Auto Control Assignment 3

1. (a) $TF(s) = \dfrac{17}{s^6+4s^5+3s^4+2s^3+s^2+4s+4}$

s^6	1	3	1	4
s^5	4	2	4	0
s^4	$\frac{5}{2}$	0	4	0
s^3	2	$-\frac{12}{5}$	0	
s^2	3	4		
s^1	-5.067			
s^0	4			

$b_1 = \frac{-1}{4}\begin{vmatrix}1 & 3\\ 4 & 2\end{vmatrix} = \frac{-(-10)}{4} = \frac{5}{2}$

$b_2 = \frac{-1}{4}\begin{vmatrix}1 & 1\\ 4 & 4\end{vmatrix} = \frac{0}{4} = 0$

$b_3 = -\frac{1}{4}\begin{vmatrix}1 & 4\\ 4 & 0\end{vmatrix} = \frac{-(-16)}{4} = 4$

$b_4 = 0$

$c_1 = -\frac{1}{\frac{5}{2}}\begin{vmatrix}4 & 2\\ \frac{5}{2} & 0\end{vmatrix} = -\frac{2\times -\frac{10}{2}}{5} = +2$

$c_2 = -\frac{1}{\frac{5}{2}}\begin{vmatrix}4 & 4\\ \frac{5}{2} & 4\end{vmatrix} = -\frac{2\times 6}{5} = \frac{12}{5}$

$c_3 = 0$

$d_1 = -\frac{1}{2}\begin{vmatrix}\frac{5}{2} & 0\\ 2 & -\frac{12}{5}\end{vmatrix} = -\frac{-6}{2} = 3$

$d_2 = -\frac{1}{2}\begin{vmatrix}\frac{5}{2} & 4\\ 2 & 0\end{vmatrix} = -\frac{-8}{2}$

$e = -\frac{1}{3}\begin{vmatrix}2 & 3\\ -\frac{12}{5} & 4\end{vmatrix} = -5.067$

$f = -\frac{1}{5.067}\begin{vmatrix}3 & 4\\ 5.067 & 0\end{vmatrix} = -\frac{5.067}{5.067}(-4) = 4$

There are 2 instances where the sign changes: ∴ There are 2 right hand poles ∴ is not stable.

(b)

$TF(s) = \dfrac{42}{s^4+2s^3+10s^2+20s+5}$

s^4	1	10	5
s^3	2	20	0
s^2	ε	5	
s^1	$-\frac{10}{\varepsilon}$		
s^0	5		

$b_1 = \frac{-1}{2}\begin{vmatrix}1 & 10\\ 2 & 20\end{vmatrix} = 0$ let $b_1 = \varepsilon$

$b_2 = -\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 0\end{vmatrix} = -\frac{1}{2}(-10) = 5$

$c_1 = -\frac{1}{\varepsilon}\begin{vmatrix}2 & 20\\ \varepsilon & 5\end{vmatrix} = \frac{10-20\varepsilon}{\varepsilon} = -\frac{10}{\varepsilon} + 20$

∴ as $\varepsilon \to 0$ $+20$ becomes negligible as $\frac{10}{\varepsilon} \to \infty$

$d_1 = \frac{\varepsilon}{10}\begin{vmatrix}\varepsilon & 5\\ -\frac{10}{\varepsilon} & 0\end{vmatrix}$

