Sol. $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

$\Leftrightarrow \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + \log_{10} 10$

$\Leftrightarrow \log_{10} [3 (4x + 1)] = \log_{10} [10 (x + 1)]$

$\Leftrightarrow 3 (4x + 1) = 10 (x + 1) \Leftrightarrow 12x + 3 = 10x + 10 \Leftrightarrow 2x = 7 \Leftrightarrow x = \frac{7}{2}.$

Ex. 7. ***Simplify :*** $\left[\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}\right]$

Sol. Given expression $= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$

$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$ $\left[\because \log_a x = \frac{1}{\log_x a}\right]$

$= 2 \log_{xyz}(xyz) = 2 \times 1 = 2.$

Ex. 8. ***If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$.*** **(C.B.I. 1997)**

Sol. $\log_{10} 50 = \log_{10}\left(\frac{100}{2}\right) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897.$

Ex. 9. ***If log 2 = 0.3010 and log 3 = 0.4771, find the values of :***

(i) log 25 ***(ii) log 4.5***

Sol. (*i*) $\log 25 = \log\left(\frac{100}{4}\right) = \log 100 - \log 4 = 2 - 2\log 2 = (2 - 2 \times 0.3010) = 1.398.$

(*ii*) $\log 4.5 = \log\left(\frac{9}{2}\right) = \log 9 - \log 2 = 2\log 3 - \log 2$

$= (2 \times 0.4771 - 0.3010) = 0.6532.$

Ex. 10. ***If log 2 = 0.30103, find the number of digits in 2^{56}.***

Sol. $\log (2^{56}) = 56 \log 2 = (56 \times 0.30103) = 16.85768.$

Its characteristic is 16. Hence, the number of digits in 2^{56} is 17.

EXERCISE 23

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The value of $\log_2 16$ is : **(M.B.A. 2003)**

(*a*) $\frac{1}{8}$ (*b*) 4 (*c*) 8 (*d*) 16.

2. The value of $\log_{343} 7$ is :

(*a*) $\frac{1}{3}$ (*b*) -3 (*c*) $-\frac{1}{3}$ (*d*) 3

3. The value of $\log_5\left(\frac{1}{125}\right)$ is :

(*a*) 3 (*b*) -3 (*c*) $\frac{1}{3}$ (*d*) $-\frac{1}{3}$

4. The value of $\log_{\sqrt{2}} 32$ is :

(*a*) $\frac{5}{2}$ (*b*) 5 (*c*) 10 (*d*) $\frac{1}{10}$

5. The value of $\log_{10} (.0001)$ is :

(*a*) $\frac{1}{4}$ (*b*) $-\frac{1}{4}$ (*c*) -4 (*d*) 4

6. The value of $\log_{(.01)} (1000)$ is :

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

7. The logarithm of 0.0625 to the base 2 is :

(a) -4 (b) -2 (c) 0.25 (d) 0.5

8. If $\log_3 x = -2$, then x is equal to :

(a) -9 (b) -6 (c) -8 (d) $\frac{1}{9}$

9. If $\log_8 x = \frac{2}{3}$, then the value of x is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 3 (d) 4

10. If $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to :

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{81}{256}$ (d) $\frac{256}{81}$

11. If $\log_x 4 = 0.4$, then the value of x is : **(Asstt. Grade, 1998)**

(a) 1 (b) 4 (c) 16 (d) 32

12. If $\log_{10000} x = -\frac{1}{4}$, then x is equal to :

(a) $\frac{1}{10}$ (b) $\frac{1}{100}$ (c) $\frac{1}{1000}$ (d) $\frac{1}{10000}$

13. If $\log_x 4 = \frac{1}{4}$, then x is equal to :

(a) 16 (b) 64 (c) 128 (d) 256

14. If $\log_x (0.1) = -\frac{1}{3}$, then the value of x is :

(a) 10 (b) 100 (c) 1000 (d) $\frac{1}{1000}$

15. If $\log_{32} x = 0.8$, then x is equal to :

(a) 25.6 (b) 16 (c) 10 (d) 12.8

16. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is : **(S.S.C. 1999)**

(a) 2^{10} (b) 2^{100} (c) 2^{1000} (d) 2^{10000}

17. The value of $\log_{(-1/3)} 81$ is equal to :

(a) -27 (b) -4 (c) 4 (d) 27

18. The value of $\log_{2\sqrt{3}} (1728)$ is :

(a) 3 (b) 5 (c) 6 (d) 9

19. $\frac{\log \sqrt{8}}{\log 8}$ is equal to : **(I.A.F. 2002)**

(a) $\frac{1}{\sqrt{8}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

20. Which of the following statements is not correct ? **(M.B.A. 2003)**

(a) $\log_{10} 10 = 1$ (b) $\log (2 + 3) = \log (2 \times 3)$

(c) $\log_{10} 1 = 0$ (d) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

21. The value of $\log_2(\log_5 625)$ is :
(*a*) 2 (*b*) 5 (*c*) 10 (*d*) 15

22. If $\log_2[\log_3(\log_2 x)] = 1$, then x is equal to :
(*a*) 0 (*b*) 12 (*c*) 128 (*d*) 512

23. The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is :
(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

24. If $a^x = b^y$, then : **(Hotel Management, 2001)**
(*a*) $\log \dfrac{a}{b} = \dfrac{x}{y}$ (*b*) $\dfrac{\log a}{\log b} = \dfrac{x}{y}$ (*c*) $\dfrac{\log a}{\log b} = \dfrac{y}{x}$ (*d*) None of these

25. log 360 is equal to :
(*a*) $2 \log 2 + 3 \log 3$ (*b*) $3 \log 2 + 2 \log 3$
(*c*) $3 \log 2 + 2 \log 3 - \log 5$ (*d*) $3 \log 2 + 2 \log 3 + \log 5$

26. The value of $\left(\dfrac{1}{3} \log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32\right)$ is :
(*a*) 0 (*b*) $\dfrac{4}{5}$ (*c*) 1 (*d*) 2

27. $2 \log_{10} 5 + \log_{10} 8 - \dfrac{1}{2} \log_{10} 4 = ?$ **(M.B.A. 2002)**
(*a*) 2 (*b*) 4 (*c*) $2 + 2 \log_{10} 2$ (*d*) $4 - 4 \log_{10} 2$

28. If $\log_a(ab) = x$, then $\log_b(ab)$ is : **(M.A.T. 2002)**
(*a*) $\dfrac{1}{x}$ (*b*) $\dfrac{x}{x+1}$ (*c*) $\dfrac{x}{1-x}$ (*d*) $\dfrac{x}{x-1}$

29. If $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, then the value of $\log(4 \cdot \sqrt[3]{63})$ is :
(*a*) $2x + \dfrac{2}{3}y - \dfrac{1}{3}z$ (*b*) $2x + \dfrac{2}{3}y + \dfrac{1}{3}z$
(*c*) $2x - \dfrac{2}{3}y + \dfrac{1}{3}z$ (*d*) $-2x + \dfrac{2}{3}y + \dfrac{1}{3}z$ **(S.S.C. 1998)**

30. If $\log_4 x + \log_2 x = 6$, then x is equal to :
(*a*) 2 (*b*) 4 (*c*) 8 (*d*) 16

31. If $\log_8 x + \log_8 \dfrac{1}{6} = \dfrac{1}{3}$, then the value of x is :
(*a*) 12 (*b*) 16 (*c*) 18 (*d*) 24

32. If $\log_{10} 125 + \log_{10} 8 = x$, then x is equal to :
(*a*) $\dfrac{1}{3}$ (*b*) .064 (*c*) -3 (*d*) 3

33. The value of $(\log_9 27 + \log_8 32)$ is :
(*a*) $\dfrac{7}{2}$ (*b*) $\dfrac{19}{6}$ (*c*) 4 (*d*) 7

34. $(\log_5 3) \times (\log_3 625)$ equals :
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

35. $(\log_5 5)(\log_4 9)(\log_3 2)$ is equal to :
(*a*) 1 (*b*) $\dfrac{3}{2}$ (*c*) 2 (*d*) 5

36. If $\log_{12} 27 = a$, then $\log_6 16$ is : **(Assistant Grade, 1998)**
(*a*) $\dfrac{3-a}{4(3+a)}$ (*b*) $\dfrac{3+a}{4(3-a)}$ (*c*) $\dfrac{4(3+a)}{(3-a)}$ (*d*) $\dfrac{4(3-a)}{(3+a)}$

37. If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to : (C.D.S. 2003)
(a) 1 (b) 3 (c) 5 (d) 10

38. If $\log_5 (x^2 + x) - \log_5 (x + 1) = 2$, then the value of x is :
(a) 5 (b) 10 (c) 25 (d) 32

39. The value of $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}\right)$ is :
(a) 0 (b) 1 (c) 5 (d) 60

40. The value of $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$ is :
(a) 2 (b) 7 (c) 8 (d) 33

41. The value of $16^{\log_4 5}$ is :
(a) $\frac{5}{64}$ (b) 5 (c) 16 (d) 25

42. If $\log x + \log y = \log (x + y)$, then :
(a) $x = y$ (b) $xy = 1$ (c) $y = \frac{x-1}{x}$ (d) $y = \frac{x}{x-1}$

43. If $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$, then :
(a) $a + b = 1$ (b) $a - b = 1$ (c) $a = b$ (d) $a^2 - b^2 = 1$

44. $\left[\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right)\right]$ is equal to :
(a) 0 (b) 1 (c) 2 (d) abc

45. $(\log_b a \times \log_c b \times \log_a c)$ is equal to :
(a) 0 (b) 1 (c) abc (d) $a + b + c$

46. $\left[\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ca)+1} + \frac{1}{(\log_c ab)+1}\right]$ is equal to :
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

47. The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right]$ is :
(a) 0 (b) 1 (c) 2 (d) 3

48. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to : (C.D.S. 2003)
(a) $-(1 + a)$ (b) $(1 + a)^{-1}$ (c) $\frac{a}{10}$ (d) $\frac{1}{10a}$

49. If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to :
(a) -1 (b) 0 (c) 1 (d) abc

50. If $\log 27 = 1.431$, then the value of $\log 9$ is : (Section Officers', 2001)
(a) 0.934 (b) 0.945 (c) 0.954 (d) 0.958

51. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is equal to : (S.S.C. 2000)
(a) $\frac{699}{301}$ (b) $\frac{1000}{301}$ (c) 0.3010 (d) 0.6990

52. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is : (S.S.C. 2001)
(a) 0.3241 (b) 0.6911 (c) 0.6990 (d) 0.7525

53. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is :
(a) 1.6020 (b) 1.9030 (c) 3.9030 (d) None of these

54. If $\log 3 = 0.477$ and $(1000)^x = 3$, then x equals : **(S.S.C. 2000)**
(a) 0.0159 (b) 0.0477 (c) 0.159 (d) 10

55. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 25$ is :
(a) 0.6020 (b) 1.2040 (c) 1.3980 (d) 1.5050

56. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, the value of $\log_5 512$ is : **(M.A.T. 2002)**
(a) 2.870 (b) 2.967 (c) 3.876 (d) 3.912

57. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10} 1.5$ is :
(a) 0.1761 (b) 0.7116 (c) 0.7161 (d) 0.7611

58. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is :
(a) 0.4471 (b) 1.4471 (c) 2.4471 (d) None of these
(S.S.C. 1999)

59. If $\log (0.57) = \bar{1}.756$, then the value of $\log 57 + \log (0.57)^3 + \log \sqrt{0.57}$ is :
(a) 0.902 (b) $\bar{2}.146$ (c) 1.902 (d) $\bar{1}.146$
(Section Officers', 2003)

60. If $\log 2 = 0.30103$, the number of digits in 2^{64} is : **(C.B.I. 1997)**
(a) 18 (b) 19 (c) 20 (d) 21

61. If $\log 2 = 0.30103$, the number of digits in 4^{50} is :
(a) 30 (b) 31 (c) 100 (d) 200

62. If $\log 2 = 0.30103$, then the number of digits in 5^{20} is :
(a) 14 (b) 16 (c) 18 (d) 25

ANSWERS

1. (b)	2. (a)	3. (b)	4. (c)	5. (c)	6. (d)	7. (a)	8. (d)
9. (d)	10. (d)	11. (d)	12. (a)	13. (d)	14. (c)	15. (b)	16. (c)
17. (b)	18. (c)	19. (c)	20. (b)	21. (a)	22. (d)	23. (a)	24. (c)
25. (d)	26. (c)	27. (a)	28. (d)	29. (b)	30. (d)	31. (a)	32. (d)
33. (b)	34. (d)	35. (a)	36. (d)	37. (b)	38. (c)	39. (b)	40. (a)
41. (d)	42. (d)	43. (a)	44. (a)	45. (b)	46. (a)	47. (a)	48. (a)
49. (c)	50. (c)	51. (b)	52. (c)	53. (b)	54. (c)	55. (c)	56. (c)
57. (a)	58. (a)	59. (a)	60. (c)	61. (b)	62. (a)		

SOLUTIONS

1. Let $\log_2 16 = n$. Then, $2^n = 16 = 2^4 \Rightarrow n = 4$.
$\therefore \log_2 16 = n$.

2. Let $\log_{343} 7 = n$. Then, $(343)^n = 7 \Leftrightarrow (7^3)^n = 7 \Leftrightarrow 3n = 1 \Leftrightarrow n = \frac{1}{3}$.
$\therefore \log_{343} 7 = \frac{1}{3}$.

3. Let $\log_5 \left(\frac{1}{125}\right) = n$. Then, $5^n = \frac{1}{125} \Leftrightarrow 5^n = 5^{-3} \Leftrightarrow n = -3$.
$\therefore \log_5 \left(\frac{1}{125}\right) = -3$.

4. Let $\log_{\sqrt{2}} 32 = n$. Then, $(\sqrt{2})^n = 32 \Leftrightarrow (2)^{n/2} = 2^5 \Leftrightarrow \frac{n}{2} = 5 \Leftrightarrow n = 10.$

$\therefore \log_{\sqrt{2}} 32 = 10.$

5. Let $\log_{10} (.0001) = n.$

Then, $10^n = .0001 \Leftrightarrow 10^n = \frac{1}{10000} = \frac{1}{10^4} \Leftrightarrow 10^n = 10^{-4} \Leftrightarrow n = -4.$

$\therefore \log_{10} (.0001) = -4.$

6. Let $\log_{(.01)} (1000) = n.$

Then, $(.01)^n = 1000 \Leftrightarrow \left(\frac{1}{100}\right)^n = 10^3 \Leftrightarrow (10^{-2})^n = 10^3 \Leftrightarrow -2n = 3 \Leftrightarrow n = -\frac{3}{2}.$

7. Let $\log_2 0.0625 = n.$

Then, $2^n = 0.0625 = \frac{625}{10000} \Leftrightarrow 2^n = \frac{1}{16} \Leftrightarrow 2^n = 2^{-4} \Leftrightarrow n = -4.$

$\therefore \log_2 0.0625 = -4.$

8. $\log_3 x = -2 \Leftrightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

9. $\log_8 x = \frac{2}{3} \Leftrightarrow x = 8^{2/3} = (2^3)^{2/3} = 2^{\left(3 \times \frac{2}{3}\right)} = 2^2 = 4.$

10. $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2} \Leftrightarrow x^{-1/2} = \frac{9}{16} \Leftrightarrow \frac{1}{\sqrt{x}} = \frac{9}{16} \Leftrightarrow \sqrt{x} = \frac{16}{9}$

$\Leftrightarrow x = \left(\frac{16}{9}\right)^2 = \frac{256}{81}.$

11. $\log_x 4 = 0.4 \Leftrightarrow \log_x 4 = \frac{4}{10} = \frac{2}{5} \Leftrightarrow x^{2/5} = 4 \Leftrightarrow x = 4^{5/2} = (2^2)^{5/2}$

$\Leftrightarrow x = 2^{\left(2 \times \frac{5}{2}\right)} = 2^5 \Leftrightarrow x = 32.$

12. $\log_{10000} x = -\frac{1}{4} \Leftrightarrow x = (10000)^{-1/4} = (10^4)^{-1/4} = 10^{-1} = \frac{1}{10}.$

13. $\log_x 4 = \frac{1}{4} \Leftrightarrow x^{1/4} = 4 \Leftrightarrow x = 4^4 = 256.$

14. $\log_x (0.1) = -\frac{1}{3} \Leftrightarrow x^{-1/3} = 0.1 \Leftrightarrow \frac{1}{x^{1/3}} = 0.1 \Leftrightarrow x^{1/3} = \frac{1}{0.1} = 10$

$\Leftrightarrow x = (10)^3 = 1000.$

15. $\log_{32} x = 0.8 \Leftrightarrow x = (32)^{0.8} = (2^5)^{4/5} = 2^4 = 16.$

16. $\log_2 x = 10 \Rightarrow x = 2^{10}.$

$\therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}.$

17. Let $\log_{(-1/3)} 81 = x$. Then, $\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3)^4 = \left(-\frac{1}{3}\right)^{-4}$

$\therefore x = -4$ *i.e.*, $\log_{(-1/3)} 81 = -4.$

18. Let $\log_{2\sqrt{3}} (1728) = x.$

Then, $(2\sqrt{3})^x = 1728 = (12)^3 = [(2\sqrt{3})^2]^3 = (2\sqrt{3})^6.$

$\therefore x = 6$, *i.e.*, $\log_{2\sqrt{3}} (1728) = 6.$

19. $\dfrac{\log \sqrt{8}}{\log 8} = \dfrac{\log (8)^{1/2}}{\log 8} = \dfrac{\frac{1}{2}\log 8}{\log 8} = \dfrac{1}{2}.$

20. (*a*) Since $\log_a a = 1$, so $\log_{10} 10 = 1$.

(*b*) $\log (2 + 3) = 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3$

$\therefore \quad \log (2 + 3) \neq \log (2 \times 3)$.

(*c*) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$.

(*d*) $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$.

So, (*b*) is incorrect.

21. Let $\log_5 625 = x$. Then, $5^x = 625 = 5^4$ or $x = 4$.

Let $\log_2 (\log_5 625) = y$. Then, $\log_2 4 = y$ or $2^y = 4 = 2^2$ or $y = 2$.

$\therefore \quad \log_2 (\log_5 625) = 2$.

