2003)**
(*a*) 1 (*b*) 3 (*c*) 5 (*d*) 7

33. Which of the following can be a product of two 3-digit numbers **3 and **8 ?
(*a*) 1010024 (*b*) 991014 (*c*) 9124 (*d*) None of these

34. A boy multiplies 987 by a certain number and obtains 559981 as his answer. If in the answer, both 9's are wrong but the other digits are correct, then the correct answer will be : **(C.B.I. 1997)**

(*a*) 553681 (*b*) 555181 (*c*) 555681 (*d*) 556581

35. When a certain number is multiplied by 13, the product consists entirely of fives. The smallest such number is : **(M.B.A. 2002)**

(*a*) 41625 (*b*) 42135 (*c*) 42515 (*d*) 42735

36. The number of digits of the smallest number, which when multiplied by 7 gives the result consisting entirely of nines, is :

(*a*) 3 (*b*) 5 (*c*) 6 (*d*) 8

37. $-95 \div 19 = ?$ **(Hotel Management, 2000)**

(*a*) -5 (*b*) -4 (*c*) 0 (*d*) 5

38. What should come in place of * mark in the following equation ? **(B.S.R.B. 1998)**

1*5$4 ÷ 148 = 78

(*a*) 1 (*b*) 4 (*c*) 6 (*d*) 8 (*e*) None of these

39. The sum of all possible two-digit numbers formed from three different one-digit natural numbers when divided by the sum of the original three numbers is equal to :

(*a*) 18 (*b*) 22 (*c*) 36 (*d*) None of these **(C.B.I. 1997)**

40. If n is a negative number, then which of the following is the least ? **(M.B.A. 2002)**

(*a*) 0 (*b*) $-n$ (*c*) $2n$ (*d*) n^2

41. If x and y are negative, then which of the following statements is / are always true ?

I. $x + y$ is positive II. xy is positive III. $x - y$ is positive. **(M.A.T. 2004)**

(*a*) I only (*b*) II only (*c*) III only (*d*) I and III only

42. If $-1 \le x \le 2$ and $1 \le y \le 3$, then least possible value of $(2y - 3x)$ is :

(*a*) 0 (*b*) -3 (*c*) -4 (*d*) -5

43. If a and b are both odd numbers, which of the following is an even number ?

(*a*) $a + b$ (*b*) $a + b + 1$ (*c*) ab (*d*) $ab + 2$

44. Which of the following is always odd ?

(*a*) Sum of two odd numbers (*b*) Difference of two odd numbers

(*c*) Product of two odd numbers (*d*) None of these

45. For the integer n, if n^3 is odd, then which of the following statements are true ?

I. n is odd. II. n^2 is odd. III. n^2 is even. **(D.M.R.C. 2003)**

(*a*) I only (*b*) II only (*c*) I and II only (*d*) I and III only

46. The least prime number is :

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

47. What is the total number of prime numbers less than 70 ?

(*a*) 17 (*b*) 18 (*c*) 19 (*d*) 20

48. The total number of even prime numbers is :

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) None of these

49. Find the sum of prime numbers lying between 60 and 75. **(R.R.B. 2000)**

(*a*) 199 (*b*) 201 (*c*) 211 (*d*) 272

50. The smallest three-digit prime number is : **(S.S.C. 2000)**

(*a*) 103 (*b*) 107 (*c*) 109 (*d*) None of these

51. Which one of the following is a prime number ?

(*a*) 161 (*b*) 221 (*c*) 373 (*d*) 437

52. The smallest value of n, for which $2n + 1$ is not a prime number, is :

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) None of these

(Hotel Management, 1997)

53. The sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is :
(a) 7 (b) 29 (c) 41 (d) 67

54. There are four prime numbers written in ascending order. The product of the first three is 385 and that of the last three is 1001. The last number is : **(S.S.C. 2003)**
(a) 11 (b) 13 (c) 17 (d) 19

55. How many numbers between 400 and 600 begin with or end with a digit of 5 ?
(a) 40 (b) 100 (c) 110 (d) 120

56. If we write all the whole numbers from 200 to 400, then how many of these contain the digit 7 once and only once ? **(Hotel Management, 2003)**
(a) 32 (b) 34 (c) 35 (d) 36

57. The unit's digit in the product $274 \times 318 \times 577 \times 313$ is :
(a) 2 (b) 3 (c) 4 (d) 5

58. The digit in unit's place of the product $81 \times 82 \times ..... \times 89$ is :
(a) 0 (b) 2 (c) 6 (d) 8

(Section Officers', 2003)

59. If the unit digit in the product $(459 \times 46 \times 28* \times 484)$ is 2, the digit in place of * is :
(a) 3 (b) 5 (c) 7 (d) None of these

60. The unit's digit in the product $(3127)^{173}$ is :
(a) 1 (b) 3 (c) 7 (d) 9

61. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is : **(L.I.C.A.A.O. 2003)**
(a) 1 (b) 2 (c) 4 (d) 6

62. The digit in the unit's place of the number represented by $(7^{95} - 3^{58})$ is :
(a) 0 (b) 4 (c) 6 (d) 7

(A.A.O. Exam, 2003)

63. If x is an even number, then x^{4n}, where n is a positive integer, will always have :
(a) zero in the unit's place (b) 6 in the unit's place
(c) either 0 or 6 in the unit's place (d) None of these

(Hotel Management, 1997)

64. The number of prime factors of $(3 \times 5)^{12} (2 \times 7)^{10} (10)^{25}$ is :
(a) 47 (b) 60 (c) 72 (d) None of these

65. $397 \times 397 + 104 \times 104 + 2 \times 397 \times 104 = ?$
(a) 250001 (b) 251001 (c) 260101 (d) 261001

66. $186 \times 186 + 159 \times 159 - 2 \times 186 \times 159 = ?$
(a) 729 (b) 1039 (c) 2019 (d) 7029

67. $(475 + 425)^2 - 4 \times 475 \times 425$ is equal to :
(a) 2500 (b) 3160 (c) 3500 (d) 3600

68. If $(64)^2 - (36)^2 = 20z$, the value of z is :
(a) 70 (b) 120 (c) 180 (d) None of these

69. $(46)^2 - (?)^2 = 4398 - 3066$ **(B.S.R.B. 1998)**
(a) 16 (b) 28 (c) 36 (d) 42

70. $\dfrac{(856+167)^2+(856-167)^2}{856\times 856+167\times 167}$ is equal to :
(a) 1 (b) 2 (c) 689 (d) 1023

71. $\dfrac{(469+174)^2-(469-174)^2}{469\times 174}$ is equal to :
(a) 2 (b) 4 (c) 295 (d) 643

72. The sum of first 45 natural numbers is :
(*a*) 1035 (*b*) 1280 (*c*) 2070 (*d*) 2140

73. The sum of even numbers between 1 and 31 is :
(*a*) 16 (*b*) 128 (*c*) 240 (*d*) 512

74. $(51 + 52 + 53 + ... + 100)$ is equal to :
(*a*) 2525 (*b*) 2975 (*c*) 3225 (*d*) 3775

75. How many numbers between 200 and 600 are divisible by 4, 5 and 6 ?
(*a*) 5 (*b*) 6 (*c*) 7 (*d*) 8

76. How many three-digit numbers are divisible by 6 in all ?
(*a*) 149 (*b*) 150 (*c*) 151 (*d*) 166

77. If $(1^2 + 2^2 + 3^2 + ... + 10^2) = 385$, then the value of $(2^2 + 4^2 + 6^2 + ... + 20^2)$ is :
(*a*) 770 (*b*) 1155 (*c*) 1540 (*d*) (385×385)

78. The value of $(11^2 + 12^2 + 13^2 + 14^2 + ... + 20^2)$ is :
(*a*) 385 (*b*) 2485 (*c*) 2870 (*d*) 3255

79. If 1*548 is divisible by 3, which of the following digits can replace * ?
(*a*) 0 (*b*) 2 (*c*) 7 (*d*) 9
(S.S.C. 1999)

80. If the number 357*25* is divisible by both 3 and 5, then the missing digits in the unit's place and the thousandth place respectively are : **(Hotel Management, 1997)**
(*a*) 0, 6 (*b*) 5, 6 (*c*) 5, 4 (*d*) None of these

81. 5*2 is a three-digit number with * as a missing digit. If the number is divisible by 6, the missing digit is :
(*a*) 2 (*b*) 3 (*c*) 6 (*d*) 7

82. What least value must be assigned to * so that the number 63576*2 is divisible by 8 ?
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

83. What least value must be given to * so that the number 451*603 is exactly divisible by 9 ?
(*a*) 2 (*b*) 5 (*c*) 7 (*d*) 8

84. How many of the following numbers are divisible by 3 but not by 9 ?
2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276
(*a*) 5 (*b*) 6 (*c*) 7 (*d*) None of these

85. Which one of the following numbers is exactly divisible by 11 ? **(C.D.S. 2003)**
(*a*) 235641 (*b*) 245642 (*c*) 315624 (*d*) 415624

86. What least value must be assigned to * so that the number 86325*6 is divisible by 11 ?
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

87. A number 476**0 is divisible by both 3 and 11. The non-zero digits in the hundredth and tenth place respectively are :
(*a*) 7, 4 (*b*) 7, 5 (*c*) 8, 5 (*d*) None of these

88. Which of the following numbers is divisible by 3, 7, 9 and 11 ?
(*a*) 639 (*b*) 2079 (*c*) 3791 (*d*) 37911

89. The value of P, when $4864 \times 9P2$ is divisible by 12, is :
(*a*) 2 (*b*) 5 (*c*) 8 (*d*) None of these

90. Which of the following numbers is exactly divisible by 24 ? **(M.B.A. 1998)**
(*a*) 35718 (*b*) 63810 (*c*) 537804 (*d*) 3125736

91. If the number 42573* is completely divisible by 72, then which of the following numbers should replace the asterisk ?

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7

92. Which of the following numbers is exactly divisible by 99 ?

(*a*) 114345 (*b*) 135792 (*c*) 913464 (*d*) 3572404

93. The digits indicated by * and $ in 3422213*$ so that this number is divisible by 99, are respectively :

(*a*) 1, 9 (*b*) 3, 7 (*c*) 4, 6 (*d*) 5, 5

94. If x and y are the two digits of the number $653xy$ such that this number is divisible by 80, then $x + y$ is equal to :

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 6

95. How many of the following numbers are divisible by 132 ?

264, 396, 462, 792, 968, 2178, 5184, 6336 **(Hotel Management, 2002)**

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 7

96. 6897 is divisible by : **(I.A.M. 2002)**

(*a*) 11 only (*b*) 19 only

(*c*) both 11 and 19 (*d*) neither 11 nor 19

97. Which of the following numbers is exactly divisible by all prime numbers between 1 and 17 ?

(*a*) 345345 (*b*) 440440 (*c*) 510510 (*d*) 515513

98. 325325 is a six-digit number. It is divisible by : **(S.S.C. 1998)**

(*a*) 7 only (*b*) 11 only (*c*) 13 only (*d*) all 7, 11 and 13

99. The number 311311311311311311311 is : **(C.D.S. 2003)**

(*a*) divisible by 3 but not by 11 (*b*) divisible by 11 but not by 3

(*c*) divisible by both 3 and 11 (*d*) neither divisible by 3 nor by 11

100. There is one number which is formed by writing one digit 6 times (*e.g.* 111111, 444444 etc.). Such a number is always divisible by :

(*a*) 7 only (*b*) 11 only (*c*) 13 only (*d*) All of these

101. A 4-digit number is formed by repeating a 2-digit number such as 2525, 3232 etc. Any number of this form is exactly divisible by : **(S.S.C. 2000)**

(*a*) 7 (*b*) 11

(*c*) 13 (*d*) smallest 3-digit prime number

102. A six-digit number is formed by repeating a three-digit number; for example, 256256 or 678678 etc. Any number of this form is always exactly divisible by :

(*a*) 7 only (*b*) 11 only (*c*) 13 only (*d*) 1001

103. The largest natural number which exactly divides the product of any four consecutive natural numbers is :

(*a*) 6 (*b*) 12 (*c*) 24 (*d*) 120

104. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is :

(*a*) 16 (*b*) 24 (*c*) 48 (*d*) 96

105. The sum of three consecutive odd numbers is always divisible by :

I. 2 II. 3 III. 5 IV. 6

(*a*) Only I (*b*) Only II (*c*) Only I and III (*d*) Only II and IV

(Hotel Management, 2003)

106. The difference between the squares of two consecutive odd integers is always divisible by : **(M.B.A. 2003)**

(*a*) 3 (*b*) 6 (*c*) 7 (*d*) 8

107. A number is multiplied by 11 and 11 is added to the product. If the resulting number is divisible by 13, the smallest original number is :

(*a*) 12 (*b*) 22 (*c*) 26 (*d*) 53

108. The sum of the digits of a 3-digit number is subtracted from the number. The resulting number is :

(*a*) divisible by 6 (*b*) divisible by 9
(*c*) divisible neither by 6 nor by 9 (*d*) divisible by both 6 and 9

109. If x and y are positive integers such that $(3x + 7y)$ is a multiple of 11, then which of the following will also be divisible by 11 ?