$= 5\times\frac{10}{\varepsilon}\times\frac{\varepsilon}{10} = 5$

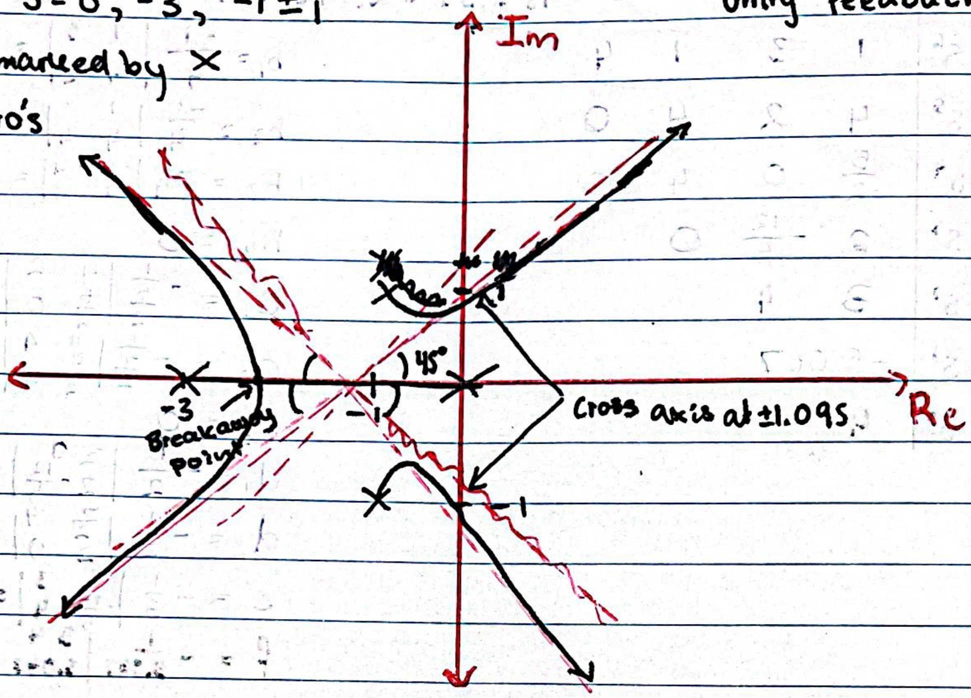
2 sign changes ∴ 2 right hand poles and unstable system!

2. (a) $G(s) = \frac{1}{s(s+3)(s^2+2s+2)}$ $G(s)H(s) = G(s)$

Poles $s = 0, -3, -1 \pm i$ Unity feedback $H(s) = 1$

Poles are marked by $\times$

No zero's



Step 2: Branches:

$1 + kG(s)H(s) = \frac{G(s)+k}{G(s)} = 0$

$(s^2+3s)(s^2+2s+2) + k = 0$ $s^4 + 5s^3 + 8s^2 + 6s + k = 0$

∴ Order 4 means 4 branches

Step 3: Symmetry Step 4: Real Axis Segments

Step 5: Starting and Ending Points

Start at OL Poles $(\times)$ and all tend to infinity!

Step 6: Behaviour at Infinity:

$\sigma_A = \frac{\sum \text{Finite Poles} - 0}{\# P - 0} = \frac{0 + (2)(-1) - 3}{4} = -\frac{5}{4}$

$\theta_A = \frac{(2n+1)\pi}{\#\text{finite poles} - 0} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $n = 0, 1, 2, 3$ $\left(\begin{matrix}\#P - 1 \\ 4 - 1 \\ 3\end{matrix}\right)$

~~Step 7: Stability Region~~

Step 7: Stability Region:

$1 + K\,G(s) = 0 \qquad \therefore s^4 + 5s^3 + 8s^2 + 6s + k = 0$

Routh Array:

s^4	1	8	k
s^3	5	6	0
s^2	$34/5$	k	
s^1	$6 - \frac{25}{34}k$		
s^0	k		

$b_1 = -\frac{1}{5}(6-40) = 34/5 \qquad b_2 = -\frac{1}{5}(-5k) = k$

$c_1 = -\frac{5}{34}\left(5k - 6 \cdot \frac{34}{5}\right) = \frac{25}{34}k + 6$

Stable when: $6 - \frac{25}{34}k = 0$

$k \approx 8.16$

$\therefore s^4 + 5s^3 + 8s^2 + 6s + 8.16 = 0$, Solving: $\pm \frac{\sqrt{30}}{5} j$, $\cancel{\frac{-28 + \sqrt{55}j}{10}}$ needs to be purely imaginary.