22. $\log_2 [\log_3 (\log_2 x)] = 1 = \log_2 2$

$\Leftrightarrow \quad \log_3 (\log_2 x) = 2 \quad \Leftrightarrow \quad \log_2 x = 3^2 = 9 \quad \Leftrightarrow \quad x = 2^9 = 512.$

23. $\log_2 \log_2 \log_3 (\log_3 27^3) = \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9]$

$= \log_2 \log_2 \log_3 (9 \log_3 3) = \log_2 \log_2 \log_3 9 \quad [\because \log_3 3 = 1]$

$= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3)$

$= \log_2 \log_2 2 = \log_2 1 = 0.$

24. $a^x = b^y \Rightarrow \log a^x = \log b^y \Rightarrow x \log a = y \log b \Rightarrow \dfrac{\log a}{\log b} = \dfrac{y}{x}.$

25. $360 = (2 \times 2 \times 2) \times (3 \times 3) \times 5.$

So, $\log 360 = \log (2^3 \times 3^2 \times 5) = \log 2^3 + \log 3^2 + \log 5 = 3 \log 2 + 2 \log 3 + \log 5.$

26. $\dfrac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32$

$= \log_{10} (125)^{1/3} - \log_{10} (4)^2 + \log_{10} 32 = \log_{10} 5 - \log_{10} 16 + \log_{10} 32$

$= \log_{10}\left(\dfrac{5 \times 32}{16}\right) = \log_{10} 10 = 1.$

27. $2\log_{10} 5 + \log_{10} 8 - \dfrac{1}{2}\log_{10} 4 = \log_{10} (5^2) + \log_{10} 8 - \log_{10} (4^{1/2})$

$= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10}\left(\dfrac{25 \times 8}{2}\right) = \log_{10} 100 = 2.$

28. $\log_a (ab) = x \quad \Leftrightarrow \quad \dfrac{\log ab}{\log a} = x \quad \Leftrightarrow \quad \dfrac{\log a + \log b}{\log a} = x$

$\Leftrightarrow \quad 1 + \dfrac{\log b}{\log a} = x \quad \Leftrightarrow \quad \dfrac{\log b}{\log a} = x - 1$

$\Leftrightarrow \quad \dfrac{\log a}{\log b} = \dfrac{1}{x-1} \quad \Leftrightarrow \quad 1 + \dfrac{\log a}{\log b} = 1 + \dfrac{1}{x-1}$

$\Leftrightarrow \quad \dfrac{\log b}{\log b} + \dfrac{\log a}{\log b} = \dfrac{x}{x-1} \quad \Leftrightarrow \quad \dfrac{\log b + \log a}{\log b} = \dfrac{x}{x-1}$

$\Leftrightarrow \quad \dfrac{\log (ab)}{\log b} = \dfrac{x}{x-1} \quad \Leftrightarrow \quad \log_b (ab) = \dfrac{x}{x-1}.$

29. $\log (4 \cdot \sqrt[3]{63}) = \log 4 + \log (\sqrt[3]{63}) = \log 4 + \log (63)^{1/3} = \log (2^2) + \log (7 \times 3^2)^{1/3}$

$= 2\log 2 + \dfrac{1}{3}\log 7 + \dfrac{2}{3}\log 3 = 2x + \dfrac{1}{3}z + \dfrac{2}{3}y.$

30. $\log_4 x + \log_2 x = 6 \Leftrightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6$

$\Leftrightarrow \frac{\log x}{2 \log 2} + \frac{\log x}{\log 2} = 6 \Leftrightarrow 3 \log x = 12 \log 2$

$\Leftrightarrow \log x = 4 \log 2 \Leftrightarrow \log x = \log (2^4) = \log 16 \Leftrightarrow x = 16.$

31. $\log_8 x + \log_8 \left(\frac{1}{6}\right) = \frac{1}{3} \Leftrightarrow \frac{\log x}{\log 8} + \frac{\log \frac{1}{6}}{\log 8} = \frac{1}{3}$

$\Leftrightarrow \log x + \log \frac{1}{6} = \frac{1}{3} \log 8 \Leftrightarrow \log x + \log \frac{1}{6} = \log (8^{1/3}) = \log 2$

$\Leftrightarrow \log x = \log 2 - \log \frac{1}{6} = \log \left(2 \times \frac{6}{1}\right) = \log 12$

$\therefore x = 12.$

32. $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10} (125 \times 8) = x$

$\Rightarrow x = \log_{10} (1000) = \log_{10} (10)^3 = 3 \log_{10} 10 = 3.$

33. Let $\log_9 27 = x$. Then, $9^x = 27 \Leftrightarrow (3^2)^x = 3^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2}.$

Let $\log_8 32 = y$. Then, $8^y = 32 \Leftrightarrow (2^3)^y = 2^5 \Leftrightarrow 3y = 5 \Leftrightarrow y = \frac{5}{3}.$

$\therefore \log_9 27 + \log_8 32 = \left(\frac{3}{2} + \frac{5}{3}\right) = \frac{19}{6}.$

34. Given expression $= \left(\frac{\log 3}{\log 5} \times \frac{\log 625}{\log 3}\right) = \frac{\log 625}{\log 5} = \frac{\log (5^4)}{\log 5} = \frac{4 \log 5}{\log 5} = 4.$

35. Given expression $= \frac{\log 9}{\log 4} \times \frac{\log 2}{\log 3}$ $[\because \log_5 5 = 1]$

$= \frac{\log 3^2}{\log 2^2} \times \frac{\log 2}{\log 3} = \frac{2 \log 3}{2 \log 2} \times \frac{\log 2}{\log 3} = 1.$

36. $\log_{12} 27 = a \Rightarrow \frac{\log 27}{\log 12} = a \Rightarrow \frac{\log 3^3}{\log (3 \times 2^2)} = a$

$\Rightarrow \frac{3 \log 3}{\log 3 + 2 \log 2} = a \Rightarrow \frac{\log 3 + 2 \log 2}{3 \log 3} = \frac{1}{a}$

$\Rightarrow \frac{\log 3}{3 \log 3} + \frac{2 \log 2}{3 \log 3} = \frac{1}{a} \Rightarrow \frac{2}{3} \frac{\log 2}{\log 3} = \frac{1}{a} - \frac{1}{3} = \left(\frac{3-a}{3a}\right)$

$\Rightarrow \frac{\log 2}{\log 3} = \left(\frac{3-a}{2a}\right) \Rightarrow \log 3 = \left(\frac{2a}{3-a}\right) \log 2.$

$\log_6 16 = \frac{\log 16}{\log 6} = \frac{\log 2^4}{\log (2 \times 3)} = \frac{4 \log 2}{\log 2 + \log 3} = \frac{4 \log 2}{\log 2 \left[1 + \left(\frac{2a}{3-a}\right)\right]}$

$= \frac{4}{\left(\frac{3+a}{3-a}\right)} = \frac{4(3-a)}{(3+a)}.$

37. $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$

$\Rightarrow \log_{10} [5 (5x + 1)] = \log_{10} [10 (x + 5)] \Rightarrow 5 (5x + 1) = 10 (x + 5)$

$\Rightarrow 5x + 1 = 2x + 10 \Rightarrow 3x = 9 \Rightarrow x = 3.$

38. $\log_5 (x^2 + x) - \log_5 (x + 1) = 2 \Rightarrow \log_5 \left(\frac{x^2 + x}{x + 1}\right) = 2$

$\Rightarrow \log_5 \left[\frac{x (x + 1)}{x + 1}\right] = 2 \Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25.$

39. Given expression $= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 = \log_{60} (3 \times 4 \times 5) = \log_{60} 60 = 1.$

40. Given expression $= \left(\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}\right)$

$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2.$

41. We know that : $a^{\log_a x} = x.$

$\therefore \quad 16^{\log_4 5} = (4^2)^{\log_4 5} = 4^{2 \log_4 5} = 4^{\log_4 (5^2)} = 4^{\log_4 25} = 25.$

42. $\log x + \log y = \log (x + y) \Rightarrow \log (x + y) = \log (xy)$

$\Rightarrow x + y = xy \Rightarrow y (x - 1) = x \Rightarrow y = \frac{x}{x - 1}.$

43. $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b) \Rightarrow \log (a + b) = \log \left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1.$

So, $a + b = 1.$

44. Given expression $= \log \left(\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab}\right) = \log 1 = 0.$

45. Given expression $= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

46. Given expression $= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} (abc) = 1.$

47. Given expression $= \log_x \left(\frac{p}{q}\right) + \log_x \left(\frac{q}{r}\right) + \log_x \left(\frac{r}{p}\right) = \log_x \left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log_x 1 = 0.$

48. $\log_{10} \left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70 = -\log_{10} (7 \times 10) = -(\log_{10} 7 + \log_{10} 10) = -(a + 1).$

49. $a = b^x,\ b = c^y,\ c = a^z \Rightarrow x = \log_b a,\ y = \log_c b,\ z = \log_a c$

$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c) \Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

50. $\log 27 = 1.431 \Rightarrow \log (3^3) = 1.431 \Rightarrow 3 \log 3 = 1.431$

$\Rightarrow \log 3 = 0.477$

$\therefore \quad \log 9 = \log (3^2) = 2 \log 3 = (2 \times 0.477) = 0.954.$

51. $\log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = \frac{10000}{3010} = \frac{1000}{301}$

52. $\log_{10} 5 = \log_{10}\left(\frac{10}{2}\right) = \log_{10} 10 - \log_{10} 2 = 1 - \log_{10} 2 = (1 - 0.3010) = 0.6990.$

53. $\log_{10} 80 = \log_{10} (8 \times 10) = \log_{10} 8 + \log_{10} 10 = \log_{10} (2^3) + 1 = 3 \log_{10} 2 + 1$
$= (3 \times 0.3010) + 1 = 1.9030.$

54. $(1000)^x = 3 \Rightarrow \log [(1000)^x] = \log 3 \Rightarrow x \log 1000 = \log 3$
$\Rightarrow x \log (10^3) = \log 3 \Rightarrow 3x \log 10 = \log 3$
$\Rightarrow 3x = \log 3 \Rightarrow x = \frac{0.477}{3} = 0.159.$

55. $\log_{10} 25 = \log_{10}\left(\frac{100}{4}\right) = \log_{10} 100 - \log_{10} 4 = 2 - 2 \log_{10} 2 = (2 - 2 \times 0.3010)$
$= (2 - 0.6020) = 1.3980.$

56. $\log_5 512 = \frac{\log 512}{\log 5} = \frac{\log 2^9}{\log\left(\frac{10}{2}\right)} = \frac{9 \log 2}{\log 10 - \log 2}$
$= \frac{(9 \times 0.3010)}{1 - 0.3010} = \frac{2.709}{0.699} = \frac{2709}{699} = 3.876.$

57. $\log_{10} (1.5) = \log_{10}\left(\frac{3}{2}\right) = \log_{10} 3 - \log_{10} 2 = (0.4771 - 0.3010) = 0.1761.$

58. $\log_{10} (2.8) = \log_{10}\left(\frac{28}{10}\right) = \log_{10} 28 - \log_{10} 10$
$= \log_{10} (7 \times 2^2) - 1 = \log_{10} 7 + 2 \log_{10} 2 - 1$
$= 0.8451 + 2 \times 0.3010 - 1 = 0.8451 + 0.602 - 1 = 0.4471.$

59. $\log (0.57) = \bar{1}.756 \Rightarrow \log 57 = 1.756$ [$\because$ mantissa will remain the same]
$\therefore \log 57 + \log (0.57)^3 + \log \sqrt{0.57}$
$= \log 57 + 3 \log\left(\frac{57}{100}\right) + \log\left(\frac{57}{100}\right)^{1/2}$
$= \log 57 + 3 \log 57 - 3 \log 100 + \frac{1}{2} \log 57 - \frac{1}{2} \log 100$
$= \frac{9}{2} \log 57 - \frac{7}{2} \log 100 = \frac{9}{2} \times 1.756 - \frac{7}{2} \times 2 = 7.902 - 7 = 0.902.$

60. $\log (2^{64}) = 64 \times \log 2 = (64 \times 0.30103) = 19.26592.$
Its characteristic is 19. Hence, the number of digits in 2^{64} is 20.

61. $\log 4^{50} = 50 \log 4 = 50 \log 2^2 = (50 \times 2) \log 2 = 100 \times \log 2 = (100 \times 0.30103) = 30.103.$
$\therefore$ Characteristic = 30. Hence, the number of digits in $4^{50} = 31$.

62. $\log 5^{20} = 20 \log 5 = 20 \times \left[\log\left(\frac{10}{2}\right)\right] = 20 (\log 10 - \log 2)$
$= 20 (1 - 0.3010) = 20 \times 0.6990 = 13.9800.$
$\therefore$ Characteristic = 13. Hence, the number of digits in 5^{20} is 14.

24. AREA

FUNDAMENTAL CONCEPTS

I. **Results on Triangles :**

1. Sum of the angles of a triangle is 180°.
2. The sum of any two sides of a triangle is greater than the third side.
3. **Pythagoras Theorem** : In a right-angled triangle,
$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2.$$
4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the ***median***.
5. The point where the three medians of a triangle meet, is called ***centroid***. The centroid divides each of the medians in the ratio 2 : 1.
6. In an isosceles triangle, the altitude from the vertex bisects the base.
7. The median of a triangle divides it into two triangles of the same area.
8. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

II. **Results on Quadrilaterals :**

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I. 1. Area of a rectangle = (Length × Breadth).

$$\therefore \quad \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}}\right) \text{ and Breadth} = \left(\frac{\text{Area}}{\text{Length}}\right)$$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square $= (\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a+b+c)$.

3. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi-perimeter $s = \frac{\Delta}{s}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus $= \frac{1}{2} \times$ (Product of diagonals).

3. Area of a trapezium $= \frac{1}{2} \times$ (sum of parallel sides) × distance between them.

VI. 1. Area of a circle $= \pi R^2$, where R is the radius.

2. Circumference of a circle $= 2\pi R$.

3. Length of an arc $= \frac{2\pi R\theta}{360}$, where θ is the central angle.

4. Area of a sector $= \frac{1}{2}(\text{arc} \times R) = \frac{\pi R^2\theta}{360}$.

VII. 1. Area of a semi-circle $= \frac{\pi R^2}{2}$.

2. Circumference of a semi-circle $= \pi R$.

SOLVED EXAMPLES

Ex. 1. ***One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.***

Sol. Other side $= \sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8$ m.

$\therefore$ Area $= (15 \times 8)\ m^2 = 120\ m^2$.

Ex. 2. ***A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.***

Sol. Let length $= 2x$ metres and breadth $= 3x$ metres.

Now, area $= \left(\frac{1}{6} \times 1000\right) m^2 = \left(\frac{5000}{3}\right) m^2$.

So, $2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right)$.

$\therefore$ Length $= 2x = \frac{100}{3}$ m $= 33\frac{1}{3}$ m and Breadth $= 3x = \left(3 \times \frac{50}{3}\right)$ m $= 50$ m.

Ex. 3. ***Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.***

Sol. Area of the carpet = Area of the room $= (13 \times 9)\ m^2 = 117\ m^2$.

Length of the carpet $= \left(\frac{\text{Area}}{\text{Width}}\right) = \left(117 \times \frac{4}{3}\right)$ m $= 156$ m.

$\therefore$ Cost of carpeting = Rs. (156×12.40) = Rs. 1934.40.

Ex. 4. ***If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.***

Sol. Let length $= x$ and breadth $= y$. Then,

$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.

Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy = 120$.

$\therefore$ Area $= xy = 120 \text{ cm}^2$.

Ex. 5. ***The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.***

Sol. Let breadth $= x$. Then, length $= 2x$. Then,

$(2x - 5)(x + 5) - 2x \times x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.

$\therefore$ Length of the rectangle $= 20$ cm.

Ex. 6. ***In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements.*** **(M.B.A. 2003)**

Sol. Let x and y be the sides of the rectangle. Then, Correct area $= xy$.

Calculated area $= \left(\frac{105}{100}x\right) \times \left(\frac{96}{100}y\right) = \frac{504}{500}xy$.

Error in measurement $= \left(\frac{504}{500}xy\right) - xy = \frac{4}{500}xy$.

$\therefore$ Error % $= \left[\frac{4}{500}xy \times \frac{1}{xy} \times 100\right]\% = \frac{4}{5}\% = 0.8\%$.

Ex. 7. ***A rectangular grassy plot 110 m by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.***

Sol. Area of the plot $= (110 \times 65) \text{ m}^2 = 7150 \text{ m}^2$.

Area of the plot excluding the path $= [(110 - 5) \times (65 - 5)] \text{ m}^2 = 6300 \text{ m}^2$.

$\therefore$ Area of the path $= (7150 - 6300) \text{ m}^2 = 850 \text{ m}^2$.

Cost of gravelling the path $= \text{Rs.} \left(850 \times \frac{80}{100}\right) = \text{Rs. } 680$.

Ex. 8. ***The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.*** **(S.S.C. 2003)**

Sol. Side of first square $= \left(\frac{40}{4}\right)$ cm $= 10$ cm;

Side of second square $= \left(\frac{32}{4}\right)$ cm $= 8$ cm.

Area of third square $= [(10)^2 - (8)^2] \text{ cm}^2 = (100 - 64) \text{ cm}^2 = 36 \text{ cm}^2$.

Side of third square $= \sqrt{36}$ cm $= 6$ cm.

$\therefore$ Required perimeter $= (6 \times 4)$ cm $= 24$ cm.

Ex. 9. ***A room 5m 55 cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.***

Sol. Area of the room $= (544 \times 374) \text{ cm}^2$.

Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.

Area of 1 tile $= (34 \times 34) \text{ cm}^2$.

$\therefore$ Number of tiles required $= \left(\frac{544 \times 374}{34 \times 34}\right) = 176$.

Ex. 10. ***Find the area of a square, one of whose diagonals is 3.8 m long.***

Sol. Area of the square $= \frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 3.8 \times 3.8\right) m^2 = 7.22\ m^2$.

Ex. 11. ***The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.*** **(Section Officers', 2003)**

Sol. Let the diagonals of the squares be $2x$ and $5x$ respectively.

$\therefore$ Ratio of their areas $= \frac{1}{2} \times (2x)^2 : \frac{1}{2} \times (5x)^2 = 4x^2 : 25x^2 = 4 : 25$.

Ex. 12. ***If each side of a square is increased by 25%, find the percentage change in its area.***

Sol. Let each side of the square be a. Then, area $= a^2$.

New side $= \frac{125a}{100} = \frac{5a}{4}$. New area $= \left(\frac{5a}{4}\right)^2 = \frac{25a^2}{16}$.

Increase in area $= \left(\frac{25a^2}{16} - a^2\right) = \frac{9a^2}{16}$.

$\therefore$ Increase % $= \left(\frac{9a^2}{16} \times \frac{1}{a^2} \times 100\right)\% = 56.25\%$.

Ex. 13. ***If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.***

Sol. Let x and y be the length and breadth of the rectangle respectively.

Then, $x - 4 = y + 3$ or $x - y = 7$ ...(*i*)

Area of the rectangle $= xy$; Area of the square $= (x - 4)(y + 3)$

$\therefore$ $(x - 4)(y + 3) = xy \iff 3x - 4y = 12$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 16$ and $y = 9$.

$\therefore$ Perimeter of the rectangle $= 2(x + y) = [2(16 + 9)]$ cm $= 50$ cm.

Ex. 14. ***A room is half as long again as it is broad. The cost of carpeting the room at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m² is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.***

Sol. Let breadth $= x$ metres, length $= \frac{3x}{2}$ metres, height = H metres.

Area of the floor $= \left(\frac{\text{Total cost of carpeting}}{\text{Rate}/m^2}\right) m^2 = \left(\frac{270}{5}\right) m^2 = 54\ m^2$.

$\therefore$ $x \times \frac{3x}{2} = 54 \iff x^2 = \left(54 \times \frac{2}{3}\right) = 36 \iff x = 6$.

So, breadth = 6 m and length $= \left(\frac{3}{2} \times 6\right)$ m = 9 m.

Now, papered area $= \left(\frac{1720}{10}\right) m^2 = 172\ m^2$.

Area of 1 door and 2 windows $= 8\ m^2$.

Total area of 4 walls $= (172 + 8)\ m^2 = 180\ m^2$.

$\therefore$ $2(9 + 6) \times H = 180 \iff H = \left(\frac{180}{30}\right) = 6$ m.

Ex. 15. ***Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.***

Sol. Let $a = 13$, $b = 14$ and $c = 15$. Then, $s = \frac{1}{2}(a + b + c) = 21.$

$\therefore$ $(s - a) = 8$, $(s - b) = 7$ and $(s - c) = 6$.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \times 8 \times 7 \times 6} = 84 \text{ cm}^2.$

Ex. 16. ***Find the area of a right-angled triangle whose base is 12 cm and hypotenuse 13 cm.***

Sol. Height of the triangle $= \sqrt{(13)^2 - (12)^2}$ cm $= \sqrt{25}$ cm $= 5$ cm.

$\therefore$ Its area $= \frac{1}{2} \times \text{Base} \times \text{Height} = \left(\frac{1}{2} \times 12 \times 5\right) \text{cm}^2 = 30 \text{ cm}^2.$

Ex. 17. ***The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.***

Sol. Area of the field $= \frac{\text{Total cost}}{\text{Rate}} = \left(\frac{333.18}{24.68}\right)$ hectares $= 13.5$ hectares

$= (13.5 \times 10000) \text{ m}^2 = 135000 \text{ m}^2.$

Let altitude $= x$ metres and base $= 3x$ metres.

Then, $\frac{1}{2} \times 3x \times x = 135000 \Leftrightarrow x^2 = 90000 \Leftrightarrow x = 300.$

$\therefore$ Base $= 900$ m and Altitude $= 300$ m.

Ex. 18. ***The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.***

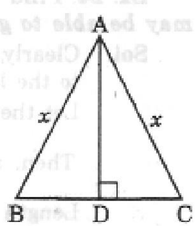
Sol. Let ABC be the isosceles triangle and AD be the altitude.