(*a*) $4x + 6y$ (*b*) $x + y + 4$ (*c*) $9x + 4y$ (*d*) $4x - 9y$

110. A 3-digit number $4a3$ is added to another 3-digit number 984 to give the four-digit number $13b7$, which is divisible by 11. Then, $(a + b)$ is :

(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 15

111. The largest number that exactly divides each number of the sequence $(1^5 - 1)$, $(2^5 - 2)$, $(3^5 - 3)$, ..., $(n^5 - n)$, ... is :

(*a*) 1 (*b*) 15 (*c*) 30 (*d*) 120

112. The greatest number by which the product of three consecutive multiples of 3 is always divisible is : **(S.S.C. 2000)**

(*a*) 54 (*b*) 81 (*c*) 162 (*d*) 243

113. The smallest number to be added to 1000 so that 45 divides the sum exactly is :

(*a*) 10 (*b*) 20 (*c*) 35 (*d*) 80

114. The smallest number that must be added to 803642 in order to obtain a multiple of 11 is : **(C.B.I. 2003)**

(*a*) 1 (*b*) 4 (*c*) 7 (*d*) 9

115. Which of the following numbers should be added to 11158 to make it exactly divisible by 77 ?

(*a*) 5 (*b*) 7 (*c*) 8 (*d*) 9

116. The least number which must be subtracted from 6709 to make it exactly divisible by 9 is : **(C.B.I. 1998)**

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

117. What least number must be subtracted from 427398 so that the remaining number is divisible by 15 ? **(Bank P.O. 2000)**

(*a*) 3 (*b*) 6 (*c*) 11 (*d*) 16

118. What least number must be subtracted from 13294 so that the remainder is exactly divisible by 97 ?

(*a*) 1 (*b*) 3 (*c*) 4 (*d*) 5

119. When the sum of two numbers is multiplied by 5, the product is divisible by 15. Which one of the following pairs of numbers satisfies the above condition ?

(*a*) 240, 335 (*b*) 250, 341 (*c*) 245, 342 (*d*) None of these

(Hotel Management, 1998)

120. The least number by which 72 must be multiplied in order to produce a multiple of 112, is :

(*a*) 6 (*b*) 12 (*c*) 14 (*d*) 18

121. The number of times 99 is subtracted from 1111 so that the remainder is less than 99, is : **(S.C.R.A. 1996)**

(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 13

122. Find the number which is nearest to 457 and is exactly divisible by 11.

(*a*) 450 (*b*) 451 (*c*) 460 (*d*) 462

(Hotel Management, 2003)

123. The number nearest to 99547 which is exactly divisible by 687 is :
(*a*) 98928 (*b*) 99479 (*c*) 99615 (*d*) 100166

124. What largest number of five digits is divisible by 99 ?
(*a*) 99909 (*b*) 99981 (*c*) 99990 (*d*) 99999

125. The smallest number of five digits exactly divisible by 476 is : **(S.S.C. 2004)**
(*a*) 10000 (*b*) 10472 (*c*) 10476 (*d*) 47600

126. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is :
(*a*) 364724 (*b*) 365387 (*c*) 365737 (*d*) 366757

127. On dividing 4150 by a certain number, the quotient is 55 and the remainder is 25. The divisor is :
(*a*) 65 (*b*) 70 (*c*) 75 (*d*) 80

128. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is : **(S.S.C. 2000)**
(*a*) 1220 (*b*) 1250 (*c*) 22030 (*d*) 220030

129. A four-digit number divisible by 7 becomes divisible by 3, when 10 is added to it. The largest such number is :
(*a*) 9947 (*b*) 9987 (*c*) 9989 (*d*) 9996

130. A number when divided by 114 leaves the remainder 21. If the same number is divided by 19, then the remainder will be : **(R.R.B. 2003)**
(*a*) 1 (*b*) 2 (*c*) 7 (*d*) 21

131. A number when divided by 296 gives a remainder 75. When the same number is divided by 37, then the remainder will be : **(C.B.I. 2003)**
(*a*) 1 (*b*) 2 (*c*) 8 (*d*) 11

132. A number when divided by 119 leaves 19 as remainder. If the same number is divided by 17, the remainder obtained is : **(Section Officers', 2001)**
(*a*) 2 (*b*) 3 (*c*) 7 (*d*) 10

133. A number when divided by 899 gives a remainder 63. If the same number is divided by 29, the remainder will be :
(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 10

134. When a number is divided by 31, the remainder is 29. When the same number is divided by 16, what will be the remainder ? **(Bank P.O. 2002)**
(*a*) 11 (*b*) 13 (*c*) 15 (*d*) Data inadequate

135. When a number is divided by 13, the remainder is 11. When the same number is divided by 17, the remainder is 9. What is the number ? **(S.B.I.P.O. 1997)**
(*a*) 339 (*b*) 349 (*c*) 369 (*d*) Data inadequate

136. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is :
(*a*) 4236 (*b*) 4306 (*c*) 4336 (*d*) 5336

137. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is : **(A.A.O. Exam, 2003)**
(*a*) 240 (*b*) 270 (*c*) 295 (*d*) 360

138. In doing a division of a question with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. The correct quotient is :
(*a*) 0 (*b*) 12 (*c*) 13 (*d*) 20 **(S.S.C. 2003)**

139. When n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4 ?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 6

140. A number when divided by 6 leaves a remainder 3. When the square of the same number is divided by 6, the remainder is : **(S.S.C. 2000)**

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

141. A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4, then the respective remainders will be :

(*a*) 1, 2 (*b*) 2, 3 (*c*) 3, 2 (*d*) 4, 1

(S.S.C. 2003)

142. A number was divided successively in order by 4, 5 and 6. The remainders were respectively 2, 3 and 4. The number is : **(C.B.I. 1997)**

(*a*) 214 (*b*) 476 (*c*) 954 (*d*) 1908

143. In dividing a number by 585, a student employed the method of short division. He divided the number successively by 5, 9 and 13 (factors of 585) and got the remainders 4, 8 and 12. If he had divided the number by 585, the remainder would have been :

(*a*) 24 (*b*) 144 (*c*) 292 (*d*) 584

(N.I.F.T. 1997)

144. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6 ?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

145. $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by : **(C.B.I. 2003)**

(*a*) 3 (*b*) 10 (*c*) 11 (*d*) 13

146. If x is a whole number, then $x^2 (x^2 - 1)$ is always divisible by : **(S.S.C. 1998)**

(*a*) 12 (*b*) 24 (*c*) $12 - x$ (*d*) multiple of 12

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*b*)	**8.** (*b*)
9. (*b*)	**10.** (*c*)	**11.** (*c*)	**12.** (*d*)	**13.** (*c*)	**14.** (*e*)	**15.** (*c*)	**16.** (*c*)
17. (*a*)	**18.** (*c*)	**19.** (*a*)	**20.** (*c*)	**21.** (*c*)	**22.** (*b*)	**23.** (*b*)	**24.** (*a*)
25. (*d*)	**26.** (*d*)	**27.** (*d*)	**28.** (*b*)	**29.** (*b*)	**30.** (*b*)	**31.** (*a*)	**32.** (*a*)
33. (*b*)	**34.** (*c*)	**35.** (*d*)	**36.** (*c*)	**37.** (*a*)	**38.** (*a*)	**39.** (*b*)	**40.** (*c*)
41. (*b*)	**42.** (*c*)	**43.** (*a*)	**44.** (*c*)	**45.** (*c*)	**46.** (*c*)	**47.** (*c*)	**48.** (*b*)
49. (*d*)	**50.** (*d*)	**51.** (*c*)	**52.** (*b*)	**53.** (*d*)	**54.** (*b*)	**55.** (*c*)	**56.** (*d*)
57. (*a*)	**58.** (*a*)	**59.** (*c*)	**60.** (*c*)	**61.** (*c*)	**62.** (*b*)	**63.** (*b*)	**64.** (*d*)
65. (*b*)	**66.** (*a*)	**67.** (*a*)	**68.** (*d*)	**69.** (*b*)	**70.** (*b*)	**71.** (*b*)	**72.** (*a*)
73. (*c*)	**74.** (*d*)	**75.** (*b*)	**76.** (*b*)	**77.** (*c*)	**78.** (*b*)	**79.** (*a*)	**80.** (*b*)
81. (*a*)	**82.** (*c*)	**83.** (*d*)	**84.** (*b*)	**85.** (*d*)	**86.** (*c*)	**87.** (*c*)	**88.** (*b*)
89. (*d*)	**90.** (*d*)	**91.** (*c*)	**92.** (*a*)	**93.** (*a*)	**94.** (*a*)	**95.** (*a*)	**96.** (*c*)
97. (*c*)	**98.** (*d*)	**99.** (*d*)	**100.** (*d*)	**101.** (*d*)	**102.** (*d*)	**103.** (*c*)	**104.** (*c*)
105. (*b*)	**106.** (*d*)	**107.** (*a*)	**108.** (*b*)	**109.** (*d*)	**110.** (*a*)	**111.** (*c*)	**112.** (*c*)
113. (*c*)	**114.** (*c*)	**115.** (*b*)	**116.** (*c*)	**117.** (*a*)	**118.** (*d*)	**119.** (*b*)	**120.** (*c*)
121. (*b*)	**122.** (*d*)	**123.** (*c*)	**124.** (*c*)	**125.** (*b*)	**126.** (*c*)	**127.** (*c*)	**128.** (*d*)
129. (*c*)	**130.** (*b*)	**131.** (*a*)	**132.** (*a*)	**133.** (*c*)	**134.** (*d*)	**135.** (*b*)	**136.** (*d*)
137. (*b*)	**138.** (*d*)	**139.** (*b*)	**140.** (*d*)	**141.** (*b*)	**142.** (*a*)	**143.** (*d*)	**144.** (*c*)
145. (*b*)	**146.** (*a*)						

SOLUTIONS

1. (Local Value) − (Face Value) = (7000 − 7) = 6993.

2. (Place Value of 7) − (Place Value of 3) = (7000 − 30) = 6970.

3. Required Sum = (100000 + 99999) = 199999.

4. Required Remainder = (10000 − 999) = 9001.

5. 5978 + 6134 + 7014 = 19126.

6. 18265 + 2736 + 41328 = 62329.

7. 39798 + 3798 + 378 = 43974.

8. $9358 - 6014 + 3127 = (9358 + 3127) - 6014 = (12485 - 6014) = 6471.$

9. $9572 - 4018 - 2164 = 9572 - (4018 + 2164) = (9572 - 6182) = 3390.$

10. Let $7589 - x = 3434$. Then, $x = (7589 - 3434) = 4155$.

11. Let $9548 + 7314 = 8362 + x$. Then, $16862 = 8362 + x \Leftrightarrow x = (16862 - 8362) = 8500$.

12. Let $7845 - x = 8461 - 3569$. Then, $7845 - x = 4892 \Leftrightarrow x = (7845 - 4892) = 2953$.

13. Let $3578 + 5729 - x486 = 5821$.

Then, $9307 - x486 = 5821 \Leftrightarrow x486 = (9307 - 5821) \Leftrightarrow x486 = 3486 \Leftrightarrow x = 3$.