root locus crosses axis at $\pm \frac{\sqrt{30}}{5} j \approx \pm 1.095 j$

Step 8: Breakaway Points

$$G(s)H(s) = \frac{N(s)}{D(s)} = \frac{1}{s^4 + 5s^3 + 8s^2 + 6s + 8.16}$$

$$\left[N(s)D'(s) - N'(s)D(s)\right]_{s=s_b} = 0$$

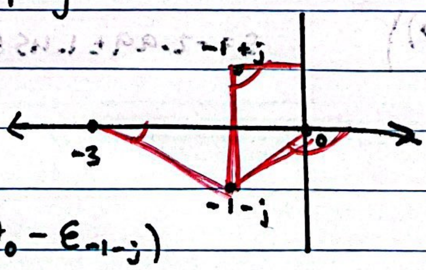
$$\cancel{-(0)(s^4 + 5s^3 + 8s^2 + 6s + 8.16)} + \underbrace{(4s_0^3 + 15s_0^2 + 16s_0 + 6)}_{} = 0 \; ; \quad s_0 = -2.288$$

Breakaway Point: $S_b = -2.288$

Step 9: Angles: $\Sigma\theta_z - \Sigma\theta_p = 180° \pm n360°$ .....

For Pole: $-1 - j$

Observed: $\varepsilon_{-3} = 0°$, $\varepsilon_0 = 180°$



$-(\theta_{-3} + \theta_{-1+j} + \theta_0 - \varepsilon_{-1-j})$

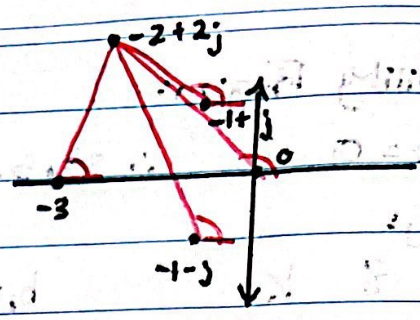
$= -\left(\tan^{-1}\left(\frac{2}{-1}\right) + 90 + (-90-45) + \varepsilon_{-1-j}\right) = 180 + 360$

$\therefore -\tan^{-1}\left(\frac{1}{-2}\right) + 90 + 135 + 180 - 360 = \varepsilon_{-1-j} \qquad \varepsilon_{-1-j} = 71.565°$

$\therefore$ symmetry along real axis

$\varepsilon_{-1+j} = -71.565°$

2 (b) (i) $s = -2+2j$

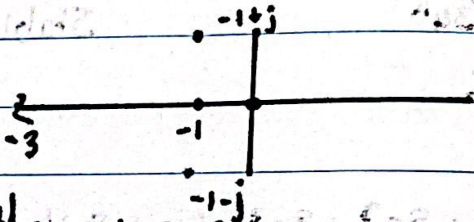


$\Sigma\theta_z - \Sigma\theta_p = 180 \pm n\,360°$

$-\left(\tan^{-1}\left(\frac{2}{1}\right) + \left(90+\tan^{-1}\left(\frac{1}{3}\right)\right) + \left(90+\tan^{-1}\left(\frac{3}{2}\right)\right) + \left(90+\tan^{-1}\left(\frac{1}{1}\right)\right)\right) = 441.87°$

$441.87° \neq 180 \pm n\,360°$ ∴ not on real axis!

(ii) $s = -1$



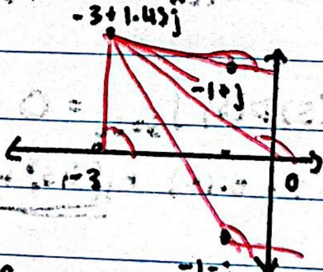
This lies on the root locus as we found in part A is runs along the real axis.

$k = \frac{1}{|G(s)H(s)|}\Big|_{s_0} = -(s^4+5s^3+8s^2+6s)\Big|_{s_0=-1}$

$k = 2$

$1+2\left(\frac{1}{s^4+5s^3+8s^2+6s}\right)$ $s = -1$, $s = -2.839$, $s = -0.58 \pm .606\,j$

iii)



$-\Sigma\theta_p = 180 + n\,360°$

$-\left(90 + \left(90+\tan^{-1}\left(\frac{3}{1.45}\right)\right) + \left(90+\tan^{-1}\left(\frac{2}{0.45}\right)\right) + \left(90+\tan^{-1}\left(\frac{2}{2.45}\right)\right)\right) \approx 540° = -180 + n\,360°$

$s = -3 + 1.45\,j$ is on the root locus!