Let AB = AC = x. Then, BC = $(32 - 2x)$.

Since, in an isosceles triangle, the altitude bisects the base, so BD = DC = $(16 - x)$.

In Δ ADC, $AC^2 = AD^2 + DC^2 \Rightarrow x^2 = (8)^2 + (16 - x)^2$

$\Rightarrow 32x = 320 \Rightarrow x = 10.$



$\therefore$ BC $= (32 - 2x) = (32 - 20)$ cm $= 12$ cm.

Hence, required area $= \left(\frac{1}{2} \times BC \times AD\right) = \left(\frac{1}{2} \times 12 \times 10\right) \text{cm}^2 = 60 \text{ cm}^2.$

Ex. 19. ***Find the length of the altitude of an equilateral triangle of side $3\sqrt{3}$ cm.***

Sol. Area of the triangle $= \frac{\sqrt{3}}{4} \times (3\sqrt{3})^2 = \frac{27\sqrt{3}}{4}$. Let the height be h.

Then, $\frac{1}{2} \times 3\sqrt{3} \times h = \frac{27\sqrt{3}}{4} \Leftrightarrow h = \frac{27\sqrt{3}}{4} \times \frac{2}{3\sqrt{3}} = \frac{9}{2} = 4.5$ cm.

Ex. 20. ***In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.***

Sol. Let the bases of the two triangles be x and y and their heights be $3h$ and $4h$ respectively. Then,

$$\frac{\frac{1}{2} \times x \times 3h}{\frac{1}{2} \times y \times 4h} = \frac{4}{3} \Leftrightarrow \frac{x}{y} = \left(\frac{4}{3} \times \frac{4}{3}\right) = \frac{16}{9}.$$

$\therefore$ Required ratio $= 16 : 9$.

Ex. 21. ***The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.***

Sol. Let the height of the parallelogram be x cm. Then, base = $(2x)$ cm.

$\therefore$ $2x \times x = 72 \Leftrightarrow 2x^2 = 72 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6.$

Hence, height of the parallelogram = 6 cm.

Ex. 22. ***Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.***

Sol. Let other diagonal = $2x$ cm.

Since diagonals of a rhombus bisect each other at right angles, we have :

$(20)^2 = (12)^2 + x^2 \Leftrightarrow x = \sqrt{(20)^2 - (12)^2} = \sqrt{256} = 16$ cm.

So, other diagonal = 32 cm.

$\therefore$ Area of rhombus $= \frac{1}{2} \times$ (Product of diagonals) $= \left(\frac{1}{2} \times 24 \times 32\right) \text{cm}^2 = 384 \text{ cm}^2.$

Ex. 23. ***The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm², find the lengths of the parallel sides.*** **(R.R.B. 2002)**

Sol. Let the two parallel sides of the trapezium be a cm and b cm.

Then, $a - b = 4$ ...(*i*)

And, $\frac{1}{2} \times (a + b) \times 19 = 475 \Leftrightarrow (a + b) = \left(\frac{475 \times 2}{19}\right) \Leftrightarrow a + b = 50$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $a = 27$, $b = 23$.

So, the two parallel sides are 27 cm and 23 cm.

Ex. 24. ***Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq. metres.*** **(M.A.T. 2003)**

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.

Let the length of the rope be R metres.

Then, $\pi R^2 = 9856 \Leftrightarrow R^2 = \left(9856 \times \frac{7}{22}\right) = 3136 \Leftrightarrow R = 56.$

$\therefore$ Length of the rope = 56 m.

Ex. 25. ***The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.***

Sol. Area = $(13.86 \times 10000) \text{ m}^2 = 138600 \text{ m}^2$.

$\pi R^2 = 138600 \Leftrightarrow R^2 = \left(138600 \times \frac{7}{22}\right) \Leftrightarrow R = 210$ m.

Circumference $= 2\pi R = \left(2 \times \frac{22}{7} \times 210\right)$ m = 1320 m.

$\therefore$ Cost of fencing = Rs. (1320×4.40) = Rs. 5808.

Ex. 26. ***The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph ?***

Sol. Distance to be covered in 1 min. $= \left(\frac{66 \times 1000}{60}\right)$ m = 1100 m.

Circumference of the wheel $= \left(2 \times \frac{22}{7} \times 0.70\right)$ m = 4.4 m.

$\therefore$ Number of revolutions per min. $= \left(\frac{1100}{4.4}\right) = 250.$

Ex. 27. *A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.*

Sol. Distance covered in one revolution $= \left(\frac{88 \times 1000}{1000}\right)$ m = 88 m.

$\therefore \quad 2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14$ m.

Ex. 28. *The inner circumference of a circular race track, 14 m wide, is 440 m. Find the radius of the outer circle.*

Sol. Let inner radius be r metres. Then, $2\pi r = 440 \Rightarrow r = \left(440 \times \frac{7}{44}\right) = 70$ m.

$\therefore$ Radius of outer circle = (70 + 14) m = 84 m.

Ex. 29. *Two concentric circles form a ring. The inner and outer circumferences of the ring are $50\frac{2}{7}$ m and $75\frac{3}{7}$ m respectively. Find the width of the ring.*

Sol. Let the inner and outer radii be r and R metres.

Then, $2\pi r = \frac{352}{7} \Rightarrow r = \left(\frac{352}{7} \times \frac{7}{22} \times \frac{1}{2}\right) = 8$ m.

$2\pi R = \frac{528}{7} \Rightarrow R = \left(\frac{528}{7} \times \frac{7}{22} \times \frac{1}{2}\right) = 12$ m.

$\therefore$ Width of the ring = (R − r) = (12 − 8) m = 4 m.

Ex. 30. *A sector of 120°, cut out from a circle, has an area of $9\frac{3}{7}$ sq. cm. Find the radius of the circle.* **(C.B.I. 1997)**

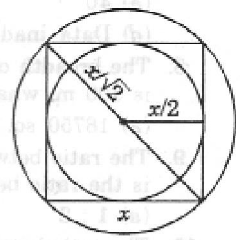
Sol. Let the radius of the circle be r cm. Then,

$$\frac{\pi r^2 \theta}{360} = \frac{66}{7} \Leftrightarrow \frac{22}{7} \times r^2 \times \frac{120}{360} = \frac{66}{7} \Leftrightarrow r^2 = \left(\frac{66}{7} \times \frac{7}{22} \times 3\right) = 9 \Leftrightarrow r = 3.$$

Hence, radius = 3 cm.

Ex. 31. *Find the ratio of the areas of the incircle and circumcircle of a square.*

Sol. Let the side of the square be x. Then, its diagonal $= \sqrt{2}\,x$.



Radius of incircle $= \frac{x}{2}$ and

radius of circumcircle $= \frac{\sqrt{2}\,x}{2} = \frac{x}{\sqrt{2}}$.

$\therefore$ Required ratio $= \left(\frac{\pi x^2}{4} : \frac{\pi x^2}{2}\right) = \frac{1}{4} : \frac{1}{2} = 1 : 2.$

Ex. 32. *If the radius of a circle is decreased by 50%, find the percentage decrease in its area.*

Sol. Let original radius = R. New radius $= \frac{50}{100} R = \frac{R}{2}$.

Original area $= \pi R^2$ and New area $= \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$.

$\therefore$ Decrease in area $= \left(\frac{3\pi R^2}{4} \times \frac{1}{\pi R^2} \times 100\right)\% = 75\%.$

EXERCISE 24A

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark* (✓) *against the correct answer* :

1. The length of a room is 5.5 m and width is 3.75 m. Find the cost of paving the floor by slabs at the rate of Rs. 800 per sq. metre. **(IGNOU, 2003)**
(*a*) Rs. 15,000 (*b*) Rs. 15,550 (*c*) Rs. 15,600 (*d*) Rs. 16,500

2. The length of a rectangle is 18 cm and its breadth is 10 cm. When the length is increased to 25 cm, what will be the breadth of the rectangle if the area remains the same ?
(*a*) 7 cm (*b*) 7.1 cm (*c*) 7.2 cm (*d*) 7.3 cm

3. A rectangular plot measuring 90 metres by 50 metres is to be enclosed by wire fencing. If the poles of the fence are kept 5 metres apart, how many poles will be needed ?
(*a*) 55 (*b*) 56 (*c*) 57 (*d*) 58

4. The length of a rectangular plot is 60% more than its breadth. If the difference between the length and the breadth of that rectangle is 24 cm, what is the area of that rectangle ? **(Bank P.O. 1998)**
(*a*) 2400 sq. cm (*b*) 2480 sq. cm (*c*) 2560 sq. cm
(*d*) Data inadequate (*e*) None of these

5. A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet ? **(M.A.T. 2003)**
(*a*) 46 (*b*) 81 (*c*) 126 (*d*) 252

6. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is : **(Section Officers', 2003)**
(*a*) 1520 m^2 (*b*) 2420 m^2 (*c*) 2480 m^2 (*d*) 2520 m^2

7. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot @ Rs. 26.50 per metre is Rs. 5300, what is the length of the plot in metres ? **(Bank P.O. 1999)**
(*a*) 40 (*b*) 50 (*c*) 120
(*d*) Data inadequate (*e*) None of these

8. The breadth of a rectangular field is 60% of its length. If the perimeter of the field is 800 m, what is the area of the field ?
(*a*) 18750 sq. m (*b*) 37500 sq. m (*c*) 40000 sq. m (*d*) 48000 sq. m

9. The ratio between the length and the perimeter of a rectangular plot is 1 : 3. What is the ratio between the length and breadth of the plot ?
(*a*) 1 : 2 (*b*) 2 : 1 (*c*) 3 : 2 (*d*) Data inadequate

10. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km / hr completes one round in 8 minutes, then the area of the park (in sq. m) is : **(S.S.C. 2003)**
(*a*) 15360 (*b*) 153600 (*c*) 30720 (*d*) 307200

11. The length of a rectangular hall is 5 m more than its breadth. The area of the hall is 750 m^2. The length of the hall is : **(S.S.C. 2004)**
(*a*) 15 m (*b*) 22.5 m (*c*) 25 m (*d*) 30 m

12. The area of a rectangle is 460 square metres. If the length is 15% more than the breadth, what is the breadth of the rectangular field ? **(Bank P.O. 2003)**
(*a*) 15 metres (*b*) 26 metres (*c*) 34.5 metres
(*d*) Cannot be determined (*e*) None of these

13. A rectangular field is to be fenced on three sides leaving a side of 20 feet uncovered. If the area of the field is 680 sq. feet, how many feet of fencing will be required ?

(*a*) 34 (*b*) 40 (*c*) 68 (*d*) 88

(R.R.B. 2002)

14. The ratio between the perimeter and the breadth of a rectangle is 5 : 1. If the area of the rectangle is 216 sq. cm, what is the length of the rectangle ?

(*a*) 16 cm (*b*) 18 cm (*c*) 24 cm

(*d*) Data inadequate (*e*) None of these **(B.S.R.B. 1998)**

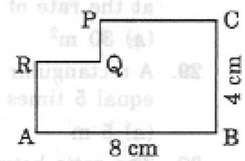
15. A farmer wishes to start a 100 sq. m rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden letting his house compound wall act as the fourth side fencing. The dimension of the garden is : **(R.R.B. 2003)**

(*a*) 15 m × 6.67 m (*b*) 20 m × 5 m (*c*) 30 m × 3.33 m (*d*) 40 m × 2.5 m

16. The sides of a rectangular field are in the ratio 3 : 4. If the area of the field is 7500 sq. m, the cost of fencing the field @ 25 paise per metre is : **(R.R.B. 2004)**

(*a*) Rs. 55.50 (*b*) Rs. 67.50 (*c*) Rs. 86.50 (*d*) Rs. 87.50

17. A rectangle of certain dimensions is chopped off from one corner of a larger rectangle as shown. AB = 8 cm and BC = 4 cm. The perimeter of the figure ABCPQRA (in cm) is : **(Asstt. Grade, 1998)**



(*a*) 24 (*b*) 28 (*c*) 36 (*d*) 48

18. A large field of 700 hectares is divided into two parts. The difference of the areas of the two parts is one-fifth of the average of the two areas. What is the area of the smaller part in hectares ?

(*a*) 225 (*b*) 280 (*c*) 300 (*d*) 315

19. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper ? **(S.S.C. 2000)**

(*a*) 140 cm^2 (*b*) 240 cm^2 (*c*) 560 cm^2 (*d*) None of these

20. A rectangular plot is half as long again as it is broad and its area is $\frac{2}{3}$ hectares. Then, its length is :

(*a*) 100 m (*b*) 33.33 m (*c*) 66.66 m (*d*) $\frac{100\sqrt{3}}{3}$ m

21. A courtyard 25 m long and 16 m broad is to be paved with bricks of dimensions 20 cm by 10 cm. The total number of bricks required is :

(*a*) 18000 (*b*) 20000 (*c*) 25000 (*d*) None of these

22. The cost of carpeting a room 18 m long with a carpet 75 cm wide at Rs. 4.50 per metre is Rs. 810. The breadth of the room is :

(*a*) 7 m (*b*) 7.5 m (*c*) 8 m (*d*) 8.5 m

23. The diagonal of the floor of a rectangular closet is $7\frac{1}{2}$ feet. The shorter side of the closet is $4\frac{1}{2}$ feet. What is the area of the closet in square feet ? **(M.B.A. 2003)**

(*a*) $5\frac{1}{4}$ (*b*) $13\frac{1}{2}$ (*c*) 27 (*d*) 37

24. The length of a rectangle is three times of its width. If the length of the diagonal is $8\sqrt{10}$ cm, then the perimeter of the rectangle is : (S.S.C. 2000)
(a) $15\sqrt{10}$ cm (b) $16\sqrt{10}$ cm (c) $24\sqrt{10}$ cm (d) 64 cm

25. The diagonal of a rectangle is thrice its smaller side. The ratio of the length to the breadth of the rectangle is :
(a) 3 : 1 (b) $\sqrt{3}$: 1 (c) $\sqrt{2}$: 1 (d) $2\sqrt{2}$: 1

26. A rectangular carpet has an area of 120 sq. metres and a perimeter of 46 metres. The length of its diagonal is :
(a) 15 m (b) 16 m (c) 17 m (d) 20 m

27. The diagonal of a rectangle is $\sqrt{41}$ cm and its area is 20 sq. cm. The perimeter of the rectangle must be : (Hotel Management, 2002)
(a) 9 cm (b) 18 cm (c) 20 cm (d) 41 cm

28. A took 15 seconds to cross a rectangular field diagonally walking at the rate of 52 m / min and B took the same time to cross the same field along its sides walking at the rate of 68 m / min. The area of the field is : (S.S.C. 2003)
(a) 30 m^2 (b) 40 m^2 (c) 50 m^2 (d) 60 m^2

29. A rectangular carpet has an area of 60 sq. m. If its diagonal and longer side together equal 5 times the shorter side, the length of the carpet is :
(a) 5 m (b) 12 m (c) 13 m (d) 14.5 m

30. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field ?
(a) 40 metres (b) 60 metres (c) 65 metres
(d) Cannot be determined (e) None of these

31. The length of a blackboard is 8 cm more than its breadth. If the length is increased by 7 cm and breadth is decreased by 4 cm, the area remains the same. The length and breadth of the blackboard (in cm) will be :
(a) 28, 20 (b) 34, 26 (c) 40, 32 (d) 56, 48

32. If the length and breadth of a rectangular room are each increased by 1 m, then the area of floor is increased by 21 sq. m. If the length is increased by 1 m and breadth is decreased by 1 m, then the area is decreased by 5 sq. m. The perimeter of the floor is : (M.B.A. 2002)
(a) 30 m (b) 32 m (c) 36 m (d) 40 m

33. The percentage increase in the area of a rectangle, if each of its sides is increased by 20%, is : (M.A.T. 2004)
(a) 40% (b) 42% (c) 44% (d) 46%

34. A rectangle has width a and length b. If the width is decreased by 20% and the length is increased by 10%, then what is the area of the new rectangle in percentage compared to 'ab' ? (R.R.B. 2002)
(a) 80% (b) 88% (c) 110% (d) 120%

35. If the length and breadth of a rectangular plot be increased by 50% and 20% respectively, then how many times will its area be increased ? (Bank P.O. 2003)
(a) $1\frac{1}{3}$ (b) 2 (c) $3\frac{2}{5}$ (d) $4\frac{1}{5}$ (e) None of these

36. A towel, when bleached, was found to have lost 20% of its length and 10% of its breadth. The percentage of decrease in area is : (N.I.F.T. 1997)
(a) 10% (b) 10.08% (c) 20% (d) 28%

37. The length of a rectangle is halved, while its breadth is tripled. What is the percentage change in area ? **(S.S.C. 2000)**
(*a*) 25% increase (*b*) 50% increase (*c*) 50% decrease (*d*) 75% decrease

38. The length of a rectangle is decreased by *r*%, and the breadth is increased by $(r + 5)$%. Find *r*, if the area of the rectangle is unaltered. **(SCMHRD, 2002)**
(*a*) 5 (*b*) 8 (*c*) 10 (*d*) 15 (*e*) 20

39. The length of a rectangle is increased by 60%. By what percent would the width have to be decreased so as to maintain the same area ? **(M.A.T. 2003)**
(*a*) $37\frac{1}{2}$% (*b*) 60% (*c*) 75% (*d*) 120%

40. If the area of a rectangular plot increases by 30% while its breadth remains same, what will be the ratio of the areas of new and old figures ? **(Bank P.O. 2003)**
(*a*) 1 : 3 (*b*) 3 : 1 (*c*) 4 : 7 (*d*) 10 : 13 (*e*) None of these

41. A typist uses a sheet measuring 20 cm by 30 cm lengthwise. If a margin of 2 cm is left on each side and a 3 cm margin on top and bottom, then percent of the page used for typing is : **(M.A.T. 1998)**
(*a*) 40 (*b*) 60 (*c*) 64 (*d*) 72

42. A room is 15 feet long and 12 feet broad. A mat has to be placed on the floor of this room leaving $1\frac{1}{2}$ feet space from the walls. What will be the cost of the mat at the rate of Rs. 3.50 per square feet ? **(R.R.B. 2002)**
(*a*) Rs. 378 (*b*) Rs. 472.50 (*c*) Rs. 496 (*d*) Rs. 630

43. What will be the cost of gardening 1 metre broad boundary around a rectangular plot having perimeter of 340 metres at the rate of Rs. 10 per square metre ?
(*a*) Rs. 1700 (*b*) Rs. 3400 (*c*) Rs. 3440
(*d*) Cannot be determined (*e*) None of these **(Bank P.O. 2003)**

44. 2 metres broad pathway is to be constructed around a rectangular plot on the inside. The area of the plot is 96 sq. m. The rate of construction is Rs. 50 per square metre. Find the total cost of the construction. **(S.B.I.P.O. 2000)**
(*a*) Rs. 2400 (*b*) Rs. 4000 (*c*) Rs. 4800
(*d*) Data inadequate (*e*) None of these

45. Within a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. How wide should the walk be ?
(*a*) 1 m (*b*) 2 m (*c*) 2.1 m (*d*) 2.5 m

46. A rectangular lawn 55 m by 35 m has two roads each 4 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of gravelling the roads at 75 paise per sq. metre is :
(*a*) Rs. 254.50 (*b*) Rs. 258 (*c*) Rs. 262.50 (*d*) Rs. 270

47. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq. m, then what is the width of the road ? **(M.A.T. 1997)**
(*a*) 2.91 m (*b*) 3 m (*c*) 5.82 m (*d*) None of these

48. A housing society has been allotted a square piece of land measuring 2550.25 sq. m. What is the side of the plot ?
(*a*) 50.25 m (*b*) 50.5 m (*c*) 50.65 m (*d*) None of these

49. The cost of cultivating a square field at the rate of Rs. 135 per hectare is Rs. 1215. The cost of putting a fence around it at the rate of 75 paise per metre would be :
(*a*) Rs. 360 (*b*) Rs. 810 (*c*) Rs. 900 (*d*) Rs. 1800