14. $6x43 - 46y9 = 1904 \Leftrightarrow 6x43 = 1904 + 46y9$ $[1 + y = 4 \Leftrightarrow y = 3]$

$\Leftrightarrow 6x43 = 1904 + 4639 = 6543$ $[\because y = 3]$

$\Leftrightarrow x = 5$.

15. We may represent the given sum, as shown.

$\therefore$ $1 + A + C - B = 12 \Leftrightarrow A + C - B = 11.$

Giving maximum values to A and C, *i.e.*,

A = 9 and C = 9, we get B = 7.

$$\begin{array}{rccc} & 1 & 1 & \\ & 5 & A & 9 \\ + & 9 & C & 6 \\ & 7 & B & 2 \\ \hline & 8 & 2 & 3 \\ \hline \end{array}$$

16. Let $x + (10 + x) + (20 + x) + (10x + 3) + (10x + 1) = 200 + 10 + x$.

Then, $22x = 176 \Leftrightarrow x = 8$.

17. $5358 \times 51 = 5358 \times (50 + 1) = (5358 \times 50) + (5358 \times 1) = (267900 + 5358) = 273258.$

18. $360 \times 17 = 360 \times (20 - 3) = (360 \times 20) - (360 \times 3) = (7200 - 1080) = 6120.$

19. $587 \times 999 = 587 \times (1000 - 1) = (587 \times 1000) - (587 \times 1) = (587000 - 587) = 586413.$

20. $469157 \times 9999 = 469157 \times (10000 - 1) = (469157 \times 10000) - (469157 \times 1)$

$= (4691570000 - 469157) = 4691100843.$

21. $8756 \times 99999 = 8756 \times (100000 - 1) = (8756 \times 100000) - (8756 \times 1)$

$= (875600000 - 8756) = 875591244.$

22. $(112 \times 5^4) = \dfrac{1120000}{2^4}$ (see the rule) $= \dfrac{1120000}{16} = 70000.$

23. $935421 \times 625 = 935421 \times 5^4 = \dfrac{9354210000}{2^4}$ (see the rule)

$= \dfrac{9354210000}{16} = 584638125.$

24. $12846 \times 593 + 12846 \times 407 = 12846 \times (593 + 407) = 12846 \times 1000 = 12846000.$

25. $(1014 \times 986) = (1000 + 14) \times (1000 - 14) = (1000)^2 - (14)^2 = 1000000 - 196 = 999804.$

26. $(1307 \times 1307) = (1307)^2 = (1300 + 7)^2 = (1690000 + 49 + 18200) = 1708249.$

27. $(1399 \times 1399) = (1399)^2 = (1400 - 1)^2 = (1400)^2 + 1^2 - 2 \times 1400 \times 1$

$= 1960000 + 1 - 2800 = 1960001 - 2800 = 1957201.$

28. $(106 \times 106 + 94 \times 94) = \frac{1}{2} \times 2(a^2 + b^2) = \frac{1}{2}[(a+b)^2 + (a-b)^2]$

$= \frac{1}{2} \cdot [(106 + 94)^2 + (106 - 94)^2] = \frac{1}{2} \cdot [(200)^2 + (12)^2]$

$= \frac{1}{2}(40000 + 144) = \frac{1}{2}(40144) = 20072.$

29. $(217 \times 217 + 183 \times 183) = \frac{1}{2} \times 2(a^2 + b^2) = \frac{1}{2} \cdot [(a+b)^2 + (a-b)^2]$

$= \frac{1}{2} \cdot [(217 + 183)^2 + (217 - 183)^2] = \frac{1}{2}[(400)^2 + (34)^2]$

$= \frac{1}{2}(160000 + 1156) = \frac{161156}{2} = 80578.$

30. $12345679 \times 72 = 12345679 \times (100 - 28) = 1234567900 - (12345679 \times 28)$

$= 1234567900 - [12345679 \times (30 - 2)]$

$= 1234567900 - 370370370 + 24691358 = 888888888.$

31. $(300 + 10x + 4) \times 4 = 1200 + 40x + 16 = (12 \times 100) + (4x + 1) \times 10 + 6$

$\therefore \; 4x + 1 = 1 \;\Leftrightarrow\; 4x = 0 \;\Leftrightarrow\; x = 0.$

32. $(1000 + N) > (1000N)$. Clearly, $N = 1$.

33. When two 3-digit numbers are multiplied, the product must contain 5 or 6 digits. So, the required number is 991014.

34. $987 = 3 \times 7 \times 47.$

So, required number must be divisible by each one of 3, 7, 47.

None of the numbers in (*a*) and (*b*) are divisible by 3, while (*d*) is not divisible by 7.

$\therefore$ Correct answer is (*c*).

35. By hit and trial, we find that a number exactly divisible by 13 and consisting entirely of fives is 555555.

On dividing 555555 by 13, we get 42735 as quotient.

$\therefore$ Required number = 42735.

36. By hit and trial, we find that a number exactly divisible by 7 and consisting entirely of nines is 999999. Number of digits in it = 6.

37. $\frac{-95}{19} = -5.$

38. Let $\frac{x}{148} = 78$. Then, $x = (148 \times 78) = 11544.$

$\therefore$ Required digit = 1.

39. Let the one-digit numbers be x, y, z.

Sum of all possible 2-digit numbers

$= (10x + y) + (10x + z) + (10y + x) + (10y + z) + (10z + x) + (10z + y) = 22(x + y + z)$

$\therefore$ Sum of all possible 2-digit numbers when divided by sum of one-digit numbers gives 22.

40. $n < 0 \;\Rightarrow\; 2n < 0, -n > 0$ and $n^2 > 0$.

$\therefore$ Least of $2n$, 0, $-n$ and n^2 is $2n$.

41. $x < 0, y < 0 \;\Rightarrow (x + y) < 0, xy > 0$ and $x - y$ may be +ve or −ve.

$\therefore$ II is always true.

42. $\left.\begin{array}{l} y \geq 1 \;\Rightarrow\; 2y \geq 2 \\ x \leq 2 \;\Rightarrow\; -3x \geq -6 \end{array}\right\} \Rightarrow (2y - 3x) \geq -4.$

43. Sum of two odd numbers is always even.

44. Product of two odd numbers is always odd.

45. n^3 is odd $\Rightarrow$ n is odd and n^2 is odd.

46. The least prime number is 2.

47. Prime numbers less than 70 are :
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61 and 67.
Their number is 19.

48. There is only one even prime number, namely 2.

49. Required sum $= (61 + 67 + 71 + 73) = 272$.

50. 100 is divisible by 2, so it is not prime.
101 is not divisible by any of the numbers 2, 3, 5, 7. So, it is prime.
Hence, the smallest 3-digit prime number is 101.

51. 161 is divisible by 7. So, it is not prime. 221 is divisible by 13. So, it is not prime.
Now, $20 > \sqrt{373}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
And, 373 is not divisible by any of them. So, 373 is prime.
Since 437 is divisible by 19, so it is not prime.

52. $(2 \times 1 + 1) = 3$, $(2 \times 2 + 1) = 5$, $(2 \times 3 + 1) = 7$, $(2 \times 4 + 1) = 9$, which is not prime.
$\therefore$ $n = 4$.

53. $x + (x + 36) + y = 100 \Leftrightarrow 2x + y = 64$.
$\therefore$ y must be even prime, which is 2.
$\therefore$ $2x + 2 = 64 \Rightarrow x = 31$.
Third prime number $= (x + 36) = (31 + 36) = 67$.

54. Let the given prime numbers be a, b, c, d. Then, $abc = 385$ and $bcd = 1001$.
$\therefore$ $\dfrac{abc}{bcd} = \dfrac{385}{1001} \Leftrightarrow \dfrac{a}{d} = \dfrac{5}{13}$. So, $a = 5$, $d = 13$.

55. Numbers satisfying the given conditions are 405, 415, 425, 435, 445, 455, 465, 475, 485, 495 and 500 to 599.
Number of such numbers $= (10 + 100) = 110$.

56. Required numbers from 200 to 300 are 207, 217, 227, 237, 247, 257, 267, 270, 271, 272, 273, 274, 275, 276, 278, 279, 287, 297. Their number is 18.
Similarly, such numbers between 300 and 400 are also 18 in number.
$\therefore$ Total number of such numbers = 36.

57. Required digit = Unit digit in $(4 \times 8 \times 7 \times 3) = 2$.

58. Required digit = Unit digit in $(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9) = 0$.

59. $(9 \times 6 \times 4) = 216$. In order to obtain 2 at the unit place, we must multiply 216 by 2 or 7.
$\therefore$ Of the given numbers, we have 7.

60. Unit digit in $(3127)^{173}$ = Unit digit in $(7)^{173}$. Now, 7^4 gives unit digit 1.
$\therefore$ $(7)^{173} = (7^4)^{43} \times 7^1$. Thus, $(7)^{173}$ gives unit digit 7.

61. Unit digit in 7^4 is 1.
$\therefore$ Unit digit in 7^{68} is 1.
Unit digit in 7^{71} is 3. [$1 \times 7 \times 7 \times 7$ gives unit digit 3]
Again, every power of 6 will give unit digit 6.
$\therefore$ Unit digit in 6^{59} is 6.
Unit digit in 3^4 is 1.
$\therefore$ Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.
$\therefore$ Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$ = Unit digit in $(3 \times 6 \times 3) = 4$.

62. Unit digit in 7^4 is 1. So, unit digit in 7^{92} is 1.

$\therefore$ Unit digit in 7^{95} is 3. [$\because$ Unit digit in $1 \times 7 \times 7 \times 7$ is 3]

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{56} is 1.

$\therefore$ Unit digit in 3^{58} is 9.

$\therefore$ Unit digit in $(7^{95} - 3^{58}) = (13 - 9) = 4$.

63. $x^{4n} = (2^4)^n$ or $(4^4)^n$ or $(6^4)^n$ or $(8^4)n$.

Clearly, the unit digit in each case is 6.

64. $(3 \times 5)^{12} \times (2 \times 7)^{10} \times (10)^{25} = (3 \times 5)^{12} \times (2 \times 7)^{10} \times (2 \times 5)^{25}$

$= 3^{12} \times 5^{12} \times 2^{10} \times 7^{10} \times 2^{25} \times 5^{25} = 2^{35} \times 3^{12} \times 5^{37} \times 7^{10}$.

Total number of prime factors $= (35 + 12 + 37 + 10) = 94$.

65. Given Exp. $= a^2 + b^2 + 2ab$, where $a = 397$ and $b = 104$

$= (a+b)^2 = (397+104)^2 = (501)^2 = (500+1)^2 = (500)^2 + 1^2 + 2 \times 500 \times 1$

$= 250000 + 1 + 1000 = 251001$.

66. Given Exp. $= a^2 + b^2 - 2ab$, where $a = 186$ and $b = 159$

$= (a-b)^2 = (186-159)^2 = (27)^2$

$= (20+7)^2 = (20)^2 + 7^2 + 2 \times 20 \times 7 = 400 + 49 + 280 = 729$.

67. Given Exp. $= (a+b)^2 - 4ab$, where $a = 475$ and $b = 425$

$= (a-b)^2 = (475-425)^2 = (50)^2 = 2500$.

68. $20z = (64)^2 - (36)^2 \Leftrightarrow 20z = (64 + 36)(64 - 36)$

$\Leftrightarrow 20z = 100 \times 28 \Leftrightarrow z = \frac{100 \times 28}{20} = 140$.

69. Let $(46)^2 - x^2 = 4398 - 3066$.

Then, $(46)^2 - x^2 = 1332 \Leftrightarrow x^2 = (46)^2 - 1332 = (2116 - 1332)$

$\Leftrightarrow x^2 = 784 \Leftrightarrow x = \sqrt{784} = 28$.

