

CIS602-02 Special Topics in CIS Computer Vision

Assignment # 1

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Problem 1) Image Manipulation (15 pts).

See the script **problem1.mlx**.

Assuming all the script and images are in same directory

Problem 2) Transformation Matrices (35 pts)

For problem a, b, c, d see the script **problem2.mlx**.

Assuming all the script and images are in same directory

b)

$$A = \begin{pmatrix} (2^{1/2})/2 & - (2^{1/2})/2 \\ (2^{1/2})/2 & (2^{1/2})/2 \end{pmatrix}$$

$$X_0 = \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

After scaling operation

$$X_0 = \begin{pmatrix} X_0 \\ Y_0/2 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}$$

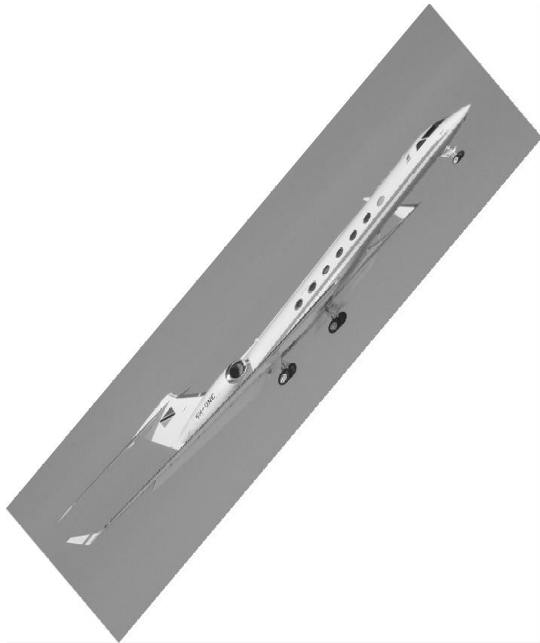
$$X_2 = AZX_0$$

c)

$T = A.Z$

$T =$

$$\begin{array}{cc} (2^{1/2})/2 & - (2^{1/2})/4 \\ (2^{1/2})/2 & (2^{1/2})/4 \end{array}$$



e)

Given:

$$R^T = R^{-1}$$

A is invertible.

$$R^T R = I$$

$$R^T A = B$$

$$R^T (R^T A) = R^T B$$

$$(R^T R) (A) = R^T B$$

$$\text{Given } R^T R = I$$

$$I A = R^T B$$

$$A = R^T B$$

Adding A^{-1} on both sides

$$A A^{-1} = R^T B A^{-1}$$

$$I = R^T B A^{-1}$$

$$Y = R^T B A^{-1}$$

Problem 3. Linear Filter (20 pts)

a) Convolve the following I and F . Assume we use zero-padding where necessary.

ANSWER:

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$I * F$ convolution

1)

$$\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} * \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= 0+0+0-2 = -2$$

2)

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} * \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= 0+0+2+0 = 2$$

3)

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= 0+0+0-1 = -1$$

4)

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$= 0-2+0-1 = -3$$

5)

$$\begin{array}{cc} 2 & 0 \\ 1 & -1 \end{array} * \begin{array}{cc} 1 & -1 \\ 1 & -1 \end{array}$$

$$= 2+0+1+1 = 4$$

6)

$$\begin{array}{cc} 0 & 1 \\ -1 & 2 \end{array} * \begin{array}{cc} 1 & -1 \\ 1 & -1 \end{array}$$

$$= 0-1-1-2 = -4$$

Result $I * F =$

-2	2	-1
-3	4	-4

(b) Note that the F given in (2) is separable; that is, it can be written as a product of two 1D filters: $F = F_1 F_2$.

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$I * F_1$ convolution

1)

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

2)

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2$$

3)

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

4)

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$$

5)

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

6)

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3$$

7)

$$\begin{array}{ccc} 0 & * & 1 \\ -1 & & 1 \end{array} = -1$$

8)

$$\begin{array}{ccc} 1 & * & 1 \\ 2 & & 1 \end{array} = 3$$

$I * F1 =$

$$\begin{array}{cccc} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{array}$$

$(I * F1) * F2$ convolution

1)

$$\begin{array}{ccc} 0 & 2 & * \\ & & 1 \end{array} \quad \begin{array}{ccc} & -1 & = \\ & & -2 \end{array}$$

2)

$$\begin{array}{ccc} 2 & 0 & * \\ & & 1 \end{array} \quad \begin{array}{ccc} & -1 & = \\ & & 2 \end{array}$$

3)

$$\begin{array}{ccc} 0 & 1 & * \\ & & 1 \end{array} \quad \begin{array}{ccc} & -1 & = \\ & & -1 \end{array}$$

4)

$$\begin{array}{ccc} 0 & 3 & * \\ & & 1 \end{array} \quad \begin{array}{ccc} & -1 & = \\ & & -3 \end{array}$$