50. The perimeters of five squares are 24 cm, 32 cm, 40 cm, 76 cm and 80 cm respectively. The perimeter of another square equal in area to the sum of the areas of these squares is : **(S.S.C. 2004)**
(*a*) 31 cm (*b*) 62 cm (*c*) 124 cm (*d*) 961 cm

51. The number of marble slabs of size 20 cm × 30 cm required to pave the floor of a square room of side 3 metres, is :
(*a*) 100 (*b*) 150 (*c*) 225 (*d*) 250

52. 50 square stone slabs of equal size were needed to cover a floor area of 72 sq. m. The length of each stone slab is : **(S.S.C. 2003)**
(*a*) 102 cm (*b*) 120 cm (*c*) 201 cm (*d*) 210 cm

53. The length and breadth of the floor of the room are 20 feet and 10 feet respectively. Square tiles of 2 feet length of different colours are to be laid on the floor. Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there ?
(*a*) 16 (*b*) 24 (*c*) 32
(*d*) 48 (*e*) None of these **(S.B.I.P.O. 2000)**

54. What is the least number of square tiles required to pave the floor of a room 15 m 17 cm long and 9 m 2 cm broad ? **(S.S.C. 2003)**
(*a*) 814 (*b*) 820 (*c*) 840 (*d*) 844

55. A rectangular room can be partitioned into two equal square rooms by a partition 7 metres long. What is the area of the rectangular room in square metres ?
(*a*) 49 (*b*) 147 (*c*) 196 (*d*) None of these

56. The perimeter of a square is 48 cm. The area of a rectangle is 4 cm^2 less than the area of the square. If the length of the rectangle is 14 cm, then its perimeter is :
(*a*) 24 cm (*b*) 48 cm (*c*) 50 cm (*d*) 54 cm
(S.S.C. 2002)

57. The area of a rectangle is thrice that of a square. If the length of the rectangle is 40 cm and its breadth is $\frac{3}{2}$ times that of the side of the square, then the side of the square is :
(*a*) 15 cm (*b*) 20 cm (*c*) 30 cm (*d*) 60 cm

58. If the perimeter of a rectangle and a square, each is equal to 80 cm and the difference of their areas is 100 sq. cm, the sides of the rectangle are :
(*a*) 25 cm, 15 cm (*b*) 28 cm, 12 cm (*c*) 30 cm, 10 cm (*d*) 35 cm, 15 cm

59. The cost of fencing a square field @ Rs. 20 per metre is Rs. 10,080. How much will it cost to lay a three metre wide pavement along the fencing inside the field @ Rs. 50 per sq. metre ?
(*a*) Rs. 37,350 (*b*) Rs. 73,800 (*c*) Rs. 77,400 (*d*) None of these

60. A park square in shape has a 3 metre wide road inside it running along its sides. The area occupied by the road is 1764 square metres. What is the perimeter along the outer edge of the road ? **(Bank P.O. 1998)**
(*a*) 576 metres (*b*) 600 metres (*c*) 640 metres
(*d*) Data inadequate (*e*) None of these

61. A man walked diagonally across a square lot. Approximately, what was the percent saved by not walking along the edges ? **(M.B.A. 2003)**
(*a*) 20 (*b*) 24 (*c*) 30 (*d*) 33

62. A man walking at the speed of 4 kmph crosses a square field diagonally in 3 minutes. The area of the field is :
(*a*) 18000 m^2 (*b*) 19000 m^2 (*c*) 20000 m^2 (*d*) 25000 m^2

63. If the length of the diagonal of a square is 20 cm, then its perimeter must be :
(a) $10\sqrt{2}$ cm (b) 40 cm (c) $40\sqrt{2}$ cm (d) 200 cm
(R.R.B. 2003)

64. The area of a square field is 69696 cm^2. Its diagonal will be equal to :
(a) 313.296 m (b) 353.296 m (c) 373.296 m (d) 393.296 m
(S.S.C. 1999)

65. What will be the length of the diagonal of that square plot whose area is equal to the area of a rectangular plot of length 45 metres and breadth 40 metres ?
(a) 42.5 metres (b) 60 metres (c) 75 metres
(d) Data inadequate (e) None of these
(Bank P.O. 1999)

66. The length of a rectangle is 20% more than its breadth. What will be the ratio of the area of a rectangle to that of a square whose side is equal to the breadth of the rectangle ?
(Bank P.O. 2000)
(a) 2 : 1 (b) 5 : 6 (c) 6 : 5
(d) Data inadequate (e) None of these

67. A square and a rectangle have equal areas. If their perimeters are p_1 and p_2 respectively, then :
(a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) None of these

68. If the perimeters of a square and a rectangle are the same, then the area A and B enclosed by them would satisfy the condition :
(a) A < B (b) A ≤ B (c) A > B (d) A ≥ B

69. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square, is :
(S.S.C. 2002)
(a) 8 cm (b) $8\sqrt{2}$ cm (c) $4\sqrt{2}$ cm (d) 16 cm

70. The ratio of the area of a square to that of the square drawn on its diagonal, is :
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5
(IGNOU, 2003)

71. The ratio of the areas of two squares, one having its diagonal double than the other, is :
(a) 2 : 1 (b) 2 : 3 (c) 3 : 1 (d) 4 : 1

72. If the ratio of areas of two squares is 225 : 256, then the ratio of their perimeters is :
(a) 225 : 256 (b) 256 : 225 (c) 15 : 16 (d) 16 : 15
(S.S.C. 2004)

73. Of the two square fields, the area of one is 1 hectare while the other one is broader by 1%. The difference in their areas is :
(a) 100 m^2 (b) 101 m^2 (c) 200 m^2 (d) 201 m^2

74. If each side of a square is increased by 50%, the ratio of the area of the resulting square to that of the given square is :
(a) 4 : 5 (b) 5 : 4 (c) 4 : 9 (d) 9 : 4

75. What happens to the area of a square when its side is halved ? Its area will :
(a) remain same (b) become half (c) become one-fourth (d) become double
(R.R.B. 2003)

76. An error of 2% in excess is made while measuring the side of a square. The percentage of error in the calculated area of the square is :
(C.D.S. 2003)
(a) 2% (b) 2.02% (c) 4% (d) 4.04%

77. If the area of a square increases by 69%, then the side of the square increases by :
(a) 13% (b) 30% (c) 39% (d) 69%
(M.A.T. 1998)

78. If the diagonal of a square is made 1.5 times, then the ratio of the areas of two squares is :
(*a*) 4 : 3 (*b*) 4 : 5 (*c*) 4 : 7 (*d*) 4 : 9

79. The length and breadth of a square are increased by 40% and 30% respectively. The area of the resulting rectangle exceeds the area of the square by :
(*a*) 35% (*b*) 42% (*c*) 62% (*d*) 82%

80. The length of one pair of opposite sides of a square is increased by 5 cm on each side; the ratio of the length and the breadth of the newly formed rectangle becomes 3 : 2. What is the area of the original square ? **(Bank P.O. 1999)**
(*a*) 25 sq. cm (*b*) 81 sq. cm (*c*) 100 sq. cm
(*d*) 225 sq. cm (*e*) None of these

81. If the side of a square is increased by 5 cm, the area increases by 165 sq. cm. The side of the square is :
(*a*) 12 cm (*b*) 13 cm (*c*) 14 cm (*d*) 15 cm

82. The difference of the areas of two squares drawn on two line segments of different lengths is 32 sq. cm. Find the length of the greater line segment if one is longer than the other by 2 cm. **(S.S.C. 2003)**
(*a*) 7 cm (*b*) 9 cm (*c*) 11 cm (*d*) 16 cm

83. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle. **(S.S.C. 2002)**
(*a*) 17 cm (*b*) 26 cm (*c*) 30 cm (*d*) 34 cm

84. A tank is 25 m long, 12 m wide and 6 m deep. The cost of plastering its walls and bottom at 75 paise per sq. m, is : **(C.B.I. 1997)**
(*a*) Rs. 456 (*b*) Rs. 458 (*c*) Rs. 558 (*d*) Rs. 568

85. The dimensions of a room are 10 m × 7 m × 5 m. There are 2 doors and 3 windows in the room. The dimensions of the doors are 1 m × 3 m. One window is of size 2 m × 1.5 m and the other two windows are of size 1 m × 1.5 m. The cost of painting the walls at Rs. 3 per m^2 is :
(*a*) Rs. 474 (*b*) Rs. 578.50 (*c*) Rs. 684 (*d*) Rs. 894

86. The cost of papering the four walls of a room is Rs. 475. Each one of the length, breadth and height of another room is double that of this room. The cost of papering the walls of this new room is :
(*a*) Rs. 712.50 (*b*) Rs. 950 (*c*) Rs. 1425 (*d*) Rs. 1900

87. The ratio of height of a room to its semi-perimeter is 2 : 5. It costs Rs. 260 to paper the walls of the room with paper 50 cm wide at Rs. 2 per metre allowing an area of 15 sq. m for doors and windows. The height of the room is :
(*a*) 2.6 m (*b*) 3.9 m (*c*) 4 m (*d*) 4.2 m

88. The base of a triangle is 15 cm and height is 12 cm. The height of another triangle of double the area having the base 20 cm is : **(S.S.C. 2002)**
(*a*) 8 cm (*b*) 9 cm (*c*) 12.5 cm (*d*) 18 cm

89. ABC is a triangle with base AB. D is a point on AB such that AB = 5 and DB = 3. What is the ratio of the area of Δ ADC to the area of Δ ABC ? **(S.S.C. 2000)**
(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 2 : 5 (*d*) 3 : 5

90. The area of a right-angled triangle is 40 times its base. What is its height ?
(*a*) 45 cm (*b*) 60 cm (*c*) 80 cm
(*d*) Data inadequate (*e*) None of these **(B.S.R.B. 1998)**

91. If the area of a triangle is 1176 cm^2 and base : corresponding altitude is 3 : 4, then the altitude of the triangle is : **(S.S.C. 2000)**
(*a*) 42 cm (*b*) 52 cm (*c*) 54 cm (*d*) 56 cm

92. The three sides of a triangle are 5 cm, 12 cm and 13 cm respectively. Then, its area is :
(a) $10\sqrt{3}$ cm^2 (b) $10\sqrt{6}$ cm^2 (c) 20 cm^2 (d) 30 cm^2

93. The sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is : **(M.A.T. 2004)**
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

94. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is : **(S.S.C. 2004)**
(a) 6 cm (b) 12 cm (c) 36 cm (d) 72 cm

95. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid-points of the sides of this triangle is : **(S.S.C. 2003)**
(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) 3 (d) 6

96. One side of a right-angled triangle is twice the other, and the hypotenuse is 10 cm. The area of the triangle is :
(a) 20 cm^2 (b) $33\frac{1}{3}$ cm^2 (c) 40 cm^2 (d) 50 cm^2

97. The perimeter of a right-angled triangle is 60 cm. Its hypotenuse is 26 cm. The area of the triangle is : **(M.B.A. 2002)**
(a) 120 cm^2 (b) 240 cm^2 (c) 390 cm^2 (d) 780 cm^2

98. If the perimeter of an isosceles right triangle is $(6 + 3\sqrt{2})$ m, then the area of the triangle is : **(M.A.T. 2003)**
(a) 4.5 m^2 (b) 5.4 m^2 (c) 9 m^2 (d) 81 m^2

99. The perimeter of a triangle is 30 cm and its area is 30 cm^2. If the largest side measures 13 cm, then what is the length of the smallest side of the triangle ?
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm **(S.S.C. 2003)**

100. If the area of an equilateral triangle is $24\sqrt{3}$ sq. cm, then its perimeter is :
(a) $2\sqrt{6}$ cm (b) $4\sqrt{6}$ cm (c) $12\sqrt{6}$ cm (d) 96 cm

101. The height of an equilateral triangle is 10 cm. Its area is : **(S.S.C. 2003)**
(a) $\frac{100}{3}$ cm^2 (b) 30 cm^2 (c) 100 cm^2 (d) $\frac{100}{\sqrt{3}}$ cm^2

102. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}$ cm, $2\sqrt{3}$ cm and $5\sqrt{3}$ cm. The perimeter (in cm) of the triangle is :
(a) 24 (b) 32 (c) 48 (d) 64

103. If x is the length of a median of an equilateral triangle, then its area is :
(a) x^2 (b) $\frac{1}{2}x^2$ (c) $\frac{\sqrt{3}}{2}x^2$ (d) $\frac{\sqrt{3}}{3}x^2$

104. If the area of a square with side a is equal to the area of a triangle with base a, then the altitude of the triangle is : **(B.S.F. 2001)**
(a) $\frac{a}{2}$ (b) a (c) $2a$ (d) $4a$

105. An equilateral triangle is described on the diagonal of a square. What is the ratio of the area of the triangle to that of the square ? **(S.S.C. 2002)**
(a) $2 : \sqrt{3}$ (b) $4 : \sqrt{3}$ (c) $\sqrt{3} : 2$ (d) $\sqrt{3} : 4$

106. What will be the ratio between the area of a rectangle and the area of a triangle with one of the sides of the rectangle as base and a vertex on the opposite side of the rectangle ? (S.B.I.P.O. 1999)
(a) 1 : 2 (b) 2 : 1 (c) 3 : 1
(d) Data inadequate (e) None of these

107. If an equilateral triangle of area X and a square of area Y have the same perimeter, then X is : (C.D.S. 2003)
(a) equal to Y (b) greater than Y
(c) less than Y (d) less than or equal to Y

108. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then the area of the triangle is :
(a) $24\sqrt{2}$ cm^2 (b) $24\sqrt{3}$ cm^2 (c) $48\sqrt{3}$ cm^2 (d) $64\sqrt{3}$ cm^2

109. The ratio of bases of two triangles is $x : y$ and that of their areas is $a : b$. Then the ratio of their corresponding altitudes will be : (S.S.C. 2004)
(a) $ax : by$ (b) $\frac{a}{x} : \frac{b}{y}$ (c) $ay : bx$ (d) $\frac{x}{a} : \frac{b}{y}$

110. If the side of an equilateral triangle is decreased by 20%, its area is decreased by :
(a) 36% (b) 40% (c) 60% (d) 64%
(C.B.I. 1997)

111. If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area ? (S.B.I.P.O. 2000)
(a) No change (b) 8% decrease (c) 16% decrease
(d) 16% increase (e) None of these

112. If every side of a triangle is doubled, the area of the new triangle is K times the area of the old one. K is equal to : (R.R.B. 2003)
(a) $\sqrt{2}$ (b) 2 (c) 3 (d) 4

113. One side of a parallelogram is 18 cm and its distance from the opposite side is 8 cm. The area of the parallelogram is :
(a) 48 cm^2 (b) 72 cm^2 (c) 100 cm^2 (d) 144 cm^2

114. A parallelogram has sides 30 m and 14 m and one of its diagonals is 40 m long. Then, its area is :
(a) 168 m^2 (b) 336 m^2 (c) 372 m^2 (d) 480 m^2

115. One diagonal of a parallelogram is 70 cm and the perpendicular distance of this diagonal from either of the outlying vertices is 27 cm. The area of the parallelogram (in sq. cm) is :
(a) 1800 (b) 1836 (c) 1890 (d) 1980

116. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is : (M.A.T. 2003)
(a) $10\sqrt{2}$ m (b) 100 m (c) $100\sqrt{2}$ m (d) 200 m

117. If a parallelogram with area P, a rectangle with area R and a triangle with area T are all constructed on the same base and all have the same altitude, then which of the following statements is false ?
(a) P = R (b) P + T = 2R (c) P = 2T (d) T = (1 / 2) R

118. The area of a rhombus is 150 cm^2. The length of one of its diagonals is 10 cm. The length of the other diagonal is : (S.S.C. 2004)
(a) 25 cm (b) 30 cm (c) 35 cm (d) 40 cm

119. One of the diagonals of a rhombus is double the other diagonal. Its area is 25 sq. cm. The sum of the diagonals is : (S.S.C. 2003)
(*a*) 10 cm (*b*) 12 cm (*c*) 15 cm (*d*) 16 cm

120. The perimeter of a rhombus is 56 m and its height is 5 m. Its area is :
(*a*) 64 sq. m (*b*) 70 sq. m (*c*) 78 sq. m (*d*) 84 sq. m

121. If the diagonals of a rhombus are 24 cm and 10 cm, the area and the perimeter of the rhombus are respectively :
(*a*) 120 cm^2, 52 cm (*b*) 120 cm^2, 64 cm (*c*) 240 cm^2, 52 cm (*d*) 240 cm^2, 64 cm

122. Each side of a rhombus is 26 cm and one of its diagonals is 48 cm long. The area of the rhombus is : (R.R.B. 2003)
(*a*) 240 cm^2 (*b*) 300 cm^2 (*c*) 360 cm^2 (*d*) 480 cm^2

123. The length of one diagonal of a rhombus is 80% of the other diagonal. The area of the rhombus is how many times the square of the length of the other diagonal ?
(*a*) $\frac{4}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{1}{4}$

124. If a square and a rhombus stand on the same base, then the ratio of the areas of the square and the rhombus is :
(*a*) greater than 1 (*b*) equal to 1 (*c*) equal to $\frac{1}{2}$ (*d*) equal to $\frac{1}{4}$

125. The two parallel sides of a trapezium are 1.5 m and 2.5 m respectively. If the perpendicular distance between them is 6.5 metres, the area of the trapezium is :
(*a*) 10 m^2 (*b*) 13 m^2 (*c*) 20 m^2 (*d*) 26 m^2

126. The area of a field in the shape of a trapezium measures 1440 m^2. The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is : (S.S.C. 2004)
(*a*) 45 m (*b*) 60 m (*c*) 75 m (*d*) 120 m

127. The cross-section of a canal is trapezium in shape. The canal is 12 m wide at the top and 8 m wide at the bottom. If the area of the cross-section is 840 sq. m, the depth of the canal is :
(*a*) 8.75 m (*b*) 42 m (*c*) 63 m (*d*) 84 m

128. The area of a circle of radius 5 is numerically what percent of its circumference ?
(*a*) 200 (*b*) 225 (*c*) 240 (*d*) 250
(S.S.C. 2000)

129. A man runs round a circular field of radius 50 m at the speed of 12 km / hr. What is the time taken by the man to take twenty rounds of the field ? (M.A.T. 1997)
(*a*) 30 min. (*b*) 32 min. (*c*) 34 min. (*d*) None of these

130. A cow is tethered in the middle of a field with a 14 feet long rope. If the cow grazes 100 sq. ft. per day, then approximately what time will be taken by the cow to graze the whole field ? (Bank P.O. 2003)
(*a*) 2 days (*b*) 6 days (*c*) 18 days
(*d*) 24 days (*e*) None of these

131. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ? (Bank P.O. 2004)
(*a*) 88 cm^2 (*b*) 154 cm^2 (*c*) 1250 cm^2
(*d*) Cannot be determined (*e*) None of these

132. The circumference of a circle, whose area is 24.64 m^2, is : (R.R.B. 2003)
(*a*) 14.64 m (*b*) 16.36 m (*c*) 17.60 m (*d*) 18.40 m

133. If the circumference and the area of a circle are numerically equal, then the diameter is equal to : (S.S.C. 2000)
(*a*) $\frac{\pi}{2}$ (*b*) 2π (*c*) 2 (*d*) 4

134. The difference between the circumference and the radius of a circle is 37 cm. The area of the circle is : **(Section Officers', 2001)**
(*a*) 111 cm^2 (*b*) 148 cm^2 (*c*) 154 cm^2 (*d*) 259 cm^2

135. The sum of areas of two circles A and B is equal to the area of a third circle C whose diameter is 30 cm. If the diameter of circle A is 18 cm, then the radius of circle B is :
(*a*) 10 cm (*b*) 12 cm (*c*) 15 cm (*d*) 18 cm

136. Between a square of perimeter 44 cm and a circle of circumference 44 cm, which figure has larger area and by how much ? **(S.S.C. 2000)**
(*a*) Both have equal area (*b*) Square, 33 cm^2
(*c*) Circle, 33 cm^2 (*d*) Square, 495 cm^2

137. A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be : **(R.R.B. 2002)**
(*a*) 3520 cm^2 (*b*) 6400 cm^2 (*c*) 7744 cm^2 (*d*) 8800 cm^2

138. A wire when bent in the form of a square encloses an area of 484 sq. cm. What will be the enclosed area when the same wire is bent into the form of a circle ?
(*a*) 462 sq. cm (*b*) 539 sq. cm (*c*) 616 sq. cm (*d*) 693 sq. cm
(S.S.C. 2002)

139. A circular wire of radius 42 cm is bent in the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is : **(S.S.C. 2004)**
(*a*) 25 cm (*b*) 30 cm (*c*) 36 cm (*d*) 60 cm

140. There is a rectangular tank of length 180 m and breadth 120 m in a circular field. If the area of the land portion of the field is 40000 m^2, what is the radius of the field ?
(*a*) 130 m (*b*) 135 m (*c*) 140 m (*d*) 145 m

141. The areas of two circular fields are in the ratio 16 : 49. If the radius of the latter is 14 m, then what is the radius of the former ? **(IGNOU, 2003)**
(*a*) 4 m (*b*) 8 m (*c*) 18 m (*d*) 32 m

142. If the ratio of areas of two circles is 4 : 9, then the ratio of their circumferences will be : **(R.R.B. 2003)**
(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 4 : 9 (*d*) 9 : 4

143. The perimeter of a circle is equal to the perimeter of a square. Then, their areas are in the ratio :
(*a*) 4 : 1 (*b*) 11 : 7 (*c*) 14 : 11 (*d*) 22 : 7

144. The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions ?
(*a*) 1492 m (*b*) 1980 m (*c*) 2530 m (*d*) 2880 m

145. The number of revolutions a wheel of diameter 40 cm makes in travelling a distance of 176 m, is : **(S.S.C. 2003)**
(*a*) 140 (*b*) 150 (*c*) 160 (*d*) 166

146. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km will be : **(R.R.B. 2003)**
(*a*) 2800 (*b*) 4000 (*c*) 5500 (*d*) 7000

147. The wheel of an engine, $7\frac{1}{2}$ metres in circumference makes 7 revolutions in 9 seconds. The speed of the train in km per hour is :
(*a*) 130 (*b*) 132 (*c*) 135 (*d*) 150

148. The wheel of a motorcycle, 70 cm in diameter, makes 40 revolutions in every 10 seconds. What is the speed of the motorcycle in km / hr ? **(R.R.B. 2002)**
(*a*) 22.32 (*b*) 27.68 (*c*) 31.68 (*d*) 36.24

149. Wheels of diameters 7 cm and 14 cm start rolling simultaneously from X and Y, which

are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller wheel is : **(M.A.T. 2003)**

(*a*) 22 cm / sec (*b*) 44 cm / sec (*c*) 66 cm / sec (*d*) 132 cm / sec

150. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions ?