70. Given Exp. $= \frac{(a+b)^2 + (a-b)^2}{(a^2+b^2)} = \frac{2(a^2+b^2)}{(a^2+b^2)} = 2$.

71. Given Exp. $= \frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4$.

72. We know that : $(1+2+3+\ldots\ldots+n) = \frac{n(n+1)}{2}$.

$\therefore \quad (1+2+3+\ldots\ldots+45) = \left(\frac{45 \times 46}{2}\right) = 1035$.

73. Required numbers are 2, 4, 6, ..., 30.

This is an A.P. containing 15 terms.

$\therefore$ Required sum $= \frac{n}{2}(\text{first term} + \text{last term}) = \frac{15}{2}(2+30) = 240$.

74. $(51 + 52 + 53 + \ldots\ldots + 100)$

$= (1 + 2 = 3 + \ldots\ldots + 100) - (1 + 2 + 3 + \ldots\ldots + 50)$

$= \left(\frac{100 \times 101}{2} - \frac{50 \times 51}{2}\right) = (5050 - 1275) = 3775$.

75. Every such number must be divisible by L.C.M. of 4, 5, 6, *i.e.* 60.
Such numbers are 240, 300, 360, 420, 480, 540.
Clearly, there are 6 such numbers.

76. Required numbers are 102, 108, 114, ..., 996.
This is an A.P. with $a = 102$ and $d = 6$.
Let the number of its terms be n. Then,
$a + (n - 1)\,d = 996 \Leftrightarrow 102 + (n - 1) \times 6 = 996 \Leftrightarrow n = 150.$

77. $2^2 + 4^2 + ... + 20^2 = (1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 + ... + (2 \times 10)^2$
$= 2^2 \times 1^2 + 2^2 \times 2^2 + 2^2 \times 3^2 + ... + 2^2 \times 10^2$
$= 2^2\,[1^2 + 2^2 + 3^2 + ... + 10^2]$
$= 4 \times \dfrac{10 \times 11 \times 21}{6} = 4 \times 385 = 1540.$

78. $11^2 + 12^2 + 13^2 + ... + 20^2$
$= (1^2 + 2^2 + 3^2 + ... + 20^2) - (1^2 + 2^2 + 3^2 + ... + 10^2)$
$= \left[\dfrac{20\,(20+1)\,(40+1)}{6} - \dfrac{10\,(10+1)\,(20+1)}{6}\right] = 2485.$

79. $1 + x + 5 + 4 + 8 = (18 + x)$. Clearly, when $x = 0$, then sum of digits is divisible by 3.

80. Let the required number be $357y25x$.
Then, for divisibility by 5, we must have $x = 0$ or $x = 5$.
Case I. When $x = 0$.
Then, sum of digits $= (22 + y)$. For divisibility by 3, $(22 + y)$ must be divisible by 3.
$\therefore$ $y = 2$ or 5 or 8.
$\therefore$ Numbers are (0, 2) or (0, 5) or (0, 8).
Case II. When $x = 5$.
Then, sum of digits $= (27 + y)$. For divisibility by 3, we must have $y = 0$ or 3 or 6 or 9.
$\therefore$ Numbers are (5, 0) or (5, 3) or (5, 6) or (5, 9).
So, correct answer is (*b*).

81. Let the number be $5x2$. Clearly, it is divisible by 2.
Now, $5 + x + 2 = (7 + x)$ must be divisible by 3. So, $x = 2$.

82. The given number is divisible by 8, if the number $6x2$ is divisible by 8.
Clearly, the least value of x is 3.

83. $(4 + 5 + 1 + x + 6 + 0 + 3) = 19 + x$. Clearly, $x = 8$.

84. Taking the sum of the digits, we have :
$S_1 = 9,\ S_2 = 12,\ S_3 = 18,\ S_4 = 9,\ S_5 = 21,\ S_6 = 12,\ S_7 = 18,\ S_8 = 21,\ S_9 = 15,\ S_{10} = 24.$
Clearly, $S_2, S_5, S_6, S_8, S_9, S_{10}$ are all divisible by 3 but not by 9.
So, the number of required numbers = 6.

85. (*a*) $(1 + 6 + 3) - (2 + 5 + 4) = 1$ (No) (*b*) $(2 + 6 + 4) - (4 + 5 + 2) = 1$ (No)
(*c*) $(4 + 6 + 1) - (2 + 5 + 3) = 1$ (No) (*d*) $(4 + 6 + 1) - (2 + 5 + 4) = 0$ (Yes).

86. $(6 + 5 + 3 + 8) - (x + 2 + 6) = (14 - x)$. Now, $(14 - x)$ is divisible by 11, when $x = 3$.

87. $(4 + 7 + 6 + x + y + 0) = [17 + (x + y)]$. Also, $(0 + x + 7) - (y + 6 + 4) = (x - y - 3)$.
Now, $[17 + (x + y)]$ must be divisible by 3 and $(x - y - 3)$ is either 0 or divisible by 11.
Clearly, $x = 8$ and $y = 5$ satisfy both the conditions.

88. (*a*) 639 is not divisible by 7. (*b*) 2079 is divisible by 3, 7, 9 and 11.
(*c*) 3791 is not divisible by 3. (*d*) 37911 is not divisible by 9.
$\therefore$ Correct answer is (*b*).

89. Since 4864 is divisible by 4, so 9P2 must be divisible by 3.

$\therefore$ (11 + P) must be divisible by 3.

$\therefore$ Least value of P is 1.

90. The required number should be divisible by 3 and 8.

(*a*) 718 is not divisible by 8. (*b*) 810 is not divisible by 8.

(*c*) 804 is not divisible by 8.

(*d*) Sum of digits = 27, which is divisible by 3.

And, 736 is divisible by 8. So, given number is divisible by 3 and 8.

91. The given number should be divisible by both 9 and 8.

$\therefore$ $(4 + 2 + 5 + 7 + 3 + x) = (21 + x)$ is divisible by 9 and $(73x)$ is divisible by 8.

$\therefore$ $x = 6$.

92. The required number should be divisible by both 9 and 11.

Clearly, 114345 is divisible by both 9 and 11. So, it is divisible by 99.

93. The given number will be divisible by 99 if it is divisible by both 9 and 11.

Now, $(3 + 4 + 2 + 2 + 2 + 1 + 3 + x + y) = 17 + (x + y)$ must be divisible by 9.

Also, $(y + 3 + 2 + 2 + 3) - (x + 1 + 2 + 4) = (y - x + 3)$ must be 0 or divisible by 11.

$\therefore$ $x + y = 10$ and $y - x + 3 = 0$.

Clearly, $x = 1$, $y = 9$ satisfy both these equations.

94. Since $653xy$ is divisible by 5 as well as 2, so $y = 0$.

Now, $653x0$ must be divisible by 8.

So, $3x0$ must be divisible by 8. This happens when $x = 2$.

$\therefore$ $x + y = (2 + 0) = 2$.

95. A number is divisible by 132, if it is divisible by each one of 11, 3 and 4.

Clearly, 968 is not divisible by 3. None of 462 and 2178 is divisible by 4.

Also, 5184 is not divisible by 11.

Each one of remaining 4 is divisible by each one of 11, 3 and 4 and therefore, by 132.

96. Clearly, 6897 is divisible by both 11 and 19.

97. None of the numbers in (*a*) and (*c*) is divisible by 2.

Number in (*b*) is not divisible by 3.

Clearly, 510510 is divisible by each prime number between 1 and 17.

98. Clearly, 325325 is divisible by all 7, 11 and 13.

99. Sum of digits = 35 and so it is not divisible by 3.

(Sum of digits at odd places) − (Sum of digits at even places) = (19 − 16) = 3, not divisible by 11.

So, the given number is neither divisible by 3 nor by 11.

100. Since 111111 is divisible by each one of 7, 11 and 13, so each one of given type of numbers is divisible by each one of 7, 11, 13, as we may write, $222222 = 2 \times 111111$, $333333 = 3 \times 111111$, etc.

101. Smallest 3-digit prime number is 101. Clearly, $2525 = 25 \times 101$; $3232 = 32 \times 101$, etc.

$\therefore$ Each such number is divisible by 101.

102. $256256 = 256 \times 1001$; $678678 = 678 \times 1001$, etc.

So, any number of this form is divisible by 1001.

103. Required number $= 1 \times 2 \times 3 \times 4 = 24$.

104. Required number $= (2 \times 4 \times 6) = 48$.

105. Let the three consecutive odd numbers be $(2x + 1)$, $(2x + 3)$ and $(2x + 5)$.

Their sum $= (6x + 9) = 3\,(2x + 3)$, which is always divisible by 3.

106. Let the two consecutive odd integers be $(2x + 1)$ and $(2x + 3)$.
Then, $(2x+3)^2 - (2x+1)^2 = (2x+3+2x+1)(2x+3-2x-1) = (4x+4) \times 2$
$= 8(x+1)$, which is always divisible by 8.

107. Let the required number be x.
Then, $(11x + 11) = 11(x+1)$ is divisible by 13. So, $x = 12$.

108. Let the 3-digit number be xyz. Then,
$(100x + 10y + z) - (x + y + z) = 99x + 9y = 9(11x + y)$, which is divisible by 9.

109. Putting $x = 5$ and $y = 1$, we get $(3x + 7y) = (3 \times 5 + 7 \times 1) = 22$, which is divisible by 11.
$\therefore$ $4x + 5y = (4 \times 5 + 5 \times 1) = 25$, which is not divisible by 11.
$x + y + 4 = (5 + 1 + 4) = 9$, which is not divisible by 11.
$9x + 4y = (9 \times 5 + 4 \times 1) = 49$, which is not divisible by 11.
$4x - 9y = (4 \times 5 - 9 \times 1) = 11$, which is divisible by 11.

110. $\left.\begin{array}{r} 4\,a\,3 \\ 9\,8\,4 \\ \hline 13\,b\,7 \end{array}\right\} \Rightarrow a + 8 = b \Rightarrow b - a = 8$
Also, $13b7$ is divisible by 11.
$\therefore$ $(7 + 3) - (b + 1) = (9 - b) \Rightarrow (9 - b) = 0 \Rightarrow b = 9$.
$\therefore$ $b = 9$ and $a = 1 \Rightarrow (a + b) = 10$.

111. Required number $= (2^5 - 2) = (32 - 2) = 30$.

112. Required number = Product of first three multiples of 3 $= (3 \times 6 \times 9) = 162$.

113. On dividing 1000 by 45, we get remainder = 10.
$\therefore$ Required number to be added $= (45 - 10) = 35$.

114. On dividing 803642 by 11, we get remainder = 4.
$\therefore$ Required number to be added $= (11 - 4) = 7$.

115. On dividing 11158 by 77, we get remainder = 70.
$\therefore$ Required number to be added $= (77 - 70) = 7$.

116. On dividing 6709 by 9, we get remainder = 4.
$\therefore$ Required number to be subtracted = 4.

117. On dividing 427398 by 15, we get remainder = 3.
$\therefore$ Required number to be subtracted = 3.

118. On dividing 13294 by 97, we get remainder = 5.
$\therefore$ Required number to be subtracted = 5.

119. Clearly, $5 \times$ (sum of numbers) is divisible by 15.
$\therefore$ Sum of numbers must be divisible by 3.
Now, $(250 + 341) = 591$ is divisible by 3. So, required pair is 250, 341.

120. Required number is divisible by 72 as well as by 112, if it is divisible by their LCM, which is 1008.
Now, 1008 when divided by 72, gives quotient = 14.
$\therefore$ Required number = 14.

121. Let it be n times. Then, $(1111 - 99n) < 99$.
By hit and trial, we find that $n = 11$.

122. On dividing 457 by 11, remainder is 6.
$\therefore$ Required number is either 451 or 462. Nearest to 456 is 462.

123. On dividing 99547 by 687, the remainder is 619, which is more than half of 687. So, we must add $(687 - 619) = 68$ to the given number.
$\therefore$ Required number $= (99547 + 68) = 99615$.

124. Largest number of 5 digits = 99999. On dividing 99999 by 99, we get 9 as remainder.
$\therefore$ Required number $= (99999 - 9) = 99990$.