$k = \frac{1}{|G(s)H(s)|}$ ..... $k = 31.32$

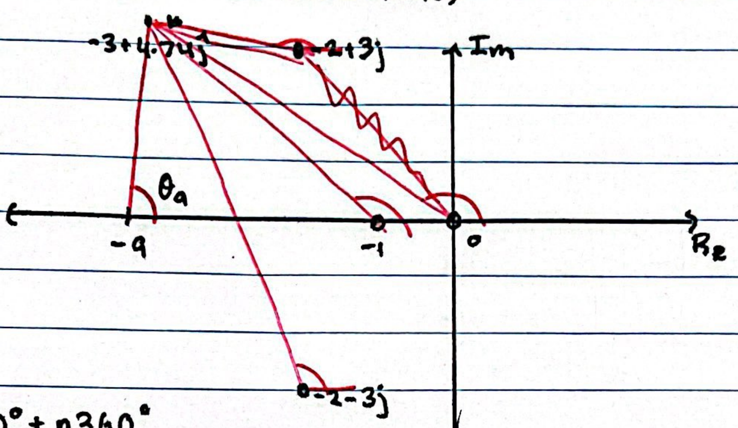
$1 + 31.32\,(G(s)H(s))$ $s = -2.99 \pm 1.456\,j$ $s = 0.993 \pm 1.608\,j$

3. PI Controller

$C(s) = \dfrac{K_{PI}(s+a)}{s}$ $s = -3 + 4.74j$

$G(s) = \dfrac{1}{s(s+1)(s^2+4s+13)} = 0, -1, -2 \pm 3j$ (Poles)   Zeros: a

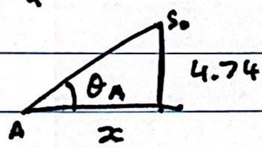
$C(s)G(s) = \dfrac{K_{PI}(s+a)}{s} \cdot \dfrac{1}{s(s+1)(s^2+4s+13)}$



$\Sigma\theta_z - \Sigma\theta_p = 180° \pm n360°$

$\theta_a - \left(90 + \tan^{-1}\left(\frac{1}{1.74}\right)\right) - \left(90 + \tan^{-1}\left(\frac{1}{7.74}\right)\right) - \left(90 + \tan^{-1}\left(\frac{2}{4.74}\right)\right) - \left(90 + \tan^{-1}\left(\frac{3}{4.74}\right)\right)$
$- \left(90 + \tan^{-1}\left(\frac{3}{4.74}\right)\right) = 180$

$\theta_a = 755.091 - 2(360) = 35.091°$



$\dfrac{4.74}{x} = \tan\theta_A$ $x = 6.7566$

$\therefore a = 9.7566$

zero is at -9.7566

Finding K_{PI} $|C(s)G(s)H(s)|_{s=s_0} = 1$

$\left| K_{PI} \dfrac{(s+9.7566)}{s} \cdot \dfrac{1}{s(s+1)(s^2+4s+13)} \right| = 1$

$K_{PI} = \dfrac{1}{\left| \dfrac{(s+9.7566)}{s} \cdot \dfrac{1}{s(s+1)(s^2+4s+13)} \right|_{s=s_0}} = \dfrac{1}{\left(\dfrac{\Pi \text{ zero lengths}}{\Pi \text{ pole lengths}}\right)}$

$= \dfrac{l_0 \times l_0 \times l_1 \times l_{-2-3j} \times l_{-2+3j}}{l_{-9.75299}} = 307.2167$

$1 + C(s)G(s) = \dfrac{307.2167(s+9.7566)}{s^2(s+1)(s^2+4s+13)} + 1 = 0$

roots are: $s = -4.8266,\ -3.00 \pm 4.74j,\ 2.917 \pm 3.349j$

3.(b) An integral compensator brings in a new pole to reduce the steady state error by changing the systems type!