(*a*) 18 (*b*) 20 (*c*) 25 (*d*) 30

151. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decametres.

(*a*) $4\frac{4}{13}$ m (*b*) $6\frac{4}{11}$ m (*c*) $12\frac{4}{11}$ m (*d*) $12\frac{8}{11}$ m **(S.S.C. 2003)**

152. The front wheels of a wagon are 2π feet in circumference and the rear wheels are 3π feet in circumference. When the front wheels have made 10 more revolutions than the rear wheels, how many feet has the wagon travelled ? **(M.B.A. 2003)**

(*a*) 30π (*b*) 60π (*c*) 90π (*d*) 150π

153. A circular ground whose diameter is 35 metres, has a 1.4 m broad garden around it. What is the area of the garden in square metres ? **(S.B.I.P.O. 1999)**

(*a*) 160.16 (*b*) 176.16 (*c*) 196.16 (*d*) Data inadequate (*e*) None of these

154. A circular garden has a circumference of 440 m. There is a 7 m wide border inside the garden along its periphery. The area of the border is :

(*a*) 2918 m^2 (*b*) 2921 m^2 (*c*) 2924 m^2 (*d*) 2926 m^2

155. The areas of two concentric circles forming a ring are 154 sq. cm and 616 sq. cm. The breadth of the ring is :

(*a*) 7 cm (*b*) 14 cm (*c*) 21 cm (*d*) 28 cm

156. A circular park has a path of uniform width around it. The difference between outer and inner circumferences of the circular path is 132 m. Its width is : **(S.S.C. 2003)**

(*a*) 20 m (*b*) 21 m (*c*) 22 m (*d*) 24 m

157. A circular swimming pool is surrounded by a concrete wall 4 ft. wide. If the area of the concrete wall surrounding the pool is $\frac{11}{25}$ that of the pool, then the radius of the pool is : **(Assistant Grade, 1998)**

(*a*) 8 ft (*b*) 16 ft (*c*) 20 ft (*d*) 30 ft

158. The ratio of the outer and the inner perimeters of a circular path is 23 : 22. If the path is 5 metres wide, the diameter of the inner circle is : **(S.S.C. 2004)**

(*a*) 55 m (*b*) 110 m (*c*) 220 m (*d*) 230 m

159. What will be the area of a semi-circle of 14 m diameter ? **(NABARD, 2002)**

(*a*) 22 m^2 (*b*) 77 m^2 (*c*) 154 m^2 (*d*) 308 m^2 (*e*) None of these

160. A semi-circular shaped window has diameter of 63 cm. Its perimeter equals :

(*a*) 126 cm (*b*) 162 cm (*c*) 198 cm (*d*) 251 cm **(S.S.C. 1999)**

161. What will be the area of a semi-circle whose perimeter is 36 cm ?

(*a*) 154 cm^2 (*b*) 168 cm^2 (*c*) 308 cm^2 (*d*) Data inadequate (*e*) None of these **(B.S.R.B. 1998)**

162. If a wire is bent into the shape of a square, then the area of the square is 81 sq. cm. When the wire is bent into a semi-circular shape, then the area of the semi-circle will be : **(S.S.C. 2002)**

(*a*) 22 cm^2 (*b*) 44 cm^2 (*c*) 77 cm^2 (*d*) 154 cm^2

163. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cm, is :
(a) 7.5 cm^2 (b) 7.75 cm^2 (c) 8.5 cm^2 (d) 8.75 cm^2
(S.S.C. 1999)

164. In a circle of radius 7 cm, an arc subtends an angle of 108° at the centre. The area of the sector is :
(a) 43.2 cm^2 (b) 44.2 cm^2 (c) 45.2 cm^2 (d) 46.2 cm^2

165. The area of the greatest circle which can be inscribed in a square whose perimeter is 120 cm, is : **(S.S.C. 2004)**
(a) $\frac{22}{7}\times\left(\frac{7}{2}\right)^2$ cm^2 (b) $\frac{22}{7}\times\left(\frac{9}{2}\right)^2$ cm^2
(c) $\frac{22}{7}\times\left(\frac{15}{2}\right)^2$ cm^2 (d) $\frac{22}{7}\times(15)^2$ cm^2

166. The area of the largest circle, that can be drawn inside a rectangle with sides 18 cm by 14 cm, is : **(S.S.C. 2000)**
(a) 49 cm^2 (b) 154 cm^2 (c) 378 cm^2 (d) 1078 cm^2

167. The area of a circle is 220 sq. cm. The area of a square inscribed in this circle will be : **(C.B.I. 1997)**
(a) 49 cm^2 (b) 70 cm^2 (c) 140 cm^2 (d) 150 cm^2

168. A square is inscribed in a circle whose radius is 4 cm. The area of the portion between the circle and the square is :
(a) $(8\pi - 16)$ (b) $(8\pi - 32)$ (c) $(16\pi - 16)$ (d) $(16\pi - 32)$

169. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is : **(C.B.I. 2003)**
(a) $50\sqrt{2}$ cm (b) $\frac{100}{\pi}$ cm (c) $\frac{50\sqrt{2}}{\pi}$ cm (d) $\frac{100\sqrt{2}}{\pi}$ cm

170. Four equal sized maximum circular plates are cut off from a square paper sheet of area 784 cm^2. The circumference of each plate is : **(S.S.C. 2003)**
(a) 22 cm (b) 44 cm (c) 66 cm (d) 88 cm

171. There are 4 semi-circular gardens on each side of a square-shaped pond with each side 21 m. The cost of fencing the entire plot at the rate of Rs 12.50 per metre is :
(a) Rs. 1560 (b) Rs. 1650 (c) Rs. 3120 (d) Rs. 3300

172. The ratio of the areas of the incircle and circumcircle of an equilateral triangle is :
(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 9

173. The radius of the circumcircle of an equilateral triangle of side 12 cm is :
(a) $\frac{4\sqrt{2}}{3}$ cm (b) $4\sqrt{2}$ cm (c) $\frac{4\sqrt{3}}{3}$ cm (d) $4\sqrt{3}$ cm

174. The area of the incircle of an equilateral triangle of side 42 cm is : **(S.S.C. 2004)**
(a) $22\sqrt{3}$ cm^2 (b) 231 cm^2 (c) 462 cm^2 (d) 924 cm^2

175. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle.
(a) 71.5 cm (b) 71.7 cm (c) 72.3 cm (d) 72.7 cm

176. The sides of a triangle are 6 cm, 11 cm and 15 cm. The radius of its incircle is :
(a) $3\sqrt{2}$ cm (b) $\frac{4\sqrt{2}}{5}$ cm (c) $\frac{5\sqrt{2}}{4}$ cm (d) $6\sqrt{2}$ cm
(M.A.T. 2001)

177. The perimeter of a triangle is 30 cm and the circumference of its incircle is 88 cm. The area of the triangle is : **(S.S.C. 2003)**
(a) 70 cm^2 (b) 140 cm^2 (c) 210 cm^2 (d) 420 cm^2

178. If in a triangle, the area is numerically equal to the perimeter, then the radius of the inscribed circle of the triangle is : **(S.S.C. 2000)**
(a) 1 (b) 1.5 (c) 2 (d) 3

179. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of the triangle, S, the area of the square and C, the area of the circle, then :
(a) $S < T < C$ (b) $T < C < S$ (c) $T < S < C$ (d) $C < S < T$
(C.D.S. 2003)

180. If an area enclosed by a circle or a square or an equilateral triangle is the same, then the maximum perimeter is possessed by : **(S.C.R.A. 1997)**
(a) circle (b) square (c) equilateral triangle
(d) triangle and square have equal perimeters greater than that of circle

181. The area of the largest triangle that can be inscribed in a semi-circle of radius r, is :
(a) r^2 (b) $2r^2$ (c) r^3 (d) $2r^3$
(Section Officers', 2001)

182. ABC is a right-angled triangle with right angle at B. If the semi-circle on AB with AB as diameter encloses an area of 81 sq. cm and the semi-circle on BC with BC as diameter encloses an area of 36 sq. cm, then the area of the semi-circle on AC with AC as diameter will be :
(a) 117 cm^2 (b) 121 cm^2 (c) 217 cm^2 (d) 221 cm^2

183. If the radius of a circle is increased by 75%, then its circumference will increase by :
(a) 25% (b) 50% (c) 75% (d) 100%
(C.D.S. 2003)

184. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, then the time required by A to go round the new path once, travelling at the same speed as before, is : **(S.S.C. 2000)**
(a) 20 min. (b) 25 min. (c) 50 min. (d) 100 min.

185. If the radius of a circle is increased by 6%, then the area is increased by :
(a) 6% (b) 12% (c) 12.36% (d) 16.64%
(D.M.R.C. 2003)

186. If the radius of a circle is diminished by 10%, then its area is diminished by :
(a) 10% (b) 19% (c) 20% (d) 36%
(Hotel Management, 2003)

187. If the radius of a circle is doubled, its area is increased by : **(C.B.I. 1998)**
(a) 100% (b) 200% (c) 300% (d) 400%.

188. If the circumference of a circle increases from 4π to 8π, what change occurs in its area ?
(a) It is halved. (b) It doubles. (c) It triples. (d) It quadruples.
(S.S.C. 2000)

189. Three circles of radius 3.5 cm are placed in such a way that each circle touches the other two. The area of the portion enclosed by the circles is : **(S.S.C. 2003)**
(a) 1.967 cm^2 (b) 1.975 cm^2 (c) 19.67 cm^2 (d) 21.21 cm^2

190. Four circular cardboard pieces, each of radius 7 cm are placed in such a way that each piece touches two other pieces. The area of the space enclosed by the four pieces is :
(a) 21 cm^2 (b) 42 cm^2 (c) 84 cm^2 (d) 168 cm^2

191. Four horses are tethered at four corners of a square plot of side 63 metres so that they just cannot reach one another. The area left ungrazed is :
(a) 675.5 m^2 (b) 780.6 m^2 (c) 785.8 m^2 (d) 850.5 m^2

ANSWERS

1. (d)	2. (c)	3. (b)	4. (c)	5. (c)	6. (d)	7. (e)	8. (b)	9. (b)
10. (b)	11. (d)	12. (e)	13. (d)	14. (b)	15. (b)	16. (d)	17. (a)	18. (d)
19. (a)	20. (a)	21. (b)	22. (b)	23. (c)	24. (d)	25. (d)	26. (c)	27. (b)
28. (d)	29. (b)	30. (a)	31. (a)	32. (d)	33. (c)	34. (b)	35. (e)	36. (d)
37. (b)	38. (e)	39. (a)	40. (e)	41. (c)	42. (a)	43. (c)	44. (d)	45. (b)
46. (b)	47. (b)	48. (b)	49. (e)	50. (c)	51. (c)	52. (b)	53. (a)	54. (a)
55. (d)	56. (b)	57. (b)	58. (c)	59. (b)	60. (b)	61. (c)	62. (c)	63. (c)
64. (c)	65. (b)	66. (c)	67. (a)	68. (c)	69. (a)	70. (a)	71. (d)	72. (c)
73. (d)	74. (d)	75. (c)	76. (d)	77. (b)	78. (d)	79. (d)	80. (c)	81. (c)
82. (b)	83. (d)	84. (c)	85. (a)	86. (d)	87. (c)	88. (d)	89. (b)	90. (c)
91. (d)	92. (d)	93. (d)	94. (d)	95. (b)	96. (a)	97. (a)	98. (a)	99. (c)
100. (c)	101. (d)	102. (c)	103. (d)	104. (c)	105. (c)	106. (b)	107. (c)	108. (d)
109. (c)	110. (a)	111. (c)	112. (d)	113. (d)	114. (b)	115. (c)	116. (d)	117. (b)
118. (b)	119. (c)	120. (b)	121. (a)	122. (d)	123. (b)	124. (b)	125. (b)	126. (c)
127. (d)	128. (d)	129. (d)	130. (b)	131. (e)	132. (c)	133. (d)	134. (c)	135. (b)
136. (c)	137. (c)	138. (c)	139. (d)	140. (c)	141. (b)	142. (a)	143. (c)	144. (b)
145. (a)	146. (d)	147. (b)	148. (c)	149. (c)	150. (c)	151. (b)	152. (b)	153. (a)
154. (d)	155. (a)	156. (b)	157. (c)	158. (c)	159. (b)	160. (b)	161. (e)	162. (c)
163. (d)	164. (d)	165. (d)	166. (b)	167. (c)	168. (d)	169. (c)	170. (b)	171. (b)
172. (c)	173. (d)	174. (c)	175. (d)	176. (c)	177. (c)	178. (c)	179. (c)	180. (c)
181. (a)	182. (a)	183. (c)	184. (c)	185. (c)	186. (b)	187. (c)	188. (d)	189. (a)
190. (b)	191. (d)							

SOLUTIONS

1. Area of the floor = (5.5×3.75) m^2 = 20.625 m^2.

   $\therefore$ Cost of paving = Rs. (800×20.625) = Rs. 16500.

2. Let the breadth be b. Then, $25 \times b = 18 \times 10 \Leftrightarrow b = \left(\frac{18 \times 10}{25}\right)$ cm = 7.2 cm.

3. Perimeter of the plot = 2 (90 + 50) = 280 m.

   $\therefore$ Number of poles = $\left(\frac{280}{5}\right)$ = 56 m.

4. Let breadth = x cm. Then, length = $\left(\frac{160}{100}x\right)$ cm = $\frac{8}{5}x$ cm.

   So, $\frac{8}{5}x - x = 24 \Leftrightarrow \frac{3}{5}x = 24 \Leftrightarrow x = \left(\frac{24 \times 5}{3}\right) = 40.$

   $\therefore$ Length = 64 cm, Breadth = 40 cm.

   Area = (64×40) cm^2 = 2560 cm^2.

5. Clearly, we have : $l = 9$ and $l + 2b = 37$ or $b = 14$.

   $\therefore$ Area = $(l \times b) = (9 \times 14)$ sq. ft. = 126 sq. ft.

6. We have : $(l - b) = 23$ and $2(l + b) = 206$ or $(l + b) = 103$.

   Solving the two equations, we get : $l = 63$ and $b = 40$.

   $\therefore$ Area = $(l \times b) = (63 \times 40)$ m^2 = 2520 m^2.

7. Let breadth = x metres. Then, length = $(x + 20)$ metres.

   Perimeter = $\left(\frac{5300}{26.50}\right)$ m = 200 m.

$\therefore \quad 2[(x+20)+x] = 200 \Leftrightarrow 2x + 20 = 100 \Leftrightarrow 2x = 80 \Leftrightarrow x = 40.$

Hence, length $= x + 20 = 60$ m.

8. Let length $= x$ metres. Then, breadth $= \left(\frac{60}{100}x\right)$ metres $= \left(\frac{3x}{5}\right)$ metres.

Perimeter $= \left[2\left(x + \frac{3x}{5}\right)\right]$ m $= \left(\frac{16x}{5}\right)$ m.

$\therefore \quad \frac{16x}{5} = 800 \Leftrightarrow x = \left(\frac{800 \times 5}{16}\right) = 250.$

So, length = 250 m; breadth = 150 m.

$\therefore$ Area $= (250 \times 150)$ m^2 = 37500 m^2.

9. $\frac{l}{2(l+b)} = \frac{1}{3} \Rightarrow 3l = 2l + 2b \Rightarrow l = 2b \Rightarrow \frac{l}{b} = \frac{2}{1} = 2:1.$

10. Perimeter = Distance covered in 8 min. $= \left(\frac{12000}{60} \times 8\right)$ m = 1600 m.

Let length $= 3x$ metres and breadth $= 2x$ metres.

Then, $2(3x + 2x) = 1600$ or $x = 160$.

$\therefore$ Length = 480 m and Breadth = 320 m.

$\therefore$ Area $= (480 \times 320)$ m^2 = 153600 m^2.

11. Let breadth $= x$ metres. Then, length $= (x + 5)$ metres.

Then, $x(x+5) = 750 \Leftrightarrow x^2 + 5x - 750 = 0 \Leftrightarrow (x+30)(x-25) = 0 \Leftrightarrow x = 25.$

$\therefore$ Length $= (x + 5) = 30$ m.

12. Let breadth $= x$ metres. Then, length $= \left(\frac{115x}{100}\right)$ metres.

$\therefore \quad x \times \frac{115x}{100} = 460 \Leftrightarrow x^2 = \left(\frac{460 \times 100}{115}\right) = 400 \Leftrightarrow x = 20.$

13. We have : $l = 20$ ft and $lb = 680$ sq. ft. So, $b = 34$ ft.

$\therefore$ Length of fencing $= (l + 2b) = (20 + 68)$ ft = 88 ft.

14. $\frac{2(l+b)}{b} = \frac{5}{1} \Rightarrow 2l + 2b = 5b \Rightarrow 3b = 2l \Rightarrow b = \frac{2}{3}l.$

Then, Area = 216 cm^2 $\Rightarrow l \times b = 216 \Rightarrow l \times \frac{2}{3}l = 216 \Rightarrow l^2 = 324 \Rightarrow l = 18$ cm.

15. We have : $2b + l = 30 \Rightarrow l = 30 - 2b.$

Area = 100 m^2 $\Rightarrow l \times b = 100 \Rightarrow b(30 - 2b) = 100 \Rightarrow b^2 - 15b + 50 = 0$

$\Rightarrow (b - 10)(b - 5) = 0 \Rightarrow b = 10$ or $b = 5$.

When $b = 10$, $l = 10$ and when $b = 5$, $l = 20$.

Since the garden is rectangular, so its dimension is 20 m $\times$ 5 m.