125. Smallest number of 5 digits = 10000.
On dividing 10000 by 476, we get remainder = 4.
$\therefore$ Required number $= [10000 + (476 - 4)] = 10472$.

126. Required number $= 999 \times 366 + 103 = (1000 - 1) \times 366 + 103 = 366000 - 366 + 103$
$= 365737$.

127. $4150 = 55 \times x + 25 \Leftrightarrow 55x = 4125 \Leftrightarrow x = \frac{4125}{55} = 75$.

128. Required number $= (555 + 445) \times 2 \times 110 + 30 = 220000 + 30 = 220030$.

129. Largest number of 4 digits = 9999. On dividing 9999 by 7, we get remainder = 3.
Largest number of 4 digits divisible by 7 is $(9999 - 3) = 9996$.
Let $(9996 - x + 10)$ be divisible by 3. By hit and trial, we find that $x = 7$.
$\therefore$ Required number $= (9996 - 7) = 9989$.

130. Number $= (114 \times Q) + 21 = 19 \times 6 \times Q + 19 + 2 = 19 \times (6Q + 1) + 2$.
$\therefore$ Required remainder = 2.

131. Number $= (296 \times Q) + 75 = (37 \times 8Q) + (37 \times 2) + 1 = 37 \times (8Q + 2) + 1$.
$\therefore$ Required remainder = 1.

132. Number $= (119 \times Q) + 19 = 17 \times (7Q) + (17 + 2) = 17 \times (7Q + 1) + 2$.
$\therefore$ Required remainder = 2.

133. Number $= (899 \times Q) + 63 = (29 \times 31 \times Q) + (29 \times 2) + 5 = 29 \times (31Q + 2) + 5$.
$\therefore$ Required remainder = 5.

134. Number $= (31 \times Q) + 29$. Given data is inadequate.

135. Given number $= 13p + 11$. And, Given number $= 17q + 9$.
$\therefore$ $13p + 11 = 17q + 9 \Leftrightarrow 17q - 13p = 2$.
By hit and trial, we find that $p = 26$ and $q = 20$.
$\therefore$ Required number $= (13 \times 26 + 11) = 349$.

136. Divisor $= (5 \times 46) = 230$. Also, $10 \times Q = 230 \Rightarrow Q = 23$. And, $R = 46$.
$\therefore$ Dividend $= (230 \times 23 + 46) = 5336$.

137. Let the smaller number be x. Then, larger number $= (1365 + x)$.
$\therefore$ $1365 + x = 6x + 15 \Leftrightarrow 5x = 1350 \Leftrightarrow x = 270$.
Hence, the required number is 270.

138. Dividend $= (12 \times 35) = 420$. Now, dividend = 420 and divisor = 21.
$\therefore$ Correct quotient $= \frac{420}{21} = 20$.

139. Let $n = 4q + 3 \Rightarrow 2n = 8q + 6 = (8q + 4) + 2 \Rightarrow 2n = 4(2q + 1) + 2$.
So, when $2n$ is divided by 4, remainder = 2.

140. Let $x = 6q + 3$. Then, $x^2 = (6q + 3)^2 = 36q^2 + 36q + 9 = 6(6q^2 + 6q + 1) + 3$.
So, when x^2 is divided by 6, remainder = 3.

141.

4	x
5	$y - 1$
	$1 - 4$

$\therefore$ $y = (5 \times 1 + 4) = 9$
$\therefore$ $x = (4y + 1) = (4 \times 9 + 1) = 37$.

Now, 37 when divided successively by 5 and 4, we get :

$$\begin{array}{r|l} 5 & 37 \\ \hline 4 & 7-2 \\ \hline & 1-3 \end{array}$$

$\therefore$ Respective remainders are 2, 3.

142.
$$\begin{array}{r|l} 4 & x \\ \hline 5 & y-2 \\ \hline 6 & z-3 \\ \hline & 1-4 \end{array}$$

$\therefore$

143.
$$\begin{array}{r|l} 5 & x \\ \hline 9 & y-4 \\ \hline 13 & z-8 \\ \hline & 1-12 \end{array}$$

$\therefore$

Now, 1169 when divided by 585 gives remainder = 584.

144. Let $n = 3q + 1$ and let $q = 2p + 1$. Then, $n = 3\,(2p + 1) + 1 = 6p + 4$.

$\therefore$ The number when divided by 6, we get remainder = 4.

145. $4^{61} + 4^{62} + 4^{63} + 4^{64} = 4^{61}\,(1 + 4 + 4^2 + 4^3) = 4^{61} \times 85 = 4^{60} \times 340$, which is clearly divisible by 10.

146. Putting $x = 2$, we get $2^2\,(2^2 - 1) = 12$. So, $x^2\,(x^2 - 1)$ is always divisible by 12.

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Factors and Multiples** : If a number a divides another number b exactly, we say that a is a ***factor*** of b. In this case, b is called a ***multiple*** of a.

II. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.)** : The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers :

1. ***Factorization Method*** : *Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.*
2. ***Division Method*** : *Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.*

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. **Least Common Multiple (L.C.M.)** : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. ***Factorization Method of Finding L.C.M.*** : *Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.*
2. ***Common Division Method (Short-cut Method) of Finding L.C.M.*** : *Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.*

IV. **Product of two numbers = Product of their H.C.F. and L.C.M.**

V. **Co-primes** : Two numbers are said to be co-primes if their H.C.F. is 1.

VI. **H.C.F. and L.C.M. of Fractions** :

1. $\text{H.C.F.} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2. $\text{L.C.M.} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$.

VII. **H.C.F. and L.C.M. of Decimal Fractions** : *In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.*

VIII. **Comparison of Fractions** : *Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.*

SOLVED EXAMPLES

Ex. 1. ***Find the H.C.F. of*** $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^3$, $2^3 \times 5^3 \times 7^2$.

Sol. The prime numbers common to given numbers are 2, 5 and 7.

$\therefore$ H.C.F. $= 2^2 \times 5 \times 7^2 = 980$.

Ex. 2. ***Find the H.C.F. of 108, 288 and 360.***

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$.

$\therefore$ H.C.F. $= 2^2 \times 3^2 = 36$.

Ex. 3. ***Find the H.C.F. of 513, 1134 and 1215.***

Sol.

```
1134 ) 1215 ( 1
       1134
       ----
         81 ) 1134 ( 14
               81
              ----
               324
               324
              ----
                ×
```

$\therefore$ H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

```
81 ) 513 ( 6
     486
     ---
      27 ) 81 ( 3
           81
           --
            ×
```

$\therefore$ H.C.F. of given numbers = 27.

Ex. 4. ***Reduce*** $\dfrac{391}{667}$ ***to lowest terms.***

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}.$$

Ex. 5. ***Find the L.C.M. of*** $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$.

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 $= 2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$.

Ex. 6. ***Find the L.C.M. of 72, 108 and 2100.***

Sol. $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$.

$\therefore$ L.C.M. $= 2^3 \times 3^3 \times 5^2 \times 7 = 37800$.

Ex. 7. ***Find the L.C.M. of 16, 24, 36 and 54.***

Sol.

2	16 – 24 – 36 – 54
2	8 – 12 – 18 – 27
2	4 – 6 – 9 – 27
3	2 – 3 – 9 – 27
3	2 – 1 – 3 – 9
	2 – 1 – 1 – 3

$\therefore$ L.C.M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432$.

Ex. 8. ***Find the H.C.F. and L.C.M. of*** $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ ***and*** $\frac{10}{27}$.

Sol. H.C.F. of given fractions $= \dfrac{\text{H.C.F. of } 2, 8, 16, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \dfrac{2}{81}$.

L.C.M. of given fractions $= \dfrac{\text{L.C.M. of } 2, 8, 16, 10}{\text{H.C.F. of } 3, 9, 81, 27} = \dfrac{80}{3}$.

Ex. 9. ***Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.***

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

$\therefore$ H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

$\therefore$ L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. ***Two numbers are in the ratio of 15 : 11. If their H.C.F. is 13, find the numbers.***

Sol. Let the required numbers be $15x$ and $11x$.

Then, their H.C.F. is x. So, $x = 13$.

$\therefore$ The numbers are $(15 \times 13$ and $11 \times 13)$ *i.e.*, 195 and 143.

Ex. 11. ***The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.***

Sol. Other number $= \left(\dfrac{11 \times 693}{77}\right) = 99$.

Ex. 12. ***Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.***

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

$\therefore$ H.C.F. $= 3^2 \times 5 = 45$.

Hence, required length = 45 cm.

Ex. 13. ***Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.***

Sol. Required number = H.C.F. of $(1657 - 6)$ and $(2037 - 5)$ = H.C.F. of 1651 and 2032

$$
\begin{array}{l}
1651 \,)\, 2032 \,(\, 1 \\
\quad\;\;\; 1651 \\
\hline
\quad\;\;\; 381 \,)\, 1651 \,(\, 4 \\
\qquad\qquad 1524 \\
\hline
\qquad\qquad 127 \,)\, 381 \,(\, 3 \\
\qquad\qquad\qquad\; 381 \\
\hline
\qquad\qquad\qquad\;\; \times
\end{array}
$$

$\therefore$ Required number = 127.

Ex. 14. ***Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.***

Sol. Required number = H.C.F. of $(132 - 62)$, $(237 - 132)$ and $(237 - 62)$

= H.C.F. of 70, 105 and 175 = 35.

Ex. 15. ***Find the least number exactly divisible by 12, 15, 20 and 27.***

Sol. Required number = L.C.M. of 12, 15, 20, 27.

$$\begin{array}{r|l} 3 & 12 - 15 - 20 - 27 \\ \hline 4 & 4 - 5 - 20 - 9 \\ \hline 5 & 1 - 5 - 5 - 9 \\ \hline & 1 - 1 - 1 - 9 \end{array}$$

$\therefore$ L.C.M. = $3 \times 4 \times 5 \times 9 = 540$.

Hence, required number = 540.

Ex. 16. ***Find the least number which when divided by 6, 7, 8, 9 and 12 leaves the same remainder 1 in each case.***

Sol. Required number = (L.C.M. of 6, 7, 8, 9, 12) + 1

$$\begin{array}{r|l} 3 & 6 - 7 - 8 - 9 - 12 \\ \hline 2 & 2 - 7 - 8 - 3 - 4 \\ \hline 2 & 1 - 7 - 4 - 3 - 2 \\ \hline & 1 - 7 - 2 - 3 - 1 \end{array}$$

$\therefore$ L.C.M. = $3 \times 2 \times 2 \times 7 \times 2 \times 3 = 504$.

Hence, required number = $(504 + 1) = 505$.

Ex. 17. ***Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.***

Sol. The largest number of four digits is 9999.

Required number must be divisible by L.C.M. of 12, 15, 18, 27 *i.e.*, 540.

On dividing 9999 by 540, we get 279 as remainder.

$\therefore$ Required number = $(9999 - 279) = 9720$.

Ex. 18. ***Find the smallest number of five digits exactly divisible by 16, 24, 36 and 54.***

Sol. Smallest number of five digits is 10000.

Required number must be divisible by L.C.M. of 16, 24, 36, 54 *i.e.*, 432.

On dividing 10000 by 432, we get 64 as remainder.

$\therefore$ Required number = $10000 + (432 - 64) = 10368$.

Ex. 19. ***Find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.***

Sol. Here, $(20 - 14) = 6$, $(25 - 19) = 6$, $(35 - 29) = 6$ and $(40 - 34) = 6$.

$\therefore$ Required number = (L.C.M. of 20, 25, 35, 40) − 6 = 1394.

Ex. 20. ***Find the least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder.***

Sol. L.C.M. of 5, 6, 7, 8 = 840.

$\therefore$ Required number is of the form $840k + 3$.

Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.

$\therefore$ Required number = $(840 \times 2 + 3) = 1683$.

Ex. 21. ***The traffic lights at three different road crossings change after every 48 sec., 72 sec. and 108 sec. respectively. If they all change simultaneously at 8 : 20 : 00 hours, then at what time will they again change simultaneously ?***

Sol. Interval of change = (L.C.M. of 48, 72, 108) sec. = 432 sec.

So, the lights will again change simultaneously after every 432 seconds *i.e.*, 7 min. 12 sec.