5)

$$\begin{array}{ccc} 3 & -1 & * \\ & & 1 \end{array} \quad \begin{array}{ccc} & -1 & = \\ & & 4 \end{array}$$

6)

$$\begin{array}{ccc} -1 & 3 & * \\ & & 1 \end{array} \quad \begin{array}{ccc} & -1 & = \\ & & -4 \end{array}$$

$(I * F1) * F2 =$

-2	2	-1
-3	4	-4

Problem 4. SVD for Image Compression (30 pts)

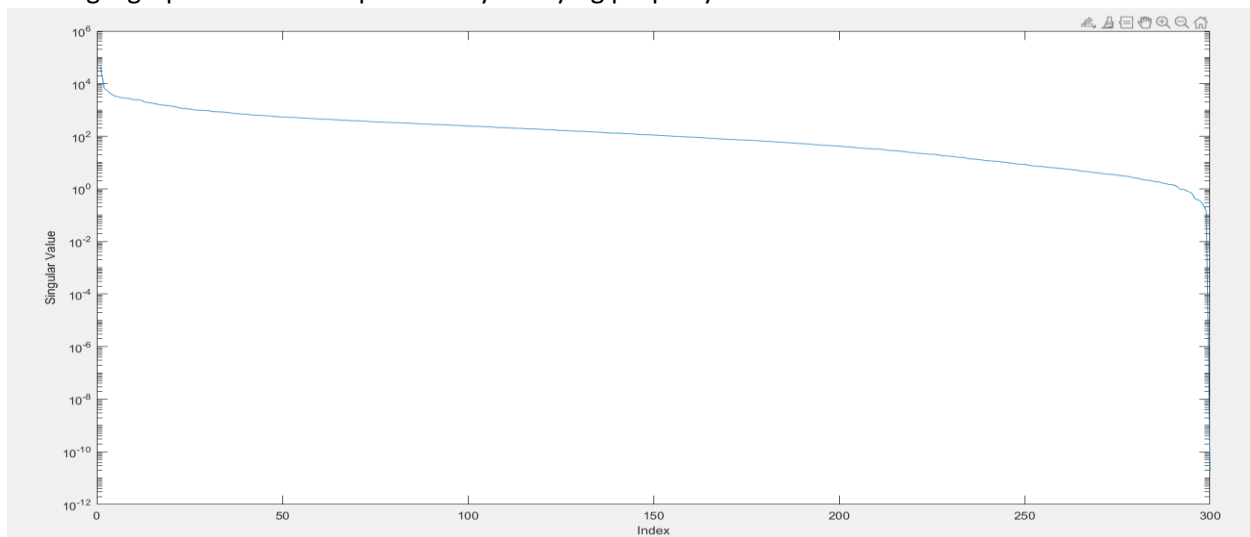
Assuming all the script and images are in same directory
Run the script problem4.mlx.

A)

Computation of top 10 largest values

```
>> flower  
  
ans =  
  
1.0e+04 *  
  
4.4672  
0.6623  
0.5166  
0.3875  
0.3328  
0.3069  
0.2853  
0.2773  
0.2651  
0.2455  
  
>> |
```

Plotting a graph shows the exponentially decaying property



b)

Verify that you can reconstruct and display the image using the three SVD matrices (note that the `svd` command returns V , not V^T). Then, perform compression by using only the top k singular values and their corresponding left/right singular vectors. Let $k = 10, 50$, and 100 . Reconstruct and print the compressed images for the three different values of k . Briefly describe what you observe.



After performing the image compression on the bit map image using values $k = 10, k = 50, k = 100$

The result shows that quality of an image increase with increase in singular k compression value as $k = 100$ image looks very much like an original

Problem 5. Instead of transmitting the original (grayscale) image, you can perform SVD compression on it and transmit only the top k singular values and the corresponding left/right singular vectors. This should be much smaller than the original image for low values of k . With this specific image, will we still save space by compressing when $k = 200$? Show why or why not.

1) By transmitting the $k = 200$ singular values along with left/right singular vectors space will not be saved.

2) when we transmit typical bitmap image into a grayscale 300×300 8-bit integer matrix should be sent.

3) so total bits required are $300 \times 300 \times 8 = 7.2 \times 10^5$ bits

4) in singular vector decomposition of the gray image this information must be 8 byte double precision number in order to reduce the substantial round off errors during the recreation of the image.

5) if the $k = 200$ values are used this indicates $(200 + 2 \times 200^2) \times 64 = 5.13 \times 10^6$ bits of information to be transmitted.

6) to save the transmitting image using this specific method we should consider a critical point k where svd quantities drops below the original information.

$$(k + 2 \cdot k^2) \cdot 8^2, 8 \cdot 300^2$$

$$K < (-1/2) \pm \sqrt{(1/4) + (300^2/4)}^{1/2}$$

$$K \leq 74$$

I.e., we should expect the reduction in the required resources on or beyond the $k \leq 74$