16. Let length $= (3x)$ metres and breadth $= (4x)$ metres.

Then, $3x \times 4x = 7500 \Leftrightarrow 12x^2 = 7500 \Leftrightarrow x^2 = 625 \Leftrightarrow x = 25.$

So, length = 75 m and breadth = 100 m.

Perimeter $= [2(75 + 100)]$ m = 350 m.

$\therefore$ Cost of fencing = Rs. (0.25×350) = Rs. 87.50.

17. Required perimeter = (AB + BC + CP + PQ + QR + RA)

= AB + BC + (CP + QR) + (PQ + RA)

= AB + BC + AB + BC = 2 (AB + BC)

= [2 (8 + 4)] cm = 24 cm.

18. Let the areas of the two parts be x and $(700 - x)$ hectares respectively. Then,

$[x-(700-x)] = \frac{1}{5}\times\left[\frac{x+(700-x)}{2}\right] \Leftrightarrow 2x - 700 = 70 \Leftrightarrow x = 385.$

So, area of smaller part = (700 − 385) hectares = 315 hectares.

19. When folded along breadth, we have : $2\left(\frac{l}{2}+b\right) = 34$ or $l + 2b = 34$ ...(*i*)

When folded along length, we have : $2\left(l+\frac{b}{2}\right) = 38$ or $2l + b = 38$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $l = 14$ and $b = 10$.

$\therefore$ Area of the paper $= (14 \times 10)\ cm^2 = 140\ cm^2$.

20. Let breadth = x metres. Then, length = $\left(\frac{3}{2}x\right)$ metres.

Area $= \left(\frac{2}{3}\times 10000\right) m^2$.

$\therefore \frac{3}{2}x \times x = \frac{2}{3}\times 10000 \Leftrightarrow x^2 = \frac{4}{9}\times 10000 \Leftrightarrow x = \frac{2}{3}\times 100.$

$\therefore$ Length $= \frac{3}{2}x = \left(\frac{3}{2}\times\frac{2}{3}\times 100\right) m = 100\ m.$

21. Number of bricks $= \left(\frac{\text{Area of courtyard}}{\text{Area of 1 brick}}\right) = \left(\frac{2500\times 1600}{20\times 10}\right) = 20000.$

22. Length of the carpet $= \left(\frac{\text{Total cost}}{\text{Rate / m}}\right) = \left(\frac{8100}{45}\right) m = 180\ m.$

Area of the room = Area of the carpet $= \left(180\times\frac{75}{100}\right) m^2 = 135\ m^2.$

$\therefore$ Breadth of the room $= \left(\frac{\text{Area}}{\text{Length}}\right) = \left(\frac{135}{18}\right) m = 7.5\ m.$

23. Other side $= \sqrt{\left(\frac{15}{2}\right)^2 - \left(\frac{9}{2}\right)^2}$ ft $= \sqrt{\frac{225}{4}-\frac{81}{4}}$ ft $= \sqrt{\frac{144}{4}}$ ft = 6 ft.

$\therefore$ Area of the closet = (6 × 4.5) sq. ft = 27 sq. ft.

24. Let breadth = x cm. Then, length = $3x$ cm.

$x^2 + (3x)^2 = (8\sqrt{10})^2 \Rightarrow 10x^2 = 640 \Rightarrow x^2 = 64 \Rightarrow x = 8.$

So, length = 24 cm and breadth = 8 cm.

$\therefore$ Perimeter = [2 (24 + 8)] cm = 64 cm.

25. $\sqrt{l^2+b^2} = 3b \Rightarrow l^2 + b^2 = 9b^2 \Rightarrow l^2 = 8b^2 \Rightarrow \frac{l^2}{b^2} = 8 \Rightarrow \frac{l}{b} = \sqrt{8} = 2\sqrt{2}.$

26. $2(l + b) = 46$ or $l + b = 23$. Also, $lb = 120$.

$\therefore$ Diagonal $= \sqrt{l^2+b^2} = \sqrt{(l+b)^2 - 2lb} = \sqrt{(23)^2 - 240} = \sqrt{289} = 17$ m.

27. $\sqrt{l^2+b^2} = \sqrt{41}$ or $l^2 + b^2 = 41$. Also, $lb = 20$.

$(l + b)^2 = (l^2 + b^2) + 2lb = 41 + 40 = 81 \Rightarrow (l + b) = 9.$

$\therefore$ Perimeter = $2(l + b)$ = 18 cm.

28. Length of diagonal $= \left(52 \times \frac{15}{60}\right)$ m $= 13$ m.

Sum of length and breadth $= \left(68 \times \frac{15}{60}\right)$ m $= 17$ m.

$\therefore \quad \sqrt{l^2 + b^2} = 13$ or $l^2 + b^2 = 169$ and $l + b = 17$.

Area $= lb = \frac{1}{2}(2\,lb) = \frac{1}{2}[(l+b)^2 - (l^2 + b^2)] = \frac{1}{2}[(17)^2 - 169] = \frac{1}{2}(289 - 169) = 60$ m^2.

29. We have : $lb = 60$ and $\sqrt{l^2 + b^2} + l = 5b$.

Now, $l^2 + b^2 = (5b - l)^2 \Rightarrow 24b^2 - 10lb = 0 \Rightarrow 24b^2 - 600 = 0$

$\Rightarrow b^2 = 25 \Rightarrow b = 5$.

$\therefore \quad l = \left(\frac{60}{5}\right)$ m $= 12$ m. So, length of the carpet $= 12$ m.

30. Let length $= (3x)$ metres and breadth $= (2x)$ metres.
Then, $(3x + 5) \times 2x = 2600 \Leftrightarrow 6x^2 + 10x - 2600 = 0$
$\Leftrightarrow 3x^2 + 5x - 1300 = 0 \Leftrightarrow (3x + 65)(x - 20) = 0 \Leftrightarrow x = 20$.
$\therefore$ Breadth $= 2x = 40$ m.

31. Let breadth $= x$ cm. Then, length $= (x + 8)$ cm.
$\therefore \quad (x + 8)\,x = (x + 15)(x - 4) \Leftrightarrow x^2 + 8x = x^2 + 11x - 60 \Leftrightarrow x = 20$.
So, length $= 28$ cm and breadth $= 20$ cm.

32. Let length $= x$ metres and breadth $= y$ metres. Then,
$(x + 1)(y + 1) - xy = 21 \Rightarrow x + y = 20$ ...(*i*)
And, $xy - [(x + 1)(y - 1)] = 5 \Leftrightarrow x - y = 6$ ...(*ii*)
Solving (*i*) and (*ii*), we get : $x = 13$ and $y = 7$.
So, length $= 13$ m and breadth $= 7$ m.
$\therefore$ Perimeter $= [2(13 + 7)]$ m $= 40$ m.

33. Let original length $= x$ metres and original breadth $= y$ metres.
Original area $= (xy)$ m^2.

New length $= \left(\frac{120}{100}x\right)$ m $= \left(\frac{6}{5}x\right)$ m; New breadth $= \left(\frac{120}{100}y\right)$ m $= \left(\frac{6}{5}y\right)$ m.

New Area $= \left(\frac{6}{5}x \times \frac{6}{5}y\right)$ m^2 $= \left(\frac{36}{25}xy\right)$ m^2.

$\therefore$ Increase% $= \left(\frac{11}{25}xy \times \frac{1}{xy} \times 100\right)\% = 44\%$.

34. New area $= \left(\frac{80}{100}a \times \frac{110}{100}b\right) = \left(\frac{4}{5} \times \frac{11}{10}ab\right) = \left(\frac{22}{25}ab\right)$.

$\therefore$ Required percentage $= \left(\frac{22}{25}ab \times \frac{1}{ab} \times 100\right)\% = 88\%$.

35. Let original length $= x$ metres and original breadth $= y$ metres.
Original area $= (xy)$ m^2.

New length $= \left(\frac{150}{100}x\right)$ m $= \left(\frac{3}{2}x\right)$ m; New breadth $= \left(\frac{120}{100}y\right)$ m $= \left(\frac{6}{5}y\right)$ m.

New area $= \left(\frac{3}{2}x \times \frac{6}{5}y\right) m^2 = \left(\frac{9}{5}xy\right) m^2$.

$\therefore$ Increase $= \left(\frac{\frac{4}{5}xy}{xy}\right) = \frac{4}{5}$ times.

36. Let original length $= x$ and original breadth $= y$.
Decrease in area $= xy - \left(\frac{80}{100}x \times \frac{90}{100}y\right) = \left(xy - \frac{18}{25}xy\right) = \frac{7}{25}xy$.

$\therefore$ Decrease% $= \left(\frac{7}{25}xy \times \frac{1}{xy} \times 100\right)\% = 28\%$.

37. Let original length $= x$ and original breadth $= y$.
Original area $= xy$.

New length $= \frac{x}{2}$; New breadth $= 3y$. New Area $= \left(\frac{x}{2} \times 3y\right) = \frac{3}{2}xy$.

$\therefore$ Increase% $= \left(\frac{1}{2}xy \times \frac{1}{xy} \times 100\right)\% = 50\%$.

38. Let original length $= x$ and original breadth $= y$.
Then, original area $= xy$.

New area $= \left[\frac{(100-r)}{100} \times x\right]\left[\frac{(105+r)}{100} \times y\right] = \left[\left(\frac{10500 - 5r - r^2}{10000}\right)xy\right]$

$\therefore \quad \left(\frac{10500 - 5r - r^2}{10000}\right)xy = xy \Leftrightarrow r^2 + 5r - 500 = 0 \Leftrightarrow (r+25)(r-20) = 0 \Leftrightarrow r = 20.$

39. Let original length $= x$ and original breadth $= y$.
Then, original area $= xy$.

New length $= \frac{160x}{100} = \frac{8x}{5}$. Let new breadth $= z$.

Then, $\frac{8x}{5} \times z = xy \Rightarrow z = \frac{5y}{8}$.

$\therefore$ Decrease in breadth $= \left(\frac{3y}{8} \times \frac{1}{y} \times 100\right)\% = 37\frac{1}{2}\%$.

40. Let original length $= x$ and original breadth $= y$.
Then, original area $= xy$.

New length $= \frac{130}{100}x = \frac{13x}{10}$. New breadth $= y$. New area $= \left(\frac{13x}{10} \times y\right) = \frac{13xy}{10}$.

$\therefore$ Required ratio $= \left(\frac{\frac{13xy}{10}}{xy}\right) = \frac{13}{10} = 13 : 10$.

41. Area of the sheet $= (20 \times 30)\ cm^2 = 600\ cm^2$.
Area used for typing $= [(20 - 4) \times (30 - 6)]\ cm^2 = 384\ cm^2$.

$\therefore$ Required percentage $= \left(\frac{384}{600} \times 100\right)\% = 64\%$.

42. Area of the mat = $[(15 - 3) \times (12 - 3)]$ sq. ft = 108 sq. ft.
$\therefore$ Cost of the mat = Rs. (108×3.50) = Rs. 378.

43. $2(l + b) = 340$ (Given).
Area of the boundary = $[(l + 2)(b + 2) - lb] = 2(l + b) + 4 = 344$.
$\therefore$ Cost of gardening = Rs. (344×10) = Rs. 3440.

44. $lb = 96$ (Given).
Area of pathway = $[(l - 4)(b - 4) - lb] = 16 - 4(l + b)$, which cannot be determined.
So, data is inadequate.

45. Let the width of walk be x metres. Then,
$(20 - 2x)(10 - 2x) = 96 \iff 4x^2 + 60x - 104 = 0 \iff x^2 + 15x - 26 = 0$
$\iff (x - 13)(x - 2) = 0 \iff x = 2 \quad [\because x \neq 13]$

46. Area of crossroads = $(55 \times 4 + 35 \times 4 - 4 \times 4)$ m^2 = 344 m^2.
$\therefore$ Cost of gravelling = Rs. $\left(344 \times \frac{75}{100}\right)$ = Rs. 258.

47. Area of the park = (60×40) m^2 = 2400 m^2. Area of the lawn = 2109 m^2.
$\therefore$ Area of the crossroads = $(2400 - 2109)$ m^2 = 291 m^2.
Let the width of the road be x metres. Then,
$60x + 40x - x^2 = 291 \iff x^2 - 100x + 291 = 0 \iff (x - 97)(x - 3) = 0$
$\iff x = 3 \quad [\because x \neq 97]$.

48. Side = $\sqrt{2550.25} = \sqrt{\frac{255025}{100}} = \frac{505}{10} = 50.5$ m.

49. Area = $\frac{\text{Total cost}}{\text{Rate}} = \left(\frac{1215}{135}\right)$ hectares = (9×10000) sq. m.
$\therefore$ Side of the square = $\sqrt{90000}$ = 300 m.
Perimeter of the field = (300×4) = 1200 m.
Cost of fencing = Rs. $\left(1200 \times \frac{3}{4}\right)$ = Rs. 900.

50. The sides of the five squares are $\left(\frac{24}{4}\right), \left(\frac{32}{4}\right), \left(\frac{40}{4}\right), \left(\frac{76}{4}\right), \left(\frac{80}{4}\right)$ *i.e.*, 6 cm, 8 cm, 10 cm, 19 cm, 20 cm.
$\therefore$ Area of the new square = $[6^2 + 8^2 + (10)^2 + (19)^2 + (20)^2]$
$= (36 + 64 + 100 + 361 + 400)$ cm^2 = 961 cm^2.
Side of the new square = $\sqrt{961}$ cm = 31 cm.
Perimeter of the new square = (4×31) cm = 124 cm.

51. Number of marbles = $\left(\frac{300 \times 300}{20 \times 20}\right) = 225$.

52. Area of each slab = $\left(\frac{72}{50}\right)$ m^2 = 1.44 m^2.
$\therefore$ Length of each slab = $\sqrt{1.44}$ m = 1.2 m = 120 cm.

53. Area left after laying black tiles = $[(20 - 4) \times (10 - 4)]$ sq. ft = 96 sq. ft.
Area under white tiles = $\left(\frac{1}{3} \times 96\right)$ sq. ft = 32 sq. ft.
Area under blue tiles = $(96 - 32)$ sq. ft = 64 sq. ft.
Number of blue tiles = $\frac{64}{(2 \times 2)} = 16$.

54. Length of largest tile = H.C.F. of 1517 cm and 902 cm = 41 cm.
Area of each tile = (41×41) cm^2.

$\therefore$ Required number of tiles = $\left(\frac{1517 \times 902}{41 \times 41}\right) = 814$.

55. Length of the room = $(7 + 7)$ m = 14 m. Breadth of the room = 7 m.

$\therefore$ Area of the room = (14×7) m^2 = 98 m^2.

56. Side of the square = 12 cm. Area of the rectangle = $[(12 \times 12) - 4]$ cm^2 = 140 cm^2.
Now, area = 140 cm^2, length = 14 cm.

$\therefore$ Breadth = $\frac{\text{area}}{\text{length}} = \frac{140}{14}$ cm = 10 cm.

Hence, Perimeter = $2\,(l + b) = 2\,(14 + 10)$ cm = 48 cm.

57. Let the side of the square be x cm. Then, its area = x^2 cm^2.
Area of the rectangle = $(3x^2)$ cm^2.

$\therefore 40 \times \frac{3}{2} \times x = 3x^2 \Leftrightarrow x = 20$.

58. Side of the square = $\frac{80}{4}$ cm = 20 cm.

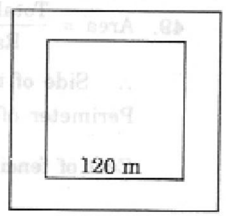
$2\,(l + b) = 80 \Rightarrow l + b = 40$. Now, $(20 \times 20) - lb = 100 \Leftrightarrow lb = 300$.

$(l - b) = \sqrt{(l + b)^2 - 4lb} = \sqrt{(40 \times 40) - (4 \times 300)} = \sqrt{400} = 20$.

Now, $l + b = 40$ and $l - b = 20 \Rightarrow l = 30$ and $b = 10$.

$\therefore$ Sides of the rectangle are 30 cm and 10 cm.

59. Perimeter = $\frac{\text{Total cost}}{\text{Cost per m}} = \frac{10080}{20}$ m = 504 m.



Side of the square = $\frac{504}{4}$ m = 126 m.

Breadth of the pavement = 3 m.

Side of inner square = $(126 - 6)$ m = 120 m.

Area of the pavement = $[(126 \times 126) - (120 \times 120)]$ m^2

$= [(126 + 120)\,(126 - 120)]$ m^2 = (246×6) m^2.

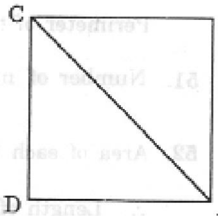
$\therefore$ Cost of pavement = Rs. $(246 \times 6 \times 50)$ = Rs. 73800.

60. Let the length of the outer edge be x metres. Then, length of the inner edge = $(x - 6)$ m.

$\therefore x^2 - (x - 6)^2 = 1764 \Leftrightarrow x^2 - (x^2 - 12x + 36) = 1764 \Leftrightarrow 12x = 1800 \Leftrightarrow x = 150$.

$\therefore$ Required perimeter = $(4x)$ m = (4×150) m = 600 m.

61. Let the side of the square be x metres.



Then, AB + BC = $2x$ metres.

AC = $\sqrt{2}\,x = (1.41x)$ m

Saving on $2x$ metres = $(0.59x)$ m.

Saving% = $\left(\frac{0.59x}{2x} \times 100\right)\%$ = 30% (approx.).

62. Speed of the man = $\left(4 \times \frac{5}{18}\right)$ m/sec = $\frac{10}{9}$ m/s.

Time taken = (3×60) sec = 180 sec.

Length of diagonal = (speed $\times$ time) = $\left(\frac{10}{9} \times 180\right)$ m = 200 m.

Area of the field = $\frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 200 \times 200\right)$ m^2 = 20000 m^2.

63. $d = \sqrt{2} \times l \Rightarrow l = \dfrac{20}{\sqrt{2}}$ cm.

$\therefore$ Perimeter $= (4l)$ cm $= \left(\dfrac{4 \times 20}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}\right)$ cm $= 40\sqrt{2}$ cm.

64. Side $= \sqrt{69696}$ cm $= 264$ cm.

$\therefore$ $d = \sqrt{2} \times \text{side} = (264 \times \sqrt{2})$ cm $= (264 \times 1.414)$ cm $= 373.296$ cm.

65. Area $= (45 \times 40)$ m^2 $\Leftrightarrow \dfrac{1}{2} \times (\text{diagonal})^2 = 1800 \Leftrightarrow$ diagonal $= 60$ m.

66. Let breadth be x metres. Then, length $= 120\%$ of $x = \left(\dfrac{120}{100}x\right) = \dfrac{6x}{5}$ m.

Required ratio $= \left(\dfrac{6x}{5} \times x \times \dfrac{1}{x \times x}\right) = 6 : 5.$

67. A square and a rectangle with equal areas will satisfy the relation $p_1 < p_2$.

68. Take a square of side 4 cm and a rectangle having $l = 6$ cm, $b = 2$ cm.

Then, perimeter of square = perimeter of rectangle.

Area of square $= 16$ cm^2, area of rectangle $= 12$ cm^2.

$\therefore$ A > B.

69. $d_1 = 4\sqrt{2}$ cm $\Rightarrow$ area $= \dfrac{1}{2} d_1^{\,2} = \dfrac{1}{2} \times (4\sqrt{2})^2 = 16$ cm^2.

Area of new square $= (2 \times 16)$ cm^2 $= 32$ cm^2.

$\therefore \dfrac{1}{2} d_2^{\,2} = 32 \Rightarrow d_2^{\,2} = 64 \Rightarrow d_2 = 8$ cm.

70. Required ratio $= \dfrac{a^2}{(\sqrt{2}\,a)^2} = \dfrac{a^2}{2a^2} = \dfrac{1}{2} = 1 : 2.$

71. Let the diagonals be $2d$ and d.

Then, ratio of their areas $= \dfrac{\frac{1}{2} \times (2d)^2}{\frac{1}{2} \times d^2} = \dfrac{4d^2}{d^2} = \dfrac{4}{1} = 4 : 1.$

72. $\dfrac{a^2}{b^2} = \dfrac{225}{256} = \dfrac{(15)^2}{(16)^2} \Leftrightarrow \dfrac{a}{b} = \dfrac{15}{16} \Leftrightarrow \dfrac{4a}{4b} = \dfrac{4 \times 15}{4 \times 16} = \dfrac{15}{16}.$

$\therefore$ Ratio of perimeters $= 15 : 16$.