Hence, next simultaneous change will take place at 8 : 27 : 12 hrs.

Ex. 22. ***Arrange the fractions*** $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ ***in the ascending order.***

Sol. L.C.M. of 18, 36, 45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}$.

Since, $155 < 170 < 172 < 177$, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$.

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$.

EXERCISE 2

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. 252 can be expressed as a product of primes as : (IGNOU, 2002)
(a) $2 \times 2 \times 3 \times 3 \times 7$ (b) $2 \times 2 \times 2 \times 3 \times 7$
(c) $3 \times 3 \times 3 \times 3 \times 7$ (d) $2 \times 3 \times 3 \times 3 \times 7$

2. Which of the following has most number of divisors ? (M.B.A. 2002)
(a) 99 (b) 101 (c) 176 (d) 182

3. A number n is said to be perfect if the sum of all its divisors (excluding n itself) is equal to n. An example of perfect number is :
(a) 6 (b) 9 (c) 15 (d) 21

4. $\frac{1095}{1168}$ when expressed in simplest form is : (M.B.A. 1998)
(a) $\frac{13}{16}$ (b) $\frac{15}{16}$ (c) $\frac{17}{26}$ (d) $\frac{25}{26}$

5. Reduce $\frac{128352}{238368}$ to its lowest terms. (IGNOU, 2003)
(a) $\frac{3}{4}$ (b) $\frac{5}{13}$ (c) $\frac{7}{13}$ (d) $\frac{9}{13}$

6. The H.C.F. of $2^2 \times 3^3 \times 5^5$, $2^3 \times 3^2 \times 5^2 \times 7$ and $2^4 \times 3^4 \times 5 \times 7^2 \times 11$ is :
(a) $2^2 \times 3^2 \times 5$ (b) $2^2 \times 3^2 \times 5 \times 7 \times 11$
(c) $2^4 \times 3^4 \times 5^5$ (d) $2^4 \times 3^4 \times 5^5 \times 7 \times 11$

7. The H.C.F. of $2^4 \times 3^2 \times 5^3 \times 7$, $2^3 \times 3^3 \times 5^2 \times 7^2$ and $3 \times 5 \times 7 \times 11$ is :
(a) 105 (b) 1155 (c) 2310 (d) 27720

8. H.C.F. of $4 \times 27 \times 3125$, $8 \times 9 \times 25 \times 7$ & $16 \times 81 \times 5 \times 11 \times 49$ is : (C.B.I. 1997)
(a) 180 (b) 360 (c) 540 (d) 1260

9. Find the highest common factor of 36 and 84. (R.R.B. 2003)
(a) 4 (b) 6 (c) 12 (d) 18

10. The H.C.F. of 204, 1190 and 1445 is :
(a) 17 (b) 18 (c) 19 (d) 21

11. Which of the following is a pair of co-primes ?
(a) (16, 62) (b) (18, 25) (c) (21, 35) (d) (23, 92)

12. The H.C.F. of 2923 and 3239 is :
(a) 37 (b) 47 (c) 73 (d) 79

13. The H.C.F. of 3556 and 3444 is :
(a) 23 (b) 25 (c) 26 (d) 28

14. The L.C.M. of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^3 \times 5^3 \times 7^2 \times 11$ is :
(a) $2^3 \times 3^2 \times 5$ (b) $2^5 \times 3^4 \times 5^3$
(c) $2^3 \times 3^2 \times 5 \times 7 \times 11$ (d) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

15. Find the lowest common multiple of 24, 36 and 40. **(R.R.B. 2003)**
(a) 120 (b) 240 (c) 360 (d) 480

16. The L.C.M. of 22, 54, 108, 135 and 198 is : **(M.B.A. 1998)**
(a) 330 (b) 1980 (c) 5940 (d) 11880

17. The L.C.M. of 148 and 185 is :
(a) 680 (b) 740 (c) 2960 (d) 3700

18. The H.C.F. of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}$ and $\frac{10}{27}$ is :
(a) $\frac{2}{3}$ (b) $\frac{2}{81}$ (c) $\frac{160}{3}$ (d) $\frac{160}{81}$

19. The H.C.F. of $\frac{9}{10}, \frac{12}{25}, \frac{18}{35}$ and $\frac{21}{40}$ is :
(a) $\frac{3}{5}$ (b) $\frac{252}{5}$ (c) $\frac{3}{2800}$ (d) $\frac{63}{700}$

20. The L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$ is :
(a) $\frac{1}{54}$ (b) $\frac{10}{27}$ (c) $\frac{20}{3}$ (d) None of these

21. The L.C.M. of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{9}{13}$ is :
(a) 36 (b) $\frac{1}{36}$ (c) $\frac{1}{1365}$ (d) $\frac{12}{455}$

22. The H.C.F. of 1.75, 5.6 and 7 is :
(a) 0.07 (b) 0.7 (c) 3.5 (d) 0.35

23. The G.C.D. of 1.08, 0.36 and 0.9 is : **(Hotel Management, 2002)**
(a) 0.03 (b) 0.9 (c) 0.18 (d) 0.108

24. The H.C.F. of 0.54, 1.8 and 7.2 is :
(a) 1.8 (b) 0.18 (c) 0.018 (d) 18

25. The L.C.M. of 3, 2.7 and 0.09 is :
(a) 2.7 (b) 0.27 (c) 0.027 (d) 27

26. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4 \times 3^5 \times 5^2 \times 7^2$. The third number is : **(S.S.C. 1999)**
(a) $2^2 \times 3^5 \times 7^2$ (b) $2^2 \times 5^3 \times 7^2$ (c) $2^5 \times 5^2 \times 7^2$ (d) $2^3 \times 3^5 \times 7^2$

27. Three numbers are in the ratio 1 : 2 : 3 and their H.C.F. is 12. The numbers are :
(a) 4, 8, 12 (b) 5, 10, 15 (c) 10, 20, 30 (d) 12, 24, 36
(Section Officers', 2001)

28. The ratio of two numbers is 3 : 4 and their H.C.F. is 4. Their L.C.M. is :
(a) 12 (b) 16 (c) 24 (d) 48
(S.S.C. 2002)

29. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are :
(a) 27, 189 (b) 81, 189 (c) 108, 108 (d) 154, 16[illegible]

30. The sum of two numbers is 528 and their H.C.F. is 33. The number of pairs of numbers satisfying the above conditions is : **(C.B.I. 1997)**
(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 12

31. The number of number-pairs lying between 40 and 100 with their H.C.F. as 15 is :
(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6

32. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are :
(*a*) 66, 78 (*b*) 70, 82 (*c*) 94, 106 (*d*) 84, 96

33. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is : **(S.S.C. 2003)**
(*a*) 101 (*b*) 107 (*c*) 111 (*d*) 185

34. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is : **(C.B.I. 2003)**
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

35. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is :
(*a*) 75 (*b*) 81 (*c*) 85 (*d*) 89
(S.S.C. 2003)

36. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is : **(S.S.C. 2003)**
(*a*) 28 (*b*) 32 (*c*) 40 (*d*) 64

37. Three numbers are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is :
(*a*) 40 (*b*) 80 (*c*) 120 (*d*) 200
(M.B.A. 2003)

38. The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is : **(Section Officers', 2001)**
(*a*) 279 (*b*) 283 (*c*) 308 (*d*) 318

39. The sum of two numbers is 2000 and their L.C.M. is 21879. The two numbers are :
(*a*) 1993, 7 (*b*) 1991, 9 (*c*) 1989, 11 (*d*) 1987, 13

40. The H.C.F. and L.C.M. of two numbers are 84 and 21 respectively. If the ratio of the two numbers is 1 : 4, then the larger of the two numbers is : **(M.A.T. 1997)**
(*a*) 12 (*b*) 48 (*c*) 84 (*d*) 108

41. The L.C.M. of two numbers is 495 and their H.C.F. is 5. If the sum of the numbers is 10, then their difference is : **(S.S.C. 1999)**
(*a*) 10 (*b*) 46 (*c*) 70 (*d*) 90

42. The product of the L.C.M. and H.C.F. of two numbers is 24. The difference of two numbers is 2. Find the numbers.
(*a*) 2 and 4 (*b*) 6 and 4 (*c*) 8 and 6 (*d*) 8 and 10

43. If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to :
(*a*) $\frac{55}{601}$ (*b*) $\frac{601}{55}$ (*c*) $\frac{11}{120}$ (*d*) $\frac{120}{11}$
(C.D.S. 2003)

44. The L.C.M. of two numbers is 45 times their H.C.F. If one of the numbers is 125 and the sum of H.C.F. and L.C.M. is 1150, the other number is :
(*a*) 215 (*b*) 220 (*c*) 225 (*d*) 235

45. The H.C.F. and L.C.M. of two numbers are 50 and 250 respectively. If the first number is divided by 2, the quotient is 50. The second number is :
(*a*) 50 (*b*) 100 (*c*) 125 (*d*) 250

46. The product of two numbers is 1320 and their H.C.F. is 6. The L.C.M. of the numbers is :
(*a*) 220 (*b*) 1314 (*c*) 1326 (*d*) 7920

47. Product of two co-prime numbers is 117. Their L.C.M. should be : (C.B.I. 1997)
(a) 1 (b) 117 (c) equal to their H.C.F. (d) cannot be calculated

48. The L.C.M. of three different numbers is 120. Which of the following cannot be their H.C.F. ?
(a) 8 (b) 12 (c) 24 (d) 35

49. The H.C.F. of two numbers is 8. Which one of the following can never be their L.C.M. ?
(a) 24 (b) 48 (c) 56 (d) 60 (S.S.C. 2000)

50. The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is : (S.S.C. 2004)
(a) 276 (b) 299 (c) 322 (d) 345

51. About the number of pairs which have 16 as their H.C.F. and 136 as their L.C.M., we can definitely say that :
(a) no such pair exists (b) only one such pair exists
(c) only two such pairs exist (d) many such pairs exist

52. The H.C.F. and L.C.M. of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is : (C.B.I. 1998)
(a) 77 (b) 88 (c) 99 (d) 110

53. Two numbers, both greater than 29, have H.C.F. 29 and L.C.M. 4147. The sum of the numbers is : (S.S.C. 2002)
(a) 666 (b) 669 (c) 696 (d) 966

54. L.C.M. of two prime numbers x and y $(x > y)$ is 161. The value of $3y - x$ is :
(a) -2 (b) -1 (c) 1 (d) 2 (S.S.C. 1999)

55. The greatest number that exactly divides 105, 1001 and 2436 is :
(a) 3 (b) 7 (c) 11 (d) 21

56. The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is : (R.R.B. 2003)
(a) 15 cm (b) 25 cm (c) 35 cm (d) 42 cm

57. Three different containers contain 496 litres, 403 litres and 713 litres of mixtures of milk and water respectively. What biggest measure can measure all the different quantities exactly ?
(a) 1 litre (b) 7 litres (c) 31 litres (d) 41 litres

58. The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is : (S.S.C. 1999)
(a) 91 (b) 910 (c) 1001 (d) 1911

59. A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose ? (N.I.F.T. 2000)
(a) 14 cms (b) 21 cms (c) 42 cms (d) None of these

60. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case. (L.I.C. 2003)
(a) 4 (b) 7 (c) 9 (d) 13

61. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is : (S.S.C. 2004)
(a) 4 (b) 5 (c) 6 (d) 8

62. The greatest number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case, is :
(a) 64 (b) 124 (c) 156 (d) 260

63. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is : (R.R.B. 2004)
(a) 123 (b) 127 (c) 235 (d) 305

64. Which of the following fractions is the largest ? (IGNOU, 2003)
(a) $\frac{7}{8}$ (b) $\frac{13}{16}$ (c) $\frac{31}{40}$ (d) $\frac{63}{80}$

65. What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30 ? (S.S.C. 2003)
(a) 196 (b) 630 (c) 1260 (d) 2520

66. The smallest fraction, which each of $\frac{6}{7}, \frac{5}{14}, \frac{10}{21}$ will divide exactly, is : (S.S.C. 1998)
(a) $\frac{30}{7}$ (b) $\frac{30}{98}$ (c) $\frac{60}{147}$ (d) $\frac{50}{294}$