73. Area = 1 hect. = 10000 sq. m $\Rightarrow$ side $= \sqrt{10000}$ m $= 100$ m.

Side of the other square = 101 m.

Difference in their areas $= [(101)^2 - (100)^2]$ m^2

$= [(101 + 100)(101 - 100)]$ m^2 $= 201$ m^2.

74. Let the sides be x cm and $\dfrac{150}{100}x = \dfrac{3x}{2}$ cm.

Required ratio $= \dfrac{\frac{9}{4}x^2}{x^2} = \dfrac{9}{4} = 9 : 4.$

75. $A_1 = x^2$ and $A_2 = \left(\dfrac{1}{2}x\right)^2 = \dfrac{1}{4}x^2 = \dfrac{1}{4}A_1.$

76. 100 cm is read as 102 cm.

$\therefore$ $A_1 = (100 \times 100)\ cm^2$ and $A_2 = (102 \times 102)\ cm^2$.

$(A_2 - A_1) = [(102)^2 - (100)^2] = (102 + 100) \times (102 - 100) = 404\ cm^2$.

$\therefore$ Percentage error $= \left(\frac{404}{100 \times 100} \times 100\right)\% = 4.04\%$.

77. Let original area $= 100\ cm^2$. Then, new area $= 169\ cm^2$.

$\Rightarrow$ Original side = 10 cm, New side = 13 cm.

Increase on 10 cm = 3 cm. Increase% $= \left(\frac{3}{10} \times 100\right)\% = 30\%$.

78. Given diagonal $= d$. New diagonal $= \frac{3}{2}d$.

Original area $= \frac{1}{2}d^2$, New area $= \frac{1}{2} \times \left(\frac{3}{2}d\right)^2 = \frac{9}{8}d^2$.

$\therefore$ Required ratio $= \frac{1}{2}d^2 : \frac{9}{8}d^2 = \frac{1}{2} : \frac{9}{8} = 4 : 9$.

79. Let length $= l$ metres and breadth $= b$ metres. Then, original area $= (lb)\ m^2$.

New length $= (140\%$ of $l)$ m $= \left(\frac{140}{100} \times l\right)$ m $= \frac{7l}{5}$ m.

New breadth $= (130\%$ of $b)$ m $= \left(\frac{130}{100} \times b\right)$ m $= \frac{13b}{10}$ m.

New area $= \left(\frac{7l}{5} \times \frac{13b}{10}\right) = \left(\frac{91}{50}lb\right) m^2$. Increase $= \left(\frac{91}{50}lb - lb\right) = \frac{41}{50}lb$.

$\therefore$ Increase% $= \left(\frac{41}{50} \times lb \times \frac{1}{lb} \times 100\right)\% = 82\%$.

80. Let original length of each side $= x$ cm. Then, its area $= (x^2)\ cm^2$.

Length of rectangle formed $= (x + 5)$ cm and its breadth $= x$ cm.

$\therefore$ $\frac{x+5}{x} = \frac{3}{2} \Leftrightarrow 2x + 10 = 3x \Leftrightarrow x = 10$.

$\therefore$ Original length of each side = 10 cm and its area $= 100\ cm^2$.

81. Let original side $= x$ cm. Then, new side $= (x + 5)$ cm.

$\therefore$ $(x + 5)^2 - x^2 = 165 \Leftrightarrow x^2 + 10x + 25 - x^2 = 165 \Leftrightarrow 10x = 140 \Leftrightarrow x = 14$.

Hence, the side of the square is 14 cm.

82. Let the lengths of the line segments be x cm and $(x + 2)$ cm.

Then, $(x + 2)^2 - x^2 = 32 \Leftrightarrow x^2 + 4x + 4 - x^2 = 32 \Leftrightarrow 4x = 28 \Leftrightarrow x = 7$.

$\therefore$ Length of longer line segment $= (7 + 2)$ cm $= 9$ cm.

83. Let the length of each side of the square be x cm.

Then, length of rectangle $= (x + 5)$ cm and its breadth $= (x - 3)$ cm.

$\therefore$ $(x + 5)(x - 3) = x^2 \Leftrightarrow x^2 + 2x - 15 = x^2 \Leftrightarrow x = \frac{15}{2}$.

$\therefore$ Length $= \left(\frac{15}{2} + 5\right)$ cm $= \frac{25}{2}$ cm, breadth $= \left(\frac{15}{2} - 3\right)$ cm $= \frac{9}{2}$ cm.

Hence, perimeter $= 2(l + b) = 2\left(\frac{25}{2} + \frac{9}{2}\right)$ cm $= 34$ cm.

84. Area to be plastered $= [2(l + b) \times h] + (l \times b)$
$= \{[2(25 + 12) \times 6] + (25 \times 12)\}\ m^2$
$= (444 + 300)\ m^2 = 744\ m^2$.

$\therefore$ Cost of plastering $= \text{Rs.}\left(744 \times \frac{75}{100}\right) = \text{Rs. } 558$.

85. Area of 4 walls $= [2(l + b) \times h] = [2(10 + 7) \times 5]\ m^2 = 170\ m^2$.
Area of 2 doors and 3 windows $= [2(1 \times 3) + (2 \times 1.5) + 2(1 \times 1.5)]\ m^2 = 12\ m^2$.
$\therefore$ Area to be painted $= (170 - 12)\ m^2 = 158\ m^2$.
Cost of painting $= \text{Rs. } (158 \times 3) = \text{Rs. } 474$.

86. $A_1 = 2(l + b) \times h$; $A_2 = 2(2l + 2b) \times 2h = 8(l + b) \times h = 4A_1$.
$\therefore$ Required cost $= \text{Rs. } (4 \times 475) = \text{Rs. } 1900$.

87. Let $h = 2x$ metres and $(l + b) = 5x$ metres.

Length of the paper $= \frac{\text{Total cost}}{\text{Rate per m}} = \frac{260}{2}\ m = 130\ m$.

Area of the paper $= \left(130 \times \frac{50}{100}\right) m^2 = 65\ m^2$.

Total area of 4 walls $= (65 + 15)\ m^2 = 80\ m^2$.
$\therefore\ 2(l + b) \times h = 80 \Leftrightarrow 2 \times 5x \times 2x = 80 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
Height of the room = 4 m.

88. $A_1 = \left(\frac{1}{2} \times 15 \times 12\right) cm^2 = 90\ cm^2$. $A_2 = 2A_1 = 180\ cm^2$.

$\therefore\ \frac{1}{2} \times 20 \times h = 180 \Leftrightarrow h = 18$ cm.

89. $a = 5$, $b = 12$ and $c = 13$. So, $s = \frac{1}{2}(5 + 12 + 13)$ cm $= 15$ cm.

$\therefore$ Area $= \sqrt{15 \times 10 \times 3 \times 2} = 30\ cm^2$.

$\frac{1}{2} \times 12 \times \text{Height} = 30 \Rightarrow \text{Height} = 5$ cm.

90. $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow 40 \times \text{Base} = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow \text{Height} = 80$ cm.

91. Let Base $= 3x$ cm and Altitude $= 4x$ cm.

Then, $\frac{1}{2} \times 3x \times 4x = 1176 \Leftrightarrow 12x^2 = 2352 \Leftrightarrow x^2 = 196 \Leftrightarrow x = 14$ cm.

$\therefore$ Altitude $= (4 \times 14)$ cm $= 56$ cm.

92. Since $5^2 + (12)^2 = (13)^2$, so, it is a right-angled triangle with
Base = 12 cm and Height = 5 cm.

$\therefore$ Area $= \left(\frac{1}{2} \times 12 \times 5\right) cm^2 = 30\ cm^2$.

93. Ratio of sides $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$.

Perimeter = 52 cm. So, sides are $\left(52 \times \frac{6}{13}\right)$ cm, $\left(52 \times \frac{4}{13}\right)$ cm and $\left(52 \times \frac{3}{13}\right)$ cm.

$\therefore\ a = 24$ cm, $b = 16$ cm, $c = 12$ cm.

$\therefore$ Length of smallest side = 12 cm.

94. Let $a = 3x$ cm, $b = 4x$ cm and $c = 5x$ cm. Then, $s = 6x$ cm.

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6x \times 3x \times 2x \times x} = (6x^2)\ \text{cm}^2.$

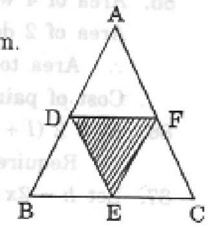
$\therefore\ 6x^2 = 216 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6.$

$\therefore\ a = 18$ cm, $b = 24$ cm and $c = 30$ cm.

Perimeter $= (18 + 24 + 30)$ cm $= 72$ cm.

95. $a = 3$ cm, $b = 4$ cm and $c = 5$ cm.

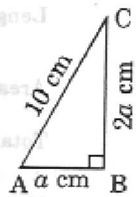
It is a right-angled triangle with base = 3 cm and height = 4 cm.



$\therefore$ Its area $= \left(\frac{1}{2} \times 3 \times 4\right)\ \text{cm}^2 = 6\ \text{cm}^2.$

Area of required triangle $= \left(\frac{1}{4} \times 6\right)\ \text{cm}^2 = \frac{3}{2}\ \text{cm}^2.$

96. Let the sides be a cm and $2a$ cm.



Then, $a^2 + (2a)^2 = (10)^2 \Leftrightarrow 5a^2 = 100 \Leftrightarrow a^2 = 20.$

$\therefore$ Area $= \left(\frac{1}{2} \times a \times 2a\right) = a^2 = 20\ \text{cm}^2.$

97. Let Base = b cm and Height = h cm.

$b + h + 26 = 60 \Leftrightarrow b + h = 34 \Leftrightarrow (b + h)^2 = (34)^2$ ...(i)

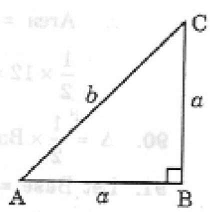
Also, $b^2 + h^2 = (26)^2$ ...(ii)

$\therefore\ (b + h)^2 - (b^2 + h^2) = (34)^2 - (26)^2 \Leftrightarrow 2bh = (34 + 26)(34 - 26) = 480$

$\Leftrightarrow bh = 240 \Leftrightarrow \frac{1}{2} bh = 120.$

$\therefore$ Area $= 120\ \text{cm}^2.$

98. Let the sides be a metres, a metres and b metres.



Then, $2a + b = 6 + 3\sqrt{2}$ and $b^2 = a^2 + a^2 = 2a^2 \Leftrightarrow b = \sqrt{2}\,a.$

$\therefore\ 2a + \sqrt{2}\,a = 6 + 3\sqrt{2} \Leftrightarrow a = 3.$

$\therefore$ Area $= \left(\frac{1}{2} \times 3 \times 3\right)\ \text{m}^2 = 4.5\ \text{m}^2.$

99. Let the smallest side be x cm.

Then, other sides are 13 cm and $(17 - x)$ cm.

Let $a = 13$, $b = x$ and $c = (17 - x)$. So, $s = 15$.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15 \times 2 \times (15 - x)(x - 2)}$

$\Leftrightarrow 30 \times (15 - x)(x - 2) = (30)^2 \Leftrightarrow (15 - x)(x - 2) = 30 \Leftrightarrow x^2 - 17x + 60 = 0$

$\Leftrightarrow (x - 12)(x - 5) = 0 \Leftrightarrow x = 12$ or $x = 5.$

$\therefore$ Smallest side = 5 cm.

100. Area of an equilateral triangle of side a cm $= \left(\frac{\sqrt{3}}{4} a^2\right)\ \text{cm}^2.$

$\therefore\ \frac{\sqrt{3}}{4} a^2 = 24\sqrt{3} \Leftrightarrow a^2 = 96 \Leftrightarrow a = 4\sqrt{6}$ cm.

$\therefore$ Perimeter $= 3a = 12\sqrt{6}$ cm.

84. Area to be plastered $= [2(l+b) \times h] + (l \times b)$
$= \{[2(25+12) \times 6] + (25 \times 12)\}\ m^2$
$= (444 + 300)\ m^2 = 744\ m^2$.

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Area of 2 doors and 3 windows $= [2(1 \times 3) + (2 \times 1.5) + 2(1 \times 1.5)]\ m^2 = 12\ m^2$.
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Area of the paper $= \left(130 \times \frac{50}{100}\right) m^2 = 65\ m^2$.

Total area of 4 walls $= (65 + 15)\ m^2 = 80\ m^2$.
$\therefore$ $2(l+b) \times h = 80 \Leftrightarrow 2 \times 5x \times 2x = 80 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
Height of the room = 4 m.

88. $A_1 = \left(\frac{1}{2} \times 15 \times 12\right) cm^2 = 90\ cm^2$. $A_2 = 2A_1 = 180\ cm^2$.

$\therefore$ $\frac{1}{2} \times 20 \times h = 180 \Leftrightarrow h = 18$ cm.

89. $a = 5$, $b = 12$ and $c = 13$. So, $s = \frac{1}{2}(5 + 12 + 13)$ cm = 15 cm.

$\therefore$ Area $= \sqrt{15 \times 10 \times 3 \times 2} = 30\ cm^2$.

$\frac{1}{2} \times 12 \times \text{Height} = 30 \Rightarrow \text{Height} = 5$ cm.

90. $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow 40 \times \text{Base} = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow \text{Height} = 80$ cm.

91. Let Base $= 3x$ cm and Altitude $= 4x$ cm.

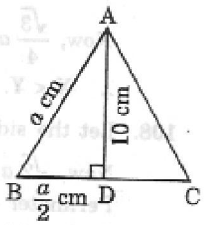
Then, $\frac{1}{2} \times 3x \times 4x = 1176 \Leftrightarrow 12x^2 = 2352 \Leftrightarrow x^2 = 196 \Leftrightarrow x = 14$ cm.
$\therefore$ Altitude $= (4 \times 14)$ cm = 56 cm.

92. Since $5^2 + (12)^2 = (13)^2$, so, it is a right-angled triangle with
Base = 12 cm and Height = 5 cm.
$\therefore$ Area $= \left(\frac{1}{2} \times 12 \times 5\right) cm^2 = 30\ cm^2$.

93. Ratio of sides $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$.

Perimeter = 52 cm. So, sides are $\left(52 \times \frac{6}{13}\right)$ cm, $\left(52 \times \frac{4}{13}\right)$ cm and $\left(52 \times \frac{3}{13}\right)$ cm.
$\therefore$ $a = 24$ cm, $b = 16$ cm, $c = 12$ cm.
$\therefore$ Length of smallest side = 12 cm.

101. Let each side be a cm.

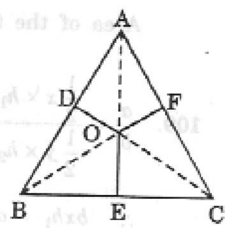


Then, $\left(\frac{a}{2}\right)^2 + (10)^2 = a^2 \Leftrightarrow \left(a^2 - \frac{a^2}{4}\right) = 100$

$\Leftrightarrow \frac{3a^2}{4} = 100 \Leftrightarrow a^2 = \frac{400}{3}.$

$\therefore$ Area $= \frac{\sqrt{3}}{4} \times a^2 = \left(\frac{\sqrt{3}}{4} \times \frac{400}{3}\right) \text{cm}^2 = \frac{100}{\sqrt{3}} \text{cm}^2.$

102. Let each side of the triangle be a cm.



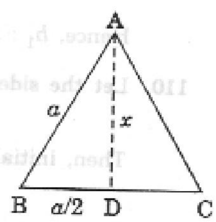
Then, ar (Δ AOB) + ar (Δ BOC) + ar (Δ AOC) = ar (Δ ABC)

$\Rightarrow \frac{1}{2} \times a \times \sqrt{3} + \frac{1}{2} \times a \times 2\sqrt{3} + \frac{1}{2} \times a \times 5\sqrt{3} = \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \frac{a}{2} \sqrt{3}\,(1 + 2 + 5) = \frac{\sqrt{3}}{4} a^2 \Rightarrow a = 16.$

$\therefore$ Perimeter $= (3 \times 16) = 48$ cm.

103. Let the side of the triangle be a. Then,



$a^2 = \left(\frac{a}{2}\right)^2 + x^2 \Leftrightarrow \frac{3a^2}{4} = x^2 \Leftrightarrow a^2 = \frac{4x^2}{3}.$

$\therefore$ Area $= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times \frac{4}{3} x^2 = \frac{x^2}{\sqrt{3}} = \frac{x^2\sqrt{3}}{3}.$

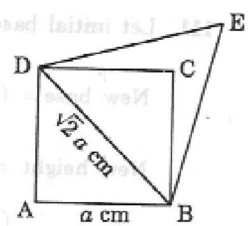
104. Area of a square with side $a = a^2$ sq. units.

Area of a triangle with base $a = \left(\frac{1}{2} \times a \times h\right)$ sq. units.

$\therefore \quad a^2 = \frac{1}{2} \times a \times h \Leftrightarrow h = 2a.$

Hence, the altitude of the triangle is $2a$.

105. Let the side of the square be a cm.



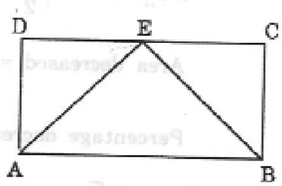
Then, the length of its diagonal $= \sqrt{2}\,a$ cm.

Area of equilateral triangle with side $\sqrt{2}\,a = \frac{\sqrt{3}}{4} \times (\sqrt{2}\,a)^2$

$= \frac{\sqrt{3}\,a^2}{2}.$

$\therefore$ Required ratio $= \frac{\sqrt{3}\,a^2}{2} : a^2 = \sqrt{3} : 2.$

106. Area of rectangle $= lb$ sq. units.



Area of the triangle $= \frac{1}{2}\,lb$ sq. units.

$\therefore$ Required ratio $= lb : \frac{1}{2}\,lb = 2 : 1.$

107. Let each side of the triangle be a cm and each side of the square be b cm.

Then, $X = \frac{\sqrt{3}}{4} a^2$ and $Y = b^2$, where $3a = 4b$, *i.e.*, $b = \frac{3a}{4}$.

$\therefore \quad X = \frac{\sqrt{3}}{4} a^2$ and $Y = \frac{9a^2}{16} \quad \left[\because\ b = \frac{3a}{4}\right]$

Now, $\frac{\sqrt{3}}{4}a^2 = \frac{1.732}{4}a^2 = 0.433\,a^2$ and $\frac{9a^2}{16} = 0.5625\,a^2$.

$\therefore$ X < Y.

108. Let the side of the square be a cm. Then, its diagonal = $\sqrt{2}\,a$ cm.

Now, $\sqrt{2}\,a = 12\sqrt{2} \Rightarrow a = 12$ cm.

Perimeter of the square = $4a$ = 48 cm. Perimeter of the equilateral triangle = 48 cm.

Each side of the triangle = 16 cm.

Area of the triangle = $\left(\frac{\sqrt{3}}{4} \times 16 \times 16\right) \text{cm}^2 = (64\sqrt{3})\ \text{cm}^2$.

109. $\frac{a}{b} = \frac{\frac{1}{2}x \times h_1}{\frac{1}{2}y \times h_2}$ $\left[\text{Ratio of areas} = \frac{a}{b},\ \text{Ratio of base} = x : y\right]$

$\therefore\ bxh_1 = ayh_2 \Leftrightarrow \frac{h_1}{h_2} = \frac{ay}{bx}$.

Hence, $h_1 : h_2 = ay : bx$.

110. Let the sides be x cm and (80% of x) cm = $\frac{4x}{5}$ cm.

Then, initial area = $\frac{\sqrt{3}}{4}x^2$, final area = $\frac{\sqrt{3}}{4}\cdot\left(\frac{4x}{5}\right)^2 = \frac{16\sqrt{3}\,x^2}{100}$.

Decrease in area = $\left(\frac{\sqrt{3}}{4}x^2 - \frac{16\sqrt{3}}{100}x^2\right) \text{cm}^2 = \frac{9\sqrt{3}\,x^2}{100}\ \text{cm}^2$.

$\therefore$ Decrease% = $\left(\frac{9\sqrt{3}\,x^2}{100} \times \frac{4}{\sqrt{3}\,x^2} \times 100\right)\% = 36\%$.

111. Let initial base = b cm and initial height = h cm. Then, initial area = $\left(\frac{1}{2}bh\right) \text{cm}^2$.

New base = (140% of b) cm = $\left(\frac{140b}{100}\right)$ cm = $\left(\frac{7b}{5}\right)$ cm.

New height = (60% of h) cm = $\left(\frac{60h}{100}\right)$ cm = $\left(\frac{3h}{5}\right)$ cm.

New area = $\left(\frac{1}{2} \times \frac{7b}{5} \times \frac{3h}{5}\right) \text{cm}^2 = \left(\frac{21}{50}bh\right) \text{cm}^2$.

Area decreased = $\left(\frac{1}{2}bh - \frac{21}{50}bh\right) \text{cm}^2 = \left(\frac{4}{50}bh\right) \text{cm}^2$.

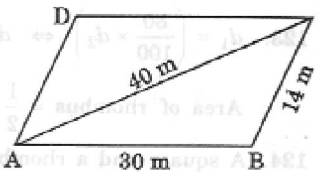
Percentage decrease = $\left(\frac{4bh}{50} \times \frac{2}{bh} \times 100\right)\% = 16\%$.

112. $A_1 = \frac{\sqrt{3}}{2}a^2$ and $A_2 = \frac{\sqrt{3}}{2}(2a)^2 = 4 \times \frac{\sqrt{3}}{2}a^2 = 4A_1$.

$\therefore$ K = 4.

113. Area of ∥gm = (Base × Height) = (18 × 8) cm^2 = 144 cm^2.

114. Let ABCD be the given ∥gm.



Area of ∥gm ABCD = 2 × (area of Δ ABC).

Now, a = 30 m, b = 14 m, c = 40 m.

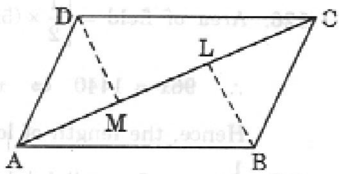
$\therefore \quad s = \frac{1}{2}(30 + 14 + 40)$ m = 42 m.

$\therefore \quad$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 12 \times 28 \times 2}\ m^2 = 168\ m^2$.

Hence, area of ∥gm ABCD = (2 × 168) m^2 = 336 m^2.

115. Let ABCD be the given ∥gm.



Let AC = 70 cm.

Draw BL ⊥ AC and DM ⊥ AC.

Then, DM = BL = 27 cm.

Area of ∥gm ABCD = ar (ΔABC) + ar (ΔACD)

$= \left[\left(\frac{1}{2} \times 70 \times 27\right) + \left(\frac{1}{2} \times 70 \times 27\right)\right]$ sq. cm = 1890 sq. cm.

116. Let the altitude of the triangle be h_1 and base of each be b.

Then, $\frac{1}{2} \times b \times h_1 = b \times h_2$, where h_2 = 100 m

$\Leftrightarrow \quad h_1 = 2h_2 = (2 \times 100)$ m = 200 m.

117. Let each have base = b and height = h. Then, P = $b \times h$, R = $b \times h$, T = $\frac{1}{2} \times b \times h$

So, P = R, P = 2T and T = $\frac{1}{2}$R are all correct statements.

118. $\frac{1}{2} d_1 \times d_2 = 150 \Leftrightarrow \frac{1}{2} \times 10 \times d_2 = 150 \Leftrightarrow d_2 = 30$ cm.

119. $\frac{1}{2} d_1 \times 2d_1 = 25 \Leftrightarrow d_1^2 = 25 \Leftrightarrow d_1 = 5$.

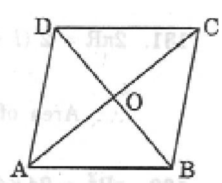
$\therefore$ Sum of lengths of diagonals = (5 + 10) cm = 15 cm.

120. Perimeter of the rhombus = 56 m. Each side of the rhombus = $\frac{56}{4}$ m = 14 m.

Height of the rhombus = 5 m.

$\therefore$ Area = (14 × 5) m^2 = 70 m^2.

121. Area $= \frac{1}{2} d_1 d_2 = \left(\frac{1}{2} \times 24 \times 10\right) cm^2 = 120\ cm^2$.



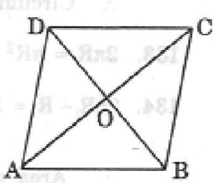
OA $= \frac{1}{2} d_1 = \left(\frac{1}{2} \times 24\right)$ cm = 12 cm.

OB $= \frac{1}{2} d_2 = \left(\frac{1}{2} \times 10\right)$ cm = 5 cm.

$AB^2 = OA^2 + OB^2 = (12)^2 + 5^2 = 169 \Leftrightarrow$ AB = 13 cm.

$\therefore$ Perimeter = (13 × 4) cm = 52 cm.

122. AB = 26 cm and AC = 48 cm $\Rightarrow$ OA $= \left(\frac{1}{2} \times 48\right)$ cm = 24 cm.



$OB^2 = AB^2 - OA^2 = (26)^2 - (24)^2 = (26 + 24)(26 - 24) = 100$

$\Rightarrow$ OB = 50 cm $\Rightarrow$ BD = 2 × OB = (2 × 50) cm = 100 cm.

$\therefore \quad$ Area $= \frac{1}{2} \times AC \times BD = \left(\frac{1}{2} \times 48 \times 100\right) cm^2 = 2400\ cm^2$.

123. $d_1 = \left(\frac{80}{100} \times d_2\right) \Leftrightarrow d_1 = \frac{4d_2}{5}$.

Area of rhombus $= \frac{1}{2} d_1 d_2 = \left(\frac{1}{2} \times \frac{4d_2}{5} \times d_2\right) = \frac{2}{5}(d_2)^2$.

124. A square and a rhombus on the same base are equal in area.

125. Area of trapezium $= \left[\frac{1}{2} \times (1.5 + 2.5) \times 6.5\right] \text{ m}^2 = 13 \text{ m}^2$.

126. Area of field $= \left[\frac{1}{2} \times (5x + 3x) \times 24\right] \text{ m}^2 = (96x) \text{ m}^2$.

$\therefore \quad 96x = 1440 \Leftrightarrow x = \frac{1440}{96} \Leftrightarrow x = 15.$

Hence, the length of longer parallel side $= (5x) = 75$ m.

127. $\frac{1}{2}$(sum of parallel sides) $\times$ depth = Its area

$\Leftrightarrow \frac{1}{2}(12 + 8) \times d = 840 \Leftrightarrow d = 84$ m.

128. Required % $= \left[\frac{\pi \times (5)^2}{2\pi \times 5} \times 100\right]\% = 250\%$.

129. Speed = 12 km/hr $= \left(12 \times \frac{5}{18}\right)$ m/s $= \frac{10}{3}$ m/s.

Distance covered $= \left(20 \times 2 \times \frac{22}{7} \times 50\right)$ m $= \frac{44000}{7}$ m.

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \left(\frac{44000}{7} \times \frac{3}{10}\right)$ s $= \left(\frac{4400 \times 3}{7} \times \frac{1}{60}\right)$ min

$= \frac{220}{7}$ min $= 31\frac{3}{7}$ min.

130. Area of the field grazed $= \left(\frac{22}{7} \times 14 \times 14\right)$ sq. ft = 616 sq. ft.

Number of days taken to graze the field $= \frac{616}{100}$ days = 6 days (approx.).

131. $2\pi R = 2(l + b) \Leftrightarrow 2\pi R = 2(26 + 18)$ cm $\Leftrightarrow R = \left(\frac{88}{2 \times 22} \times 7\right) = 14$ cm.

$\therefore$ Area of the circle $= \pi R^2 = \left(\frac{22}{7} \times 14 \times 14\right) \text{ cm}^2 = 616 \text{ cm}^2$.

132. $\pi R^2 = 24.64 \Leftrightarrow R^2 = \left(\frac{24.64}{22} \times 7\right) = 7.84 \Leftrightarrow R = \sqrt{7.84} = 2.8$ cm.

$\therefore$ Circumference $= \left(2 \times \frac{22}{7} \times 2.8\right)$ cm = 17.60 m.

133. $2\pi R = \pi R^2 \Leftrightarrow R = 2 \Leftrightarrow 2R = 4$. Hence, diameter = 4.

134. $2\pi R - R = 37 \Leftrightarrow \left(\frac{44}{7} - 1\right) R = 37 \Leftrightarrow R = 7$.

$\therefore$ Area of the circle $= \left(\frac{22}{7} \times 7 \times 7\right) \text{ cm}^2 = 154 \text{ cm}^2$.

135. $\pi R_1^2 + \pi R_2^2 = \pi R_3^2 \Leftrightarrow R_1^2 + R_2^2 = R_3^2 \Leftrightarrow (9)^2 + R_2^2 = (15)^2$
$\Leftrightarrow R_2^2 = (15)^2 - (9)^2 = 144 \Leftrightarrow R_2 = 12$ cm.

136. Side of the square $= \frac{44}{4}$ cm $= 11$ cm.

Area of the square $= (11 \times 11)$ cm^2 $= 121$ cm^2.

$2\pi R = 44 \Leftrightarrow 2 \times \frac{22}{7} \times R = 44 \Leftrightarrow R = 7$ cm.

Area of circle $= \pi R^2 = \left(\frac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

$\therefore$ Area of circle is larger by 33 cm^2.

137. Length of wire $= 2\pi \times R = \left(2 \times \frac{22}{7} \times 56\right)$ cm $= 352$ cm.

Side of the square $= \frac{352}{4}$ cm $= 88$ cm.

Area of the square $= (88 \times 88)$ cm^2 $= 7744$ cm^2.

138. Side of the square $= \sqrt{484}$ cm $= 22$ cm. Perimeter of the square $= (22 \times 4)$ cm $= 88$ cm.

$2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14$ cm.

$\therefore$ Required area $= \pi R^2 = \left(\frac{22}{7} \times 14 \times 14\right)$ cm^2 $= 616$ cm^2.

139. Length of wire $= 2\pi R = \left(2 \times \frac{22}{7} \times 42\right)$ cm $= 264$ cm.

Perimeter of rectangle $= 2\,(6x + 5x)$ cm $= 22x$ cm.

$\therefore$ $22x = 264 \Leftrightarrow x = 12$.

Smaller side $= (5 \times 12)$ cm $= 60$ cm.

140. Total area of the field $= [(180 \times 120) + 40000]$ m^2
$= (21600 + 40000)$ m^2 $= 61600$ m^2.

$\therefore$ $\pi R^2 = 61600 \Leftrightarrow R^2 = \left(61600 \times \frac{7}{22}\right) = (400 \times 7 \times 7)$ m

$\Leftrightarrow R = (20 \times 7)$ m $= 140$ m.

141. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{16}{49} \Leftrightarrow \frac{R_1^2}{(14 \times 14)} = \frac{16}{49}$

$\Leftrightarrow R_1^2 = \frac{14 \times 14 \times 16}{49} \Leftrightarrow R_1 = \frac{14 \times 4}{7} = 8$ m.

142. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{4}{9} \Leftrightarrow \frac{R_1^2}{R_2^2} = \frac{4}{9} \Leftrightarrow \frac{R_1}{R_2} = \frac{2}{3} \Leftrightarrow \frac{2\pi R_1}{2\pi R_2} = \frac{R_1}{R_2} = \frac{2}{3}$.

$\therefore$ Required ratio $= 2 : 3$.

143. Let the radius of the given circle be R cm and the side of the square be a cm.

Then, $2\pi R = 4a \Leftrightarrow \frac{R}{a} = \frac{2}{\pi}$.

$\therefore$ Ratio of their areas $= \frac{\pi R^2}{a^2} = \left(\pi \times \frac{4}{\pi^2}\right) = \left(\frac{4}{22} \times 7\right) = \frac{14}{11} = 14 : 11$.

144. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times 0.63\right)$ m $= \frac{99}{25}$ m.

Distance covered in 500 revolutions $= \left(\frac{99}{25} \times 500\right)$ m $= 1980$ m.

145. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times 20\right)$ cm $= \frac{880}{7}$ cm.

Required number of revolutions $= \left(17600 \times \frac{7}{880}\right) = 140$.

146. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times \frac{25}{100}\right)$ m $= \frac{11}{7}$ m.

$\therefore$ Required number of revolutions $= \left(11000 \times \frac{7}{11}\right) = 7000$.

147. Distance covered in 9 sec $= \left(2 \times \frac{22}{7} \times \frac{15}{2} \times 7\right)$ m $= 330$ m.

Distance covered in 1 sec $= \frac{330}{9}$ m $= \frac{110}{3}$ m.

$\therefore$ Required speed $= \left(\frac{110}{3} \times \frac{18}{5}\right)$ km/hr $= 132$ km/hr.

148. Distance covered in 10 sec $= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times 40\right)$ m $= 88$ m.

Distance covered in 1 sec $= \frac{88}{10}$ m $= 8.8$ m.

$\therefore$ Speed $= 8.8$ m/s $= \left(8.8 \times \frac{18}{5}\right)$ km/hr $= 31.68$ km/hr.

149. Let each wheel make x revolutions per sec. Then,

$$\left[\left(2\pi \times \frac{7}{2} \times x\right) + (2\pi \times 7 \times x)\right] \times 10 = 1980$$

$$\Leftrightarrow \left(\frac{22}{7} \times 7 \times x\right) + \left(2 \times \frac{22}{7} \times 7 \times x\right) = 198 \quad \Leftrightarrow \quad 66x = 198 \quad \Leftrightarrow \quad x = 3.$$

Distance moved by smaller wheel in 3 revolutions $= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 3\right)$ cm $= 66$ cm.

$\therefore$ Speed of smaller wheel $= \frac{66}{3}$ m/s $= 22$ m/s.

150. Distance covered by smaller wheel in 1 revolution $= (2\pi \times 15)$ cm $= (30\pi)$ cm.
Distance covered by larger wheel in 1 revolution $= (2\pi \times 25)$ cm $= (50\pi)$ cm.

Let $k \times 30\pi = 15 \times 50\pi$. Then, $k = \left(\frac{15 \times 50\pi}{30\pi}\right) = 25$.

$\therefore$ Required number of revolutions $= 25$.

151. Let the diameter of the wheel be d metres.
Distance covered in 1 revolution $= (\pi d)$ m.
Distance covered in 113 revolutions $= (113\pi d)$ m.

$\therefore$ $113 \times \frac{22}{7} \times d = 226 \times 10 \Leftrightarrow d = \left(226 \times 10 \times \frac{7}{22} \times \frac{1}{113}\right)$ m $= 6\frac{4}{11}$ m.

152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x + 10)$ revolutions.
$(x + 10) \times 3\pi = x \times 2\pi \iff 3x + 30 = 2x \iff x = 30.$
Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground = 17.5 m. Radius of inner circle = (17.5 − 1.4) m = 16.1 m.

$$\text{Area of the garden} = \pi \times [(17.5)^2 - (16.1)^2]\ m^2 = \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right] m^2$$
$$= \left(\frac{22}{7} \times 33.6 \times 1.4\right) m^2 = 147.84\ m^2.$$

154. $2\pi R = 440 \iff 2 \times \frac{22}{7} \times R = 440 \iff R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius = (70 − 7) m = 63 m.

$$\text{Area of the border} = \pi[(70)^2 - (63)^2]\ m^2$$
$$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right] m^2 = 2926\ m^2.$$

155. $\pi R_1^2 = 616 \iff R_1^2 = \left(616 \times \frac{7}{22}\right) = 196 \iff R_1 = 14$ cm.

$\pi R_2^2 = 154 \iff R_2^2 = \left(154 \times \frac{7}{22}\right) = 49 \iff R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm $= (14 - 7)$ cm $= 7$ cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \iff 2\pi(R_1 - R_2) = 132 \iff (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

$\therefore$ Required width = 21 m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall = (R + 4) ft.

$$\text{Area of the concrete wall} = \pi[(R + 4)^2 - R^2]\ \text{sq. ft}$$
$$= [\pi(R + 4 + R)(R + 4 - R)]\ \text{sq. ft} = 8\pi(R + 2)\ \text{sq. ft.}$$

$$8\pi(R + 2) = \frac{11}{25}\pi R^2 \iff 11R^2 = 200(R + 2) \iff 11R^2 - 200R - 400 = 0$$
$$\iff 11R^2 - 220R + 20R - 400 = 0$$
$$\iff 11R(R - 20) + 20(R - 20) = 0$$
$$\iff (R - 20)(11R + 20) = 0 \iff R = 20.$$

$\therefore$ Radius of the pool = 20 ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \iff \frac{R_1}{R_2} = \frac{23}{22} \iff R_1 = \frac{23}{22}R_2.$

Also, $R_1 - R_2 = 5$ m $\iff \frac{23R_2}{22} - R_2 = 5 \iff R_2 = 110.$

$\therefore$ Diameter of inner circle = (2 × 110) m = 220 m.

159. Area of the semi-circle $= \frac{1}{2}\pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) m^2 = 77\ m^2.$

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm $= (99 + 63)$ cm $= 162$ cm.

152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x + 10)$ revolutions.

$(x + 10) \times 3\pi = x \times 2\pi \Leftrightarrow 3x + 30 = 2x \Leftrightarrow x = 30.$

Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground = 17.5 m. Radius of inner circle = (17.5 − 1.4) m = 16.1 m.

$$\text{Area of the garden} = \pi \times [(17.5)^2 - (16.1)^2]\ m^2 = \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right] m^2$$

$$= \left(\frac{22}{7} \times 33.6 \times 1.4\right) m^2 = 147.84\ m^2.$$

154. $2\pi R = 440 \Leftrightarrow 2 \times \frac{22}{7} \times R = 440 \Leftrightarrow R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius = (70 − 7) m = 63 m.

$$\text{Area of the border} = \pi [(70)^2 - (63)^2]\ m^2$$

$$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right] m^2 = 2926\ m^2.$$

155. $\pi R_1^2 = 616 \Leftrightarrow R_1^2 = \left(616 \times \frac{7}{22}\right) = 196 \Leftrightarrow R_1 = 14$ cm.

$\pi R_2^2 = 154 \Leftrightarrow R_2^2 = \left(154 \times \frac{7}{22}\right) = 49 \Leftrightarrow R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm $= (14 - 7)$ cm $= 7$ cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \Leftrightarrow 2\pi (R_1 - R_2) = 132 \Leftrightarrow (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

∴ Required width = 21 m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall = (R + 4) ft.

$$\text{Area of the concrete wall} = \pi [(R + 4)^2 - R^2]\ \text{sq. ft}$$

$$= [\pi (R + 4 + R)(R + 4 - R)]\ \text{sq. ft} = 8\pi (R + 2)\ \text{sq. ft}.$$

$$8\pi (R + 2) = \frac{11}{25}\pi R^2 \Leftrightarrow 11R^2 = 200(R + 2) \Leftrightarrow 11R^2 - 200R - 400 = 0$$

$$\Leftrightarrow 11R^2 - 220R + 20R - 400 = 0$$

$$\Leftrightarrow 11R(R - 20) + 20(R - 20) = 0$$

$$\Leftrightarrow (R - 20)(11R + 20) = 0 \Leftrightarrow R = 20.$$

∴ Radius of the pool = 20 ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \Leftrightarrow \frac{R_1}{R_2} = \frac{23}{22} \Leftrightarrow R_1 = \frac{23}{22} R_2.$

Also, $R_1 - R_2 = 5$ m $\Leftrightarrow \frac{23R_2}{22} - R_2 = 5 \Leftrightarrow R_2 = 110.$

∴ Diameter of inner circle = (2 × 110) m = 220 m.

159. Area of the semi-circle $= \frac{1}{2}\pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) m^2 = 77\ m^2.$

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm $= (99 + 63)$ cm $= 162$ cm.