67. The least number of five digits which is exactly divisible by 12, 15 and 18, is :
(a) 10010 (b) 10015 (c) 10020 (d) 10080

68. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is :
(a) 9000 (b) 9400 (c) 9600 (d) 9800 (S.S.C. 2002)

69. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is : (Hotel Management, 2003)
(a) 3 (b) 13 (c) 23 (d) 33

70. The least number which is a perfect square and is divisible by each of the numbers 16, 20 and 24, is :
(a) 1600 (b) 3600 (c) 6400 (d) 14400

71. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is : (L.I.C. 2003)
(a) 1008 (b) 1015 (c) 1022 (d) 1032

72. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54, is :
(a) 427 (b) 859 (c) 869 (d) 4320

73. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is : (R.R.B. 2003)
(a) 504 (b) 536 (c) 544 (d) 548

74. The largest four-digit number which when divided by 4, 7 or 13 leaves a remainder of 3 in each case, is :
(a) 8739 (b) 9831 (c) 9834 (d) 9893

75. Let the least number of six digits, which when divided by 4, 6, 10 and 15, leaves in each case the same remainder of 2, be N. The sum of the digits in N is : (S.S.C. 2003)
(a) 3 (b) 4 (c) 5 (d) 6

76. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is : (A.A.O. Exam, 2003)
(a) 74 (b) 94 (c) 184 (d) 364

77. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is : (C.B.I. 1997)
(a) 11115 (b) 15110 (c) 15120 (d) 15210

78. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves remainders 7, 10 and 13 respectively.
(a) 3002 (b) 3013 (c) 3024 (d) 3036

79. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is : (L.I.C.A.A.O. 2003)
(a) 1677 (b) 1683 (c) 2523 (d) 3363

80. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case, but when divided by 7 leaves no remainder.
(a) 17004 (b) 18000 (c) 18002 (d) 18004

81. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?
(a) 4 (b) 10 (c) 15 (d) 16

82. Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at :
(a) 12 midnight (b) 3 a.m. (c) 6 a.m. (d) 9 a.m.

83. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point ? **(S.S.C. 2003)**
(a) 26 minutes 18 seconds (b) 42 minutes 36 seconds
(c) 45 minutes (d) 46 minutes 12 seconds

ANSWERS

1. (a)	2. (c)	3. (a)	4. (b)	5. (c)	6. (a)	7. (a)	8. (a)	9. (c)
10. (a)	11. (b)	12. (d)	13. (d)	14. (d)	15. (c)	16. (c)	17. (b)	18. (b)
19. (c)	20. (c)	21. (a)	22. (d)	23. (c)	24. (b)	25. (d)	26. (a)	27. (d)
28. (d)	29. (a)	30. (a)	31. (b)	32. (d)	33. (c)	34. (b)	35. (c)	36. (c)
37. (a)	38. (c)	39. (c)	40. (c)	41. (a)	42. (b)	43. (c)	44. (c)	45. (c)
46. (a)	47. (b)	48. (d)	49. (d)	50. (c)	51. (a)	52. (a)	53. (c)	54. (a)
55. (b)	56. (c)	57. (c)	58. (a)	59. (b)	60. (a)	61. (a)	62. (a)	63. (b)
64. (a)	65. (b)	66. (a)	67. (d)	68. (c)	69. (c)	70. (b)	71. (b)	72. (b)
73. (d)	74. (b)	75. (c)	76. (d)	77. (b)	78. (b)	79. (b)	80. (d)	81. (d)
82. (d)	83. (d)							

SOLUTIONS

1. Clearly, $252 = 2 \times 2 \times 3 \times 3 \times 7$.

2. $99 = 1 \times 3 \times 3 \times 11$; $101 = 1 \times 101$;
$176 = 1 \times 2 \times 2 \times 2 \times 2 \times 11$; $182 = 1 \times 2 \times 7 \times 13$.
So, divisors of 99 are 1, 3, 9, 11, 33 and 99;
divisors of 101 are 1 and 101;
divisors of 176 are 1, 2, 4, 8, 16, 22, 44, 88 and 176;
divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182.
Hence, 176 has the most number of divisors.

3.

n	*Divisors excluding n*	*Sum of divisors*
6	1, 2, 3	6
9	1, 3	4
15	1, 3, 5	9
21	1, 3, 7	11

Clearly, 6 is a perfect number.

4. 1095) 1168 (1
1095
73) 1095 (15
73
365
365
×

So, H.C.F. of 1095 and 1168 = 73.

$\therefore \dfrac{1095}{1168} = \dfrac{1095 \div 73}{1168 \div 73} = \dfrac{15}{16}.$

5. 128352) 238368 (1
128352
110016) 128352 (1
110016
18336) 110016 (6
110016
×

So, H.C.F. of 128352 and 238368 = 18336.

$\therefore \dfrac{128352}{238368} = \dfrac{128352 \div 18336}{238368 \div 18336} = \dfrac{7}{13}.$

6. H.C.F. = Product of lowest powers of common factors = $2^2 \times 3^2 \times 5$.

7. H.C.F. = Product of lowest powers of common factors = $3 \times 5 \times 7 = 105$.

8. $4 \times 27 \times 3125 = 2^2 \times 3^3 \times 5^5$; $8 \times 9 \times 25 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$;
$16 \times 81 \times 5 \times 11 \times 49 = 2^4 \times 3^4 \times 5 \times 7^2 \times 11$.
$\therefore$ H.C.F. $= 2^2 \times 3^2 \times 5 = 180$.

9. $36 = 2^2 \times 3^2$; $84 = 2^2 \times 3 \times 7$.
$\therefore$ H.C.F. $= 2^2 \times 3 = 12$.

10. $204 = 2^2 \times 3 \times 17$; $1190 = 2 \times 5 \times 7 \times 17$; $1445 = 5 \times 17^2$.
$\therefore$ H.C.F. = 17.

11. H.C.F. of 18 and 25 is 1. So, they are co-primes.

12. 2923) 3239 (1
2923
316) 2923 (9
2844
79) 316 (4
316
×

$\therefore$ H.C.F. = 79.

13. 3444) 3556 (1
3444
112) 3444 (30
3360
84) 112 (1
84
28) 84 (3
84
×

$\therefore$ H.C.F. = 28.

14. L.C.M. = Product of highest powers of prime factors = $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$.

15.

2	24 – 36 – 40
2	12 – 18 – 20
2	6 – 9 – 10
3	3 – 9 – 5
	1 – 3 – 5

L.C.M. = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

16.

2	22 – 54 – 108 – 135 – 198
3	11 – 27 – 54 – 135 – 99
3	11 – 9 – 18 – 45 – 33
3	11 – 3 – 6 – 15 – 11
11	11 – 1 – 2 – 5 – 11
	1 – 1 – 2 – 5 – 1

L.C.M. = $2 \times 3 \times 3 \times 3 \times 11 \times 2 \times 5 = 5940$.

17. H.C.F. of 148 and 185 is 37.

$\therefore$ L.C.M. $= \left(\dfrac{148 \times 185}{37}\right) = 740.$

18. Required H.C.F. $= \dfrac{\text{H.C.F. of } 2, 8, 64, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \dfrac{2}{81}.$

19. Required H.C.F. = $\dfrac{\text{H.C.F. of } 9, 12, 18, 21}{\text{L.C.M. of } 10, 25, 35, 40} = \dfrac{3}{2800}$.

20. Required L.C.M. = $\dfrac{\text{L.C.M. of } 1, 5, 2, 4}{\text{H.C.F. of } 3, 6, 9, 27} = \dfrac{20}{3}$.

21. Required L.C.M. = $\dfrac{\text{L.C.M. of } 2, 3, 4, 9}{\text{H.C.F. of } 3, 5, 7, 13} = \dfrac{36}{1} = 36$.

22. Given numbers with two decimal places are : 1.75, 5.60 and 7.00. Without decimal places, these numbers are : 175, 560 and 700, whose H.C.F. is 35.
$\therefore$ H.C.F. of given numbers = 0.35.

23. Given numbers are 1.08, 0.36 and 0.90. H.C.F. of 108, 36 and 90 is 18.
$\therefore$ H.C.F. of given numbers = 0.18.

24. Given numbers are 0.54, 1.80 and 7.20. H.C.F. of 54, 180 and 720 is 18.
$\therefore$ H.C.F. of given numbers = 0.18.

25. Given numbers are 3.00, 2.70 and 0.09. L.C.M. of 300, 270 and 9 is 2700.
$\therefore$ L.C.M. of given numbers = 27.00 = 27.

26. $3240 = 2^3 \times 3^4 \times 5$; $3600 = 2^4 \times 3^2 \times 5^2$; H.C.F. = $36 = 2^2 \times 3^2$.
Since H.C.F. is the product of lowest powers of common factors, so the third number must have $(2^2 \times 3^2)$ as its factor.
Since L.C.M. is the product of highest powers of common prime factors, so the third number must have 3^5 and 7^2 as its factors.
$\therefore$ Third number = $2^2 \times 3^5 \times 7^2$.

27. Let the required numbers be x, $2x$ and $3x$. Then, their H.C.F. = x. So, $x = 12$.
$\therefore$ The numbers are 12, 24 and 36.

28. Let the numbers be $3x$ and $4x$. Then, their H.C.F. = x. So, $x = 4$.
So, the numbers are 12 and 16.
L.C.M. of 12 and 16 = 48.

29. Let the required numbers be $27a$ and $27b$. Then, $27a + 27b = 216 \Rightarrow a + b = 8$.
Now, co-primes with sum 8 are (1, 7) and (3, 5).
$\therefore$ Required numbers are $(27 \times 1, 27 \times 7)$ and $(27 \times 3, 27 \times 5)$ *i.e.*, (27, 189) and (81, 135).
Out of these, the given one in the answer is the pair (27, 189).

30. Let the required numbers be $33a$ and $33b$. Then, $33a + 33b = 528 \Rightarrow a + b = 16$.
Now, co-primes with sum 16 are (1, 15), (3, 13), (5, 11) and (7, 9).
$\therefore$ Required numbers are $(33 \times 1, 33 \times 15)$, $(33 \times 3, 33 \times 13)$, $(33 \times 5, 33 \times 11)$, $(33 \times 7, 33 \times 9)$.
The number of such pairs is 4.

31. Numbers with H.C.F. 15 must contain 15 as a factor.
Now, multiples of 15 between 40 and 100 are 45, 60, 75 and 90.
$\therefore$ Number-pairs with H.C.F. 15 are (45, 60), (45, 75), (60, 75) and (75, 90).
[$\because$ H.C.F. of (60, 90) is 30 and that of (45, 90) is 45]
Clearly, there are 4 such pairs.

32. Out of the given numbers, the two with H.C.F. 12 and difference 12 are 84 and 96.

33. Let the numbers be $37a$ and $37b$. Then, $37a \times 37b = 4107 \Rightarrow ab = 3$.
Now, co-primes with product 3 are (1, 3).
So, the required numbers are $(37 \times 1, 37 \times 3)$ *i.e.*, (1, 111).
$\therefore$ Greater number = 111.

34. Let the numbers be $13a$ and $13b$. Then, $13a \times 13b = 2028 \Rightarrow ab = 12$.
Now, co-primes with product 12 are (1, 12) and (3, 4).
So, the required numbers are $(13 \times 1, 13 \times 12)$ and $(13 \times 3, 13 \times 4)$.
Clearly, there are 2 such pairs.

35. Since the numbers are co-prime, they contain only 1 as the common factor.
Also, the given two products have the middle number in common.
So, middle number = H.C.F. of 551 and 1073 = 29;
First number $= \left(\frac{551}{29}\right) = 19$; Third number $= \left(\frac{1073}{29}\right) = 37$.
$\therefore$ Required sum $= (19 + 29 + 37) = 85$.

36. Let the numbers be $2x$ and $3x$. Then, their L.C.M. $= 6x$. So, $6x = 48$ or $x = 8$.
$\therefore$ The numbers are 16 and 24.
Hence, required sum $= (16 + 24) = 40$.

37. Let the numbers be $3x$, $4x$ and $5x$. Then, their L.C.M. $= 60x$. So, $60x = 2400$ or $x = 40$.
$\therefore$ The numbers are (3×40), (4×40) and (5×40).
Hence, required H.C.F. = 40.

38. Other number $= \left(\frac{11 \times 7700}{275}\right) = 308$.

39. Let the numbers be x and $(2000 - x)$. Then, their L.C.M. $= x(2000 - x)$.
So, $x(2000 - x) = 21879 \Leftrightarrow x^2 - 2000x + 21879 = 0$
$\Leftrightarrow (x - 1989)(x - 11) = 0 \Leftrightarrow x = 1989$ or $x = 11$.
Hence, the numbers are 1989 and 11.

40. Let the numbers be x and $4x$. Then, $x \times 4x = 84 \times 21 \Leftrightarrow x^2 = \left(\frac{84 \times 21}{4}\right) \Leftrightarrow x = 21$.
Hence, larger number $= 4x = 84$.

41. Let the numbers be x and $(100 - x)$.
Then, $x(100 - x) = 5 \times 495 \Leftrightarrow x^2 - 100x + 2475 = 0$
$\Leftrightarrow (x - 55)(x - 45) = 0 \Leftrightarrow x = 55$ or $x = 45$.
$\therefore$ The numbers are 45 and 55.
Required difference $= (55 - 45) = 10$.

42. Let the numbers be x and $(x + 2)$.
Then, $x(x + 2) = 24 \Leftrightarrow x^2 + 2x - 24 = 0 \Leftrightarrow (x - 4)(x + 6) = 0 \Leftrightarrow x = 4$.
So, the numbers are 4 and 6.

43. Let the numbers be a and b. Then, $a + b = 55$ and $ab = 5 \times 120 = 600$.
$\therefore$ Required sum $= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{55}{600} = \frac{11}{120}$.

44. Let H.C.F. be h and L.C.M. be l. Then, $l = 45h$ and $l + h = 1150$.
$\therefore 45h + h = 1150$ or $h = 25$. So, $l = (1150 - 25) = 1125$.
Hence, other number $= \left(\frac{25 \times 1125}{125}\right) = 225$.

45. First number $= (50 \times 2) = 100$. Second number $= \left(\frac{50 \times 250}{100}\right) = 125$.

46. L.C.M. $= \frac{\text{Product of numbers}}{\text{H.C.F.}} = \frac{1320}{6} = 220$.

47. H.C.F of co-prime numbers is 1. So, L.C.M. $= \frac{117}{1} = 117$.

48. Since H.C.F. is always a factor of L.C.M., we cannot have three numbers with H.C.F. 35 and L.C.M. 120.

49. H.C.F. of two numbers divides their L.C.M. exactly. Clearly, 8 is not a factor of 60.

50. Clearly, the numbers are (23×13) and (23×14).

$\therefore$ Larger number $= (23 \times 14) = 322$.

51. Since 16 is not a factor of 136, it follows that there does not exist any pair of numbers with H.C.F. 16 and L.C.M. 136.

52. Product of numbers $= 11 \times 385 = 4235$.

Let the numbers be $11a$ and $11b$. Then, $11a \times 11b = 4235 \Rightarrow ab = 35$.

Now, co-primes with product 35 are (1, 35) and (5, 7).

So, the numbers are $(11 \times 1, 11 \times 35)$ and $(11 \times 5, 11 \times 7)$.

Since one number lies between 75 and 125, the suitable pair is (55, 77).

Hence, required number = 77.

53. Product of numbers $= 29 \times 4147$.

Let the numbers be $29a$ and $29b$. Then, $29a \times 29b = (24 \times 4147) \Rightarrow ab = 143$.

Now, co-primes with product 143 are (1, 143) and (11, 13).

So, the numbers are $(29 \times 1, 29 \times 143)$ and $(29 \times 11, 29 \times 13)$.

Since both numbers are greater than 29, the suitable pair is $(29 \times 11, 29 \times 13)$ *i.e.*, (319, 377).

$\therefore$ Required sum $= (319 + 377) = 696$.

54. H.C.F. of two prime numbers is 1. Product of numbers $= (1 \times 161) = 161$.

Let the numbers be a and b. Then, $ab = 161$.

Now, co-primes with product 161 are (1, 161) and (7, 23).

Since x and y are prime numbers and $x > y$, we have $x = 23$ and $y = 7$.

$\therefore$ $3y - x = (3 \times 7) - 23 = -2$.

55. H.C.F. of 2436 and 1001 is 7. Also, H.C.F. of 105 and 7 is 7.

$\therefore$ H.C.F. of 105, 1001 and 2436 is 7.

56. Required length = H.C.F. of 700 cm, 385 cm and 1295 cm = 35 cm.

57. Required measurement = (H.C.F. of 496, 403, 713) litres = 31 litres.

58. Required number of students = H.C.F. of 1001 and 910 = 91.

59. Largest size of the tile = H.C.F. of 378 cm and 525 cm = 21 cm.

60. Required number = H.C.F. of $(91 - 43)$, $(183 - 91)$ and $(183 - 43)$

= H.C.F. of 48, 92 and 140 = 4.

61. N = H.C.F. of $(4665 - 1305)$, $(6905 - 4665)$ and $(6905 - 1305)$

= H.C.F. of 3360, 2240 and 5600 = 1120.

Sum of digits in N $= (1 + 1 + 2 + 0) = 4$.

62. Required number = H.C.F. of $(1356 - 12)$, $(1868 - 12)$ and $(2764 - 12)$

= H.C.F. of 1344, 1856 and 2752 = 64.

63. Required number = H.C.F. of $(1657 - 6)$ and $(2037 - 5)$

= H.C.F. of 1651 and 2032 = 127.

64. L.C.M. of 8, 16, 40 and 80 = 80.

$\frac{7}{8} = \frac{70}{80}$; $\frac{13}{16} = \frac{65}{80}$; $\frac{31}{40} = \frac{62}{80}$.

Since, $\frac{70}{80} > \frac{63}{80} > \frac{65}{80} > \frac{62}{80}$, so $\frac{7}{8} > \frac{63}{80} > \frac{13}{16} > \frac{31}{40}$.

So, $\frac{7}{8}$ is the largest.

65. L.C.M. of 12, 18, 21, 30

2	12	18	21	30
3	6	9	21	15
	2	3	7	5

$= 2 \times 3 \times 2 \times 3 \times 7 \times 5 = 1260.$

$\therefore$ Required number $= (1260 \div 2) = 630.$

66. Required fraction = L.C.M. of $\frac{6}{7}, \frac{5}{14}, \frac{10}{21} = \frac{\text{L.C.M. of } 6, 5, 10}{\text{H.C.F. of } 7, 14, 21} = \frac{30}{7}.$

67. Least number of 5 digits is 10000. L.C.M. of 12, 15 and 18 is 180.
On dividing 10000 by 180, the remainder is 100.
$\therefore$ Required number $= 10000 + (180 - 100) = 10080.$

68. Greatest number of 4 digits is 9999. L.C.M. of 15, 25, 40 and 75 is 600.
On dividing 9999 by 600, the remainder is 399.
$\therefore$ Required number $= (9999 - 399) = 9600.$

69. L.C.M. of 5, 6, 4 and 3 = 60. On dividing 2497 by 60, the remainder is 37.
$\therefore$ Number to be added $= (60 - 37) = 23.$

70. The least number divisible by 16, 20, 24
$= \text{L.C.M. of } 16, 20, 24 = 240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5.$
To make it a perfect square, it must be multiplied by 3×5.
$\therefore$ Required number $= 240 \times 3 \times 5 = 3600.$

71. Required number = (L.C.M. of 12, 16, 18, 21, 28) + 7 = 1008 + 7 = 1015.

72. Required number = (L.C.M. of 24, 32, 36, 54) − 5 = 864 − 5 = 859.

73. Required number = (L.C.M. of 12, 15, 20, 54) + 8 = 540 + 8 = 548.

74. Greatest number of 4 digits is 9999. L.C.M. of 4, 7 and 13 = 364.
On dividing 9999 by 364, remainder obtained is 171.
$\therefore$ Greatest number of 4 digits divisible by 4, 7 and 13 $= (9999 - 171) = 9828.$
Hence, required number $= (9828 + 3) = 9831.$

75. Least number of 6 digits is 100000. L.C.M. of 4, 6, 10 and 15 = 60.
On dividing 100000 by 60, the remainder obtained is 40.
$\therefore$ Least number of 6 digits divisible by 4, 6, 10 and 15 $= 100000 + (60 - 40) = 100020.$
$\therefore$ N $= (100020 + 2) = 100022$. Sum of digits in N $= (1 + 2 + 2) = 5.$

76. L.C.M. of 6, 9, 15 and 18 is 90.
Let required number be $90k + 4$, which is a multiple of 7.
Least value of k for which $(90k + 4)$ is divisible by 7 is $k = 4$.
$\therefore$ Required number $= 90 \times 4 + 4 = 364.$

77. Here $(48 - 38) = 10, (60 - 50) = 10, (72 - 62) = 10, (108 - 98) = 10$ & $(140 - 130) = 10.$
$\therefore$ Required number = (L.C.M. of 48, 60, 72, 108, 140) − 10 = 15120 − 10 = 15110.

78. Here $(18 - 7) = 11, (21 - 10) = 11$ and $(24 - 13) = 11$. L.C.M. of 18, 21 and 24 is 504.
Let required number be $504k - 11$.
Least value of k for which $(504k - 11)$ is divisible by 23 is $k = 6$.
$\therefore$ Required number $= 504 \times 6 - 11 = 3024 - 11 = 3013.$

79. L.C.M. of 5, 6, 7, 8 = 840.
$\therefore$ Required number is of the form $840k + 3$.
Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.
$\therefore$ Required number $= (840 \times 2 + 3) = 1683.$

80. L.C.M. of 16, 18, 20, 25 = 3600. Required number is of the form $3600k + 4$.
Least value of k for which $(3600k + 4)$ is divisible by 7 is $k = 5$.
$\therefore$ Required number $= (3600 \times 5 + 4) = 18004.$

81. L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

So, the bells will toll together after every 120 seconds, *i.e.*, 2 minutes.

In 30 minutes, they will toll together $\left[\left(\frac{30}{2}\right)+1\right]$ = 16 times.

82. Interval after which the devices will beep together

= (L.C.M. of 30, 60, 90, 105) min. = 1260 min. = 21 hrs.

So, the devices will again beep together 21 hrs. after 12 noon *i.e.*, at 9 a.m.

83. L.C.M. of 252, 308 and 198 = 2772.

So, A, B and C will again meet at the starting point in 2772 sec. *i.e.*, 46 min. 12 sec.

5. **Dividing a Decimal Fraction By a Decimal Fraction** : Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, $\frac{0.00066}{0.11} = \frac{0.00066 \times 100}{0.11 \times 100} = \frac{0.066}{11} = .006.$

V. Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions $\frac{3}{5}, \frac{6}{7}$ and $\frac{7}{9}$ in descending order.

Now, $\frac{3}{5} = 0.6$, $\frac{6}{7} = 0.857$, $\frac{7}{9} = 0.777.....$

Since $0.857 > 0.777 ..... > 0.6$, so $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$.

VI. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a ***recurring decimal***.

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.

Thus, $\frac{1}{3} = 0.333 ..... = 0.\dot{3}$; $\frac{22}{7} = 3.142857142857 ..... = 3.\overline{142857}$.

Pure Recurring Decimal : A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

Thus, $0.\dot{5} = \frac{5}{9}$; $0.\overline{53} = \frac{53}{99}$; $0.\overline{067} = \frac{67}{999}$; etc.

Mixed Recurring Decimal : A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., $0.17333 ..... = 0.17\overline{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus, $0.1\overline{6} = \frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$; $0.22\overline{73} = \frac{2273-22}{9900} = \frac{2251}{9900}$.

VII. Some Basic Formulae :

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$.
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$.
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$.
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.